Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.1.2-g-cos-^p-a+b-sin-^m

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [653]. This is test number [70].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (653)	% 0. (0)
Mathematica	% 97.7 (638)	% 2.3 (15)
Maple	% 86.06 (562)	% 13.94 (91)
Maxima	% 39.36 (257)	% 60.64 (396)
Fricas	% 54.82 (358)	% 45.18 (295)
Sympy	% 13.02 (85)	% 86.98 (568)
Giac	% 41.35 (270)	% 58.65 (383)

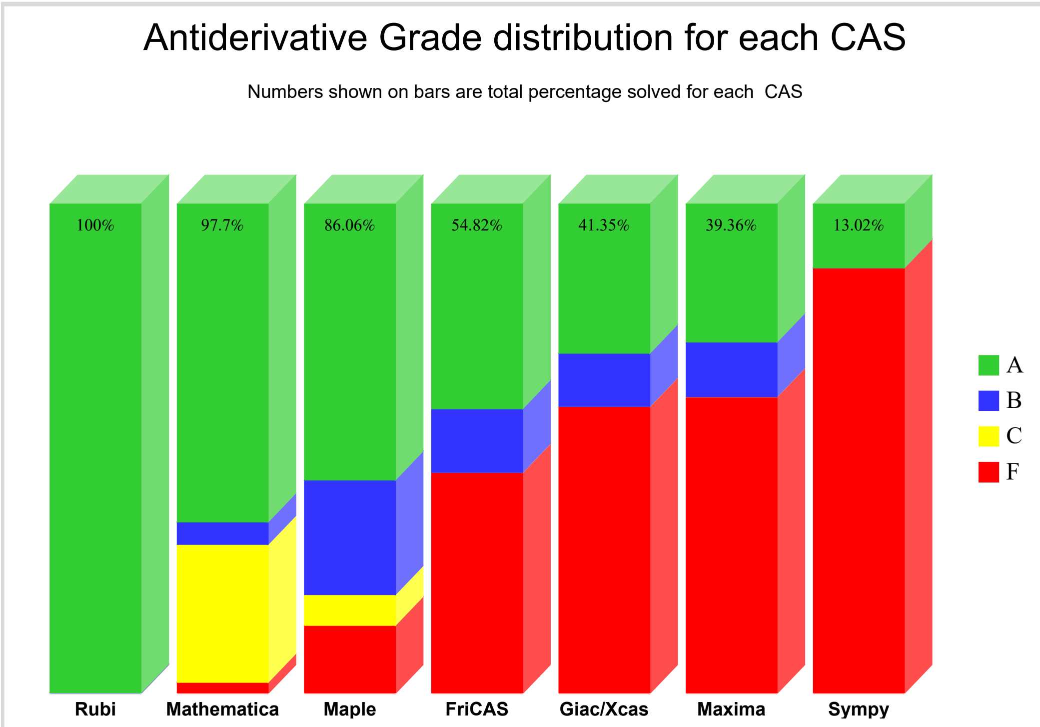
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

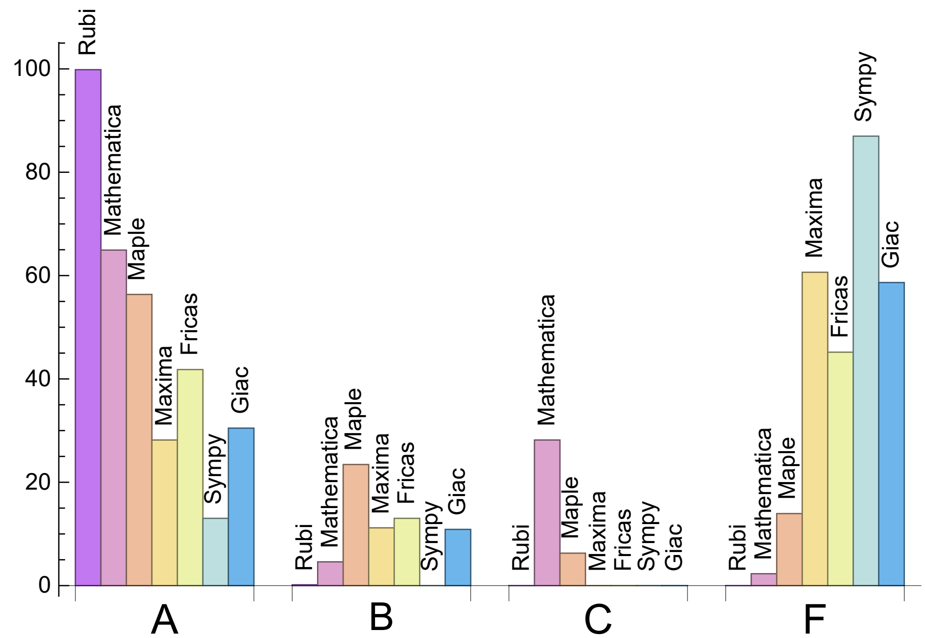
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.85	0.15	0.	0.
Mathematica	64.93	4.59	28.18	2.3
Maple	56.36	23.43	6.28	13.94
Maxima	28.18	11.18	0.	60.64
Fricas	41.81	13.02	0.	45.18
Sympy	13.02	0.	0.	86.98
Giac	30.47	10.87	0.	58.65

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.26	157.61	1.	121.	1.
Mathematica	2.41	256.69	1.53	82.	0.9
Maple	3.17	4781.18	9.49	173.5	1.42
Maxima	1.1	210.06	2.15	131.	1.44
Fricas	2.66	649.59	4.86	265.5	3.28
Sympy	24.27	378.34	5.94	172.	3.08
Giac	1.5	341.46	2.55	161.	1.75

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {615, 648}

Mathematica {112, 330, 352, 353, 446, 456, 467, 469, 470, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 628}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

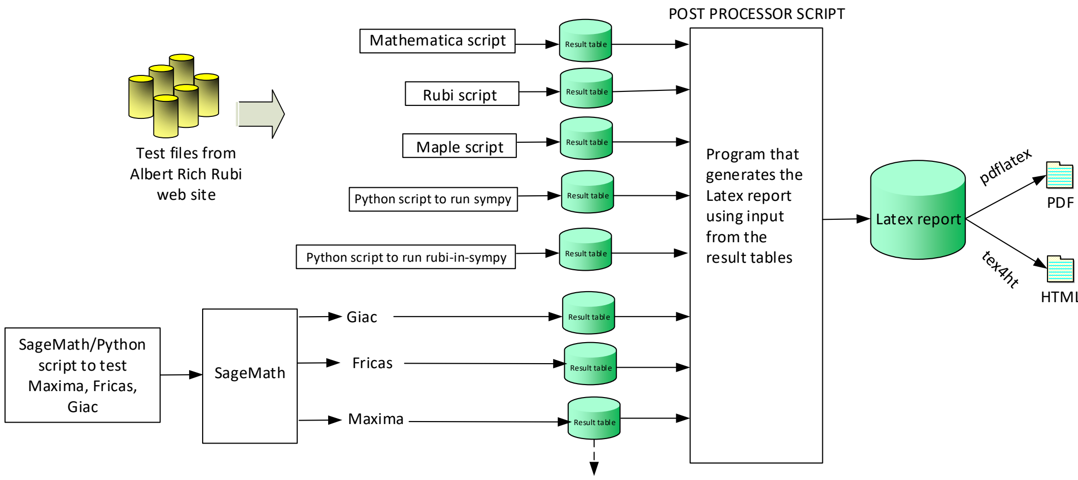
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

B grade: { 615 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 48, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 114, 115, 116, 117, 118, 119, 120, 121, 123, 127, 128, 129, 130, 131, 132, 133, 134, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 152, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 175, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 196, 198, 200, 202, 276, 277, 278, 279, 285, 286, 287, 288, 294, 295, 296, 297, 301, 302, 303, 304, 308, 309, 310, 311, 312, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 347, 350, 351, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 403, 404, 406, 407, 408, 409, 410, 411, 412, 413, 414, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 447, 448, 449, 450, 451, 452, 453, 454, 455, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 522, 523, 524, 525, 526, 527, 528, 529, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 616, 623, 624, 625, 626, 627, 628, 630, 631, 632, 633, 634, 635, 636, 647, 648, 649, 650 }

B grade: { 18, 32, 45, 53, 55, 68, 88, 122, 135, 151, 348, 349, 389, 402, 405, 415, 431, 432, 433, 444, 445, 446, 456, 457, 467, 469, 470, 619, 620, 621 }

C grade: { 34, 47, 49, 79, 108, 109, 110, 111, 112, 113, 124, 125, 126, 137, 138, 148, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 181, 192, 193, 194, 195, 197, 199, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 289, 290, 291, 292, 293, 298, 299, 300, 305, 306, 307, 313, 314, 315, 330, 338, 352, 353, 519, 520, 521, 530, 531, 532, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618 }

F grade: { 622, 629, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 651, 652, 653}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 24, 25, 26, 27, 29, 31, 32, 33, 34, 38, 44, 45, 46, 52, 54, 56, 57, 58, 59, 61, 63, 65, 67, 68, 69, 70, 71, 72, 73, 74, 76, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 214, 215, 216, 217, 218, 219, 220, 224, 225, 226, 227, 228, 233, 234, 235, 236, 237, 238, 243, 244, 245, 246, 247, 253, 254, 255, 256, 257, 263, 264, 265, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 316, 317, 318, 319, 320, 346, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 406, 407, 408, 409, 410, 411, 412, 415, 416, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 434, 437, 438, 439, 440, 441, 442, 443, 446, 447, 449, 450, 451, 452, 453, 454, 455, 461, 462, 463, 464, 465, 466, 473, 474, 475, 476, 477, 483, 484, 485, 494, 495, 496, 506, 507, 508, 509, 510, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 539, 541, 542, 543, 544, 545, 551, 552, 559, 560, 568, 569, 633 }

B grade: { 21, 23, 28, 30, 35, 36, 37, 39, 40, 41, 42, 43, 47, 48, 49, 50, 51, 53, 55, 60, 62, 64, 66, 75, 77, 81, 90, 203, 211, 212, 213, 221, 222, 223, 229, 230, 231, 232, 239, 240, 241, 242, 248, 249, 250, 251, 252, 258, 259, 260, 261, 262, 266, 267, 268, 269, 270, 271, 272, 292, 293, 314, 315, 391, 405, 413, 414, 417, 418, 424, 431, 432, 433, 435, 436, 444, 445, 448, 456, 457, 458, 459, 460, 467, 468, 469, 470, 471, 472, 478, 479, 480, 481, 482, 486, 487, 488, 489, 490, 491, 492, 493, 497, 498, 499, 500, 501, 502, 503, 504, 505, 511, 512, 513, 514, 515, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 536, 537, 538, 540, 546, 547, 548, 549, 550, 553, 554, 555, 556, 557, 558, 561, 562, 563, 564, 565, 566, 567, 570, 571, 572, 573, 615 }

C grade: { 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614 }

F grade: { 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 37, 38, 39, 40, 42, 44, 45, 46, 47, 48, 49, 50, 52, 54, 56, 57, 59, 61, 63, 65, 67, 68, 69, 70, 72, 74, 78, 80, 82, 83, 85, 87, 91, 95, 96, 98, 100, 101, 103, 105, 107, 114, 116, 118, 120, 127, 129, 131, 139, 141, 143, 145, 160, 162, 169, 171, 173, 175, 183, 185, 187, 189, 191, 363, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 449, 450, 451, 452, 453, 461, 462, 464, 473, 474, 475, 483, 484, 485, 494, 495, 496, 506, 507, 508, 516, 517, 518, 527, 528, 529

B grade: { 30, 36, 41, 43, 51, 53, 55, 58, 60, 62, 64, 66, 71, 73, 75, 76, 77, 79, 81, 84, 86, 88, 89, 90, 92, 93, 94, 97, 99, 122, 133, 135, 147, 149, 151, 156, 158, 276, 277, 278, 279, 285, 286, 287, 288, 294, 295, 296, 297, 301, 302, 303, 304, 308, 309, 310, 311, 312, 316, 317, 318, 319, 320, 367, 368, 369, 413, 414, 454, 455, 463, 465, 466 }

C grade: { }

F grade: { 102, 104, 106, 108, 109, 110, 111, 112, 113, 115, 117, 119, 121, 123, 124, 125, 126, 128, 130, 132, 134, 136, 137, 138, 140, 142, 144, 146, 148, 150, 152, 153, 154, 155, 157, 159, 161, 163, 164, 165, 166, 167, 168, 170, 172, 174, 176, 177, 178, 179, 180, 181, 182, 184, 186, 188, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 289, 290, 291, 292, 293, 298, 299, 300, 305, 306, 307, 313, 314, 315, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 370, 371, 372, 373, 374, 375, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 456, 457, 458, 459, 460, 467, 468, 469, 470, 471, 472, 476, 477, 478, 479, 480, 481, 482, 486, 487, 488, 489, 490, 491, 492, 493, 497, 498, 499, 500, 501, 502, 503, 504, 505, 509, 510, 511, 512, 513, 514, 515, 519, 520, 521, 522, 523, 524, 525, 526, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 40, 42, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 82, 84, 85, 86, 87, 91, 94, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 130, 132, 133, 134, 135, 136, 138, 139, 140, 142, 144, 146, 147, 148, 149, 150, 152, 154, 155, 156, 157, 158, 160, 162, 163, 165, 166, 167, 168, 169, 171, 173, 175, 176, 178, 179, 180, 181, 183, 185, 187, 189, 192, 193, 194, 195, 276, 277, 278, 279, 285, 286, 287, 288, 295, 296, 297, 301, 302, 303, 304, 309, 310, 311, 312, 318, 319, 320, 343, 344, 345, 346, 360, 361, 362, 363, 368, 369, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 444, 445, 446, 447, 448, 449, 450, 451, 456, 458, 461, 473, 474, 475, 483, 484, 494, 506, 507, 508, 516, 517, 518, 527, 528, 632, 633 }

B grade: { 18, 23, 32, 36, 39, 41, 43, 45, 76, 81, 83, 88, 89, 90, 92, 93, 95, 96, 98, 109, 124, 129, 131, 137, 141, 143, 145, 151, 153, 159, 161, 164, 170, 172, 174, 177, 182, 184, 186, 188, 190, 191, 294, 308, 316, 317, 367, 389, 402, 413, 414, 415, 442, 443, 452, 453, 454, 455, 457, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 476, 477, 485, 487, 495, 496, 497, 498, 499, 529, 530, 630, 631 }

C grade: { }

F grade: { 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 289, 290, 291, 292, 293, 298, 299, 300, 305, 306, 307, 313, 314, 315, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 364, 365, 366, 370, 371, 372, 373, 374, 375, 478, 479, 480, 481, 482, 486, 488, 489, 490, 491, 492, 493, 500, 501, 502, 503, 504, 505, 509, 510, 511, 512, 513, 514, 515, 519, 520, 521, 522, 523, 524, 525, 526, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 18, 27, 28, 29, 30, 31, 32, 41, 42, 43, 44, 45, 52, 53, 54, 55, 56, 66, 67, 68, 69, 80, 81, 82, 89, 91, 93, 95, 107, 118, 120, 162, 175, 189, 191, 346, 376, 377, 378, 382, 383, 387, 388, 389, 393, 394, 395, 400, 401, 402, 406, 407, 413, 414, 415, 419, 427, 439, 440, 451, 452, 461, 462, 463, 464, 475, 484, 485, 508, 518, 528, 529, 633 }

B grade: { }

C grade: { }

F grade: { 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 33, 34, 35, 36, 37, 38, 39, 40, 46, 47, 48, 49, 50, 51, 57, 58, 59, 60, 61, 62, 63, 64, 65, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 84, 85, 86, 87, 88, 90, 92, 94, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 384, 385, 386, 390, 391, 392, 396, 397, 398, 399, 403, 404, 405, 408, 409, 410, 411, 412, 416, 417, 418, 420, 421, 422, 423, 424, 425, 426, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 453, 454, 455, 456, 457, 458, 459, 460, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 509, 510, 511, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 524, 525, 526, 527, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 27, 29, 31, 32, 34, 36, 38, 39, 40, 42, 44, 45, 46, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 105, 107, 156, 158, 160, 162, 163, 165, 167, 169, 171, 173, 175, 176, 178, 180, 183, 185, 187, 189, 191, 192, 194, 346, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 415, 416, 417, 418, 419, 421, 422, 423, 425, 426, 427, 428, 429, 430, 432, 433, 434, 437, 438, 439, 440, 441, 442, 443, 445, 446, 449, 450, 451, 452, 453, 454, 455, 458, 461, 462, 463, 464, 473, 474, 475, 506, 507, 508, 509, 510, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 633 }

B grade: { 8, 19, 26, 28, 30, 33, 35, 37, 41, 43, 48, 50, 60, 76, 78, 90, 91, 108, 109, 121, 132, 146, 157, 159, 161, 164, 166, 168, 170, 172, 174, 182, 184, 186, 188, 190, 343, 344, 345, 386, 399, 411, 412, 413, 414, 420, 424, 431, 435, 436, 444, 447, 448, 456, 457, 459, 460, 465, 466, 467, 468, 469, 470, 471, 472, 511, 521, 532, 630, 631, 632 }

C grade: { }

F grade: { 102, 104, 106, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 177, 179, 181, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 512, 513, 514, 515, 522, 523, 524, 525, 526, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	74	56	124	161	105	159
normalized size	1	1.	0.85	0.64	1.43	1.85	1.21	1.83
time (sec)	N/A	0.056	0.046	0.029	0.949	1.714	12.651	1.184

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	57	62	85	167	172	144
normalized size	1	1.	0.66	0.71	0.98	1.92	1.98	1.66
time (sec)	N/A	0.061	0.16	0.026	0.973	1.733	8.813	1.184

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	60	46	95	128	83	119
normalized size	1	1.	0.94	0.72	1.48	2.	1.3	1.86
time (sec)	N/A	0.044	0.023	0.025	0.953	1.624	4.317	1.188

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	62	52	65	132	124	104
normalized size	1	1.	0.95	0.8	1.	2.03	1.91	1.6
time (sec)	N/A	0.045	0.082	0.024	0.95	1.644	2.343	1.19

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	36	65	97	82	65
normalized size	1	1.	0.98	0.8	1.44	2.16	1.82	1.44
time (sec)	N/A	0.037	0.017	0.025	0.943	1.64	1.205	1.113

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	41	50	96	71	63
normalized size	1	1.	1.07	0.95	1.16	2.23	1.65	1.47
time (sec)	N/A	0.033	0.048	0.025	0.959	1.658	0.686	1.116

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	28	39	25	27	62	34	34
normalized size	1	1.27	1.77	1.14	1.23	2.82	1.55	1.55
time (sec)	N/A	0.016	0.013	0.009	0.941	1.611	0.22	1.12

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	26	16	20	39	0	50
normalized size	1	1.	1.53	0.94	1.18	2.29	0.	2.94
time (sec)	N/A	0.02	0.014	0.029	0.945	1.712	0.	1.206

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	31	104	0	26
normalized size	1	1.	1.	1.04	1.35	4.52	0.	1.13
time (sec)	N/A	0.032	0.013	0.069	0.946	1.605	0.	1.144

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	52	54	57	166	0	73
normalized size	1	1.	1.33	1.38	1.46	4.26	0.	1.87
time (sec)	N/A	0.042	0.018	0.082	0.946	1.773	0.	1.209

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	47	131	0	89
normalized size	1	1.	0.93	0.86	1.07	2.98	0.	2.02
time (sec)	N/A	0.036	0.06	0.084	0.953	1.624	0.	1.173

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	68	74	116	350	0	124
normalized size	1	1.	0.81	0.88	1.38	4.17	0.	1.48
time (sec)	N/A	0.064	0.081	0.085	0.958	1.953	0.	1.242

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	171	129	155	216	398	166
normalized size	1	1.	1.36	1.02	1.23	1.71	3.16	1.32
time (sec)	N/A	0.107	1.558	0.037	0.965	1.806	14.668	1.152

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	58	99	128	176	202	158
normalized size	1	1.	0.87	1.48	1.91	2.63	3.01	2.36
time (sec)	N/A	0.061	0.08	0.039	0.951	1.76	8.321	1.141

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	151	109	120	181	287	143
normalized size	1	1.	1.48	1.07	1.18	1.77	2.81	1.4
time (sec)	N/A	0.093	0.562	0.036	0.958	1.68	5.303	1.159

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	46	79	76	136	129	76
normalized size	1	1.	1.02	1.76	1.69	3.02	2.87	1.69
time (sec)	N/A	0.046	0.082	0.036	0.947	1.678	2.528	1.14

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	131	87	88	144	180	97
normalized size	1	1.	1.68	1.12	1.13	1.85	2.31	1.24
time (sec)	N/A	0.089	0.303	0.036	0.943	1.675	1.373	1.131

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	47	21	27	101	53	27
normalized size	1	1.	2.14	0.95	1.23	4.59	2.41	1.23
time (sec)	N/A	0.024	0.021	0.013	0.932	1.736	0.614	1.191

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	29	53	41	73	0	123
normalized size	1	1.	0.85	1.56	1.21	2.15	0.	3.62
time (sec)	N/A	0.042	0.019	0.042	0.94	1.648	0.	1.187

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	75	47	63	167	0	45
normalized size	1	1.	1.97	1.24	1.66	4.39	0.	1.18
time (sec)	N/A	0.082	0.056	0.046	1.416	1.64	0.	1.15

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	32	75	24	36	0	41
normalized size	1	1.	1.6	3.75	1.2	1.8	0.	2.05
time (sec)	N/A	0.038	0.138	0.062	0.933	1.541	0.	1.178

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	58	63	70	235	0	73
normalized size	1	1.	0.92	1.	1.11	3.73	0.	1.16
time (sec)	N/A	0.069	0.009	0.065	0.953	1.677	0.	1.208

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	144	96	306	0	104
normalized size	1	1.	0.88	2.25	1.5	4.78	0.	1.62
time (sec)	N/A	0.066	0.067	0.067	0.953	1.71	0.	1.216

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	82	93	104	207	0	143
normalized size	1	1.	1.28	1.45	1.62	3.23	0.	2.23
time (sec)	N/A	0.056	0.011	0.103	0.976	1.534	0.	1.174

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	85	190	146	505	0	161
normalized size	1	1.	0.78	1.74	1.34	4.63	0.	1.48
time (sec)	N/A	0.089	0.143	0.108	0.971	1.649	0.	1.2

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	110	121	132	278	0	231
normalized size	1	1.	1.34	1.48	1.61	3.39	0.	2.82
time (sec)	N/A	0.059	0.023	0.112	0.975	1.688	0.	1.189

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	181	163	190	258	439	212
normalized size	1	1.	1.18	1.06	1.23	1.68	2.85	1.38
time (sec)	N/A	0.154	1.995	0.043	0.982	1.87	24.358	1.187

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	58	133	146	207	270	181
normalized size	1	1.	0.87	1.99	2.18	3.09	4.03	2.7
time (sec)	N/A	0.066	0.096	0.042	0.95	1.809	13.793	1.204

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	161	143	155	212	335	166
normalized size	1	1.	1.24	1.1	1.19	1.63	2.58	1.28
time (sec)	N/A	0.14	0.793	0.042	0.967	1.762	8.628	1.166

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	113	111	171	172	111
normalized size	1	1.	0.96	2.51	2.47	3.8	3.82	2.47
time (sec)	N/A	0.047	0.145	0.042	0.948	1.727	4.839	1.165

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	141	121	123	181	226	120
normalized size	1	1.	1.33	1.14	1.16	1.71	2.13	1.13
time (sec)	N/A	0.124	0.424	0.039	0.962	1.764	2.696	1.143

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	65	21	27	131	94	27
normalized size	1	1.	2.95	0.95	1.23	5.95	4.27	1.23
time (sec)	N/A	0.025	0.028	0.015	0.946	1.689	1.325	1.15

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	41	69	58	108	0	173
normalized size	1	1.	0.79	1.33	1.12	2.08	0.	3.33
time (sec)	N/A	0.047	0.029	0.047	0.966	1.655	0.	1.161

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	55	87	92	230	0	123
normalized size	1	1.	1.1	1.74	1.84	4.6	0.	2.46
time (sec)	N/A	0.136	0.033	0.05	1.449	1.627	0.	1.167

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	59	128	45	109	0	124
normalized size	1	1.	1.48	3.2	1.12	2.72	0.	3.1
time (sec)	N/A	0.052	0.041	0.069	0.951	1.693	0.	1.201

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	28	120	105	236	0	51
normalized size	1	1.	0.9	3.87	3.39	7.61	0.	1.65
time (sec)	N/A	0.085	0.025	0.07	0.967	1.602	0.	1.161

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	35	146	38	73	0	85
normalized size	1	1.	1.52	6.35	1.65	3.17	0.	3.7
time (sec)	N/A	0.04	0.234	0.074	0.969	1.612	0.	1.195

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	110	171	139	367	0	116
normalized size	1	1.	1.2	1.86	1.51	3.99	0.	1.26
time (sec)	N/A	0.093	0.02	0.075	0.972	1.604	0.	1.158

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	67	238	130	446	0	122
normalized size	1	1.	0.77	2.74	1.49	5.13	0.	1.4
time (sec)	N/A	0.072	0.101	0.08	0.958	1.707	0.	1.244

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	134	217	165	274	0	186
normalized size	1	1.	1.35	2.19	1.67	2.77	0.	1.88
time (sec)	N/A	0.083	0.011	0.082	0.956	1.65	0.	1.153

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	58	513	234	400	558	296
normalized size	1	1.	0.87	7.66	3.49	5.97	8.33	4.42
time (sec)	N/A	0.084	0.422	0.053	0.947	2.041	120.691	1.283

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	211	535	458	450	1280	281
normalized size	1	1.	0.74	1.87	1.6	1.57	4.48	0.98
time (sec)	N/A	0.403	3.16	0.051	0.996	2.079	86.414	1.268

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	463	181	352	445	181
normalized size	1	1.	0.96	10.29	4.02	7.82	9.89	4.02
time (sec)	N/A	0.047	1.043	0.052	0.949	1.952	56.598	1.205

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	191	480	431	390	1018	235
normalized size	1	1.	0.73	1.83	1.65	1.49	3.89	0.9
time (sec)	N/A	0.374	1.514	0.05	0.988	1.906	40.576	1.244

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	147	21	27	304	148	27
normalized size	1	1.	6.68	0.95	1.23	13.82	6.73	1.23
time (sec)	N/A	0.024	0.089	0.02	0.937	1.849	19.877	1.205

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	95	149	147	302	0	389
normalized size	1	1.	0.59	0.92	0.91	1.86	0.	2.4
time (sec)	N/A	0.077	0.177	0.076	0.95	1.839	0.	1.279

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	55	389	447	632	0	312
normalized size	1	1.	0.27	1.94	2.22	3.14	0.	1.55
time (sec)	N/A	0.342	0.057	0.066	1.458	1.875	0.	1.211

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	111	345	131	328	0	371
normalized size	1	1.	0.92	2.85	1.08	2.71	0.	3.07
time (sec)	N/A	0.094	0.259	0.112	0.959	1.88	0.	1.266

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	59	478	420	639	0	270
normalized size	1	1.	0.33	2.67	2.35	3.57	0.	1.51
time (sec)	N/A	0.319	0.053	0.11	1.466	1.72	0.	1.235

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	73	503	128	344	0	328
normalized size	1	1.	0.66	4.57	1.16	3.13	0.	2.98
time (sec)	N/A	0.091	0.445	0.116	0.96	1.831	0.	1.239

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	141	245	348	126	0	154
normalized size	1	1.	1.93	3.36	4.77	1.73	0.	2.11
time (sec)	N/A	0.068	0.784	0.06	1.444	1.603	0.	1.163

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	46	45	63	93	779	63
normalized size	1	1.	0.98	0.96	1.34	1.98	16.57	1.34
time (sec)	N/A	0.056	0.093	0.05	0.931	1.672	126.403	1.146

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	119	141	211	92	697	101
normalized size	1	1.	2.43	2.88	4.31	1.88	14.22	2.06
time (sec)	N/A	0.055	0.328	0.048	1.429	1.522	20.811	1.167

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	24	28	34	61	158	34
normalized size	1	1.	0.75	0.88	1.06	1.91	4.94	1.06
time (sec)	N/A	0.046	0.038	0.015	0.938	1.65	7.862	1.156

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	97	43	70	38	119	46
normalized size	1	1.	5.11	2.26	3.68	2.	6.26	2.42
time (sec)	N/A	0.043	0.125	0.	1.406	1.625	3.609	1.303

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	19	24	39	24	26
normalized size	1	1.	1.	1.19	1.5	2.44	1.5	1.62
time (sec)	N/A	0.026	0.011	0.012	0.932	1.693	0.545	1.336

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	54	63	163	0	78
normalized size	1	1.	0.81	1.46	1.7	4.41	0.	2.11
time (sec)	N/A	0.051	0.039	0.	0.938	1.636	0.	1.569

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	45	70	174	131	0	90
normalized size	1	1.	1.07	1.67	4.14	3.12	0.	2.14
time (sec)	N/A	0.052	0.055	0.	0.948	1.641	0.	1.133

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	75	90	123	336	0	130
normalized size	1	1.	0.97	1.17	1.6	4.36	0.	1.69
time (sec)	N/A	0.076	0.094	0.	0.942	1.753	0.	1.187

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	66	130	397	194	0	161
normalized size	1	1.	1.06	2.1	6.4	3.13	0.	2.6
time (sec)	N/A	0.059	0.097	0.059	0.967	1.644	0.	1.142

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	97	126	176	400	0	157
normalized size	1	1.	0.81	1.05	1.47	3.33	0.	1.31
time (sec)	N/A	0.107	0.149	0.059	0.948	1.679	0.	1.181

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	151	415	531	161	0	242
normalized size	1	1.	1.45	3.99	5.11	1.55	0.	2.33
time (sec)	N/A	0.112	1.12	0.082	1.466	2.002	0.	1.142

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	46	45	63	122	0	63
normalized size	1	1.	0.98	0.96	1.34	2.6	0.	1.34
time (sec)	N/A	0.052	0.16	0.068	0.938	1.933	0.	1.146

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	131	279	360	130	0	171
normalized size	1	1.	1.64	3.49	4.5	1.62	0.	2.14
time (sec)	N/A	0.1	0.721	0.067	1.434	1.848	0.	1.122

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	34	19	47	92	0	47
normalized size	1	1.	1.48	0.83	2.04	4.	0.	2.04
time (sec)	N/A	0.043	0.056	0.062	0.94	1.901	0.	1.131

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	109	142	189	89	461	99
normalized size	1	1.	1.95	2.54	3.38	1.59	8.23	1.77
time (sec)	N/A	0.085	0.171	0.072	1.435	1.986	169.455	1.125

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	26	33	41	68	180	73
normalized size	1	1.	0.81	1.03	1.28	2.12	5.62	2.28
time (sec)	N/A	0.049	0.033	0.065	0.924	1.919	1.57	1.16

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	104	41	76	151	143	45
normalized size	1	1.	3.06	1.21	2.24	4.44	4.21	1.32
time (sec)	N/A	0.043	0.18	0.076	1.439	1.802	8.428	1.126

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	31	21	27	45	32	27
normalized size	1	1.	1.48	1.	1.29	2.14	1.52	1.29
time (sec)	N/A	0.026	0.07	0.015	0.961	1.851	0.991	1.151

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	38	72	97	279	0	96
normalized size	1	1.	0.63	1.2	1.62	4.65	0.	1.6
time (sec)	N/A	0.058	0.092	0.071	0.964	2.08	0.	1.188

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	53	98	275	200	0	126
normalized size	1	1.	0.75	1.38	3.87	2.82	0.	1.77
time (sec)	N/A	0.093	0.076	0.069	0.989	1.921	0.	1.157

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	85	108	146	466	0	143
normalized size	1	1.	0.82	1.04	1.4	4.48	0.	1.38
time (sec)	N/A	0.082	0.12	0.087	1.144	2.204	0.	1.195

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	78	158	535	261	0	196
normalized size	1	1.	0.84	1.7	5.75	2.81	0.	2.11
time (sec)	N/A	0.098	0.063	0.095	1.002	2.005	0.	1.151

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	137	144	225	527	0	170
normalized size	1	1.	0.94	0.99	1.54	3.61	0.	1.16
time (sec)	N/A	0.11	0.338	0.086	0.977	2.184	0.	1.186

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	141	313	419	163	0	189
normalized size	1	1.	1.37	3.04	4.07	1.58	0.	1.83
time (sec)	N/A	0.109	1.094	0.083	1.479	1.982	0.	1.164

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	44	19	61	119	0	61
normalized size	1	1.	1.91	0.83	2.65	5.17	0.	2.65
time (sec)	N/A	0.043	0.158	0.069	0.97	1.908	0.	1.173

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	121	177	248	122	0	119
normalized size	1	1.	1.57	2.3	3.22	1.58	0.	1.55
time (sec)	N/A	0.1	0.486	0.084	1.45	1.983	0.	1.156

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	38	49	55	100	0	155
normalized size	1	1.	0.76	0.98	1.1	2.	0.	3.1
time (sec)	N/A	0.05	0.05	0.069	0.942	2.011	0.	1.168

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	59	64	188	203	0	108
normalized size	1	1.	1.2	1.31	3.84	4.14	0.	2.2
time (sec)	N/A	0.085	0.038	0.084	1.436	1.953	0.	1.182

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	58	37	50	105	450	47
normalized size	1	1.	1.49	0.95	1.28	2.69	11.54	1.21
time (sec)	N/A	0.051	0.063	0.072	0.947	1.808	1.868	1.175

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	28	55	134	238	298	49
normalized size	1	1.	1.04	2.04	4.96	8.81	11.04	1.81
time (sec)	N/A	0.039	0.017	0.089	0.975	1.765	29.251	1.15

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	33	21	27	82	51	27
normalized size	1	1.	1.5	0.95	1.23	3.73	2.32	1.23
time (sec)	N/A	0.025	0.057	0.014	0.944	1.831	1.669	1.193

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	61	90	132	409	0	109
normalized size	1	1.	0.74	1.1	1.61	4.99	0.	1.33
time (sec)	N/A	0.063	0.083	0.082	0.962	2.094	0.	1.202

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	63	130	419	270	0	161
normalized size	1	1.	0.64	1.31	4.23	2.73	0.	1.63
time (sec)	N/A	0.135	0.098	0.081	0.987	1.908	0.	1.166

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	95	126	197	595	0	157
normalized size	1	1.	0.75	1.	1.56	4.72	0.	1.25
time (sec)	N/A	0.095	0.168	0.102	0.959	1.832	0.	1.198

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	85	190	651	331	0	231
normalized size	1	1.	0.69	1.54	5.29	2.69	0.	1.88
time (sec)	N/A	0.145	0.114	0.109	1.028	1.71	0.	1.184

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	145	162	254	659	0	184
normalized size	1	1.	0.85	0.95	1.49	3.85	0.	1.08
time (sec)	N/A	0.128	0.517	0.112	0.962	1.915	0.	1.194

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	275	146	398	655	0	134
normalized size	1	1.	2.17	1.15	3.13	5.16	0.	1.06
time (sec)	N/A	0.182	6.072	0.125	1.516	1.718	0.	1.207

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	55	100	194	2032	92
normalized size	1	1.	0.78	1.53	2.78	5.39	56.44	2.56
time (sec)	N/A	0.046	0.103	0.105	0.956	1.653	54.313	1.186

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	36	145	506	614	0	169
normalized size	1	1.	0.62	2.5	8.72	10.59	0.	2.91
time (sec)	N/A	0.08	0.083	0.131	1.023	1.546	0.	1.203

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	58	43	126	247	1658	185
normalized size	1	1.	0.89	0.66	1.94	3.8	25.51	2.85
time (sec)	N/A	0.059	0.124	0.115	0.954	1.687	53.215	1.167

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	58	175	622	760	0	204
normalized size	1	1.	0.49	1.48	5.27	6.44	0.	1.73
time (sec)	N/A	0.168	0.082	0.141	1.037	1.621	0.	1.19

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	33	130	261	2020	38
normalized size	1	1.	0.96	0.73	2.89	5.8	44.89	0.84
time (sec)	N/A	0.053	0.165	0.122	0.957	1.76	53.88	1.16

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	78	205	738	896	0	239
normalized size	1	1.	0.43	1.12	4.03	4.9	0.	1.31
time (sec)	N/A	0.272	0.122	0.148	1.066	1.727	0.	1.181

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	33	21	27	263	128	27
normalized size	1	1.	1.5	0.95	1.23	11.95	5.82	1.23
time (sec)	N/A	0.025	0.228	0.02	0.965	1.725	52.717	1.154

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	122	180	288	1052	0	177
normalized size	1	1.	0.63	0.93	1.48	5.42	0.	0.91
time (sec)	N/A	0.112	0.773	0.122	0.969	1.963	0.	1.185

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	113	280	999	624	0	336
normalized size	1	1.	0.46	1.14	4.08	2.55	0.	1.37
time (sec)	N/A	0.404	0.342	0.098	1.15	1.87	0.	1.219

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	175	216	335	1241	0	224
normalized size	1	1.	0.74	0.91	1.41	5.21	0.	0.94
time (sec)	N/A	0.171	1.769	0.135	0.98	2.167	0.	1.237

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	125	340	1169	695	0	406
normalized size	1	1.	0.45	1.22	4.19	2.49	0.	1.46
time (sec)	N/A	0.419	0.419	0.144	1.171	2.013	0.	1.194

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	195	252	412	1332	0	251
normalized size	1	1.	0.69	0.89	1.45	4.69	0.	0.88
time (sec)	N/A	0.216	2.614	0.144	0.989	2.264	0.	1.235

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	74	57	97	254	0	104
normalized size	1	1.	0.76	0.59	1.	2.62	0.	1.07
time (sec)	N/A	0.083	4.238	0.091	0.948	1.748	0.	2.169

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	99	75	0	494	0	0
normalized size	1	1.	0.78	0.59	0.	3.89	0.	0.
time (sec)	N/A	0.258	3.909	0.122	0.	1.685	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	64	41	74	189	0	81
normalized size	1	1.	0.88	0.56	1.01	2.59	0.	1.11
time (sec)	N/A	0.073	0.999	0.086	0.966	1.72	0.	1.741

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	89	65	0	367	0	0
normalized size	1	1.	0.94	0.68	0.	3.86	0.	0.
time (sec)	N/A	0.179	0.76	0.109	0.	1.684	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	54	31	51	124	0	58
normalized size	1	1.	1.1	0.63	1.04	2.53	0.	1.18
time (sec)	N/A	0.065	0.239	0.086	0.961	1.591	0.	2.022

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	79	55	0	246	0	0
normalized size	1	1.	1.25	0.87	0.	3.9	0.	0.
time (sec)	N/A	0.111	0.169	0.102	0.	1.693	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	44	21	27	69	58	27
normalized size	1	1.	1.83	0.88	1.12	2.88	2.42	1.12
time (sec)	N/A	0.033	0.081	0.008	0.958	1.582	0.484	1.521

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	95	32	0	258	0	167
normalized size	1	1.	2.38	0.8	0.	6.45	0.	4.18
time (sec)	N/A	0.06	0.102	0.052	0.	1.725	0.	1.366

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	106	83	0	431	0	394
normalized size	1	1.	1.47	1.15	0.	5.99	0.	5.47
time (sec)	N/A	0.079	0.222	0.113	0.	1.755	0.	1.418

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	271	90	0	274	0	0
normalized size	1	1.	2.85	0.95	0.	2.88	0.	0.
time (sec)	N/A	0.123	0.349	0.145	0.	1.719	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	302	153	0	517	0	0
normalized size	1	1.	2.2	1.12	0.	3.77	0.	0.
time (sec)	N/A	0.159	0.401	0.12	0.	1.767	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	179	118	0	338	0	0
normalized size	1	1.	1.2	0.79	0.	2.27	0.	0.
time (sec)	N/A	0.203	0.463	0.211	0.	1.862	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	191	244	0	583	0	0
normalized size	1	1.	0.97	1.24	0.	2.96	0.	0.
time (sec)	N/A	0.294	0.621	0.152	0.	1.895	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	61	57	97	313	0	0
normalized size	1	1.	0.63	0.59	1.	3.23	0.	0.
time (sec)	N/A	0.086	0.389	0.096	0.969	1.83	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	79	87	0	622	0	0
normalized size	1	1.	0.5	0.55	0.	3.91	0.	0.
time (sec)	N/A	0.302	0.624	0.112	0.	1.793	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	51	41	74	242	0	0
normalized size	1	1.	0.7	0.56	1.01	3.32	0.	0.
time (sec)	N/A	0.076	0.159	0.082	0.965	1.703	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	69	77	0	471	0	0
normalized size	1	1.	0.54	0.61	0.	3.71	0.	0.
time (sec)	N/A	0.235	0.154	0.113	0.	1.687	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	41	31	51	171	252	0
normalized size	1	1.	0.84	0.63	1.04	3.49	5.14	0.
time (sec)	N/A	0.068	0.084	0.079	0.948	1.675	131.176	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	59	67	0	336	0	0
normalized size	1	1.	0.62	0.71	0.	3.54	0.	0.
time (sec)	N/A	0.167	0.14	0.113	0.	1.625	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	27	104	90	0
normalized size	1	1.	1.	0.88	1.12	4.33	3.75	0.
time (sec)	N/A	0.034	0.041	0.009	0.942	1.575	26.235	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	60	49	0	196	0	910
normalized size	1	1.	0.97	0.79	0.	3.16	0.	14.68
time (sec)	N/A	0.068	0.101	0.075	0.	1.697	0.	8.416

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	67	37	132	63	0	0
normalized size	1	1.	2.58	1.42	5.08	2.42	0.	0.
time (sec)	N/A	0.058	0.158	0.083	1.632	1.611	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	72	70	0	266	0	0
normalized size	1	1.	0.99	0.96	0.	3.64	0.	0.
time (sec)	N/A	0.112	0.25	0.102	0.	1.697	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	130	107	0	585	0	0
normalized size	1	1.	1.21	1.	0.	5.47	0.	0.
time (sec)	N/A	0.135	0.382	0.135	0.	1.762	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	44	101	0	417	0	0
normalized size	1	1.	0.35	0.8	0.	3.28	0.	0.
time (sec)	N/A	0.181	0.071	0.188	0.	1.768	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	288	172	0	667	0	0
normalized size	1	1.	1.7	1.02	0.	3.95	0.	0.
time (sec)	N/A	0.221	0.421	0.132	0.	1.957	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	51	41	74	296	0	0
normalized size	1	1.	0.7	0.56	1.01	4.05	0.	0.
time (sec)	N/A	0.074	0.202	0.082	0.955	1.685	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	79	87	0	595	0	0
normalized size	1	1.	0.5	0.55	0.	3.74	0.	0.
time (sec)	N/A	0.293	0.309	0.112	0.	1.689	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	41	31	51	217	0	0
normalized size	1	1.	0.84	0.63	1.04	4.43	0.	0.
time (sec)	N/A	0.067	0.123	0.082	0.953	1.777	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	69	77	0	433	0	0
normalized size	1	1.	0.54	0.61	0.	3.41	0.	0.
time (sec)	N/A	0.224	0.265	0.106	0.	1.641	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	27	146	0	0
normalized size	1	1.	1.	0.88	1.12	6.08	0.	0.
time (sec)	N/A	0.034	0.057	0.007	0.943	1.609	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	73	66	0	235	0	1515
normalized size	1	1.	0.85	0.77	0.	2.73	0.	17.62
time (sec)	N/A	0.075	0.18	0.086	0.	1.728	0.	18.874

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	45	258	99	0	0
normalized size	1	1.	0.65	0.82	4.69	1.8	0.	0.
time (sec)	N/A	0.117	4.059	0.089	1.645	1.606	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	75	66	0	263	0	0
normalized size	1	1.	1.09	0.96	0.	3.81	0.	0.
time (sec)	N/A	0.11	0.368	0.107	0.	1.751	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	69	47	248	111	0	0
normalized size	1	1.	2.3	1.57	8.27	3.7	0.	0.
time (sec)	N/A	0.059	5.149	0.082	1.65	1.625	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	110	107	0	387	0	0
normalized size	1	1.	1.07	1.04	0.	3.76	0.	0.
time (sec)	N/A	0.172	0.278	0.164	0.	1.747	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	129	120	0	709	0	0
normalized size	1	1.	0.93	0.86	0.	5.1	0.	0.
time (sec)	N/A	0.197	5.296	0.126	0.	1.803	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	44	113	0	549	0	0
normalized size	1	1.	0.28	0.71	0.	3.45	0.	0.
time (sec)	N/A	0.242	0.092	0.234	0.	1.757	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	64	57	97	410	0	0
normalized size	1	1.	0.66	0.59	1.	4.23	0.	0.
time (sec)	N/A	0.079	0.602	0.131	0.945	1.903	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	102	107	0	859	0	0
normalized size	1	1.	0.46	0.48	0.	3.85	0.	0.
time (sec)	N/A	0.43	0.549	0.119	0.	1.763	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	54	41	74	335	0	0
normalized size	1	1.	0.74	0.56	1.01	4.59	0.	0.
time (sec)	N/A	0.075	0.27	0.09	0.955	1.67	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	92	97	0	683	0	0
normalized size	1	1.	0.48	0.51	0.	3.58	0.	0.
time (sec)	N/A	0.365	0.233	0.116	0.	1.69	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	31	51	250	0	0
normalized size	1	1.	0.9	0.63	1.04	5.1	0.	0.
time (sec)	N/A	0.066	0.133	0.082	0.957	1.654	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	82	87	0	522	0	0
normalized size	1	1.	0.52	0.55	0.	3.28	0.	0.
time (sec)	N/A	0.292	0.204	0.11	0.	1.655	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	27	176	0	0
normalized size	1	1.	1.	0.88	1.12	7.33	0.	0.
time (sec)	N/A	0.033	0.081	0.007	0.95	1.636	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	85	83	0	274	0	3368
normalized size	1	1.	0.77	0.75	0.	2.49	0.	30.62
time (sec)	N/A	0.085	0.316	0.096	0.	1.67	0.	42.406

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	48	55	320	134	0	0
normalized size	1	1.	0.54	0.62	3.6	1.51	0.	0.
time (sec)	N/A	0.174	5.477	0.101	1.672	1.668	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	42	83	0	297	0	0
normalized size	1	1.	0.46	0.91	0.	3.26	0.	0.
time (sec)	N/A	0.127	0.089	0.158	0.	1.685	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	82	57	432	142	0	0
normalized size	1	1.	1.34	0.93	7.08	2.33	0.	0.
time (sec)	N/A	0.113	5.295	0.102	2.475	1.624	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	108	75	0	382	0	0
normalized size	1	1.	1.02	0.71	0.	3.6	0.	0.
time (sec)	N/A	0.171	0.291	0.145	0.	1.777	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	69	47	365	142	0	0
normalized size	1	1.	2.3	1.57	12.17	4.73	0.	0.
time (sec)	N/A	0.057	5.272	0.106	1.67	1.629	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	120	144	0	501	0	0
normalized size	1	1.	0.89	1.07	0.	3.71	0.	0.
time (sec)	N/A	0.232	0.541	0.218	0.	1.801	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	139	139	0	828	0	0
normalized size	1	1.	0.81	0.81	0.	4.84	0.	0.
time (sec)	N/A	0.268	5.496	0.143	0.	2.212	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	44	129	0	663	0	0
normalized size	1	1.	0.23	0.68	0.	3.47	0.	0.
time (sec)	N/A	0.312	0.101	0.26	0.	2.245	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	388	205	0	917	0	0
normalized size	1	1.	1.67	0.88	0.	3.94	0.	0.
time (sec)	N/A	0.354	5.662	0.147	0.	2.402	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	61	57	379	228	0	97
normalized size	1	1.	0.63	0.59	3.91	2.35	0.	1.
time (sec)	N/A	0.076	0.289	0.089	0.97	1.916	0.	1.306

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	59	64	0	432	0	497
normalized size	1	1.	0.62	0.67	0.	4.55	0.	5.23
time (sec)	N/A	0.17	0.246	0.125	0.	1.895	0.	2.194

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	51	41	216	165	0	74
normalized size	1	1.	0.7	0.56	2.96	2.26	0.	1.01
time (sec)	N/A	0.067	0.136	0.079	0.954	1.842	0.	1.23

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	54	0	306	0	346
normalized size	1	1.	0.78	0.86	0.	4.86	0.	5.49
time (sec)	N/A	0.111	0.06	0.111	0.	1.92	0.	1.984

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	34	31	101	104	0	51
normalized size	1	1.	0.69	0.63	2.06	2.12	0.	1.04
time (sec)	N/A	0.063	0.064	0.088	0.954	1.862	0.	1.155

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	44	0	194	0	193
normalized size	1	1.	1.	1.47	0.	6.47	0.	6.43
time (sec)	N/A	0.051	0.065	0.089	0.	1.822	0.	1.828

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	46	32	27
normalized size	1	1.	1.	0.95	1.23	2.09	1.45	1.23
time (sec)	N/A	0.03	0.025	0.008	0.946	1.78	0.84	1.136

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	39	54	0	252	0	80
normalized size	1	1.	0.65	0.9	0.	4.2	0.	1.33
time (sec)	N/A	0.062	0.049	0.102	0.	2.166	0.	1.133

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	118	130	0	549	0	566
normalized size	1	1.	1.16	1.27	0.	5.38	0.	5.55
time (sec)	N/A	0.093	0.265	0.131	0.	2.326	0.	2.258

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	42	107	0	396	0	143
normalized size	1	1.	0.36	0.92	0.	3.41	0.	1.23
time (sec)	N/A	0.133	0.065	0.185	0.	2.278	0.	1.109

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	117	231	0	624	0	1006
normalized size	1	1.	0.72	1.43	0.	3.85	0.	6.21
time (sec)	N/A	0.218	0.585	0.181	0.	2.438	0.	2.611

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	44	135	0	462	0	194
normalized size	1	1.	0.25	0.77	0.	2.64	0.	1.11
time (sec)	N/A	0.263	0.08	0.244	0.	2.568	0.	1.112

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	140	308	0	695	0	1438
normalized size	1	1.	0.63	1.39	0.	3.14	0.	6.51
time (sec)	N/A	0.357	0.662	0.184	0.	2.729	0.	4.421

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	54	57	97	204	0	97
normalized size	1	1.	0.56	0.59	1.	2.1	0.	1.
time (sec)	N/A	0.082	0.226	0.089	0.974	2.293	0.	1.256

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	57	0	375	0	421
normalized size	1	1.	0.78	0.9	0.	5.95	0.	6.68
time (sec)	N/A	0.116	0.184	0.178	0.	2.126	0.	2.154

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	44	41	74	142	0	74
normalized size	1	1.	0.6	0.56	1.01	1.95	0.	1.01
time (sec)	N/A	0.075	0.096	0.078	0.963	2.249	0.	1.203

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	42	47	0	257	0	269
normalized size	1	1.	1.4	1.57	0.	8.57	0.	8.97
time (sec)	N/A	0.058	0.054	0.105	0.	2.204	0.	2.012

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	32	29	49	78	0	49
normalized size	1	1.	0.68	0.62	1.04	1.66	0.	1.04
time (sec)	N/A	0.067	0.05	0.08	0.965	2.202	0.	1.156

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	84	94	0	536	0	257
normalized size	1	1.	1.11	1.24	0.	7.05	0.	3.38
time (sec)	N/A	0.081	0.146	0.134	0.	2.261	0.	1.905

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	78	46	27
normalized size	1	1.	1.	0.95	1.23	3.55	2.09	1.23
time (sec)	N/A	0.034	0.03	0.004	0.949	2.232	2.864	1.134

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	41	71	0	355	0	103
normalized size	1	1.	0.46	0.8	0.	3.99	0.	1.16
time (sec)	N/A	0.077	0.068	0.085	0.	2.349	0.	1.12

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	224	202	0	652	0	0
normalized size	1	1.	1.67	1.51	0.	4.87	0.	0.
time (sec)	N/A	0.162	0.264	0.148	0.	2.331	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	42	124	0	502	0	178
normalized size	1	1.	0.28	0.83	0.	3.35	0.	1.19
time (sec)	N/A	0.202	0.068	0.166	0.	2.505	0.	1.141

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	334	289	0	729	0	0
normalized size	1	1.	1.71	1.48	0.	3.74	0.	0.
time (sec)	N/A	0.291	0.325	0.184	0.	2.489	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	44	152	0	572	0	225
normalized size	1	1.	0.21	0.72	0.	2.71	0.	1.07
time (sec)	N/A	0.343	0.091	0.234	0.	2.572	0.	1.14

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	444	367	0	805	0	0
normalized size	1	1.	1.73	1.43	0.	3.14	0.	0.
time (sec)	N/A	0.429	1.406	0.178	0.	2.893	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	59	67	0	568	0	648
normalized size	1	1.	0.62	0.71	0.	5.98	0.	6.82
time (sec)	N/A	0.195	0.663	0.16	0.	2.274	0.	2.475

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	64	67	120	247	0	120
normalized size	1	1.	0.53	0.55	0.99	2.04	0.	0.99
time (sec)	N/A	0.09	0.29	0.102	0.942	2.302	0.	1.313

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	49	57	0	431	0	497
normalized size	1	1.	0.78	0.9	0.	6.84	0.	7.89
time (sec)	N/A	0.119	0.335	0.12	0.	2.23	0.	2.327

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	54	57	97	176	0	97
normalized size	1	1.	0.56	0.59	1.	1.81	0.	1.
time (sec)	N/A	0.083	0.175	0.161	0.937	2.269	0.	1.247

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	42	47	0	309	0	344
normalized size	1	1.	1.4	1.57	0.	10.3	0.	11.47
time (sec)	N/A	0.057	0.111	0.104	0.	2.151	0.	2.169

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	44	41	74	111	0	74
normalized size	1	1.	0.62	0.58	1.04	1.56	0.	1.04
time (sec)	N/A	0.075	0.071	0.081	0.952	2.159	0.	1.204

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	96	112	0	593	0	344
normalized size	1	1.	0.89	1.04	0.	5.49	0.	3.19
time (sec)	N/A	0.144	0.179	0.15	0.	2.354	0.	2.07

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	30	29	57	104	267	49
normalized size	1	1.	0.67	0.64	1.27	2.31	5.93	1.09
time (sec)	N/A	0.068	0.054	0.087	0.943	2.133	30.471	1.169

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	100	123	0	666	0	396
normalized size	1	1.	1.33	1.64	0.	8.88	0.	5.28
time (sec)	N/A	0.082	0.229	0.119	0.	2.31	0.	2.188

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	27	116	65	27
normalized size	1	1.	1.	0.88	1.12	4.83	2.71	1.12
time (sec)	N/A	0.035	0.04	0.006	0.935	2.148	30.179	1.118

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	41	88	0	452	0	130
normalized size	1	1.	0.36	0.78	0.	4.	0.	1.15
time (sec)	N/A	0.089	0.084	0.097	0.	2.475	0.	1.126

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	284	266	0	755	0	0
normalized size	1	1.	1.7	1.59	0.	4.52	0.	0.
time (sec)	N/A	0.23	0.396	0.187	0.	2.449	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	42	141	0	601	0	201
normalized size	1	1.	0.23	0.76	0.	3.25	0.	1.09
time (sec)	N/A	0.275	0.09	0.194	0.	2.531	0.	1.143

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	394	355	0	833	0	0
normalized size	1	1.	1.69	1.52	0.	3.58	0.	0.
time (sec)	N/A	0.364	0.516	0.192	0.	2.615	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	98	249	0	0	0	0
normalized size	1	1.	0.79	2.01	0.	0.	0.	0.
time (sec)	N/A	0.084	0.584	0.454	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	264	214	0	0	0	0
normalized size	1	1.	2.78	2.25	0.	0.	0.	0.
time (sec)	N/A	0.067	2.609	0.411	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	75	179	0	0	0	0
normalized size	1	1.	0.79	1.88	0.	0.	0.	0.
time (sec)	N/A	0.065	0.364	0.352	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	260	120	0	0	0	0
normalized size	1	1.	4.13	1.9	0.	0.	0.	0.
time (sec)	N/A	0.046	0.958	0.303	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	48	103	0	0	0	0
normalized size	1	1.	0.79	1.69	0.	0.	0.	0.
time (sec)	N/A	0.046	20.95	0.22	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	188	117	0	0	0	0
normalized size	1	1.	2.07	1.29	0.	0.	0.	0.
time (sec)	N/A	0.068	0.915	0.488	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	86	189	0	0	0	0
normalized size	1	1.	0.89	1.95	0.	0.	0.	0.
time (sec)	N/A	0.065	0.429	0.609	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	144	304	0	0	0	0
normalized size	1	1.	1.14	2.41	0.	0.	0.	0.
time (sec)	N/A	0.086	1.223	0.962	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	66	295	0	0	0	0
normalized size	1	1.	0.39	1.76	0.	0.	0.	0.
time (sec)	N/A	0.144	0.102	0.452	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	66	260	0	0	0	0
normalized size	1	1.	0.48	1.9	0.	0.	0.	0.
time (sec)	N/A	0.122	0.113	0.433	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	66	203	0	0	0	0
normalized size	1	1.	0.48	1.48	0.	0.	0.	0.
time (sec)	N/A	0.125	0.074	0.46	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	66	188	0	0	0	0
normalized size	1	1.	0.63	1.79	0.	0.	0.	0.
time (sec)	N/A	0.092	0.044	0.388	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	64	152	0	0	0	0
normalized size	1	1.	0.61	1.45	0.	0.	0.	0.
time (sec)	N/A	0.095	0.033	0.328	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	64	120	0	0	0	0
normalized size	1	1.	0.75	1.41	0.	0.	0.	0.
time (sec)	N/A	0.132	0.062	0.483	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	66	193	0	0	0	0
normalized size	1	1.	0.74	2.17	0.	0.	0.	0.
time (sec)	N/A	0.129	0.06	0.619	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	66	305	0	0	0	0
normalized size	1	1.	0.52	2.4	0.	0.	0.	0.
time (sec)	N/A	0.181	0.077	1.003	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	66	375	0	0	0	0
normalized size	1	1.	0.58	3.29	0.	0.	0.	0.
time (sec)	N/A	0.091	0.113	1.218	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	66	488	0	0	0	0
normalized size	1	1.	0.46	3.37	0.	0.	0.	0.
time (sec)	N/A	0.112	0.161	1.622	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	66	321	0	0	0	0
normalized size	1	1.	0.33	1.58	0.	0.	0.	0.
time (sec)	N/A	0.211	0.074	0.595	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	66	264	0	0	0	0
normalized size	1	1.	0.39	1.55	0.	0.	0.	0.
time (sec)	N/A	0.189	0.114	0.494	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	66	251	0	0	0	0
normalized size	1	1.	0.38	1.46	0.	0.	0.	0.
time (sec)	N/A	0.188	0.073	0.449	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	66	214	0	0	0	0
normalized size	1	1.	0.47	1.53	0.	0.	0.	0.
time (sec)	N/A	0.147	0.044	0.452	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	64	178	0	0	0	0
normalized size	1	1.	0.47	1.31	0.	0.	0.	0.
time (sec)	N/A	0.147	0.031	0.447	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	64	146	0	0	0	0
normalized size	1	1.	0.6	1.38	0.	0.	0.	0.
time (sec)	N/A	0.198	0.054	0.667	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	66	219	0	0	0	0
normalized size	1	1.	0.6	1.99	0.	0.	0.	0.
time (sec)	N/A	0.202	0.053	0.744	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	66	332	0	0	0	0
normalized size	1	1.	0.52	2.61	0.	0.	0.	0.
time (sec)	N/A	0.202	0.09	1.153	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	66	401	0	0	0	0
normalized size	1	1.	0.52	3.16	0.	0.	0.	0.
time (sec)	N/A	0.198	0.083	1.273	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	66	514	0	0	0	0
normalized size	1	1.	0.4	3.12	0.	0.	0.	0.
time (sec)	N/A	0.243	0.139	1.977	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	66	295	0	0	0	0
normalized size	1	1.	0.31	1.4	0.	0.	0.	0.
time (sec)	N/A	0.248	0.103	0.504	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	66	258	0	0	0	0
normalized size	1	1.	0.37	1.45	0.	0.	0.	0.
time (sec)	N/A	0.201	0.067	0.519	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	64	222	0	0	0	0
normalized size	1	1.	0.36	1.25	0.	0.	0.	0.
time (sec)	N/A	0.213	0.06	0.615	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	64	190	0	0	0	0
normalized size	1	1.	0.41	1.22	0.	0.	0.	0.
time (sec)	N/A	0.234	0.084	0.767	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	66	263	0	0	0	0
normalized size	1	1.	0.43	1.73	0.	0.	0.	0.
time (sec)	N/A	0.224	0.075	0.776	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	66	332	0	0	0	0
normalized size	1	1.	0.52	2.61	0.	0.	0.	0.
time (sec)	N/A	0.199	0.099	1.299	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	66	401	0	0	0	0
normalized size	1	1.	0.52	3.16	0.	0.	0.	0.
time (sec)	N/A	0.198	0.118	1.585	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	66	514	0	0	0	0
normalized size	1	1.	0.39	3.04	0.	0.	0.	0.
time (sec)	N/A	0.25	0.147	2.043	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	66	583	0	0	0	0
normalized size	1	1.	0.39	3.45	0.	0.	0.	0.
time (sec)	N/A	0.259	0.211	2.272	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	66	251	0	0	0	0
normalized size	1	1.	0.5	1.9	0.	0.	0.	0.
time (sec)	N/A	0.117	0.168	0.563	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	66	216	0	0	0	0
normalized size	1	1.	0.65	2.14	0.	0.	0.	0.
time (sec)	N/A	0.095	0.092	0.598	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	66	181	0	0	0	0
normalized size	1	1.	0.65	1.79	0.	0.	0.	0.
time (sec)	N/A	0.095	0.079	0.516	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	66	122	0	0	0	0
normalized size	1	1.	0.97	1.79	0.	0.	0.	0.
time (sec)	N/A	0.075	0.099	0.591	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	110	0	0	0	0
normalized size	1	1.	1.	1.67	0.	0.	0.	0.
time (sec)	N/A	0.076	0.075	0.345	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	66	115	0	0	0	0
normalized size	1	1.	0.89	1.55	0.	0.	0.	0.
time (sec)	N/A	0.068	0.042	0.831	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	64	190	0	0	0	0
normalized size	1	1.	0.82	2.44	0.	0.	0.	0.
time (sec)	N/A	0.072	0.04	0.914	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	63	304	0	0	0	0
normalized size	1	1.	0.56	2.71	0.	0.	0.	0.
time (sec)	N/A	0.097	0.056	1.286	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	66	375	0	0	0	0
normalized size	1	1.	0.59	3.35	0.	0.	0.	0.
time (sec)	N/A	0.094	0.061	1.511	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	66	488	0	0	0	0
normalized size	1	1.	0.46	3.41	0.	0.	0.	0.
time (sec)	N/A	0.114	0.083	2.212	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	66	203	0	0	0	0
normalized size	1	1.	0.46	1.4	0.	0.	0.	0.
time (sec)	N/A	0.11	0.189	0.56	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	66	190	0	0	0	0
normalized size	1	1.	0.58	1.67	0.	0.	0.	0.
time (sec)	N/A	0.092	0.09	0.658	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	66	155	0	0	0	0
normalized size	1	1.	0.59	1.38	0.	0.	0.	0.
time (sec)	N/A	0.097	0.077	0.434	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	66	120	0	0	0	0
normalized size	1	1.	0.84	1.52	0.	0.	0.	0.
time (sec)	N/A	0.078	0.094	0.717	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	66	193	0	0	0	0
normalized size	1	1.	0.8	2.33	0.	0.	0.	0.
time (sec)	N/A	0.075	0.077	1.057	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	66	303	0	0	0	0
normalized size	1	1.	0.57	2.61	0.	0.	0.	0.
time (sec)	N/A	0.119	0.042	1.456	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	64	372	0	0	0	0
normalized size	1	1.	0.55	3.21	0.	0.	0.	0.
time (sec)	N/A	0.127	0.045	1.764	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	66	488	0	0	0	0
normalized size	1	1.	0.44	3.25	0.	0.	0.	0.
time (sec)	N/A	0.157	0.068	2.269	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	66	557	0	0	0	0
normalized size	1	1.	0.44	3.71	0.	0.	0.	0.
time (sec)	N/A	0.162	0.073	2.612	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	66	670	0	0	0	0
normalized size	1	1.	0.36	3.7	0.	0.	0.	0.
time (sec)	N/A	0.183	0.106	3.316	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	66	251	0	0	0	0
normalized size	1	1.	0.39	1.49	0.	0.	0.	0.
time (sec)	N/A	0.179	0.37	0.658	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	66	216	0	0	0	0
normalized size	1	1.	0.48	1.57	0.	0.	0.	0.
time (sec)	N/A	0.152	0.229	0.662	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	66	181	0	0	0	0
normalized size	1	1.	0.5	1.37	0.	0.	0.	0.
time (sec)	N/A	0.151	0.178	0.763	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	66	146	0	0	0	0
normalized size	1	1.	0.64	1.42	0.	0.	0.	0.
time (sec)	N/A	0.155	0.098	0.987	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	66	219	0	0	0	0
normalized size	1	1.	0.62	2.05	0.	0.	0.	0.
time (sec)	N/A	0.136	0.08	1.101	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	66	330	0	0	0	0
normalized size	1	1.	0.56	2.8	0.	0.	0.	0.
time (sec)	N/A	0.131	0.084	1.941	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	66	401	0	0	0	0
normalized size	1	1.	0.56	3.4	0.	0.	0.	0.
time (sec)	N/A	0.134	0.066	1.905	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	66	512	0	0	0	0
normalized size	1	1.	0.43	3.35	0.	0.	0.	0.
time (sec)	N/A	0.173	0.043	2.585	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	66	580	0	0	0	0
normalized size	1	1.	0.43	3.79	0.	0.	0.	0.
time (sec)	N/A	0.184	0.049	3.264	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	66	696	0	0	0	0
normalized size	1	1.	0.35	3.72	0.	0.	0.	0.
time (sec)	N/A	0.225	0.062	4.014	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	66	225	0	0	0	0
normalized size	1	1.	0.37	1.25	0.	0.	0.	0.
time (sec)	N/A	0.175	0.38	0.931	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	66	190	0	0	0	0
normalized size	1	1.	0.44	1.28	0.	0.	0.	0.
time (sec)	N/A	0.153	0.22	1.056	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	66	263	0	0	0	0
normalized size	1	1.	0.46	1.81	0.	0.	0.	0.
time (sec)	N/A	0.152	0.172	1.204	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	66	332	0	0	0	0
normalized size	1	1.	0.55	2.77	0.	0.	0.	0.
time (sec)	N/A	0.134	0.091	2.181	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	66	401	0	0	0	0
normalized size	1	1.	0.55	3.34	0.	0.	0.	0.
time (sec)	N/A	0.134	0.078	2.124	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	66	514	0	0	0	0
normalized size	1	1.	0.43	3.34	0.	0.	0.	0.
time (sec)	N/A	0.182	0.088	2.808	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	66	583	0	0	0	0
normalized size	1	1.	0.43	3.79	0.	0.	0.	0.
time (sec)	N/A	0.185	0.074	3.526	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	66	694	0	0	0	0
normalized size	1	1.	0.35	3.63	0.	0.	0.	0.
time (sec)	N/A	0.228	0.046	3.932	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	66	762	0	0	0	0
normalized size	1	1.	0.35	3.99	0.	0.	0.	0.
time (sec)	N/A	0.242	0.058	4.301	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	66	878	0	0	0	0
normalized size	1	1.	0.29	3.9	0.	0.	0.	0.
time (sec)	N/A	0.299	0.088	5.447	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	269	241	0	0	0	0
normalized size	1	1.	1.14	1.02	0.	0.	0.	0.
time (sec)	N/A	0.357	0.921	0.236	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	195	213	0	0	0	0
normalized size	1	1.	1.01	1.1	0.	0.	0.	0.
time (sec)	N/A	0.27	0.856	0.155	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	108	142	0	0	0	0
normalized size	1	1.	0.67	0.88	0.	0.	0.	0.
time (sec)	N/A	0.209	0.463	0.109	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	34	177	95	0	0
normalized size	1	1.	1.	1.	5.21	2.79	0.	0.
time (sec)	N/A	0.068	0.117	0.123	1.553	2.562	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	46	44	278	128	0	0
normalized size	1	1.	0.62	0.59	3.76	1.73	0.	0.
time (sec)	N/A	0.146	0.233	0.119	1.583	2.576	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	56	54	381	155	0	0
normalized size	1	1.	0.49	0.47	3.31	1.35	0.	0.
time (sec)	N/A	0.224	0.328	0.124	1.598	2.432	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	74	70	482	188	0	0
normalized size	1	1.	0.48	0.45	3.13	1.22	0.	0.
time (sec)	N/A	0.307	0.758	0.138	1.622	2.166	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	78	314	0	0	0	0
normalized size	1	1.	0.24	0.98	0.	0.	0.	0.
time (sec)	N/A	0.564	0.167	0.201	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	78	288	0	0	0	0
normalized size	1	1.	0.28	1.04	0.	0.	0.	0.
time (sec)	N/A	0.475	0.132	0.17	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	77	262	0	0	0	0
normalized size	1	1.	0.32	1.08	0.	0.	0.	0.
time (sec)	N/A	0.358	0.117	0.166	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	75	228	0	0	0	0
normalized size	1	1.	0.38	1.15	0.	0.	0.	0.
time (sec)	N/A	0.284	0.105	0.145	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	75	323	0	0	0	0
normalized size	1	1.	0.36	1.54	0.	0.	0.	0.
time (sec)	N/A	0.293	0.115	0.12	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	177	112	0	0
normalized size	1	1.	1.	0.94	4.92	3.11	0.	0.
time (sec)	N/A	0.073	0.107	0.089	1.578	2.786	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	72	44	279	177	0	0
normalized size	1	1.	0.97	0.59	3.77	2.39	0.	0.
time (sec)	N/A	0.148	0.129	0.096	1.579	2.542	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	105	54	379	215	0	0
normalized size	1	1.	0.93	0.48	3.35	1.9	0.	0.
time (sec)	N/A	0.23	0.196	0.102	1.608	2.297	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	74	70	482	248	0	0
normalized size	1	1.	0.49	0.46	3.17	1.63	0.	0.
time (sec)	N/A	0.309	0.248	0.112	1.667	2.38	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	77	344	0	0	0	0
normalized size	1	1.	0.24	1.07	0.	0.	0.	0.
time (sec)	N/A	0.543	0.283	0.208	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	78	317	0	0	0	0
normalized size	1	1.	0.27	1.11	0.	0.	0.	0.
time (sec)	N/A	0.437	0.122	0.183	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	76	284	0	0	0	0
normalized size	1	1.	0.31	1.15	0.	0.	0.	0.
time (sec)	N/A	0.359	0.103	0.169	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	75	445	0	0	0	0
normalized size	1	1.	0.31	1.86	0.	0.	0.	0.
time (sec)	N/A	0.364	0.154	0.128	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	77	545	0	0	0	0
normalized size	1	1.	0.38	2.67	0.	0.	0.	0.
time (sec)	N/A	0.296	0.189	0.137	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	177	265	0	0
normalized size	1	1.	1.	0.94	4.92	7.36	0.	0.
time (sec)	N/A	0.075	0.173	0.092	1.59	2.667	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	54	44	279	185	0	0
normalized size	1	1.	0.71	0.58	3.67	2.43	0.	0.
time (sec)	N/A	0.148	0.181	0.095	1.589	2.599	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	64	54	381	254	0	0
normalized size	1	1.	0.57	0.48	3.37	2.25	0.	0.
time (sec)	N/A	0.223	0.217	0.105	1.596	2.497	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	74	70	482	298	0	0
normalized size	1	1.	0.49	0.47	3.21	1.99	0.	0.
time (sec)	N/A	0.305	0.282	0.106	1.656	2.124	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	77	239	0	0	0	0
normalized size	1	1.	0.32	0.98	0.	0.	0.	0.
time (sec)	N/A	0.367	0.189	0.149	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	77	212	0	0	0	0
normalized size	1	1.	0.38	1.06	0.	0.	0.	0.
time (sec)	N/A	0.279	0.116	0.125	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	77	141	0	0	0	0
normalized size	1	1.	0.46	0.83	0.	0.	0.	0.
time (sec)	N/A	0.196	0.083	0.095	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	34	176	107	0	0
normalized size	1	1.	1.	1.	5.18	3.15	0.	0.
time (sec)	N/A	0.06	0.064	0.102	1.588	2.435	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	46	44	284	177	0	0
normalized size	1	1.	0.61	0.58	3.74	2.33	0.	0.
time (sec)	N/A	0.133	0.103	0.107	1.602	2.622	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	56	54	387	211	0	0
normalized size	1	1.	0.49	0.47	3.37	1.83	0.	0.
time (sec)	N/A	0.209	0.099	0.112	1.625	2.264	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	66	70	490	240	0	0
normalized size	1	1.	0.43	0.45	3.18	1.56	0.	0.
time (sec)	N/A	0.288	0.178	0.121	1.653	2.06	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	80	266	0	0	0	0
normalized size	1	1.	0.32	1.08	0.	0.	0.	0.
time (sec)	N/A	0.372	0.125	0.148	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	80	232	0	0	0	0
normalized size	1	1.	0.37	1.08	0.	0.	0.	0.
time (sec)	N/A	0.287	0.184	0.113	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	80	321	0	0	0	0
normalized size	1	1.	0.34	1.36	0.	0.	0.	0.
time (sec)	N/A	0.362	0.123	0.101	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	49	34	177	250	0	0
normalized size	1	1.	1.36	0.94	4.92	6.94	0.	0.
time (sec)	N/A	0.064	0.072	0.105	1.564	2.877	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	59	44	285	181	0	0
normalized size	1	1.	0.78	0.58	3.75	2.38	0.	0.
time (sec)	N/A	0.13	0.11	0.113	1.577	2.811	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	56	54	397	252	0	0
normalized size	1	1.	0.49	0.47	3.45	2.19	0.	0.
time (sec)	N/A	0.21	0.104	0.093	1.606	2.825	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	66	70	504	289	0	0
normalized size	1	1.	0.43	0.45	3.27	1.88	0.	0.
time (sec)	N/A	0.289	0.172	0.104	1.653	2.85	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	76	80	609	323	0	0
normalized size	1	1.	0.39	0.41	3.16	1.67	0.	0.
time (sec)	N/A	0.372	0.269	0.114	1.691	2.667	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	80	282	0	0	0	0
normalized size	1	1.	0.31	1.08	0.	0.	0.	0.
time (sec)	N/A	0.469	0.168	0.146	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	80	443	0	0	0	0
normalized size	1	1.	0.33	1.85	0.	0.	0.	0.
time (sec)	N/A	0.366	0.115	0.117	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	80	545	0	0	0	0
normalized size	1	1.	0.37	2.5	0.	0.	0.	0.
time (sec)	N/A	0.296	0.128	0.115	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	49	34	177	174	0	0
normalized size	1	1.	1.36	0.94	4.92	4.83	0.	0.
time (sec)	N/A	0.071	0.107	0.087	1.59	3.423	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	59	44	279	378	0	0
normalized size	1	1.	0.78	0.58	3.67	4.97	0.	0.
time (sec)	N/A	0.131	0.102	0.11	1.59	3.311	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	69	54	387	251	0	0
normalized size	1	1.	0.6	0.47	3.37	2.18	0.	0.
time (sec)	N/A	0.202	0.142	0.115	1.62	3.103	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	66	70	504	328	0	0
normalized size	1	1.	0.43	0.45	3.27	2.13	0.	0.
time (sec)	N/A	0.297	0.138	0.112	1.664	2.891	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	76	80	609	366	0	0
normalized size	1	1.	0.39	0.41	3.16	1.9	0.	0.
time (sec)	N/A	0.375	0.331	0.115	1.69	2.988	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	77	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.153	0.142	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	77	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.108	0.124	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	77	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.077	0.113	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	77	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.075	0.118	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.077	0.112	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.087	0.11	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	94	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.199	8.375	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	94	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.105	3.378	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	94	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.103	2.616	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	245	0	0	0	0	0
normalized size	1	1.	2.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	1.34	0.907	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	94	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.165	0.133	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.16	0.26	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.153	0.32	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.174	1.833	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	102	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.223	0.107	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	102	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.186	0.098	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	101	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.184	0.098	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	310	0	0	0	0	0
normalized size	1	1.	3.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	3.714	0.108	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.111	0.091	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	101	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	0.168	0.09	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	102	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.132	0.091	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	112	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.19	0.882	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	89	0	0	408	0	698
normalized size	1	1.	0.82	0.	0.	3.74	0.	6.4
time (sec)	N/A	0.086	0.698	3.554	0.	3.095	0.	1.103

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	68	0	0	263	0	397
normalized size	1	1.	0.84	0.	0.	3.25	0.	4.9
time (sec)	N/A	0.069	0.322	1.564	0.	2.766	0.	1.098

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	52	0	0	153	0	185
normalized size	1	1.	0.95	0.	0.	2.78	0.	3.36
time (sec)	N/A	0.058	0.112	0.837	0.	2.672	0.	1.091

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	0	72	80	35
normalized size	1	1.	1.	1.04	0.	2.77	3.08	1.35
time (sec)	N/A	0.028	0.032	0.001	0.	2.291	2.644	1.072

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	63	0	0	0	0	0
normalized size	1	1.	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.094	0.54	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	111	0	0	0	0	0
normalized size	1	1.	2.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.338	0.095	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	163	0	0	0	0	0
normalized size	1	1.	3.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.71	0.134	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	78	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.123	1.293	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.094	0.749	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	5807	0	0	0	0	0
normalized size	1	1.	79.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	25.916	0.082	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	307	0	0	0	0	0
normalized size	1	1.	3.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	13.104	0.103	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.196	0.096	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.129	0.091	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.083	0.098	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	83	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.077	0.093	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.108	0.085	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.115	0.087	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	101	0	0	247	0	0
normalized size	1	1.	0.5	0.	0.	1.23	0.	0.
time (sec)	N/A	0.321	0.196	0.171	0.	2.379	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	76	0	0	184	0	0
normalized size	1	1.	0.54	0.	0.	1.3	0.	0.
time (sec)	N/A	0.222	0.137	0.157	0.	2.331	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	53	0	0	144	0	0
normalized size	1	1.	0.6	0.	0.	1.62	0.	0.
time (sec)	N/A	0.124	0.121	0.148	0.	2.371	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	88	93	0	0
normalized size	1	1.	1.	0.	2.59	2.74	0.	0.
time (sec)	N/A	0.051	0.053	0.144	1.572	2.288	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	108	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.129	0.355	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.254	0.15	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	113	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.25	0.152	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	105	0	842	813	0	0
normalized size	1	1.	0.7	0.	5.61	5.42	0.	0.
time (sec)	N/A	0.24	0.434	0.914	1.666	2.462	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	72	0	474	440	0	0
normalized size	1	1.	0.77	0.	5.04	4.68	0.	0.
time (sec)	N/A	0.141	0.219	0.935	1.568	2.374	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	194	197	0	0
normalized size	1	1.	0.98	0.	4.41	4.48	0.	0.
time (sec)	N/A	0.055	0.154	1.25	1.507	2.292	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.067	0.773	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	76	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.137	0.802	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	96	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.208	0.928	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	96	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.181	0.927	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	90	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.085	0.42	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.225	0.766	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	46	95	128	83	119
normalized size	1	1.	1.	0.77	1.58	2.13	1.38	1.98
time (sec)	N/A	0.042	0.023	0.027	0.949	2.313	3.768	1.081

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	65	97	82	65
normalized size	1	1.	1.	0.82	1.48	2.2	1.86	1.48
time (sec)	N/A	0.032	0.013	0.022	0.953	2.128	1.084	1.079

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	28	39	25	27	62	34	34
normalized size	1	1.27	1.77	1.14	1.23	2.82	1.55	1.55
time (sec)	N/A	0.016	0.011	0.008	0.942	2.043	0.21	1.063

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	26	34	47	97	0	50
normalized size	1	1.	0.6	0.79	1.09	2.26	0.	1.16
time (sec)	N/A	0.04	0.011	0.026	0.95	2.298	0.	1.109

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	52	54	72	178	0	74
normalized size	1	1.	1.27	1.32	1.76	4.34	0.	1.8
time (sec)	N/A	0.038	0.02	0.031	0.956	2.188	0.	1.13

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	68	74	105	219	0	95
normalized size	1	1.	1.11	1.21	1.72	3.59	0.	1.56
time (sec)	N/A	0.044	0.162	0.033	0.965	2.273	0.	1.141

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	62	52	65	132	124	104
normalized size	1	1.	0.95	0.8	1.	2.03	1.91	1.6
time (sec)	N/A	0.045	0.11	0.021	0.963	2.252	2.227	1.077

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	41	50	96	71	63
normalized size	1	1.	1.07	0.95	1.16	2.23	1.65	1.47
time (sec)	N/A	0.033	0.059	0.02	0.964	2.166	0.61	1.07

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	31	53	0	45
normalized size	1	1.	1.	1.04	1.35	2.3	0.	1.96
time (sec)	N/A	0.031	0.011	0.028	0.956	2.057	0.	1.097

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	47	92	0	103
normalized size	1	1.	0.93	0.86	1.07	2.09	0.	2.34
time (sec)	N/A	0.036	0.079	0.033	0.967	1.988	0.	1.092

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	53	48	65	127	0	162
normalized size	1	1.	0.88	0.8	1.08	2.12	0.	2.7
time (sec)	N/A	0.04	0.179	0.032	0.951	2.095	0.	1.102

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	104	98	143	211	202	184
normalized size	1	1.	1.05	0.99	1.44	2.13	2.04	1.86
time (sec)	N/A	0.089	0.205	0.043	0.96	2.238	8.259	1.089

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	56	78	99	163	129	108
normalized size	1	1.	0.73	1.01	1.29	2.12	1.68	1.4
time (sec)	N/A	0.071	0.116	0.042	0.962	2.22	2.596	1.085

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	46	21	27	109	53	27
normalized size	1	1.	2.09	0.95	1.23	4.95	2.41	1.23
time (sec)	N/A	0.027	0.014	0.015	0.941	2.185	0.576	1.08

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	72	81	159	0	84
normalized size	1	1.	0.89	1.18	1.33	2.61	0.	1.38
time (sec)	N/A	0.082	0.063	0.043	0.963	2.279	0.	1.129

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	113	118	105	221	0	116
normalized size	1	1.	1.92	2.	1.78	3.75	0.	1.97
time (sec)	N/A	0.061	0.943	0.055	0.957	2.231	0.	1.15

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	166	165	155	275	0	159
normalized size	1	1.	1.68	1.67	1.57	2.78	0.	1.61
time (sec)	N/A	0.088	0.731	0.058	0.968	2.323	0.	1.147

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	141	128	154	270	398	219
normalized size	1	1.	0.97	0.88	1.05	1.85	2.73	1.5
time (sec)	N/A	0.134	0.375	0.043	0.976	2.46	13.741	1.102

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	133	108	119	217	287	166
normalized size	1	1.	1.15	0.93	1.03	1.87	2.47	1.43
time (sec)	N/A	0.116	0.193	0.043	0.963	2.303	4.727	1.098

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	85	86	86	167	180	103
normalized size	1	1.	0.99	1.	1.	1.94	2.09	1.2
time (sec)	N/A	0.095	0.237	0.042	0.969	2.262	1.318	1.085

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	55	46	62	104	0	85
normalized size	1	1.	1.12	0.94	1.27	2.12	0.	1.73
time (sec)	N/A	0.048	0.059	0.036	1.465	2.092	0.	1.115

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	105	62	69	122	0	138
normalized size	1	1.	1.4	0.83	0.92	1.63	0.	1.84
time (sec)	N/A	0.096	0.322	0.051	0.973	2.127	0.	1.111

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	84	92	103	173	0	244
normalized size	1	1.	0.82	0.89	1.	1.68	0.	2.37
time (sec)	N/A	0.101	0.433	0.056	0.977	2.106	0.	1.12

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	110	120	131	225	0	351
normalized size	1	1.	0.85	0.93	1.02	1.74	0.	2.72
time (sec)	N/A	0.123	0.836	0.062	0.972	2.208	0.	1.12

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	120	135	194	281	280	250
normalized size	1	1.	0.83	0.94	1.35	1.95	1.94	1.74
time (sec)	N/A	0.133	0.544	0.057	0.949	2.38	13.326	1.114

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	56	115	135	221	178	151
normalized size	1	1.	0.73	1.49	1.75	2.87	2.31	1.96
time (sec)	N/A	0.08	0.148	0.055	0.965	2.254	4.59	1.107

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	57	21	27	158	97	27
normalized size	1	1.	2.59	0.95	1.23	7.18	4.41	1.23
time (sec)	N/A	0.026	0.069	0.019	0.955	2.243	1.226	1.095

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	67	108	123	221	0	126
normalized size	1	1.	0.84	1.35	1.54	2.76	0.	1.58
time (sec)	N/A	0.105	0.118	0.054	0.945	2.443	0.	1.145

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	176	154	132	273	0	154
normalized size	1	1.	1.59	1.39	1.19	2.46	0.	1.39
time (sec)	N/A	0.134	1.305	0.067	0.967	2.369	0.	1.167

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	318	195	184	332	0	188
normalized size	1	1.	3.38	2.07	1.96	3.53	0.	2.
time (sec)	N/A	0.08	4.113	0.072	0.95	2.4	0.	1.167

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	182	145	158	279	348	234
normalized size	1	1.	1.15	0.92	1.	1.77	2.2	1.48
time (sec)	N/A	0.217	0.385	0.053	0.96	2.519	8.278	1.103

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	107	123	126	232	236	153
normalized size	1	1.	0.82	0.94	0.96	1.77	1.8	1.17
time (sec)	N/A	0.194	0.54	0.05	0.962	2.235	2.595	1.09

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	68	89	95	154	0	166
normalized size	1	1.	0.86	1.13	1.2	1.95	0.	2.1
time (sec)	N/A	0.07	0.307	0.046	1.445	2.508	0.	1.109

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	136	122	108	176	0	173
normalized size	1	1.	1.62	1.45	1.29	2.1	0.	2.06
time (sec)	N/A	0.088	0.413	0.068	0.955	2.422	0.	1.111

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	190	173	142	232	0	328
normalized size	1	1.	1.41	1.28	1.05	1.72	0.	2.43
time (sec)	N/A	0.192	0.623	0.085	0.972	2.395	0.	1.105

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	245	219	167	279	0	483
normalized size	1	1.	1.48	1.33	1.01	1.69	0.	2.93
time (sec)	N/A	0.207	0.906	0.082	0.969	2.36	0.	1.131

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	299	265	196	347	0	639
normalized size	1	1.	1.56	1.38	1.02	1.81	0.	3.33
time (sec)	N/A	0.221	1.593	0.118	0.981	2.385	0.	1.139

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	120	530	420	879	614	626
normalized size	1	1.	0.83	3.68	2.92	6.1	4.26	4.35
time (sec)	N/A	0.221	2.061	0.086	0.957	3.134	122.701	1.209

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	56	480	315	740	493	367
normalized size	1	1.	0.73	6.23	4.09	9.61	6.4	4.77
time (sec)	N/A	0.151	0.885	0.085	0.972	3.026	60.16	1.164

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	137	21	27	582	168	27
normalized size	1	1.	6.23	0.95	1.23	26.45	7.64	1.23
time (sec)	N/A	0.026	0.359	0.029	0.944	3.103	20.96	1.129

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	227	465	428	784	0	510
normalized size	1	1.	0.93	1.9	1.75	3.2	0.	2.08
time (sec)	N/A	0.182	0.217	0.103	0.956	3.422	0.	1.172

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	366	645	436	894	0	551
normalized size	1	1.	1.29	2.27	1.54	3.15	0.	1.94
time (sec)	N/A	0.242	2.357	0.127	0.972	3.369	0.	1.194

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	514	760	470	879	0	579
normalized size	1	1.	1.61	2.38	1.47	2.75	0.	1.81
time (sec)	N/A	0.304	4.075	0.139	0.981	3.37	0.	1.214

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	457	497	454	780	1115	491
normalized size	1	1.	1.08	1.17	1.07	1.84	2.64	1.16
time (sec)	N/A	1.217	1.053	0.082	1.011	3.262	44.476	1.17

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	313	406	470	651	0	1079
normalized size	1	1.	0.9	1.16	1.35	1.87	0.	3.09
time (sec)	N/A	0.563	1.125	0.066	1.482	2.812	0.	1.171

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	414	495	443	633	0	923
normalized size	1	1.	1.12	1.34	1.2	1.72	0.	2.5
time (sec)	N/A	0.646	1.164	0.134	1.471	2.92	0.	1.183

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	472	544	425	660	0	895
normalized size	1	1.	1.24	1.43	1.12	1.73	0.	2.35
time (sec)	N/A	0.724	1.292	0.126	1.468	3.014	0.	1.199

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	479	567	419	732	0	980
normalized size	1	1.	1.19	1.4	1.04	1.81	0.	2.43
time (sec)	N/A	0.821	1.481	0.14	1.49	2.908	0.	1.194

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	313	662	425	795	0	1204
normalized size	1	1.	1.33	2.81	1.8	3.37	0.	5.1
time (sec)	N/A	0.385	4.5	0.135	1.005	3.15	0.	1.191

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	103	163	146	250	0	162
normalized size	1	1.	0.87	1.38	1.24	2.12	0.	1.37
time (sec)	N/A	0.105	0.195	0.036	0.964	2.713	0.	1.125

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	72	74	128	0	76
normalized size	1	1.	0.89	1.18	1.21	2.1	0.	1.25
time (sec)	N/A	0.066	0.067	0.033	0.944	2.545	0.	1.104

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	42	41	26
normalized size	1	1.	1.	1.06	1.33	2.33	2.28	1.44
time (sec)	N/A	0.027	0.007	0.013	0.933	2.374	0.618	1.101

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	64	76	86	158	0	96
normalized size	1	1.	0.85	1.01	1.15	2.11	0.	1.28
time (sec)	N/A	0.083	0.055	0.043	0.949	2.662	0.	1.141

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	170	164	188	366	0	239
normalized size	1	1.	1.38	1.33	1.53	2.98	0.	1.94
time (sec)	N/A	0.164	0.59	0.067	0.961	3.266	0.	1.172

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	266	305	375	579	0	448
normalized size	1	1.	1.36	1.56	1.92	2.97	0.	2.3
time (sec)	N/A	0.255	0.953	0.061	0.985	4.306	0.	1.162

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	2843	1055	0	1121	0	670
normalized size	1	1.	15.12	5.61	0.	5.96	0.	3.56
time (sec)	N/A	0.462	6.293	0.058	0.	3.073	0.	1.126

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	1685	450	0	768	0	305
normalized size	1	1.	13.27	3.54	0.	6.05	0.	2.4
time (sec)	N/A	0.252	6.132	0.045	0.	2.729	0.	1.114

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	398	142	0	498	0	128
normalized size	1	1.	5.69	2.03	0.	7.11	0.	1.83
time (sec)	N/A	0.114	2.217	0.037	0.	2.859	0.	1.127

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	152	117	0	684	0	144
normalized size	1	1.	1.81	1.39	0.	8.14	0.	1.71
time (sec)	N/A	0.094	0.293	0.046	0.	2.841	0.	1.119

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	202	270	0	1027	0	369
normalized size	1	1.	1.47	1.97	0.	7.5	0.	2.69
time (sec)	N/A	0.251	1.262	0.059	0.	2.804	0.	1.134

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	370	525	0	1482	0	788
normalized size	1	1.	1.88	2.66	0.	7.52	0.	4.
time (sec)	N/A	0.495	2.549	0.063	0.	3.284	0.	1.163

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	235	305	257	571	0	339
normalized size	1	1.	1.28	1.66	1.4	3.1	0.	1.84
time (sec)	N/A	0.173	0.518	0.076	0.955	3.248	0.	1.113

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	127	174	157	359	0	203
normalized size	1	1.	1.06	1.45	1.31	2.99	0.	1.69
time (sec)	N/A	0.101	0.635	0.079	0.953	2.816	0.	1.122

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	78	82	188	316	123
normalized size	1	1.	0.83	1.24	1.3	2.98	5.02	1.95
time (sec)	N/A	0.065	0.038	0.075	0.943	2.805	2.135	1.107

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	27	45	51	27
normalized size	1	1.	1.	1.05	1.35	2.25	2.55	1.35
time (sec)	N/A	0.027	0.024	0.023	0.933	2.38	1.141	1.102

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	102	101	159	443	0	198
normalized size	1	1.	0.98	0.97	1.53	4.26	0.	1.9
time (sec)	N/A	0.115	0.206	0.091	1.057	3.2	0.	1.144

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	222	192	371	867	0	329
normalized size	1	1.	1.25	1.08	2.1	4.9	0.	1.86
time (sec)	N/A	0.209	1.736	0.108	0.978	3.825	0.	1.166

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	406	331	682	1195	0	621
normalized size	1	1.	1.51	1.23	2.54	4.44	0.	2.31
time (sec)	N/A	0.321	6.113	0.118	1.032	5.571	0.	1.183

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	3695	1021	0	1364	0	633
normalized size	1	1.	19.76	5.46	0.	7.29	0.	3.39
time (sec)	N/A	0.371	6.542	0.093	0.	3.249	0.	1.125

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	2447	385	0	938	0	317
normalized size	1	1.	19.12	3.01	0.	7.33	0.	2.48
time (sec)	N/A	0.213	6.263	0.081	0.	2.922	0.	1.119

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	494	153	0	852	0	170
normalized size	1	1.	5.88	1.82	0.	10.14	0.	2.02
time (sec)	N/A	0.11	5.302	0.072	0.	2.766	0.	1.108

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	162	222	0	1207	0	366
normalized size	1	1.	1.25	1.71	0.	9.28	0.	2.82
time (sec)	N/A	0.208	1.102	0.082	0.	2.819	0.	1.139

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	336	370	0	1728	0	576
normalized size	1	1.	1.74	1.92	0.	8.95	0.	2.98
time (sec)	N/A	0.367	1.909	0.118	0.	3.381	0.	1.143

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	282	320	270	693	0	331
normalized size	1	1.	1.48	1.68	1.42	3.65	0.	1.74
time (sec)	N/A	0.159	0.661	0.098	0.953	3.43	0.	1.136

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	143	183	177	473	0	192
normalized size	1	1.	1.13	1.44	1.39	3.72	0.	1.51
time (sec)	N/A	0.105	0.96	0.093	0.956	2.923	0.	1.147

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	55	85	103	257	670	84
normalized size	1	1.	0.76	1.18	1.43	3.57	9.31	1.17
time (sec)	N/A	0.072	0.128	0.092	0.955	2.773	2.741	1.124

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	96	73	27
normalized size	1	1.	1.	0.95	1.23	4.36	3.32	1.23
time (sec)	N/A	0.026	0.024	0.026	0.942	2.47	2.035	1.108

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	135	166	301	999	0	327
normalized size	1	1.	0.93	1.14	2.08	6.89	0.	2.26
time (sec)	N/A	0.154	0.55	0.119	0.971	3.655	0.	1.164

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	283	258	591	1577	0	558
normalized size	1	1.	1.25	1.14	2.62	6.98	0.	2.47
time (sec)	N/A	0.277	4.105	0.141	1.029	5.523	0.	1.699

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	388	398	979	1995	0	776
normalized size	1	1.	1.18	1.21	2.98	6.08	0.	2.37
time (sec)	N/A	0.42	2.611	0.142	1.044	9.126	0.	1.206

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	3905	1060	0	1669	0	617
normalized size	1	1.	19.82	5.38	0.	8.47	0.	3.13
time (sec)	N/A	0.367	6.583	0.121	0.	3.823	0.	1.163

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	2657	560	0	1515	0	367
normalized size	1	1.	19.12	4.03	0.	10.9	0.	2.64
time (sec)	N/A	0.208	6.281	0.103	0.	3.506	0.	1.158

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	93	443	0	1107	0	279
normalized size	1	1.	0.81	3.85	0.	9.63	0.	2.43
time (sec)	N/A	0.128	0.306	0.095	0.	3.638	0.	1.144

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	193	705	0	1983	0	520
normalized size	1	1.	1.01	3.67	0.	10.33	0.	2.71
time (sec)	N/A	0.391	3.055	0.092	0.	4.312	0.	1.18

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	380	854	0	2643	0	840
normalized size	1	1.	1.44	3.23	0.	10.01	0.	3.18
time (sec)	N/A	0.64	2.811	0.151	0.	4.681	0.	1.228

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	171	208	377	882	2718	290
normalized size	1	1.	0.83	1.	1.82	4.26	13.13	1.4
time (sec)	N/A	0.171	1.022	0.154	1.013	4.181	69.054	1.444

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	107	127	278	705	2181	158
normalized size	1	1.	0.76	0.9	1.97	5.	15.47	1.12
time (sec)	N/A	0.109	0.285	0.159	1.001	4.105	65.657	1.442

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	54	67	204	572	2632	70
normalized size	1	1.	0.7	0.87	2.65	7.43	34.18	0.91
time (sec)	N/A	0.072	0.195	0.155	0.989	4.197	63.344	1.43

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	482	167	27
normalized size	1	1.	1.	0.95	1.23	21.91	7.59	1.23
time (sec)	N/A	0.027	0.078	0.047	0.964	4.182	55.018	1.397

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	365	699	1566	7699	0	1364
normalized size	1	1.	0.95	1.82	4.07	20.	0.	3.54
time (sec)	N/A	0.528	2.541	0.253	1.248	26.269	0.	1.546

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	527	527	770	804	2255	9441	0	1791
normalized size	1	1.	1.46	1.53	4.28	17.91	0.	3.4
time (sec)	N/A	0.736	6.723	0.276	1.433	44.71	0.	1.531

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	491	491	6586	9454	0	8992	0	3140
normalized size	1	1.	13.41	19.25	0.	18.31	0.	6.4
time (sec)	N/A	1.274	8.549	0.227	0.	10.695	0.	1.589

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	552	6933	0	5125	0	2228
normalized size	1	1.	1.36	17.03	0.	12.59	0.	5.47
time (sec)	N/A	0.792	6.043	0.211	0.	6.256	0.	1.558

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	411	411	1167	9171	0	6137	0	2608
normalized size	1	1.	2.84	22.31	0.	14.93	0.	6.35
time (sec)	N/A	0.791	6.076	0.208	0.	6.729	0.	1.545

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	422	422	1896	11250	0	7096	0	2979
normalized size	1	1.	4.49	26.66	0.	16.82	0.	7.06
time (sec)	N/A	0.746	6.191	0.206	0.	7.423	0.	2.097

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	529	494	7675	0	9913	0	3524
normalized size	1	1.	0.93	14.51	0.	18.74	0.	6.66
time (sec)	N/A	1.765	4.966	0.216	0.	10.099	0.	1.855

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	653	653	597	7823	0	11641	0	4113
normalized size	1	1.	0.91	11.98	0.	17.83	0.	6.3
time (sec)	N/A	2.137	5.452	0.354	0.	13.997	0.	2.78

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	117	126	157	348	0	234
normalized size	1	1.	0.76	0.82	1.02	2.26	0.	1.52
time (sec)	N/A	0.117	0.324	0.346	0.969	3.587	0.	1.452

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	55	82	192	0	105
normalized size	1	1.	0.7	0.66	0.99	2.31	0.	1.27
time (sec)	N/A	0.082	0.123	0.295	0.951	3.14	0.	1.657

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	27	51	83	27
normalized size	1	1.	1.	0.88	1.12	2.12	3.46	1.12
time (sec)	N/A	0.036	0.016	0.007	0.942	2.759	0.518	1.074

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	63	0	4132	0	0
normalized size	1	1.	1.	0.85	0.	55.84	0.	0.
time (sec)	N/A	0.116	0.053	0.291	0.	6.071	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	143	185	0	4968	0	0
normalized size	1	1.	1.15	1.49	0.	40.06	0.	0.
time (sec)	N/A	0.167	0.667	0.408	0.	8.35	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	224	509	0	0	0	0
normalized size	1	1.	1.08	2.46	0.	0.	0.	0.
time (sec)	N/A	0.323	1.483	0.717	0.	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	233	1189	0	0	0	0
normalized size	1	1.	0.78	3.99	0.	0.	0.	0.
time (sec)	N/A	0.565	0.839	0.542	0.	0.	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	185	792	0	0	0	0
normalized size	1	1.	0.86	3.68	0.	0.	0.	0.
time (sec)	N/A	0.261	0.798	0.389	0.	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	127	617	0	0	0	0
normalized size	1	1.	0.85	4.14	0.	0.	0.	0.
time (sec)	N/A	0.171	2.785	0.486	0.	0.	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	270	1259	0	0	0	0
normalized size	1	1.	1.09	5.08	0.	0.	0.	0.
time (sec)	N/A	0.371	3.295	0.571	0.	0.	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	131	126	157	462	0	0
normalized size	1	1.	0.85	0.82	1.02	3.	0.	0.
time (sec)	N/A	0.123	0.707	0.462	0.95	3.3	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	55	82	265	314	0
normalized size	1	1.	0.7	0.66	0.99	3.19	3.78	0.
time (sec)	N/A	0.091	0.182	0.22	0.961	2.493	134.232	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	27	123	116	0
normalized size	1	1.	1.	0.88	1.12	5.12	4.83	0.
time (sec)	N/A	0.039	0.021	0.006	0.949	2.358	27.408	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	89	218	0	0	0	0
normalized size	1	1.	0.95	2.32	0.	0.	0.	0.
time (sec)	N/A	0.167	0.096	0.378	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	121	279	0	4775	0	0
normalized size	1	1.	0.93	2.15	0.	36.73	0.	0.
time (sec)	N/A	0.274	0.682	0.385	0.	5.458	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	297	409	0	0	0	0
normalized size	1	1.	1.58	2.18	0.	0.	0.	0.
time (sec)	N/A	0.318	2.487	0.474	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	278	1355	0	0	0	0
normalized size	1	1.	0.84	4.12	0.	0.	0.	0.
time (sec)	N/A	0.692	1.058	0.607	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	222	943	0	0	0	0
normalized size	1	1.	0.9	3.82	0.	0.	0.	0.
time (sec)	N/A	0.462	1.01	0.432	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	163	633	0	0	0	0
normalized size	1	1.	0.97	3.77	0.	0.	0.	0.
time (sec)	N/A	0.203	0.635	0.614	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	211	937	0	0	0	0
normalized size	1	1.	0.97	4.3	0.	0.	0.	0.
time (sec)	N/A	0.438	2.338	0.568	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	364	1519	0	0	0	0
normalized size	1	1.	1.1	4.6	0.	0.	0.	0.
time (sec)	N/A	0.698	6.25	0.843	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	113	126	157	554	0	0
normalized size	1	1.	0.73	0.82	1.02	3.6	0.	0.
time (sec)	N/A	0.12	0.572	0.454	0.954	2.615	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	55	82	342	0	0
normalized size	1	1.	0.7	0.66	0.99	4.12	0.	0.
time (sec)	N/A	0.091	0.086	0.224	0.951	2.254	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	27	176	0	0
normalized size	1	1.	1.	0.88	1.12	7.33	0.	0.
time (sec)	N/A	0.037	0.028	0.007	0.933	2.119	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	105	312	0	4676	0	0
normalized size	1	1.	0.9	2.67	0.	39.97	0.	0.
time (sec)	N/A	0.232	0.156	0.368	0.	12.449	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	147	356	0	4980	0	0
normalized size	1	1.	0.95	2.3	0.	32.13	0.	0.
time (sec)	N/A	0.27	0.825	0.446	0.	5.172	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	307	538	0	5315	0	0
normalized size	1	1.	1.54	2.7	0.	26.71	0.	0.
time (sec)	N/A	0.282	3.262	0.809	0.	5.744	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	321	1619	0	0	0	0
normalized size	1	1.	0.81	4.07	0.	0.	0.	0.
time (sec)	N/A	0.937	1.214	0.595	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	239	1190	0	0	0	0
normalized size	1	1.	0.8	3.98	0.	0.	0.	0.
time (sec)	N/A	0.669	0.991	0.488	0.	0.	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	203	1039	0	0	0	0
normalized size	1	1.	1.	5.12	0.	0.	0.	0.
time (sec)	N/A	0.28	0.868	0.685	0.	0.	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	259	1249	0	0	0	0
normalized size	1	1.	1.09	5.25	0.	0.	0.	0.
time (sec)	N/A	0.394	3.428	0.605	0.	0.	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	351	1360	0	0	0	0
normalized size	1	1.	1.09	4.22	0.	0.	0.	0.
time (sec)	N/A	0.672	6.259	0.66	0.	0.	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	439	338	1888	0	0	0	0
normalized size	1	1.	0.77	4.3	0.	0.	0.	0.
time (sec)	N/A	0.944	4.459	6.417	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	118	126	216	270	0	217
normalized size	1	1.	0.78	0.83	1.42	1.78	0.	1.43
time (sec)	N/A	0.114	0.288	0.241	0.981	2.137	0.	1.211

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	58	55	101	135	0	97
normalized size	1	1.	0.72	0.68	1.25	1.67	0.	1.2
time (sec)	N/A	0.085	0.076	0.169	0.967	1.982	0.	1.1

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	46	54	27
normalized size	1	1.	1.	0.95	1.23	2.09	2.45	1.23
time (sec)	N/A	0.035	0.013	0.005	0.964	1.959	1.023	1.095

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	62	0	0	0	101
normalized size	1	1.	1.	0.84	0.	0.	0.	1.36
time (sec)	N/A	0.094	0.054	0.263	0.	0.	0.	1.08

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	176	218	0	0	0	286
normalized size	1	1.	1.22	1.51	0.	0.	0.	1.99
time (sec)	N/A	0.305	0.494	0.487	0.	0.	0.	1.106

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	244	618	0	0	0	566
normalized size	1	1.	1.06	2.69	0.	0.	0.	2.46
time (sec)	N/A	0.389	1.869	0.816	0.	0.	0.	1.133

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	219	942	0	0	0	0
normalized size	1	1.	0.89	3.81	0.	0.	0.	0.
time (sec)	N/A	0.365	1.037	0.547	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	145	462	0	0	0	0
normalized size	1	1.	0.83	2.64	0.	0.	0.	0.
time (sec)	N/A	0.188	0.75	0.419	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	177	640	0	0	0	0
normalized size	1	1.	0.97	3.5	0.	0.	0.	0.
time (sec)	N/A	0.203	0.638	0.585	0.	0.	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	306	1314	0	0	0	0
normalized size	1	1.	1.05	4.52	0.	0.	0.	0.
time (sec)	N/A	0.439	4.038	1.633	0.	0.	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	116	116	167	302	0	184
normalized size	1	1.	0.77	0.77	1.11	2.01	0.	1.23
time (sec)	N/A	0.121	0.253	0.224	0.975	2.352	0.	1.107

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	54	90	161	0	80
normalized size	1	1.	0.72	0.68	1.14	2.04	0.	1.01
time (sec)	N/A	0.094	0.067	0.24	0.971	2.027	0.	1.099

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	78	56	27
normalized size	1	1.	1.	0.95	1.23	3.55	2.55	1.23
time (sec)	N/A	0.038	0.014	0.004	0.946	1.933	3.595	1.082

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	91	99	0	0	0	161
normalized size	1	1.	0.87	0.94	0.	0.	0.	1.53
time (sec)	N/A	0.158	0.079	0.442	0.	0.	0.	1.088

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	221	250	0	0	0	401
normalized size	1	1.	1.19	1.34	0.	0.	0.	2.16
time (sec)	N/A	0.334	1.106	0.545	0.	0.	0.	1.116

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	324	649	0	0	0	733
normalized size	1	1.	1.14	2.29	0.	0.	0.	2.58
time (sec)	N/A	0.521	2.195	0.749	0.	0.	0.	1.193

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	273	1195	0	0	0	0
normalized size	1	1.	0.87	3.82	0.	0.	0.	0.
time (sec)	N/A	0.539	1.475	0.601	0.	0.	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	187	797	0	0	0	0
normalized size	1	1.	0.82	3.48	0.	0.	0.	0.
time (sec)	N/A	0.333	1.05	0.471	0.	0.	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	125	434	0	0	0	0
normalized size	1	1.	0.78	2.71	0.	0.	0.	0.
time (sec)	N/A	0.187	2.732	0.549	0.	0.	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	205	1062	0	0	0	0
normalized size	1	1.	0.82	4.23	0.	0.	0.	0.
time (sec)	N/A	0.365	1.657	0.788	0.	0.	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	348	1646	0	0	0	0
normalized size	1	1.	0.97	4.58	0.	0.	0.	0.
time (sec)	N/A	0.613	2.975	3.27	0.	0.	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	117	116	165	344	0	184
normalized size	1	1.	0.78	0.77	1.1	2.29	0.	1.23
time (sec)	N/A	0.122	0.295	0.332	0.953	2.955	0.	1.112

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	56	55	86	216	304	78
normalized size	1	1.	0.71	0.7	1.09	2.73	3.85	0.99
time (sec)	N/A	0.094	0.054	0.241	0.944	2.364	30.673	1.117

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	27	130	87	27
normalized size	1	1.	1.	0.88	1.12	5.42	3.62	1.12
time (sec)	N/A	0.042	0.019	0.004	0.943	2.26	29.481	1.095

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	94	130	0	7309	0	227
normalized size	1	1.	0.68	0.94	0.	52.58	0.	1.63
time (sec)	N/A	0.243	0.072	0.437	0.	8.074	0.	1.087

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	245	283	0	0	0	512
normalized size	1	1.	1.06	1.23	0.	0.	0.	2.22
time (sec)	N/A	0.416	0.856	0.784	0.	0.	0.	1.157

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	339	296	682	0	0	0	861
normalized size	1	1.	0.87	2.01	0.	0.	0.	2.54
time (sec)	N/A	0.645	3.361	0.795	0.	0.	0.	1.219

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	356	2253	0	0	0	0
normalized size	1	1.	0.93	5.87	0.	0.	0.	0.
time (sec)	N/A	0.761	1.762	0.596	0.	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	244	1642	0	0	0	0
normalized size	1	1.	0.83	5.6	0.	0.	0.	0.
time (sec)	N/A	0.513	1.187	0.592	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	174	1047	0	0	0	0
normalized size	1	1.	0.79	4.74	0.	0.	0.	0.
time (sec)	N/A	0.32	1.027	0.497	0.	0.	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	167	864	0	0	0	0
normalized size	1	1.	0.76	3.95	0.	0.	0.	0.
time (sec)	N/A	0.263	1.009	0.535	0.	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	241	1653	0	0	0	0
normalized size	1	1.	0.74	5.09	0.	0.	0.	0.
time (sec)	N/A	0.609	1.846	3.105	0.	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	341	2585	0	0	0	0
normalized size	1	1.	0.8	6.08	0.	0.	0.	0.
time (sec)	N/A	0.876	2.438	4.317	0.	0.	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	104	259	0	0	0	0
normalized size	1	1.	0.84	2.09	0.	0.	0.	0.
time (sec)	N/A	0.092	0.825	0.99	0.	0.	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	79	222	0	0	0	0
normalized size	1	1.	0.83	2.34	0.	0.	0.	0.
time (sec)	N/A	0.073	0.482	1.102	0.	0.	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	79	185	0	0	0	0
normalized size	1	1.	0.83	1.95	0.	0.	0.	0.
time (sec)	N/A	0.071	0.464	1.019	0.	0.	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	123	0	0	0	0
normalized size	1	1.	0.89	1.95	0.	0.	0.	0.
time (sec)	N/A	0.051	0.101	0.753	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	106	0	0	0	0
normalized size	1	1.	0.82	1.74	0.	0.	0.	0.
time (sec)	N/A	0.052	0.196	0.594	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	54	119	0	0	0	0
normalized size	1	1.	0.59	1.31	0.	0.	0.	0.
time (sec)	N/A	0.074	0.116	1.112	0.	0.	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	55	193	0	0	0	0
normalized size	1	1.	0.57	1.99	0.	0.	0.	0.
time (sec)	N/A	0.072	0.146	1.668	0.	0.	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	70	310	0	0	0	0
normalized size	1	1.	0.56	2.46	0.	0.	0.	0.
time (sec)	N/A	0.091	0.325	2.526	0.	0.	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	160	473	0	0	0	0
normalized size	1	1.	0.85	2.52	0.	0.	0.	0.
time (sec)	N/A	0.191	1.908	1.214	0.	0.	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	113	408	0	0	0	0
normalized size	1	1.	0.76	2.74	0.	0.	0.	0.
time (sec)	N/A	0.167	0.889	1.271	0.	0.	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	115	343	0	0	0	0
normalized size	1	1.	0.77	2.3	0.	0.	0.	0.
time (sec)	N/A	0.164	1.141	1.028	0.	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	80	251	0	0	0	0
normalized size	1	1.	0.73	2.3	0.	0.	0.	0.
time (sec)	N/A	0.13	0.312	1.132	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	75	210	0	0	0	0
normalized size	1	1.	0.69	1.93	0.	0.	0.	0.
time (sec)	N/A	0.129	0.392	0.777	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	71	197	0	0	0	0
normalized size	1	1.	0.63	1.74	0.	0.	0.	0.
time (sec)	N/A	0.135	0.233	1.283	0.	0.	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	72	333	0	0	0	0
normalized size	1	1.	0.61	2.8	0.	0.	0.	0.
time (sec)	N/A	0.139	0.256	1.688	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	105	564	0	0	0	0
normalized size	1	1.	0.66	3.52	0.	0.	0.	0.
time (sec)	N/A	0.172	0.536	3.381	0.	0.	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	205	618	0	0	0	0
normalized size	1	1.	0.86	2.61	0.	0.	0.	0.
time (sec)	N/A	0.308	2.016	2.566	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	150	534	0	0	0	0
normalized size	1	1.	0.76	2.71	0.	0.	0.	0.
time (sec)	N/A	0.286	1.385	2.238	0.	0.	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	153	450	0	0	0	0
normalized size	1	1.	0.78	2.28	0.	0.	0.	0.
time (sec)	N/A	0.289	1.351	1.996	0.	0.	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	101	339	0	0	0	0
normalized size	1	1.	0.65	2.17	0.	0.	0.	0.
time (sec)	N/A	0.241	0.606	1.639	0.	0.	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	94	279	0	0	0	0
normalized size	1	1.	0.62	1.84	0.	0.	0.	0.
time (sec)	N/A	0.24	0.771	1.382	0.	0.	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	98	248	0	0	0	0
normalized size	1	1.	0.61	1.55	0.	0.	0.	0.
time (sec)	N/A	0.241	0.396	1.937	0.	0.	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	103	384	0	0	0	0
normalized size	1	1.	0.63	2.34	0.	0.	0.	0.
time (sec)	N/A	0.249	0.684	1.881	0.	0.	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	126	618	0	0	0	0
normalized size	1	1.	0.67	3.3	0.	0.	0.	0.
time (sec)	N/A	0.264	0.705	3.974	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	140	750	0	0	0	0
normalized size	1	1.	0.74	3.99	0.	0.	0.	0.
time (sec)	N/A	0.267	0.638	4.631	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	251	863	0	0	0	0
normalized size	1	1.	0.82	2.83	0.	0.	0.	0.
time (sec)	N/A	0.551	4.813	2.684	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	209	776	0	0	0	0
normalized size	1	1.	0.81	3.01	0.	0.	0.	0.
time (sec)	N/A	0.508	2.103	2.417	0.	0.	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	189	639	0	0	0	0
normalized size	1	1.	0.73	2.48	0.	0.	0.	0.
time (sec)	N/A	0.509	2.72	2.106	0.	0.	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	137	525	0	0	0	0
normalized size	1	1.	0.65	2.5	0.	0.	0.	0.
time (sec)	N/A	0.442	1.09	1.846	0.	0.	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	130	412	0	0	0	0
normalized size	1	1.	0.62	1.96	0.	0.	0.	0.
time (sec)	N/A	0.445	1.091	1.437	0.	0.	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	135	378	0	0	0	0
normalized size	1	1.	0.62	1.73	0.	0.	0.	0.
time (sec)	N/A	0.438	0.575	2.007	0.	0.	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	137	575	0	0	0	0
normalized size	1	1.	0.63	2.66	0.	0.	0.	0.
time (sec)	N/A	0.445	1.159	2.206	0.	0.	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	152	874	0	0	0	0
normalized size	1	1.	0.64	3.69	0.	0.	0.	0.
time (sec)	N/A	0.465	0.568	4.376	0.	0.	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	177	1067	0	0	0	0
normalized size	1	1.	0.73	4.43	0.	0.	0.	0.
time (sec)	N/A	0.463	0.88	5.161	0.	0.	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	219	1416	0	0	0	0
normalized size	1	1.	0.83	5.36	0.	0.	0.	0.
time (sec)	N/A	0.479	1.597	7.477	0.	0.	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	531	531	2035	3711	0	0	0	0
normalized size	1	1.	3.83	6.99	0.	0.	0.	0.
time (sec)	N/A	1.908	27.999	3.802	0.	0.	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	446	446	834	2126	0	0	0	0
normalized size	1	1.	1.87	4.77	0.	0.	0.	0.
time (sec)	N/A	1.286	27.126	2.586	0.	0.	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	461	461	1955	2329	0	0	0	0
normalized size	1	1.	4.24	5.05	0.	0.	0.	0.
time (sec)	N/A	1.323	29.086	3.155	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	384	384	709	1131	0	0	0	0
normalized size	1	1.	1.85	2.95	0.	0.	0.	0.
time (sec)	N/A	0.87	21.434	2.72	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	397	397	233	1266	0	0	0	0
normalized size	1	1.	0.59	3.19	0.	0.	0.	0.
time (sec)	N/A	0.882	4.703	2.537	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	292	361	682	0	0	0	0
normalized size	1	1.	1.24	2.34	0.	0.	0.	0.
time (sec)	N/A	0.582	15.969	1.838	0.	0.	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	558	678	0	0	0	0
normalized size	1	1.	1.87	2.27	0.	0.	0.	0.
time (sec)	N/A	0.574	16.756	2.056	0.	0.	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	411	411	791	1103	0	0	0	0
normalized size	1	1.	1.92	2.68	0.	0.	0.	0.
time (sec)	N/A	0.928	21.773	3.057	0.	0.	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	434	434	1192	1083	0	0	0	0
normalized size	1	1.	2.75	2.5	0.	0.	0.	0.
time (sec)	N/A	0.999	24.786	3.79	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	486	486	881	2399	0	0	0	0
normalized size	1	1.	1.81	4.94	0.	0.	0.	0.
time (sec)	N/A	1.328	6.757	5.316	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	543	543	2030	19829	0	0	0	0
normalized size	1	1.	3.74	36.52	0.	0.	0.	0.
time (sec)	N/A	1.519	27.726	10.645	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	459	459	835	20346	0	0	0	0
normalized size	1	1.	1.82	44.33	0.	0.	0.	0.
time (sec)	N/A	1.12	26.893	8.373	0.	0.	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	473	473	1956	14392	0	0	0	0
normalized size	1	1.	4.14	30.43	0.	0.	0.	0.
time (sec)	N/A	1.118	27.333	8.384	0.	0.	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	390	390	371	13221	0	0	0	0
normalized size	1	1.	0.95	33.9	0.	0.	0.	0.
time (sec)	N/A	0.819	38.814	6.415	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	404	404	614	9301	0	0	0	0
normalized size	1	1.	1.52	23.02	0.	0.	0.	0.
time (sec)	N/A	0.892	12.493	6.523	0.	0.	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	422	422	787	7033	0	0	0	0
normalized size	1	1.	1.86	16.67	0.	0.	0.	0.
time (sec)	N/A	0.868	16.337	6.78	0.	0.	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	429	429	1181	4457	0	0	0	0
normalized size	1	1.	2.75	10.39	0.	0.	0.	0.
time (sec)	N/A	0.898	24.19	6.516	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	492	492	777	8216	0	0	0	0
normalized size	1	1.	1.58	16.7	0.	0.	0.	0.
time (sec)	N/A	1.219	6.639	10.079	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	514	514	1258	6022	0	0	0	0
normalized size	1	1.	2.45	11.72	0.	0.	0.	0.
time (sec)	N/A	1.309	25.339	13.628	0.	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	574	574	949	10743	0	0	0	0
normalized size	1	1.	1.65	18.72	0.	0.	0.	0.
time (sec)	N/A	1.623	6.875	21.724	0.	0.	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	575	575	932	111631	0	0	0	0
normalized size	1	1.	1.62	194.14	0.	0.	0.	0.
time (sec)	N/A	1.415	25.563	29.625	0.	0.	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	589	589	2024	85607	0	0	0	0
normalized size	1	1.	3.44	145.34	0.	0.	0.	0.
time (sec)	N/A	1.487	27.536	32.438	0.	0.	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	483	483	777	85489	0	0	0	0
normalized size	1	1.	1.61	177.	0.	0.	0.	0.
time (sec)	N/A	1.075	26.124	25.289	0.	0.	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	497	497	1954	65216	0	0	0	0
normalized size	1	1.	3.93	131.22	0.	0.	0.	0.
time (sec)	N/A	1.079	26.424	24.696	0.	0.	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	505	505	831	63272	0	0	0	0
normalized size	1	1.	1.65	125.29	0.	0.	0.	0.
time (sec)	N/A	1.105	24.158	26.309	0.	0.	0.	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	519	519	1211	45147	0	0	0	0
normalized size	1	1.	2.33	86.99	0.	0.	0.	0.
time (sec)	N/A	1.138	24.3	27.009	0.	0.	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	514	514	837	36688	0	0	0	0
normalized size	1	1.	1.63	71.38	0.	0.	0.	0.
time (sec)	N/A	1.187	26.462	27.897	0.	0.	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	520	520	1226	25322	0	0	0	0
normalized size	1	1.	2.36	48.7	0.	0.	0.	0.
time (sec)	N/A	1.226	24.949	24.928	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	596	596	922	46134	0	0	0	0
normalized size	1	1.	1.55	77.41	0.	0.	0.	0.
time (sec)	N/A	1.593	6.824	46.141	0.	0.	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	614	614	1308	32645	0	0	0	0
normalized size	1	1.	2.13	53.17	0.	0.	0.	0.
time (sec)	N/A	1.721	24.001	62.991	0.	0.	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	685	685	1014	49016	0	0	0	0
normalized size	1	1.	1.48	71.56	0.	0.	0.	0.
time (sec)	N/A	2.028	6.95	90.928	0.	0.	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	671	671	2102	300244	0	0	0	0
normalized size	1	1.	3.13	447.46	0.	0.	0.	0.
time (sec)	N/A	1.832	27.709	111.302	0.	0.	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	557	557	937	180834	0	0	0	0
normalized size	1	1.	1.68	324.66	0.	0.	0.	0.
time (sec)	N/A	1.364	27.061	78.119	0.	0.	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	571	571	2020	144252	0	0	0	0
normalized size	1	1.	3.54	252.63	0.	0.	0.	0.
time (sec)	N/A	1.378	27.157	91.461	0.	0.	0.	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	591	591	900	237416	0	0	0	0
normalized size	1	1.	1.52	401.72	0.	0.	0.	0.
time (sec)	N/A	1.43	26.948	84.159	0.	0.	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	597	597	1263	192036	0	0	0	0
normalized size	1	1.	2.12	321.67	0.	0.	0.	0.
time (sec)	N/A	1.52	24.97	96.503	0.	0.	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	574	574	892	179434	0	0	0	0
normalized size	1	1.	1.55	312.6	0.	0.	0.	0.
time (sec)	N/A	1.447	26.819	95.938	0.	0.	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	592	592	1263	138380	0	0	0	0
normalized size	1	1.	2.13	233.75	0.	0.	0.	0.
time (sec)	N/A	1.471	24.58	94.995	0.	0.	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	579	579	900	112960	0	0	0	0
normalized size	1	1.	1.55	195.09	0.	0.	0.	0.
time (sec)	N/A	1.532	6.731	88.013	0.	0.	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	593	593	1276	85165	0	0	0	0
normalized size	1	1.	2.15	143.62	0.	0.	0.	0.
time (sec)	N/A	1.57	25.24	86.842	0.	0.	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	674	674	996	150599	0	0	0	0
normalized size	1	1.	1.48	223.44	0.	0.	0.	0.
time (sec)	N/A	1.952	7.028	152.143	0.	0.	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	C	B	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	183	374	117	442	0	0	0	0
normalized size	1	2.04	0.64	2.42	0.	0.	0.	0.
time (sec)	N/A	0.428	0.313	0.514	0.	0.	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	290	0	0	0	0	0
normalized size	1	1.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.37	54.716	3.557	0.	0.	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	285	0	0	0	0	0
normalized size	1	1.	1.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	1.023	2.566	0.	0.	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	240	0	0	0	0	0
normalized size	1	1.	2.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.935	0.875	0.	0.	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	3819	0	0	0	0	0
normalized size	1	1.	24.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	20.06	0.628	0.	0.	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	4793	0	0	0	0	0
normalized size	1	1.	28.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	25.565	0.526	0.	0.	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	7904	0	0	0	0	0
normalized size	1	1.	46.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	28.951	0.744	0.	0.	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	170	170	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	66.861	4.677	0.	0.	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	187	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	7.819	0.176	0.	0.	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	187	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.821	0.132	0.	0.	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	187	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	1.083	0.128	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	185	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	1.095	0.131	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	185	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	2.92	0.129	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	187	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	3.071	0.124	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	2.363	0.863	0.	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	459	0	0	1801	0	4832
normalized size	1	1.	1.81	0.	0.	7.09	0.	19.02
time (sec)	N/A	0.164	6.117	0.3	0.	2.693	0.	1.176

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	169	0	0	825	0	1904
normalized size	1	1.	1.01	0.	0.	4.94	0.	11.4
time (sec)	N/A	0.112	0.914	0.238	0.	2.069	0.	1.14

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	74	0	0	311	0	504
normalized size	1	1.	0.8	0.	0.	3.38	0.	5.48
time (sec)	N/A	0.072	0.263	0.204	0.	1.443	0.	1.119

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	0	80	99	35
normalized size	1	1.	1.	1.04	0.	3.08	3.81	1.35
time (sec)	N/A	0.027	0.025	0.009	0.	1.408	2.647	1.105

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	99	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.114	0.632	0.	0.	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	157	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.226	0.567	0.319	0.	0.	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	260	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.421	4.068	0.563	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	4.117	0.174	0.	0.	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	5.861	0.142	0.	0.	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	2.083	0.089	0.	0.	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	3.881	0.229	0.	0.	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	56.661	0.121	0.	0.	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	5.844	0.105	0.	0.	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	1.986	0.109	0.	0.	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	1.757	0.102	0.	0.	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	1.974	0.101	0.	0.	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	2.134	0.101	0.	0.	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	598	598	826	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	1.018	6.094	0.202	0.	0.	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	311	420	319	0	0	0	0	0
normalized size	1	1.35	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.512	5.053	0.178	0.	0.	0.	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	168	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.293	0.941	0.181	0.	0.	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	132	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.372	0.175	0.	0.	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	1.582	0.389	0.	0.	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	5.254	0.184	0.	0.	0.	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	4.235	0.181	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [594] had the largest ratio of [0.52]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	19	0.105
2	A	5	3	1.	19	0.158
3	A	3	2	1.	19	0.105
4	A	4	3	1.	19	0.158
5	A	3	2	1.	19	0.105
6	A	3	3	1.	19	0.158
7	A	2	1	1.27	17	0.059
8	A	2	2	1.	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	3	3	1.	19	0.158
10	A	4	3	1.	19	0.158
11	A	3	2	1.	19	0.105
12	A	4	3	1.	19	0.158
13	A	6	4	1.	21	0.19
14	A	3	2	1.	21	0.095
15	A	5	4	1.	21	0.19
16	A	3	2	1.	21	0.095
17	A	4	4	1.	21	0.19
18	A	2	2	1.	19	0.105
19	A	3	2	1.	19	0.105
20	A	3	3	1.	21	0.143
21	A	2	2	1.	21	0.095
22	A	3	3	1.	21	0.143
23	A	4	3	1.	21	0.143
24	A	3	2	1.	21	0.095
25	A	4	3	1.	21	0.143
26	A	3	2	1.	21	0.095
27	A	7	4	1.	21	0.19
28	A	3	2	1.	21	0.095
29	A	6	4	1.	21	0.19
30	A	3	2	1.	21	0.095
31	A	5	4	1.	21	0.19
32	A	2	2	1.	19	0.105
33	A	3	2	1.	19	0.105
34	A	4	4	1.	21	0.19
35	A	3	2	1.	21	0.095
36	A	2	2	1.	21	0.095
37	A	2	2	1.	21	0.095
38	A	4	3	1.	21	0.143
39	A	4	3	1.	21	0.143
40	A	4	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
41	A	3	2	1.	21	0.095
42	A	11	4	1.	21	0.19
43	A	3	2	1.	21	0.095
44	A	10	4	1.	21	0.19
45	A	2	2	1.	19	0.105
46	A	3	2	1.	19	0.105
47	A	9	6	1.	21	0.286
48	A	3	2	1.	21	0.095
49	A	8	6	1.	21	0.286
50	A	3	2	1.	21	0.095
51	A	4	3	1.	21	0.143
52	A	3	2	1.	21	0.095
53	A	3	3	1.	21	0.143
54	A	2	1	1.	21	0.048
55	A	2	2	1.	21	0.095
56	A	2	2	1.	19	0.105
57	A	4	3	1.	19	0.158
58	A	3	3	1.	21	0.143
59	A	4	3	1.	21	0.143
60	A	3	2	1.	21	0.095
61	A	4	3	1.	21	0.143
62	A	5	4	1.	21	0.19
63	A	3	2	1.	21	0.095
64	A	4	4	1.	21	0.19
65	A	2	2	1.	21	0.095
66	A	3	3	1.	21	0.143
67	A	3	2	1.	21	0.095
68	A	2	2	1.	21	0.095
69	A	2	2	1.	19	0.105
70	A	4	3	1.	19	0.158
71	A	4	3	1.	21	0.143
72	A	4	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
73	A	4	2	1.	21	0.095
74	A	4	3	1.	21	0.143
75	A	5	4	1.	21	0.19
76	A	2	2	1.	21	0.095
77	A	4	4	1.	21	0.19
78	A	3	2	1.	21	0.095
79	A	3	3	1.	21	0.143
80	A	3	2	1.	21	0.095
81	A	1	1	1.	21	0.048
82	A	2	2	1.	19	0.105
83	A	4	3	1.	19	0.158
84	A	5	3	1.	21	0.143
85	A	4	3	1.	21	0.143
86	A	5	2	1.	21	0.095
87	A	4	3	1.	21	0.143
88	A	5	2	1.	21	0.095
89	A	2	2	1.	21	0.095
90	A	2	2	1.	21	0.095
91	A	3	2	1.	21	0.095
92	A	4	2	1.	21	0.095
93	A	3	2	1.	21	0.095
94	A	6	2	1.	21	0.095
95	A	2	2	1.	19	0.105
96	A	4	3	1.	19	0.158
97	A	10	3	1.	21	0.143
98	A	4	3	1.	21	0.143
99	A	10	2	1.	21	0.095
100	A	4	3	1.	21	0.143
101	A	3	2	1.	23	0.087
102	A	4	2	1.	23	0.087
103	A	3	2	1.	23	0.087
104	A	3	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
105	A	3	2	1.	23	0.087
106	A	2	2	1.	23	0.087
107	A	2	2	1.	21	0.095
108	A	3	3	1.	21	0.143
109	A	3	3	1.	23	0.13
110	A	5	5	1.	23	0.217
111	A	5	5	1.	23	0.217
112	A	7	6	1.	23	0.261
113	A	7	6	1.	23	0.261
114	A	3	2	1.	23	0.087
115	A	5	2	1.	23	0.087
116	A	3	2	1.	23	0.087
117	A	4	2	1.	23	0.087
118	A	3	2	1.	23	0.087
119	A	3	2	1.	23	0.087
120	A	2	2	1.	21	0.095
121	A	4	4	1.	21	0.19
122	A	1	1	1.	23	0.043
123	A	4	4	1.	23	0.174
124	A	4	3	1.	23	0.13
125	A	6	5	1.	23	0.217
126	A	6	5	1.	23	0.217
127	A	3	2	1.	23	0.087
128	A	5	2	1.	23	0.087
129	A	3	2	1.	23	0.087
130	A	4	2	1.	23	0.087
131	A	2	2	1.	21	0.095
132	A	5	4	1.	21	0.19
133	A	2	2	1.	23	0.087
134	A	4	4	1.	23	0.174
135	A	1	1	1.	23	0.043
136	A	5	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
137	A	5	3	1.	23	0.13
138	A	7	5	1.	23	0.217
139	A	3	2	1.	23	0.087
140	A	7	2	1.	23	0.087
141	A	3	2	1.	23	0.087
142	A	6	2	1.	23	0.087
143	A	3	2	1.	23	0.087
144	A	5	2	1.	23	0.087
145	A	2	2	1.	21	0.095
146	A	6	4	1.	21	0.19
147	A	3	2	1.	23	0.087
148	A	5	5	1.	23	0.217
149	A	2	2	1.	23	0.087
150	A	5	5	1.	23	0.217
151	A	1	1	1.	23	0.043
152	A	6	4	1.	23	0.174
153	A	6	3	1.	23	0.13
154	A	8	5	1.	23	0.217
155	A	8	5	1.	23	0.217
156	A	3	2	1.	23	0.087
157	A	3	2	1.	23	0.087
158	A	3	2	1.	23	0.087
159	A	2	2	1.	23	0.087
160	A	3	2	1.	23	0.087
161	A	1	1	1.	23	0.043
162	A	2	2	1.	21	0.095
163	A	4	4	1.	21	0.19
164	A	4	4	1.	23	0.174
165	A	6	5	1.	23	0.217
166	A	6	5	1.	23	0.217
167	A	8	6	1.	23	0.261
168	A	8	5	1.	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
169	A	3	2	1.	23	0.087
170	A	2	2	1.	23	0.087
171	A	3	2	1.	23	0.087
172	A	1	1	1.	23	0.043
173	A	3	2	1.	23	0.087
174	A	3	3	1.	23	0.13
175	A	2	2	1.	21	0.095
176	A	5	4	1.	21	0.19
177	A	5	5	1.	23	0.217
178	A	7	6	1.	23	0.261
179	A	7	5	1.	23	0.217
180	A	9	6	1.	23	0.261
181	A	9	5	1.	23	0.217
182	A	3	2	1.	23	0.087
183	A	3	2	1.	23	0.087
184	A	2	2	1.	23	0.087
185	A	3	2	1.	23	0.087
186	A	1	1	1.	23	0.043
187	A	3	2	1.	23	0.087
188	A	4	3	1.	23	0.13
189	A	3	2	1.	23	0.087
190	A	3	3	1.	23	0.13
191	A	2	2	1.	21	0.095
192	A	6	4	1.	21	0.19
193	A	6	5	1.	23	0.217
194	A	8	6	1.	23	0.261
195	A	8	5	1.	23	0.217
196	A	5	4	1.	23	0.174
197	A	4	4	1.	23	0.174
198	A	4	4	1.	23	0.174
199	A	3	3	1.	23	0.13
200	A	3	3	1.	23	0.13

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	4	1.	23	0.174
202	A	4	4	1.	23	0.174
203	A	5	4	1.	23	0.174
204	A	6	5	1.	25	0.2
205	A	5	5	1.	25	0.2
206	A	5	5	1.	25	0.2
207	A	4	4	1.	25	0.16
208	A	4	4	1.	25	0.16
209	A	4	4	1.	25	0.16
210	A	4	4	1.	25	0.16
211	A	5	5	1.	25	0.2
212	A	4	4	1.	25	0.16
213	A	5	4	1.	25	0.16
214	A	7	5	1.	25	0.2
215	A	6	5	1.	25	0.2
216	A	6	5	1.	25	0.2
217	A	5	4	1.	25	0.16
218	A	5	4	1.	25	0.16
219	A	5	5	1.	25	0.2
220	A	5	5	1.	25	0.2
221	A	5	5	1.	25	0.2
222	A	5	5	1.	25	0.2
223	A	6	5	1.	25	0.2
224	A	7	5	1.	25	0.2
225	A	6	4	1.	25	0.16
226	A	6	4	1.	25	0.16
227	A	6	5	1.	25	0.2
228	A	6	5	1.	25	0.2
229	A	5	4	1.	25	0.16
230	A	5	4	1.	25	0.16
231	A	6	6	1.	25	0.24
232	A	6	6	1.	25	0.24

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
233	A	5	4	1.	25	0.16
234	A	4	4	1.	25	0.16
235	A	4	4	1.	25	0.16
236	A	3	3	1.	25	0.12
237	A	3	3	1.	25	0.12
238	A	3	3	1.	25	0.12
239	A	3	3	1.	25	0.12
240	A	4	4	1.	25	0.16
241	A	4	4	1.	25	0.16
242	A	5	4	1.	25	0.16
243	A	5	4	1.	25	0.16
244	A	4	4	1.	25	0.16
245	A	4	4	1.	25	0.16
246	A	3	3	1.	25	0.12
247	A	3	3	1.	25	0.12
248	A	4	4	1.	25	0.16
249	A	4	4	1.	25	0.16
250	A	5	5	1.	25	0.2
251	A	5	5	1.	25	0.2
252	A	6	5	1.	25	0.2
253	A	6	5	1.	25	0.2
254	A	5	5	1.	25	0.2
255	A	5	5	1.	25	0.2
256	A	4	4	1.	25	0.16
257	A	4	4	1.	25	0.16
258	A	4	4	1.	25	0.16
259	A	4	4	1.	25	0.16
260	A	5	4	1.	25	0.16
261	A	5	4	1.	25	0.16
262	A	6	5	1.	25	0.2
263	A	6	4	1.	25	0.16
264	A	5	4	1.	25	0.16

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	5	4	1.	25	0.16
266	A	4	3	1.	25	0.12
267	A	4	3	1.	25	0.12
268	A	5	5	1.	25	0.2
269	A	5	5	1.	25	0.2
270	A	6	4	1.	25	0.16
271	A	6	4	1.	25	0.16
272	A	7	5	1.	25	0.2
273	A	8	8	1.	27	0.296
274	A	7	7	1.	27	0.259
275	A	6	6	1.	27	0.222
276	A	1	1	1.	27	0.037
277	A	2	2	1.	27	0.074
278	A	3	2	1.	27	0.074
279	A	4	2	1.	27	0.074
280	A	10	9	1.	27	0.333
281	A	9	8	1.	27	0.296
282	A	8	7	1.	27	0.259
283	A	7	7	1.	27	0.259
284	A	7	7	1.	27	0.259
285	A	1	1	1.	27	0.037
286	A	2	2	1.	27	0.074
287	A	3	2	1.	27	0.074
288	A	4	2	1.	27	0.074
289	A	10	8	1.	27	0.296
290	A	9	7	1.	27	0.259
291	A	8	7	1.	27	0.259
292	A	8	8	1.	27	0.296
293	A	7	7	1.	27	0.259
294	A	1	1	1.	27	0.037
295	A	2	2	1.	27	0.074
296	A	3	2	1.	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
297	A	4	2	1.	27	0.074
298	A	8	8	1.	27	0.296
299	A	7	7	1.	27	0.259
300	A	6	6	1.	27	0.222
301	A	1	1	1.	27	0.037
302	A	2	2	1.	27	0.074
303	A	3	2	1.	27	0.074
304	A	4	2	1.	27	0.074
305	A	8	8	1.	27	0.296
306	A	7	7	1.	27	0.259
307	A	8	8	1.	27	0.296
308	A	1	1	1.	27	0.037
309	A	2	2	1.	27	0.074
310	A	3	2	1.	27	0.074
311	A	4	2	1.	27	0.074
312	A	5	2	1.	27	0.074
313	A	9	9	1.	27	0.333
314	A	8	8	1.	27	0.296
315	A	7	7	1.	27	0.259
316	A	1	1	1.	27	0.037
317	A	2	2	1.	27	0.074
318	A	3	2	1.	27	0.074
319	A	4	2	1.	27	0.074
320	A	5	2	1.	27	0.074
321	A	3	3	1.	27	0.111
322	A	3	3	1.	27	0.111
323	A	3	3	1.	27	0.111
324	A	3	3	1.	27	0.111
325	A	3	3	1.	27	0.111
326	A	3	3	1.	27	0.111
327	A	2	2	1.	23	0.087
328	A	2	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	2	2	1.	23	0.087
330	A	2	2	1.	21	0.095
331	A	2	2	1.	23	0.087
332	A	2	2	1.	23	0.087
333	A	2	2	1.	23	0.087
334	A	2	2	1.	23	0.087
335	A	3	3	1.	25	0.12
336	A	3	3	1.	25	0.12
337	A	3	3	1.	25	0.12
338	A	3	3	1.	25	0.12
339	A	3	3	1.	25	0.12
340	A	3	3	1.	25	0.12
341	A	3	3	1.	25	0.12
342	A	3	3	1.	23	0.13
343	A	3	2	1.	21	0.095
344	A	3	2	1.	21	0.095
345	A	3	2	1.	21	0.095
346	A	2	2	1.	19	0.105
347	A	2	2	1.	19	0.105
348	A	2	2	1.	21	0.095
349	A	2	2	1.	21	0.095
350	A	3	3	1.	21	0.143
351	A	3	3	1.	21	0.143
352	A	3	3	1.	21	0.143
353	A	3	3	1.	21	0.143
354	A	3	3	1.	25	0.12
355	A	3	3	1.	25	0.12
356	A	3	3	1.	25	0.12
357	A	3	3	1.	25	0.12
358	A	3	3	1.	25	0.12
359	A	3	3	1.	25	0.12
360	A	4	2	1.	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
361	A	3	2	1.	27	0.074
362	A	2	2	1.	27	0.074
363	A	1	1	1.	27	0.037
364	A	3	3	1.	25	0.12
365	A	3	3	1.	27	0.111
366	A	3	3	1.	27	0.111
367	A	3	2	1.	27	0.074
368	A	2	2	1.	27	0.074
369	A	1	1	1.	27	0.037
370	A	3	3	1.	27	0.111
371	A	3	3	1.	27	0.111
372	A	4	4	1.	27	0.148
373	A	4	4	1.	27	0.148
374	A	4	4	1.	25	0.16
375	A	4	4	1.	27	0.148
376	A	4	3	1.	19	0.158
377	A	3	2	1.	19	0.105
378	A	2	1	1.27	17	0.059
379	A	4	3	1.	17	0.176
380	A	3	3	1.	19	0.158
381	A	4	4	1.	19	0.21
382	A	4	3	1.	19	0.158
383	A	3	3	1.	19	0.158
384	A	3	3	1.	19	0.158
385	A	3	2	1.	19	0.105
386	A	3	2	1.	19	0.105
387	A	4	3	1.	21	0.143
388	A	3	2	1.	21	0.095
389	A	2	2	1.	19	0.105
390	A	6	4	1.	19	0.21
391	A	3	3	1.	21	0.143
392	A	4	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
393	A	6	4	1.	21	0.19
394	A	5	4	1.	21	0.19
395	A	4	4	1.	21	0.19
396	A	3	2	1.	21	0.095
397	A	4	4	1.	21	0.19
398	A	4	3	1.	21	0.143
399	A	4	3	1.	21	0.143
400	A	3	2	1.	21	0.095
401	A	3	2	1.	21	0.095
402	A	2	2	1.	19	0.105
403	A	6	4	1.	19	0.21
404	A	6	5	1.	21	0.238
405	A	4	4	1.	21	0.19
406	A	6	5	1.	21	0.238
407	A	5	5	1.	21	0.238
408	A	2	2	1.	21	0.095
409	A	5	5	1.	21	0.238
410	A	5	5	1.	21	0.238
411	A	5	4	1.	21	0.19
412	A	5	4	1.	21	0.19
413	A	3	2	1.	21	0.095
414	A	3	2	1.	21	0.095
415	A	2	2	1.	19	0.105
416	A	6	4	1.	19	0.21
417	A	7	5	1.	21	0.238
418	A	8	6	1.	21	0.286
419	A	10	5	1.	21	0.238
420	A	7	3	1.	21	0.143
421	A	7	4	1.	21	0.19
422	A	7	4	1.	21	0.19
423	A	7	4	1.	21	0.19
424	A	10	6	1.	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
425	A	3	2	1.	21	0.095
426	A	3	2	1.	21	0.095
427	A	2	2	1.	19	0.105
428	A	6	4	1.	19	0.21
429	A	4	3	1.	21	0.143
430	A	5	4	1.	21	0.19
431	A	7	6	1.	21	0.286
432	A	6	6	1.	21	0.286
433	A	5	5	1.	21	0.238
434	A	5	5	1.	21	0.238
435	A	6	6	1.	21	0.286
436	A	7	6	1.	21	0.286
437	A	3	2	1.	21	0.095
438	A	3	2	1.	21	0.095
439	A	3	2	1.	21	0.095
440	A	2	2	1.	19	0.105
441	A	4	3	1.	19	0.158
442	A	4	3	1.	21	0.143
443	A	5	4	1.	21	0.19
444	A	7	6	1.	21	0.286
445	A	6	6	1.	21	0.286
446	A	5	5	1.	21	0.238
447	A	6	6	1.	21	0.286
448	A	7	6	1.	21	0.286
449	A	3	2	1.	21	0.095
450	A	3	2	1.	21	0.095
451	A	3	2	1.	21	0.095
452	A	2	2	1.	19	0.105
453	A	4	3	1.	19	0.158
454	A	4	3	1.	21	0.143
455	A	5	4	1.	21	0.19
456	A	7	7	1.	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
457	A	6	6	1.	21	0.286
458	A	6	6	1.	21	0.286
459	A	7	7	1.	21	0.333
460	A	8	7	1.	21	0.333
461	A	3	2	1.	21	0.095
462	A	3	2	1.	21	0.095
463	A	3	2	1.	21	0.095
464	A	2	2	1.	19	0.105
465	A	4	3	1.	19	0.158
466	A	4	3	1.	21	0.143
467	A	11	7	1.	21	0.333
468	A	11	7	1.	21	0.333
469	A	11	7	1.	21	0.333
470	A	11	6	1.	21	0.286
471	A	12	7	1.	21	0.333
472	A	13	7	1.	21	0.333
473	A	3	2	1.	23	0.087
474	A	3	2	1.	23	0.087
475	A	2	2	1.	21	0.095
476	A	5	4	1.	21	0.19
477	A	6	5	1.	23	0.217
478	A	7	6	1.	23	0.261
479	A	8	8	1.	23	0.348
480	A	7	7	1.	23	0.304
481	A	7	7	1.	23	0.304
482	A	7	7	1.	23	0.304
483	A	3	2	1.	23	0.087
484	A	3	2	1.	23	0.087
485	A	2	2	1.	21	0.095
486	A	6	5	1.	21	0.238
487	A	6	5	1.	23	0.217
488	A	7	6	1.	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
489	A	8	7	1.	23	0.304
490	A	7	7	1.	23	0.304
491	A	6	6	1.	23	0.261
492	A	7	7	1.	23	0.304
493	A	8	7	1.	23	0.304
494	A	3	2	1.	23	0.087
495	A	3	2	1.	23	0.087
496	A	2	2	1.	21	0.095
497	A	7	6	1.	21	0.286
498	A	7	6	1.	23	0.261
499	A	7	6	1.	23	0.261
500	A	9	8	1.	23	0.348
501	A	8	8	1.	23	0.348
502	A	7	7	1.	23	0.304
503	A	7	7	1.	23	0.304
504	A	8	8	1.	23	0.348
505	A	9	8	1.	23	0.348
506	A	3	2	1.	23	0.087
507	A	3	2	1.	23	0.087
508	A	2	2	1.	21	0.095
509	A	5	4	1.	21	0.19
510	A	6	5	1.	23	0.217
511	A	7	6	1.	23	0.261
512	A	7	7	1.	23	0.304
513	A	6	6	1.	23	0.261
514	A	6	6	1.	23	0.261
515	A	7	7	1.	23	0.304
516	A	3	2	1.	23	0.087
517	A	3	2	1.	23	0.087
518	A	2	2	1.	21	0.095
519	A	6	5	1.	21	0.238
520	A	7	6	1.	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
521	A	8	7	1.	23	0.304
522	A	8	7	1.	23	0.304
523	A	7	7	1.	23	0.304
524	A	6	6	1.	23	0.261
525	A	7	7	1.	23	0.304
526	A	8	7	1.	23	0.304
527	A	3	2	1.	23	0.087
528	A	3	2	1.	23	0.087
529	A	2	2	1.	21	0.095
530	A	7	6	1.	21	0.286
531	A	8	6	1.	23	0.261
532	A	9	7	1.	23	0.304
533	A	9	8	1.	23	0.348
534	A	8	8	1.	23	0.348
535	A	7	7	1.	23	0.304
536	A	7	7	1.	23	0.304
537	A	8	8	1.	23	0.348
538	A	9	8	1.	23	0.348
539	A	5	4	1.	23	0.174
540	A	4	4	1.	23	0.174
541	A	4	4	1.	23	0.174
542	A	3	3	1.	23	0.13
543	A	3	3	1.	23	0.13
544	A	4	4	1.	23	0.174
545	A	4	4	1.	23	0.174
546	A	5	4	1.	23	0.174
547	A	6	5	1.	25	0.2
548	A	5	5	1.	25	0.2
549	A	5	5	1.	25	0.2
550	A	4	4	1.	25	0.16
551	A	4	4	1.	25	0.16
552	A	4	4	1.	25	0.16

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
553	A	4	4	1.	25	0.16
554	A	5	5	1.	25	0.2
555	A	7	6	1.	25	0.24
556	A	6	6	1.	25	0.24
557	A	6	6	1.	25	0.24
558	A	5	5	1.	25	0.2
559	A	5	5	1.	25	0.2
560	A	5	5	1.	25	0.2
561	A	5	5	1.	25	0.2
562	A	5	5	1.	25	0.2
563	A	5	5	1.	25	0.2
564	A	8	6	1.	25	0.24
565	A	7	6	1.	25	0.24
566	A	7	6	1.	25	0.24
567	A	6	5	1.	25	0.2
568	A	6	5	1.	25	0.2
569	A	6	5	1.	25	0.2
570	A	6	5	1.	25	0.2
571	A	6	6	1.	25	0.24
572	A	6	6	1.	25	0.24
573	A	6	5	1.	25	0.2
574	A	15	12	1.	25	0.48
575	A	14	12	1.	25	0.48
576	A	14	12	1.	25	0.48
577	A	13	11	1.	25	0.44
578	A	13	11	1.	25	0.44
579	A	9	7	1.	25	0.28
580	A	9	7	1.	25	0.28
581	A	13	11	1.	25	0.44
582	A	13	11	1.	25	0.44
583	A	14	12	1.	25	0.48
584	A	15	12	1.	25	0.48

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
585	A	14	12	1.	25	0.48
586	A	14	12	1.	25	0.48
587	A	13	11	1.	25	0.44
588	A	13	11	1.	25	0.44
589	A	13	11	1.	25	0.44
590	A	13	11	1.	25	0.44
591	A	14	12	1.	25	0.48
592	A	14	12	1.	25	0.48
593	A	15	12	1.	25	0.48
594	A	15	13	1.	25	0.52
595	A	15	13	1.	25	0.52
596	A	14	12	1.	25	0.48
597	A	14	12	1.	25	0.48
598	A	14	12	1.	25	0.48
599	A	14	12	1.	25	0.48
600	A	14	12	1.	25	0.48
601	A	14	12	1.	25	0.48
602	A	15	13	1.	25	0.52
603	A	15	13	1.	25	0.52
604	A	16	13	1.	25	0.52
605	A	16	13	1.	25	0.52
606	A	15	12	1.	25	0.48
607	A	15	12	1.	25	0.48
608	A	15	13	1.	25	0.52
609	A	15	13	1.	25	0.52
610	A	15	12	1.	25	0.48
611	A	15	12	1.	25	0.48
612	A	15	12	1.	25	0.48
613	A	15	12	1.	25	0.48
614	A	16	13	1.	25	0.52
615	B	2	2	2.04	27	0.074
616	A	4	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
617	A	3	3	1.	23	0.13
618	A	2	2	1.	21	0.095
619	A	1	1	1.	23	0.043
620	A	1	1	1.	23	0.043
621	A	1	1	1.	23	0.043
622	A	1	1	1.	23	0.043
623	A	2	2	1.	25	0.08
624	A	2	2	1.	25	0.08
625	A	2	2	1.	25	0.08
626	A	2	2	1.	25	0.08
627	A	2	2	1.	25	0.08
628	A	2	2	1.	25	0.08
629	A	2	2	1.	23	0.087
630	A	3	2	1.	21	0.095
631	A	3	2	1.	21	0.095
632	A	3	2	1.	21	0.095
633	A	2	2	1.	19	0.105
634	A	5	3	1.	19	0.158
635	A	6	4	1.	21	0.19
636	A	7	5	1.	21	0.238
637	A	2	2	1.	21	0.095
638	A	2	2	1.	21	0.095
639	A	2	2	1.	21	0.095
640	A	2	2	1.	21	0.095
641	A	2	2	1.	25	0.08
642	A	2	2	1.	25	0.08
643	A	2	2	1.	25	0.08
644	A	2	2	1.	25	0.08
645	A	2	2	1.	25	0.08
646	A	2	2	1.	25	0.08
647	A	9	7	1.	27	0.259
648	A	5	5	1.35	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
649	A	3	3	1.	27	0.111
650	A	1	1	1.	27	0.037
651	A	2	2	1.	25	0.08
652	A	2	2	1.	27	0.074
653	A	2	2	1.	27	0.074

Chapter 3

Listing of integrals

3.1 $\int \cos^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$-\frac{(a\sin(c+dx)+a)^8}{8a^7d}+\frac{6(a\sin(c+dx)+a)^7}{7a^6d}-\frac{2(a\sin(c+dx)+a)^6}{a^5d}+\frac{8(a\sin(c+dx)+a)^5}{5a^4d}$$

```
[Out] (8*(a + a*Sin[c + d*x])^5)/(5*a^4*d) - (2*(a + a*Sin[c + d*x])^6)/(a^5*d) +
 (6*(a + a*Sin[c + d*x])^7)/(7*a^6*d) - (a + a*Sin[c + d*x])^8/(8*a^7*d)
```

Rubi [A] time = 0.0555053, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 43}

$$-\frac{(a\sin(c+dx)+a)^8}{8a^7d}+\frac{6(a\sin(c+dx)+a)^7}{7a^6d}-\frac{2(a\sin(c+dx)+a)^6}{a^5d}+\frac{8(a\sin(c+dx)+a)^5}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^5)/(5*a^4*d) - (2*(a + a*Sin[c + d*x])^6)/(a^5*d) +
 (6*(a + a*Sin[c + d*x])^7)/(7*a^6*d) - (a + a*Sin[c + d*x])^8/(8*a^7*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a-x)^3(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^7d}\\ &= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^4-12a^2(a+x)^5+6a(a+x)^6-(a+x)^7\right)dx, x, a\sin(c+dx\right.}{a^7d}\\ &= \frac{8(a+a\sin(c+dx))^5}{5a^4d}-\frac{2(a+a\sin(c+dx))^6}{a^5d}+\frac{6(a+a\sin(c+dx))^7}{7a^6d}-\frac{(a+a\sin}{8a}\end{aligned}$$

Mathematica [A] time = 0.0464023, size = 74, normalized size = 0.85

$$-\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d + (3*
a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Maple [A] time = 0.029, size = 56, normalized size = 0.6

$$\frac{1}{d}\left(-\frac{a(\cos(dx+c))^8}{8}+\frac{a\sin(dx+c)}{7}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6\,(\cos(dx+c))^4}{5}+\frac{8\,(\cos(dx+c))^2}{5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/8*a*cos(d*x+c)^8+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(
d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.94879, size = 124, normalized size = 1.43

$$-\frac{35\,a\sin\left(dx+c\right)^{8}+40\,a\sin\left(dx+c\right)^{7}-140\,a\sin\left(dx+c\right)^{6}-168\,a\sin\left(dx+c\right)^{5}+210\,a\sin\left(dx+c\right)^{4}+280\,a\sin\left(dx+\right.}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/280*(35*a*sin(d*x + c)^8 + 40*a*sin(d*x + c)^7 - 140*a*sin(d*x + c)^6 -
168*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 + 280*a*sin(d*x + c)^3 - 140*a*
sin(d*x + c)^2 - 280*a*sin(d*x + c))/d
```

Fricas [A] time = 1.71391, size = 161, normalized size = 1.85

$$-\frac{35\,a\cos\left(dx+c\right)^{8}-8\left(5\,a\cos\left(dx+c\right)^{6}+6\,a\cos\left(dx+c\right)^{4}+8\,a\cos\left(dx+c\right)^{2}+16\,a\right)\sin\left(dx+c\right)}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/280*(35*a*cos(d*x + c)^8 - 8*(5*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 +
8*a*cos(d*x + c)^2 + 16*a)*sin(d*x + c))/d
```

Sympy [A] time = 12.6511, size = 105, normalized size = 1.21

$$\begin{cases} \frac{16a\sin^{7}\left(c+dx\right)}{35d}+\frac{8a\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{5d}+\frac{2a\sin^{3}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d}+\frac{a\sin\left(c+dx\right)\cos^{6}\left(c+dx\right)}{d}-\frac{a\cos^{8}\left(c+dx\right)}{8d} & \text{for}\: d \neq 0 \\ x\left(a\sin\left(c\right)+a\right)\cos^{7}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**7/(35*d) + 8*a*sin(c + d*x)**5*cos(c + d*x)**
2/(5*d) + 2*a*sin(c + d*x)**3*cos(c + d*x)**4/d + a*sin(c + d*x)*cos(c + d*
x)**6/d - a*cos(c + d*x)**8/(8*d), Ne(d, 0)), (x*(a*sin(c) + a)*cos(c)**7,
True))
```

Giac [A] time = 1.18415, size = 159, normalized size = 1.83

$$-\frac{a\cos(8\,dx+8\,c)}{1024\,d}-\frac{a\cos(6\,dx+6\,c)}{128\,d}-\frac{7\,a\cos(4\,dx+4\,c)}{256\,d}-\frac{7\,a\cos(2\,dx+2\,c)}{128\,d}+\frac{a\sin(7\,dx+7\,c)}{448\,d}+\frac{7\,a\sin(5\,dx+5}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1024*a*cos(8*d*x + 8*c)/d - 1/128*a*cos(6*d*x + 6*c)/d - 7/256*a*cos(4*d
*x + 4*c)/d - 7/128*a*cos(2*d*x + 2*c)/d + 1/448*a*sin(7*d*x + 7*c)/d + 7/3
20*a*sin(5*d*x + 5*c)/d + 7/64*a*sin(3*d*x + 3*c)/d + 35/64*a*sin(d*x + c)/
d
```

3.2 $\int \cos^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$-\frac{a\cos^7(c+dx)}{7d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}$$

```
[Out] (5*a*x)/16 - (a*Cos[c + d*x]^7)/(7*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16
*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c +
d*x])/(6*d)
```

Rubi [A] time = 0.0610808, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2669, 2635, 8}

$$-\frac{a\cos^7(c+dx)}{7d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/16 - (a*Cos[c + d*x]^7)/(7*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16
*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c +
d*x])/(6*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+a\sin(c+dx))\,dx &= -\frac{a\cos^7(c+dx)}{7d}+a\int \cos^6(c+dx)\,dx\\ &= -\frac{a\cos^7(c+dx)}{7d}+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}(5a)\int \cos^4(c+dx)\,dx\\ &= -\frac{a\cos^7(c+dx)}{7d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{8}(5\\ &= -\frac{a\cos^7(c+dx)}{7d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a\,c}{}\\ &= \frac{5ax}{16}-\frac{a\cos^7(c+dx)}{7d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}\end{aligned}$$

Mathematica [A] time = 0.160345, size = 57, normalized size = 0.66

$$\frac{a\left(7(45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx))+60c+60dx)-192\cos^7(c+dx)\right)}{1344d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-192*Cos[c + d*x]^7 + 7*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*
(c + d*x)] + Sin[6*(c + d*x)])))/(1344*d)
```

Maple [A] time = 0.026, size = 62, normalized size = 0.7

$$\frac{1}{d}\left(-\frac{a(\cos(dx+c))^7}{7}+a\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/7*a*cos(d*x+c)^7+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x
+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 0.973284, size = 85, normalized size = 0.98

$$-\frac{192\,a\cos\left(dx+c\right)^{7}+7\,\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a}{1344\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/1344*(192*a*cos(d*x + c)^7 + 7*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

Fricas [A] time = 1.73268, size = 167, normalized size = 1.92

$$-\frac{48\,a\cos\left(dx+c\right)^{7}-105\,adx-7\,\left(8\,a\cos\left(dx+c\right)^{5}+10\,a\cos\left(dx+c\right)^{3}+15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{336\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/336*(48*a*cos(d*x + c)^7 - 105*a*d*x - 7*(8*a*cos(d*x + c)^5 + 10*a*cos(
d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 8.81304, size = 172, normalized size = 1.98

$$\begin{cases} \frac{5ax\sin^{6}(c+dx)}{16}+\frac{15ax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{15ax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{5ax\cos^{6}(c+dx)}{16}+\frac{5a\sin^{5}(c+dx)\cos(c+dx)}{16d}+\frac{5a\sin^{3}(c+dx)\cos^{3}(\end{cases}$$
$$x\left(a\sin(c)+a\right)\cos^{6}(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/16
 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a*sin(c + d*x)**3*cos(c + d*
x)**3/(6*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - a*cos(c + d*x)**7/
```

```
(7*d), Ne(d, 0)), (x*(a*sin(c) + a)*cos(c)**6, True))
```

Giac [A] time = 1.18363, size = 144, normalized size = 1.66

$$\frac{5}{16}\,ax - \frac{a\cos\left(7\,dx + 7\,c\right)}{448\,d} - \frac{a\cos\left(5\,dx + 5\,c\right)}{64\,d} - \frac{3\,a\cos\left(3\,dx + 3\,c\right)}{64\,d} - \frac{5\,a\cos\left(dx + c\right)}{64\,d} + \frac{a\sin\left(6\,dx + 6\,c\right)}{192\,d} + \frac{3\,a\sin\left(4\,dx\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/16*a*x - 1/448*a*cos(7*d*x + 7*c)/d - 1/64*a*cos(5*d*x + 5*c)/d - 3/64*a*
cos(3*d*x + 3*c)/d - 5/64*a*cos(d*x + c)/d + 1/192*a*sin(6*d*x + 6*c)/d + 3
/64*a*sin(4*d*x + 4*c)/d + 15/64*a*sin(2*d*x + 2*c)/d
```

3.3 $\int \cos^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=64

$$\frac{(a\sin(c+dx)+a)^6}{6a^5d}-\frac{4(a\sin(c+dx)+a)^5}{5a^4d}+\frac{(a\sin(c+dx)+a)^4}{a^3d}$$

```
[Out] (a + a*Sin[c + d*x])^4/(a^3*d) - (4*(a + a*Sin[c + d*x])^5)/(5*a^4*d) + (a
+ a*Sin[c + d*x])^6/(6*a^5*d)
```

Rubi [A] time = 0.0444546, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 43}

$$\frac{(a\sin(c+dx)+a)^6}{6a^5d}-\frac{4(a\sin(c+dx)+a)^5}{5a^4d}+\frac{(a\sin(c+dx)+a)^4}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a + a*Sin[c + d*x])^4/(a^3*d) - (4*(a + a*Sin[c + d*x])^5)/(5*a^4*d) + (a
+ a*Sin[c + d*x])^6/(6*a^5*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^5 d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^3-4a(a+x)^4+(a+x)^5\right)dx, x, a\sin(c+dx)\right)}{a^5 d}\\ &= \frac{(a+a\sin(c+dx))^4}{a^3 d}-\frac{4(a+a\sin(c+dx))^5}{5a^4 d}+\frac{(a+a\sin(c+dx))^6}{6a^5 d}\end{aligned}$$

Mathematica [A] time = 0.0226898, size = 60, normalized size = 0.94

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^6)/(6*d) + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.025, size = 46, normalized size = 0.7

$$\frac{1}{d}\left(-\frac{a(\cos(dx+c))^6}{6}+\frac{a\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/6*a*cos(d*x+c)^6+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+
c))
```

Maxima [A] time = 0.952699, size = 95, normalized size = 1.48

$$\frac{5\,a\sin(dx+c)^6+6\,a\sin(dx+c)^5-15\,a\sin(dx+c)^4-20\,a\sin(dx+c)^3+15\,a\sin(dx+c)^2+30\,a\sin(dx+c)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(5*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 - 15*a*sin(d*x + c)^4 - 20*a*
sin(d*x + c)^3 + 15*a*sin(d*x + c)^2 + 30*a*sin(d*x + c))/d
```

Fricas [A] time = 1.62355, size = 128, normalized size = 2.

$$-\frac{5\,a\cos\left(dx+c\right)^{6}-2\left(3\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(5*a*cos(d*x + c)^6 - 2*(3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*
a)*sin(d*x + c))/d
```

Sympy [A] time = 4.31667, size = 83, normalized size = 1.3

$$\begin{cases} \frac{8a\sin^{5}\left(c+dx\right)}{15d}+\frac{4a\sin^{3}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{3d}+\frac{a\sin\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d}-\frac{a\cos^{6}\left(c+dx\right)}{6d} & \text{for}\: d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)\cos^{5}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + a*sin(c + d*x)*cos(c + d*x)**4/d - a*cos(c + d*x)**6/(6*d), Ne(d,
0)), (x*(a*sin(c) + a)*cos(c)**5, True))
```

Giac [A] time = 1.18751, size = 119, normalized size = 1.86

$$-\frac{a\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{a\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{5\,a\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*a*cos(6*d*x + 6*c)/d - 1/32*a*cos(4*d*x + 4*c)/d - 5/64*a*cos(2*d*x
+ 2*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 5/8*a*si
n(d*x + c)/d
```

3.4 $\int \cos^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

```
[Out] (3*a*x)/8 - (a*Cos[c + d*x]^5)/(5*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.0453441, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2669, 2635, 8}

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (a*Cos[c + d*x]^5)/(5*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(c+dx)(a+a\sin(c+dx))\,dx &= -\frac{a\cos^5(c+dx)}{5d}+a\int \cos^4(c+dx)\,dx\\
&= -\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3a)\int \cos^2(c+dx)\,dx\\
&= -\frac{a\cos^5(c+dx)}{5d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{8}(3\\
&= \frac{3ax}{8}-\frac{a\cos^5(c+dx)}{5d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}
\end{aligned}
$$

Mathematica [A] time = 0.0822159, size = 62, normalized size = 0.95

$$\frac{3a(c+dx)}{8d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}-\frac{a\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) - (a*Cos[c + d*x]^5)/(5*d) + (a*Sin[2*(c + d*x)])/(4*
d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.024, size = 52, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{a\left(\cos\left(dx+c\right)\right)^5}{5}+a\left(\frac{\sin\left(dx+c\right)}{4}\left(\left(\cos\left(dx+c\right)\right)^3+\frac{3\cos\left(dx+c\right)}{2}\right)+\frac{3\,dx}{8}+\frac{3\,c}{8}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/5*a*cos(d*x+c)^5+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/
8*d*x+3/8*c))
```

Maxima [A] time = 0.950086, size = 65, normalized size = 1.

$$-\frac{32\,a\cos\left(dx+c\right)^5-5\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/160*(32*a*cos(d*x + c)^5 - 5*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2
*d*x + 2*c))*a)/d
```

Fricas [A] time = 1.64397, size = 132, normalized size = 2.03

$$-\frac{8\,a\cos\left(dx+c\right)^{5}-15\,adx-5\,\left(2\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/40*(8*a*cos(d*x + c)^5 - 15*a*d*x - 5*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x
+ c))*sin(d*x + c))/d
```

Sympy [A] time = 2.34263, size = 124, normalized size = 1.91

$$\begin{cases} \frac{3ax\sin^{4}(c+dx)}{8}+\frac{3ax\sin^{2}(c+dx)\cos^{2}(c+dx)}{4}+\frac{3ax\cos^{4}(c+dx)}{8}+\frac{3a\sin^{3}(c+dx)\cos(c+dx)}{8d}+\frac{5a\sin(c+dx)\cos^{3}(c+dx)}{8d}-\frac{a\cos^{5}(c+dx)}{5d} & \text{for } d \ne \\ x\left(a\sin(c)+a\right)\cos^{4}(c) & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) - a*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x
*(a*sin(c) + a)*cos(c)**4, True))
```

Giac [A] time = 1.19006, size = 104, normalized size = 1.6

$$\frac{3}{8}\,ax-\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}+\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x - 1/80*a*cos(5*d*x + 5*c)/d - 1/16*a*cos(3*d*x + 3*c)/d - 1/8*a*cos
(d*x + c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

3.5 $\int \cos^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=45

$$\frac{2(a\sin(c+dx)+a)^3}{3a^2d}-\frac{(a\sin(c+dx)+a)^4}{4a^3d}$$

```
[Out] (2*(a + a*Sin[c + d*x])^3)/(3*a^2*d) - (a + a*Sin[c + d*x])^4/(4*a^3*d)
```

Rubi [A] time = 0.0365742, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^3}{3a^2d}-\frac{(a\sin(c+dx)+a)^4}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^3)/(3*a^2*d) - (a + a*Sin[c + d*x])^4/(4*a^3*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(a+x)^2-(a+x)^3\right)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{2(a+a\sin(c+dx))^3}{3a^2d} - \frac{(a+a\sin(c+dx))^4}{4a^3d}$$

Mathematica [A] time = 0.0173511, size = 44, normalized size = 0.98

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^4)/(4*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.025, size = 36, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{a(\cos(dx+c))^4}{4}+\frac{a\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/4*a*cos(d*x+c)^4+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.942905, size = 65, normalized size = 1.44

$$-\frac{3\,a\sin(dx+c)^4+4\,a\sin(dx+c)^3-6\,a\sin(dx+c)^2-12\,a\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*a*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

Fricas [A] time = 1.64002, size = 97, normalized size = 2.16

$$-\frac{3\,a\cos\left(dx+c\right)^4 - 4\left(a\cos\left(dx+c\right)^2 + 2\,a\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a*cos(d*x + c)^4 - 4*(a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

Sympy [A] time = 1.20515, size = 82, normalized size = 1.82

$$\begin{cases} \frac{a\sin^4\left(c+dx\right)}{4d} + \frac{2a\sin^3\left(c+dx\right)}{3d} + \frac{a\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{2d} + \frac{a\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d} & \text{for}\ d \neq 0 \\ x\left(a\sin\left(c\right) + a\right)\cos^3\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)**4/(4*d) + 2*a*sin(c + d*x)**3/(3*d) + a*sin(c +
d*x)**2*cos(c + d*x)**2/(2*d) + a*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0))
, (x*(a*sin(c) + a)*cos(c)**3, True))
```

Giac [A] time = 1.11346, size = 65, normalized size = 1.44

$$-\frac{3\,a\sin\left(dx+c\right)^4 + 4\,a\sin\left(dx+c\right)^3 - 6\,a\sin\left(dx+c\right)^2 - 12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*a*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

3.6 $\int \cos^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$-\frac{a\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[Out] (a*x)/2 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0330301, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2669, 2635, 8}

$$-\frac{a\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/2 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sin(c+dx))\,dx &= -\frac{a\cos^3(c+dx)}{3d}+a\int \cos^2(c+dx)\,dx\\ &= -\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx\\ &= \frac{ax}{2}-\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0484589, size = 46, normalized size = 1.07

$$\frac{a(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}-\frac{a\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) - (a*Cos[c + d*x]^3)/(3*d) + (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.025, size = 41, normalized size = 1.

$$\frac{1}{d}\left(-\frac{a\left(\cos\left(dx+c\right)\right)^{3}}{3}+a\left(\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/3*a*cos(d*x+c)^3+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.959197, size = 50, normalized size = 1.16

$$-\frac{4\,a\cos\left(dx+c\right)^{3}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*a*cos(d*x + c)^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a)/d
```

Fricas [A] time = 1.65802, size = 96, normalized size = 2.23

$$-\frac{2\,a\cos\left(dx+c\right)^{3}-3\,adx-3\,a\cos\left(dx+c\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*cos(d*x + c)^3 - 3*a*d*x - 3*a*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [A] time = 0.685955, size = 71, normalized size = 1.65

$$\begin{cases} \frac{ax\sin^{2}\left(c+dx\right)}{2}+\frac{ax\cos^{2}\left(c+dx\right)}{2}+\frac{a\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}-\frac{a\cos^{3}\left(c+dx\right)}{3d} & \text{for}\ d\neq 0\\ x\left(a\sin\left(c\right)+a\right)\cos^{2}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*c
os(c + d*x)/(2*d) - a*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)*c
os(c)**2, True))
```

Giac [A] time = 1.1155, size = 63, normalized size = 1.47

$$\frac{1}{2}\,ax-\frac{a\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{a\cos\left(dx+c\right)}{4\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x - 1/12*a*cos(3*d*x + 3*c)/d - 1/4*a*cos(d*x + c)/d + 1/4*a*sin(2*d*
x + 2*c)/d
```

3.7 $\int \cos(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=22

$$\frac{(a\sin(c+dx)+a)^2}{2ad}$$

```
[Out] (a + a*Sin[c + d*x])^2/(2*a*d)
```

Rubi [A] time = 0.0163306, antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2667}

$$\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{a\sin(c+dx)}{d}+\frac{a\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0133389, size = 39, normalized size = 1.77

$$-\frac{a\cos^2(c+dx)}{2d}+\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^2)/(2*d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.009, size = 25, normalized size = 1.1

$$\frac{1}{d}\left(\frac{(\sin(dx+c))^2 a}{2}+a\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/2*sin(d*x+c)^2*a+a*sin(d*x+c))
```

Maxima [A] time = 0.94099, size = 27, normalized size = 1.23

$$\frac{(a\sin(dx+c)+a)^2}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(a*sin(d*x + c) + a)^2/(a*d)
```

Fricas [A] time = 1.61103, size = 62, normalized size = 2.82

$$-\frac{a\cos(dx+c)^2-2\,a\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)^2 - 2*a*sin(d*x + c))/d
```

Sympy [A] time = 0.220222, size = 34, normalized size = 1.55

$$\begin{cases} \frac{a\sin^2(c+dx)}{2d} + \frac{a\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)**2/(2*d) + a*sin(c + d*x)/d, Ne(d, 0)), (x*(a*sin
(c) + a)*cos(c), True))
```

Giac [A] time = 1.11971, size = 34, normalized size = 1.55

$$\frac{a\sin(dx+c)^2 + 2a\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(a*sin(d*x + c)^2 + 2*a*sin(d*x + c))/d
```

3.8 $\int \sec(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=17

$$-\frac{a\log(1-\sin(c+dx))}{d}$$

```
[Out] -((a*Log[1 - Sin[c + d*x]])/d)
```

Rubi [A] time = 0.0199603, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2667, 31}

$$-\frac{a\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Log[1 - Sin[c + d*x]])/d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))\,dx &= \frac{a\,\text{Subst}\left(\int \frac{1}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{a\log(1-\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0140097, size = 26, normalized size = 1.53

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d
```

Maple [A] time = 0.029, size = 16, normalized size = 0.9

$$-\frac{a\ln\left(\sin\left(dx+c\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a*ln(sin(d*x+c)-1)
```

Maxima [A] time = 0.945267, size = 20, normalized size = 1.18

$$-\frac{a\log\left(\sin\left(dx+c\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -a*log(sin(d*x + c) - 1)/d
```

Fricas [A] time = 1.7122, size = 39, normalized size = 2.29

$$-\frac{a\log\left(-\sin\left(dx+c\right)+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -a*log(-sin(d*x + c) + 1)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int \sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x), x) + Integral(sec(c + d*x), x))
```

Giac [B] time = 1.20579, size = 50, normalized size = 2.94

$$\frac{a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)-2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*a*log(abs(tan(1/2*d*x + 1/2*c) - 1))
)/d
```

3.9 $\int \sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=23

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}$$

```
[Out] (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.0321409, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2669, 3767, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{a\sec(c+dx)}{d} + a\int \sec^2(c+dx)\,dx$$
$$= \frac{a\sec(c+dx)}{d} - \frac{a\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}$$
$$= \frac{a\sec(c+dx)}{d} + \frac{a\tan(c+dx)}{d}$$

Mathematica [A] time = 0.0131422, size = 23, normalized size = 1.

$$\frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.069, size = 24, normalized size = 1.

$$\frac{1}{d}\left(\frac{a}{\cos(dx+c)} + a\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(a/cos(d*x+c)+a*tan(d*x+c))
```

Maxima [A] time = 0.946107, size = 31, normalized size = 1.35

$$\frac{a\tan(dx+c) + \frac{a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*tan(d*x + c) + a/cos(d*x + c))/d
```

Fricas [A] time = 1.60502, size = 104, normalized size = 4.52

$$\frac{a\cos(dx+c)+a\sin(dx+c)+a}{d\cos(dx+c)-d\sin(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*cos(d*x + c) + a*sin(d*x + c) + a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sin(c+dx)\sec^2(c+dx)\,dx+\int\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x)
)
```

Giac [A] time = 1.14352, size = 26, normalized size = 1.13

$$-\frac{2a}{d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*a/(d*(tan(1/2*d*x + 1/2*c) - 1))
```

3.10 $\int \sec^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=39

$$\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + a^2/(2*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0417776, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2667, 44, 206}

$$\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + a^2/(2*d*(a - a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sec^3(c+dx)(a+a\sin(c+dx))\,dx &= \frac{a^3\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^2}+\frac{1}{2a\left(a^2-x^2\right)}\right)dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c+dx)\right)}{2d}\\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2}{2d(a-a\sin(c+dx))}
\end{aligned}
$$

Mathematica [A] time = 0.0175316, size = 52, normalized size = 1.33

$$\frac{a\sec^2(c+dx)}{2d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]^2)/(2*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.082, size = 54, normalized size = 1.4

$$\frac{a}{2\,d\,(\cos(dx+c))^2}+\frac{a\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/d*a/cos(d*x+c)^2+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+ta
n(d*x+c))
```

Maxima [A] time = 0.945554, size = 57, normalized size = 1.46

$$\frac{a\log\left(\sin\left(dx+c\right)+1\right)-a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,a}{\sin(dx+c)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*a/(sin(d*x + c)
- 1))/d
```

Fricas [A] time = 1.77265, size = 166, normalized size = 4.26

$$\frac{\left(a\sin\left(dx+c\right)-a\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a\sin\left(dx+c\right)-a\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,a}{4\,\left(d\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((a*sin(d*x + c) - a)*log(sin(d*x + c) + 1) - (a*sin(d*x + c) - a)*log(
-sin(d*x + c) + 1) - 2*a)/(d*sin(d*x + c) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sin\left(c+dx\right)\sec^{3}\left(c+dx\right)\,dx+\int\sec^{3}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x)
)
```

Giac [A] time = 1.20889, size = 73, normalized size = 1.87

$$\frac{a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{a\sin(dx+c)-3\,a}{\sin(dx+c)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(a*log(abs(sin(d*x + c) + 1)) - a*log(abs(sin(d*x + c) - 1)) + (a*sin(d
*x + c) - 3*a)/(sin(d*x + c) - 1))/d
```

3.11 $\int \sec^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}$$

```
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.036244, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2669, 3767}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sec^4(c+dx)(a+a\sin(c+dx))\,dx &= \frac{a\sec^3(c+dx)}{3d} + a\int \sec^4(c+dx)\,dx \\
&= \frac{a\sec^3(c+dx)}{3d} - \frac{a\,\mathrm{Subst}\left(\int \left(1+x^2\right)dx, x, -\tan(c+dx)\right)}{d} \\
&= \frac{a\sec^3(c+dx)}{3d} + \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A] time = 0.0600332, size = 41, normalized size = 0.93

$$
\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d} + \frac{a\sec^3(c+dx)}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.084, size = 38, normalized size = 0.9

$$
\frac{1}{d}\left(\frac{a}{3\,(\cos(dx+c))^3} - a\left(-\frac{2}{3} - \frac{(\sec(dx+c))^2}{3}\right)\tan(dx+c)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a/cos(d*x+c)^3-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 0.952829, size = 47, normalized size = 1.07

$$
\frac{\left(\tan(dx+c)^3 + 3\,\tan(dx+c)\right)a + \frac{a}{\cos(dx+c)^3}}{3\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a + a/cos(d*x + c)^3)/d
```

Fricas [A] time = 1.62443, size = 131, normalized size = 2.98

$$-\frac{2\,a\cos\left(dx+c\right)^{2}+2\,a\sin\left(dx+c\right)-a}{3\,\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*cos(d*x + c)^2 + 2*a*sin(d*x + c) - a)/(d*cos(d*x + c)*sin(d*x +
c) - d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17314, size = 89, normalized size = 2.02

$$-\frac{\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*a/(tan(1/2*d*x + 1/2*c) + 1) + (9*a*tan(1/2*d*x + 1/2*c)^2 - 12*a*t
an(1/2*d*x + 1/2*c) + 7*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

3.12 $\int \sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=84

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/
(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.063694, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2667, 44, 206}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/
(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+a\sin(c+dx))\,dx &= \frac{a^5\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^5\,\mathrm{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^3}+\frac{1}{4a^3(a-x)^2}+\frac{1}{8a^3(a+x)^2}+\frac{3}{8a^3\left(a^2-x^2\right)}\right) dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a+a\sin(c+dx))}+\frac{\left(3a^2\right)\mathrm{Su}}{}\\ &= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{}{8d(a-}\end{aligned}$$

Mathematica [A] time = 0.0807652, size = 68, normalized size = 0.81

$$\frac{a\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^4)/(4*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(Ar
cTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A] time = 0.085, size = 74, normalized size = 0.9

$$\frac{a}{4\,d\,(\cos(dx+c))^4}+\frac{a\tan(dx+c)\,(\sec(dx+c))^3}{4\,d}+\frac{3\,a\sec(dx+c)\tan(dx+c)}{8\,d}+\frac{3\,a\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a/cos(d*x+c)^4+1/4/d*a*tan(d*x+c)*sec(d*x+c)^3+3/8*a*sec(d*x+c)*tan(d
*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.957914, size = 116, normalized size = 1.38

$$\frac{3\,a\log\left(\sin\left(dx+c\right)+1\right)-3\,a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,a\sin\left(dx+c\right)^{2}-3\,a\sin\left(dx+c\right)-2\,a\right)}{\sin\left(dx+c\right)^{3}-\sin\left(dx+c\right)^{2}-\sin\left(dx+c\right)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(3*a*log(sin(d*x + c) + 1) - 3*a*log(sin(d*x + c) - 1) - 2*(3*a*sin(d*
x + c)^2 - 3*a*sin(d*x + c) - 2*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d
*x + c) + 1))/d
```

Fricas [A] time = 1.95307, size = 350, normalized size = 4.17

$$-\frac{6\,a\cos\left(dx+c\right)^{2}-3\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right.\right.}{16\left(d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(6*a*cos(d*x + c)^2 - 3*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x +
c)^2)*log(sin(d*x + c) + 1) + 3*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x
+ c)^2)*log(-sin(d*x + c) + 1) + 6*a*sin(d*x + c) - 2*a)/(d*cos(d*x + c)^2*
sin(d*x + c) - d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24214, size = 124, normalized size = 1.48

$$\frac{6\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-6\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,(3\,a\sin(dx+c)+5\,a)}{\sin(dx+c)+1}+\frac{9\,a\sin(dx+c)^{2}-26\,a\sin(dx+c)+21\,a}{(\sin(dx+c)-1)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(6*a*log(abs(sin(d*x + c) + 1)) - 6*a*log(abs(sin(d*x + c) - 1)) - 2*(
3*a*sin(d*x + c) + 5*a)/(sin(d*x + c) + 1) + (9*a*sin(d*x + c)^2 - 26*a*sin
(d*x + c) + 21*a)/(sin(d*x + c) - 1)^2)/d
```

3.13 $\int \cos^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$-\frac{9a^2\cos^7(c+dx)}{56d}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{8d}+\frac{3a^2\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{15a^2\sin(c+dx)\cos^3(c+dx)}{64d}$$

```
[Out] (45*a^2*x)/128 - (9*a^2*Cos[c + d*x]^7)/(56*d) + (45*a^2*Cos[c + d*x]*Sin[c
 + d*x])/(128*d) + (15*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (3*a^2*Cos
[c + d*x]^5*Sin[c + d*x])/(16*d) - (Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x])
)/(8*d)
```

Rubi [A] time = 0.106751, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2678, 2669, 2635, 8}

$$-\frac{9a^2\cos^7(c+dx)}{56d}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{8d}+\frac{3a^2\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{15a^2\sin(c+dx)\cos^3(c+dx)}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (45*a^2*x)/128 - (9*a^2*Cos[c + d*x]^7)/(56*d) + (45*a^2*Cos[c + d*x]*Sin[c
 + d*x])/(128*d) + (15*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (3*a^2*Cos
[c + d*x]^5*Sin[c + d*x])/(16*d) - (Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x])
)/(8*d)
```


Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+a\sin(c+dx))^2\,dx &= -\frac{\cos^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{8d}+\frac{1}{8}(9a)\int \cos^6(c+dx)(a+a\sin(c+dx))\,d\\ &= -\frac{9a^2\cos^7(c+dx)}{56d}-\frac{\cos^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{8d}+\frac{1}{8}\left(9a^2\right)\int \cos^6(c+dx\\ &= -\frac{9a^2\cos^7(c+dx)}{56d}+\frac{3a^2\cos^5(c+dx)\sin(c+dx)}{16d}-\frac{\cos^7(c+dx)\left(a^2+a^2\sin(c\right.}{8d}\\ &= -\frac{9a^2\cos^7(c+dx)}{56d}+\frac{15a^2\cos^3(c+dx)\sin(c+dx)}{64d}+\frac{3a^2\cos^5(c+dx)\sin(c+dx}{16d}\\ &= -\frac{9a^2\cos^7(c+dx)}{56d}+\frac{45a^2\cos(c+dx)\sin(c+dx)}{128d}+\frac{15a^2\cos^3(c+dx)\sin(c+d}{64d}\\ &= \frac{45a^2x}{128}-\frac{9a^2\cos^7(c+dx)}{56d}+\frac{45a^2\cos(c+dx)\sin(c+dx)}{128d}+\frac{15a^2\cos^3(c+dx)\sin}{64d}\end{aligned}$$

Mathematica [A] time = 1.55813, size = 171, normalized size = 1.36

$$-\frac{a^2\left(630\sqrt{1-\sin(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\left(112\sin^8(c+dx)+144\sin^7(c+dx)-424\sin^6(c+dx)\right.}{896d(\sin(c+dx)-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Cos[c + d*x]^7*(630*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - S
in[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(256 - 837*Sin[c + d*x] - 187*Sin[c +
```

```
 d*x]^2 + 978*Sin[c + d*x]^3 + 558*Sin[c + d*x]^4 - 600*Sin[c + d*x]^5 - 42
4*Sin[c + d*x]^6 + 144*Sin[c + d*x]^7 + 112*Sin[c + d*x]^8)))/(896*d*(-1 +
Sin[c + d*x])^4*(1 + Sin[c + d*x])^(7/2))
```

Maple [A] time = 0.037, size = 129, normalized size = 1.

$$\frac{1}{d}\left(a^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^7}{8}+\frac{\sin(dx+c)}{48}\left((\cos(dx+c))^5+\frac{5\,(\cos(dx+c))^3}{4}+\frac{15\,\cos(dx+c)}{8}\right)+\frac{5\,dx}{128}+\frac{5\,c}{128}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-2/7*a^2*cos(d*x+c)^7+a^2*(1/
6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*
c))
```

Maxima [A] time = 0.964566, size = 155, normalized size = 1.23

$$-\frac{6144\,a^2\cos(dx+c)^7-7\left(64\,\sin(2\,dx+2\,c)^3+120\,dx+120\,c-3\,\sin(8\,dx+8\,c)-24\,\sin(4\,dx+4\,c)\right)a^2+112\left(4\,\sin\right.}{21504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/21504*(6144*a^2*cos(d*x + c)^7 - 7*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 12
0*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a^2 + 112*(4*sin(2*d*x + 2*
c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2)/d
```

Fricas [A] time = 1.80596, size = 216, normalized size = 1.71

$$-\frac{256\,a^2\cos(dx+c)^7-315\,a^2dx+7\left(16\,a^2\cos(dx+c)^7-24\,a^2\cos(dx+c)^5-30\,a^2\cos(dx+c)^3-45\,a^2\cos(dx+c)\right)}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/896*(256*a^2*cos(d*x + c)^7 - 315*a^2*d*x + 7*(16*a^2*cos(d*x + c)^7 - 2
4*a^2*cos(d*x + c)^5 - 30*a^2*cos(d*x + c)^3 - 45*a^2*cos(d*x + c))*sin(d*x
 + c))/d
```

Sympy [A] time = 14.6679, size = 398, normalized size = 3.16

$$\begin{cases} \frac{5a^2x\sin^8(c+dx)}{128} + \frac{5a^2x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{5a^2x\sin^6(c+dx)}{16} + \frac{15a^2x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{15a^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{5a^2x\sin^2(c+dx}{32} \\ x(a\sin(c)+a)^2\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**8/128 + 5*a**2*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + 5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c
+ d*x)**4/64 + 15*a**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 5*a**2*x*sin(
c + d*x)**2*cos(c + d*x)**6/32 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/
16 + 5*a**2*x*cos(c + d*x)**8/128 + 5*a**2*x*cos(c + d*x)**6/16 + 5*a**2*si
n(c + d*x)**7*cos(c + d*x)/(128*d) + 55*a**2*sin(c + d*x)**5*cos(c + d*x)**
3/(384*d) + 5*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 73*a**2*sin(c + d*
x)**3*cos(c + d*x)**5/(384*d) + 5*a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - 5*a**2*sin(c + d*x)*cos(c + d*x)**7/(128*d) + 11*a**2*sin(c + d*x)*cos(
c + d*x)**5/(16*d) - 2*a**2*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a*sin(c)
+ a)**2*cos(c)**6, True))
```

Giac [A] time = 1.15243, size = 166, normalized size = 1.32

$$\frac{45}{128}a^2x - \frac{a^2\cos(7dx+7c)}{224d} - \frac{a^2\cos(5dx+5c)}{32d} - \frac{3a^2\cos(3dx+3c)}{32d} - \frac{5a^2\cos(dx+c)}{32d} - \frac{a^2\sin(8dx+8c)}{1024d} + \frac{5a^2\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 45/128*a^2*x - 1/224*a^2*cos(7*d*x + 7*c)/d - 1/32*a^2*cos(5*d*x + 5*c)/d -
 3/32*a^2*cos(3*d*x + 3*c)/d - 5/32*a^2*cos(d*x + c)/d - 1/1024*a^2*sin(8*d
*x + 8*c)/d + 5/128*a^2*sin(4*d*x + 4*c)/d + 1/4*a^2*sin(2*d*x + 2*c)/d
```

3.14 $\int \cos^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=67

$$\frac{(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(a\sin(c+dx)+a)^6}{3a^4d}+\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

```
[Out] (4*(a + a*Sin[c + d*x])^5)/(5*a^3*d) - (2*(a + a*Sin[c + d*x])^6)/(3*a^4*d)
 + (a + a*Sin[c + d*x])^7/(7*a^5*d)
```

Rubi [A] time = 0.0610944, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(a\sin(c+dx)+a)^6}{3a^4d}+\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^5)/(5*a^3*d) - (2*(a + a*Sin[c + d*x])^6)/(3*a^4*d)
 + (a + a*Sin[c + d*x])^7/(7*a^5*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^4 - 4a(a+x)^5 + (a+x)^6\right)\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{4(a+a\sin(c+dx))^5}{5a^3 d} - \frac{2(a+a\sin(c+dx))^6}{3a^4 d} + \frac{(a+a\sin(c+dx))^7}{7a^5 d}$$

Mathematica [A] time = 0.0795505, size = 58, normalized size = 0.87

$$-\frac{a^2(\sin(c+dx)+1)^2\left(15\sin^2(c+dx)-40\sin(c+dx)+29\right)\cos^6(c+dx)}{105d(\sin(c+dx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Cos[c + d*x]^6*(1 + Sin[c + d*x])^2*(29 - 40*Sin[c + d*x] + 15*Sin[c
+ d*x]^2))/(105*d*(-1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.039, size = 99, normalized size = 1.5

$$\frac{1}{d}\left(a^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^6}{7} + \frac{\sin(dx+c)}{35}\left(\frac{8}{3} + (\cos(dx+c))^4 + \frac{4(\cos(dx+c))^2}{3}\right)\right) - \frac{a^2(\cos(dx+c))^6}{3} + \frac{a^2\,\mathrm{si}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-1/3*a^2*cos(d*x+c)^6+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.950871, size = 128, normalized size = 1.91

$$\frac{15\,a^2\sin(dx+c)^7 + 35\,a^2\sin(dx+c)^6 - 21\,a^2\sin(dx+c)^5 - 105\,a^2\sin(dx+c)^4 - 35\,a^2\sin(dx+c)^3 + 105\,a^2\sin(dx+}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*a^2*sin(d*x + c)^7 + 35*a^2*sin(d*x + c)^6 - 21*a^2*sin(d*x + c)^
5 - 105*a^2*sin(d*x + c)^4 - 35*a^2*sin(d*x + c)^3 + 105*a^2*sin(d*x + c)^2
 + 105*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.75967, size = 176, normalized size = 2.63

$$-\frac{35\,a^2\cos\left(dx+c\right)^6+\left(15\,a^2\cos\left(dx+c\right)^6-24\,a^2\cos\left(dx+c\right)^4-32\,a^2\cos\left(dx+c\right)^2-64\,a^2\right)\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(35*a^2*cos(d*x + c)^6 + (15*a^2*cos(d*x + c)^6 - 24*a^2*cos(d*x + c
)^4 - 32*a^2*cos(d*x + c)^2 - 64*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 8.32056, size = 202, normalized size = 3.01

$$\begin{cases}\frac{8a^2\sin^7(c+dx)}{105d}+\frac{a^2\sin^6(c+dx)}{3d}+\frac{4a^2\sin^5(c+dx)\cos^2(c+dx)}{15d}+\frac{8a^2\sin^5(c+dx)}{15d}+\frac{a^2\sin^4(c+dx)\cos^2(c+dx)}{d}+\frac{a^2\sin^3(c+dx)\cos^4(c+dx)}{3d}+\frac{4a^2\mathrm{si}}{} \\ x\left(a\sin\left(c\right)+a\right)^2\cos^5\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**7/(105*d) + a**2*sin(c + d*x)**6/(3*d) + 4*
a**2*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + 8*a**2*sin(c + d*x)**5/(15*d)
 + a**2*sin(c + d*x)**4*cos(c + d*x)**2/d + a**2*sin(c + d*x)**3*cos(c + d*
x)**4/(3*d) + 4*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + a**2*sin(c + d
*x)**2*cos(c + d*x)**4/d + a**2*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)),
(x*(a*sin(c) + a)**2*cos(c)**5, True))
```

Giac [A] time = 1.14066, size = 158, normalized size = 2.36

$$-\frac{a^2\cos(6\,dx+6\,c)}{96\,d}-\frac{a^2\cos(4\,dx+4\,c)}{16\,d}-\frac{5\,a^2\cos(2\,dx+2\,c)}{32\,d}-\frac{a^2\sin(7\,dx+7\,c)}{448\,d}+\frac{a^2\sin(5\,dx+5\,c)}{320\,d}+\frac{19\,a^2\sin(3\,d}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*a^2*cos(6*d*x + 6*c)/d - 1/16*a^2*cos(4*d*x + 4*c)/d - 5/32*a^2*cos(2
*d*x + 2*c)/d - 1/448*a^2*sin(7*d*x + 7*c)/d + 1/320*a^2*sin(5*d*x + 5*c)/d
 + 19/192*a^2*sin(3*d*x + 3*c)/d + 45/64*a^2*sin(d*x + c)/d
```

3.15 $\int \cos^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=102

$$-\frac{7a^2\cos^5(c+dx)}{30d}-\frac{\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{6d}+\frac{7a^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{16d}+$$

```
[Out] (7*a^2*x)/16 - (7*a^2*Cos[c + d*x]^5)/(30*d) + (7*a^2*Cos[c + d*x]*Sin[c +
d*x])/(16*d) + (7*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (Cos[c + d*x]^5
*(a^2 + a^2*Sin[c + d*x]))/(6*d)
```

Rubi [A] time = 0.0925856, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2678, 2669, 2635, 8}

$$-\frac{7a^2\cos^5(c+dx)}{30d}-\frac{\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{6d}+\frac{7a^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{16d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (7*a^2*x)/16 - (7*a^2*Cos[c + d*x]^5)/(30*d) + (7*a^2*Cos[c + d*x]*Sin[c +
d*x])/(16*d) + (7*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (Cos[c + d*x]^5
*(a^2 + a^2*Sin[c + d*x]))/(6*d)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+a\sin(c+dx))^2\,dx &= -\frac{\cos^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{6d}+\frac{1}{6}(7a)\int \cos^4(c+dx)(a+a\sin(c+dx))\,d\\ &= -\frac{7a^2\cos^5(c+dx)}{30d}-\frac{\cos^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{6d}+\frac{1}{6}\left(7a^2\right)\int \cos^4(c+dx\\ &= -\frac{7a^2\cos^5(c+dx)}{30d}+\frac{7a^2\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{\cos^5(c+dx)\left(a^2+a^2\sin(c\right.}{6d}\\ &= -\frac{7a^2\cos^5(c+dx)}{30d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{7a^2\cos^3(c+dx)\sin(c+dx)}{24d}\\ &= \frac{7a^2x}{16}-\frac{7a^2\cos^5(c+dx)}{30d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{7a^2\cos^3(c+dx)\sin(c}{24d}\end{aligned}$$

Mathematica [A] time = 0.561608, size = 151, normalized size = 1.48

$$-\frac{a^2\left(\sqrt{\sin(c+dx)+1}\left(40\sin^6(c+dx)+56\sin^5(c+dx)-106\sin^4(c+dx)-182\sin^3(c+dx)+57\sin^2(c+dx)+231\,\mathrm{si}\right.\right.}{240d(\sin(c+dx)-1)^3(\sin(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Cos[c + d*x]^5*(-210*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 -
Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-96 + 231*Sin[c + d*x] + 57*Sin[c +
 d*x]^2 - 182*Sin[c + d*x]^3 - 106*Sin[c + d*x]^4 + 56*Sin[c + d*x]^5 + 40*
Sin[c + d*x]^6)))/(240*d*(-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^(5/2))
```

Maple [A] time = 0.036, size = 109, normalized size = 1.1

$$\frac{1}{d}\left(a^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^5}{6}+\frac{\sin(dx+c)}{24}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{dx}{16}+\frac{c}{16}\right)-\frac{2a^2(\cos(dx+c))^5}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)-2/5*a^2*cos(d*x+c)^5+a^2*(1/4*(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 0.957817, size = 120, normalized size = 1.18

$$-\frac{384\,a^2\cos(dx+c)^5-5\left(4\sin(2\,dx+2\,c)^3+12\,dx+12\,c-3\sin(4\,dx+4\,c)\right)a^2-30\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/960*(384*a^2*cos(d*x + c)^5 - 5*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c -
3*sin(4*d*x + 4*c))*a^2 - 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*
x + 2*c))*a^2)/d
```

Fricas [A] time = 1.6799, size = 181, normalized size = 1.77

$$-\frac{96\,a^2\cos(dx+c)^5-105\,a^2dx+5\left(8\,a^2\cos(dx+c)^5-14\,a^2\cos(dx+c)^3-21\,a^2\cos(dx+c)\right)\sin(dx+c)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/240*(96*a^2*cos(d*x + c)^5 - 105*a^2*d*x + 5*(8*a^2*cos(d*x + c)^5 - 14*
a^2*cos(d*x + c)^3 - 21*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 5.30316, size = 287, normalized size = 2.81

$$\begin{cases} \frac{a^2 x \sin^6(c+dx)}{16} + \frac{3a^2 x \sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3a^2 x \sin^4(c+dx)}{8} + \frac{3a^2 x \sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3a^2 x \sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{a^2 x \cos^6(c+dx)}{16} + \\ x\,(a\sin(c)+a)^2\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/16 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x
)**4/16 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + a**2*x*cos(c + d*x)*
*6/16 + 3*a**2*x*cos(c + d*x)**4/8 + a**2*sin(c + d*x)**5*cos(c + d*x)/(16*
d) + a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 3*a**2*sin(c + d*x)**3*co
s(c + d*x)/(8*d) - a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 5*a**2*sin(c
+ d*x)*cos(c + d*x)**3/(8*d) - 2*a**2*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x*
(a*sin(c) + a)**2*cos(c)**4, True))
```

Giac [A] time = 1.15864, size = 143, normalized size = 1.4

$$\frac{7}{16}a^2x - \frac{a^2\cos(5\,dx+5\,c)}{40\,d} - \frac{a^2\cos(3\,dx+3\,c)}{8\,d} - \frac{a^2\cos(dx+c)}{4\,d} - \frac{a^2\sin(6\,dx+6\,c)}{192\,d} + \frac{a^2\sin(4\,dx+4\,c)}{64\,d} + \frac{17\,a^2\sin(\ldots}{64\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 7/16*a^2*x - 1/40*a^2*cos(5*d*x + 5*c)/d - 1/8*a^2*cos(3*d*x + 3*c)/d - 1/4
*a^2*cos(d*x + c)/d - 1/192*a^2*sin(6*d*x + 6*c)/d + 1/64*a^2*sin(4*d*x + 4
*c)/d + 17/64*a^2*sin(2*d*x + 2*c)/d
```

3.16 $\int \cos^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=45

$$\frac{(a\sin(c+dx)+a)^4}{2a^2d}-\frac{(a\sin(c+dx)+a)^5}{5a^3d}$$

```
[Out] (a + a*Sin[c + d*x])^4/(2*a^2*d) - (a + a*Sin[c + d*x])^5/(5*a^3*d)
```

Rubi [A] time = 0.046204, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a\sin(c+dx)+a)^4}{2a^2d}-\frac{(a\sin(c+dx)+a)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a + a*Sin[c + d*x])^4/(2*a^2*d) - (a + a*Sin[c + d*x])^5/(5*a^3*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(a+x)^3-(a+x)^4\right)\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{(a+a\sin(c+dx))^4}{2a^2 d} - \frac{(a+a\sin(c+dx))^5}{5a^3 d}$$

Mathematica [A] time = 0.081595, size = 46, normalized size = 1.02

$$-\frac{a^2\sin(c+dx)\left(2\sin^4(c+dx)+5\sin^3(c+dx)-10\sin(c+dx)-10\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Sin[c + d*x]*(-10 - 10*Sin[c + d*x] + 5*Sin[c + d*x]^3 + 2*Sin[c + d*
x]^4))/(10*d)
```

Maple [A] time = 0.036, size = 79, normalized size = 1.8

$$\frac{1}{d}\left(a^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^4}{5}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{15}\right)-\frac{a^2(\cos(dx+c))^4}{2}+\frac{a^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/
2*a^2*cos(d*x+c)^4+1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.94727, size = 76, normalized size = 1.69

$$-\frac{2\,a^2\sin(dx+c)^5+5\,a^2\sin(dx+c)^4-10\,a^2\sin(dx+c)^2-10\,a^2\sin(dx+c)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(2*a^2*sin(d*x + c)^5 + 5*a^2*sin(d*x + c)^4 - 10*a^2*sin(d*x + c)^2
- 10*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.67835, size = 136, normalized size = 3.02

$$-\frac{5\,a^2\cos\left(dx+c\right)^4+2\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2-4\,a^2\right)\sin\left(dx+c\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/10*(5*a^2*cos(d*x + c)^4 + 2*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2
- 4*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 2.52839, size = 129, normalized size = 2.87

$$\begin{cases}\frac{2a^2\sin^5(c+dx)}{15d}+\frac{a^2\sin^4(c+dx)}{2d}+\frac{a^2\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)\cos^2(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{d} & \text{for } d \ne \\ x\left(a\sin\left(c\right)+a\right)^2\cos^3\left(c\right) & \text{other}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*a**2*sin(c + d*x)**5/(15*d) + a**2*sin(c + d*x)**4/(2*d) + a**
2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 2*a**2*sin(c + d*x)**3/(3*d) + a*
*2*sin(c + d*x)**2*cos(c + d*x)**2/d + a**2*sin(c + d*x)*cos(c + d*x)**2/d,
 Ne(d, 0)), (x*(a*sin(c) + a)**2*cos(c)**3, True))
```

Giac [A] time = 1.13963, size = 76, normalized size = 1.69

$$-\frac{2\,a^2\sin\left(dx+c\right)^5+5\,a^2\sin\left(dx+c\right)^4-10\,a^2\sin\left(dx+c\right)^2-10\,a^2\sin\left(dx+c\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*a^2*sin(d*x + c)^5 + 5*a^2*sin(d*x + c)^4 - 10*a^2*sin(d*x + c)^2
- 10*a^2*sin(d*x + c))/d
```

3.17 $\int \cos^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=78

$$-\frac{5a^2\cos^3(c+dx)}{12d}-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}+\frac{5a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{5a^2x}{8}$$

```
[Out] (5*a^2*x)/8 - (5*a^2*Cos[c + d*x]^3)/(12*d) + (5*a^2*Cos[c + d*x]*Sin[c + d
*x])/(8*d) - (Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x]))/(4*d)
```

Rubi [A] time = 0.0885705, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2678, 2669, 2635, 8}

$$-\frac{5a^2\cos^3(c+dx)}{12d}-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}+\frac{5a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{5a^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/8 - (5*a^2*Cos[c + d*x]^3)/(12*d) + (5*a^2*Cos[c + d*x]*Sin[c + d
*x])/(8*d) - (Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x]))/(4*d)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sin(c+dx))^2\,dx &= -\frac{\cos^3(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{4d}+\frac{1}{4}(5a)\int \cos^2(c+dx)(a+a\sin(c+dx))\,d\\ &= -\frac{5a^2\cos^3(c+dx)}{12d}-\frac{\cos^3(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{4d}+\frac{1}{4}\left(5a^2\right)\int \cos^2(c+dx\\ &= -\frac{5a^2\cos^3(c+dx)}{12d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{8d}-\frac{\cos^3(c+dx)\left(a^2+a^2\sin(c+\right.}{4d}\\ &= \frac{5a^2x}{8}-\frac{5a^2\cos^3(c+dx)}{12d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{8d}-\frac{\cos^3(c+dx)\left(a^2+a^2\,\mathrm{s}\right.}{4d}\end{aligned}$$

Mathematica [A] time = 0.302669, size = 131, normalized size = 1.68

$$-\frac{a^2\left(30\sqrt{1-\sin(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\left(6\sin^4(c+dx)+10\sin^3(c+dx)-7\sin^2(c+dx)-25\,\mathrm{si}\right.\right.}{24d(\sin(c+dx)-1)^2(\sin(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Cos[c + d*x]^3*(30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Si
n[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(16 - 25*Sin[c + d*x] - 7*Sin[c + d*x]
^2 + 10*Sin[c + d*x]^3 + 6*Sin[c + d*x]^4)))/(24*d*(-1 + Sin[c + d*x])^2*(1
 + Sin[c + d*x])^(3/2))
```

Maple [A] time = 0.036, size = 87, normalized size = 1.1

$$\frac{1}{d}\left(a^2\left(-\frac{(\cos(dx+c))^3\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{2a^2(\cos(dx+c))^3}{3}+a^2\left(\frac{\cos(dx+c)\sin(d}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)-2/3*a^2*cos(d*x+c)^3+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.942792, size = 88, normalized size = 1.13

$$-\frac{64\,a^2\cos\left(dx+c\right)^3-3\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^2-24\,\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^2}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/96*(64*a^2*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2 - 24*
(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2)/d
```

Fricas [A] time = 1.67521, size = 144, normalized size = 1.85

$$-\frac{16\,a^2\cos\left(dx+c\right)^3-15\,a^2dx+3\,\left(2\,a^2\cos\left(dx+c\right)^3-5\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/24*(16*a^2*cos(d*x + c)^3 - 15*a^2*d*x + 3*(2*a^2*cos(d*x + c)^3 - 5*a^2
*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.37272, size = 180, normalized size = 2.31

$$\begin{cases}\frac{a^2x\sin^4\left(c+dx\right)}{8}+\frac{a^2x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{a^2x\sin^2\left(c+dx\right)}{2}+\frac{a^2x\cos^4\left(c+dx\right)}{8}+\frac{a^2x\cos^2\left(c+dx\right)}{2}+\frac{a^2\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{a^2\sin\left(c+dx\right)\mathrm{co}}{8d}\\ x\left(a\sin\left(c\right)+a\right)^2\cos^2\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**4/8 + a**2*x*sin(c + d*x)**2*cos(c + d*x)**
2/4 + a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**4/8 + a**2*x*cos(c +
d*x)**2/2 + a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a**2*sin(c + d*x)*cos
(c + d*x)**3/(8*d) + a**2*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*a**2*cos(c +
d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*cos(c)**2, True))
```

Giac [A] time = 1.13125, size = 97, normalized size = 1.24

$$\frac{5}{8}\,a^{2}x-\frac{a^{2}\cos\left(3\,dx+3\,c\right)}{6\,d}-\frac{a^{2}\cos\left(dx+c\right)}{2\,d}-\frac{a^{2}\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a^{2}\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/8*a^2*x - 1/6*a^2*cos(3*d*x + 3*c)/d - 1/2*a^2*cos(d*x + c)/d - 1/32*a^2*
sin(4*d*x + 4*c)/d + 1/4*a^2*sin(2*d*x + 2*c)/d
```

3.18 $\int \cos(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=22

$$\frac{(a\sin(c+dx)+a)^3}{3ad}$$

```
[Out] (a + a*Sin[c + d*x])^3/(3*a*d)
```

Rubi [A] time = 0.0237991, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 32}

$$\frac{(a\sin(c+dx)+a)^3}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a + a*Sin[c + d*x])^3/(3*a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{(a+a\sin(c+dx))^3}{3ad}\end{aligned}$$

Mathematica [B] time = 0.0206811, size = 47, normalized size = 2.14

$$\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)/d + (a^2*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.013, size = 21, normalized size = 1.

$$\frac{(a+a\sin(dx+c))^3}{3\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/3*(a+a*sin(d*x+c))^3/d/a
```

Maxima [A] time = 0.93159, size = 27, normalized size = 1.23

$$\frac{(a\sin(dx+c)+a)^3}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*sin(d*x + c) + a)^3/(a*d)
```

Fricas [B] time = 1.73584, size = 101, normalized size = 4.59

$$-\frac{3\,a^2\cos(dx+c)^2+\left(a^2\cos(dx+c)^2-4\,a^2\right)\sin(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*cos(d*x + c)^2 + (a^2*cos(d*x + c)^2 - 4*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 0.614323, size = 53, normalized size = 2.41

$$\begin{cases} \frac{a^2 \sin^3(c+dx)}{3d} + \frac{a^2 \sin^2(c+dx)}{d} + \frac{a^2 \sin(c+dx)}{d} & \text{for } d \neq 0 \\ x\,(a\sin(c)+a)^2 \cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)**2/d + a**2*sin(c
 + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**2*cos(c), True))
```

Giac [A] time = 1.19124, size = 27, normalized size = 1.23

$$\frac{(a\sin(dx+c)+a)^3}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(a*sin(d*x + c) + a)^3/(a*d)
```

3.19 $\int \sec(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=34

$$-\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\log(1-\sin(c+dx))}{d}$$

```
[Out] (-2*a^2*Log[1 - Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/d
```

Rubi [A] time = 0.0416049, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 43}

$$-\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Log[1 - Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/d
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a\,\mathrm{Subst}\left(\int \frac{a+x}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\mathrm{Subst}\left(\int \left(-1+\frac{2a}{a-x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{2a^2\log(1-\sin(c+dx))}{d} - \frac{a^2\sin(c+dx)}{d}$$

Mathematica [A] time = 0.019072, size = 29, normalized size = 0.85

$$\frac{a^2(-\sin(c+dx)-2\log(1-\sin(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-2*Log[1 - Sin[c + d*x]] - Sin[c + d*x]))/d
```

Maple [A] time = 0.042, size = 53, normalized size = 1.6

$$2\,\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d} - 2\,\frac{a^2\ln(\cos(dx+c))}{d} - \frac{a^2\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))-2/d*a^2*ln(cos(d*x+c))-a^2*sin(d*x+c)/d
```

Maxima [A] time = 0.939936, size = 41, normalized size = 1.21

$$-\frac{2\,a^2\log(\sin(dx+c)-1)+a^2\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*a^2*log(sin(d*x + c) - 1) + a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.64804, size = 73, normalized size = 2.15

$$-\frac{2\,a^2\log\left(-\sin\left(dx+c\right)+1\right)+a^2\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x), x) + Integral(sin(c + d*x)**2*s
ec(c + d*x), x) + Integral(sec(c + d*x), x))
```

Giac [B] time = 1.18736, size = 123, normalized size = 3.62

$$\frac{2\left(a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-2\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*a^2*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) - (a^2*tan(1/2*d*x + 1/2*c)^2 + a^2*tan(1/2*d*x + 1/2*c) + a^2)/(tan
(1/2*d*x + 1/2*c)^2 + 1))/d
```

3.20 $\int \sec^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=38

$$\frac{2a^4\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}-a^2x$$

```
[Out] -(a^2*x) + (2*a^4*Cos[c + d*x])/(d*(a^2 - a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0817753, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2670, 2680, 8}

$$\frac{2a^4\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (2*a^4*Cos[c + d*x])/(d*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \sec^2(c+dx)(a+a\sin(c+dx))^2\,dx &= a^4\int \frac{\cos^2(c+dx)}{(a-a\sin(c+dx))^2}\,dx \\
&= \frac{2a^4\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)} - a^2\int 1\,dx \\
&= -a^2x + \frac{2a^4\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] time = 0.0561172, size = 75, normalized size = 1.97

$$
\frac{2a^2\sqrt{\sin(c+dx)+1}\left(\sqrt{1-\sin(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\right)\sec(c+dx)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sec[c + d*x]*Sqrt[1 + Sin[c + d*x]]*(ArcSin[Sqrt[1 - Sin[c + d*x]]/S
qrt[2]]*Sqrt[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]))/d
```

Maple [A] time = 0.046, size = 47, normalized size = 1.2

$$
\frac{1}{d}\left(a^2(\tan(dx+c)-dx-c)+2\,\frac{a^2}{\cos(dx+c)}+a^2\tan(dx+c)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a^2/cos(d*x+c)+a^2*tan(d*x+c))
```

Maxima [A] time = 1.41559, size = 63, normalized size = 1.66

$$
-\frac{(dx+c-\tan(dx+c))a^2-a^2\tan(dx+c)-\frac{2\,a^2}{\cos(dx+c)}}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a^2 - a^2*tan(d*x + c) - 2*a^2/cos(d*x + c))/d
```

Fricas [A] time = 1.63995, size = 167, normalized size = 4.39

$$-\frac{a^2dx - 2\,a^2 + \left(a^2dx - 2\,a^2\right)\cos\left(dx + c\right) - \left(a^2dx + 2\,a^2\right)\sin\left(dx + c\right)}{d\cos\left(dx + c\right) - d\sin\left(dx + c\right) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x - 2*a^2 + (a^2*d*x - 2*a^2)*cos(d*x + c) - (a^2*d*x + 2*a^2)*sin(
d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\sin\left(c + dx\right)\sec^2\left(c + dx\right)dx + \int \sin^2\left(c + dx\right)\sec^2\left(c + dx\right)dx + \int \sec^2\left(c + dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

Giac [A] time = 1.1502, size = 45, normalized size = 1.18

$$-\frac{(dx + c)a^2 + \frac{4\,a^2}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)*a^2 + 4*a^2/(tan(1/2*d*x + 1/2*c) - 1))/d
```

3.21 $\int \sec^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=20

$$\frac{a^3}{d(a-a\sin(c+dx))}$$

```
[Out] a^3/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0376204, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$\frac{a^3}{d(a-a\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^3/(d*(a - a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3}{d(a-a\sin(c+dx))}$$

Mathematica [A] time = 0.138102, size = 32, normalized size = 1.6

$$\frac{a^2}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2)
```

Maple [B] time = 0.062, size = 75, normalized size = 3.8

$$\frac{a^2(\sin(dx+c))^3}{2d(\cos(dx+c))^2}+\frac{a^2\sin(dx+c)}{2d}+\frac{a^2}{d(\cos(dx+c))^2}+\frac{a^2\sec(dx+c)\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*a^2*sin(d*x+c)/d+1/d*a^2/cos(d*x+c)
^2+1/2/d*a^2*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.932716, size = 24, normalized size = 1.2

$$-\frac{a^2}{d(\sin(dx+c)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -a^2/(d*(sin(d*x + c) - 1))
```

Fricas [A] time = 1.54091, size = 36, normalized size = 1.8

$$-\frac{a^2}{d\sin(dx+c)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -a^2/(d*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17787, size = 41, normalized size = 2.05

$$\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*a^2*tan(1/2*d*x + 1/2*c)/(d*(tan(1/2*d*x + 1/2*c) - 1)^2)
```

3.22 $\int \sec^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=63

$$\frac{a^4\cos(c+dx)}{3d\left(a^2-a^2\sin(c+dx)\right)}+\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}$$

```
[Out] (a^4*Cos[c + d*x])/(3*d*(a - a*Sin[c + d*x])^2) + (a^4*Cos[c + d*x])/(3*d*(
a^2 - a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0691645, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2670, 2650, 2648}

$$\frac{a^4\cos(c+dx)}{3d\left(a^2-a^2\sin(c+dx)\right)}+\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^4*Cos[c + d*x])/(3*d*(a - a*Sin[c + d*x])^2) + (a^4*Cos[c + d*x])/(3*d*(
a^2 - a^2*Sin[c + d*x]))
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sin(c+dx))^2\,dx &= a^4\int \frac{1}{(a-a\sin(c+dx))^2}\,dx\\ &= \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{1}{3}a^3\int \frac{1}{a-a\sin(c+dx)}\,dx\\ &= \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{a^3\cos(c+dx)}{3d(a-a\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0086958, size = 58, normalized size = 0.92

$$-\frac{a^2\tan^3(c+dx)}{3d}+\frac{2a^2\sec^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/d - (a^2*T
an[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.065, size = 63, normalized size = 1.

$$\frac{1}{d}\left(\frac{a^2(\sin(dx+c))^3}{3(\cos(dx+c))^3}+\frac{2a^2}{3(\cos(dx+c))^3}-a^2\left(-\frac{2}{3}-\frac{(\sec(dx+c))^2}{3}\right)\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*a^2*sin(d*x+c)^3/cos(d*x+c)^3+2/3*a^2/cos(d*x+c)^3-a^2*(-2/3-1/3*s
ec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 0.953012, size = 70, normalized size = 1.11

$$\frac{a^2\tan\left(dx+c\right)^3+\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2+\frac{2\,a^2}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 2*a^2/cos
(d*x + c)^3)/d
```

Fricas [A] time = 1.67741, size = 235, normalized size = 3.73

$$-\frac{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2-\left(a^2\cos\left(dx+c\right)-a^2\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2 - (a^2*cos(d*x + c) - a
^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*
d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2081, size = 73, normalized size = 1.16

$$-\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a^{2}\right)}{3\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c) + 2*a^2)/(d
*(tan(1/2*d*x + 1/2*c) - 1)^3)
```

3.23 $\int \sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=64

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{a^3}{4d(a-a\sin(c+dx))}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + a^3/
(4*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0663574, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{a^3}{4d(a-a\sin(c+dx))}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + a^3/
(4*d*(a - a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{a^5\operatorname{Subst}\left(\int \frac{1}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^5\operatorname{Subst}\left(\int \left(\frac{1}{2a(a-x)^3}+\frac{1}{4a^2(a-x)^2}+\frac{1}{4a^2(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{a^3}{4d(a-a\sin(c+dx))}+\frac{a^3\operatorname{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c\right.}{4d}\\ &= \frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{a^3}{4d(a-a\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0671311, size = 56, normalized size = 0.88

$$\frac{a^2(\sin(c+dx)+1)^2\sec^4(c+dx)\left(-\sin(c+dx)+(\sin(c+dx)-1)^2\tanh^{-1}(\sin(c+dx))+2\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c + d*x]^4*(2 + ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])^2 - Sin[
c + d*x])*(1 + Sin[c + d*x])^2)/(4*d)
```

Maple [B] time = 0.067, size = 144, normalized size = 2.3

$$\frac{a^2(\sin(dx+c))^3}{4d(\cos(dx+c))^4}+\frac{a^2(\sin(dx+c))^3}{8d(\cos(dx+c))^2}+\frac{a^2\sin(dx+c)}{8d}+\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{4d}+\frac{a^2}{2d(\cos(dx+c))^4}+\frac{a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/8
*a^2*sin(d*x+c)/d+1/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2/cos(d*x+c)^
4+1/4/d*a^2*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^2*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.953411, size = 96, normalized size = 1.5

$$\frac{a^2 \log\left(\sin\left(dx+c\right)+1\right) - a^2 \log\left(\sin\left(dx+c\right)-1\right) - \frac{2\left(a^2 \sin(dx+c) - 2\,a^2\right)}{\sin(dx+c)^2 - 2\,\sin(dx+c) + 1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*log(sin(d*x + c) + 1) - a^2*log(sin(d*x + c) - 1) - 2*(a^2*sin(d*x
 + c) - 2*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [B] time = 1.70987, size = 306, normalized size = 4.78

$$\frac{2\,a^2 \sin\left(dx+c\right) - 4\,a^2 + \left(a^2 \cos\left(dx+c\right)^2 + 2\,a^2 \sin\left(dx+c\right) - 2\,a^2\right) \log\left(\sin\left(dx+c\right)+1\right) - \left(a^2 \cos\left(dx+c\right)^2 + 2\,a^2 \sin\right.}{8\left(d \cos\left(dx+c\right)^2 + 2\,d \sin\left(dx+c\right) - 2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a^2*sin(d*x + c) - 4*a^2 + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c)
- 2*a^2)*log(sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c) -
 2*a^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21577, size = 104, normalized size = 1.62

$$\frac{2\,a^2\log\left(|\sin\left(dx+c\right)+1|\right)-2\,a^2\log\left(|\sin\left(dx+c\right)-1|\right)+\frac{3\,a^2\sin(dx+c)^2-10\,a^2\sin(dx+c)+11\,a^2}{(\sin(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) - 2*a^2*log(abs(sin(d*x + c) - 1)) +
 (3*a^2*sin(d*x + c)^2 - 10*a^2*sin(d*x + c) + 11*a^2)/(sin(d*x + c) - 1)^2
)/d
```

3.24 $\int \sec^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=64

$$\frac{a^2\tan^3(c+dx)}{5d}+\frac{3a^2\tan(c+dx)}{5d}+\frac{2\sec^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{5d}$$

```
[Out] (2*Sec[c + d*x]^5*(a^2 + a^2*Sin[c + d*x]))/(5*d) + (3*a^2*Tan[c + d*x])/(5
*d) + (a^2*Tan[c + d*x]^3)/(5*d)
```

Rubi [A] time = 0.0562102, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2676, 3767}

$$\frac{a^2\tan^3(c+dx)}{5d}+\frac{3a^2\tan(c+dx)}{5d}+\frac{2\sec^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^5*(a^2 + a^2*Sin[c + d*x]))/(5*d) + (3*a^2*Tan[c + d*x])/(5
*d) + (a^2*Tan[c + d*x]^3)/(5*d)
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{2\sec^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{5d} + \frac{1}{5}\left(3a^2\right)\int \sec^4(c+dx)\,dx$$
$$= \frac{2\sec^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{5d} - \frac{\left(3a^2\right)\text{Subst}\left(\int\left(1+x^2\right)dx, x, -\tan(c+a\right.}{5d}$$
$$= \frac{2\sec^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{5d} + \frac{3a^2\tan(c+dx)}{5d} + \frac{a^2\tan^3(c+dx)}{5d}$$

Mathematica [A] time = 0.0106957, size = 82, normalized size = 1.28

$$\frac{2a^2\tan^5(c+dx)}{5d} + \frac{2a^2\sec^5(c+dx)}{5d} - \frac{a^2\tan^3(c+dx)\sec^2(c+dx)}{d} + \frac{a^2\tan(c+dx)\sec^4(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^4*Tan[c + d*x])/d - (a^2*S
ec[c + d*x]^2*Tan[c + d*x]^3)/d + (2*a^2*Tan[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.103, size = 93, normalized size = 1.5

$$\frac{1}{d}\left(a^2\left(\frac{(\sin(dx+c))^3}{5(\cos(dx+c))^5} + \frac{2(\sin(dx+c))^3}{15(\cos(dx+c))^3}\right) + \frac{2a^2}{5(\cos(dx+c))^5} - a^2\left(-\frac{8}{15} - \frac{(\sec(dx+c))^4}{5} - \frac{4(\sec(dx+c))^2}{15}\right)\tan(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+2/5
*a^2/cos(d*x+c)^5-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)
)
```

Maxima [A] time = 0.975744, size = 104, normalized size = 1.62

$$\frac{\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)a^2 + \left(3\tan(dx+c)^5 + 5\tan(dx+c)^3\right)a^2 + \frac{6a^2}{\cos(dx+c)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + (3*tan
(d*x + c)^5 + 5*tan(d*x + c)^3)*a^2 + 6*a^2/cos(d*x + c)^5)/d
```

Fricas [A] time = 1.53434, size = 207, normalized size = 3.23

$$-\frac{4\,a^2\cos\left(dx+c\right)^2-2\,a^2-\left(2\,a^2\cos\left(dx+c\right)^2-3\,a^2\right)\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)^3+2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/5*(4*a^2*cos(d*x + c)^2 - 2*a^2 - (2*a^2*cos(d*x + c)^2 - 3*a^2)*sin(d*x
 + c))/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17407, size = 143, normalized size = 2.23

$$-\frac{\frac{5\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-90\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21\,a^2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(5*a^2/(tan(1/2*d*x + 1/2*c) + 1) + (35*a^2*tan(1/2*d*x + 1/2*c)^4 -
90*a^2*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*tan(1/2*d*x + 1/2*c)^2 - 70*a^2*tan
(1/2*d*x + 1/2*c) + 21*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^5)/d
```

3.25 $\int \sec^7(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=109

$$\frac{a^5}{12d(a-a\sin(c+dx))^3}+\frac{a^4}{8d(a-a\sin(c+dx))^2}+\frac{3a^3}{16d(a-a\sin(c+dx))}-\frac{a^3}{16d(a\sin(c+dx)+a)}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^5/(12*d*(a - a*Sin[c + d*x])^3) + a^4
/(8*d*(a - a*Sin[c + d*x])^2) + (3*a^3)/(16*d*(a - a*Sin[c + d*x])) - a^3/(
16*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0886078, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$\frac{a^5}{12d(a-a\sin(c+dx))^3}+\frac{a^4}{8d(a-a\sin(c+dx))^2}+\frac{3a^3}{16d(a-a\sin(c+dx))}-\frac{a^3}{16d(a\sin(c+dx)+a)}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^5/(12*d*(a - a*Sin[c + d*x])^3) + a^4
/(8*d*(a - a*Sin[c + d*x])^2) + (3*a^3)/(16*d*(a - a*Sin[c + d*x])) - a^3/(
16*d*(a + a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^7\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\,\mathrm{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^4}+\frac{1}{4a^3(a-x)^3}+\frac{3}{16a^4(a-x)^2}+\frac{1}{16a^4(a+x)^2}+\frac{1}{4a^4\left(a^2-x^2\right)}\right)dx, x, a\sin\ldots}{d}$$

$$= \frac{a^5}{12d(a-a\sin(c+dx))^3}+\frac{a^4}{8d(a-a\sin(c+dx))^2}+\frac{3a^3}{16d(a-a\sin(c+dx))}-\frac{\ldots}{16d\ldots}$$

$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^5}{12d(a-a\sin(c+dx))^3}+\frac{a^4}{8d(a-a\sin(c+dx))^2}+\frac{\ldots}{16\ldots}$$

Mathematica [A] time = 0.142689, size = 85, normalized size = 0.78

$$-\frac{a^2(\sin(c+dx)+1)^2\sec^6(c+dx)\left(-3\sin^3(c+dx)+6\sin^2(c+dx)-\sin(c+dx)+3(\sin(c+dx)+1)(\sin(c+dx)-1)^3\ldots\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Sec[c + d*x]^6*(1 + Sin[c + d*x])^2*(-4 - Sin[c + d*x] + 6*Sin[c + d*
x]^2 - 3*Sin[c + d*x]^3 + 3*ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])^3*(1
+ Sin[c + d*x])))/(12*d)
```

Maple [A] time = 0.108, size = 190, normalized size = 1.7

$$\frac{a^2\left(\sin(dx+c)\right)^3}{6d\left(\cos(dx+c)\right)^6}+\frac{a^2\left(\sin(dx+c)\right)^3}{8d\left(\cos(dx+c)\right)^4}+\frac{a^2\left(\sin(dx+c)\right)^3}{16d\left(\cos(dx+c)\right)^2}+\frac{a^2\sin(dx+c)}{16d}+\frac{a^2\ln\left(\sec(dx+c)+\tan(dx+c)\right)}{4d}+\frac{\ldots}{3\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/6/d*a^2*sin(d*x+c)^3/cos(d*x+c)^6+1/8/d*a^2*sin(d*x+c)^3/cos(d*x+c)^4+1/1
6/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/16*a^2*sin(d*x+c)/d+1/4/d*a^2*ln(sec(d*
x+c)+tan(d*x+c))+1/3/d*a^2/cos(d*x+c)^6+1/6/d*a^2*tan(d*x+c)*sec(d*x+c)^5+5
/24/d*a^2*tan(d*x+c)*sec(d*x+c)^3+5/16/d*a^2*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.97087, size = 146, normalized size = 1.34

$$\frac{3\,a^2\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,a^2\sin(dx+c)^3-6\,a^2\sin(dx+c)^2+a^2\sin(dx+c)+4\,a^2\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^3+2\,\sin(dx+c)-1}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a^2*log(sin(d*x + c) + 1) - 3*a^2*log(sin(d*x + c) - 1) - 2*(3*a^2*
sin(d*x + c)^3 - 6*a^2*sin(d*x + c)^2 + a^2*sin(d*x + c) + 4*a^2)/(sin(d*x
+ c)^4 - 2*sin(d*x + c)^3 + 2*sin(d*x + c) - 1))/d
```

Fricas [A] time = 1.64871, size = 505, normalized size = 4.63

$$-\frac{12\,a^2\cos\left(dx+c\right)^2-4\,a^2-3\left(a^2\cos\left(dx+c\right)^4+2\,a^2\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\,a^2\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+\right.}{24\left(d\cos\left(dx+c\right)^4+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*a^2*cos(d*x + c)^2 - 4*a^2 - 3*(a^2*cos(d*x + c)^4 + 2*a^2*cos(d*
x + c)^2*sin(d*x + c) - 2*a^2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 3*(a^
2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2*sin(d*x + c) - 2*a^2*cos(d*x + c)^2
)*log(-sin(d*x + c) + 1) - 2*(3*a^2*cos(d*x + c)^2 - 4*a^2)*sin(d*x + c))/(
d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2*sin(d*x + c) - 2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20027, size = 161, normalized size = 1.48

$$\frac{6\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-6\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{3\left(2\,a^2\sin(dx+c)+3\,a^2\right)}{\sin(dx+c)+1}+\frac{11\,a^2\sin(dx+c)^3-42\,a^2\sin(dx+c)^2+57\,a^2\sin(dx+c)-30\,a^2}{(\sin(dx+c)-1)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/48*(6*a^2*log(abs(sin(d*x + c) + 1)) - 6*a^2*log(abs(sin(d*x + c) - 1)) -
 3*(2*a^2*sin(d*x + c) + 3*a^2)/(sin(d*x + c) + 1) + (11*a^2*sin(d*x + c)^3
 - 42*a^2*sin(d*x + c)^2 + 57*a^2*sin(d*x + c) - 30*a^2)/(sin(d*x + c) - 1)
^3)/d
```

3.26 $\int \sec^8(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=82

$$\frac{a^2\tan^5(c+dx)}{7d}+\frac{10a^2\tan^3(c+dx)}{21d}+\frac{5a^2\tan(c+dx)}{7d}+\frac{2\sec^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{7d}$$

```
[Out] (2*Sec[c + d*x]^7*(a^2 + a^2*Sin[c + d*x]))/(7*d) + (5*a^2*Tan[c + d*x])/(7
*d) + (10*a^2*Tan[c + d*x]^3)/(21*d) + (a^2*Tan[c + d*x]^5)/(7*d)
```

Rubi [A] time = 0.058504, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2676, 3767}

$$\frac{a^2\tan^5(c+dx)}{7d}+\frac{10a^2\tan^3(c+dx)}{21d}+\frac{5a^2\tan(c+dx)}{7d}+\frac{2\sec^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^7*(a^2 + a^2*Sin[c + d*x]))/(7*d) + (5*a^2*Tan[c + d*x])/(7
*d) + (10*a^2*Tan[c + d*x]^3)/(21*d) + (a^2*Tan[c + d*x]^5)/(7*d)
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{2\sec^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{7d}+\frac{1}{7}\left(5a^2\right)\int \sec^6(c+dx)\,dx$$
$$= \frac{2\sec^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{7d}-\frac{\left(5a^2\right)\mathrm{Subst}\left(\int\left(1+2x^2+x^4\right)dx,x,-\tan\right.}{7d}$$
$$= \frac{2\sec^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{7d}+\frac{5a^2\tan(c+dx)}{7d}+\frac{10a^2\tan^3(c+dx)}{21d}+\frac{a^2}{}$$

Mathematica [A] time = 0.0234543, size = 110, normalized size = 1.34

$$-\frac{8a^2\tan^7(c+dx)}{21d}+\frac{2a^2\sec^7(c+dx)}{7d}-\frac{5a^2\tan^3(c+dx)\sec^4(c+dx)}{3d}+\frac{4a^2\tan^5(c+dx)\sec^2(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sec[c + d*x]^7)/(7*d) + (a^2*Sec[c + d*x]^6*Tan[c + d*x])/d - (5*a^2
*Sec[c + d*x]^4*Tan[c + d*x]^3)/(3*d) + (4*a^2*Sec[c + d*x]^2*Tan[c + d*x]^
5)/(3*d) - (8*a^2*Tan[c + d*x]^7)/(21*d)
```

Maple [A] time = 0.112, size = 121, normalized size = 1.5

$$\frac{1}{d}\left(a^2\left(\frac{(\sin(dx+c))^3}{7(\cos(dx+c))^7}+\frac{4(\sin(dx+c))^3}{35(\cos(dx+c))^5}+\frac{8(\sin(dx+c))^3}{105(\cos(dx+c))^3}\right)+\frac{2a^2}{7(\cos(dx+c))^7}-a^2\left(-\frac{16}{35}-\frac{(\sec(dx+c))^6}{7}-\frac{6}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/10
5*sin(d*x+c)^3/cos(d*x+c)^3)+2/7*a^2/cos(d*x+c)^7-a^2*(-16/35-1/7*sec(d*x+c
)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 0.974856, size = 132, normalized size = 1.61

$$\frac{\left(15\tan(dx+c)^7+42\tan(dx+c)^5+35\tan(dx+c)^3\right)a^2+3\left(5\tan(dx+c)^7+21\tan(dx+c)^5+35\tan(dx+c)^3+\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*((15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^2 + 3*
(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c)
)*a^2 + 30*a^2/cos(d*x + c)^7)/d
```

Fricas [A] time = 1.68832, size = 278, normalized size = 3.39

$$-\frac{16\,a^2\cos\left(dx+c\right)^4-8\,a^2\cos\left(dx+c\right)^2-2\,a^2-\left(8\,a^2\cos\left(dx+c\right)^4-12\,a^2\cos\left(dx+c\right)^2-5\,a^2\right)\sin\left(dx+c\right)}{21\,\left(d\cos\left(dx+c\right)^5+2\,d\cos\left(dx+c\right)^3\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/21*(16*a^2*cos(d*x + c)^4 - 8*a^2*cos(d*x + c)^2 - 2*a^2 - (8*a^2*cos(d*
x + c)^4 - 12*a^2*cos(d*x + c)^2 - 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5 +
 2*d*cos(d*x + c)^3*sin(d*x + c) - 2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18891, size = 231, normalized size = 2.82

$$-\frac{\frac{7\left(9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}+\frac{273\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2450\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2870\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^7}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/168*(7*(9*a^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2*tan(1/2*d*x + 1/2*c) + 8*a
^2)/(tan(1/2*d*x + 1/2*c) + 1)^3 + (273*a^2*tan(1/2*d*x + 1/2*c)^6 - 1155*a
^2*tan(1/2*d*x + 1/2*c)^5 + 2450*a^2*tan(1/2*d*x + 1/2*c)^4 - 2870*a^2*tan(
1/2*d*x + 1/2*c)^3 + 2037*a^2*tan(1/2*d*x + 1/2*c)^2 - 791*a^2*tan(1/2*d*x
+ 1/2*c) + 152*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^7)/d
```

3.27 $\int \cos^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=154

$$-\frac{11a^3\cos^7(c+dx)}{56d}-\frac{11\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{72d}+\frac{11a^3\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{55a^3\sin(c+dx)\cos^3(c+}{192d}$$

```
[Out] (55*a^3*x)/128 - (11*a^3*Cos[c + d*x]^7)/(56*d) + (55*a^3*Cos[c + d*x]*Sin[
c + d*x])/(128*d) + (55*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (11*a^3*
Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (a*Cos[c + d*x]^7*(a + a*Sin[c + d*x]
)^2)/(9*d) - (11*Cos[c + d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(72*d)
```

Rubi [A] time = 0.153738, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2678, 2669, 2635, 8}

$$-\frac{11a^3\cos^7(c+dx)}{56d}-\frac{11\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{72d}+\frac{11a^3\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{55a^3\sin(c+dx)\cos^3(c+}{192d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (55*a^3*x)/128 - (11*a^3*Cos[c + d*x]^7)/(56*d) + (55*a^3*Cos[c + d*x]*Sin[
c + d*x])/(128*d) + (55*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (11*a^3*
Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (a*Cos[c + d*x]^7*(a + a*Sin[c + d*x]
)^2)/(9*d) - (11*Cos[c + d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(72*d)
```


Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+a\sin(c+dx))^3\,dx &= -\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}+\frac{1}{9}(11a)\int \cos^6(c+dx)(a+a\sin(c+dx))^2\\ &= -\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}-\frac{11\cos^7(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{72d}+\frac{1}{8}\\ &= -\frac{11a^3\cos^7(c+dx)}{56d}-\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}-\frac{11\cos^7(c+dx)\left(a^3+\right.}{72d}\\ &= -\frac{11a^3\cos^7(c+dx)}{56d}+\frac{11a^3\cos^5(c+dx)\sin(c+dx)}{48d}-\frac{a\cos^7(c+dx)(a+a\sin(c}{9d}\\ &= -\frac{11a^3\cos^7(c+dx)}{56d}+\frac{55a^3\cos^3(c+dx)\sin(c+dx)}{192d}+\frac{11a^3\cos^5(c+dx)\sin(c+}{48d}\\ &= -\frac{11a^3\cos^7(c+dx)}{56d}+\frac{55a^3\cos(c+dx)\sin(c+dx)}{128d}+\frac{55a^3\cos^3(c+dx)\sin(c+}{192d}\\ &= \frac{55a^3x}{128}-\frac{11a^3\cos^7(c+dx)}{56d}+\frac{55a^3\cos(c+dx)\sin(c+dx)}{128d}+\frac{55a^3\cos^3(c+dx)\,\mathrm{s}}{192d}\end{aligned}$$

Mathematica [A] time = 1.99511, size = 181, normalized size = 1.18

$$-\frac{a^3\left(6930\sqrt{1-\sin(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\left(896\sin^9(c+dx)+2128\sin^8(c+dx)-2000\sin^7(c+\right.\right.}{8064d(\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Cos[c + d*x]^7*(6930*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 -
Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(3712 - 8311*Sin[c + d*x] - 5641*Sin
[c + d*x]^2 + 7174*Sin[c + d*x]^3 + 11514*Sin[c + d*x]^4 - 1224*Sin[c + d*x
]^5 - 8248*Sin[c + d*x]^6 - 2000*Sin[c + d*x]^7 + 2128*Sin[c + d*x]^8 + 896
*Sin[c + d*x]^9)))/(8064*d*(-1 + Sin[c + d*x])^4*(1 + Sin[c + d*x])^(7/2))
```

Maple [A] time = 0.043, size = 163, normalized size = 1.1

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^7}{9}-\frac{2(\cos(dx+c))^7}{63}\right)+3a^3\left(-1/8\sin(dx+c)(\cos(dx+c))^7+1/48\left((\cos(dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+3*a^3*(-1/8*sin
(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*s
in(d*x+c)+5/128*d*x+5/128*c)-3/7*a^3*cos(d*x+c)^7+a^3*(1/6*(cos(d*x+c)^5+5/
4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 0.98203, size = 190, normalized size = 1.23

$$-\frac{27648a^3\cos(dx+c)^7-1024\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a^3-63\left(64\sin(2dx+2c)^3+120dx+120c-3\sin\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/64512*(27648*a^3*cos(d*x + c)^7 - 1024*(7*cos(d*x + c)^9 - 9*cos(d*x + c
)^7)*a^3 - 63*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c)
 - 24*sin(4*d*x + 4*c))*a^3 + 336*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3)/d
```

Fricas [A] time = 1.87002, size = 258, normalized size = 1.68

$$\frac{896a^3\cos(dx+c)^9-4608a^3\cos(dx+c)^7+3465a^3dx-21\left(144a^3\cos(dx+c)^7-88a^3\cos(dx+c)^5-110a^3\cos(dx\right.}{8064d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8064*(896*a^3*cos(d*x + c)^9 - 4608*a^3*cos(d*x + c)^7 + 3465*a^3*d*x - 2
1*(144*a^3*cos(d*x + c)^7 - 88*a^3*cos(d*x + c)^5 - 110*a^3*cos(d*x + c)^3
- 165*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 24.3576, size = 439, normalized size = 2.85

$$\begin{cases} \frac{15a^3x\sin^8(c+dx)}{128} + \frac{15a^3x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{5a^3x\sin^6(c+dx)}{16} + \frac{45a^3x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{15a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15a^3x\sin^2(c \\ x\left(a\sin(c)+a\right)^3\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((15*a**3*x*sin(c + d*x)**8/128 + 15*a**3*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 5*a**3*x*sin(c + d*x)**6/16 + 45*a**3*x*sin(c + d*x)**4*cos(
c + d*x)**4/64 + 15*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*a**3*x*s
in(c + d*x)**2*cos(c + d*x)**6/32 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)*
*4/16 + 15*a**3*x*cos(c + d*x)**8/128 + 5*a**3*x*cos(c + d*x)**6/16 + 15*a*
*3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*a**3*sin(c + d*x)**5*cos(c + d
*x)**3/(128*d) + 5*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 73*a**3*sin(c
 + d*x)**3*cos(c + d*x)**5/(128*d) + 5*a**3*sin(c + d*x)**3*cos(c + d*x)**3
/(6*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 15*a**3*sin(c + d*x)*
cos(c + d*x)**7/(128*d) + 11*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 2*a
**3*cos(c + d*x)**9/(63*d) - 3*a**3*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a
*sin(c) + a)**3*cos(c)**6, True))
```

Giac [A] time = 1.18716, size = 212, normalized size = 1.38

$$\frac{55}{128}a^3x + \frac{a^3\cos(9\,dx+9\,c)}{2304\,d} - \frac{9\,a^3\cos(7\,dx+7\,c)}{1792\,d} - \frac{3\,a^3\cos(5\,dx+5\,c)}{64\,d} - \frac{29\,a^3\cos(3\,dx+3\,c)}{192\,d} - \frac{33\,a^3\cos(dx+c)}{128\,d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 55/128*a^3*x + 1/2304*a^3*cos(9*d*x + 9*c)/d - 9/1792*a^3*cos(7*d*x + 7*c)/
d - 3/64*a^3*cos(5*d*x + 5*c)/d - 29/192*a^3*cos(3*d*x + 3*c)/d - 33/128*a^
3*cos(d*x + c)/d - 3/1024*a^3*sin(8*d*x + 8*c)/d - 1/96*a^3*sin(6*d*x + 6*c
)/d + 3/128*a^3*sin(4*d*x + 4*c)/d + 9/32*a^3*sin(2*d*x + 2*c)/d
```

3.28 $\int \cos^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=67

$$\frac{(a\sin(c+dx)+a)^8}{8a^5d}-\frac{4(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(a\sin(c+dx)+a)^6}{3a^3d}$$

```
[Out] (2*(a + a*Sin[c + d*x])^6)/(3*a^3*d) - (4*(a + a*Sin[c + d*x])^7)/(7*a^4*d)
 + (a + a*Sin[c + d*x])^8/(8*a^5*d)
```

Rubi [A] time = 0.0656316, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a\sin(c+dx)+a)^8}{8a^5d}-\frac{4(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(a\sin(c+dx)+a)^6}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^6)/(3*a^3*d) - (4*(a + a*Sin[c + d*x])^7)/(7*a^4*d)
 + (a + a*Sin[c + d*x])^8/(8*a^5*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^5\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^5 - 4a(a+x)^6 + (a+x)^7\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{2(a+a\sin(c+dx))^6}{3a^3 d} - \frac{4(a+a\sin(c+dx))^7}{7a^4 d} + \frac{(a+a\sin(c+dx))^8}{8a^5 d}$$

Mathematica [A] time = 0.0961315, size = 58, normalized size = 0.87

$$-\frac{a^3(\sin(c+dx)+1)^3\left(21\sin^2(c+dx) - 54\sin(c+dx) + 37\right)\cos^6(c+dx)}{168d(\sin(c+dx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Cos[c + d*x]^6*(1 + Sin[c + d*x])^3*(37 - 54*Sin[c + d*x] + 21*Sin[c
+ d*x]^2))/(168*d*(-1 + Sin[c + d*x])^3)
```

Maple [B] time = 0.042, size = 133, normalized size = 2.

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^6}{8} - \frac{(\cos(dx+c))^6}{24}\right) + 3a^3\left(-1/7\sin(dx+c)(\cos(dx+c))^6 + 1/35\left(8/3 + (\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+3*a^3*(-1/7*sin
(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1
/2*a^3*cos(d*x+c)^6+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.950401, size = 146, normalized size = 2.18

$$\frac{21\,a^3\sin(dx+c)^8 + 72\,a^3\sin(dx+c)^7 + 28\,a^3\sin(dx+c)^6 - 168\,a^3\sin(dx+c)^5 - 210\,a^3\sin(dx+c)^4 + 56\,a^3\sin(dx}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/168*(21*a^3*sin(d*x + c)^8 + 72*a^3*sin(d*x + c)^7 + 28*a^3*sin(d*x + c)^
6 - 168*a^3*sin(d*x + c)^5 - 210*a^3*sin(d*x + c)^4 + 56*a^3*sin(d*x + c)^3
 + 252*a^3*sin(d*x + c)^2 + 168*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.80867, size = 207, normalized size = 3.09

$$\frac{21\,a^{3}\cos\left(dx+c\right)^{8}-112\,a^{3}\cos\left(dx+c\right)^{6}-8\left(9\,a^{3}\cos\left(dx+c\right)^{6}-6\,a^{3}\cos\left(dx+c\right)^{4}-8\,a^{3}\cos\left(dx+c\right)^{2}-16\,a^{3}\right)\sin\left(dx+\right.}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/168*(21*a^3*cos(d*x + c)^8 - 112*a^3*cos(d*x + c)^6 - 8*(9*a^3*cos(d*x +
c)^6 - 6*a^3*cos(d*x + c)^4 - 8*a^3*cos(d*x + c)^2 - 16*a^3)*sin(d*x + c))/
d
```

Sympy [A] time = 13.7934, size = 270, normalized size = 4.03

$$\begin{cases}\frac{a^{3}\sin^{8}\left(c+dx\right)}{24d}+\frac{8a^{3}\sin^{7}\left(c+dx\right)}{35d}+\frac{a^{3}\sin^{6}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{6d}+\frac{a^{3}\sin^{6}\left(c+dx\right)}{2d}+\frac{4a^{3}\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{5d}+\frac{8a^{3}\sin^{5}\left(c+dx\right)}{15d}+\frac{a^{3}\sin^{4}\left(c+dx\right)\cos^{4}}{4d} \\ x\left(a\sin\left(c\right)+a\right)^{3}\cos^{5}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**8/(24*d) + 8*a**3*sin(c + d*x)**7/(35*d) + a*
*3*sin(c + d*x)**6*cos(c + d*x)**2/(6*d) + a**3*sin(c + d*x)**6/(2*d) + 4*a
**3*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 8*a**3*sin(c + d*x)**5/(15*d) +
 a**3*sin(c + d*x)**4*cos(c + d*x)**4/(4*d) + 3*a**3*sin(c + d*x)**4*cos(c
+ d*x)**2/(2*d) + a**3*sin(c + d*x)**3*cos(c + d*x)**4/d + 4*a**3*sin(c + d
*x)**3*cos(c + d*x)**2/(3*d) + 3*a**3*sin(c + d*x)**2*cos(c + d*x)**4/(2*d)
 + a**3*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a*sin(c) + a)**3*cos
(c)**5, True))
```

Giac [B] time = 1.20354, size = 181, normalized size = 2.7

$$\frac{a^3\cos(8\,dx+8\,c)}{1024\,d}-\frac{5\,a^3\cos(6\,dx+6\,c)}{384\,d}-\frac{25\,a^3\cos(4\,dx+4\,c)}{256\,d}-\frac{33\,a^3\cos(2\,dx+2\,c)}{128\,d}-\frac{3\,a^3\sin(7\,dx+7\,c)}{448\,d}-\frac{a^3\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1024*a^3*cos(8*d*x + 8*c)/d - 5/384*a^3*cos(6*d*x + 6*c)/d - 25/256*a^3*c
os(4*d*x + 4*c)/d - 33/128*a^3*cos(2*d*x + 2*c)/d - 3/448*a^3*sin(7*d*x + 7
*c)/d - 1/64*a^3*sin(5*d*x + 5*c)/d + 17/192*a^3*sin(3*d*x + 3*c)/d + 55/64
*a^3*sin(d*x + c)/d
```

3.29 $\int \cos^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=130

$$-\frac{3a^3\cos^5(c+dx)}{10d}-\frac{3\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{14d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{9a^3\sin(c+dx)\cos(c+dx)}{16d}+$$

```
[Out] (9*a^3*x)/16 - (3*a^3*Cos[c + d*x]^5)/(10*d) + (9*a^3*Cos[c + d*x]*Sin[c +
d*x])/(16*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (a*Cos[c + d*x]^
5*(a + a*Sin[c + d*x])^2)/(7*d) - (3*Cos[c + d*x]^5*(a^3 + a^3*Sin[c + d*x]
))/(14*d)
```

Rubi [A] time = 0.140113, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2678, 2669, 2635, 8}

$$-\frac{3a^3\cos^5(c+dx)}{10d}-\frac{3\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{14d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{9a^3\sin(c+dx)\cos(c+dx)}{16d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (9*a^3*x)/16 - (3*a^3*Cos[c + d*x]^5)/(10*d) + (9*a^3*Cos[c + d*x]*Sin[c +
d*x])/(16*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (a*Cos[c + d*x]^
5*(a + a*Sin[c + d*x])^2)/(7*d) - (3*Cos[c + d*x]^5*(a^3 + a^3*Sin[c + d*x]
))/(14*d)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(c+dx)(a+a\sin(c+dx))^3\,dx &= -\frac{a\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{1}{7}(9a)\int \cos^4(c+dx)(a+a\sin(c+dx))^{\cdots} \\
&= -\frac{a\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}-\frac{3\cos^5(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{14d}+\frac{1}{2}\cdots \\
&= -\frac{3a^3\cos^5(c+dx)}{10d}-\frac{a\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}-\frac{3\cos^5(c+dx)\left(a^3+\cdots\right.}{14d} \\
&= -\frac{3a^3\cos^5(c+dx)}{10d}+\frac{3a^3\cos^3(c+dx)\sin(c+dx)}{8d}-\frac{a\cos^5(c+dx)(a+a\sin(c\cdots}{7d} \\
&= -\frac{3a^3\cos^5(c+dx)}{10d}+\frac{9a^3\cos(c+dx)\sin(c+dx)}{16d}+\frac{3a^3\cos^3(c+dx)\sin(c+dx\cdots}{8d} \\
&= \frac{9a^3x}{16}-\frac{3a^3\cos^5(c+dx)}{10d}+\frac{9a^3\cos(c+dx)\sin(c+dx)}{16d}+\frac{3a^3\cos^3(c+dx)\sin(\cdots}{8d}
\end{aligned}
$$

Mathematica [A] time = 0.793144, size = 161, normalized size = 1.24

$$
-\frac{a^3\left(\sqrt{\sin(c+dx)+1}\left(80\sin^7(c+dx)+200\sin^6(c+dx)-72\sin^5(c+dx)-558\sin^4(c+dx)-306\sin^3(c+dx)+411\cdots\right.\right.}{560d(\sin(c+dx)-1)^3(\sin(c+\cdots}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Cos[c + d*x]^5*(-630*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 -
Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-368 + 613*Sin[c + d*x] + 411*Sin[c
```

```
 + d*x]^2 - 306*Sin[c + d*x]^3 - 558*Sin[c + d*x]^4 - 72*Sin[c + d*x]^5 + 2
00*Sin[c + d*x]^6 + 80*Sin[c + d*x]^7)))/(560*d*(-1 + Sin[c + d*x])^3*(1 +
Sin[c + d*x])^(5/2))
```

Maple [A] time = 0.042, size = 143, normalized size = 1.1

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^5}{7}-\frac{2\,(\cos(dx+c))^5}{35}\right)+3\,a^3\left(-1/6\,\sin(dx+c)(\cos(dx+c))^5+1/24\,\left((\cos(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*a^3*(-1/6*sin
(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x
+1/16*c)-3/5*a^3*cos(d*x+c)^5+a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*
x+c)+3/8*d*x+3/8*c))
```

Maple [A] time = 0.966801, size = 155, normalized size = 1.19

$$-\frac{1344\,a^3\cos(dx+c)^5-64\left(5\,\cos(dx+c)^7-7\,\cos(dx+c)^5\right)a^3-35\left(4\,\sin(2\,dx+2\,c)^3+12\,dx+12\,c-3\,\sin(4\,dx+4\right.}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2240*(1344*a^3*cos(d*x + c)^5 - 64*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)
*a^3 - 35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^3 -
 70*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

Fricas [A] time = 1.76231, size = 212, normalized size = 1.63

$$\frac{80\,a^3\cos(dx+c)^7-448\,a^3\cos(dx+c)^5+315\,a^3dx-35\left(8\,a^3\cos(dx+c)^5-6\,a^3\cos(dx+c)^3-9\,a^3\cos(dx+c)\right)\sin(}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(80*a^3*cos(d*x + c)^7 - 448*a^3*cos(d*x + c)^5 + 315*a^3*d*x - 35*(8
*a^3*cos(d*x + c)^5 - 6*a^3*cos(d*x + c)^3 - 9*a^3*cos(d*x + c))*sin(d*x +
c))/d
```

Sympy [A] time = 8.62774, size = 335, normalized size = 2.58

$$\begin{cases} \frac{3a^3x\sin^6(c+dx)}{16} + \frac{9a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3a^3x\sin^4(c+dx)}{8} + \frac{9a^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^3x\cos^6(c+dx)}{16} \\ x\left(a\sin(c)+a\right)^3\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**6/16 + 9*a**3*x*sin(c + d*x)**4*cos(c + d
*x)**2/16 + 3*a**3*x*sin(c + d*x)**4/8 + 9*a**3*x*sin(c + d*x)**2*cos(c + d
*x)**4/16 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**3*x*cos(c + d
*x)**6/16 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**3*sin(c + d*x)**5*cos(c + d*x
)/(16*d) + a**3*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) + 3*a**3*sin(c + d*x)
**3*cos(c + d*x)/(8*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**
3*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 5*a**3*sin(c + d*x)*cos(c + d*x)**3
/(8*d) - 2*a**3*cos(c + d*x)**7/(35*d) - 3*a**3*cos(c + d*x)**5/(5*d), Ne(d
, 0)), (x*(a*sin(c) + a)**3*cos(c)**4, True))
```

Giac [A] time = 1.16627, size = 166, normalized size = 1.28

$$\frac{9}{16}a^3x + \frac{a^3\cos(7\,dx+7\,c)}{448\,d} - \frac{11\,a^3\cos(5\,dx+5\,c)}{320\,d} - \frac{13\,a^3\cos(3\,dx+3\,c)}{64\,d} - \frac{27\,a^3\cos(dx+c)}{64\,d} - \frac{a^3\sin(6\,dx+6\,c)}{64\,d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 9/16*a^3*x + 1/448*a^3*cos(7*d*x + 7*c)/d - 11/320*a^3*cos(5*d*x + 5*c)/d -
 13/64*a^3*cos(3*d*x + 3*c)/d - 27/64*a^3*cos(d*x + c)/d - 1/64*a^3*sin(6*d
*x + 6*c)/d - 1/64*a^3*sin(4*d*x + 4*c)/d + 19/64*a^3*sin(2*d*x + 2*c)/d
```

3.30 $\int \cos^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=45

$$\frac{2(a\sin(c+dx)+a)^5}{5a^2d}-\frac{(a\sin(c+dx)+a)^6}{6a^3d}$$

```
[Out] (2*(a + a*Sin[c + d*x])^5)/(5*a^2*d) - (a + a*Sin[c + d*x])^6/(6*a^3*d)
```

Rubi [A] time = 0.0471518, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^5}{5a^2d}-\frac{(a\sin(c+dx)+a)^6}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^5)/(5*a^2*d) - (a + a*Sin[c + d*x])^6/(6*a^3*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(a+x)^4-(a+x)^5\right)\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{2(a+a\sin(c+dx))^5}{5a^2 d} - \frac{(a+a\sin(c+dx))^6}{6a^3 d}$$

Mathematica [A] time = 0.1451, size = 43, normalized size = 0.96

$$-\frac{a^3(5\sin(c+dx)-7)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^{10}}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^10*(-7 + 5*Sin[c + d*x]))/(30*d
)
```

Maple [B] time = 0.042, size = 113, normalized size = 2.5

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^4}{6}-\frac{(\cos(dx+c))^4}{12}\right)+3\,a^3\left(-1/5\,\sin(dx+c)(\cos(dx+c))^4+1/15\,\left(2+(\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a^3*(-1/5*sin
(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/4*a^3*cos(d*x+c)^4
+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 0.947696, size = 111, normalized size = 2.47

$$-\frac{5\,a^3\sin(dx+c)^6+18\,a^3\sin(dx+c)^5+15\,a^3\sin(dx+c)^4-20\,a^3\sin(dx+c)^3-45\,a^3\sin(dx+c)^2-30\,a^3\sin(dx+}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/30*(5*a^3*sin(d*x + c)^6 + 18*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4
 - 20*a^3*sin(d*x + c)^3 - 45*a^3*sin(d*x + c)^2 - 30*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.72699, size = 171, normalized size = 3.8

$$\frac{5\,a^3\cos\left(dx+c\right)^6-30\,a^3\cos\left(dx+c\right)^4-2\left(9\,a^3\cos\left(dx+c\right)^4-8\,a^3\cos\left(dx+c\right)^2-16\,a^3\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(5*a^3*cos(d*x + c)^6 - 30*a^3*cos(d*x + c)^4 - 2*(9*a^3*cos(d*x + c)^
4 - 8*a^3*cos(d*x + c)^2 - 16*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 4.83883, size = 172, normalized size = 3.82

$$\begin{cases}\frac{a^3\sin^6\left(c+dx\right)}{12d}+\frac{2a^3\sin^5\left(c+dx\right)}{5d}+\frac{a^3\sin^4\left(c+dx\right)\cos^2\left(c+dx\right)}{4d}+\frac{3a^3\sin^4\left(c+dx\right)}{4d}+\frac{a^3\sin^3\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{2a^3\sin^3\left(c+dx\right)}{3d}+\frac{3a^3\sin^2\left(c+dx\right)\cos^2}{2d} \\ x\left(a\sin\left(c\right)+a\right)^3\cos^3\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**6/(12*d) + 2*a**3*sin(c + d*x)**5/(5*d) + a**
3*sin(c + d*x)**4*cos(c + d*x)**2/(4*d) + 3*a**3*sin(c + d*x)**4/(4*d) + a*
*3*sin(c + d*x)**3*cos(c + d*x)**2/d + 2*a**3*sin(c + d*x)**3/(3*d) + 3*a**
3*sin(c + d*x)**2*cos(c + d*x)**2/(2*d) + a**3*sin(c + d*x)*cos(c + d*x)**2
/d, Ne(d, 0)), (x*(a*sin(c) + a)**3*cos(c)**3, True))
```

Giac [B] time = 1.1651, size = 111, normalized size = 2.47

$$-\frac{5\,a^3\sin\left(dx+c\right)^6+18\,a^3\sin\left(dx+c\right)^5+15\,a^3\sin\left(dx+c\right)^4-20\,a^3\sin\left(dx+c\right)^3-45\,a^3\sin\left(dx+c\right)^2-30\,a^3\sin\left(dx+c\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/30*(5*a^3*sin(d*x + c)^6 + 18*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4
 - 20*a^3*sin(d*x + c)^3 - 45*a^3*sin(d*x + c)^2 - 30*a^3*sin(d*x + c))/d
```

3.31 $\int \cos^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=106

$$-\frac{7a^3\cos^3(c+dx)}{12d}-\frac{7\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{20d}+\frac{7a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^3x}{8}-\frac{a\cos^3(c+dx)(a\sin(c+}{5d}$$

```
[Out] (7*a^3*x)/8 - (7*a^3*Cos[c + d*x]^3)/(12*d) + (7*a^3*Cos[c + d*x]*Sin[c + d
*x])/(8*d) - (a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(5*d) - (7*Cos[c + d
*x]^3*(a^3 + a^3*Sin[c + d*x]))/(20*d)
```

Rubi [A] time = 0.12388, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2678, 2669, 2635, 8}

$$-\frac{7a^3\cos^3(c+dx)}{12d}-\frac{7\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{20d}+\frac{7a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^3x}{8}-\frac{a\cos^3(c+dx)(a\sin(c+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (7*a^3*x)/8 - (7*a^3*Cos[c + d*x]^3)/(12*d) + (7*a^3*Cos[c + d*x]*Sin[c + d
*x])/(8*d) - (a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(5*d) - (7*Cos[c + d
*x]^3*(a^3 + a^3*Sin[c + d*x]))/(20*d)
```


Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sin(c+dx))^3\,dx &= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}+\frac{1}{5}(7a)\int \cos^2(c+dx)(a+a\sin(c+dx))^2\\&= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}-\frac{7\cos^3(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{20d}+\frac{1}{4}\\&= -\frac{7a^3\cos^3(c+dx)}{12d}-\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}-\frac{7\cos^3(c+dx)\left(a^3+\right.}{20d}\\&= -\frac{7a^3\cos^3(c+dx)}{12d}+\frac{7a^3\cos(c+dx)\sin(c+dx)}{8d}-\frac{a\cos^3(c+dx)(a+a\sin(c+}{5d}\\&= \frac{7a^3x}{8}-\frac{7a^3\cos^3(c+dx)}{12d}+\frac{7a^3\cos(c+dx)\sin(c+dx)}{8d}-\frac{a\cos^3(c+dx)(a+a}{5d}\end{aligned}$$

Mathematica [A] time = 0.42379, size = 141, normalized size = 1.33

$$-\frac{a^3\left(210\sqrt{1-\sin(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\left(24\sin^5(c+dx)+66\sin^4(c+dx)+22\sin^3(c+dx)-\right.\right.}{120d(\sin(c+dx)-1)^2(\sin(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Cos[c + d*x]^3*(210*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - S
in[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(136 - 151*Sin[c + d*x] - 97*Sin[c +
d*x]^2 + 22*Sin[c + d*x]^3 + 66*Sin[c + d*x]^4 + 24*Sin[c + d*x]^5)))/(120*
d*(-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^(3/2))
```

Maple [A] time = 0.039, size = 121, normalized size = 1.1

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^3}{5}-\frac{2\,(\cos(dx+c))^3}{15}\right)+3\,a^3\left(-1/4\,(\cos(dx+c))^3\sin(dx+c)+1/8\,\cos(dx+c)\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3*a^3*(-1/4*cos
(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-a^3*cos(d*x+c
)^3+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.962166, size = 123, normalized size = 1.16

$$-\frac{480\,a^3\cos(dx+c)^3-32\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)a^3-45\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^3-120\,(2\,dx+2\,c+s}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/480*(480*a^3*cos(d*x + c)^3 - 32*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a
^3 - 45*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^3 - 120*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^3)/d
```

Fricas [A] time = 1.76386, size = 181, normalized size = 1.71

$$\frac{24\,a^3\cos(dx+c)^5-160\,a^3\cos(dx+c)^3+105\,a^3dx-15\left(6\,a^3\cos(dx+c)^3-7\,a^3\cos(dx+c)\right)\sin(dx+c)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(24*a^3*cos(d*x + c)^5 - 160*a^3*cos(d*x + c)^3 + 105*a^3*d*x - 15*(6
*a^3*cos(d*x + c)^3 - 7*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 2.6959, size = 226, normalized size = 2.13

$$\begin{cases} \frac{3a^3x\sin^4{(c+dx)}}{8} + \frac{3a^3x\sin^2{(c+dx)}\cos^2{(c+dx)}}{4} + \frac{a^3x\sin^2{(c+dx)}}{2} + \frac{3a^3x\cos^4{(c+dx)}}{8} + \frac{a^3x\cos^2{(c+dx)}}{2} + \frac{3a^3\sin^3{(c+dx)}\cos{(c+dx)}}{8d} - \frac{a^3\sin^2{(c+\ldots} \\ x(a\sin{(c)}+a)^3\cos^2{(c)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**4/8 + a**3*x*co
s(c + d*x)**2/2 + 3*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a**3*sin(c +
d*x)**2*cos(c + d*x)**3/(3*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) +
 a**3*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*a**3*cos(c + d*x)**5/(15*d) - a**
3*cos(c + d*x)**3/d, Ne(d, 0)), (x*(a*sin(c) + a)**3*cos(c)**2, True))
```

Giac [A] time = 1.14277, size = 120, normalized size = 1.13

$$\frac{7}{8}a^3x + \frac{a^3\cos(5\,dx+5\,c)}{80\,d} - \frac{13\,a^3\cos(3\,dx+3\,c)}{48\,d} - \frac{7\,a^3\cos(dx+c)}{8\,d} - \frac{3\,a^3\sin(4\,dx+4\,c)}{32\,d} + \frac{a^3\sin(2\,dx+2\,c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 7/8*a^3*x + 1/80*a^3*cos(5*d*x + 5*c)/d - 13/48*a^3*cos(3*d*x + 3*c)/d - 7/
8*a^3*cos(d*x + c)/d - 3/32*a^3*sin(4*d*x + 4*c)/d + 1/4*a^3*sin(2*d*x + 2*
c)/d
```

3.32 $\int \cos(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=22

$$\frac{(a\sin(c+dx)+a)^4}{4ad}$$

```
[Out] (a + a*Sin[c + d*x])^4/(4*a*d)
```

Rubi [A] time = 0.0253023, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 32}

$$\frac{(a\sin(c+dx)+a)^4}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a + a*Sin[c + d*x])^4/(4*a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{(a+a\sin(c+dx))^4}{4ad}\end{aligned}$$

Mathematica [B] time = 0.027618, size = 65, normalized size = 2.95

$$\frac{a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/
d + (a^3*Sin[c + d*x]^4)/(4*d)
```

Maple [A] time = 0.015, size = 21, normalized size = 1.

$$\frac{(a+a\sin(dx+c))^4}{4\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4*(a+a*sin(d*x+c))^4/d/a
```

Maxima [A] time = 0.94555, size = 27, normalized size = 1.23

$$\frac{(a\sin(dx+c)+a)^4}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a*sin(d*x + c) + a)^4/(a*d)
```

Fricas [B] time = 1.68935, size = 131, normalized size = 5.95

$$\frac{a^3\cos(dx+c)^4-8\,a^3\cos(dx+c)^2-4\left(a^3\cos(dx+c)^2-2\,a^3\right)\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a^3*cos(d*x + c)^4 - 8*a^3*cos(d*x + c)^2 - 4*(a^3*cos(d*x + c)^2 - 2*
a^3)*sin(d*x + c))/d
```

Sympy [A] time = 1.3255, size = 94, normalized size = 4.27

$$\begin{cases} \frac{a^3 \sin^3(c+dx)}{d} - \frac{a^3 \sin^2(c+dx)\cos^2(c+dx)}{2d} + \frac{3a^3\sin^2(c+dx)}{2d} + \frac{a^3\sin(c+dx)}{d} - \frac{a^3\cos^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x\left(a\sin(c)+a\right)^3\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**3/d - a**3*sin(c + d*x)**2*cos(c + d*x)**2/(2
*d) + 3*a**3*sin(c + d*x)**2/(2*d) + a**3*sin(c + d*x)/d - a**3*cos(c + d*x
)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*cos(c), True))
```

Giac [A] time = 1.15015, size = 27, normalized size = 1.23

$$\frac{\left(a\sin(dx+c)+a\right)^4}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*sin(d*x + c) + a)^4/(a*d)
```

3.33 $\int \sec(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=52

$$-\frac{a^3\sin^2(c+dx)}{2d}-\frac{3a^3\sin(c+dx)}{d}-\frac{4a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] (-4*a^3*Log[1 - Sin[c + d*x]])/d - (3*a^3*Sin[c + d*x])/d - (a^3*Sin[c + d*
x]^2)/(2*d)
```

Rubi [A] time = 0.0469152, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 43}

$$-\frac{a^3\sin^2(c+dx)}{2d}-\frac{3a^3\sin(c+dx)}{d}-\frac{4a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*a^3*Log[1 - Sin[c + d*x]])/d - (3*a^3*Sin[c + d*x])/d - (a^3*Sin[c + d*
x]^2)/(2*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{a\,\mathrm{Subst}\left(\int \frac{(a+x)^2}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\mathrm{Subst}\left(\int \left(-3a+\frac{4a^2}{a-x}-x\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{4a^3\log(1-\sin(c+dx))}{d} - \frac{3a^3\sin(c+dx)}{d} - \frac{a^3\sin^2(c+dx)}{2d}$$

Mathematica [A] time = 0.0290341, size = 41, normalized size = 0.79

$$\frac{a^3\left(-\frac{1}{2}\sin^2(c+dx)-3\sin(c+dx)-4\log(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-4*Log[1 - Sin[c + d*x]] - 3*Sin[c + d*x] - Sin[c + d*x]^2/2))/d
```

Maple [A] time = 0.047, size = 69, normalized size = 1.3

$$-\frac{a^3(\sin(dx+c))^2}{2d} - 4\frac{a^3\ln(\cos(dx+c))}{d} - 3\frac{a^3\sin(dx+c)}{d} + 4\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2*a^3*sin(d*x+c)^2/d-4/d*a^3*ln(cos(d*x+c))-3*a^3*sin(d*x+c)/d+4/d*a^3*l
n(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.96589, size = 58, normalized size = 1.12

$$-\frac{a^3\sin(dx+c)^2 + 8a^3\log(\sin(dx+c)-1) + 6a^3\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^3*sin(d*x + c)^2 + 8*a^3*log(sin(d*x + c) - 1) + 6*a^3*sin(d*x + c)
)/d
```

Fricas [A] time = 1.65545, size = 108, normalized size = 2.08

$$\frac{a^3 \cos(dx+c)^2 - 8\,a^3 \log(-\sin(dx+c)+1) - 6\,a^3 \sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*cos(d*x + c)^2 - 8*a^3*log(-sin(d*x + c) + 1) - 6*a^3*sin(d*x + c)
)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3 \left(\int 3\sin(c+dx)\sec(c+dx)\,dx + \int 3\sin^2(c+dx)\sec(c+dx)\,dx + \int \sin^3(c+dx)\sec(c+dx)\,dx + \int \sec(c+dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*sec(c + d*x), x) + Integral(3*sin(c + d*x)**2
*sec(c + d*x), x) + Integral(sin(c + d*x)**3*sec(c + d*x), x) + Integral(se
c(c + d*x), x))
```

Giac [B] time = 1.16096, size = 173, normalized size = 3.33

$$\frac{2\left(2\,a^3 \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right) - 4\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{3\,a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 3\,a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 7\,a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(2*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 4*a^3*log(abs(tan(1/2*d*x + 1/2*
c) - 1)) - (3*a^3*tan(1/2*d*x + 1/2*c)^4 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 + 7
*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 3*a^3)/(tan(1/2*
d*x + 1/2*c)^2 + 1)^2)/d
```

3.34 $\int \sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=50

$$\frac{3a^3\cos(c+dx)}{d}+\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}-3a^3x$$

```
[Out] -3*a^3*x + (3*a^3*Cos[c + d*x])/d + (2*a^5*Cos[c + d*x]^3)/(d*(a - a*Sin[c
+ d*x])^2)
```

Rubi [A] time = 0.136254, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2670, 2680, 2682, 8}

$$\frac{3a^3\cos(c+dx)}{d}+\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}-3a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -3*a^3*x + (3*a^3*Cos[c + d*x])/d + (2*a^5*Cos[c + d*x]^3)/(d*(a - a*Sin[c
+ d*x])^2)
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \sec^2(c+dx)(a+a\sin(c+dx))^3\,dx &= a^6\int \frac{\cos^4(c+dx)}{(a-a\sin(c+dx))^3}\,dx\\
&= \frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}-\left(3a^4\right)\int \frac{\cos^2(c+dx)}{a-a\sin(c+dx)}\,dx\\
&= \frac{3a^3\cos(c+dx)}{d}+\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}-\left(3a^3\right)\int 1\,dx\\
&= -3a^3x+\frac{3a^3\cos(c+dx)}{d}+\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}
\end{aligned}
$$

Mathematica [C] time = 0.0333694, size = 55, normalized size = 1.1

$$
\frac{4\sqrt{2}a^3\sqrt{\sin(c+dx)+1}\sec(c+dx)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*Sqrt[2]*a^3*Hypergeometric2F1[-3/2, -1/2, 1/2, (1 - Sin[c + d*x])/2]*Sec
[c + d*x]*Sqrt[1 + Sin[c + d*x]])/d
```

Maple [A] time = 0.05, size = 87, normalized size = 1.7

$$
\frac{1}{d}\left(a^3\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)+3a^3(\tan(dx+c)-dx-c)+3\frac{a^3}{\cos(dx+c)}+a^3\tan(dx+c
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(tan(d
*x+c)-d*x-c)+3*a^3/cos(d*x+c)+a^3*tan(d*x+c))
```

Maxima [A] time = 1.44948, size = 92, normalized size = 1.84

$$-\frac{3\,(dx+c-\tan{(dx+c)})a^3 - a^3\left(\frac{1}{\cos(dx+c)}+\cos{(dx+c)}\right) - a^3\tan{(dx+c)} - \frac{3\,a^3}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(3*(d*x + c - tan(d*x + c))*a^3 - a^3*(1/cos(d*x + c) + cos(d*x + c)) - a^
3*tan(d*x + c) - 3*a^3/cos(d*x + c))/d
```

Fricas [A] time = 1.62659, size = 230, normalized size = 4.6

$$-\frac{3\,a^3dx - a^3\cos{(dx+c)}^2 - 4\,a^3 + \left(3\,a^3dx - 5\,a^3\right)\cos{(dx+c)} - \left(3\,a^3dx - a^3\cos{(dx+c)} + 4\,a^3\right)\sin{(dx+c)}}{d\cos{(dx+c)} - d\sin{(dx+c)} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*a^3*d*x - a^3*cos(d*x + c)^2 - 4*a^3 + (3*a^3*d*x - 5*a^3)*cos(d*x + c)
 - (3*a^3*d*x - a^3*cos(d*x + c) + 4*a^3)*sin(d*x + c))/(d*cos(d*x + c) - d
*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16698, size = 123, normalized size = 2.46

$$-\frac{3\,(dx+c)a^3+\frac{2\left(4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(d*x + c)*a^3 + 2*(4*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2
*c) + 5*a^3)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x
 + 1/2*c) - 1))/d
```

3.35 $\int \sec^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=40

$$\frac{2a^4}{d(a-a\sin(c+dx))}+\frac{a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] (a^3*Log[1 - Sin[c + d*x]])/d + (2*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0522718, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{2a^4}{d(a-a\sin(c+dx))}+\frac{a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[1 - Sin[c + d*x]])/d + (2*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sec^3(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{a^3\,\text{Subst}\left(\int \frac{a+x}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{a^3\,\text{Subst}\left(\int \left(\frac{2a}{(a-x)^2}+\frac{1}{-a+x}\right)dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{a^3\log(1-\sin(c+dx))}{d} + \frac{2a^4}{d(a-a\sin(c+dx))}
\end{aligned}
$$

Mathematica [A] time = 0.0410921, size = 59, normalized size = 1.48

$$\frac{a^3(1-\sin(c+dx))(\sin(c+dx)+1)\sec^2(c+dx)\left(\frac{2}{1-\sin(c+dx)}+\log(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(Log[1 - Sin[c + d*x]] + 2/(1 - Sin[c + d*x]))*(1 - Sin
[c + d*x])*(1 + Sin[c + d*x]))/d
```

Maple [B] time = 0.069, size = 128, normalized size = 3.2

$$\frac{a^3(\tan(dx+c))^2}{2d} + \frac{a^3\ln(\cos(dx+c))}{d} + \frac{3a^3(\sin(dx+c))^3}{2d(\cos(dx+c))^2} + \frac{3a^3\sin(dx+c)}{2d} - \frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d*a^3*tan(d*x+c)^2+1/d*a^3*ln(cos(d*x+c))+3/2/d*a^3*sin(d*x+c)^3/cos(d*
x+c)^2+3/2*a^3*sin(d*x+c)/d-1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*a^3/cos
(d*x+c)^2+1/2/d*a^3*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.95084, size = 45, normalized size = 1.12

$$\frac{a^3\log(\sin(dx+c)-1) - \frac{2a^3}{\sin(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (a^3*log(sin(d*x + c) - 1) - 2*a^3/(sin(d*x + c) - 1))/d
```

Fricas [A] time = 1.6929, size = 109, normalized size = 2.72

$$-\frac{2\,a^{3}-\left(a^{3}\sin\left(dx+c\right)-a^{3}\right)\log\left(-\sin\left(dx+c\right)+1\right)}{d\sin\left(dx+c\right)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(2*a^3 - (a^3*sin(d*x + c) - a^3)*log(-sin(d*x + c) + 1))/(d*sin(d*x + c)
- d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20148, size = 124, normalized size = 3.1

$$-\frac{a^{3}\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)-2\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{3\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^{3}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*a^3*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (3*a^3*tan(1/2*d*x + 1/2*c)^2 - 10*a^3*tan(1/2*d*x + 1/2*c) + 3*a^3
)/(tan(1/2*d*x + 1/2*c) - 1)^2)/d
```

3.36 $\int \sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=31

$$\frac{a^6\cos^3(c+dx)}{3d(a-a\sin(c+dx))^3}$$

```
[Out] (a^6*Cos[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^3)
```

Rubi [A] time = 0.0847633, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2670, 2671}

$$\frac{a^6\cos^3(c+dx)}{3d(a-a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^6*Cos[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^3)
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^3\,dx = a^6\int \frac{\cos^2(c+dx)}{(a-a\sin(c+dx))^3}\,dx$$
$$= \frac{a^6\cos^3(c+dx)}{3d(a-a\sin(c+dx))^3}$$

Mathematica [A] time = 0.0248598, size = 28, normalized size = 0.9

$$\frac{a^3(\sin(c+dx)+1)^3\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c + d*x]^3*(1 + Sin[c + d*x])^3)/(3*d)
```

Maple [B] time = 0.07, size = 120, normalized size = 3.9

$$\frac{1}{d}\left(a^3\left(\frac{(\sin(dx+c))^4}{3\,(\cos(dx+c))^3}-\frac{(\sin(dx+c))^4}{3\,\cos(dx+c)}-\frac{\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}\right)+\frac{a^3(\sin(dx+c))^3}{(\cos(dx+c))^3}+\frac{a^3}{(\cos(dx+c))^3}-a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+
sin(d*x+c)^2)*cos(d*x+c))+a^3*sin(d*x+c)^3/cos(d*x+c)^3+a^3/cos(d*x+c)^3-a^
3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [B] time = 0.967453, size = 105, normalized size = 3.39

$$\frac{3\,a^3\tan(dx+c)^3+\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^3-\frac{\left(3\,\cos(dx+c)^2-1\right)a^3}{\cos(dx+c)^3}+\frac{3\,a^3}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - (3*cos(
d*x + c)^2 - 1)*a^3/cos(d*x + c)^3 + 3*a^3/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.60187, size = 236, normalized size = 7.61

$$\frac{a^3\cos(dx+c)^2 - a^3\cos(dx+c) - 2a^3 - \left(a^3\cos(dx+c) + 2a^3\right)\sin(dx+c)}{3\left(d\cos(dx+c)^2 - d\cos(dx+c) + (d\cos(dx+c) + 2d)\sin(dx+c) - 2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3 - (a^3*cos(d*x + c) + 2*
a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2
*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16137, size = 51, normalized size = 1.65

$$-\frac{2\left(3a^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a^3\right)}{3d\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^3*tan(1/2*d*x + 1/2*c)^2 + a^3)/(d*(tan(1/2*d*x + 1/2*c) - 1)^3)
```

3.37 $\int \sec^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=23

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}$$

```
[Out] a^5/(2*d*(a - a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.0399265, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^5/(2*d*(a - a*Sin[c + d*x])^2)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{1}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5}{2d(a-a\sin(c+dx))^2}$$

Mathematica [A] time = 0.233885, size = 35, normalized size = 1.52

$$\frac{a^3}{2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3/(2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4)
```

Maple [B] time = 0.074, size = 146, normalized size = 6.4

$$\frac{a^3(\sin(dx+c))^4}{4d(\cos(dx+c))^4}+\frac{3a^3(\sin(dx+c))^3}{4d(\cos(dx+c))^4}+\frac{3a^3(\sin(dx+c))^3}{8d(\cos(dx+c))^2}+\frac{3a^3\sin(dx+c)}{8d}+\frac{3a^3}{4d(\cos(dx+c))^4}+\frac{a^3\tan(dx+c)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^4/cos(d*x+c)^4+3/4/d*a^3*sin(d*x+c)^3/cos(d*x+c)^4+3/8
/d*a^3*sin(d*x+c)^3/cos(d*x+c)^2+3/8*a^3*sin(d*x+c)/d+3/4/d*a^3/cos(d*x+c)^
4+1/4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^3*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.969195, size = 38, normalized size = 1.65

$$\frac{a^3}{2\left(\sin(dx+c)^2-2\sin(dx+c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3/((sin(d*x + c)^2 - 2*sin(d*x + c) + 1)*d)
```

Fricas [A] time = 1.61176, size = 73, normalized size = 3.17

$$-\frac{a^{3}}{2\left(d\cos\left(dx+c\right)^{2}+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*a^3/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19545, size = 85, normalized size = 3.7

$$\frac{2\left(a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*
x + 1/2*c))/(d*(tan(1/2*d*x + 1/2*c) - 1)^4)
```

3.38 $\int \sec^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=92

$$\frac{2a^6\cos(c+dx)}{15d\left(a^3-a^3\sin(c+dx)\right)}+\frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3}+\frac{2a^5\cos(c+dx)}{15d(a-a\sin(c+dx))^2}$$

```
[Out] (a^6*Cos[c + d*x])/(5*d*(a - a*Sin[c + d*x])^3) + (2*a^5*Cos[c + d*x])/(15*
d*(a - a*Sin[c + d*x])^2) + (2*a^6*Cos[c + d*x])/(15*d*(a^3 - a^3*Sin[c + d
*x]))
```

Rubi [A] time = 0.0934226, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2670, 2650, 2648}

$$\frac{2a^6\cos(c+dx)}{15d\left(a^3-a^3\sin(c+dx)\right)}+\frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3}+\frac{2a^5\cos(c+dx)}{15d(a-a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (a^6*Cos[c + d*x])/(5*d*(a - a*Sin[c + d*x])^3) + (2*a^5*Cos[c + d*x])/(15*
d*(a - a*Sin[c + d*x])^2) + (2*a^6*Cos[c + d*x])/(15*d*(a^3 - a^3*Sin[c + d
*x]))
```


Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sec^6(c+dx)(a+a\sin(c+dx))^3\,dx &= a^6\int \frac{1}{(a-a\sin(c+dx))^3}\,dx \\
&= \frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3}+\frac{1}{5}\left(2a^5\right)\int \frac{1}{(a-a\sin(c+dx))^2}\,dx \\
&= \frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3}+\frac{2a^5\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{1}{15}\left(2a^4\right)\int \frac{1}{a-a\sin(c+dx}\ldots \\
&= \frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3}+\frac{2a^5\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{2a^4\cos(c+dx)}{15d(a-a\sin(c+dx))}
\end{aligned}
$$

Mathematica [A] time = 0.0196873, size = 110, normalized size = 1.2

$$
\frac{2a^3\tan^5(c+dx)}{15d}+\frac{7a^3\sec^5(c+dx)}{15d}+\frac{a^3\tan^2(c+dx)\sec^3(c+dx)}{3d}-\frac{a^3\tan^3(c+dx)\sec^2(c+dx)}{3d}+\frac{a^3\tan(c+dx)\,\mathrm{se}\ldots}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*Sec[c + d*x]^5)/(15*d) + (a^3*Sec[c + d*x]^4*Tan[c + d*x])/d + (a^3*
Sec[c + d*x]^3*Tan[c + d*x]^2)/(3*d) - (a^3*Sec[c + d*x]^2*Tan[c + d*x]^3)/
(3*d) + (2*a^3*Tan[c + d*x]^5)/(15*d)
```

Maple [A] time = 0.075, size = 171, normalized size = 1.9

$$
\frac{1}{d}\left(a^3\left(\frac{(\sin(dx+c))^4}{5\,(\cos(dx+c))^5}+\frac{(\sin(dx+c))^4}{15\,(\cos(dx+c))^3}-\frac{(\sin(dx+c))^4}{15\,\cos(dx+c)}-\frac{\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{15}\right)+3\,a^3\left(1/5\,\frac{(\sin(d\ldots}{(\cos(d\ldots}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/15
*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/5*sin(d
*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+3/5*a^3/cos(d*x+c)^5-a
^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 0.971747, size = 139, normalized size = 1.51

$$\frac{\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a^{3}+3\left(3\,\tan\left(dx+c\right)^{5}+5\,\tan\left(dx+c\right)^{3}\right)a^{3}-\frac{\left(5\,\cos(dx+c)^{2}-3\right)a^{3}}{\cos(dx+c)^{5}}+\cdots}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 3*(3*t
an(d*x + c)^5 + 5*tan(d*x + c)^3)*a^3 - (5*cos(d*x + c)^2 - 3)*a^3/cos(d*x
+ c)^5 + 9*a^3/cos(d*x + c)^5)/d
```

Fricas [A] time = 1.60439, size = 367, normalized size = 3.99

$$\frac{2\,a^{3}\cos\left(dx+c\right)^{3}-4\,a^{3}\cos\left(dx+c\right)^{2}-9\,a^{3}\cos\left(dx+c\right)-3\,a^{3}+\left(2\,a^{3}\cos\left(dx+c\right)^{2}+6\,a^{3}\cos\left(dx+c\right)-3\,a^{3}\right)\sin\left(dx+c\cdots\right.}{15\left(d\cos\left(dx+c\right)^{3}+3\,d\cos\left(dx+c\right)^{2}-2\,d\cos\left(dx+c\right)-\left(d\cos\left(dx+c\right)^{2}-2\,d\cos\left(dx+c\right)-4\,d\right)\sin\left(dx+c\right)-4\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(2*a^3*cos(d*x + c)^3 - 4*a^3*cos(d*x + c)^2 - 9*a^3*cos(d*x + c) - 3*
a^3 + (2*a^3*cos(d*x + c)^2 + 6*a^3*cos(d*x + c) - 3*a^3)*sin(d*x + c))/(d*
cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - 2*d*cos(d*x + c) - (d*cos(d*x + c)^2
- 2*d*cos(d*x + c) - 4*d)*sin(d*x + c) - 4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15781, size = 116, normalized size = 1.26

$$-\frac{2\left(15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-30\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+40\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-20\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^{3}\right)}{15\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^3*tan(1/2*d*x + 1/2*c)^4 - 30*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*a
^3*tan(1/2*d*x + 1/2*c)^2 - 20*a^3*tan(1/2*d*x + 1/2*c) + 7*a^3)/(d*(tan(1/
2*d*x + 1/2*c) - 1)^5)
```

3.39 $\int \sec^7(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=87

$$\frac{a^6}{6d(a-a\sin(c+dx))^3}+\frac{a^5}{8d(a-a\sin(c+dx))^2}+\frac{a^4}{8d(a-a\sin(c+dx))}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{8d}$$

```
[Out] (a^3*ArcTanh[Sin[c + d*x]])/(8*d) + a^6/(6*d*(a - a*Sin[c + d*x])^3) + a^5/
(8*d*(a - a*Sin[c + d*x])^2) + a^4/(8*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0724086, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$\frac{a^6}{6d(a-a\sin(c+dx))^3}+\frac{a^5}{8d(a-a\sin(c+dx))^2}+\frac{a^4}{8d(a-a\sin(c+dx))}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*ArcTanh[Sin[c + d*x]])/(8*d) + a^6/(6*d*(a - a*Sin[c + d*x])^3) + a^5/
(8*d*(a - a*Sin[c + d*x])^2) + a^4/(8*d*(a - a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^7(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{a^7\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^7\,\mathrm{Subst}\left(\int \left(\frac{1}{2a(a-x)^4}+\frac{1}{4a^2(a-x)^3}+\frac{1}{8a^3(a-x)^2}+\frac{1}{8a^3(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^6}{6d(a-a\sin(c+dx))^3}+\frac{a^5}{8d(a-a\sin(c+dx))^2}+\frac{a^4}{8d(a-a\sin(c+dx))}+\frac{a^4\,\mathrm{S}}{}\\ &= \frac{a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^6}{6d(a-a\sin(c+dx))^3}+\frac{a^5}{8d(a-a\sin(c+dx))^2}+\frac{}{8d}\end{aligned}$$

Mathematica [A] time = 0.100502, size = 67, normalized size = 0.77

$$-\frac{a^3(\sin(c+dx)+1)^3\sec^6(c+dx)\left(-3\sin^2(c+dx)+9\sin(c+dx)+3(\sin(c+dx)-1)^3\tanh^{-1}(\sin(c+dx))-10\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Sec[c + d*x]^6*(1 + Sin[c + d*x])^3*(-10 + 3*ArcTanh[Sin[c + d*x]]*(-
1 + Sin[c + d*x])^3 + 9*Sin[c + d*x] - 3*Sin[c + d*x]^2))/(24*d)
```

Maple [B] time = 0.08, size = 238, normalized size = 2.7

$$\frac{a^3(\sin(dx+c))^4}{6d(\cos(dx+c))^6}+\frac{a^3(\sin(dx+c))^4}{12d(\cos(dx+c))^4}+\frac{a^3(\sin(dx+c))^3}{2d(\cos(dx+c))^6}+\frac{3a^3(\sin(dx+c))^3}{8d(\cos(dx+c))^4}+\frac{3a^3(\sin(dx+c))^3}{16d(\cos(dx+c))^2}+\frac{3a^3\sin(d}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/6/d*a^3*sin(d*x+c)^4/cos(d*x+c)^6+1/12/d*a^3*sin(d*x+c)^4/cos(d*x+c)^4+1/
2/d*a^3*sin(d*x+c)^3/cos(d*x+c)^6+3/8/d*a^3*sin(d*x+c)^3/cos(d*x+c)^4+3/16/
d*a^3*sin(d*x+c)^3/cos(d*x+c)^2+3/16*a^3*sin(d*x+c)/d+1/8/d*a^3*ln(sec(d*x+
c)+tan(d*x+c))+1/2/d*a^3/cos(d*x+c)^6+1/6/d*a^3*tan(d*x+c)*sec(d*x+c)^5+5/2
4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+5/16/d*a^3*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 0.957635, size = 130, normalized size = 1.49

$$\frac{3\,a^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a^3\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,a^3\sin(dx+c)^2-9\,a^3\sin(dx+c)+10\,a^3\right)}{\sin(dx+c)^3-3\,\sin(dx+c)^2+3\,\sin(dx+c)-1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^3*log(sin(d*x + c) + 1) - 3*a^3*log(sin(d*x + c) - 1) - 2*(3*a^3*
sin(d*x + c)^2 - 9*a^3*sin(d*x + c) + 10*a^3)/(sin(d*x + c)^3 - 3*sin(d*x +
 c)^2 + 3*sin(d*x + c) - 1))/d
```

Fricas [B] time = 1.70695, size = 446, normalized size = 5.13

$$\frac{6\,a^3\cos\left(dx+c\right)^2+18\,a^3\sin\left(dx+c\right)-26\,a^3+3\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3-\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)\right)\log\left(\sin\right.}{48\left(3\,d\cos\left(dx+c\right)^2-\left(d\cos\left(dx+c\right)^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(6*a^3*cos(d*x + c)^2 + 18*a^3*sin(d*x + c) - 26*a^3 + 3*(3*a^3*cos(d*
x + c)^2 - 4*a^3 - (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c))*log(sin(d*x +
 c) + 1) - 3*(3*a^3*cos(d*x + c)^2 - 4*a^3 - (a^3*cos(d*x + c)^2 - 4*a^3)*s
in(d*x + c))*log(-sin(d*x + c) + 1))/(3*d*cos(d*x + c)^2 - (d*cos(d*x + c)^
2 - 4*d)*sin(d*x + c) - 4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24394, size = 122, normalized size = 1.4

$$\frac{6\,a^3\log\left(|\sin\left(dx+c\right)+1|\right)-6\,a^3\log\left(|\sin\left(dx+c\right)-1|\right)+\frac{11\,a^3\sin(dx+c)^3-45\,a^3\sin(dx+c)^2+69\,a^3\sin(dx+c)-51\,a^3}{(\sin(dx+c)-1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*a^3*log(abs(sin(d*x + c) + 1)) - 6*a^3*log(abs(sin(d*x + c) - 1)) +
 (11*a^3*sin(d*x + c)^3 - 45*a^3*sin(d*x + c)^2 + 69*a^3*sin(d*x + c) - 51*
a^3)/(sin(d*x + c) - 1)^3)/d
```

3.40 $\int \sec^8(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=99

$$\frac{3a^3\tan^5(c+dx)}{35d}+\frac{2a^3\tan^3(c+dx)}{7d}+\frac{3a^3\tan(c+dx)}{7d}+\frac{3a^3\sec^5(c+dx)}{35d}+\frac{2a\sec^7(c+dx)(a\sin(c+dx)+a)^2}{7d}$$

```
[Out] (3*a^3*Sec[c + d*x]^5)/(35*d) + (2*a*Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2)
/(7*d) + (3*a^3*Tan[c + d*x])/(7*d) + (2*a^3*Tan[c + d*x]^3)/(7*d) + (3*a^3
*Tan[c + d*x]^5)/(35*d)
```

Rubi [A] time = 0.083488, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2676, 2669, 3767}

$$\frac{3a^3\tan^5(c+dx)}{35d}+\frac{2a^3\tan^3(c+dx)}{7d}+\frac{3a^3\tan(c+dx)}{7d}+\frac{3a^3\sec^5(c+dx)}{35d}+\frac{2a\sec^7(c+dx)(a\sin(c+dx)+a)^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*a^3*Sec[c + d*x]^5)/(35*d) + (2*a*Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2)
/(7*d) + (3*a^3*Tan[c + d*x])/(7*d) + (2*a^3*Tan[c + d*x]^3)/(7*d) + (3*a^3
*Tan[c + d*x]^5)/(35*d)
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{2a\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{1}{7}\left(3a^2\right)\int \sec^6(c+dx)(a+a\sin(c+dx)$$
$$= \frac{3a^3\sec^5(c+dx)}{35d}+\frac{2a\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{1}{7}\left(3a^3\right)\int \sec^6(c+d$$
$$= \frac{3a^3\sec^5(c+dx)}{35d}+\frac{2a\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d}-\frac{\left(3a^3\right)\text{Subst}\left(\int\left(1+\right.\right.}{}$$
$$= \frac{3a^3\sec^5(c+dx)}{35d}+\frac{2a\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{3a^3\tan(c+dx)}{7d}+\frac{2a}{}$$

Mathematica [A] time = 0.0114133, size = 134, normalized size = 1.35

$$-\frac{8a^3\tan^7(c+dx)}{35d}+\frac{13a^3\sec^7(c+dx)}{35d}+\frac{a^3\tan^2(c+dx)\sec^5(c+dx)}{5d}-\frac{a^3\tan^3(c+dx)\sec^4(c+dx)}{d}+\frac{4a^3\tan^5(c+d}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*a^3*Sec[c + d*x]^7)/(35*d) + (a^3*Sec[c + d*x]^6*Tan[c + d*x])/d + (a^3
*Sec[c + d*x]^5*Tan[c + d*x]^2)/(5*d) - (a^3*Sec[c + d*x]^4*Tan[c + d*x]^3)
/d + (4*a^3*Sec[c + d*x]^2*Tan[c + d*x]^5)/(5*d) - (8*a^3*Tan[c + d*x]^7)/(
35*d)
```

Maple [B] time = 0.082, size = 217, normalized size = 2.2

$$\frac{1}{d}\left(a^3\left(\frac{(\sin(dx+c))^4}{7(\cos(dx+c))^7}+\frac{3(\sin(dx+c))^4}{35(\cos(dx+c))^5}+\frac{(\sin(dx+c))^4}{35(\cos(dx+c))^3}-\frac{(\sin(dx+c))^4}{35\cos(dx+c)}-\frac{\left(2+(\sin(dx+c))^2\right)\cos(dx+c}{35}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35
*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^
2)*cos(d*x+c))+3*a^3*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d
*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+3/7*a^3/cos(d*x+c)^7-a^3*(-16/35-1
/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 0.955943, size = 165, normalized size = 1.67

$$\frac{\left(15\,\tan\left(dx+c\right)^{7}+42\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}\right)a^{3}+\left(5\,\tan\left(dx+c\right)^{7}+21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+3\cdots\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/35*((15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^3 + (5*
tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*a
^3 - (7*cos(d*x + c)^2 - 5)*a^3/cos(d*x + c)^7 + 15*a^3/cos(d*x + c)^7)/d
```

Fricas [A] time = 1.65027, size = 274, normalized size = 2.77

$$\frac{8\,a^{3}\cos\left(dx+c\right)^{4}-36\,a^{3}\cos\left(dx+c\right)^{2}+15\,a^{3}+4\left(6\,a^{3}\cos\left(dx+c\right)^{2}-5\,a^{3}\right)\sin\left(dx+c\right)}{35\left(3\,d\cos\left(dx+c\right)^{3}-4\,d\cos\left(dx+c\right)-\left(d\cos\left(dx+c\right)^{3}-4\,d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/35*(8*a^3*cos(d*x + c)^4 - 36*a^3*cos(d*x + c)^2 + 15*a^3 + 4*(6*a^3*cos(
d*x + c)^2 - 5*a^3)*sin(d*x + c))/(3*d*cos(d*x + c)^3 - 4*d*cos(d*x + c) -
(d*cos(d*x + c)^3 - 4*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15263, size = 186, normalized size = 1.88

$$-\frac{\frac{35\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{525\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1960\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4025\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-4480\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3143\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1176\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35*a^3/(tan(1/2*d*x + 1/2*c) + 1) + (525*a^3*tan(1/2*d*x + 1/2*c)^6
 - 1960*a^3*tan(1/2*d*x + 1/2*c)^5 + 4025*a^3*tan(1/2*d*x + 1/2*c)^4 - 4480
*a^3*tan(1/2*d*x + 1/2*c)^3 + 3143*a^3*tan(1/2*d*x + 1/2*c)^2 - 1176*a^3*ta
n(1/2*d*x + 1/2*c) + 243*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^7)/d
```

3.41 $\int \cos^5(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=67

$$\frac{(a\sin(c+dx)+a)^{13}}{13a^5d}-\frac{(a\sin(c+dx)+a)^{12}}{3a^4d}+\frac{4(a\sin(c+dx)+a)^{11}}{11a^3d}$$

```
[Out] (4*(a + a*Sin[c + d*x])^11)/(11*a^3*d) - (a + a*Sin[c + d*x])^12/(3*a^4*d)
+ (a + a*Sin[c + d*x])^13/(13*a^5*d)
```

Rubi [A] time = 0.084119, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a\sin(c+dx)+a)^{13}}{13a^5d}-\frac{(a\sin(c+dx)+a)^{12}}{3a^4d}+\frac{4(a\sin(c+dx)+a)^{11}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^11)/(11*a^3*d) - (a + a*Sin[c + d*x])^12/(3*a^4*d)
+ (a + a*Sin[c + d*x])^13/(13*a^5*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^8\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{10}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{10}-4a(a+x)^{11}+(a+x)^{12}\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{4(a+a\sin(c+dx))^{11}}{11a^3d} - \frac{(a+a\sin(c+dx))^{12}}{3a^4d} + \frac{(a+a\sin(c+dx))^{13}}{13a^5d}$$

Mathematica [A] time = 0.421885, size = 58, normalized size = 0.87

$$-\frac{a^8(\sin(c+dx)+1)^8\left(33\sin^2(c+dx)-77\sin(c+dx)+46\right)\cos^6(c+dx)}{429d(\sin(c+dx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -(a^8*Cos[c + d*x]^6*(1 + Sin[c + d*x])^8*(46 - 77*Sin[c + d*x] + 33*Sin[c
+ d*x]^2))/(429*d*(-1 + Sin[c + d*x])^3)
```

Maple [B] time = 0.053, size = 513, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/13*sin(d*x+c)^7*cos(d*x+c)^6-7/143*sin(d*x+c)^5*cos(d*x+c)^6-3
5/1287*sin(d*x+c)^3*cos(d*x+c)^6-5/429*sin(d*x+c)*cos(d*x+c)^6+1/429*(8/3+c
os(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+8*a^8*(-1/12*sin(d*x+c)^6*cos(d*x
+c)^6-1/20*sin(d*x+c)^4*cos(d*x+c)^6-1/40*sin(d*x+c)^2*cos(d*x+c)^6-1/120*c
os(d*x+c)^6)+28*a^8*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^3*cos(
d*x+c)^6-5/231*sin(d*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*cos(d*x+
c)^2)*sin(d*x+c))+56*a^8*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2
*cos(d*x+c)^6-1/60*cos(d*x+c)^6)+70*a^8*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/2
1*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x
+c))+56*a^8*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+28*a^8*(-1/7
*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c
```

```
))-4/3*a^8*cos(d*x+c)^6+1/5*a^8*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x
+c))
```

Maxima [B] time = 0.947403, size = 234, normalized size = 3.49

$$\frac{33\,a^8\sin(dx+c)^{13}+286\,a^8\sin(dx+c)^{12}+1014\,a^8\sin(dx+c)^{11}+1716\,a^8\sin(dx+c)^{10}+715\,a^8\sin(dx+c)^9-2574\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/429*(33*a^8*sin(d*x + c)^13 + 286*a^8*sin(d*x + c)^12 + 1014*a^8*sin(d*x
+ c)^11 + 1716*a^8*sin(d*x + c)^10 + 715*a^8*sin(d*x + c)^9 - 2574*a^8*sin(
d*x + c)^8 - 5148*a^8*sin(d*x + c)^7 - 3432*a^8*sin(d*x + c)^6 + 1287*a^8*s
in(d*x + c)^5 + 4290*a^8*sin(d*x + c)^4 + 3718*a^8*sin(d*x + c)^3 + 1716*a^
8*sin(d*x + c)^2 + 429*a^8*sin(d*x + c))/d
```

Fricas [B] time = 2.04122, size = 400, normalized size = 5.97

$$\frac{286\,a^8\cos(dx+c)^{12}-3432\,a^8\cos(dx+c)^{10}+10296\,a^8\cos(dx+c)^8-9152\,a^8\cos(dx+c)^6+\left(33\,a^8\cos(dx+c)^{12}-12\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/429*(286*a^8*cos(d*x + c)^12 - 3432*a^8*cos(d*x + c)^10 + 10296*a^8*cos(d
*x + c)^8 - 9152*a^8*cos(d*x + c)^6 + (33*a^8*cos(d*x + c)^12 - 1212*a^8*co
s(d*x + c)^10 + 6280*a^8*cos(d*x + c)^8 - 8512*a^8*cos(d*x + c)^6 + 768*a^8
*cos(d*x + c)^4 + 1024*a^8*cos(d*x + c)^2 + 2048*a^8)*sin(d*x + c))/d
```

Sympy [A] time = 120.691, size = 558, normalized size = 8.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((8*a**8*sin(c + d*x)**13/(1287*d) + 4*a**8*sin(c + d*x)**11*cos(c
 + d*x)**2/(99*d) + 32*a**8*sin(c + d*x)**11/(99*d) + a**8*sin(c + d*x)**9*
cos(c + d*x)**4/(9*d) + 16*a**8*sin(c + d*x)**9*cos(c + d*x)**2/(9*d) + 16*
a**8*sin(c + d*x)**9/(9*d) + 4*a**8*sin(c + d*x)**7*cos(c + d*x)**4/d + 8*a
**8*sin(c + d*x)**7*cos(c + d*x)**2/d + 32*a**8*sin(c + d*x)**7/(15*d) - 4*
a**8*sin(c + d*x)**6*cos(c + d*x)**6/(3*d) + 14*a**8*sin(c + d*x)**5*cos(c
+ d*x)**4/d + 112*a**8*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + 8*a**8*sin(
c + d*x)**5/(15*d) - a**8*sin(c + d*x)**4*cos(c + d*x)**8/d - 28*a**8*sin(c
 + d*x)**4*cos(c + d*x)**6/(3*d) + 28*a**8*sin(c + d*x)**3*cos(c + d*x)**4/
(3*d) + 4*a**8*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - 2*a**8*sin(c + d*x)*
*2*cos(c + d*x)**10/(5*d) - 14*a**8*sin(c + d*x)**2*cos(c + d*x)**8/(3*d) -
 28*a**8*sin(c + d*x)**2*cos(c + d*x)**6/(3*d) + a**8*sin(c + d*x)*cos(c +
d*x)**4/d - a**8*cos(c + d*x)**12/(15*d) - 14*a**8*cos(c + d*x)**10/(15*d)
- 7*a**8*cos(c + d*x)**8/(3*d) - 4*a**8*cos(c + d*x)**6/(3*d), Ne(d, 0)), (
x*(a*sin(c) + a)**8*cos(c)**5, True))
```

Giac [B] time = 1.28251, size = 296, normalized size = 4.42

$$\frac{a^8\cos(12\,dx+12\,c)}{3072\,d}-\frac{3\,a^8\cos(10\,dx+10\,c)}{256\,d}+\frac{27\,a^8\cos(8\,dx+8\,c)}{512\,d}+\frac{155\,a^8\cos(6\,dx+6\,c)}{768\,d}-\frac{475\,a^8\cos(4\,dx+4\,c)}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/3072*a^8*cos(12*d*x + 12*c)/d - 3/256*a^8*cos(10*d*x + 10*c)/d + 27/512*a
^8*cos(8*d*x + 8*c)/d + 155/768*a^8*cos(6*d*x + 6*c)/d - 475/1024*a^8*cos(4
*d*x + 4*c)/d - 323/128*a^8*cos(2*d*x + 2*c)/d + 1/53248*a^8*sin(13*d*x + 1
3*c)/d - 115/45056*a^8*sin(11*d*x + 11*c)/d + 205/6144*a^8*sin(9*d*x + 9*c)
/d - 7/2048*a^8*sin(7*d*x + 7*c)/d - 2033/4096*a^8*sin(5*d*x + 5*c)/d - 613
7/12288*a^8*sin(3*d*x + 3*c)/d + 4845/1024*a^8*sin(d*x + c)/d
```

3.42 $\int \cos^4(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=286

$$-\frac{4199a^8\cos^5(c+dx)}{1920d}+\frac{4199a^8\sin(c+dx)\cos^3(c+dx)}{1536d}-\frac{323a^3\cos^5(c+dx)(a\sin(c+dx)+a)^5}{1320d}-\frac{19a^2\cos^5(c+dx)(a}{132}$$

```
[Out] (4199*a^8*x)/1024 - (4199*a^8*Cos[c + d*x]^5)/(1920*d) + (4199*a^8*Cos[c +
d*x]*Sin[c + d*x])/(1024*d) + (4199*a^8*Cos[c + d*x]^3*Sin[c + d*x])/(1536*
d) - (323*a^3*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^5)/(1320*d) - (19*a^2*Cos
[c + d*x]^5*(a + a*Sin[c + d*x])^6)/(132*d) - (a*Cos[c + d*x]^5*(a + a*Sin[
c + d*x])^7)/(12*d) - (4199*a^2*Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x])^3)/
(6336*d) - (323*Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x])^4)/(792*d) - (4199*
Cos[c + d*x]^5*(a^4 + a^4*Sin[c + d*x])^2)/(4032*d) - (4199*Cos[c + d*x]^5*
(a^8 + a^8*Sin[c + d*x]))/(2688*d)
```

Rubi [A] time = 0.403442, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2678, 2669, 2635, 8}

$$-\frac{4199a^8\cos^5(c+dx)}{1920d}+\frac{4199a^8\sin(c+dx)\cos^3(c+dx)}{1536d}-\frac{323a^3\cos^5(c+dx)(a\sin(c+dx)+a)^5}{1320d}-\frac{19a^2\cos^5(c+dx)(a}{132}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^8,x]
```


```
[Out] (4199*a^8*x)/1024 - (4199*a^8*Cos[c + d*x]^5)/(1920*d) + (4199*a^8*Cos[c +
d*x]*Sin[c + d*x])/(1024*d) + (4199*a^8*Cos[c + d*x]^3*Sin[c + d*x])/(1536*
d) - (323*a^3*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^5)/(1320*d) - (19*a^2*Cos
[c + d*x]^5*(a + a*Sin[c + d*x])^6)/(132*d) - (a*Cos[c + d*x]^5*(a + a*Sin[
c + d*x])^7)/(12*d) - (4199*a^2*Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x])^3)/
(6336*d) - (323*Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x])^4)/(792*d) - (4199*
Cos[c + d*x]^5*(a^4 + a^4*Sin[c + d*x])^2)/(4032*d) - (4199*Cos[c + d*x]^5*
(a^8 + a^8*Sin[c + d*x]))/(2688*d)
```


Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
```

```
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^8\,dx = -\frac{a\cos^5(c+dx)(a+a\sin(c+dx))^7}{12d} + \frac{1}{12}(19a)\int \cos^4(c+dx)(a+a\sin(c+dx)$$
$$= -\frac{19a^2\cos^5(c+dx)(a+a\sin(c+dx))^6}{132d} - \frac{a\cos^5(c+dx)(a+a\sin(c+dx))^7}{12d} + \frac{1}{13}$$
$$= -\frac{323a^3\cos^5(c+dx)(a+a\sin(c+dx))^5}{1320d} - \frac{19a^2\cos^5(c+dx)(a+a\sin(c+dx))^6}{132d}$$
$$= -\frac{323a^3\cos^5(c+dx)(a+a\sin(c+dx))^5}{1320d} - \frac{19a^2\cos^5(c+dx)(a+a\sin(c+dx))^6}{132d}$$
$$= -\frac{4199a^5\cos^5(c+dx)(a+a\sin(c+dx))^3}{6336d} - \frac{323a^3\cos^5(c+dx)(a+a\sin(c+dx)}{1320d}$$
$$= -\frac{4199a^5\cos^5(c+dx)(a+a\sin(c+dx))^3}{6336d} - \frac{323a^3\cos^5(c+dx)(a+a\sin(c+dx)}{1320d}$$
$$= -\frac{4199a^5\cos^5(c+dx)(a+a\sin(c+dx))^3}{6336d} - \frac{323a^3\cos^5(c+dx)(a+a\sin(c+dx)}{1320d}$$
$$= -\frac{4199a^8\cos^5(c+dx)}{1920d} - \frac{4199a^5\cos^5(c+dx)(a+a\sin(c+dx))^3}{6336d} - \frac{323a^3\cos^5(c}{}$$
$$= -\frac{4199a^8\cos^5(c+dx)}{1920d} + \frac{4199a^8\cos^3(c+dx)\sin(c+dx)}{1536d} - \frac{4199a^5\cos^5(c+dx)(}{633}$$
$$= -\frac{4199a^8\cos^5(c+dx)}{1920d} + \frac{4199a^8\cos(c+dx)\sin(c+dx)}{1024d} + \frac{4199a^8\cos^3(c+dx)\,\mathrm{s}}{1536d}$$
$$= \frac{4199a^8x}{1024} - \frac{4199a^8\cos^5(c+dx)}{1920d} + \frac{4199a^8\cos(c+dx)\sin(c+dx)}{1024d} + \frac{4199a^8\cos^3}{}$$

Mathematica [A] time = 3.16037, size = 211, normalized size = 0.74

$$-\frac{a^8\left(\sqrt{\sin(c+dx)+1}\left(295680\sin^{12}(c+dx)+2284800\sin^{11}(c+dx)+6969984\sin^{10}(c+dx)+9086336\sin^9(c+dx)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -(a^8*Cos[c + d*x]^5*(-29099070*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt
[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-22470656 + 11469281*Sin[c + d
*x] + 13958687*Sin[c + d*x]^2 + 20459158*Sin[c + d*x]^3 + 14283114*Sin[c +
d*x]^4 - 8321928*Sin[c + d*x]^5 - 26346616*Sin[c + d*x]^6 - 20428112*Sin[c
+ d*x]^7 - 1239728*Sin[c + d*x]^8 + 9086336*Sin[c + d*x]^9 + 6969984*Sin[c
```

```
+ d*x]^10 + 2284800*Sin[c + d*x]^11 + 295680*Sin[c + d*x]^12)))/(3548160*d*
(-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^(5/2))
```

Maple [B] time = 0.051, size = 535, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/12*sin(d*x+c)^7*cos(d*x+c)^5-7/120*sin(d*x+c)^5*cos(d*x+c)^5-7
/192*sin(d*x+c)^3*cos(d*x+c)^5-7/384*sin(d*x+c)*cos(d*x+c)^5+7/1536*(cos(d*
x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)+8*a^8*(-1/11*sin(d*x
+c)^6*cos(d*x+c)^5-2/33*sin(d*x+c)^4*cos(d*x+c)^5-8/231*sin(d*x+c)^2*cos(d*
x+c)^5-16/1155*cos(d*x+c)^5)+28*a^8*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*s
in(d*x+c)^3*cos(d*x+c)^5-1/32*sin(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3
/2*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+56*a^8*(-1/9*sin(d*x+c)^4*cos(
d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5)+70*a^8*(-1/8*si
n(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2
*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+56*a^8*(-1/7*sin(d*x+c)^2*cos(d*
x+c)^5-2/35*cos(d*x+c)^5)+28*a^8*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*
x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)-8/5*a^8*cos(d*x+c)^5+a^8
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maple [A] time = 0.995742, size = 458, normalized size = 1.6

$$-\frac{45416448\,a^{8}\cos\left(dx+c\right)^{5}-196608\left(105\,\cos\left(dx+c\right)^{11}-385\,\cos\left(dx+c\right)^{9}+495\,\cos\left(dx+c\right)^{7}-231\,\cos\left(dx+c\right)^{5}\right)a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/28385280*(45416448*a^8*cos(d*x + c)^5 - 196608*(105*cos(d*x + c)^11 - 38
5*cos(d*x + c)^9 + 495*cos(d*x + c)^7 - 231*cos(d*x + c)^5)*a^8 + 5046272*(
35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*a^8 - 45416448*(
5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^8 + 231*(384*sin(2*d*x + 2*c)^5 + 20
*sin(4*d*x + 4*c)^3 - 840*d*x - 840*c - 15*sin(8*d*x + 8*c) + 240*sin(4*d*x
```

```
 + 4*c))*a^8 + 77616*(32*sin(2*d*x + 2*c)^5 - 120*d*x - 120*c - 5*sin(8*d*x
 + 8*c) + 40*sin(4*d*x + 4*c))*a^8 - 4139520*(4*sin(2*d*x + 2*c)^3 + 12*d*x
 + 12*c - 3*sin(4*d*x + 4*c))*a^8 - 1940400*(24*d*x + 24*c + sin(8*d*x + 8*
c) - 8*sin(4*d*x + 4*c))*a^8 - 887040*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8
*sin(2*d*x + 2*c))*a^8)/d
```

Fricas [A] time = 2.0788, size = 450, normalized size = 1.57

$$\frac{2580480\,a^8\cos\left(dx+c\right)^{11} - 31539200\,a^8\cos\left(dx+c\right)^9 + 97320960\,a^8\cos\left(dx+c\right)^7 - 90832896\,a^8\cos\left(dx+c\right)^5 + 145495}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/3548160*(2580480*a^8*cos(d*x + c)^11 - 31539200*a^8*cos(d*x + c)^9 + 9732
0960*a^8*cos(d*x + c)^7 - 90832896*a^8*cos(d*x + c)^5 + 14549535*a^8*d*x +
231*(1280*a^8*cos(d*x + c)^11 - 47744*a^8*cos(d*x + c)^9 + 253488*a^8*cos(d
*x + c)^7 - 359624*a^8*cos(d*x + c)^5 + 41990*a^8*cos(d*x + c)^3 + 62985*a^
8*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 86.4144, size = 1280, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((7*a**8*x*sin(c + d*x)**12/1024 + 21*a**8*x*sin(c + d*x)**10*cos(
c + d*x)**2/512 + 21*a**8*x*sin(c + d*x)**10/64 + 105*a**8*x*sin(c + d*x)**
8*cos(c + d*x)**4/1024 + 105*a**8*x*sin(c + d*x)**8*cos(c + d*x)**2/64 + 10
5*a**8*x*sin(c + d*x)**8/64 + 35*a**8*x*sin(c + d*x)**6*cos(c + d*x)**6/256
 + 105*a**8*x*sin(c + d*x)**6*cos(c + d*x)**4/32 + 105*a**8*x*sin(c + d*x)*
*6*cos(c + d*x)**2/16 + 7*a**8*x*sin(c + d*x)**6/4 + 105*a**8*x*sin(c + d*x
)**4*cos(c + d*x)**8/1024 + 105*a**8*x*sin(c + d*x)**4*cos(c + d*x)**6/32 +
 315*a**8*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 21*a**8*x*sin(c + d*x)**4*
cos(c + d*x)**2/4 + 3*a**8*x*sin(c + d*x)**4/8 + 21*a**8*x*sin(c + d*x)**2*
cos(c + d*x)**10/512 + 105*a**8*x*sin(c + d*x)**2*cos(c + d*x)**8/64 + 105*
```

```
a**8*x*sin(c + d*x)**2*cos(c + d*x)**6/16 + 21*a**8*x*sin(c + d*x)**2*cos(c
 + d*x)**4/4 + 3*a**8*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 7*a**8*x*cos(c
+ d*x)**12/1024 + 21*a**8*x*cos(c + d*x)**10/64 + 105*a**8*x*cos(c + d*x)**
8/64 + 7*a**8*x*cos(c + d*x)**6/4 + 3*a**8*x*cos(c + d*x)**4/8 + 7*a**8*sin
(c + d*x)**11*cos(c + d*x)/(1024*d) + 119*a**8*sin(c + d*x)**9*cos(c + d*x)
**3/(3072*d) + 21*a**8*sin(c + d*x)**9*cos(c + d*x)/(64*d) - 281*a**8*sin(c
 + d*x)**7*cos(c + d*x)**5/(2560*d) + 49*a**8*sin(c + d*x)**7*cos(c + d*x)*
*3/(32*d) + 105*a**8*sin(c + d*x)**7*cos(c + d*x)/(64*d) - 8*a**8*sin(c + d
*x)**6*cos(c + d*x)**5/(5*d) - 231*a**8*sin(c + d*x)**5*cos(c + d*x)**7/(25
60*d) - 14*a**8*sin(c + d*x)**5*cos(c + d*x)**5/(5*d) + 385*a**8*sin(c + d*
x)**5*cos(c + d*x)**3/(64*d) + 7*a**8*sin(c + d*x)**5*cos(c + d*x)/(4*d) -
48*a**8*sin(c + d*x)**4*cos(c + d*x)**7/(35*d) - 56*a**8*sin(c + d*x)**4*co
s(c + d*x)**5/(5*d) - 119*a**8*sin(c + d*x)**3*cos(c + d*x)**9/(3072*d) - 4
9*a**8*sin(c + d*x)**3*cos(c + d*x)**7/(32*d) - 385*a**8*sin(c + d*x)**3*co
s(c + d*x)**5/(64*d) + 14*a**8*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) + 3*a*
*8*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 64*a**8*sin(c + d*x)**2*cos(c + d*x
)**9/(105*d) - 32*a**8*sin(c + d*x)**2*cos(c + d*x)**7/(5*d) - 56*a**8*sin(
c + d*x)**2*cos(c + d*x)**5/(5*d) - 7*a**8*sin(c + d*x)*cos(c + d*x)**11/(1
024*d) - 21*a**8*sin(c + d*x)*cos(c + d*x)**9/(64*d) - 105*a**8*sin(c + d*x
)*cos(c + d*x)**7/(64*d) - 7*a**8*sin(c + d*x)*cos(c + d*x)**5/(4*d) + 5*a*
*8*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 128*a**8*cos(c + d*x)**11/(1155*d)
- 64*a**8*cos(c + d*x)**9/(45*d) - 16*a**8*cos(c + d*x)**7/(5*d) - 8*a**8*c
os(c + d*x)**5/(5*d), Ne(d, 0)), (x*(a*sin(c) + a)**8*cos(c)**4, True))
```

Giac [A] time = 1.26822, size = 281, normalized size = 0.98

$$\frac{4199}{1024}a^8x + \frac{a^8\cos(11\,dx + 11\,c)}{1408\,d} - \frac{31\,a^8\cos(9\,dx + 9\,c)}{1152\,d} + \frac{139\,a^8\cos(7\,dx + 7\,c)}{896\,d} + \frac{171\,a^8\cos(5\,dx + 5\,c)}{640\,d} - \frac{323\,a^8\cos}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 4199/1024*a^8*x + 1/1408*a^8*cos(11*d*x + 11*c)/d - 31/1152*a^8*cos(9*d*x +
 9*c)/d + 139/896*a^8*cos(7*d*x + 7*c)/d + 171/640*a^8*cos(5*d*x + 5*c)/d -
 323/192*a^8*cos(3*d*x + 3*c)/d - 323/64*a^8*cos(d*x + c)/d + 1/24576*a^8*s
in(12*d*x + 12*c)/d - 29/5120*a^8*sin(10*d*x + 10*c)/d + 673/8192*a^8*sin(8
*d*x + 8*c)/d - 361/3072*a^8*sin(6*d*x + 6*c)/d - 8721/8192*a^8*sin(4*d*x +
 4*c)/d + 323/512*a^8*sin(2*d*x + 2*c)/d
```

3.43 $\int \cos^3(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=45

$$\frac{(a\sin(c+dx)+a)^{10}}{5a^2d}-\frac{(a\sin(c+dx)+a)^{11}}{11a^3d}$$

```
[Out] (a + a*Sin[c + d*x])^10/(5*a^2*d) - (a + a*Sin[c + d*x])^11/(11*a^3*d)
```

Rubi [A] time = 0.0471371, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a\sin(c+dx)+a)^{10}}{5a^2d}-\frac{(a\sin(c+dx)+a)^{11}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a + a*Sin[c + d*x])^10/(5*a^2*d) - (a + a*Sin[c + d*x])^11/(11*a^3*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))^8\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^9\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(a+x)^9-(a+x)^{10}\right)\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{(a+a\sin(c+dx))^{10}}{5a^2 d} - \frac{(a+a\sin(c+dx))^{11}}{11a^3 d}$$

Mathematica [A] time = 1.0431, size = 43, normalized size = 0.96

$$-\frac{a^8(5\sin(c+dx)-6)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^{20}}{55d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -(a^8*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^20*(-6 + 5*Sin[c + d*x]))/(55*d
)
```

Maple [B] time = 0.052, size = 463, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/11*sin(d*x+c)^7*cos(d*x+c)^4-7/99*sin(d*x+c)^5*cos(d*x+c)^4-5/
99*sin(d*x+c)^3*cos(d*x+c)^4-1/33*sin(d*x+c)*cos(d*x+c)^4+1/99*(2+cos(d*x+c
)^2)*sin(d*x+c))+8*a^8*(-1/10*sin(d*x+c)^6*cos(d*x+c)^4-3/40*sin(d*x+c)^4*c
os(d*x+c)^4-1/20*sin(d*x+c)^2*cos(d*x+c)^4-1/40*cos(d*x+c)^4)+28*a^8*(-1/9*
sin(d*x+c)^5*cos(d*x+c)^4-5/63*sin(d*x+c)^3*cos(d*x+c)^4-1/21*sin(d*x+c)*co
s(d*x+c)^4+1/63*(2+cos(d*x+c)^2)*sin(d*x+c))+56*a^8*(-1/8*sin(d*x+c)^4*cos(
d*x+c)^4-1/12*sin(d*x+c)^2*cos(d*x+c)^4-1/24*cos(d*x+c)^4)+70*a^8*(-1/7*sin
(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(2+cos(d*x+c)^2)*s
in(d*x+c))+56*a^8*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+28*a^8
*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-2*a^8*cos(
```

```
d*x+c)^4+1/3*a^8*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 0.949323, size = 181, normalized size = 4.02

$$-\frac{5\,a^8\sin\left(dx+c\right)^{11}+44\,a^8\sin\left(dx+c\right)^{10}+165\,a^8\sin\left(dx+c\right)^9+330\,a^8\sin\left(dx+c\right)^8+330\,a^8\sin\left(dx+c\right)^7-462\,a^8\sin}{55\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/55*(5*a^8*sin(d*x + c)^11 + 44*a^8*sin(d*x + c)^10 + 165*a^8*sin(d*x + c
)^9 + 330*a^8*sin(d*x + c)^8 + 330*a^8*sin(d*x + c)^7 - 462*a^8*sin(d*x + c
)^5 - 660*a^8*sin(d*x + c)^4 - 495*a^8*sin(d*x + c)^3 - 220*a^8*sin(d*x + c
)^2 - 55*a^8*sin(d*x + c))/d
```

Fricas [B] time = 1.95231, size = 352, normalized size = 7.82

$$\frac{44\,a^8\cos\left(dx+c\right)^{10}-550\,a^8\cos\left(dx+c\right)^8+1760\,a^8\cos\left(dx+c\right)^6-1760\,a^8\cos\left(dx+c\right)^4+\left(5\,a^8\cos\left(dx+c\right)^{10}-190\,a^8\,c\right.}{55\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/55*(44*a^8*cos(d*x + c)^10 - 550*a^8*cos(d*x + c)^8 + 1760*a^8*cos(d*x +
c)^6 - 1760*a^8*cos(d*x + c)^4 + (5*a^8*cos(d*x + c)^10 - 190*a^8*cos(d*x +
 c)^8 + 1040*a^8*cos(d*x + c)^6 - 1568*a^8*cos(d*x + c)^4 + 256*a^8*cos(d*x
 + c)^2 + 512*a^8)*sin(d*x + c))/d
```

Sympy [A] time = 56.5975, size = 445, normalized size = 9.89

$$\begin{cases} \frac{2a^8\sin^{11}(c+dx)}{99d}+\frac{a^8\sin^9(c+dx)\cos^2(c+dx)}{9d}+\frac{8a^8\sin^9(c+dx)}{9d}+\frac{4a^8\sin^7(c+dx)\cos^2(c+dx)}{d}+\frac{4a^8\sin^7(c+dx)}{d}-\frac{2a^8\sin^6(c+dx)\cos^4(c+dx)}{d}+\frac{14a^8}{} \\ x\left(a\sin\left(c\right)+a\right)^8\cos^3\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((2*a**8*sin(c + d*x)**11/(99*d) + a**8*sin(c + d*x)**9*cos(c + d*
x)**2/(9*d) + 8*a**8*sin(c + d*x)**9/(9*d) + 4*a**8*sin(c + d*x)**7*cos(c +
 d*x)**2/d + 4*a**8*sin(c + d*x)**7/d - 2*a**8*sin(c + d*x)**6*cos(c + d*x)
**4/d + 14*a**8*sin(c + d*x)**5*cos(c + d*x)**2/d + 56*a**8*sin(c + d*x)**5
/(15*d) - 2*a**8*sin(c + d*x)**4*cos(c + d*x)**6/d - 14*a**8*sin(c + d*x)**
4*cos(c + d*x)**4/d + 2*a**8*sin(c + d*x)**4/d + 28*a**8*sin(c + d*x)**3*co
s(c + d*x)**2/(3*d) + 2*a**8*sin(c + d*x)**3/(3*d) - a**8*sin(c + d*x)**2*c
os(c + d*x)**8/d - 28*a**8*sin(c + d*x)**2*cos(c + d*x)**6/(3*d) - 14*a**8*
sin(c + d*x)**2*cos(c + d*x)**4/d + 4*a**8*sin(c + d*x)**2*cos(c + d*x)**2/
d + a**8*sin(c + d*x)*cos(c + d*x)**2/d - a**8*cos(c + d*x)**10/(5*d) - 7*a
**8*cos(c + d*x)**8/(3*d) - 14*a**8*cos(c + d*x)**6/(3*d), Ne(d, 0)), (x*(a
*sin(c) + a)**8*cos(c)**3, True))
```

Giac [B] time = 1.2046, size = 181, normalized size = 4.02

$$-\frac{5\,a^8\sin\left(dx+c\right)^{11}+44\,a^8\sin\left(dx+c\right)^{10}+165\,a^8\sin\left(dx+c\right)^{9}+330\,a^8\sin\left(dx+c\right)^{8}+330\,a^8\sin\left(dx+c\right)^{7}-462\,a^8\sin}{55\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/55*(5*a^8*sin(d*x + c)^11 + 44*a^8*sin(d*x + c)^10 + 165*a^8*sin(d*x + c
)^9 + 330*a^8*sin(d*x + c)^8 + 330*a^8*sin(d*x + c)^7 - 462*a^8*sin(d*x + c
)^5 - 660*a^8*sin(d*x + c)^4 - 495*a^8*sin(d*x + c)^3 - 220*a^8*sin(d*x + c
)^2 - 55*a^8*sin(d*x + c))/d
```

3.44 $\int \cos^2(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=262

$$-\frac{2431a^8\cos^3(c+dx)}{384d}-\frac{17a^3\cos^3(c+dx)(a\sin(c+dx)+a)^5}{48d}-\frac{17a^2\cos^3(c+dx)(a\sin(c+dx)+a)^6}{90d}-\frac{2431a^2\cos^3(c+$$

```
[Out] (2431*a^8*x)/256 - (2431*a^8*Cos[c + d*x]^3)/(384*d) + (2431*a^8*Cos[c + d*
x]*Sin[c + d*x])/(256*d) - (17*a^3*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^5)/(
48*d) - (17*a^2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^6)/(90*d) - (a*Cos[c +
d*x]^3*(a + a*Sin[c + d*x])^7)/(10*d) - (2431*a^2*Cos[c + d*x]^3*(a^2 + a^2
*Sin[c + d*x])^3)/(2016*d) - (221*Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x])^4
)/(336*d) - (2431*Cos[c + d*x]^3*(a^4 + a^4*Sin[c + d*x])^2)/(1120*d) - (24
31*Cos[c + d*x]^3*(a^8 + a^8*Sin[c + d*x]))/(640*d)
```

Rubi [A] time = 0.374343, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2678, 2669, 2635, 8}

$$-\frac{2431a^8\cos^3(c+dx)}{384d}-\frac{17a^3\cos^3(c+dx)(a\sin(c+dx)+a)^5}{48d}-\frac{17a^2\cos^3(c+dx)(a\sin(c+dx)+a)^6}{90d}-\frac{2431a^2\cos^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^8,x]
```


```
[Out] (2431*a^8*x)/256 - (2431*a^8*Cos[c + d*x]^3)/(384*d) + (2431*a^8*Cos[c + d*
x]*Sin[c + d*x])/(256*d) - (17*a^3*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^5)/(
48*d) - (17*a^2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^6)/(90*d) - (a*Cos[c +
d*x]^3*(a + a*Sin[c + d*x])^7)/(10*d) - (2431*a^2*Cos[c + d*x]^3*(a^2 + a^2
*Sin[c + d*x])^3)/(2016*d) - (221*Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x])^4
)/(336*d) - (2431*Cos[c + d*x]^3*(a^4 + a^4*Sin[c + d*x])^2)/(1120*d) - (24
31*Cos[c + d*x]^3*(a^8 + a^8*Sin[c + d*x]))/(640*d)
```


Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
```

```
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^8\,dx = -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^7}{10d} + \frac{1}{10}(17a)\int \cos^2(c+dx)(a+a\sin(c+dx)$$
$$= -\frac{17a^2\cos^3(c+dx)(a+a\sin(c+dx))^6}{90d} - \frac{a\cos^3(c+dx)(a+a\sin(c+dx))^7}{10d} + \frac{1}{6}$$
$$= -\frac{17a^3\cos^3(c+dx)(a+a\sin(c+dx))^5}{48d} - \frac{17a^2\cos^3(c+dx)(a+a\sin(c+dx))^6}{90d}$$
$$= -\frac{17a^3\cos^3(c+dx)(a+a\sin(c+dx))^5}{48d} - \frac{17a^2\cos^3(c+dx)(a+a\sin(c+dx))^6}{90d}$$
$$= -\frac{2431a^5\cos^3(c+dx)(a+a\sin(c+dx))^3}{2016d} - \frac{17a^3\cos^3(c+dx)(a+a\sin(c+dx)}{48d}$$
$$= -\frac{2431a^5\cos^3(c+dx)(a+a\sin(c+dx))^3}{2016d} - \frac{17a^3\cos^3(c+dx)(a+a\sin(c+dx)}{48d}$$
$$= -\frac{2431a^5\cos^3(c+dx)(a+a\sin(c+dx))^3}{2016d} - \frac{17a^3\cos^3(c+dx)(a+a\sin(c+dx)}{48d}$$
$$= -\frac{2431a^8\cos^3(c+dx)}{384d} - \frac{2431a^5\cos^3(c+dx)(a+a\sin(c+dx))^3}{2016d} - \frac{17a^3\cos^3(c+}{}$$
$$= -\frac{2431a^8\cos^3(c+dx)}{384d} + \frac{2431a^8\cos(c+dx)\sin(c+dx)}{256d} - \frac{2431a^5\cos^3(c+dx)(a}{2016}$$
$$= \frac{2431a^8x}{256} - \frac{2431a^8\cos^3(c+dx)}{384d} + \frac{2431a^8\cos(c+dx)\sin(c+dx)}{256d} - \frac{2431a^5\cos^3}{}$$

Mathematica [A] time = 1.51426, size = 191, normalized size = 0.73

$$-\frac{a^8\left(1531530\sqrt{1-\sin(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\left(8064\sin^{10}(c+dx)+63616\sin^9(c+dx)+209552\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -(a^8*Cos[c + d*x]^3*(1531530*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1
 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(1193984 - 508859*Sin[c + d*x] -
410693*Sin[c + d*x]^2 - 543442*Sin[c + d*x]^3 - 492846*Sin[c + d*x]^4 - 130
728*Sin[c + d*x]^5 + 257704*Sin[c + d*x]^6 + 353648*Sin[c + d*x]^7 + 209552
*Sin[c + d*x]^8 + 63616*Sin[c + d*x]^9 + 8064*Sin[c + d*x]^10)))/(80640*d*(
-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^(3/2))
```

Maple [A] time = 0.05, size = 480, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/10*sin(d*x+c)^7*cos(d*x+c)^3-7/80*sin(d*x+c)^5*cos(d*x+c)^3-7/
96*sin(d*x+c)^3*cos(d*x+c)^3-7/128*cos(d*x+c)^3*sin(d*x+c)+7/256*cos(d*x+c)
*sin(d*x+c)+7/256*d*x+7/256*c)+8*a^8*(-1/9*sin(d*x+c)^6*cos(d*x+c)^3-2/21*s
in(d*x+c)^4*cos(d*x+c)^3-8/105*sin(d*x+c)^2*cos(d*x+c)^3-16/315*cos(d*x+c)^
3)+28*a^8*(-1/8*sin(d*x+c)^5*cos(d*x+c)^3-5/48*sin(d*x+c)^3*cos(d*x+c)^3-5/
64*cos(d*x+c)^3*sin(d*x+c)+5/128*cos(d*x+c)*sin(d*x+c)+5/128*d*x+5/128*c)+5
6*a^8*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c)^3-8/105*
cos(d*x+c)^3)+70*a^8*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d
*x+c)+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+56*a^8*(-1/5*sin(d*x+c)^2
*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+28*a^8*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*c
os(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-8/3*a^8*cos(d*x+c)^3+a^8*(1/2*cos(d*x+c
)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.987901, size = 431, normalized size = 1.65

$$-\frac{1720320\,a^8\cos(dx+c)^3 - 16384\left(35\cos(dx+c)^9 - 135\cos(dx+c)^7 + 189\cos(dx+c)^5 - 105\cos(dx+c)^3\right)a^8 + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/645120*(1720320*a^8*cos(d*x + c)^3 - 16384*(35*cos(d*x + c)^9 - 135*cos(
d*x + c)^7 + 189*cos(d*x + c)^5 - 105*cos(d*x + c)^3)*a^8 + 344064*(15*cos(
d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^8 - 2408448*(3*cos(d*
x + c)^5 - 5*cos(d*x + c)^3)*a^8 - 21*(96*sin(2*d*x + 2*c)^5 - 640*sin(2*d*
x + 2*c)^3 + 840*d*x + 840*c - 45*sin(8*d*x + 8*c) - 120*sin(4*d*x + 4*c))*
a^8 + 5880*(64*sin(2*d*x + 2*c)^3 - 120*d*x - 120*c + 3*sin(8*d*x + 8*c) +
24*sin(4*d*x + 4*c))*a^8 + 235200*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3
*sin(4*d*x + 4*c))*a^8 - 564480*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^8 - 1612
80*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^8)/d
```

Fricas [A] time = 1.90641, size = 390, normalized size = 1.49

$$\frac{71680\,a^8\cos(dx+c)^9 - 921600\,a^8\cos(dx+c)^7 + 3096576\,a^8\cos(dx+c)^5 - 3440640\,a^8\cos(dx+c)^3 + 765765\,a^8dx +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/80640*(71680*a^8*cos(d*x + c)^9 - 921600*a^8*cos(d*x + c)^7 + 3096576*a^8
*cos(d*x + c)^5 - 3440640*a^8*cos(d*x + c)^3 + 765765*a^8*d*x + 63*(128*a^8
*cos(d*x + c)^9 - 4976*a^8*cos(d*x + c)^7 + 28328*a^8*cos(d*x + c)^5 - 4651
0*a^8*cos(d*x + c)^3 + 12155*a^8*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 40.576, size = 1018, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((7*a**8*x*sin(c + d*x)**10/256 + 35*a**8*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 35*a**8*x*sin(c + d*x)**8/32 + 35*a**8*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 35*a**8*x*sin(c + d*x)**6*cos(c + d*x)**2/8 + 35*a**8*x
*sin(c + d*x)**6/8 + 35*a**8*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 105*a*
*8*x*sin(c + d*x)**4*cos(c + d*x)**4/16 + 105*a**8*x*sin(c + d*x)**4*cos(c
+ d*x)**2/8 + 7*a**8*x*sin(c + d*x)**4/2 + 35*a**8*x*sin(c + d*x)**2*cos(c
+ d*x)**8/256 + 35*a**8*x*sin(c + d*x)**2*cos(c + d*x)**6/8 + 105*a**8*x*si
n(c + d*x)**2*cos(c + d*x)**4/8 + 7*a**8*x*sin(c + d*x)**2*cos(c + d*x)**2
+ a**8*x*sin(c + d*x)**2/2 + 7*a**8*x*cos(c + d*x)**10/256 + 35*a**8*x*cos(
c + d*x)**8/32 + 35*a**8*x*cos(c + d*x)**6/8 + 7*a**8*x*cos(c + d*x)**4/2 +
 a**8*x*cos(c + d*x)**2/2 + 7*a**8*sin(c + d*x)**9*cos(c + d*x)/(256*d) - 7
9*a**8*sin(c + d*x)**7*cos(c + d*x)**3/(384*d) + 35*a**8*sin(c + d*x)**7*co
s(c + d*x)/(32*d) - 8*a**8*sin(c + d*x)**6*cos(c + d*x)**3/(3*d) - 7*a**8*s
in(c + d*x)**5*cos(c + d*x)**5/(30*d) - 511*a**8*sin(c + d*x)**5*cos(c + d*
x)**3/(96*d) + 35*a**8*sin(c + d*x)**5*cos(c + d*x)/(8*d) - 16*a**8*sin(c +
 d*x)**4*cos(c + d*x)**5/(5*d) - 56*a**8*sin(c + d*x)**4*cos(c + d*x)**3/(3
*d) - 49*a**8*sin(c + d*x)**3*cos(c + d*x)**7/(384*d) - 385*a**8*sin(c + d*
x)**3*cos(c + d*x)**5/(96*d) - 35*a**8*sin(c + d*x)**3*cos(c + d*x)**3/(3*d
```

```
) + 7*a**8*sin(c + d*x)**3*cos(c + d*x)/(2*d) - 64*a**8*sin(c + d*x)**2*cos
(c + d*x)**7/(35*d) - 224*a**8*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) - 56*
a**8*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 7*a**8*sin(c + d*x)*cos(c + d*
x)**9/(256*d) - 35*a**8*sin(c + d*x)*cos(c + d*x)**7/(32*d) - 35*a**8*sin(c
 + d*x)*cos(c + d*x)**5/(8*d) - 7*a**8*sin(c + d*x)*cos(c + d*x)**3/(2*d) +
 a**8*sin(c + d*x)*cos(c + d*x)/(2*d) - 128*a**8*cos(c + d*x)**9/(315*d) -
64*a**8*cos(c + d*x)**7/(15*d) - 112*a**8*cos(c + d*x)**5/(15*d) - 8*a**8*c
os(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)**8*cos(c)**2, True))
```

Giac [A] time = 1.24364, size = 235, normalized size = 0.9

$$\frac{2431}{256}\,a^8x+\frac{a^8\cos\left(9\,dx+9\,c\right)}{288\,d}-\frac{33\,a^8\cos\left(7\,dx+7\,c\right)}{224\,d}+\frac{51\,a^8\cos\left(5\,dx+5\,c\right)}{40\,d}-\frac{17\,a^8\cos\left(3\,dx+3\,c\right)}{8\,d}-\frac{221\,a^8\cos\left(dx\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2431/256*a^8*x + 1/288*a^8*cos(9*d*x + 9*c)/d - 33/224*a^8*cos(7*d*x + 7*c)
/d + 51/40*a^8*cos(5*d*x + 5*c)/d - 17/8*a^8*cos(3*d*x + 3*c)/d - 221/16*a^
8*cos(d*x + c)/d + 1/5120*a^8*sin(10*d*x + 10*c)/d - 59/2048*a^8*sin(8*d*x
+ 8*c)/d + 527/1024*a^8*sin(6*d*x + 6*c)/d - 561/256*a^8*sin(4*d*x + 4*c)/d
 - 663/512*a^8*sin(2*d*x + 2*c)/d
```

3.45 $\int \cos(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=22

$$\frac{(a\sin(c+dx)+a)^9}{9ad}$$

```
[Out] (a + a*Sin[c + d*x])^9/(9*a*d)
```

Rubi [A] time = 0.0243345, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 32}

$$\frac{(a\sin(c+dx)+a)^9}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a + a*Sin[c + d*x])^9/(9*a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^8\,dx &= \frac{\text{Subst}\left(\int (a+x)^8\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{(a+a\sin(c+dx))^9}{9ad}\end{aligned}$$

Mathematica [B] time = 0.0894273, size = 147, normalized size = 6.68

$$\frac{a^8\sin^9(c+dx)}{9d}+\frac{a^8\sin^8(c+dx)}{d}+\frac{4a^8\sin^7(c+dx)}{d}+\frac{28a^8\sin^6(c+dx)}{3d}+\frac{14a^8\sin^5(c+dx)}{d}+\frac{14a^8\sin^4(c+dx)}{d}+\frac{28}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a^8*Sin[c + d*x])/d + (4*a^8*Sin[c + d*x]^2)/d + (28*a^8*Sin[c + d*x]^3)/(
3*d) + (14*a^8*Sin[c + d*x]^4)/d + (14*a^8*Sin[c + d*x]^5)/d + (28*a^8*Sin[
c + d*x]^6)/(3*d) + (4*a^8*Sin[c + d*x]^7)/d + (a^8*Sin[c + d*x]^8)/d + (a^
8*Sin[c + d*x]^9)/(9*d)
```

Maple [A] time = 0.02, size = 21, normalized size = 1.

$$\frac{(a+a\sin(dx+c))^9}{9\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/9*(a+a*sin(d*x+c))^9/d/a
```

Maxima [A] time = 0.93696, size = 27, normalized size = 1.23

$$\frac{(a\sin(dx+c)+a)^9}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/9*(a*sin(d*x + c) + a)^9/(a*d)
```

Fricas [B] time = 1.84925, size = 304, normalized size = 13.82

$$\frac{9\,a^8\cos\left(dx+c\right)^8-120\,a^8\cos\left(dx+c\right)^6+432\,a^8\cos\left(dx+c\right)^4-576\,a^8\cos\left(dx+c\right)^2+\left(a^8\cos\left(dx+c\right)^8-40\,a^8\cos\left(dx+\right.\right.}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a^8*cos(d*x + c)^8 - 120*a^8*cos(d*x + c)^6 + 432*a^8*cos(d*x + c)^4
 - 576*a^8*cos(d*x + c)^2 + (a^8*cos(d*x + c)^8 - 40*a^8*cos(d*x + c)^6 + 2
40*a^8*cos(d*x + c)^4 - 448*a^8*cos(d*x + c)^2 + 256*a^8)*sin(d*x + c))/d
```

Sympy [A] time = 19.8771, size = 148, normalized size = 6.73

$$\begin{cases}\frac{a^8\sin^9\left(c+dx\right)}{9d}+\frac{a^8\sin^8\left(c+dx\right)}{d}+\frac{4a^8\sin^7\left(c+dx\right)}{d}+\frac{28a^8\sin^6\left(c+dx\right)}{3d}+\frac{14a^8\sin^5\left(c+dx\right)}{d}+\frac{14a^8\sin^4\left(c+dx\right)}{d}+\frac{28a^8\sin^3\left(c+dx\right)}{3d}+\frac{4a^8\sin^2\left(c+dx\right)}{d}+ \\ x\left(a\sin\left(c\right)+a\right)^8\cos\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((a**8*sin(c + d*x)**9/(9*d) + a**8*sin(c + d*x)**8/d + 4*a**8*sin
(c + d*x)**7/d + 28*a**8*sin(c + d*x)**6/(3*d) + 14*a**8*sin(c + d*x)**5/d
+ 14*a**8*sin(c + d*x)**4/d + 28*a**8*sin(c + d*x)**3/(3*d) + 4*a**8*sin(c
+ d*x)**2/d + a**8*sin(c + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**8*cos(c),
True))
```

Giac [A] time = 1.20482, size = 27, normalized size = 1.23

$$\frac{\left(a\sin\left(dx+c\right)+a\right)^9}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/9*(a*sin(d*x + c) + a)^9/(a*d)
```

3.46 $\int \sec(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=162

$$-\frac{64a^8\sin(c+dx)}{d}-\frac{16a^5(a\sin(c+dx)+a)^3}{3d}-\frac{4a^3(a\sin(c+dx)+a)^5}{5d}-\frac{a^2(a\sin(c+dx)+a)^6}{3d}-\frac{2\left(a^2\sin(c+dx)+a\right.}{d}$$

```
[Out] (-128*a^8*Log[1 - Sin[c + d*x]])/d - (64*a^8*Sin[c + d*x])/d - (16*a^5*(a +
 a*Sin[c + d*x])^3)/(3*d) - (4*a^3*(a + a*Sin[c + d*x])^5)/(5*d) - (a^2*(a
+ a*Sin[c + d*x])^6)/(3*d) - (a*(a + a*Sin[c + d*x])^7)/(7*d) - (2*(a^2 + a
^2*Sin[c + d*x])^4)/d - (16*(a^4 + a^4*Sin[c + d*x])^2)/d
```

Rubi [A] time = 0.0768911, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 43}

$$-\frac{64a^8\sin(c+dx)}{d}-\frac{16a^5(a\sin(c+dx)+a)^3}{3d}-\frac{4a^3(a\sin(c+dx)+a)^5}{5d}-\frac{a^2(a\sin(c+dx)+a)^6}{3d}-\frac{2\left(a^2\sin(c+dx)+a\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-128*a^8*Log[1 - Sin[c + d*x]])/d - (64*a^8*Sin[c + d*x])/d - (16*a^5*(a +
 a*Sin[c + d*x])^3)/(3*d) - (4*a^3*(a + a*Sin[c + d*x])^5)/(5*d) - (a^2*(a
+ a*Sin[c + d*x])^6)/(3*d) - (a*(a + a*Sin[c + d*x])^7)/(7*d) - (2*(a^2 + a
^2*Sin[c + d*x])^4)/d - (16*(a^4 + a^4*Sin[c + d*x])^2)/d
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^8\,dx = \frac{a\,\mathrm{Subst}\left(\int \frac{(a+x)^7}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\mathrm{Subst}\left(\int \left(-64a^6 + \frac{128a^7}{a-x} - 32a^5(a+x) - 16a^4(a+x)^2 - 8a^3(a+x)^3 - 4a^2(a+$$
$$= -\frac{128a^8\log(1-\sin(c+dx))}{d} - \frac{64a^8\sin(c+dx)}{d} - \frac{16a^5(a+a\sin(c+dx))^3}{3d} - \frac{4a^3(}{}$$

Mathematica [A] time = 0.176694, size = 95, normalized size = 0.59

$$\frac{a^8\left(-\frac{1}{7}\sin^7(c+dx) - \frac{4}{3}\sin^6(c+dx) - \frac{29}{5}\sin^5(c+dx) - 16\sin^4(c+dx) - 33\sin^3(c+dx) - 60\sin^2(c+dx) - 127\sin(c+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a^8*(-128*Log[1 - Sin[c + d*x]] - 127*Sin[c + d*x] - 60*Sin[c + d*x]^2 - 3
3*Sin[c + d*x]^3 - 16*Sin[c + d*x]^4 - (29*Sin[c + d*x]^5)/5 - (4*Sin[c + d
*x]^6)/3 - Sin[c + d*x]^7/7))/d
```

Maple [A] time = 0.076, size = 149, normalized size = 0.9

$$-128\,\frac{a^8\ln(\cos(dx+c))}{d} + 128\,\frac{a^8\ln(\sec(dx+c)+\tan(dx+c))}{d} - \frac{a^8(\sin(dx+c))^7}{7\,d} - \frac{4\,a^8(\sin(dx+c))^6}{3\,d} - \frac{29\,a^8(\sin(}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^8,x)
```

```
[Out] -128/d*a^8*ln(cos(d*x+c))+128/d*a^8*ln(sec(d*x+c)+tan(d*x+c))-1/7/d*a^8*sin
(d*x+c)^7-4/3/d*a^8*sin(d*x+c)^6-29/5*a^8*sin(d*x+c)^5/d-16*a^8*sin(d*x+c)^
4/d-33*a^8*sin(d*x+c)^3/d-60*a^8*sin(d*x+c)^2/d-127*a^8*sin(d*x+c)/d
```

Maxima [A] time = 0.950359, size = 147, normalized size = 0.91

$$-\frac{15\,a^{8}\sin\left(dx+c\right)^{7}+140\,a^{8}\sin\left(dx+c\right)^{6}+609\,a^{8}\sin\left(dx+c\right)^{5}+1680\,a^{8}\sin\left(dx+c\right)^{4}+3465\,a^{8}\sin\left(dx+c\right)^{3}+6300\,a}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/105*(15*a^8*sin(d*x + c)^7 + 140*a^8*sin(d*x + c)^6 + 609*a^8*sin(d*x +
c)^5 + 1680*a^8*sin(d*x + c)^4 + 3465*a^8*sin(d*x + c)^3 + 6300*a^8*sin(d*x
 + c)^2 + 13440*a^8*log(sin(d*x + c) - 1) + 13335*a^8*sin(d*x + c))/d
```

Fricas [A] time = 1.83921, size = 302, normalized size = 1.86

$$\frac{140\,a^{8}\cos\left(dx+c\right)^{6}-2100\,a^{8}\cos\left(dx+c\right)^{4}+10080\,a^{8}\cos\left(dx+c\right)^{2}-13440\,a^{8}\log\left(-\sin\left(dx+c\right)+1\right)+3\left(5\,a^{8}\cos\left(dx\right.\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(140*a^8*cos(d*x + c)^6 - 2100*a^8*cos(d*x + c)^4 + 10080*a^8*cos(d*x
 + c)^2 - 13440*a^8*log(-sin(d*x + c) + 1) + 3*(5*a^8*cos(d*x + c)^6 - 218*
a^8*cos(d*x + c)^4 + 1576*a^8*cos(d*x + c)^2 - 5808*a^8)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27936, size = 389, normalized size = 2.4

$$2\left(6720\,a^{8}\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)-13440\,a^{8}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{17424\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}+13335\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/105*(6720*a^8*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 13440*a^8*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - (17424*a^8*tan(1/2*d*x + 1/2*c)^14 + 13335*a^8*tan(1/
2*d*x + 1/2*c)^13 + 134568*a^8*tan(1/2*d*x + 1/2*c)^12 + 93870*a^8*tan(1/2*
d*x + 1/2*c)^11 + 442344*a^8*tan(1/2*d*x + 1/2*c)^10 + 265209*a^8*tan(1/2*d
*x + 1/2*c)^9 + 780640*a^8*tan(1/2*d*x + 1/2*c)^8 + 370308*a^8*tan(1/2*d*x
+ 1/2*c)^7 + 780640*a^8*tan(1/2*d*x + 1/2*c)^6 + 265209*a^8*tan(1/2*d*x + 1
/2*c)^5 + 442344*a^8*tan(1/2*d*x + 1/2*c)^4 + 93870*a^8*tan(1/2*d*x + 1/2*c
)^3 + 134568*a^8*tan(1/2*d*x + 1/2*c)^2 + 13335*a^8*tan(1/2*d*x + 1/2*c) +
17424*a^8)/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/d
```

3.47 $\int \sec^2(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=201

$$\frac{1001a^8\cos^5(c+dx)}{10d}+\frac{143a^{16}\cos^7(c+dx)}{2d\left(a^8-a^8\sin(c+dx)\right)}+\frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7}+\frac{286a^{14}\cos^9(c+dx)}{3d\left(a^2-a^2\sin(c+dx)\right)^3}+\frac{26a^{13}\cos^{11}(c+a}{d(a-a\sin(c+dx}$$

```
[Out] (-3003*a^8*x)/16 + (1001*a^8*Cos[c + d*x]^5)/(10*d) - (3003*a^8*Cos[c + d*x
]*Sin[c + d*x])/(16*d) - (1001*a^8*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) + (2*
a^15*Cos[c + d*x]^13)/(d*(a - a*Sin[c + d*x])^7) + (26*a^13*Cos[c + d*x]^11
)/(d*(a - a*Sin[c + d*x])^5) + (286*a^14*Cos[c + d*x]^9)/(3*d*(a^2 - a^2*Si
n[c + d*x])^3) + (143*a^16*Cos[c + d*x]^7)/(2*d*(a^8 - a^8*Sin[c + d*x]))
```

Rubi [A] time = 0.341912, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2670, 2680, 2679, 2682, 2635, 8}

$$\frac{1001a^8\cos^5(c+dx)}{10d}+\frac{143a^{16}\cos^7(c+dx)}{2d\left(a^8-a^8\sin(c+dx)\right)}+\frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7}+\frac{286a^{14}\cos^9(c+dx)}{3d\left(a^2-a^2\sin(c+dx)\right)^3}+\frac{26a^{13}\cos^{11}(c+a}{d(a-a\sin(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-3003*a^8*x)/16 + (1001*a^8*Cos[c + d*x]^5)/(10*d) - (3003*a^8*Cos[c + d*x
]*Sin[c + d*x])/(16*d) - (1001*a^8*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) + (2*
a^15*Cos[c + d*x]^13)/(d*(a - a*Sin[c + d*x])^7) + (26*a^13*Cos[c + d*x]^11
)/(d*(a - a*Sin[c + d*x])^5) + (286*a^14*Cos[c + d*x]^9)/(3*d*(a^2 - a^2*Si
n[c + d*x])^3) + (143*a^16*Cos[c + d*x]^7)/(2*d*(a^8 - a^8*Sin[c + d*x]))
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \sec^2(c+dx)(a+a\sin(c+dx))^8\,dx &= a^{16}\int \frac{\cos^{14}(c+dx)}{(a-a\sin(c+dx))^8}\,dx \\
&= \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} - \left(13a^{14}\right)\int \frac{\cos^{12}(c+dx)}{(a-a\sin(c+dx))^6}\,dx \\
&= \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} + \frac{26a^{13}\cos^{11}(c+dx)}{d(a-a\sin(c+dx))^5} - \left(143a^{12}\right)\int \frac{\cos^{10}(c+dx)}{(a-a\sin(c+dx))^4} \\
&= \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} + \frac{26a^{13}\cos^{11}(c+dx)}{d(a-a\sin(c+dx))^5} + \frac{286a^{11}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^3} - \left(429a\right. \\
&= \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} + \frac{26a^{13}\cos^{11}(c+dx)}{d(a-a\sin(c+dx))^5} + \frac{286a^{11}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^3} + \frac{143}{2d\left(a^2\right.} \\
&= \frac{1001a^8\cos^5(c+dx)}{10d} + \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} + \frac{26a^{13}\cos^{11}(c+dx)}{d(a-a\sin(c+dx))^5} + \frac{286a^{11}}{3d(a-} \\
&= \frac{1001a^8\cos^5(c+dx)}{10d} - \frac{1001a^8\cos^3(c+dx)\sin(c+dx)}{8d} + \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} \\
&= \frac{1001a^8\cos^5(c+dx)}{10d} - \frac{3003a^8\cos(c+dx)\sin(c+dx)}{16d} - \frac{1001a^8\cos^3(c+dx)\sin}{8d} \\
&= -\frac{3003a^8x}{16} + \frac{1001a^8\cos^5(c+dx)}{10d} - \frac{3003a^8\cos(c+dx)\sin(c+dx)}{16d} - \frac{1001a^8\,\mathrm{c}}{}
\end{aligned}
$$

Mathematica [C] time = 0.0573164, size = 55, normalized size = 0.27

$$
\frac{128\sqrt{2}a^8\sqrt{\sin(c+dx)+1}\sec(c+dx)\,{}_2F_1\left(-\frac{13}{2},-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (128*Sqrt[2]*a^8*Hypergeometric2F1[-13/2, -1/2, 1/2, (1 - Sin[c + d*x])/2]*
Sec[c + d*x]*Sqrt[1 + Sin[c + d*x]])/d
```

Maple [B] time = 0.066, size = 389, normalized size = 1.9

$$
\frac{1}{d}\left(a^8\left(\frac{(\sin(dx+c))^9}{\cos(dx+c)} + \left((\sin(dx+c))^7 + \frac{7\,(\sin(dx+c))^5}{6} + \frac{35\,(\sin(dx+c))^3}{24} + \frac{35\,\sin(dx+c)}{16}\right)\cos(dx+c) - \frac{35\,d}{16}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(sin(d*x+c)^9/cos(d*x+c)+(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(
d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)-35/16*d*x-35/16*c)+8*a^8*(sin(d*x+c)^
8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+
c))+28*a^8*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin
(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+56*a^8*(sin(d*x+c)^6/cos(d*x+c)+(8/3+s
in(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+70*a^8*(sin(d*x+c)^5/cos(d*x+c)+(
sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+56*a^8*(sin(d*x+c)^4
/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+28*a^8*(tan(d*x+c)-d*x-c)+8*a^8/co
s(d*x+c)+a^8*tan(d*x+c))
```

Maxima [A] time = 1.45832, size = 447, normalized size = 2.22

$$\frac{384\left(\cos\left(dx+c\right)^{5}-5\,\cos\left(dx+c\right)^{3}+\frac{5}{\cos(dx+c)}+15\,\cos\left(dx+c\right)\right)a^{8}-4480\left(\cos\left(dx+c\right)^{3}-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)a^{8}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/240*(384*(cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 5/cos(d*x + c) + 15*cos(d*x
 + c))*a^8 - 4480*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^8 -
5*(105*d*x + 105*c - (87*tan(d*x + c)^5 + 136*tan(d*x + c)^3 + 57*tan(d*x +
 c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1) - 48*tan(d*
x + c))*a^8 - 840*(15*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan
(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 8*tan(d*x + c))*a^8 - 8400*(3*d*x + 3
*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^8 - 6720*(d*x +
c - tan(d*x + c))*a^8 + 13440*a^8*(1/cos(d*x + c) + cos(d*x + c)) + 240*a^8
*tan(d*x + c) + 1920*a^8/cos(d*x + c))/d
```

Fricas [A] time = 1.87544, size = 632, normalized size = 3.14

$$\frac{40\,a^{8}\cos\left(dx+c\right)^{7}+384\,a^{8}\cos\left(dx+c\right)^{6}-1526\,a^{8}\cos\left(dx+c\right)^{5}-6400\,a^{8}\cos\left(dx+c\right)^{4}+11865\,a^{8}\cos\left(dx+c\right)^{3}-45045}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="fricas")

[Out] 1/240*(40*a^8*cos(d*x + c)^7 + 384*a^8*cos(d*x + c)^6 - 1526*a^8*cos(d*x + c)^5 - 6400*a^8*cos(d*x + c)^4 + 11865*a^8*cos(d*x + c)^3 - 45045*a^8*d*x + 46080*a^8*cos(d*x + c)^2 + 30720*a^8 - 15*(3003*a^8*d*x - 4027*a^8)*cos(d*x + c) + (40*a^8*cos(d*x + c)^6 - 344*a^8*cos(d*x + c)^5 - 1870*a^8*cos(d*x + c)^4 + 4530*a^8*cos(d*x + c)^3 + 45045*a^8*d*x + 16395*a^8*cos(d*x + c)^2 - 29685*a^8*cos(d*x + c) + 30720*a^8)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**8,x)

[Out] Timed out

Giac [A] time = 1.21141, size = 312, normalized size = 1.55

$$-\frac{45045\,(dx+c)a^8 + \frac{61440\,a^8}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1} + \frac{2\left(14565\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 28800\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10} + 50855\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} - 174720\,a^8\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="giac")

[Out] -1/240*(45045*(d*x + c)*a^8 + 61440*a^8/(tan(1/2*d*x + 1/2*c) - 1) + 2*(14565*a^8*tan(1/2*d*x + 1/2*c)^11 - 28800*a^8*tan(1/2*d*x + 1/2*c)^10 + 50855*a^8*tan(1/2*d*x + 1/2*c)^9 - 174720*a^8*tan(1/2*d*x + 1/2*c)^8 + 36930*a^8*tan(1/2*d*x + 1/2*c)^7 - 400640*a^8*tan(1/2*d*x + 1/2*c)^6 - 36930*a^8*tan(1/2*d*x + 1/2*c)^5 - 426240*a^8*tan(1/2*d*x + 1/2*c)^4 - 50855*a^8*tan(1/2*d*x + 1/2*c)^3 - 211584*a^8*tan(1/2*d*x + 1/2*c)^2 - 14565*a^8*tan(1/2*d*x + 1/2*c) - 40064*a^8)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d

3.48 $\int \sec^3(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=121

$$\frac{a^8\sin^5(c+dx)}{5d}+\frac{2a^8\sin^4(c+dx)}{d}+\frac{10a^8\sin^3(c+dx)}{d}+\frac{36a^8\sin^2(c+dx)}{d}+\frac{64a^9}{d(a-a\sin(c+dx))}+\frac{129a^8\sin(c+dx)}{d}+$$

```
[Out] (192*a^8*Log[1 - Sin[c + d*x]])/d + (129*a^8*Sin[c + d*x])/d + (36*a^8*Sin[
c + d*x]^2)/d + (10*a^8*Sin[c + d*x]^3)/d + (2*a^8*Sin[c + d*x]^4)/d + (a^8
*Sin[c + d*x]^5)/(5*d) + (64*a^9)/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0940941, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{a^8\sin^5(c+dx)}{5d}+\frac{2a^8\sin^4(c+dx)}{d}+\frac{10a^8\sin^3(c+dx)}{d}+\frac{36a^8\sin^2(c+dx)}{d}+\frac{64a^9}{d(a-a\sin(c+dx))}+\frac{129a^8\sin(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (192*a^8*Log[1 - Sin[c + d*x]])/d + (129*a^8*Sin[c + d*x])/d + (36*a^8*Sin[
c + d*x]^2)/d + (10*a^8*Sin[c + d*x]^3)/d + (2*a^8*Sin[c + d*x]^4)/d + (a^8
*Sin[c + d*x]^5)/(5*d) + (64*a^9)/(d*(a - a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^8\,dx = \frac{a^3 \operatorname{Subst}\left(\int \frac{(a+x)^6}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3 \operatorname{Subst}\left(\int \left(129a^4+\frac{64a^6}{(a-x)^2}-\frac{192a^5}{a-x}+72a^3x+30a^2x^2+8ax^3+x^4\right)dx, x, a\sin$$
$$= \frac{192a^8\log(1-\sin(c+dx))}{d}+\frac{129a^8\sin(c+dx)}{d}+\frac{36a^8\sin^2(c+dx)}{d}+\frac{10a^8\sin$$

Mathematica [A] time = 0.259012, size = 111, normalized size = 0.92

$$\frac{a^8(1-\sin(c+dx))(\sin(c+dx)+1)\sec^2(c+dx)\left(\frac{1}{5}\sin^5(c+dx)+2\sin^4(c+dx)+10\sin^3(c+dx)+36\sin^2(c+dx)+1}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c + d*x]^2*(1 - Sin[c + d*x])*(1 + Sin[c + d*x])*(192*Log[1 - Sin[
c + d*x]] + 64/(1 - Sin[c + d*x]) + 129*Sin[c + d*x] + 36*Sin[c + d*x]^2 +
10*Sin[c + d*x]^3 + 2*Sin[c + d*x]^4 + Sin[c + d*x]^5/5))/d
```

Maple [B] time = 0.112, size = 345, normalized size = 2.9

$$192\,\frac{a^8\ln(\cos(dx+c))}{d}-192\,\frac{a^8\ln(\sec(dx+c)+\tan(dx+c))}{d}+68\,\frac{a^8(\sin(dx+c))^2}{d}+\frac{385\,a^8\sin(dx+c)}{2\,d}+\frac{a^8(\sin(}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 192/d*a^8*ln(cos(d*x+c))-192/d*a^8*ln(sec(d*x+c)+tan(d*x+c))+68*a^8*sin(d*x
+c)^2/d+385/2*a^8*sin(d*x+c)/d+1/2/d*a^8*sin(d*x+c)^7+4/d*a^8*sin(d*x+c)^6+
147/10*a^8*sin(d*x+c)^5/d+34*a^8*sin(d*x+c)^4/d+119/2*a^8*sin(d*x+c)^3/d+28
/d*a^8*tan(d*x+c)^2+4/d*a^8/cos(d*x+c)^2+4/d*a^8*sin(d*x+c)^8/cos(d*x+c)^2+
14/d*a^8*sin(d*x+c)^7/cos(d*x+c)^2+28/d*a^8*sin(d*x+c)^6/cos(d*x+c)^2+35/d*
a^8*sin(d*x+c)^5/cos(d*x+c)^2+14/d*a^8*sin(d*x+c)^3/cos(d*x+c)^2+1/2/d*a^8*
```

```
sec(d*x+c)*tan(d*x+c)+1/2/d*a^8*sin(d*x+c)^9/cos(d*x+c)^2
```

Maxima [A] time = 0.959338, size = 131, normalized size = 1.08

$$\frac{a^8 \sin (dx+c)^5 + 10\, a^8 \sin (dx+c)^4 + 50\, a^8 \sin (dx+c)^3 + 180\, a^8 \sin (dx+c)^2 + 960\, a^8 \log (\sin (dx+c) - 1) + 645\, a^8 \sin}{5\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/5*(a^8*sin(d*x + c)^5 + 10*a^8*sin(d*x + c)^4 + 50*a^8*sin(d*x + c)^3 + 1
80*a^8*sin(d*x + c)^2 + 960*a^8*log(sin(d*x + c) - 1) + 645*a^8*sin(d*x + c
) - 320*a^8/(sin(d*x + c) - 1))/d
```

Fricas [A] time = 1.87999, size = 328, normalized size = 2.71

$$-\frac{4\, a^8 \cos (dx+c)^6 - 172\, a^8 \cos (dx+c)^4 + 2192\, a^8 \cos (dx+c)^2 - 1119\, a^8 - 3840 \left(a^8 \sin (dx+c) - a^8\right) \log (-\sin (dx+c}{20\, (d \sin (dx+c) - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/20*(4*a^8*cos(d*x + c)^6 - 172*a^8*cos(d*x + c)^4 + 2192*a^8*cos(d*x + c
)^2 - 1119*a^8 - 3840*(a^8*sin(d*x + c) - a^8)*log(-sin(d*x + c) + 1) - (36
*a^8*cos(d*x + c)^4 - 592*a^8*cos(d*x + c)^2 - 2399*a^8)*sin(d*x + c))/(d*s
in(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**8,x)
```

[Out] Timed out

Giac [B] time = 1.26611, size = 371, normalized size = 3.07

$$-\frac{2\left(480\,a^8\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-960\,a^8\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{160\left(9\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-20\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,a^8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/5*(480*a^8*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 960*a^8*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) + 160*(9*a^8*tan(1/2*d*x + 1/2*c)^2 - 20*a^8*tan(1/2*d*x +
1/2*c) + 9*a^8)/(tan(1/2*d*x + 1/2*c) - 1)^2 - (1096*a^8*tan(1/2*d*x + 1/2*
c)^10 + 645*a^8*tan(1/2*d*x + 1/2*c)^9 + 5840*a^8*tan(1/2*d*x + 1/2*c)^8 +
2780*a^8*tan(1/2*d*x + 1/2*c)^7 + 12120*a^8*tan(1/2*d*x + 1/2*c)^6 + 4286*a
^8*tan(1/2*d*x + 1/2*c)^5 + 12120*a^8*tan(1/2*d*x + 1/2*c)^4 + 2780*a^8*tan
(1/2*d*x + 1/2*c)^3 + 5840*a^8*tan(1/2*d*x + 1/2*c)^2 + 645*a^8*tan(1/2*d*x
 + 1/2*c) + 1096*a^8)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.49 $\int \sec^4(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=179

$$-\frac{385a^8\cos^3(c+dx)}{4d}-\frac{231a^{16}\cos^5(c+dx)}{4d\left(a^8-a^8\sin(c+dx)\right)}+\frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{66a^{14}\cos^7(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)^3}-\frac{22a^{13}\cos^9(c+dx}{3d(a-a\sin(c+dx}$$

```
[Out] (1155*a^8*x)/8 - (385*a^8*Cos[c + d*x]^3)/(4*d) + (1155*a^8*Cos[c + d*x]*Si
n[c + d*x])/(8*d) + (2*a^15*Cos[c + d*x]^11)/(3*d*(a - a*Sin[c + d*x])^7) -
 (22*a^13*Cos[c + d*x]^9)/(3*d*(a - a*Sin[c + d*x])^5) - (66*a^14*Cos[c + d
*x]^7)/(d*(a^2 - a^2*Sin[c + d*x])^3) - (231*a^16*Cos[c + d*x]^5)/(4*d*(a^8
 - a^8*Sin[c + d*x]))
```

Rubi [A] time = 0.318657, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2670, 2680, 2679, 2682, 2635, 8}

$$-\frac{385a^8\cos^3(c+dx)}{4d}-\frac{231a^{16}\cos^5(c+dx)}{4d\left(a^8-a^8\sin(c+dx)\right)}+\frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{66a^{14}\cos^7(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)^3}-\frac{22a^{13}\cos^9(c+dx}{3d(a-a\sin(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (1155*a^8*x)/8 - (385*a^8*Cos[c + d*x]^3)/(4*d) + (1155*a^8*Cos[c + d*x]*Si
n[c + d*x])/(8*d) + (2*a^15*Cos[c + d*x]^11)/(3*d*(a - a*Sin[c + d*x])^7) -
 (22*a^13*Cos[c + d*x]^9)/(3*d*(a - a*Sin[c + d*x])^5) - (66*a^14*Cos[c + d
*x]^7)/(d*(a^2 - a^2*Sin[c + d*x])^3) - (231*a^16*Cos[c + d*x]^5)/(4*d*(a^8
 - a^8*Sin[c + d*x]))
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sin(c+dx))^8\,dx &= a^{16}\int \frac{\cos^{12}(c+dx)}{(a-a\sin(c+dx))^8}\,dx\\
&= \frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{1}{3}\left(11a^{14}\right)\int \frac{\cos^{10}(c+dx)}{(a-a\sin(c+dx))^6}\,dx\\
&= \frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{22a^{13}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^5}+\left(33a^{12}\right)\int \frac{\cos^8(c+dx)}{(a-a\sin(c+dx))^4}\\
&= \frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{22a^{13}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^5}-\frac{66a^{11}\cos^7(c+dx)}{d(a-a\sin(c+dx))^3}+\left(231a^1\right.\\
&= \frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{22a^{13}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^5}-\frac{66a^{11}\cos^7(c+dx)}{d(a-a\sin(c+dx))^3}-\frac{231a}{4d\left(a^2\right.}\\
&= -\frac{385a^8\cos^3(c+dx)}{4d}+\frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{22a^{13}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^5}-\frac{66a^{11}}{d(a-a}\\
&= -\frac{385a^8\cos^3(c+dx)}{4d}+\frac{1155a^8\cos(c+dx)\sin(c+dx)}{8d}+\frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}\\
&= \frac{1155a^8x}{8}-\frac{385a^8\cos^3(c+dx)}{4d}+\frac{1155a^8\cos(c+dx)\sin(c+dx)}{8d}+\frac{2a^{15}\cos^{11}(}{3d(a-a\sin(}\end{aligned}$$

Mathematica [C] time = 0.0528785, size = 59, normalized size = 0.33

$$\frac{64\sqrt{2}a^8(\sin(c+dx)+1)^{3/2}\sec^3(c+dx)\,{}_2F_1\left(-\frac{11}{2},-\frac{3}{2};-\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (64*Sqrt[2]*a^8*Hypergeometric2F1[-11/2, -3/2, -1/2, (1 - Sin[c + d*x])/2]*
Sec[c + d*x]^3*(1 + Sin[c + d*x])^(3/2))/(3*d)
```

Maple [B] time = 0.11, size = 478, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(1/3*sin(d*x+c)^9/cos(d*x+c)^3-2*sin(d*x+c)^9/cos(d*x+c)-2*(sin(d*
x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/
8*d*x+35/8*c)+8*a^8*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x
+c)-5/3*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+2
8*a^8*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d
*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+56*a^8*
(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/
3*sin(d*x+c)^2)*cos(d*x+c))+70*a^8*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+56*a
^8*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*
x+c)^2)*cos(d*x+c))+28/3*a^8*sin(d*x+c)^3/cos(d*x+c)^3+8/3*a^8/cos(d*x+c)^3
-a^8*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.46647, size = 420, normalized size = 2.35

$$\frac{224\,a^8\tan\left(dx+c\right)^3+64\left(\cos\left(dx+c\right)^3-\frac{9\,\cos(dx+c)^2-1}{\cos(dx+c)^3}-9\,\cos\left(dx+c\right)\right)a^8+\left(8\,\tan\left(dx+c\right)^3+105\,dx+105\,c-\frac{3\left(13\,\mathrm{t}\right.}{\mathrm{tan}(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/24*(224*a^8*tan(d*x + c)^3 + 64*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/
cos(d*x + c)^3 - 9*cos(d*x + c))*a^8 + (8*tan(d*x + c)^3 + 105*d*x + 105*c
- 3*(13*tan(d*x + c)^3 + 11*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^
2 + 1) - 72*tan(d*x + c))*a^8 + 112*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*t
an(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x + c))*a^8 + 560*(tan(d*x + c)
^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^8 + 8*(tan(d*x + c)^3 + 3*tan(d*x + c)
)*a^8 - 448*a^8*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)) -
448*(3*cos(d*x + c)^2 - 1)*a^8/cos(d*x + c)^3 + 64*a^8/cos(d*x + c)^3)/d
```

Fricas [A] time = 1.71977, size = 639, normalized size = 3.57

$$-\frac{6\,a^8\cos\left(dx+c\right)^6-52\,a^8\cos\left(dx+c\right)^5-317\,a^8\cos\left(dx+c\right)^4+1286\,a^8\cos\left(dx+c\right)^3+6930\,a^8dx+512\,a^8-\left(3465\,a^8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^8*cos(d*x + c)^6 - 52*a^8*cos(d*x + c)^5 - 317*a^8*cos(d*x + c)^
4 + 1286*a^8*cos(d*x + c)^3 + 6930*a^8*d*x + 512*a^8 - (3465*a^8*d*x + 5641
*a^8)*cos(d*x + c)^2 + (3465*a^8*d*x - 6674*a^8)*cos(d*x + c) - (6*a^8*cos(
d*x + c)^5 + 58*a^8*cos(d*x + c)^4 - 259*a^8*cos(d*x + c)^3 + 6930*a^8*d*x
- 1545*a^8*cos(d*x + c)^2 - 512*a^8 + (3465*a^8*d*x - 7186*a^8)*cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d
)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23457, size = 270, normalized size = 1.51

$$\frac{3465\,(dx+c)a^8 + \frac{1024\left(6\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3} + \frac{2\left(369\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1728\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+393\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/24*(3465*(d*x + c)*a^8 + 1024*(6*a^8*tan(1/2*d*x + 1/2*c)^2 - 15*a^8*tan(
1/2*d*x + 1/2*c) + 7*a^8)/(tan(1/2*d*x + 1/2*c) - 1)^3 + 2*(369*a^8*tan(1/2
*d*x + 1/2*c)^7 - 1728*a^8*tan(1/2*d*x + 1/2*c)^6 + 393*a^8*tan(1/2*d*x + 1
/2*c)^5 - 5568*a^8*tan(1/2*d*x + 1/2*c)^4 - 393*a^8*tan(1/2*d*x + 1/2*c)^3
- 5696*a^8*tan(1/2*d*x + 1/2*c)^2 - 369*a^8*tan(1/2*d*x + 1/2*c) - 1856*a^8
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.50 $\int \sec^5(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=110

$$-\frac{a^8\sin^3(c+dx)}{3d}-\frac{4a^8\sin^2(c+dx)}{d}+\frac{16a^{10}}{d(a-a\sin(c+dx))^2}-\frac{80a^9}{d(a-a\sin(c+dx))}-\frac{31a^8\sin(c+dx)}{d}-\frac{80a^8\log(1-\text{si}}{d}$$

```
[Out] (-80*a^8*Log[1 - Sin[c + d*x]])/d - (31*a^8*Sin[c + d*x])/d - (4*a^8*Sin[c
+ d*x]^2)/d - (a^8*Sin[c + d*x]^3)/(3*d) + (16*a^10)/(d*(a - a*Sin[c + d*x]
)^2) - (80*a^9)/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.091071, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$-\frac{a^8\sin^3(c+dx)}{3d}-\frac{4a^8\sin^2(c+dx)}{d}+\frac{16a^{10}}{d(a-a\sin(c+dx))^2}-\frac{80a^9}{d(a-a\sin(c+dx))}-\frac{31a^8\sin(c+dx)}{d}-\frac{80a^8\log(1-\text{si}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-80*a^8*Log[1 - Sin[c + d*x]])/d - (31*a^8*Sin[c + d*x])/d - (4*a^8*Sin[c
+ d*x]^2)/d - (a^8*Sin[c + d*x]^3)/(3*d) + (16*a^10)/(d*(a - a*Sin[c + d*x]
)^2) - (80*a^9)/(d*(a - a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^8\,dx = \frac{a^5\operatorname{Subst}\left(\int \frac{(a+x)^5}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\operatorname{Subst}\left(\int \left(-31a^2+\frac{32a^5}{(a-x)^3}-\frac{80a^4}{(a-x)^2}+\frac{80a^3}{a-x}-8ax-x^2\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{80a^8\log(1-\sin(c+dx))}{d}-\frac{31a^8\sin(c+dx)}{d}-\frac{4a^8\sin^2(c+dx)}{d}-\frac{a^8\sin^3(c+}{3d}$$

Mathematica [A] time = 0.445311, size = 73, normalized size = 0.66

$$\frac{a^8\left(-\frac{1}{3}\sin^3(c+dx)-4\sin^2(c+dx)-31\sin(c+dx)+\frac{16(5\sin(c+dx)-4)}{(\sin(c+dx)-1)^2}-80\log(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a^8*(-80*Log[1 - Sin[c + d*x]] - 31*Sin[c + d*x] - 4*Sin[c + d*x]^2 - Sin[
c + d*x]^3/3 + (16*(-4 + 5*Sin[c + d*x]))/(-1 + Sin[c + d*x])^2))/d
```

Maple [B] time = 0.116, size = 503, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^8,x)
```

```
[Out] 7/d*a^8*sin(d*x+c)^7/cos(d*x+c)^4+35/2/d*a^8*sin(d*x+c)^5/cos(d*x+c)^4+7/d*
a^8*sin(d*x+c)^3/cos(d*x+c)^4+1/4/d*a^8*tan(d*x+c)*sec(d*x+c)^3+14/d*a^8*si
n(d*x+c)^4/cos(d*x+c)^4+1/4/d*a^8*sin(d*x+c)^9/cos(d*x+c)^4+2/d*a^8*sin(d*x
+c)^8/cos(d*x+c)^4-80/d*a^8*ln(cos(d*x+c))+80/d*a^8*ln(sec(d*x+c)+tan(d*x+c
))-5/8/d*a^8*sin(d*x+c)^7-4/d*a^8*sin(d*x+c)^6-12*a^8*sin(d*x+c)^2/d-665/24
*a^8*sin(d*x+c)^3/d-6*a^8*sin(d*x+c)^4/d-91/8*a^8*sin(d*x+c)^5/d-637/8*a^8*
sin(d*x+c)/d+2/d*a^8/cos(d*x+c)^4+14/d*a^8*tan(d*x+c)^4-28/d*a^8*tan(d*x+c)
^2-4/d*a^8*sin(d*x+c)^8/cos(d*x+c)^2-21/2/d*a^8*sin(d*x+c)^7/cos(d*x+c)^2-3
```

```
5/4/d*a^8*sin(d*x+c)^5/cos(d*x+c)^2+7/2/d*a^8*sin(d*x+c)^3/cos(d*x+c)^2+3/8
/d*a^8*sec(d*x+c)*tan(d*x+c)-5/8/d*a^8*sin(d*x+c)^9/cos(d*x+c)^2
```

Maxima [A] time = 0.959797, size = 128, normalized size = 1.16

$$-\frac{a^{8} \sin\left(dx+c\right)^{3}+12\, a^{8} \sin\left(dx+c\right)^{2}+240\, a^{8} \log\left(\sin\left(dx+c\right)-1\right)+93\, a^{8} \sin\left(dx+c\right)-\frac{48\left(5\, a^{8} \sin(dx+c)-4\, a^{8}\right)}{\sin(dx+c)^{2}-2\, \sin(dx+c)+1}}{3\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/3*(a^8*sin(d*x + c)^3 + 12*a^8*sin(d*x + c)^2 + 240*a^8*log(sin(d*x + c)
 - 1) + 93*a^8*sin(d*x + c) - 48*(5*a^8*sin(d*x + c) - 4*a^8)/(sin(d*x + c)
^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.8313, size = 344, normalized size = 3.13

$$\frac{10\, a^{8} \cos\left(dx+c\right)^{4}+160\, a^{8} \cos\left(dx+c\right)^{2}+16\, a^{8}-240\left(a^{8} \cos\left(dx+c\right)^{2}+2\, a^{8} \sin\left(dx+c\right)-2\, a^{8}\right) \log\left(-\sin\left(dx+c\right)+\right.}{3\left(d \cos\left(dx+c\right)^{2}+2\, d \sin\left(dx+c\right)-2\, d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/3*(10*a^8*cos(d*x + c)^4 + 160*a^8*cos(d*x + c)^2 + 16*a^8 - 240*(a^8*cos
(d*x + c)^2 + 2*a^8*sin(d*x + c) - 2*a^8)*log(-sin(d*x + c) + 1) + (a^8*cos
(d*x + c)^4 - 72*a^8*cos(d*x + c)^2 - 64*a^8)*sin(d*x + c))/(d*cos(d*x + c)
^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23866, size = 328, normalized size = 2.98

$$\frac{2\left(120\,a^8\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-240\,a^8\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{220\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+93\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+684\,a^8\tan\left(\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/3*(120*a^8*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 240*a^8*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - (220*a^8*tan(1/2*d*x + 1/2*c)^6 + 93*a^8*tan(1/2*d*x + 1/2
*c)^5 + 684*a^8*tan(1/2*d*x + 1/2*c)^4 + 190*a^8*tan(1/2*d*x + 1/2*c)^3 + 6
84*a^8*tan(1/2*d*x + 1/2*c)^2 + 93*a^8*tan(1/2*d*x + 1/2*c) + 220*a^8)/(tan
(1/2*d*x + 1/2*c)^2 + 1)^3 + 4*(125*a^8*tan(1/2*d*x + 1/2*c)^4 - 536*a^8*ta
n(1/2*d*x + 1/2*c)^3 + 846*a^8*tan(1/2*d*x + 1/2*c)^2 - 536*a^8*tan(1/2*d*x
 + 1/2*c) + 125*a^8)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

3.51 $\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

```
[Out] (3*x)/(8*a) + Cos[c + d*x]^5/(5*a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(8*a*d
) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.0682715, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2682, 2635, 8}

$$\frac{\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(8*a) + Cos[c + d*x]^5/(5*a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(8*a*d
) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\cos^5(c+dx)}{5ad}+\frac{\int \cos^4(c+dx)\,dx}{a}\\ &= \frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{3\int \cos^2(c+dx)\,dx}{4a}\\ &= \frac{\cos^5(c+dx)}{5ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{3\int 1\,dx}{8a}\\ &= \frac{3x}{8a}+\frac{\cos^5(c+dx)}{5ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}\end{aligned}$$

Mathematica [A] time = 0.783714, size = 141, normalized size = 1.93

$$-\frac{\left(30\sqrt{1-\sin(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\left(8\sin^5(c+dx)-18\sin^4(c+dx)-6\sin^3(c+dx)+41\sin^2(\right.\right.}{40ad(\sin(c+dx)-1)^4(\sin(c+dx)+1)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Cos[c + d*x]^7*(30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c
+ d*x]] + Sqrt[1 + Sin[c + d*x]]*(-8 - 17*Sin[c + d*x] + 41*Sin[c + d*x]^2
- 6*Sin[c + d*x]^3 - 18*Sin[c + d*x]^4 + 8*Sin[c + d*x]^5)))/(40*a*d*(-1 +
Sin[c + d*x])^4*(1 + Sin[c + d*x])^(7/2))
```

Maple [B] time = 0.06, size = 245, normalized size = 3.4

$$-\frac{5}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-5}+2\,\frac{(\tan(1/2\,dx+c/2))^8}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^5}-\frac{1}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] -5/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9+2/d/a/(1+tan(1/2*d
*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8-1/2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(
1/2*d*x+1/2*c)^7+4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4+1/2/
```

```
d/a/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3+5/4/d/a/(1+tan(1/2*d*x+
1/2*c)^2)^5*tan(1/2*d*x+1/2*c)+2/5/d/a/(1+tan(1/2*d*x+1/2*c)^2)^5+3/4/a/d*a
rctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.44425, size = 348, normalized size = 4.77

$$\frac{\frac{\frac{25\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{80\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{10\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{40\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{25\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+8}{a+\frac{5\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5\,a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{15\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/20*((25*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 80*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 10*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 40*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 25*sin(d*x + c)^9
/(cos(d*x + c) + 1)^9 + 8)/(a + 5*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
0*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 + 5*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.60331, size = 126, normalized size = 1.73

$$\frac{8\,\cos(dx+c)^5+15\,dx+5\left(2\,\cos(dx+c)^3+3\,\cos(dx+c)\right)\sin(dx+c)}{40\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*cos(d*x + c)^5 + 15*d*x + 5*(2*cos(d*x + c)^3 + 3*cos(d*x + c))*sin
(d*x + c))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16256, size = 154, normalized size = 2.11

$$\frac{\frac{15\,(dx+c)}{a} - \frac{2\left(25\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} - 40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8} + 10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} - 80\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4} - 10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} - 25\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}a}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*(d*x + c)/a - 2*(25*tan(1/2*d*x + 1/2*c)^9 - 40*tan(1/2*d*x + 1/2*
c)^8 + 10*tan(1/2*d*x + 1/2*c)^7 - 80*tan(1/2*d*x + 1/2*c)^4 - 10*tan(1/2*d
*x + 1/2*c)^3 - 25*tan(1/2*d*x + 1/2*c) - 8)/((tan(1/2*d*x + 1/2*c)^2 + 1)^
5*a))/d
```

3.52 $\int \frac{\cos^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=47

$$\frac{(a-a\sin(c+dx))^4}{4a^5d}-\frac{2(a-a\sin(c+dx))^3}{3a^4d}$$

```
[Out] (-2*(a - a*Sin[c + d*x])^3)/(3*a^4*d) + (a - a*Sin[c + d*x])^4/(4*a^5*d)
```

Rubi [A] time = 0.0562803, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a-a\sin(c+dx))^4}{4a^5d}-\frac{2(a-a\sin(c+dx))^3}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*(a - a*Sin[c + d*x])^3)/(3*a^4*d) + (a - a*Sin[c + d*x])^4/(4*a^5*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int (a-x)^2(a+x)\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(2a(a-x)^2-(a-x)^3\right)\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= -\frac{2(a-a\sin(c+dx))^3}{3a^4 d}+\frac{(a-a\sin(c+dx))^4}{4a^5 d}$$

Mathematica [A] time = 0.0932865, size = 46, normalized size = 0.98

$$\frac{\sin(c+dx)\left(3\sin^3(c+dx)-4\sin^2(c+dx)-6\sin(c+dx)+12\right)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(12 - 6*Sin[c + d*x] - 4*Sin[c + d*x]^2 + 3*Sin[c + d*x]^3))/
(12*a*d)
```

Maple [A] time = 0.05, size = 45, normalized size = 1.

$$\frac{1}{da}\left(\frac{(\sin(dx+c))^4}{4}-\frac{(\sin(dx+c))^3}{3}-\frac{(\sin(dx+c))^2}{2}+\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/4*sin(d*x+c)^4-1/3*sin(d*x+c)^3-1/2*sin(d*x+c)^2+sin(d*x+c))
```

Maxima [A] time = 0.930849, size = 63, normalized size = 1.34

$$\frac{3\,\sin(dx+c)^4-4\,\sin(dx+c)^3-6\,\sin(dx+c)^2+12\,\sin(dx+c)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*sin(d*x + c)^4 - 4*sin(d*x + c)^3 - 6*sin(d*x + c)^2 + 12*sin(d*x +
 c))/(a*d)
```

Fricas [A] time = 1.67223, size = 93, normalized size = 1.98

$$\frac{3 \cos(dx+c)^4 + 4\left(\cos(dx+c)^2 + 2\right)\sin(dx+c)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(d*x + c)^4 + 4*(cos(d*x + c)^2 + 2)*sin(d*x + c))/(a*d)
```

Sympy [A] time = 126.403, size = 779, normalized size = 16.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-7*tan(c/2 + d*x/2)**8/(15*a*d*tan(c/2 + d*x/2)**8 + 60*a*d*tan(
c/2 + d*x/2)**6 + 90*a*d*tan(c/2 + d*x/2)**4 + 60*a*d*tan(c/2 + d*x/2)**2 +
 15*a*d) + 30*tan(c/2 + d*x/2)**7/(15*a*d*tan(c/2 + d*x/2)**8 + 60*a*d*tan(
c/2 + d*x/2)**6 + 90*a*d*tan(c/2 + d*x/2)**4 + 60*a*d*tan(c/2 + d*x/2)**2 +
 15*a*d) - 58*tan(c/2 + d*x/2)**6/(15*a*d*tan(c/2 + d*x/2)**8 + 60*a*d*tan(
c/2 + d*x/2)**6 + 90*a*d*tan(c/2 + d*x/2)**4 + 60*a*d*tan(c/2 + d*x/2)**2 +
 15*a*d) + 50*tan(c/2 + d*x/2)**5/(15*a*d*tan(c/2 + d*x/2)**8 + 60*a*d*tan(
c/2 + d*x/2)**6 + 90*a*d*tan(c/2 + d*x/2)**4 + 60*a*d*tan(c/2 + d*x/2)**2 +
 15*a*d) - 42*tan(c/2 + d*x/2)**4/(15*a*d*tan(c/2 + d*x/2)**8 + 60*a*d*tan(
c/2 + d*x/2)**6 + 90*a*d*tan(c/2 + d*x/2)**4 + 60*a*d*tan(c/2 + d*x/2)**2 +
 15*a*d) + 50*tan(c/2 + d*x/2)**3/(15*a*d*tan(c/2 + d*x/2)**8 + 60*a*d*tan(
c/2 + d*x/2)**6 + 90*a*d*tan(c/2 + d*x/2)**4 + 60*a*d*tan(c/2 + d*x/2)**2 +
 15*a*d) - 58*tan(c/2 + d*x/2)**2/(15*a*d*tan(c/2 + d*x/2)**8 + 60*a*d*tan(
c/2 + d*x/2)**6 + 90*a*d*tan(c/2 + d*x/2)**4 + 60*a*d*tan(c/2 + d*x/2)**2 +
 15*a*d) + 30*tan(c/2 + d*x/2)/(15*a*d*tan(c/2 + d*x/2)**8 + 60*a*d*tan(c/2
```

```
 + d*x/2)**6 + 90*a*d*tan(c/2 + d*x/2)**4 + 60*a*d*tan(c/2 + d*x/2)**2 + 15
*a*d) - 7/(15*a*d*tan(c/2 + d*x/2)**8 + 60*a*d*tan(c/2 + d*x/2)**6 + 90*a*d
*tan(c/2 + d*x/2)**4 + 60*a*d*tan(c/2 + d*x/2)**2 + 15*a*d), Ne(d, 0)), (x*
cos(c)**5/(a*sin(c) + a), True))
```

Giac [A] time = 1.14575, size = 63, normalized size = 1.34

$$\frac{3\sin\left(dx+c\right)^{4}-4\sin\left(dx+c\right)^{3}-6\sin\left(dx+c\right)^{2}+12\sin\left(dx+c\right)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*sin(d*x + c)^4 - 4*sin(d*x + c)^3 - 6*sin(d*x + c)^2 + 12*sin(d*x +
 c))/(a*d)
```

3.53 $\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

```
[Out] x/(2*a) + Cos[c + d*x]^3/(3*a*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.0550841, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2682, 2635, 8}

$$\frac{\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(2*a) + Cos[c + d*x]^3/(3*a*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\cos^3(c+dx)}{3ad}+\frac{\int \cos^2(c+dx)\,dx}{a}\\ &= \frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}\end{aligned}$$

Mathematica [B] time = 0.32849, size = 119, normalized size = 2.43

$$-\frac{\left(\sqrt{\sin(c+dx)+1}\left(2\sin^3(c+dx)-5\sin^2(c+dx)+\sin(c+dx)+2\right)-6\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}\right)\cos^5(c+}{6ad(\sin(c+dx)-1)^3(\sin(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Cos[c + d*x]^5*(-6*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c
+ d*x]] + Sqrt[1 + Sin[c + d*x]]*(2 + Sin[c + d*x] - 5*Sin[c + d*x]^2 + 2*S
in[c + d*x]^3)))/(6*a*d*(-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^(5/2))
```

Maple [B] time = 0.048, size = 141, normalized size = 2.9

$$-\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-3}+2\,\frac{(\tan(1/2\,dx+c/2))^4}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+2/d/a/(1+tan(1/2*d*x
+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4+1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*
d*x+1/2*c)+2/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3+1/a/d*arctan(tan(1/2*d*x+1/2*
c))
```

Maxima [B] time = 1.42872, size = 211, normalized size = 4.31

$$\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+2}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2)/(a + 3*a*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6) + 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/
d
```

Fricas [A] time = 1.52181, size = 92, normalized size = 1.88

$$\frac{2\cos(dx+c)^3+3dx+3\cos(dx+c)\sin(dx+c)}{6ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(d*x + c)^3 + 3*d*x + 3*cos(d*x + c)*sin(d*x + c))/(a*d)
```

Sympy [A] time = 20.8105, size = 697, normalized size = 14.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((6*d*x*tan(c/2 + d*x/2)**6/(12*a*d*tan(c/2 + d*x/2)**6 + 36*a*d*t
an(c/2 + d*x/2)**4 + 36*a*d*tan(c/2 + d*x/2)**2 + 12*a*d) + 18*d*x*tan(c/2
+ d*x/2)**4/(12*a*d*tan(c/2 + d*x/2)**6 + 36*a*d*tan(c/2 + d*x/2)**4 + 36*a
```

```
*d*tan(c/2 + d*x/2)**2 + 12*a*d) + 18*d*x*tan(c/2 + d*x/2)**2/(12*a*d*tan(c
/2 + d*x/2)**6 + 36*a*d*tan(c/2 + d*x/2)**4 + 36*a*d*tan(c/2 + d*x/2)**2 +
12*a*d) + 6*d*x/(12*a*d*tan(c/2 + d*x/2)**6 + 36*a*d*tan(c/2 + d*x/2)**4 +
36*a*d*tan(c/2 + d*x/2)**2 + 12*a*d) - 3*tan(c/2 + d*x/2)**6/(12*a*d*tan(c/
2 + d*x/2)**6 + 36*a*d*tan(c/2 + d*x/2)**4 + 36*a*d*tan(c/2 + d*x/2)**2 + 1
2*a*d) - 12*tan(c/2 + d*x/2)**5/(12*a*d*tan(c/2 + d*x/2)**6 + 36*a*d*tan(c/
2 + d*x/2)**4 + 36*a*d*tan(c/2 + d*x/2)**2 + 12*a*d) + 15*tan(c/2 + d*x/2)*
*4/(12*a*d*tan(c/2 + d*x/2)**6 + 36*a*d*tan(c/2 + d*x/2)**4 + 36*a*d*tan(c/
2 + d*x/2)**2 + 12*a*d) - 9*tan(c/2 + d*x/2)**2/(12*a*d*tan(c/2 + d*x/2)**6
 + 36*a*d*tan(c/2 + d*x/2)**4 + 36*a*d*tan(c/2 + d*x/2)**2 + 12*a*d) + 12*t
an(c/2 + d*x/2)/(12*a*d*tan(c/2 + d*x/2)**6 + 36*a*d*tan(c/2 + d*x/2)**4 +
36*a*d*tan(c/2 + d*x/2)**2 + 12*a*d) + 5/(12*a*d*tan(c/2 + d*x/2)**6 + 36*a
*d*tan(c/2 + d*x/2)**4 + 36*a*d*tan(c/2 + d*x/2)**2 + 12*a*d), Ne(d, 0)), (
x*cos(c)**4/(a*sin(c) + a), True))
```

Giac [A] time = 1.16654, size = 101, normalized size = 2.06

$$\frac{\frac{3(dx+c)}{a} - \frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3 a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^5 - 6*tan(1/2*d*x + 1/2*c)^4
 - 3*tan(1/2*d*x + 1/2*c) - 2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

3.54 $\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

```
[Out] Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rubi [A] time = 0.0457101, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2667}

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int (a-x)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.0384828, size = 24, normalized size = 0.75

$$-\frac{(\sin(c+dx)-2)\sin(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((-2 + Sin[c + d*x])*Sin[c + d*x])/(2*a*d)
```

Maple [A] time = 0.015, size = 28, normalized size = 0.9

$$-\frac{1}{da}\left(\frac{(\sin(dx+c))^2}{2}-\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d/a*(1/2*sin(d*x+c)^2-sin(d*x+c))
```

Maxima [A] time = 0.938048, size = 34, normalized size = 1.06

$$-\frac{\sin(dx+c)^2-2\sin(dx+c)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

Fricas [A] time = 1.65015, size = 61, normalized size = 1.91

$$\frac{\cos(dx+c)^2+2\sin(dx+c)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(d*x + c)^2 + 2*sin(d*x + c))/(a*d)
```

Sympy [A] time = 7.86154, size = 158, normalized size = 4.94

$$\begin{cases} \frac{2\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{2\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for}\ d \neq 0 \\ \frac{x\cos^{3}(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*tan(c/2 +
 d*x/2)**2 + a*d) - 2*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*
tan(c/2 + d*x/2)**2 + a*d) + 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**4 +
2*a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) + a), T
rue))
```

Giac [A] time = 1.1559, size = 34, normalized size = 1.06

$$-\frac{\sin(dx+c)^{2} - 2\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

3.55 $\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=19

$$\frac{\cos(c+dx)}{ad}+\frac{x}{a}$$

```
[Out] x/a + Cos[c + d*x]/(a*d)
```

Rubi [A] time = 0.0430879, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2682, 8}

$$\frac{\cos(c+dx)}{ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + Cos[c + d*x]/(a*d)
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\cos(c+dx)}{ad}+\frac{\int 1\,dx}{a}\\ &= \frac{x}{a}+\frac{\cos(c+dx)}{ad}\end{aligned}$$

Mathematica [B] time = 0.124871, size = 97, normalized size = 5.11

$$-\frac{\left(2\sqrt{1-\sin(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+(\sin(c+dx)-1)\sqrt{\sin(c+dx)+1}\right)\cos^3(c+dx)}{ad(\sin(c+dx)-1)^2(\sin(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]^3*(2*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c
+ d*x]] + (-1 + Sin[c + d*x])*Sqrt[1 + Sin[c + d*x]]))/(a*d*(-1 + Sin[c + d
*x])^2*(1 + Sin[c + d*x])^(3/2)))
```

Maple [B] time = 0., size = 43, normalized size = 2.3

$$2\,\frac{1}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)}+2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/a/d/(1+tan(1/2*d*x+1/2*c)^2)+2/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.40579, size = 70, normalized size = 3.68

$$\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2))/d
```

Fricas [A] time = 1.62544, size = 38, normalized size = 2.

$$\frac{dx + \cos(dx + c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x + cos(d*x + c))/(a*d)
```

Sympy [A] time = 3.60863, size = 119, normalized size = 6.26

$$\begin{cases} \frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{dx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{1}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) + d*x/(a
*d*tan(c/2 + d*x/2)**2 + a*d) - tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**
2 + a*d) + 1/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*cos(c)**2/(a*si
n(c) + a), True))
```

Giac [A] time = 1.30259, size = 46, normalized size = 2.42

$$\frac{\frac{dx+c}{a} + \frac{2}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a + 2/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

3.56 $\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=16

$$\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] Log[1 + Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0255872, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 31}

$$\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\log(1+\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.0107324, size = 16, normalized size = 1.

$$\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a*d)
```

Maple [A] time = 0.012, size = 19, normalized size = 1.2

$$\frac{\ln(a+a\sin(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d*ln(a+a*sin(d*x+c))/a
```

Maxima [A] time = 0.932045, size = 24, normalized size = 1.5

$$\frac{\log(a\sin(dx+c)+a)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(a*sin(d*x + c) + a)/(a*d)
```

Fricas [A] time = 1.69334, size = 39, normalized size = 2.44

$$\frac{\log\left(\sin\left(dx+c\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] log(sin(d*x + c) + 1)/(a*d)
```

Sympy [A] time = 0.545384, size = 24, normalized size = 1.5

$$\begin{cases} \frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad} & \text{for}\ d \neq 0 \\ \frac{x\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((log(sin(c + d*x) + 1)/(a*d), Ne(d, 0)), (x*cos(c)/(a*sin(c) + a)
, True))
```

Giac [A] time = 1.33604, size = 26, normalized size = 1.62

$$\frac{\log\left(|a\sin\left(dx+c\right)+a|\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(a*sin(d*x + c) + a))/(a*d)
```

3.57 $\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{1}{2d(a\sin(c+dx)+a)}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - 1/(2*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0505354, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2667, 44, 206}

$$\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{1}{2d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - 1/(2*d*(a + a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{(a-x)(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a\,\mathrm{Subst}\left(\int \left(\frac{1}{2a(a+x)^2}+\frac{1}{2a\left(a^2-x^2\right)}\right)dx, x, a\sin(c+dx)\right)}{d}\\
&= -\frac{1}{2d(a+a\sin(c+dx))}+\frac{\mathrm{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c+dx)\right)}{2d}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{1}{2d(a+a\sin(c+dx))}
\end{aligned}
$$

Mathematica [A] time = 0.039131, size = 30, normalized size = 0.81

$$\frac{\tanh^{-1}(\sin(c+dx))-\frac{1}{\sin(c+dx)+1}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]] - (1 + Sin[c + d*x])^(-1))/(2*a*d)
```

Maple [A] time = 0., size = 54, normalized size = 1.5

$$-\frac{\ln(\sin(dx+c)-1)}{4\,da}-\frac{1}{2\,da\,(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{4\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/a/d*ln(sin(d*x+c)-1)-1/2/a/d/(1+sin(d*x+c))+1/4*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 0.938124, size = 63, normalized size = 1.7

$$\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c)-1)}{a} - \frac{2}{a\sin(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(log(sin(d*x + c) + 1)/a - log(sin(d*x + c) - 1)/a - 2/(a*sin(d*x + c)
+ a))/d
```

Fricas [A] time = 1.63636, size = 163, normalized size = 4.41

$$\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1) - (\sin(dx+c)+1)\log(-\sin(dx+c)+1) - 2}{4\,(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (sin(d*x + c) + 1)*log(-sin
(d*x + c) + 1) - 2)/(a*d*sin(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.56881, size = 78, normalized size = 2.11

$$\frac{\frac{\log\left(|\sin\left(dx+c\right)+1|\right)}{a} - \frac{\log\left(|\sin\left(dx+c\right)-1|\right)}{a} - \frac{\sin\left(dx+c\right)+3}{a\left(\sin\left(dx+c\right)+1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c) - 1))/a - (sin(d*x
 + c) + 3)/(a*(sin(d*x + c) + 1)))/d
```

3.58 $\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=42

$$\frac{2\tan(c+dx)}{3ad}-\frac{\sec(c+dx)}{3d(a\sin(c+dx)+a)}$$

```
[Out] -Sec[c + d*x]/(3*d*(a + a*Sin[c + d*x])) + (2*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.0522441, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2672, 3767, 8}

$$\frac{2\tan(c+dx)}{3ad}-\frac{\sec(c+dx)}{3d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Sec[c + d*x]/(3*d*(a + a*Sin[c + d*x])) + (2*Tan[c + d*x])/(3*a*d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))} + \frac{2\int \sec^2(c+dx)\,dx}{3a}$$
$$= -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))} - \frac{2\,\mathrm{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{3ad}$$
$$= -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))} + \frac{2\tan(c+dx)}{3ad}$$

Mathematica [A] time = 0.0547872, size = 45, normalized size = 1.07

$$\frac{2\tan(c+dx)-\cos(2(c+dx))\sec(c+dx)}{3ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-(Cos[2*(c + d*x)]*Sec[c + d*x]) + 2*Tan[c + d*x])/(3*a*d*(1 + Sin[c + d*x
]))
```

Maple [A] time = 0., size = 70, normalized size = 1.7

$$2\frac{1}{da}\left(-1/4\left(\tan(1/2\,dx+c/2)-1\right)^{-1}-1/3\left(\tan(1/2\,dx+c/2)+1\right)^{-3}+1/2\left(\tan(1/2\,dx+c/2)+1\right)^{-2}-3/4\left(\tan(1/2\,dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/d/a*(-1/4/(tan(1/2*d*x+1/2*c)-1)-1/3/(tan(1/2*d*x+1/2*c)+1)^3+1/2/(tan(1/
2*d*x+1/2*c)+1)^2-3/4/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 0.948407, size = 174, normalized size = 4.14

$$\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/((a + 2*a*sin(d*x + c)/(cos(
d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4)*d)
```

Fricas [A] time = 1.6413, size = 131, normalized size = 3.12

$$-\frac{2\cos\left(dx+c\right)^{2}-2\sin\left(dx+c\right)-1}{3\left(ad\cos\left(dx+c\right)\sin\left(dx+c\right)+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(d*x + c)^2 - 2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)*sin(d*x + c)
 + a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^{2}(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.13329, size = 90, normalized size = 2.14

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) + (9*tan(1/2*d*x + 1/2*c)^2 + 12*tan
(1/2*d*x + 1/2*c) + 7)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

3.59 $\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=77

$$-\frac{a}{8d(a\sin(c+dx)+a)^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{1}{4d(a\sin(c+dx)+a)}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) + 1/(8*d*(a - a*Sin[c + d*x])) - a/(8*d*(
a + a*Sin[c + d*x])^2) - 1/(4*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0757208, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$-\frac{a}{8d(a\sin(c+dx)+a)^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{1}{4d(a\sin(c+dx)+a)}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) + 1/(8*d*(a - a*Sin[c + d*x])) - a/(8*d*(
a + a*Sin[c + d*x])^2) - 1/(4*d*(a + a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{a^3\operatorname{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^3\operatorname{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2}+\frac{1}{4a^2(a+x)^3}+\frac{1}{4a^3(a+x)^2}+\frac{3}{8a^3\left(a^2-x^2\right)}\right)dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{1}{8d(a-a\sin(c+dx))}-\frac{a}{8d(a+a\sin(c+dx))^2}-\frac{1}{4d(a+a\sin(c+dx))}+\frac{3\operatorname{Subst}\left(\int \frac{1}{a^2-x^2}\,dx\right.}{8d}\\
&= \frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{a}{8d(a+a\sin(c+dx))^2}-\frac{1}{4d(a+a\sin(c+dx}\end{aligned}$$

Mathematica [A] time = 0.0936595, size = 75, normalized size = 0.97

$$-\frac{\sec^2(c+dx)\left(-3\sin^2(c+dx)-3\sin(c+dx)+3(\sin(c+dx)-1)(\sin(c+dx)+1)^2\tanh^{-1}(\sin(c+dx))+2\right)}{8ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]^2*(2 - 3*Sin[c + d*x] - 3*Sin[c + d*x]^2 + 3*ArcTanh[Sin[c +
 d*x]]*(-1 + Sin[c + d*x])*(1 + Sin[c + d*x])^2))/(8*a*d*(1 + Sin[c + d*x])
)
```

Maple [A] time = 0., size = 90, normalized size = 1.2

$$-\frac{1}{8\,da\,(\sin(dx+c)-1)}-\frac{3\,\ln(\sin(dx+c)-1)}{16\,da}-\frac{1}{8\,da\,(1+\sin(dx+c))^2}-\frac{1}{4\,da\,(1+\sin(dx+c))}+\frac{3\,\ln(1+\sin(dx}{16\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/8/a/d/(sin(d*x+c)-1)-3/16/a/d*ln(sin(d*x+c)-1)-1/8/a/d/(1+sin(d*x+c))^2-
1/4/a/d/(1+sin(d*x+c))+3/16*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 0.942087, size = 123, normalized size = 1.6

$$-\frac{\frac{2\left(3 \sin(dx+c)^2+3 \sin(dx+c)-2\right)}{a \sin(dx+c)^3+a \sin(dx+c)^2-a \sin(dx+c)-a}-\frac{3 \log(\sin(dx+c)+1)}{a}+\frac{3 \log(\sin(dx+c)-1)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*sin(d*x + c)^3 + a*sin(
d*x + c)^2 - a*sin(d*x + c) - a) - 3*log(sin(d*x + c) + 1)/a + 3*log(sin(d*
x + c) - 1)/a)/d
```

Fricas [A] time = 1.75276, size = 336, normalized size = 4.36

$$-\frac{6\,\cos\left(dx+c\right)^2-3\left(\cos\left(dx+c\right)^2\sin\left(dx+c\right)+\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(\cos\left(dx+c\right)^2\sin\left(dx+c\right)+\cos\right.}{16\left(ad\cos\left(dx+c\right)^2\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(6*cos(d*x + c)^2 - 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*
log(sin(d*x + c) + 1) + 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*lo
g(-sin(d*x + c) + 1) - 6*sin(d*x + c) - 2)/(a*d*cos(d*x + c)^2*sin(d*x + c)
 + a*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.18656, size = 130, normalized size = 1.69

$$\frac{\frac{6 \log\left(|\sin(dx+c)+1|\right)}{a} - \frac{6 \log\left(|\sin(dx+c)-1|\right)}{a} + \frac{2\,(3 \sin(dx+c)-5)}{a(\sin(dx+c)-1)} - \frac{9 \sin(dx+c)^2+26 \sin(dx+c)+21}{a(\sin(dx+c)+1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(6*log(abs(sin(d*x + c) + 1))/a - 6*log(abs(sin(d*x + c) - 1))/a + 2*(
3*sin(d*x + c) - 5)/(a*(sin(d*x + c) - 1)) - (9*sin(d*x + c)^2 + 26*sin(d*x
 + c) + 21)/(a*(sin(d*x + c) + 1)^2))/d
```

3.60 $\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{4\tan^3(c+dx)}{15ad}+\frac{4\tan(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{5d(a\sin(c+dx)+a)}$$

```
[Out] -Sec[c + d*x]^3/(5*d*(a + a*Sin[c + d*x])) + (4*Tan[c + d*x])/(5*a*d) + (4*
Tan[c + d*x]^3)/(15*a*d)
```

Rubi [A] time = 0.0593119, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2672, 3767}

$$\frac{4\tan^3(c+dx)}{15ad}+\frac{4\tan(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{5d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Sec[c + d*x]^3/(5*d*(a + a*Sin[c + d*x])) + (4*Tan[c + d*x])/(5*a*d) + (4*
Tan[c + d*x]^3)/(15*a*d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\sec^3(c+dx)}{5d(a+a\sin(c+dx))} + \frac{4\int \sec^4(c+dx)\,dx}{5a}$$
$$= -\frac{\sec^3(c+dx)}{5d(a+a\sin(c+dx))} - \frac{4\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{5ad}$$
$$= -\frac{\sec^3(c+dx)}{5d(a+a\sin(c+dx))} + \frac{4\tan(c+dx)}{5ad} + \frac{4\tan^3(c+dx)}{15ad}$$

Mathematica [A] time = 0.0969105, size = 66, normalized size = 1.06

$$-\frac{\sec^3(c+dx)(-2(3\sin(c+dx)+\sin(3(c+dx)))+2\cos(2(c+dx))+\cos(4(c+dx)))}{15ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]^3*(2*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] - 2*(3*Sin[c + d*x]
 + Sin[3*(c + d*x)])))/(15*a*d*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.059, size = 130, normalized size = 2.1

$$2\,\frac{1}{da}\left(-1/12\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-3}-1/8\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-2}-\frac{5}{16\,\tan\left(1/2\,dx+c/2\right)-16}-1/5\,\left(\tan\left(1/2\,dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/d/a*(-1/12/(tan(1/2*d*x+1/2*c)-1)^3-1/8/(tan(1/2*d*x+1/2*c)-1)^2-5/16/(ta
n(1/2*d*x+1/2*c)-1)-1/5/(tan(1/2*d*x+1/2*c)+1)^5+1/2/(tan(1/2*d*x+1/2*c)+1)
^4-5/6/(tan(1/2*d*x+1/2*c)+1)^3+3/4/(tan(1/2*d*x+1/2*c)+1)^2-11/16/(tan(1/2
*d*x+1/2*c)+1))
```

Maxima [B] time = 0.966609, size = 397, normalized size = 6.4

$$\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{21\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{13\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{25\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{5\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-3\right)}{15\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/15*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 21*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 13*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 25*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 - 5*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3)/((a + 2*a*
sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A] time = 1.64399, size = 194, normalized size = 3.13

$$-\frac{8\,\cos\left(dx+c\right)^{4}-4\,\cos\left(dx+c\right)^{2}-4\left(2\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)-1}{15\left(ad\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + 1)*sin(d
*x + c) - 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec^{4}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [B] time = 1.14159, size = 161, normalized size = 2.6

$$-\frac{\frac{5\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}+\frac{165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+480\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+400\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+113}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(5*(15*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 13)/(a*(ta
n(1/2*d*x + 1/2*c) - 1)^3) + (165*tan(1/2*d*x + 1/2*c)^4 + 480*tan(1/2*d*x
+ 1/2*c)^3 + 650*tan(1/2*d*x + 1/2*c)^2 + 400*tan(1/2*d*x + 1/2*c) + 113)/(
a*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.61 $\int \frac{\sec^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=120

$$-\frac{a^2}{24d(a\sin(c+dx)+a)^3}+\frac{a}{32d(a-a\sin(c+dx))^2}-\frac{3a}{32d(a\sin(c+dx)+a)^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{3}{16d(a\sin(c+dx)$$

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(16*a*d) + a/(32*d*(a - a*Sin[c + d*x])^2) + 1/(8
*d*(a - a*Sin[c + d*x])) - a^2/(24*d*(a + a*Sin[c + d*x])^3) - (3*a)/(32*d*
(a + a*Sin[c + d*x])^2) - 3/(16*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.106839, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$-\frac{a^2}{24d(a\sin(c+dx)+a)^3}+\frac{a}{32d(a-a\sin(c+dx))^2}-\frac{3a}{32d(a\sin(c+dx)+a)^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{3}{16d(a\sin(c+dx)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(16*a*d) + a/(32*d*(a - a*Sin[c + d*x])^2) + 1/(8
*d*(a - a*Sin[c + d*x])) - a^2/(24*d*(a + a*Sin[c + d*x])^3) - (3*a)/(32*d*
(a + a*Sin[c + d*x])^2) - 3/(16*d*(a + a*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{1}{16a^4(a-x)^3}+\frac{1}{8a^5(a-x)^2}+\frac{1}{8a^3(a+x)^4}+\frac{3}{16a^4(a+x)^3}+\frac{3}{16a^5(a+x)^2}+\frac{5}{16a^5\left(a^2-x^2\right)}\right) dx, x, a\sin\right.}{d}$$
$$= \frac{a}{32d(a-a\sin(c+dx))^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{a^2}{24d(a+a\sin(c+dx))^3}-\frac{3a}{32d(a+a\sin(c-}$$
$$= \frac{5\tanh^{-1}(\sin(c+dx))}{16ad}+\frac{a}{32d(a-a\sin(c+dx))^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{a^2}{24d(a+a\sin(c+}$$

Mathematica [A] time = 0.149027, size = 97, normalized size = 0.81

$$\frac{\sec^4(c+dx)\left(-15\sin^4(c+dx)-15\sin^3(c+dx)+25\sin^2(c+dx)+25\sin(c+dx)+15(\sin(c+dx)-1)^2(\sin(c+dx)+\right.}{48ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^4*(-8 + 25*Sin[c + d*x] + 25*Sin[c + d*x]^2 - 15*Sin[c + d*x]
^3 - 15*Sin[c + d*x]^4 + 15*ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])^2*(1
+ Sin[c + d*x])^3))/(48*a*d*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.059, size = 126, normalized size = 1.1

$$\frac{1}{32\,da\,(\sin(dx+c)-1)^2}-\frac{1}{8\,da\,(\sin(dx+c)-1)}-\frac{5\,\ln(\sin(dx+c)-1)}{32\,da}-\frac{1}{24\,da\,(1+\sin(dx+c))^3}-\frac{3}{32\,da\,(1+\sin(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/32/d/a/(sin(d*x+c)-1)^2-1/8/a/d/(sin(d*x+c)-1)-5/32/a/d*ln(sin(d*x+c)-1)-
1/24/d/a/(1+sin(d*x+c))^3-3/32/a/d/(1+sin(d*x+c))^2-3/16/a/d/(1+sin(d*x+c))
+5/32*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 0.947961, size = 176, normalized size = 1.47

$$-\frac{\frac{2\left(15\sin(dx+c)^4+15\sin(dx+c)^3-25\sin(dx+c)^2-25\sin(dx+c)+8\right)}{a\sin(dx+c)^5+a\sin(dx+c)^4-2a\sin(dx+c)^3-2a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{15\log(\sin(dx+c)+1)}{a}+\frac{15\log(\sin(dx+c)-1)}{a}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*sin(d*x + c)^4 + 15*sin(d*x + c)^3 - 25*sin(d*x + c)^2 - 25*si
n(d*x + c) + 8)/(a*sin(d*x + c)^5 + a*sin(d*x + c)^4 - 2*a*sin(d*x + c)^3 -
 2*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 15*log(sin(d*x + c) + 1)/a + 15
*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.67871, size = 400, normalized size = 3.33

$$-\frac{30\cos(dx+c)^4-10\cos(dx+c)^2-15\left(\cos(dx+c)^4\sin(dx+c)+\cos(dx+c)^4\right)\log(\sin(dx+c)+1)+15(\cos(dx-}{96(ad\cos(dx+c)^4\sin(dx+c)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/96*(30*cos(d*x + c)^4 - 10*cos(d*x + c)^2 - 15*(cos(d*x + c)^4*sin(d*x +
 c) + cos(d*x + c)^4)*log(sin(d*x + c) + 1) + 15*(cos(d*x + c)^4*sin(d*x +
c) + cos(d*x + c)^4)*log(-sin(d*x + c) + 1) - 10*(3*cos(d*x + c)^2 + 2)*sin
(d*x + c) - 4)/(a*d*cos(d*x + c)^4*sin(d*x + c) + a*d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^5(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.18127, size = 157, normalized size = 1.31

$$\frac{\frac{30\log(|\sin(dx+c)+1|)}{a}-\frac{30\log(|\sin(dx+c)-1|)}{a}+\frac{3\left(15\sin(dx+c)^2-38\sin(dx+c)+25\right)}{a(\sin(dx+c)-1)^2}-\frac{55\sin(dx+c)^3+201\sin(dx+c)^2+255\sin(dx+c)+117}{a(\sin(dx+c)+1)^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(30*log(abs(sin(d*x + c) + 1))/a - 30*log(abs(sin(d*x + c) - 1))/a +
3*(15*sin(d*x + c)^2 - 38*sin(d*x + c) + 25)/(a*(sin(d*x + c) - 1)^2) - (55
*sin(d*x + c)^3 + 201*sin(d*x + c)^2 + 255*sin(d*x + c) + 117)/(a*(sin(d*x
+ c) + 1)^3))/d
```

3.62 $\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{7\cos^5(c+dx)}{30a^2d}+\frac{\cos^7(c+dx)}{6d\left(a^2\sin(c+dx)+a^2\right)}+\frac{7\sin(c+dx)\cos^3(c+dx)}{24a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{7x}{16a^2}$$

```
[Out] (7*x)/(16*a^2) + (7*Cos[c + d*x]^5)/(30*a^2*d) + (7*Cos[c + d*x]*Sin[c + d*
x])/(16*a^2*d) + (7*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^2*d) + Cos[c + d*x]^
7/(6*d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.11249, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2679, 2682, 2635, 8}

$$\frac{7\cos^5(c+dx)}{30a^2d}+\frac{\cos^7(c+dx)}{6d\left(a^2\sin(c+dx)+a^2\right)}+\frac{7\sin(c+dx)\cos^3(c+dx)}{24a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{7x}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (7*x)/(16*a^2) + (7*Cos[c + d*x]^5)/(30*a^2*d) + (7*Cos[c + d*x]*Sin[c + d*
x])/(16*a^2*d) + (7*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^2*d) + Cos[c + d*x]^
7/(6*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
```

```
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\cos^7(c+dx)}{6d\left(a^2+a^2\sin(c+dx)\right)} + \frac{7\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx}{6a}$$
$$= \frac{7\cos^5(c+dx)}{30a^2d} + \frac{\cos^7(c+dx)}{6d\left(a^2+a^2\sin(c+dx)\right)} + \frac{7\int \cos^4(c+dx)\,dx}{6a^2}$$
$$= \frac{7\cos^5(c+dx)}{30a^2d} + \frac{7\cos^3(c+dx)\sin(c+dx)}{24a^2d} + \frac{\cos^7(c+dx)}{6d\left(a^2+a^2\sin(c+dx)\right)} + \frac{7\int \cos^2(c+dx)\,}{8a^2}$$
$$= \frac{7\cos^5(c+dx)}{30a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{7\cos^3(c+dx)\sin(c+dx)}{24a^2d} + \frac{\cos^7(c+d}{6d\left(a^2+a^2\sin(\right.}$$
$$= \frac{7x}{16a^2} + \frac{7\cos^5(c+dx)}{30a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{7\cos^3(c+dx)\sin(c+dx)}{24a^2d} + \frac{\mathrm{co}}{6d\left(a^2+\right.}$$

Mathematica [A] time = 1.12024, size = 151, normalized size = 1.45

$$-\frac{\left(\sqrt{\sin(c+dx)+1}\left(40\sin^6(c+dx)-136\sin^5(c+dx)+86\sin^4(c+dx)+202\sin^3(c+dx)-327\sin^2(c+dx)+39\sin\right.\right.}{240a^2d(\sin(c+dx)-1)^5(\sin(c+dx)+1)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Cos[c + d*x]^9*(-210*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[
c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(96 + 39*Sin[c + d*x] - 327*Sin[c + d*x]
```

```
^2 + 202*Sin[c + d*x]^3 + 86*Sin[c + d*x]^4 - 136*Sin[c + d*x]^5 + 40*Sin[c
 + d*x]^6)))/(240*a^2*d*(-1 + Sin[c + d*x])^5*(1 + Sin[c + d*x])^(9/2))
```

Maple [B] time = 0.082, size = 415, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -9/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11+4/d/a^2/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^10-89/24/d/a^2/(1+tan(1/2*d*x+1/2*c)
^2)^6*tan(1/2*d*x+1/2*c)^9+4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1
/2*c)^8+11/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7+8/d/a^2/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^6-11/4/d/a^2/(1+tan(1/2*d*x+1
/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5+8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*
d*x+1/2*c)^4+89/24/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+4/
5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^2+9/8/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)+4/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6+7
/8/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.46637, size = 531, normalized size = 5.11

$$\frac{\frac{\frac{135\sin(dx+c)}{\cos(dx+c)+1}+\frac{96\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{445\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{960\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{330\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{960\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{330\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{480\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{445\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{480\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{135}{(\mathrm{co}}}{a^2+\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/120*((135*sin(d*x + c)/(cos(d*x + c) + 1) + 96*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 445*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 960*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 330*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 960*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 + 330*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 480
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 445*sin(d*x + c)^9/(cos(d*x + c) + 1
)^9 + 480*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 135*sin(d*x + c)^11/(cos(
```

```
d*x + c) + 1)^11 + 96)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x + c)^6/(cos(d*x
+ c) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^2*sin(d*x +
c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) +
105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 2.00226, size = 161, normalized size = 1.55

$$\frac{96\cos(dx+c)^5+105\,dx-5\left(8\cos(dx+c)^5-14\cos(dx+c)^3-21\cos(dx+c)\right)\sin(dx+c)}{240\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(96*cos(d*x + c)^5 + 105*d*x - 5*(8*cos(d*x + c)^5 - 14*cos(d*x + c)^
3 - 21*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14215, size = 242, normalized size = 2.33

$$\frac{\frac{105\,(dx+c)}{a^2}-\frac{2\left(135\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-480\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+445\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-480\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-330\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-960\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+330\,t}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6a^2}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(105*(d*x + c)/a^2 - 2*(135*tan(1/2*d*x + 1/2*c)^11 - 480*tan(1/2*d*x
 + 1/2*c)^10 + 445*tan(1/2*d*x + 1/2*c)^9 - 480*tan(1/2*d*x + 1/2*c)^8 - 33
0*tan(1/2*d*x + 1/2*c)^7 - 960*tan(1/2*d*x + 1/2*c)^6 + 330*tan(1/2*d*x + 1
/2*c)^5 - 960*tan(1/2*d*x + 1/2*c)^4 - 445*tan(1/2*d*x + 1/2*c)^3 - 96*tan(
1/2*d*x + 1/2*c)^2 - 135*tan(1/2*d*x + 1/2*c) - 96)/((tan(1/2*d*x + 1/2*c)^
2 + 1)^6*a^2))/d
```

3.63 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$\frac{(a-a\sin(c+dx))^5}{5a^7d}-\frac{(a-a\sin(c+dx))^4}{2a^6d}$$

```
[Out] -(a - a*Sin[c + d*x])^4/(2*a^6*d) + (a - a*Sin[c + d*x])^5/(5*a^7*d)
```

Rubi [A] time = 0.0520617, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{(a-a\sin(c+dx))^5}{5a^7d}-\frac{(a-a\sin(c+dx))^4}{2a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a - a*Sin[c + d*x])^4/(2*a^6*d) + (a - a*Sin[c + d*x])^5/(5*a^7*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(a-x)^3-(a-x)^4\right)\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= -\frac{(a-a\sin(c+dx))^4}{2a^6d}+\frac{(a-a\sin(c+dx))^5}{5a^7d}$$

Mathematica [A] time = 0.159816, size = 46, normalized size = 0.98

$$-\frac{\sin(c+dx)\left(2\sin^4(c+dx)-5\sin^3(c+dx)+10\sin(c+dx)-10\right)}{10a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sin[c + d*x]*(-10 + 10*Sin[c + d*x] - 5*Sin[c + d*x]^3 + 2*Sin[c + d*x]^4
))/(10*a^2*d)
```

Maple [A] time = 0.068, size = 45, normalized size = 1.

$$\frac{1}{da^2}\left(-\frac{(\sin(dx+c))^5}{5}+\frac{(\sin(dx+c))^4}{2}-(\sin(dx+c))^2+\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/5*sin(d*x+c)^5+1/2*sin(d*x+c)^4-sin(d*x+c)^2+sin(d*x+c))
```

Maxima [A] time = 0.937682, size = 63, normalized size = 1.34

$$-\frac{2\sin(dx+c)^5-5\sin(dx+c)^4+10\sin(dx+c)^2-10\sin(dx+c)}{10\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(2*sin(d*x + c)^5 - 5*sin(d*x + c)^4 + 10*sin(d*x + c)^2 - 10*sin(d*x
 + c))/(a^2*d)
```

Fricas [A] time = 1.93265, size = 122, normalized size = 2.6

$$\frac{5\cos(dx+c)^4 - 2\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 - 4\right)\sin(dx+c)}{10\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/10*(5*cos(d*x + c)^4 - 2*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - 4)*sin(d*x
+ c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14583, size = 63, normalized size = 1.34

$$-\frac{2\sin(dx+c)^5 - 5\sin(dx+c)^4 + 10\sin(dx+c)^2 - 10\sin(dx+c)}{10\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*sin(d*x + c)^5 - 5*sin(d*x + c)^4 + 10*sin(d*x + c)^2 - 10*sin(d*x
 + c))/(a^2*d)
```

3.64 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=80

$$\frac{5\cos^3(c+dx)}{12a^2d}+\frac{\cos^5(c+dx)}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{5x}{8a^2}$$

```
[Out] (5*x)/(8*a^2) + (5*Cos[c + d*x]^3)/(12*a^2*d) + (5*Cos[c + d*x]*Sin[c + d*x
])/(8*a^2*d) + Cos[c + d*x]^5/(4*d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.100451, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2679, 2682, 2635, 8}

$$\frac{5\cos^3(c+dx)}{12a^2d}+\frac{\cos^5(c+dx)}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{5x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*x)/(8*a^2) + (5*Cos[c + d*x]^3)/(12*a^2*d) + (5*Cos[c + d*x]*Sin[c + d*x
])/(8*a^2*d) + Cos[c + d*x]^5/(4*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\cos^5(c+dx)}{4d\left(a^2+a^2\sin(c+dx)\right)}+\frac{5\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx}{4a}\\ &= \frac{5\cos^3(c+dx)}{12a^2d}+\frac{\cos^5(c+dx)}{4d\left(a^2+a^2\sin(c+dx)\right)}+\frac{5\int \cos^2(c+dx)\,dx}{4a^2}\\ &= \frac{5\cos^3(c+dx)}{12a^2d}+\frac{5\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^5(c+dx)}{4d\left(a^2+a^2\sin(c+dx)\right)}+\frac{5\int 1\,dx}{8a^2}\\ &= \frac{5x}{8a^2}+\frac{5\cos^3(c+dx)}{12a^2d}+\frac{5\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^5(c+dx)}{4d\left(a^2+a^2\sin(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.721368, size = 131, normalized size = 1.64

$$-\frac{\left(30\sqrt{1-\sin(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\left(6\sin^4(c+dx)-22\sin^3(c+dx)+25\sin^2(c+dx)+7\sin\right.\right.}{24a^2d(\sin(c+dx)-1)^4(\sin(c+dx)+1)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Cos[c + d*x]^7*(30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c
+ d*x]] + Sqrt[1 + Sin[c + d*x]]*(-16 + 7*Sin[c + d*x] + 25*Sin[c + d*x]^2
- 22*Sin[c + d*x]^3 + 6*Sin[c + d*x]^4)))/(24*a^2*d*(-1 + Sin[c + d*x])^4*(
1 + Sin[c + d*x])^(7/2))
```

Maple [B] time = 0.067, size = 279, normalized size = 3.5

$$-\frac{3}{4\,da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-4}+4\,\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^{6}}{da^{2}\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}\right)^{4}}-\frac{11}{4\,da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}\left(1+\left(\mathrm{ta}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -3/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+4/d/a^2/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6-11/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)
^4*tan(1/2*d*x+1/2*c)^5+4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*
c)^4+11/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+4/3/d/a^2/(
1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2+3/4/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^4*tan(1/2*d*x+1/2*c)+4/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4+5/4/d/a^2*
arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.43447, size = 360, normalized size = 4.5

$$\frac{\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{33\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{48\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{33\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{48\,\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{9\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+16}{a^{2}+\frac{4\,a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4\,a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}+\frac{15\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 48*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 48*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 - 9*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 16)/(a^2 + 4
*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*x + c)^8/
(cos(d*x + c) + 1)^8) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.84829, size = 130, normalized size = 1.62

$$\frac{16\,\cos\left(dx+c\right)^{3}+15\,dx-3\left(2\,\cos\left(dx+c\right)^{3}-5\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*cos(d*x + c)^3 + 15*d*x - 3*(2*cos(d*x + c)^3 - 5*cos(d*x + c))*si
n(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12186, size = 171, normalized size = 2.14

$$\frac{\frac{15\,(dx+c)}{a^2} - \frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 + 33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*(d*x + c)/a^2 - 2*(9*tan(1/2*d*x + 1/2*c)^7 - 48*tan(1/2*d*x + 1/2
*c)^6 + 33*tan(1/2*d*x + 1/2*c)^5 - 48*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*
d*x + 1/2*c)^3 - 16*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) - 16)/(
(tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^2))/d
```

3.65 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=23

$$-\frac{(a-a\sin(c+dx))^3}{3a^5d}$$

```
[Out] -(a - a*Sin[c + d*x])^3/(3*a^5*d)
```

Rubi [A] time = 0.0431779, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$-\frac{(a-a\sin(c+dx))^3}{3a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a - a*Sin[c + d*x])^3/(3*a^5*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\mathrm{Subst}\left(\int (a-x)^2\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= -\frac{(a-a\sin(c+dx))^3}{3a^5 d}$$

Mathematica [A] time = 0.0561295, size = 34, normalized size = 1.48

$$\frac{\sin(c+dx)\left(\sin^2(c+dx)-3\sin(c+dx)+3\right)}{3a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]*(3 - 3*Sin[c + d*x] + Sin[c + d*x]^2))/(3*a^2*d)
```

Maple [A] time = 0.062, size = 19, normalized size = 0.8

$$\frac{(\sin(dx+c)-1)^3}{3\,da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/3/d/a^2*(sin(d*x+c)-1)^3
```

Maxima [A] time = 0.939798, size = 47, normalized size = 2.04

$$\frac{\sin(dx+c)^3 - 3\,\sin(dx+c)^2 + 3\,\sin(dx+c)}{3\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 3*sin(d*x + c))/(a^2*d)
```

Fricas [A] time = 1.90076, size = 92, normalized size = 4.

$$\frac{3\cos(dx+c)^2-\left(\cos(dx+c)^2-4\right)\sin(dx+c)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*cos(d*x + c)^2 - (cos(d*x + c)^2 - 4)*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1311, size = 47, normalized size = 2.04

$$\frac{\sin(dx+c)^3-3\sin(dx+c)^2+3\sin(dx+c)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 3*sin(d*x + c))/(a^2*d)
```

3.66 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=56

$$\frac{3\cos(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{2d\left(a^2\sin(c+dx)+a^2\right)}+\frac{3x}{2a^2}$$

```
[Out] (3*x)/(2*a^2) + (3*Cos[c + d*x])/(2*a^2*d) + Cos[c + d*x]^3/(2*d*(a^2 + a^2
*Sin[c + d*x]))
```

Rubi [A] time = 0.0847453, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2679, 2682, 8}

$$\frac{3\cos(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{2d\left(a^2\sin(c+dx)+a^2\right)}+\frac{3x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*x)/(2*a^2) + (3*Cos[c + d*x])/(2*a^2*d) + Cos[c + d*x]^3/(2*d*(a^2 + a^2
*Sin[c + d*x]))
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\cos^3(c+dx)}{2d\left(a^2+a^2\sin(c+dx)\right)} + \frac{3\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx}{2a}$$
$$= \frac{3\cos(c+dx)}{2a^2d} + \frac{\cos^3(c+dx)}{2d\left(a^2+a^2\sin(c+dx)\right)} + \frac{3\int 1\,dx}{2a^2}$$
$$= \frac{3x}{2a^2} + \frac{3\cos(c+dx)}{2a^2d} + \frac{\cos^3(c+dx)}{2d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.171297, size = 109, normalized size = 1.95

$$-\frac{\left(\sqrt{\sin(c+dx)+1}\left(\sin^2(c+dx)-5\sin(c+dx)+4\right)-6\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}\right)\cos^5(c+dx)}{2a^2d(\sin(c+dx)-1)^3(\sin(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Cos[c + d*x]^5*(-6*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c
+ d*x]] + Sqrt[1 + Sin[c + d*x]]*(4 - 5*Sin[c + d*x] + Sin[c + d*x]^2)))/(2
*a^2*d*(-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^(5/2))
```

Maple [B] time = 0.072, size = 142, normalized size = 2.5

$$\frac{1}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}+4\,\frac{(\tan(1/2\,dx+c/2))^2}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}-\frac{1}{da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+4/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2-1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan
(1/2*d*x+1/2*c)+4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2+3/d/a^2*arctan(tan(1/2*d
*x+1/2*c))
```

Maxima [B] time = 1.43535, size = 189, normalized size = 3.38

$$-\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-4}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((sin(d*x + c)/(cos(d*x + c) + 1) - 4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 4)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.986, size = 89, normalized size = 1.59

$$\frac{3\,dx-\cos\left(dx+c\right)\sin\left(dx+c\right)+4\,\cos\left(dx+c\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*d*x - cos(d*x + c)*sin(d*x + c) + 4*cos(d*x + c))/(a^2*d)
```

Sympy [A] time = 169.455, size = 461, normalized size = 8.23

$$\begin{cases}\frac{33dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{22a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+44a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+22a^2d}+\frac{66dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{22a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+44a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+22a^2d}+\frac{33dx}{22a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+44a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+22a^2d} \\ \frac{x\cos^4(c)}{(a\sin(c)+a)^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((33*d*x*tan(c/2 + d*x/2)**4/(22*a**2*d*tan(c/2 + d*x/2)**4 + 44*a
**2*d*tan(c/2 + d*x/2)**2 + 22*a**2*d) + 66*d*x*tan(c/2 + d*x/2)**2/(22*a**
2*d*tan(c/2 + d*x/2)**4 + 44*a**2*d*tan(c/2 + d*x/2)**2 + 22*a**2*d) + 33*d
*x/(22*a**2*d*tan(c/2 + d*x/2)**4 + 44*a**2*d*tan(c/2 + d*x/2)**2 + 22*a**2
*d) - 42*tan(c/2 + d*x/2)**4/(22*a**2*d*tan(c/2 + d*x/2)**4 + 44*a**2*d*tan
(c/2 + d*x/2)**2 + 22*a**2*d) + 22*tan(c/2 + d*x/2)**3/(22*a**2*d*tan(c/2 +
 d*x/2)**4 + 44*a**2*d*tan(c/2 + d*x/2)**2 + 22*a**2*d) + 4*tan(c/2 + d*x/2
)**2/(22*a**2*d*tan(c/2 + d*x/2)**4 + 44*a**2*d*tan(c/2 + d*x/2)**2 + 22*a*
*2*d) - 22*tan(c/2 + d*x/2)/(22*a**2*d*tan(c/2 + d*x/2)**4 + 44*a**2*d*tan(
c/2 + d*x/2)**2 + 22*a**2*d) + 46/(22*a**2*d*tan(c/2 + d*x/2)**4 + 44*a**2*
d*tan(c/2 + d*x/2)**2 + 22*a**2*d), Ne(d, 0)), (x*cos(c)**4/(a*sin(c) + a)*
*2, True))
```

Giac [A] time = 1.12535, size = 99, normalized size = 1.77

$$\frac{\frac{3\,(dx+c)}{a^2} + \frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(d*x + c)/a^2 + 2*(tan(1/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*c)^2
 - tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

3.67 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=32

$$\frac{2\log(\sin(c+dx)+1)}{a^2d}-\frac{\sin(c+dx)}{a^2d}$$

```
[Out] (2*Log[1 + Sin[c + d*x]])/(a^2*d) - Sin[c + d*x]/(a^2*d)
```

Rubi [A] time = 0.0494738, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{2\log(\sin(c+dx)+1)}{a^2d}-\frac{\sin(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Log[1 + Sin[c + d*x]])/(a^2*d) - Sin[c + d*x]/(a^2*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{2a}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{2\log(1+\sin(c+dx))}{a^2d} - \frac{\sin(c+dx)}{a^2d}$$

Mathematica [A] time = 0.0331975, size = 26, normalized size = 0.81

$$-\frac{\sin(c+dx) - 2\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((-2*Log[1 + Sin[c + d*x]] + Sin[c + d*x])/(a^2*d))
```

Maple [A] time = 0.065, size = 33, normalized size = 1.

$$2\,\frac{\ln(1+\sin(dx+c))}{a^2d} - \frac{\sin(dx+c)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2*ln(1+sin(d*x+c))/a^2/d-sin(d*x+c)/a^2/d
```

Maxima [A] time = 0.924143, size = 41, normalized size = 1.28

$$\frac{\frac{2\log(\sin(dx+c)+1)}{a^2} - \frac{\sin(dx+c)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*log(sin(d*x + c) + 1)/a^2 - sin(d*x + c)/a^2)/d
```

Fricas [A] time = 1.91854, size = 68, normalized size = 2.12

$$\frac{2 \log \left(\sin \left(dx + c\right) + 1\right) - \sin \left(dx + c\right)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (2*log(sin(d*x + c) + 1) - sin(d*x + c))/(a^2*d)
```

Sympy [A] time = 1.57043, size = 180, normalized size = 5.62

$$\begin{cases} \frac{2 \log \left(\sin \left(c+dx\right)+1\right) \sin \left(c+dx\right)}{a^2 d \sin \left(c+dx\right)+a^2 d} + \frac{2 \log \left(\sin \left(c+dx\right)+1\right)}{a^2 d \sin \left(c+dx\right)+a^2 d} + \frac{\sin^3 \left(c+dx\right)}{a^2 d \sin \left(c+dx\right)+a^2 d} - \frac{\sin^2 \left(c+dx\right)}{a^2 d \sin \left(c+dx\right)+a^2 d} + \frac{\sin \left(c+dx\right) \cos^2 \left(c+dx\right)}{a^2 d \sin \left(c+dx\right)+a^2 d} + \frac{2}{a^2 d \sin \left(c+dx\right)+a^2 d} \\ \frac{x \cos^3 \left(c\right)}{\left(a \sin \left(c\right)+a\right)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*log(sin(c + d*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2
*d) + 2*log(sin(c + d*x) + 1)/(a**2*d*sin(c + d*x) + a**2*d) + sin(c + d*x)
**3/(a**2*d*sin(c + d*x) + a**2*d) - sin(c + d*x)**2/(a**2*d*sin(c + d*x) +
 a**2*d) + sin(c + d*x)*cos(c + d*x)**2/(a**2*d*sin(c + d*x) + a**2*d) + 2/
(a**2*d*sin(c + d*x) + a**2*d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) + a)**2,
True))
```

Giac [A] time = 1.15992, size = 73, normalized size = 2.28

$$-\frac{\frac{2 \log\left(\frac{|a \sin(dx+c)+a|}{(a \sin(dx+c)+a)^2 |a|}\right)}{a^2} + \frac{a \sin(dx+c)+a}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a^2 + (a*s
in(d*x + c) + a)/a^3)/d
```

3.68 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=34

$$-\frac{2\cos(c+dx)}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{x}{a^2}$$

```
[Out] -(x/a^2) - (2*Cos[c + d*x])/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0427926, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2680, 8}

$$-\frac{2\cos(c+dx)}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*Cos[c + d*x])/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{2\cos(c+dx)}{d\left(a^2+a^2\sin(c+dx)\right)} - \frac{\int 1\,dx}{a^2}$$
$$= -\frac{x}{a^2} - \frac{2\cos(c+dx)}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [B] time = 0.180156, size = 104, normalized size = 3.06

$$\frac{2\left(\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}(\sin(c+dx)+1)+\sqrt{\sin(c+dx)+1}(\sin(c+dx)-1)\right)\cos^3(c+dx)}{a^2d(\sin(c+dx)-1)^2(\sin(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Cos[c + d*x]^3*((-1 + Sin[c + d*x])*Sqrt[1 + Sin[c + d*x]] + ArcSin[Sqrt
[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])))/(a^
2*d*(-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^(5/2))
```

Maple [A] time = 0.076, size = 41, normalized size = 1.2

$$-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^2}-4\,\frac{1}{da^2\left(\tan\left(1/2\,dx+c/2\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-4/d/a^2/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.43919, size = 76, normalized size = 2.24

$$-\frac{2\left(\frac{2}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2*(2/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1)) + arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.80153, size = 151, normalized size = 4.44

$$-\frac{dx + (dx + 2)\cos(dx + c) + (dx - 2)\sin(dx + c) + 2}{a^2 d\cos(dx + c) + a^2 d\sin(dx + c) + a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(d*x + (d*x + 2)*cos(d*x + c) + (d*x - 2)*sin(d*x + c) + 2)/(a^2*d*cos(d*x
 + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [A] time = 8.42848, size = 143, normalized size = 4.21

$$\begin{cases} -\frac{5dx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{5a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+5a^2d} - \frac{5dx}{5a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+5a^2d} + \frac{12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{5a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+5a^2d} - \frac{8}{5a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+5a^2d} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{(a\sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-5*d*x*tan(c/2 + d*x/2)/(5*a**2*d*tan(c/2 + d*x/2) + 5*a**2*d) -
 5*d*x/(5*a**2*d*tan(c/2 + d*x/2) + 5*a**2*d) + 12*tan(c/2 + d*x/2)/(5*a**2
*d*tan(c/2 + d*x/2) + 5*a**2*d) - 8/(5*a**2*d*tan(c/2 + d*x/2) + 5*a**2*d),
 Ne(d, 0)), (x*cos(c)**2/(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.12626, size = 45, normalized size = 1.32

$$-\frac{\frac{dx+c}{a^2}+\frac{4}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)/a^2 + 4/(a^2*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.69 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}$$

```
[Out] -(1/(d*(a^2 + a^2*Sin[c + d*x])))
```

Rubi [A] time = 0.0261668, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 32}

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(1/(d*(a^2 + a^2*Sin[c + d*x])))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.0704079, size = 31, normalized size = 1.48

$$-\frac{1}{a^2 d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(1/(a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))
```

Maple [A] time = 0.015, size = 21, normalized size = 1.

$$-\frac{1}{d\left(a+a\sin\left(dx+c\right)\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+a*sin(d*x+c))/a
```

Maxima [A] time = 0.960756, size = 27, normalized size = 1.29

$$-\frac{1}{(a\sin\left(dx+c\right)+a)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/((a*sin(d*x + c) + a)*a*d)
```

Fricas [A] time = 1.85078, size = 45, normalized size = 2.14

$$-\frac{1}{a^2 d \sin(dx+c) + a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [A] time = 0.990904, size = 32, normalized size = 1.52

$$\begin{cases} -\frac{1}{a^2 d \sin(c+dx)+a^2 d} & \text{for}\ d \neq 0 \\ \frac{x \cos(c)}{(a \sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-1/(a**2*d*sin(c + d*x) + a**2*d), Ne(d, 0)), (x*cos(c)/(a*sin(c
) + a)**2, True))
```

Giac [A] time = 1.15113, size = 27, normalized size = 1.29

$$-\frac{1}{(a \sin(dx+c)+a)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/((a*sin(d*x + c) + a)*a*d)
```

3.70 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=60

$$-\frac{1}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{1}{4d(a\sin(c+dx)+a)^2}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) - 1/(4*d*(a + a*Sin[c + d*x])^2) - 1/(4*d*(
a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0582109, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2667, 44, 206}

$$-\frac{1}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{1}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) - 1/(4*d*(a + a*Sin[c + d*x])^2) - 1/(4*d*(
a^2 + a^2*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{(a-x)(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a\,\mathrm{Subst}\left(\int \left(\frac{1}{2a(a+x)^3}+\frac{1}{4a^2(a+x)^2}+\frac{1}{4a^2(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}\\
&= -\frac{1}{4d(a+a\sin(c+dx))^2}-\frac{1}{4d\left(a^2+a^2\sin(c+dx)\right)}+\frac{\mathrm{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c+dx)\right)}{4ad}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{1}{4d(a+a\sin(c+dx))^2}-\frac{1}{4d\left(a^2+a^2\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [A] time = 0.0923803, size = 38, normalized size = 0.63

$$\frac{\tanh^{-1}(\sin(c+dx))-\frac{\sin(c+dx)+2}{(\sin(c+dx)+1)^2}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (ArcTanh[Sin[c + d*x]] - (2 + Sin[c + d*x])/(1 + Sin[c + d*x])^2)/(4*a^2*d)
```

Maple [A] time = 0.071, size = 72, normalized size = 1.2

$$-\frac{\ln(\sin(dx+c)-1)}{8\,da^2}-\frac{1}{4\,da^2\,(1+\sin(dx+c))^2}-\frac{1}{4\,da^2\,(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{8\,da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/8/d/a^2*ln(sin(d*x+c)-1)-1/4/d/a^2/(1+sin(d*x+c))^2-1/4/d/a^2/(1+sin(d*x
+c))+1/8*ln(1+sin(d*x+c))/a^2/d
```

Maxima [A] time = 0.963566, size = 97, normalized size = 1.62

$$-\frac{\frac{2\,(\sin(dx+c)+2)}{a^2\sin(dx+c)^2+2\,a^2\sin(dx+c)+a^2}-\frac{\log(\sin(dx+c)+1)}{a^2}+\frac{\log(\sin(dx+c)-1)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2*(sin(d*x + c) + 2)/(a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x + c) + a^2)
- log(sin(d*x + c) + 1)/a^2 + log(sin(d*x + c) - 1)/a^2)/d
```

Fricas [A] time = 2.08026, size = 279, normalized size = 4.65

$$\frac{\left(\cos\left(dx+c\right)^2-2\,\sin\left(dx+c\right)-2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(\cos\left(dx+c\right)^2-2\,\sin\left(dx+c\right)-2\right)\log\left(-\sin\left(dx+c\right)+1\right)+2}{8\left(a^2d\cos\left(dx+c\right)^2-2\,a^2d\sin\left(dx+c\right)-2\,a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - (cos(d*x
 + c)^2 - 2*sin(d*x + c) - 2)*log(-sin(d*x + c) + 1) + 2*sin(d*x + c) + 4)/
(a^2*d*cos(d*x + c)^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.18831, size = 96, normalized size = 1.6

$$\frac{\frac{2\,\log\left(|\sin\left(dx+c\right)+1|\right)}{a^{2}}-\frac{2\,\log\left(|\sin\left(dx+c\right)-1|\right)}{a^{2}}-\frac{3\,\sin\left(dx+c\right)^{2}+10\,\sin\left(dx+c\right)+11}{a^{2}\left(\sin\left(dx+c\right)+1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(sin(d*x + c) + 1))/a^2 - 2*log(abs(sin(d*x + c) - 1))/a^2 -
 (3*sin(d*x + c)^2 + 10*sin(d*x + c) + 11)/(a^2*(sin(d*x + c) + 1)^2))/d
```

3.71 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan(c+dx)}{5a^2d}-\frac{\sec(c+dx)}{5d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\sec(c+dx)}{5d(a\sin(c+dx)+a)^2}$$

```
[Out] -Sec[c + d*x]/(5*d*(a + a*Sin[c + d*x])^2) - Sec[c + d*x]/(5*d*(a^2 + a^2*S
in[c + d*x])) + (2*Tan[c + d*x])/(5*a^2*d)
```

Rubi [A] time = 0.09312, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2672, 3767, 8}

$$\frac{2\tan(c+dx)}{5a^2d}-\frac{\sec(c+dx)}{5d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\sec(c+dx)}{5d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Sec[c + d*x]/(5*d*(a + a*Sin[c + d*x])^2) - Sec[c + d*x]/(5*d*(a^2 + a^2*S
in[c + d*x])) + (2*Tan[c + d*x])/(5*a^2*d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}+\frac{3\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx}{5a}$$
$$= -\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}-\frac{\sec(c+dx)}{5d\left(a^2+a^2\sin(c+dx)\right)}+\frac{2\int \sec^2(c+dx)\,dx}{5a^2}$$
$$= -\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}-\frac{\sec(c+dx)}{5d\left(a^2+a^2\sin(c+dx)\right)}-\frac{2\,\mathrm{Subst}(\int 1\,dx,x,-\tan(c+dx))}{5a^2d}$$
$$= -\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}-\frac{\sec(c+dx)}{5d\left(a^2+a^2\sin(c+dx)\right)}+\frac{2\tan(c+dx)}{5a^2d}$$

Mathematica [A] time = 0.0756553, size = 53, normalized size = 0.75

$$-\frac{\sec(c+dx)(-5\sin(c+dx)+\sin(3(c+dx))+4\cos(2(c+dx)))}{10a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]*(4*Cos[2*(c + d*x)] - 5*Sin[c + d*x] + Sin[3*(c + d*x)]))/(1
0*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.069, size = 98, normalized size = 1.4

$$2\,\frac{1}{da^2}\left(-1/8\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-1}-2/5\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-5}+\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-4}-3/2\,\left(\tan\left(1/2\,dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(-1/8/(tan(1/2*d*x+1/2*c)-1)-2/5/(tan(1/2*d*x+1/2*c)+1)^5+1/(tan(1/
2*d*x+1/2*c)+1)^4-3/2/(tan(1/2*d*x+1/2*c)+1)^3+5/4/(tan(1/2*d*x+1/2*c)+1)^2
```

```
-7/8/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 0.98925, size = 275, normalized size = 3.87

$$-\frac{2\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{10\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{5\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+2\right)}{5\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2/5*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 - 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 5*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 2)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 - 4*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x
+ c) + 1)^6)*d)
```

Fricas [A] time = 1.92073, size = 200, normalized size = 2.82

$$\frac{4\cos(dx+c)^2+\left(2\cos(dx+c)^2-3\right)\sin(dx+c)-2}{5\left(a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)\sin(dx+c)-2\,a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5*(4*cos(d*x + c)^2 + (2*cos(d*x + c)^2 - 3)*sin(d*x + c) - 2)/(a^2*d*cos
(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.15695, size = 126, normalized size = 1.77

$$-\frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+90\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+70\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) + (35*tan(1/2*d*x + 1/2*c)^4 + 90
*tan(1/2*d*x + 1/2*c)^3 + 120*tan(1/2*d*x + 1/2*c)^2 + 70*tan(1/2*d*x + 1/2
*c) + 21)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.72 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}-\frac{3}{16d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{a}{12d(a\sin(c+dx)+a)^3}-\frac{1}{8d(a\sin(c+dx$$

```
[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) - a/(12*d*(a + a*Sin[c + d*x])^3) - 1/(8*d*
(a + a*Sin[c + d*x])^2) + 1/(16*d*(a^2 - a^2*Sin[c + d*x])) - 3/(16*d*(a^2
+ a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0820373, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}-\frac{3}{16d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{a}{12d(a\sin(c+dx)+a)^3}-\frac{1}{8d(a\sin(c+dx$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) - a/(12*d*(a + a*Sin[c + d*x])^3) - 1/(8*d*
(a + a*Sin[c + d*x])^2) + 1/(16*d*(a^2 - a^2*Sin[c + d*x])) - 3/(16*d*(a^2
+ a^2*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{a^3\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{16a^4(a-x)^2}+\frac{1}{4a^2(a+x)^4}+\frac{1}{4a^3(a+x)^3}+\frac{3}{16a^4(a+x)^2}+\frac{1}{4a^4(a^2-x^2)}\right) dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{a}{12d(a+a\sin(c+dx))^3}-\frac{1}{8d(a+a\sin(c+dx))^2}+\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}-\frac{}{16d\left(a^2+\right.}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{a}{12d(a+a\sin(c+dx))^3}-\frac{1}{8d(a+a\sin(c+dx))^2}+\frac{1}{16d\left(a^2-a^2\sin\right.}$$

Mathematica [A] time = 0.119599, size = 85, normalized size = 0.82

$$-\frac{\sec^2(c+dx)\left(-3\sin^3(c+dx)-6\sin^2(c+dx)-\sin(c+dx)+3(\sin(c+dx)-1)(\sin(c+dx)+1)^3\tanh^{-1}(\sin(c+dx)\right.}{12a^2d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]^2*(4 - Sin[c + d*x] - 6*Sin[c + d*x]^2 - 3*Sin[c + d*x]^3 +
3*ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])*(1 + Sin[c + d*x])^3))/(12*a^2*
d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.087, size = 108, normalized size = 1.

$$-\frac{1}{16\,da^2(\sin(dx+c)-1)}-\frac{\ln(\sin(dx+c)-1)}{8\,da^2}-\frac{1}{12\,da^2(1+\sin(dx+c))^3}-\frac{1}{8\,da^2(1+\sin(dx+c))^2}-\frac{3}{16\,da^2(1+\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/16/d/a^2/(sin(d*x+c)-1)-1/8/d/a^2*ln(sin(d*x+c)-1)-1/12/d/a^2/(1+sin(d*x
+c))^3-1/8/d/a^2/(1+sin(d*x+c))^2-3/16/d/a^2/(1+sin(d*x+c))+1/8*ln(1+sin(d*
x+c))/a^2/d
```

Maxima [A] time = 1.14414, size = 146, normalized size = 1.4

$$-\frac{\frac{2\left(3\sin(dx+c)^{3}+6\sin(dx+c)^{2}+\sin(dx+c)-4\right)}{a^{2}\sin(dx+c)^{4}+2a^{2}\sin(dx+c)^{3}-2a^{2}\sin(dx+c)-a^{2}}-\frac{3\log(\sin(dx+c)+1)}{a^{2}}+\frac{3\log(\sin(dx+c)-1)}{a^{2}}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/24*(2*(3*sin(d*x + c)^3 + 6*sin(d*x + c)^2 + sin(d*x + c) - 4)/(a^2*sin(
d*x + c)^4 + 2*a^2*sin(d*x + c)^3 - 2*a^2*sin(d*x + c) - a^2) - 3*log(sin(d
*x + c) + 1)/a^2 + 3*log(sin(d*x + c) - 1)/a^2)/d
```

Fricas [A] time = 2.20367, size = 466, normalized size = 4.48

$$\frac{12\cos(dx+c)^{2}+3\left(\cos(dx+c)^{4}-2\cos(dx+c)^{2}\sin(dx+c)-2\cos(dx+c)^{2}\right)\log(\sin(dx+c)+1)-3\left(\cos(dx+c\right.}{24\left(a^{2}d\cos(dx+c)^{4}-2a^{2}d\cos(dx+c)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*cos(d*x + c)^2 + 3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2*sin(d*x + c)
 - 2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^4 - 2*cos(d*x
+ c)^2*sin(d*x + c) - 2*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(3*cos(d
*x + c)^2 - 4)*sin(d*x + c) - 4)/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x +
c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.19462, size = 143, normalized size = 1.38

$$\frac{\frac{6\log(|\sin(dx+c)+1|)}{a^2} - \frac{6\log(|\sin(dx+c)-1|)}{a^2} + \frac{3(2\sin(dx+c)-3)}{a^2(\sin(dx+c)-1)} - \frac{11\sin(dx+c)^3+42\sin(dx+c)^2+57\sin(dx+c)+30}{a^2(\sin(dx+c)+1)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/48*(6*log(abs(sin(d*x + c) + 1))/a^2 - 6*log(abs(sin(d*x + c) - 1))/a^2 +
 3*(2*sin(d*x + c) - 3)/(a^2*(sin(d*x + c) - 1)) - (11*sin(d*x + c)^3 + 42*
sin(d*x + c)^2 + 57*sin(d*x + c) + 30)/(a^2*(sin(d*x + c) + 1)^3))/d
```

3.73 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=93

$$\frac{4\tan^3(c+dx)}{21a^2d}+\frac{4\tan(c+dx)}{7a^2d}-\frac{\sec^3(c+dx)}{7d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\sec^3(c+dx)}{7d(a\sin(c+dx)+a)^2}$$

```
[Out] -Sec[c + d*x]^3/(7*d*(a + a*Sin[c + d*x])^2) - Sec[c + d*x]^3/(7*d*(a^2 + a
^2*Sin[c + d*x])) + (4*Tan[c + d*x])/(7*a^2*d) + (4*Tan[c + d*x]^3)/(21*a^2
*d)
```

Rubi [A] time = 0.0975248, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2672, 3767}

$$\frac{4\tan^3(c+dx)}{21a^2d}+\frac{4\tan(c+dx)}{7a^2d}-\frac{\sec^3(c+dx)}{7d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\sec^3(c+dx)}{7d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] -Sec[c + d*x]^3/(7*d*(a + a*Sin[c + d*x])^2) - Sec[c + d*x]^3/(7*d*(a^2 + a
^2*Sin[c + d*x])) + (4*Tan[c + d*x])/(7*a^2*d) + (4*Tan[c + d*x]^3)/(21*a^2
*d)
```


Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= -\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}+\frac{5\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx}{7a}\\
&= -\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}-\frac{\sec^3(c+dx)}{7d\left(a^2+a^2\sin(c+dx)\right)}+\frac{4\int \sec^4(c+dx)\,dx}{7a^2}\\
&= -\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}-\frac{\sec^3(c+dx)}{7d\left(a^2+a^2\sin(c+dx)\right)}-\frac{4\,\mathrm{Subst}\left(\int\left(1+x^2\right)\,dx,x,-\tan(c+\right.}{7a^2d}\\
&= -\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}-\frac{\sec^3(c+dx)}{7d\left(a^2+a^2\sin(c+dx)\right)}+\frac{4\tan(c+dx)}{7a^2d}+\frac{4\tan^3(c+dx)}{21a^2d}
\end{aligned}
$$

Mathematica [A] time = 0.0634137, size = 78, normalized size = 0.84

$$
-\frac{\left(8\sin^5(c+dx)+16\sin^4(c+dx)-4\sin^3(c+dx)-24\sin^2(c+dx)-9\sin(c+dx)+6\right)\sec^3(c+dx)}{21a^2d(\sin(c+dx)+1)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]^3*(6 - 9*Sin[c + d*x] - 24*Sin[c + d*x]^2 - 4*Sin[c + d*x]^3
 + 16*Sin[c + d*x]^4 + 8*Sin[c + d*x]^5))/(21*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.095, size = 158, normalized size = 1.7

$$
2\,\frac{1}{da^2}\left(-1/24\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-3}-1/16\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-2}-3/16\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-1}-2/7\,\left(\tan(1/2\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(-1/24/(tan(1/2*d*x+1/2*c)-1)^3-1/16/(tan(1/2*d*x+1/2*c)-1)^2-3/16/
(tan(1/2*d*x+1/2*c)-1)-2/7/(tan(1/2*d*x+1/2*c)+1)^7+1/(tan(1/2*d*x+1/2*c)+1
)^6-2/(tan(1/2*d*x+1/2*c)+1)^5+5/2/(tan(1/2*d*x+1/2*c)+1)^4-55/24/(tan(1/2*
d*x+1/2*c)+1)^3+23/16/(tan(1/2*d*x+1/2*c)+1)^2-13/16/(tan(1/2*d*x+1/2*c)+1)
```

)

Maxima [B] time = 1.00202, size = 535, normalized size = 5.75

$$-\frac{2\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1} - \frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{76\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{28\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{42\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{56\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{28\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{42\sin(dx+c}{(\cos(dx+c)+}\right.}{21\left(a^2 + \frac{4a^2\sin(dx+c)}{\cos(dx+c)+1} + \frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{4a^2\mathrm{si}}{(\cos(d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2/21*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 76*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 28*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 42*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 56*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 - 28*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 42*sin(d*x
+ c)^8/(cos(d*x + c) + 1)^8 - 21*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 6)/(
(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x + c)^8/(cos(d*x +
 c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*sin(d*x + c)^1
0/(cos(d*x + c) + 1)^10)*d)
```

Fricas [A] time = 2.00504, size = 261, normalized size = 2.81

$$\frac{16\cos(dx+c)^4 - 8\cos(dx+c)^2 + \left(8\cos(dx+c)^4 - 12\cos(dx+c)^2 - 5\right)\sin(dx+c) - 2}{21\left(a^2d\cos(dx+c)^5 - 2a^2d\cos(dx+c)^3\sin(dx+c) - 2a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/21*(16*cos(d*x + c)^4 - 8*cos(d*x + c)^2 + (8*cos(d*x + c)^4 - 12*cos(d*x
 + c)^2 - 5)*sin(d*x + c) - 2)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x + c)
^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^4(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.15149, size = 196, normalized size = 2.11

$$-\frac{\frac{7\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+8\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}+\frac{273\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+1155\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+2450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2870\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2037\tan\left(\frac{1}{2}d\right.}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/168*(7*(9*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 8)/(a^2*(ta
n(1/2*d*x + 1/2*c) - 1)^3) + (273*tan(1/2*d*x + 1/2*c)^6 + 1155*tan(1/2*d*x
 + 1/2*c)^5 + 2450*tan(1/2*d*x + 1/2*c)^4 + 2870*tan(1/2*d*x + 1/2*c)^3 + 2
037*tan(1/2*d*x + 1/2*c)^2 + 791*tan(1/2*d*x + 1/2*c) + 152)/(a^2*(tan(1/2*
d*x + 1/2*c) + 1)^7))/d
```

3.74 $\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=146

$$-\frac{a^2}{32d(a\sin(c+dx)+a)^4}+\frac{5}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{15\tanh^{-1}(\sin(c+dx))}{64a^2d}-\frac{}{16d(a\sin(}$$

```
[Out] (15*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + 1/(64*d*(a - a*Sin[c + d*x])^2) - a
^2/(32*d*(a + a*Sin[c + d*x])^4) - a/(16*d*(a + a*Sin[c + d*x])^3) - 3/(32*
d*(a + a*Sin[c + d*x])^2) + 5/(64*d*(a^2 - a^2*Sin[c + d*x])) - 5/(32*d*(a^
2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.10989, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$-\frac{a^2}{32d(a\sin(c+dx)+a)^4}+\frac{5}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{15\tanh^{-1}(\sin(c+dx))}{64a^2d}-\frac{}{16d(a\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + 1/(64*d*(a - a*Sin[c + d*x])^2) - a
^2/(32*d*(a + a*Sin[c + d*x])^4) - a/(16*d*(a + a*Sin[c + d*x])^3) - 3/(32*
d*(a + a*Sin[c + d*x])^2) + 5/(64*d*(a^2 - a^2*Sin[c + d*x])) - 5/(32*d*(a^
2 + a^2*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^3}+\frac{5}{64a^6(a-x)^2}+\frac{1}{8a^3(a+x)^5}+\frac{3}{16a^4(a+x)^4}+\frac{3}{16a^5(a+x)^3}+\frac{5}{32a^6(a+x)^2}+\frac{15}{64a^6(a^2\ldots}\right.\right.}{d}$$

$$= \frac{1}{64d(a-a\sin(c+dx))^2}-\frac{a^2}{32d(a+a\sin(c+dx))^4}-\frac{a}{16d(a+a\sin(c+dx))^3}-\frac{3}{32d(a+a\,\text{si}\ldots}$$

$$= \frac{15\tanh^{-1}(\sin(c+dx))}{64a^2d}+\frac{1}{64d(a-a\sin(c+dx))^2}-\frac{a^2}{32d(a+a\sin(c+dx))^4}-\frac{a}{16d(a+a\,\text{si}\ldots}$$

Mathematica [A] time = 0.337791, size = 137, normalized size = 0.94

$$\frac{(1-\sin(c+dx))^2(\sin(c+dx)+1)^2\sec^4(c+dx)\left(\frac{5}{64(1-\sin(c+dx))}-\frac{5}{32(\sin(c+dx)+1)}+\frac{1}{64(1-\sin(c+dx))^2}-\frac{3}{32(\sin(c+dx)+1)^2}-\frac{\ldots}{16(\sin\ldots}\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^4*(1 - Sin[c + d*x])^2*(1 + Sin[c + d*x])^2*((15*ArcTanh[Sin[
c + d*x]])/64 + 1/(64*(1 - Sin[c + d*x])^2) + 5/(64*(1 - Sin[c + d*x])) - 1
/(32*(1 + Sin[c + d*x])^4) - 1/(16*(1 + Sin[c + d*x])^3) - 3/(32*(1 + Sin[c
 + d*x])^2) - 5/(32*(1 + Sin[c + d*x]))))/(a^2*d)
```

Maple [A] time = 0.086, size = 144, normalized size = 1.

$$\frac{1}{64\,da^2\left(\sin\left(dx+c\right)-1\right)^2}-\frac{5}{64\,da^2\left(\sin\left(dx+c\right)-1\right)}-\frac{15\,\ln\left(\sin\left(dx+c\right)-1\right)}{128\,da^2}-\frac{1}{32\,da^2\left(1+\sin\left(dx+c\right)\right)^4}-\frac{1}{16\,da^2\left(1+s\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/64/d/a^2/(sin(d*x+c)-1)^2-5/64/d/a^2/(sin(d*x+c)-1)-15/128/d/a^2*ln(sin(d
*x+c)-1)-1/32/d/a^2/(1+sin(d*x+c))^4-1/16/d/a^2/(1+sin(d*x+c))^3-3/32/d/a^2
/(1+sin(d*x+c))^2-5/32/d/a^2/(1+sin(d*x+c))+15/128*ln(1+sin(d*x+c))/a^2/d
```

Maxima [A] time = 0.977011, size = 225, normalized size = 1.54

$$-\frac{\frac{2\left(15\,\sin(dx+c)^5+30\,\sin(dx+c)^4-10\,\sin(dx+c)^3-50\,\sin(dx+c)^2-17\,\sin(dx+c)+16\right)}{a^2\sin(dx+c)^6+2\,a^2\sin(dx+c)^5-a^2\sin(dx+c)^4-4\,a^2\sin(dx+c)^3-a^2\sin(dx+c)^2+2\,a^2\sin(dx+c)+a^2}-\frac{15\,\log(\sin(dx+c)+1)}{a^2}+\frac{15\,\log(\sin(dx+c)-1)}{a^2}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/128*(2*(15*sin(d*x + c)^5 + 30*sin(d*x + c)^4 - 10*sin(d*x + c)^3 - 50*s
in(d*x + c)^2 - 17*sin(d*x + c) + 16)/(a^2*sin(d*x + c)^6 + 2*a^2*sin(d*x +
 c)^5 - a^2*sin(d*x + c)^4 - 4*a^2*sin(d*x + c)^3 - a^2*sin(d*x + c)^2 + 2*
a^2*sin(d*x + c) + a^2) - 15*log(sin(d*x + c) + 1)/a^2 + 15*log(sin(d*x + c
) - 1)/a^2)/d
```

Fricas [A] time = 2.18425, size = 527, normalized size = 3.61

$$\frac{60\,\cos\left(dx+c\right)^4-20\,\cos\left(dx+c\right)^2+15\left(\cos\left(dx+c\right)^6-2\,\cos\left(dx+c\right)^4\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)^4\right)\log\left(\sin\left(dx+c\right)+\right.}{128\left(a^2d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/128*(60*cos(d*x + c)^4 - 20*cos(d*x + c)^2 + 15*(cos(d*x + c)^6 - 2*cos(d
*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - 15*(cos(
d*x + c)^6 - 2*cos(d*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(-sin(d*x
 + c) + 1) + 2*(15*cos(d*x + c)^4 - 20*cos(d*x + c)^2 - 12)*sin(d*x + c) -
8)/(a^2*d*cos(d*x + c)^6 - 2*a^2*d*cos(d*x + c)^4*sin(d*x + c) - 2*a^2*d*co
s(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18629, size = 170, normalized size = 1.16

$$\frac{\frac{60\log\left(|\sin(dx+c)+1|\right)}{a^2}-\frac{60\log\left(|\sin(dx+c)-1|\right)}{a^2}+\frac{2\left(45\sin(dx+c)^2-110\sin(dx+c)+69\right)}{a^2(\sin(dx+c)-1)^2}-\frac{125\sin(dx+c)^4+580\sin(dx+c)^3+1038\sin(dx+c)^2+868\sin(dx+}{a^2(\sin(dx+c)+1)^4}}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/512*(60*log(abs(sin(d*x + c) + 1))/a^2 - 60*log(abs(sin(d*x + c) - 1))/a^
2 + 2*(45*sin(d*x + c)^2 - 110*sin(d*x + c) + 69)/(a^2*(sin(d*x + c) - 1)^2
) - (125*sin(d*x + c)^4 + 580*sin(d*x + c)^3 + 1038*sin(d*x + c)^2 + 868*si
n(d*x + c) + 301)/(a^2*(sin(d*x + c) + 1)^4))/d
```

3.75 $\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{7\cos^5(c+dx)}{15a^3d}+\frac{7\sin(c+dx)\cos^3(c+dx)}{12a^3d}+\frac{7\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{7x}{8a^3}+\frac{2\cos^7(c+dx)}{3ad(a\sin(c+dx)+a)^2}$$

```
[Out] (7*x)/(8*a^3) + (7*Cos[c + d*x]^5)/(15*a^3*d) + (7*Cos[c + d*x]*Sin[c + d*x
])/(8*a^3*d) + (7*Cos[c + d*x]^3*Sin[c + d*x])/(12*a^3*d) + (2*Cos[c + d*x]
^7)/(3*a*d*(a + a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.109426, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2680, 2682, 2635, 8}

$$\frac{7\cos^5(c+dx)}{15a^3d}+\frac{7\sin(c+dx)\cos^3(c+dx)}{12a^3d}+\frac{7\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{7x}{8a^3}+\frac{2\cos^7(c+dx)}{3ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*x)/(8*a^3) + (7*Cos[c + d*x]^5)/(15*a^3*d) + (7*Cos[c + d*x]*Sin[c + d*x
])/(8*a^3*d) + (7*Cos[c + d*x]^3*Sin[c + d*x])/(12*a^3*d) + (2*Cos[c + d*x]
^7)/(3*a*d*(a + a*Sin[c + d*x])^2)
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{2\cos^7(c+dx)}{3ad(a+a\sin(c+dx))^2}+\frac{7\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx}{3a^2}\\
&= \frac{7\cos^5(c+dx)}{15a^3d}+\frac{2\cos^7(c+dx)}{3ad(a+a\sin(c+dx))^2}+\frac{7\int \cos^4(c+dx)\,dx}{3a^3}\\
&= \frac{7\cos^5(c+dx)}{15a^3d}+\frac{7\cos^3(c+dx)\sin(c+dx)}{12a^3d}+\frac{2\cos^7(c+dx)}{3ad(a+a\sin(c+dx))^2}+\frac{7\int \cos^2(c+dx)\,d}{4a^3}\\
&= \frac{7\cos^5(c+dx)}{15a^3d}+\frac{7\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{7\cos^3(c+dx)\sin(c+dx)}{12a^3d}+\frac{2\cos^7(c+}{3ad(a+a\sin(c}\\
&= \frac{7x}{8a^3}+\frac{7\cos^5(c+dx)}{15a^3d}+\frac{7\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{7\cos^3(c+dx)\sin(c+dx)}{12a^3d}+\frac{2\cos}{3ad(a+}
\end{aligned}
$$

Mathematica [A] time = 1.09445, size = 141, normalized size = 1.37

$$
-\frac{\left(\sqrt{\sin(c+dx)+1}\left(24\sin^5(c+dx)-114\sin^4(c+dx)+202\sin^3(c+dx)-127\sin^2(c+dx)-121\sin(c+dx)+136\right)-\right.}{120a^3d(\sin(c+dx)-1)^5(\sin(c+dx)+1)^{9/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Cos[c + d*x]^9*(-210*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[
c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(136 - 121*Sin[c + d*x] - 127*Sin[c + d*
x]^2 + 202*Sin[c + d*x]^3 - 114*Sin[c + d*x]^4 + 24*Sin[c + d*x]^5)))/(120*
a^3*d*(-1 + Sin[c + d*x])^5*(1 + Sin[c + d*x])^(9/2))
```

Maple [B] time = 0.083, size = 313, normalized size = 3.

$$-\frac{1}{4\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-5}+6\,\frac{(\tan(1/2\,dx+c/2))^8}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^5}-\frac{13}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\mathrm{ta}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^9+6/d/a^3/(1+tan(1
/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^8-13/2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)
^5*tan(1/2*d*x+1/2*c)^7+16/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2
*c)^6+20/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4+13/2/d/a^3
/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3+16/3/d/a^3/(1+tan(1/2*d*x+
1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2+1/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1
/2*d*x+1/2*c)+34/15/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^5+7/4/d/a^3*arctan(tan(1
/2*d*x+1/2*c))
```

Maxima [B] time = 1.47899, size = 419, normalized size = 4.07

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{320\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{390\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{400\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{960\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{390\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{360\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{15\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+136}{a^3+\frac{5a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 320*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 390*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 400*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 960*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 390*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 360*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 15*s
in(d*x + c)^9/(cos(d*x + c) + 1)^9 + 136)/(a^3 + 5*a^3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
+ a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 105*arctan(sin(d*x + c)/(cos
(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.98222, size = 163, normalized size = 1.58

$$-\frac{24\cos(dx+c)^5-160\cos(dx+c)^3-105\,dx+15\left(6\cos(dx+c)^3-7\cos(dx+c)\right)\sin(dx+c)}{120\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/120*(24*cos(d*x + c)^5 - 160*cos(d*x + c)^3 - 105*d*x + 15*(6*cos(d*x +
c)^3 - 7*cos(d*x + c))*sin(d*x + c))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16394, size = 189, normalized size = 1.83

$$\frac{\frac{105\,(dx+c)}{a^3}-\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-360\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+390\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-960\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-400\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-390\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-320\tan}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^5a^3}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(105*(d*x + c)/a^3 - 2*(15*tan(1/2*d*x + 1/2*c)^9 - 360*tan(1/2*d*x +
 1/2*c)^8 + 390*tan(1/2*d*x + 1/2*c)^7 - 960*tan(1/2*d*x + 1/2*c)^6 - 400*t
an(1/2*d*x + 1/2*c)^4 - 390*tan(1/2*d*x + 1/2*c)^3 - 320*tan(1/2*d*x + 1/2*
c)^2 - 15*tan(1/2*d*x + 1/2*c) - 136)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^3))
/d
```

3.76 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=23

$$-\frac{(a-a\sin(c+dx))^4}{4a^7d}$$

```
[Out] -(a - a*Sin[c + d*x])^4/(4*a^7*d)
```

Rubi [A] time = 0.0428404, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$-\frac{(a-a\sin(c+dx))^4}{4a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a - a*Sin[c + d*x])^4/(4*a^7*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\mathrm{Subst}\left(\int (a-x)^3\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= -\frac{(a-a\sin(c+dx))^4}{4a^7d}$$

Mathematica [A] time = 0.158463, size = 44, normalized size = 1.91

$$-\frac{\sin(c+dx)\left(\sin^3(c+dx)-4\sin^2(c+dx)+6\sin(c+dx)-4\right)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Sin[c + d*x]*(-4 + 6*Sin[c + d*x] - 4*Sin[c + d*x]^2 + Sin[c + d*x]^3))/(
4*a^3*d)
```

Maple [A] time = 0.069, size = 19, normalized size = 0.8

$$-\frac{(\sin(dx+c)-1)^4}{4\,da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/4/d/a^3*(sin(d*x+c)-1)^4
```

Maxima [B] time = 0.970327, size = 61, normalized size = 2.65

$$-\frac{\sin(dx+c)^4-4\,\sin(dx+c)^3+6\,\sin(dx+c)^2-4\,\sin(dx+c)}{4\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(sin(d*x + c)^4 - 4*sin(d*x + c)^3 + 6*sin(d*x + c)^2 - 4*sin(d*x + c)
)/(a^3*d)
```

Fricas [B] time = 1.9082, size = 119, normalized size = 5.17

$$-\frac{\cos\left(dx+c\right)^{4}-8\,\cos\left(dx+c\right)^{2}+4\left(\cos\left(dx+c\right)^{2}-2\right)\sin\left(dx+c\right)}{4\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x +
c))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17269, size = 61, normalized size = 2.65

$$-\frac{\sin\left(dx+c\right)^{4}-4\,\sin\left(dx+c\right)^{3}+6\,\sin\left(dx+c\right)^{2}-4\,\sin\left(dx+c\right)}{4\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(sin(d*x + c)^4 - 4*sin(d*x + c)^3 + 6*sin(d*x + c)^2 - 4*sin(d*x + c)
)/(a^3*d)
```

3.77 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=77

$$\frac{5\cos^3(c+dx)}{3a^3d}+\frac{5\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{5x}{2a^3}+\frac{2\cos^5(c+dx)}{ad(a\sin(c+dx)+a)^2}$$

```
[Out] (5*x)/(2*a^3) + (5*Cos[c + d*x]^3)/(3*a^3*d) + (5*Cos[c + d*x]*Sin[c + d*x]
)/(2*a^3*d) + (2*Cos[c + d*x]^5)/(a*d*(a + a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.0999435, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2680, 2682, 2635, 8}

$$\frac{5\cos^3(c+dx)}{3a^3d}+\frac{5\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{5x}{2a^3}+\frac{2\cos^5(c+dx)}{ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*x)/(2*a^3) + (5*Cos[c + d*x]^3)/(3*a^3*d) + (5*Cos[c + d*x]*Sin[c + d*x]
)/(2*a^3*d) + (2*Cos[c + d*x]^5)/(a*d*(a + a*Sin[c + d*x])^2)
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{2\cos^5(c+dx)}{ad(a+a\sin(c+dx))^2}+\frac{5\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx}{a^2}\\
&= \frac{5\cos^3(c+dx)}{3a^3d}+\frac{2\cos^5(c+dx)}{ad(a+a\sin(c+dx))^2}+\frac{5\int \cos^2(c+dx)\,dx}{a^3}\\
&= \frac{5\cos^3(c+dx)}{3a^3d}+\frac{5\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{2\cos^5(c+dx)}{ad(a+a\sin(c+dx))^2}+\frac{5\int 1\,dx}{2a^3}\\
&= \frac{5x}{2a^3}+\frac{5\cos^3(c+dx)}{3a^3d}+\frac{5\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{2\cos^5(c+dx)}{ad(a+a\sin(c+dx))^2}
\end{aligned}
$$

Mathematica [A] time = 0.485727, size = 121, normalized size = 1.57

$$
-\frac{\left(30\sqrt{1-\sin(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\sin(c+dx)+1}\left(2\sin^3(c+dx)-11\sin^2(c+dx)+31\sin(c+dx)-22\right)\right)\mathrm{co}}{6a^3d(\sin(c+dx)-1)^4(\sin(c+dx)+1)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Cos[c + d*x]^7*(30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c
+ d*x]] + Sqrt[1 + Sin[c + d*x]]*(-22 + 31*Sin[c + d*x] - 11*Sin[c + d*x]^2
 + 2*Sin[c + d*x]^3)))/(6*a^3*d*(-1 + Sin[c + d*x])^4*(1 + Sin[c + d*x])^(7
/2))
```

Maple [B] time = 0.084, size = 177, normalized size = 2.3

$$
3\,\frac{(\tan(1/2\,dx+c/2))^5}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+6\,\frac{(\tan(1/2\,dx+c/2))^4}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+16\,\frac{(\tan(1/2\,dx+c/2))^2}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}-3\,\frac{\tan}{da^3\left(1+(\mathrm{t}\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+6/d/a^3/(1+tan(1/2*
d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4+16/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*ta
n(1/2*d*x+1/2*c)^2-3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)+22
/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3+5/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.44955, size = 248, normalized size = 3.22

$$-\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{9\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-22}{a^3+\frac{3a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 18*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 9*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 - 22)/(a^3 + 3*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6) - 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.98291, size = 122, normalized size = 1.58

$$-\frac{2\cos(dx+c)^3-15dx+9\cos(dx+c)\sin(dx+c)-24\cos(dx+c)}{6a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*cos(d*x + c)^3 - 15*d*x + 9*cos(d*x + c)*sin(d*x + c) - 24*cos(d*x
+ c))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15625, size = 119, normalized size = 1.55

$$\frac{\frac{15\,(dx+c)}{a^3}+\frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+22\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)/a^3 + 2*(9*tan(1/2*d*x + 1/2*c)^5 + 18*tan(1/2*d*x + 1/2*
c)^4 + 48*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 22)/((tan(1/2*d
*x + 1/2*c)^2 + 1)^3*a^3))/d
```

3.78 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=50

$$\frac{\sin^2(c+dx)}{2a^3d}-\frac{3\sin(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] (4*Log[1 + Sin[c + d*x]])/(a^3*d) - (3*Sin[c + d*x])/(a^3*d) + Sin[c + d*x]
^2/(2*a^3*d)
```

Rubi [A] time = 0.0499715, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{\sin^2(c+dx)}{2a^3d}-\frac{3\sin(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*Log[1 + Sin[c + d*x]])/(a^3*d) - (3*Sin[c + d*x])/(a^3*d) + Sin[c + d*x]
^2/(2*a^3*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a+x+\frac{4a^2}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{4\log(1+\sin(c+dx))}{a^3d} - \frac{3\sin(c+dx)}{a^3d} + \frac{\sin^2(c+dx)}{2a^3d}$$

Mathematica [A] time = 0.0500167, size = 38, normalized size = 0.76

$$\frac{\sin^2(c+dx) - 6\sin(c+dx) + 8\log(\sin(c+dx)+1)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (8*Log[1 + Sin[c + d*x]] - 6*Sin[c + d*x] + Sin[c + d*x]^2)/(2*a^3*d)
```

Maple [A] time = 0.069, size = 49, normalized size = 1.

$$4\,\frac{\ln(1+\sin(dx+c))}{a^3d} - 3\,\frac{\sin(dx+c)}{a^3d} + \frac{(\sin(dx+c))^2}{2\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4*ln(1+sin(d*x+c))/a^3/d-3*sin(d*x+c)/a^3/d+1/2*sin(d*x+c)^2/a^3/d
```

Maxima [A] time = 0.942104, size = 55, normalized size = 1.1

$$\frac{\frac{\sin(dx+c)^2-6\,\sin(dx+c)}{a^3} + \frac{8\,\log(\sin(dx+c)+1)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((sin(d*x + c)^2 - 6*sin(d*x + c))/a^3 + 8*log(sin(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 2.01086, size = 100, normalized size = 2.

$$-\frac{\cos\left(dx+c\right)^{2}-8\,\log\left(\sin\left(dx+c\right)+1\right)+6\,\sin\left(dx+c\right)}{2\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(cos(d*x + c)^2 - 8*log(sin(d*x + c) + 1) + 6*sin(d*x + c))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16831, size = 155, normalized size = 3.1

$$-\frac{2\left(\frac{2\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)}{a^{3}}-\frac{4\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{3}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2*(2*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 4*log(abs(tan(1/2*d*x + 1/2*c)
+ 1))/a^3 - (3*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*x + 1/2*c)^3 + 7*tan(1/
2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 3)/((tan(1/2*d*x + 1/2*c)^2 + 1
)^2*a^3))/d
```

3.79 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^3} \, dx$

Optimal. Leaf size=49

$$-\frac{3\cos(c+dx)}{a^3 d}-\frac{3x}{a^3}-\frac{2\cos^3(c+dx)}{ad(a\sin(c+dx)+a)^2}$$

```
[Out] (-3*x)/a^3 - (3*Cos[c + d*x])/(a^3*d) - (2*Cos[c + d*x]^3)/(a*d*(a + a*Sin[
c + d*x])^2)
```

Rubi [A] time = 0.0850498, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2680, 2682, 8}

$$-\frac{3\cos(c+dx)}{a^3 d}-\frac{3x}{a^3}-\frac{2\cos^3(c+dx)}{ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*x)/a^3 - (3*Cos[c + d*x])/(a^3*d) - (2*Cos[c + d*x]^3)/(a*d*(a + a*Sin[
c + d*x])^2)
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= -\frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2} - \frac{3\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx}{a^2} \\
&= -\frac{3\cos(c+dx)}{a^3d} - \frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2} - \frac{3\int 1\,dx}{a^3} \\
&= -\frac{3x}{a^3} - \frac{3\cos(c+dx)}{a^3d} - \frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2}
\end{aligned}
$$

Mathematica [C] time = 0.0383827, size = 59, normalized size = 1.2

$$
-\frac{\cos^5(c+dx)\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\sqrt{2}a^3d(\sin(c+dx)+1)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Cos[c + d*x]^5*Hypergeometric2F1[3/2, 5/2, 7/2, (1 - Sin[c + d*x])/2])/(5
*Sqrt[2]*a^3*d*(1 + Sin[c + d*x])^(5/2))
```

Maple [A] time = 0.084, size = 64, normalized size = 1.3

$$
-2\,\frac{1}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)} - 6\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da^3} - 8\,\frac{1}{da^3\,(\tan(1/2\,dx+c/2)+1)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a^3*arctan(tan(1/2*d*x+1/2*c))-8/d/a^
3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.43639, size = 188, normalized size = 3.84

$$-\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+5}{a^3+\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2*((sin(d*x + c)/(cos(d*x + c) + 1) + 4*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 5)/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c) + 1) + a^3*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 3*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.95341, size = 203, normalized size = 4.14

$$-\frac{3\,dx+(3\,dx+5)\cos(dx+c)+\cos(dx+c)^2+(3\,dx+\cos(dx+c)-4)\sin(dx+c)+4}{a^3d\cos(dx+c)+a^3d\sin(dx+c)+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*d*x + (3*d*x + 5)*cos(d*x + c) + cos(d*x + c)^2 + (3*d*x + cos(d*x + c)
 - 4)*sin(d*x + c) + 4)/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18172, size = 108, normalized size = 2.2

$$-\frac{\frac{3\,(dx+c)}{a^3}+\frac{2\left(4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(d*x + c)/a^3 + 2*(4*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 5)
/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) +
 1)*a^3))/d
```

3.80 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=39

$$-\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] -(Log[1 + Sin[c + d*x]]/(a^3*d)) - 2/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.0505272, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$-\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Log[1 + Sin[c + d*x]]/(a^3*d)) - 2/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{2a}{(a+x)^2}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= -\frac{\log(1+\sin(c+dx))}{a^3d} - \frac{2}{d\left(a^3+a^3\sin(c+dx)\right)}$$

Mathematica [A] time = 0.063166, size = 58, normalized size = 1.49

$$-\frac{\sin(c+dx)\log(\sin(c+dx)+1)+\log(\sin(c+dx)+1)+2}{a^3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((2 + Log[1 + Sin[c + d*x]] + Log[1 + Sin[c + d*x]]*Sin[c + d*x])/(a^3*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))
```

Maple [A] time = 0.072, size = 37, normalized size = 1.

$$-2\,\frac{1}{da^3\,(1+\sin(dx+c))}-\frac{\ln(1+\sin(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/d/a^3/(1+sin(d*x+c))-ln(1+sin(d*x+c))/a^3/d
```

Maxima [A] time = 0.947007, size = 50, normalized size = 1.28

$$-\frac{\frac{2}{a^3\sin(dx+c)+a^3}+\frac{\log(\sin(dx+c)+1)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(2/(a^3*sin(d*x + c) + a^3) + log(sin(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 1.80848, size = 105, normalized size = 2.69

$$-\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1)+2}{a^3d\sin(dx+c)+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) + 2)/(a^3*d*sin(d*x + c) + a^3*d
)
```

Sympy [A] time = 1.86801, size = 450, normalized size = 11.54

$$\begin{cases} -\frac{2\log(\sin(c+dx)+1)\sin^2(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{4\log(\sin(c+dx)+1)\sin(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{2\log(\sin(c+dx)+1)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} + \frac{\sin^4(c+}{2a^3d\sin^2(c+dx)+4a^3} \\ \frac{x\cos^3(c)}{(a\sin(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-2*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)*
*2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 4*log(sin(c + d*x) + 1)*sin(c + d*
x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 2*log(si
n(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*
d) + sin(c + d*x)**4/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*
a**3*d) + 2*sin(c + d*x)**3/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*
x) + 2*a**3*d) + sin(c + d*x)**2*cos(c + d*x)**2/(2*a**3*d*sin(c + d*x)**2
+ 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 2*sin(c + d*x)**2/(2*a**3*d*sin(c + d
*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 2*sin(c + d*x)*cos(c + d*x)**2
/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 1/(2*a**3*
d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d), Ne(d, 0)), (x*cos(c)
```

```
**3/(a*sin(c) + a)**3, True))
```

Giac [A] time = 1.17549, size = 47, normalized size = 1.21

$$-\frac{\frac{\log(|\sin(dx+c)+1|)}{a^3}+\frac{2}{a^3(\sin(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(log(abs(sin(d*x + c) + 1))/a^3 + 2/(a^3*(sin(d*x + c) + 1)))/d
```

3.81 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=27

$$-\frac{\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^3}$$

```
[Out] -Cos[c + d*x]^3/(3*d*(a + a*Sin[c + d*x])^3)
```

Rubi [A] time = 0.038762, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2671}

$$-\frac{\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Cos[c + d*x]^3/(3*d*(a + a*Sin[c + d*x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx=-\frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3}$$

Mathematica [A] time = 0.0170647, size = 28, normalized size = 1.04

$$-\frac{\cos^3(c+dx)}{3a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Cos[c + d*x]^3/(3*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [B] time = 0.089, size = 55, normalized size = 2.

$$2\,\frac{1}{da^3}\left(2\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-2}-4/3\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-3}-\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(2/(tan(1/2*d*x+1/2*c)+1)^2-4/3/(tan(1/2*d*x+1/2*c)+1)^3-1/(tan(1/2
*d*x+1/2*c)+1))
```

Maxima [B] time = 0.974754, size = 134, normalized size = 4.96

$$-\frac{2\left(\frac{3\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{3\left(a^3+\frac{3\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/3*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/((a^3 + 3*a^3*sin(d*x + c)
/(cos(d*x + c) + 1) + 3*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3)*d)
```

Fricas [B] time = 1.76474, size = 238, normalized size = 8.81

$$-\frac{\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2}{3\left(a^3d\cos\left(dx+c\right)^2-a^3d\cos\left(dx+c\right)-2\,a^3d-\left(a^3d\cos\left(dx+c\right)+2\,a^3d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(d*x + c)^2 + (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)/
(a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (a^3*d*cos(d*x + c)
+ 2*a^3*d)*sin(d*x + c))
```

Sympy [A] time = 29.2513, size = 298, normalized size = 11.04

$$\begin{cases} \frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{3}d\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{3}d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{3}d} - \frac{3\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{3}d\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{3}d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{3}d} + \frac{}{3a^{3}d\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)+} \\ \frac{x\cos^{2}(c)}{(a\sin(c)+a)^{3}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**3/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*tan
(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) - 3*tan(c/2 + d*x/
2)**2/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3
*d*tan(c/2 + d*x/2) + 3*a**3*d) + 3*tan(c/2 + d*x/2)/(3*a**3*d*tan(c/2 + d*
x/2)**3 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3
*d) - 1/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 9*a*
*3*d*tan(c/2 + d*x/2) + 3*a**3*d), Ne(d, 0)), (x*cos(c)**2/(a*sin(c) + a)**
3, True))
```

Giac [A] time = 1.14986, size = 49, normalized size = 1.81

$$-\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)}{3\,a^{3}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) + 1)^3)
```

3.82 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^3} dx$

Optimal. Leaf size=22

$$-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] -1/(2*a*d*(a + a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.0246114, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 32}

$$-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/(2*a*d*(a + a*Sin[c + d*x])^2)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{1}{2ad(a+a\sin(c+dx))^2}$$

Mathematica [A] time = 0.0569452, size = 33, normalized size = 1.5

$$-\frac{1}{2a^3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/(2*a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A] time = 0.014, size = 21, normalized size = 1.

$$-\frac{1}{2\,da\,(a+a\sin(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/2/a/d/(a+a*sin(d*x+c))^2
```

Maxima [A] time = 0.94394, size = 27, normalized size = 1.23

$$-\frac{1}{2\,(a\sin(dx+c)+a)^2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2/((a*sin(d*x + c) + a)^2*a*d)
```

Fricas [A] time = 1.83076, size = 82, normalized size = 3.73

$$\frac{1}{2\left(a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\sin\left(dx+c\right)-2\,a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

Sympy [A] time = 1.66904, size = 51, normalized size = 2.32

$$\begin{cases} -\frac{1}{2a^{3}d\sin^{2}\left(c+dx\right)+4a^{3}d\sin\left(c+dx\right)+2a^{3}d} & \text{for}\ d\neq 0 \\ \frac{x\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-1/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d)
, Ne(d, 0)), (x*cos(c)/(a*sin(c) + a)**3, True))
```

Giac [A] time = 1.19321, size = 27, normalized size = 1.23

$$-\frac{1}{2\left(a\sin\left(dx+c\right)+a\right)^{2}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2/((a*sin(d*x + c) + a)^2*a*d)
```

3.83 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{1}{8ad(a\sin(c+dx)+a)^2}-\frac{1}{6d(a\sin(c+dx)+a)^3}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(8*a^3*d) - 1/(6*d*(a + a*Sin[c + d*x])^3) - 1/(8*a*d
*(a + a*Sin[c + d*x])^2) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.0631045, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2667, 44, 206}

$$-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{1}{8ad(a\sin(c+dx)+a)^2}-\frac{1}{6d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(8*a^3*d) - 1/(6*d*(a + a*Sin[c + d*x])^3) - 1/(8*a*d
*(a + a*Sin[c + d*x])^2) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{a\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^4}+\frac{1}{4a^2(a+x)^3}+\frac{1}{8a^3(a+x)^2}+\frac{1}{8a^3(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{1}{6d(a+a\sin(c+dx))^3}-\frac{1}{8ad(a+a\sin(c+dx))^2}-\frac{1}{8d\left(a^3+a^3\sin(c+dx)\right)}+\frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{1}{6d(a+a\sin(c+dx))^3}-\frac{1}{8ad(a+a\sin(c+dx))^2}-\frac{1}{8d\left(a^3+a^3\sin(\right.}$$

Mathematica [A] time = 0.083286, size = 61, normalized size = 0.74

$$\frac{-\frac{1}{8(\sin(c+dx)+1)}-\frac{1}{8(\sin(c+dx)+1)^2}-\frac{1}{6(\sin(c+dx)+1)^3}+\frac{1}{8}\tanh^{-1}(\sin(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]/8 - 1/(6*(1 + Sin[c + d*x])^3) - 1/(8*(1 + Sin[c + d
*x])^2) - 1/(8*(1 + Sin[c + d*x])))/(a^3*d)
```

Maple [A] time = 0.082, size = 90, normalized size = 1.1

$$-\frac{\ln(\sin(dx+c)-1)}{16\,da^3}-\frac{1}{6\,da^3\,(1+\sin(dx+c))^3}-\frac{1}{8\,da^3\,(1+\sin(dx+c))^2}-\frac{1}{8\,da^3\,(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx}{16\,da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/16/d/a^3*ln(sin(d*x+c)-1)-1/6/d/a^3/(1+sin(d*x+c))^3-1/8/d/a^3/(1+sin(d*
x+c))^2-1/8/d/a^3/(1+sin(d*x+c))+1/16*ln(1+sin(d*x+c))/a^3/d
```

Maxima [A] time = 0.96238, size = 132, normalized size = 1.61

$$-\frac{\frac{2\left(3\sin(dx+c)^2+9\sin(dx+c)+10\right)}{a^3\sin(dx+c)^3+3a^3\sin(dx+c)^2+3a^3\sin(dx+c)+a^3}-\frac{3\log(\sin(dx+c)+1)}{a^3}+\frac{3\log(\sin(dx+c)-1)}{a^3}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(3*sin(d*x + c)^2 + 9*sin(d*x + c) + 10)/(a^3*sin(d*x + c)^3 + 3*a
^3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) - 3*log(sin(d*x + c) + 1)/a^3
 + 3*log(sin(d*x + c) - 1)/a^3)/d
```

Fricas [B] time = 2.0941, size = 409, normalized size = 4.99

$$-\frac{6\cos(dx+c)^2-3\left(3\cos(dx+c)^2+\left(\cos(dx+c)^2-4\right)\sin(dx+c)-4\right)\log(\sin(dx+c)+1)+3\left(3\cos(dx+c)^2+\left(\cdots\right.\right.}{48\left(3a^3d\cos(dx+c)^2-4a^3d+\left(a^3d\cos(dx+c)^2-4a\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(6*cos(d*x + c)^2 - 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*
x + c) - 4)*log(sin(d*x + c) + 1) + 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 -
 4)*sin(d*x + c) - 4)*log(-sin(d*x + c) + 1) - 18*sin(d*x + c) - 26)/(3*a^3
*d*cos(d*x + c)^2 - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 4*a^3*d)*sin(d*x + c)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x)
 + 1), x)/a**3
```

Giac [A] time = 1.20196, size = 109, normalized size = 1.33

$$\frac{\frac{6\log(|\sin(dx+c)+1|)}{a^3}-\frac{6\log(|\sin(dx+c)-1|)}{a^3}-\frac{11\sin(dx+c)^3+45\sin(dx+c)^2+69\sin(dx+c)+51}{a^3(\sin(dx+c)+1)^3}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(sin(d*x + c) + 1))/a^3 - 6*log(abs(sin(d*x + c) - 1))/a^3 -
 (11*sin(d*x + c)^3 + 45*sin(d*x + c)^2 + 69*sin(d*x + c) + 51)/(a^3*(sin(d
*x + c) + 1)^3))/d
```

3.84 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=99

$$\frac{8\tan(c+dx)}{35a^3d}-\frac{4\sec(c+dx)}{35d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4\sec(c+dx)}{35ad(a\sin(c+dx)+a)^2}-\frac{\sec(c+dx)}{7d(a\sin(c+dx)+a)^3}$$

```
[Out] -Sec[c + d*x]/(7*d*(a + a*Sin[c + d*x])^3) - (4*Sec[c + d*x])/(35*a*d*(a +
a*Sin[c + d*x])^2) - (4*Sec[c + d*x])/(35*d*(a^3 + a^3*Sin[c + d*x])) + (8*
Tan[c + d*x])/(35*a^3*d)
```

Rubi [A] time = 0.134848, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2672, 3767, 8}

$$\frac{8\tan(c+dx)}{35a^3d}-\frac{4\sec(c+dx)}{35d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4\sec(c+dx)}{35ad(a\sin(c+dx)+a)^2}-\frac{\sec(c+dx)}{7d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Sec[c + d*x]/(7*d*(a + a*Sin[c + d*x])^3) - (4*Sec[c + d*x])/(35*a*d*(a +
a*Sin[c + d*x])^2) - (4*Sec[c + d*x])/(35*d*(a^3 + a^3*Sin[c + d*x])) + (8*
Tan[c + d*x])/(35*a^3*d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3}+\frac{4\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{7a}\\
&= -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3}-\frac{4\sec(c+dx)}{35ad(a+a\sin(c+dx))^2}+\frac{12\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx}{35a^2}\\
&= -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3}-\frac{4\sec(c+dx)}{35ad(a+a\sin(c+dx))^2}-\frac{4\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)}+\frac{8\int \sec^2
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3}-\frac{4\sec(c+dx)}{35ad(a+a\sin(c+dx))^2}-\frac{4\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)}-\frac{8\,\mathrm{Subst}}{}\\
&= -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3}-\frac{4\sec(c+dx)}{35ad(a+a\sin(c+dx))^2}-\frac{4\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)}+\frac{8\tan(c}{35a^3}
\end{aligned}
$$

Mathematica [A] time = 0.0977653, size = 63, normalized size = 0.64

$$
\frac{\sec(c+dx)(14\sin(c+dx)-6\sin(3(c+dx))-14\cos(2(c+dx))+\cos(4(c+dx)))}{35a^3d(\sin(c+dx)+1)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(-14*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] + 14*Sin[c + d*x] -
6*Sin[3*(c + d*x)]))/(35*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.081, size = 130, normalized size = 1.3

$$
2\frac{1}{da^3}\left(-1/16\,(\tan(1/2\,dx+c/2)-1)^{-1}-4/7\,(\tan(1/2\,dx+c/2)+1)^{-7}+2\,(\tan(1/2\,dx+c/2)+1)^{-6}-\frac{19}{5\,(\tan(1/2\,dx+\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(-1/16/(tan(1/2*d*x+1/2*c)-1)-4/7/(tan(1/2*d*x+1/2*c)+1)^7+2/(tan(1
/2*d*x+1/2*c)+1)^6-19/5/(tan(1/2*d*x+1/2*c)+1)^5+9/2/(tan(1/2*d*x+1/2*c)+1)
^4-15/4/(tan(1/2*d*x+1/2*c)+1)^3+17/8/(tan(1/2*d*x+1/2*c)+1)^2-15/16/(tan(1
/2*d*x+1/2*c)+1))
```

Maxima [B] time = 0.98741, size = 419, normalized size = 4.23

$$-\frac{2\left(\frac{43\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{105\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{175\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{105\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{35\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+13\right)}{35\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/35*(43*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 105*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 175*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 105*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 35*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 13)/((a^
3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^3*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6
*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/(cos(d*x + c)
 + 1)^8)*d)
```

Fricas [A] time = 1.90802, size = 270, normalized size = 2.73

$$-\frac{8\cos(dx+c)^4-36\cos(dx+c)^2-4\left(6\cos(dx+c)^2-5\right)\sin(dx+c)+15}{35\left(3a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)+\left(a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(8*cos(d*x + c)^4 - 36*cos(d*x + c)^2 - 4*(6*cos(d*x + c)^2 - 5)*sin(
d*x + c) + 15)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(
```

```
d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.16622, size = 161, normalized size = 1.63

$$-\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}+\frac{525\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+1960\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+4025\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+4480\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3143\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1176\tan\left(\frac{1}{2}\right.}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) + (525*tan(1/2*d*x + 1/2*c)^6 +
 1960*tan(1/2*d*x + 1/2*c)^5 + 4025*tan(1/2*d*x + 1/2*c)^4 + 4480*tan(1/2*d
*x + 1/2*c)^3 + 3143*tan(1/2*d*x + 1/2*c)^2 + 1176*tan(1/2*d*x + 1/2*c) + 2
43)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

3.85 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=126

$$\frac{1}{32d\left(a^3-a^3\sin(c+dx)\right)}-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{5\tanh^{-1}(\sin(c+dx))}{32a^3d}-\frac{a}{16d(a\sin(c+dx)+a)^4}-\frac{1}{12d(a\sin(c+a}$$

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(32*a^3*d) - a/(16*d*(a + a*Sin[c + d*x])^4) - 1/
(12*d*(a + a*Sin[c + d*x])^3) - 3/(32*a*d*(a + a*Sin[c + d*x])^2) + 1/(32*d
*(a^3 - a^3*Sin[c + d*x])) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.0946766, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$\frac{1}{32d\left(a^3-a^3\sin(c+dx)\right)}-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{5\tanh^{-1}(\sin(c+dx))}{32a^3d}-\frac{a}{16d(a\sin(c+dx)+a)^4}-\frac{1}{12d(a\sin(c+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (5*ArcTanh[Sin[c + d*x]])/(32*a^3*d) - a/(16*d*(a + a*Sin[c + d*x])^4) - 1/
(12*d*(a + a*Sin[c + d*x])^3) - 3/(32*a*d*(a + a*Sin[c + d*x])^2) + 1/(32*d
*(a^3 - a^3*Sin[c + d*x])) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))
```


Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\,\mathrm{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^2} + \frac{1}{4a^2(a+x)^5} + \frac{1}{4a^3(a+x)^4} + \frac{3}{16a^4(a+x)^3} + \frac{1}{8a^5(a+x)^2} + \frac{5}{32a^5\left(a^2-x^2\right)}\right) dx, x, a\,s\right.}{d}$$

$$= -\frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{12d(a+a\sin(c+dx))^3} - \frac{3}{32ad(a+a\sin(c+dx))^2} + \frac{}{32d\left(a^3 -\right.}$$

$$= \frac{5\tanh^{-1}(\sin(c+dx))}{32a^3d} - \frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{12d(a+a\sin(c+dx))^3} - \frac{3}{32ad(a+a\,\mathrm{si}}$$

Mathematica [A] time = 0.168287, size = 95, normalized size = 0.75

$$-\frac{\sec^2(c+dx)\left(-15\sin^4(c+dx) - 45\sin^3(c+dx) - 35\sin^2(c+dx) + 15\sin(c+dx) + 15(\sin(c+dx)-1)(\sin(c+dx)+\right.}{96a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Sec[c + d*x]^2*(32 + 15*Sin[c + d*x] - 35*Sin[c + d*x]^2 - 45*Sin[c + d*x
]^3 - 15*Sin[c + d*x]^4 + 15*ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])*(1 +
 Sin[c + d*x])^4))/(96*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.102, size = 126, normalized size = 1.

$$-\frac{1}{32\,da^3\,(\sin(dx+c)-1)} - \frac{5\,\ln(\sin(dx+c)-1)}{64\,da^3} - \frac{1}{16\,da^3\,(1+\sin(dx+c))^4} - \frac{1}{12\,da^3\,(1+\sin(dx+c))^3} - \frac{}{32\,da^3\,(1-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/32/d/a^3/(sin(d*x+c)-1)-5/64/d/a^3*ln(sin(d*x+c)-1)-1/16/d/a^3/(1+sin(d*
x+c))^4-1/12/d/a^3/(1+sin(d*x+c))^3-3/32/d/a^3/(1+sin(d*x+c))^2-1/8/d/a^3/(
1+sin(d*x+c))+5/64*ln(1+sin(d*x+c))/a^3/d
```

Maxima [A] time = 0.959049, size = 197, normalized size = 1.56

$$-\frac{\frac{2\left(15\sin(dx+c)^4+45\sin(dx+c)^3+35\sin(dx+c)^2-15\sin(dx+c)-32\right)}{a^3\sin(dx+c)^5+3a^3\sin(dx+c)^4+2a^3\sin(dx+c)^3-2a^3\sin(dx+c)^2-3a^3\sin(dx+c)-a^3}-\frac{15\log(\sin(dx+c)+1)}{a^3}+\frac{15\log(\sin(dx+c)-1)}{a^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(2*(15*sin(d*x + c)^4 + 45*sin(d*x + c)^3 + 35*sin(d*x + c)^2 - 15*s
in(d*x + c) - 32)/(a^3*sin(d*x + c)^5 + 3*a^3*sin(d*x + c)^4 + 2*a^3*sin(d*
x + c)^3 - 2*a^3*sin(d*x + c)^2 - 3*a^3*sin(d*x + c) - a^3) - 15*log(sin(d*
x + c) + 1)/a^3 + 15*log(sin(d*x + c) - 1)/a^3)/d
```

Fricas [A] time = 1.83161, size = 595, normalized size = 4.72

$$-\frac{30\cos(dx+c)^4-130\cos(dx+c)^2-15\left(3\cos(dx+c)^4-4\cos(dx+c)^2+\left(\cos(dx+c)^4-4\cos(dx+c)^2\right)\sin(dx+\cdots}{192\left(3a^3d\cos(dx+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(30*cos(d*x + c)^4 - 130*cos(d*x + c)^2 - 15*(3*cos(d*x + c)^4 - 4*c
os(d*x + c)^2 + (cos(d*x + c)^4 - 4*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d
*x + c) + 1) + 15*(3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + (cos(d*x + c)^4 -
4*cos(d*x + c)^2)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 30*(3*cos(d*x + c)
^2 - 2)*sin(d*x + c) + 36)/(3*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2
 + (a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.19805, size = 157, normalized size = 1.25

$$\frac{\frac{60\log(|\sin(dx+c)+1|)}{a^3} - \frac{60\log(|\sin(dx+c)-1|)}{a^3} + \frac{12(5\sin(dx+c)-7)}{a^3(\sin(dx+c)-1)} - \frac{125\sin(dx+c)^4+596\sin(dx+c)^3+1110\sin(dx+c)^2+996\sin(dx+c)+405}{a^3(\sin(dx+c)+1)^4}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(60*log(abs(sin(d*x + c) + 1))/a^3 - 60*log(abs(sin(d*x + c) - 1))/a^
3 + 12*(5*sin(d*x + c) - 7)/(a^3*(sin(d*x + c) - 1)) - (125*sin(d*x + c)^4
+ 596*sin(d*x + c)^3 + 1110*sin(d*x + c)^2 + 996*sin(d*x + c) + 405)/(a^3*(
sin(d*x + c) + 1)^4))/d
```

3.86 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=123

$$\frac{8\tan^3(c+dx)}{63a^3d}+\frac{8\tan(c+dx)}{21a^3d}-\frac{2\sec^3(c+dx)}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{2\sec^3(c+dx)}{21ad(a\sin(c+dx)+a)^2}-\frac{\sec^3(c+dx)}{9d(a\sin(c+dx)+a)^3}$$

```
[Out] -Sec[c + d*x]^3/(9*d*(a + a*Sin[c + d*x])^3) - (2*Sec[c + d*x]^3)/(21*a*d*(
a + a*Sin[c + d*x])^2) - (2*Sec[c + d*x]^3)/(21*d*(a^3 + a^3*Sin[c + d*x]))
 + (8*Tan[c + d*x])/(21*a^3*d) + (8*Tan[c + d*x]^3)/(63*a^3*d)
```

Rubi [A] time = 0.145029, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2672, 3767}

$$\frac{8\tan^3(c+dx)}{63a^3d}+\frac{8\tan(c+dx)}{21a^3d}-\frac{2\sec^3(c+dx)}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{2\sec^3(c+dx)}{21ad(a\sin(c+dx)+a)^2}-\frac{\sec^3(c+dx)}{9d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Sec[c + d*x]^3/(9*d*(a + a*Sin[c + d*x])^3) - (2*Sec[c + d*x]^3)/(21*a*d*(
a + a*Sin[c + d*x])^2) - (2*Sec[c + d*x]^3)/(21*d*(a^3 + a^3*Sin[c + d*x]))
 + (8*Tan[c + d*x])/(21*a^3*d) + (8*Tan[c + d*x]^3)/(63*a^3*d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}+\frac{2\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{3a}\\
&= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{2\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}+\frac{10\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx}{21a^2}\\
&= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{2\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}-\frac{2\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+dx)\right)}+\frac{8\int \sec^4
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{2\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}-\frac{2\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+dx)\right)}-\frac{8\,\mathrm{Subst}}{}\\
&= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{2\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}-\frac{2\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+dx)\right)}+\frac{8\tan(c}{21a^3}
\end{aligned}
$$

Mathematica [A] time = 0.113553, size = 85, normalized size = 0.69

$$
\frac{\sec^3(c+dx)(36\sin(c+dx)+2\sin(3(c+dx))-6\sin(5(c+dx))-27\cos(2(c+dx))-12\cos(4(c+dx))+\cos(6(c+dx}{126a^3d(\sin(c+dx)+1)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-27*Cos[2*(c + d*x)] - 12*Cos[4*(c + d*x)] + Cos[6*(c + d*
x)] + 36*Sin[c + d*x] + 2*Sin[3*(c + d*x)] - 6*Sin[5*(c + d*x)]))/(126*a^3*
d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.109, size = 190, normalized size = 1.5

$$
2\,\frac{1}{da^3}\left(-1/48\,\left(\tan(1/2\,dx+c/2)-1\right)^{-3}-1/32\,\left(\tan(1/2\,dx+c/2)-1\right)^{-2}-\frac{7}{64\,\tan(1/2\,dx+c/2)-64}-4/9\,\left(\tan(1/2\,dx\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(-1/48/(tan(1/2*d*x+1/2*c)-1)^3-1/32/(tan(1/2*d*x+1/2*c)-1)^2-7/64/
(tan(1/2*d*x+1/2*c)-1)-4/9/(tan(1/2*d*x+1/2*c)+1)^9+2/(tan(1/2*d*x+1/2*c)+1
)^8-34/7/(tan(1/2*d*x+1/2*c)+1)^7+23/3/(tan(1/2*d*x+1/2*c)+1)^6-35/4/(tan(1
/2*d*x+1/2*c)+1)^5+59/8/(tan(1/2*d*x+1/2*c)+1)^4-19/4/(tan(1/2*d*x+1/2*c)+1
)^3+9/4/(tan(1/2*d*x+1/2*c)+1)^2-57/64/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.02829, size = 651, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/63*(51*sin(d*x + c)/(cos(d*x + c) + 1) + 39*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 235*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 450*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 306*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 294*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 378*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 63*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 - 273*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
 - 189*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 63*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 + 19)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 - 27*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)
^9 - 12*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(
cos(d*x + c) + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [A] time = 1.70969, size = 331, normalized size = 2.69

$$-\frac{16\,\cos\left(dx+c\right)^{6}-72\,\cos\left(dx+c\right)^{4}+30\,\cos\left(dx+c\right)^{2}-2\left(24\,\cos\left(dx+c\right)^{4}-20\,\cos\left(dx+c\right)^{2}-7\right)\sin\left(dx+c\right)+7}{63\left(3\,a^{3}d\cos\left(dx+c\right)^{5}-4\,a^{3}d\cos\left(dx+c\right)^{3}+\left(a^{3}d\cos\left(dx+c\right)^{5}-4\,a^{3}d\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/63*(16*cos(d*x + c)^6 - 72*cos(d*x + c)^4 + 30*cos(d*x + c)^2 - 2*(24*co
s(d*x + c)^4 - 20*cos(d*x + c)^2 - 7)*sin(d*x + c) + 7)/(3*a^3*d*cos(d*x +
```

```
c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c
)^3)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.18373, size = 231, normalized size = 1.88

$$-\frac{\frac{21\left(21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-36\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+19\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}+\frac{3591\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+19656\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+56196\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+95760\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+10773}{a^3\left(\tan\left(\frac{1}{2}dx+\right.\right.}}{2016\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2016*(21*(21*tan(1/2*d*x + 1/2*c)^2 - 36*tan(1/2*d*x + 1/2*c) + 19)/(a^3
*(tan(1/2*d*x + 1/2*c) - 1)^3) + (3591*tan(1/2*d*x + 1/2*c)^8 + 19656*tan(1
/2*d*x + 1/2*c)^7 + 56196*tan(1/2*d*x + 1/2*c)^6 + 95760*tan(1/2*d*x + 1/2*
c)^5 + 107730*tan(1/2*d*x + 1/2*c)^4 + 79464*tan(1/2*d*x + 1/2*c)^3 + 38484
*tan(1/2*d*x + 1/2*c)^2 + 10944*tan(1/2*d*x + 1/2*c) + 1615)/(a^3*(tan(1/2*
d*x + 1/2*c) + 1)^9))/d
```

3.87 $\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=171

$$-\frac{a^2}{40d(a\sin(c+dx)+a)^5}+\frac{3}{64d\left(a^3-a^3\sin(c+dx)\right)}-\frac{15}{128d\left(a^3\sin(c+dx)+a^3\right)}+\frac{21\tanh^{-1}(\sin(c+dx))}{128a^3d}-\frac{}{64d(a\sin}$$

```
[Out] (21*ArcTanh[Sin[c + d*x]])/(128*a^3*d) + 1/(128*a*d*(a - a*Sin[c + d*x])^2)
 - a^2/(40*d*(a + a*Sin[c + d*x])^5) - (3*a)/(64*d*(a + a*Sin[c + d*x])^4)
- 1/(16*d*(a + a*Sin[c + d*x])^3) - 5/(64*a*d*(a + a*Sin[c + d*x])^2) + 3/(
64*d*(a^3 - a^3*Sin[c + d*x])) - 15/(128*d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.128418, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$-\frac{a^2}{40d(a\sin(c+dx)+a)^5}+\frac{3}{64d\left(a^3-a^3\sin(c+dx)\right)}-\frac{15}{128d\left(a^3\sin(c+dx)+a^3\right)}+\frac{21\tanh^{-1}(\sin(c+dx))}{128a^3d}-\frac{}{64d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (21*ArcTanh[Sin[c + d*x]])/(128*a^3*d) + 1/(128*a*d*(a - a*Sin[c + d*x])^2)
 - a^2/(40*d*(a + a*Sin[c + d*x])^5) - (3*a)/(64*d*(a + a*Sin[c + d*x])^4)
- 1/(16*d*(a + a*Sin[c + d*x])^3) - 5/(64*a*d*(a + a*Sin[c + d*x])^2) + 3/(
64*d*(a^3 - a^3*Sin[c + d*x])) - 15/(128*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{1}{64a^6(a-x)^3} + \frac{3}{64a^7(a-x)^2} + \frac{1}{8a^3(a+x)^6} + \frac{3}{16a^4(a+x)^5} + \frac{3}{16a^5(a+x)^4} + \frac{5}{32a^6(a+x)^3} + \frac{15}{128a^7(a\cdots}\right.\right.}{d}$$

$$= \frac{1}{128ad(a-a\sin(c+dx))^2} - \frac{a^2}{40d(a+a\sin(c+dx))^5} - \frac{3a}{64d(a+a\sin(c+dx))^4} - \frac{\cdots}{16d(a+a\cdots}$$

$$= \frac{21\tanh^{-1}(\sin(c+dx))}{128a^3d} + \frac{1}{128ad(a-a\sin(c+dx))^2} - \frac{a^2}{40d(a+a\sin(c+dx))^5} - \frac{\cdots}{64d(a+a\cdots}$$

Mathematica [A] time = 0.516994, size = 145, normalized size = 0.85

$$\frac{\sec^4(c+dx)\left(-105\sin^6(c+dx) - 315\sin^5(c+dx) - 140\sin^4(c+dx) + 420\sin^3(c+dx) + 469\sin^2(c+dx) + 7\sin(c+\cdots\right.}{640a^3d(\sin(c+\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^4*(-176 + 105*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^10 + 7*Sin[c + d*x] +
469*Sin[c + d*x]^2 + 420*Sin[c + d*x]^3 - 140*Sin[c + d*x]^4 - 315*Sin[c +
d*x]^5 - 105*Sin[c + d*x]^6))/(640*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.112, size = 162, normalized size = 1.

$$\frac{1}{128\,da^3\left(\sin\left(dx+c\right)-1\right)^2}-\frac{3}{64\,da^3\left(\sin\left(dx+c\right)-1\right)}-\frac{21\,\ln\left(\sin\left(dx+c\right)-1\right)}{256\,da^3}-\frac{1}{40\,da^3\left(1+\sin\left(dx+c\right)\right)^5}-\frac{}{64\,da^3\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/128/d/a^3/(sin(d*x+c)-1)^2-3/64/d/a^3/(sin(d*x+c)-1)-21/256/d/a^3*ln(sin(
d*x+c)-1)-1/40/d/a^3/(1+sin(d*x+c))^5-3/64/d/a^3/(1+sin(d*x+c))^4-1/16/d/a^
3/(1+sin(d*x+c))^3-5/64/d/a^3/(1+sin(d*x+c))^2-15/128/d/a^3/(1+sin(d*x+c))+
21/256*ln(1+sin(d*x+c))/a^3/d
```

Maxima [A] time = 0.961971, size = 254, normalized size = 1.49

$$-\frac{\frac{2\left(105\,\sin(dx+c)^6+315\,\sin(dx+c)^5+140\,\sin(dx+c)^4-420\,\sin(dx+c)^3-469\,\sin(dx+c)^2-7\,\sin(dx+c)+176\right)}{a^3\sin(dx+c)^7+3\,a^3\sin(dx+c)^6+a^3\sin(dx+c)^5-5\,a^3\sin(dx+c)^4-5\,a^3\sin(dx+c)^3+a^3\sin(dx+c)^2+3\,a^3\sin(dx+c)+a^3}-\frac{105\,\log(\sin(dx+c)+1)}{a^3}+\frac{105\,\log(\sin(}{a^3}}{1280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(2*(105*sin(d*x + c)^6 + 315*sin(d*x + c)^5 + 140*sin(d*x + c)^4 -
420*sin(d*x + c)^3 - 469*sin(d*x + c)^2 - 7*sin(d*x + c) + 176)/(a^3*sin(d*
x + c)^7 + 3*a^3*sin(d*x + c)^6 + a^3*sin(d*x + c)^5 - 5*a^3*sin(d*x + c)^4
 - 5*a^3*sin(d*x + c)^3 + a^3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) -
105*log(sin(d*x + c) + 1)/a^3 + 105*log(sin(d*x + c) - 1)/a^3)/d
```

Fricas [A] time = 1.91453, size = 659, normalized size = 3.85

$$-\frac{210\,\cos\left(dx+c\right)^6-910\,\cos\left(dx+c\right)^4+252\,\cos\left(dx+c\right)^2-105\left(3\,\cos\left(dx+c\right)^6-4\,\cos\left(dx+c\right)^4+\left(\cos\left(dx+c\right)^6-4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/1280*(210*cos(d*x + c)^6 - 910*cos(d*x + c)^4 + 252*cos(d*x + c)^2 - 105
*(3*cos(d*x + c)^6 - 4*cos(d*x + c)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^4)
*sin(d*x + c))*log(sin(d*x + c) + 1) + 105*(3*cos(d*x + c)^6 - 4*cos(d*x +
c)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^4)*sin(d*x + c))*log(-sin(d*x + c)
+ 1) - 14*(45*cos(d*x + c)^4 - 30*cos(d*x + c)^2 - 16)*sin(d*x + c) + 96)/(
3*a^3*d*cos(d*x + c)^6 - 4*a^3*d*cos(d*x + c)^4 + (a^3*d*cos(d*x + c)^6 - 4
*a^3*d*cos(d*x + c)^4)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19368, size = 184, normalized size = 1.08

$$\frac{\frac{420\,\log\left(|\sin(dx+c)+1|\right)}{a^3} - \frac{420\,\log\left(|\sin(dx+c)-1|\right)}{a^3} + \frac{10\left(63\,\sin(dx+c)^2-150\,\sin(dx+c)+91\right)}{a^3(\sin(dx+c)-1)^2} - \frac{959\,\sin(dx+c)^5+5395\,\sin(dx+c)^4+12390\,\sin(dx+c)^3+14710}{a^3(\sin(dx+c)+1)^5}}{5120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/5120*(420*log(abs(sin(d*x + c) + 1))/a^3 - 420*log(abs(sin(d*x + c) - 1))
/a^3 + 10*(63*sin(d*x + c)^2 - 150*sin(d*x + c) + 91)/(a^3*(sin(d*x + c) -
1)^2) - (959*sin(d*x + c)^5 + 5395*sin(d*x + c)^4 + 12390*sin(d*x + c)^3 +
14710*sin(d*x + c)^2 + 9275*sin(d*x + c) + 2647)/(a^3*(sin(d*x + c) + 1)^5)
)/d
```

3.88 $\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=127

$$\frac{2\cos^5(c+dx)}{5a^3d(a\sin(c+dx)+a)^5}-\frac{2\cos^3(c+dx)}{3a^2d\left(a^2\sin(c+dx)+a^2\right)^3}+\frac{2\cos(c+dx)}{d\left(a^8\sin(c+dx)+a^8\right)}+\frac{x}{a^8}-\frac{2\cos^7(c+dx)}{7ad(a\sin(c+dx)+a)^7}$$

```
[Out] x/a^8 - (2*Cos[c + d*x]^7)/(7*a*d*(a + a*Sin[c + d*x])^7) + (2*Cos[c + d*x]
^5)/(5*a^3*d*(a + a*Sin[c + d*x])^5) - (2*Cos[c + d*x]^3)/(3*a^2*d*(a^2 + a
^2*Sin[c + d*x])^3) + (2*Cos[c + d*x])/(d*(a^8 + a^8*Sin[c + d*x]))
```

Rubi [A] time = 0.182357, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2680, 8}

$$\frac{2\cos^5(c+dx)}{5a^3d(a\sin(c+dx)+a)^5}-\frac{2\cos^3(c+dx)}{3a^2d\left(a^2\sin(c+dx)+a^2\right)^3}+\frac{2\cos(c+dx)}{d\left(a^8\sin(c+dx)+a^8\right)}+\frac{x}{a^8}-\frac{2\cos^7(c+dx)}{7ad(a\sin(c+dx)+a)^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] x/a^8 - (2*Cos[c + d*x]^7)/(7*a*d*(a + a*Sin[c + d*x])^7) + (2*Cos[c + d*x]
^5)/(5*a^3*d*(a + a*Sin[c + d*x])^5) - (2*Cos[c + d*x]^3)/(3*a^2*d*(a^2 + a
^2*Sin[c + d*x])^3) + (2*Cos[c + d*x])/(d*(a^8 + a^8*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7} - \frac{\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{a^2}$$
$$= -\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7} + \frac{2\cos^5(c+dx)}{5a^3d(a+a\sin(c+dx))^5} + \frac{\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx}{a^4}$$
$$= -\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7} + \frac{2\cos^5(c+dx)}{5a^3d(a+a\sin(c+dx))^5} - \frac{2\cos^3(c+dx)}{3a^5d(a+a\sin(c+dx))^3} - \frac{\int \frac{\cos^2}{(a+a\sin}}{}$$
$$= -\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7} + \frac{2\cos^5(c+dx)}{5a^3d(a+a\sin(c+dx))^5} - \frac{2\cos^3(c+dx)}{3a^5d(a+a\sin(c+dx))^3} + \frac{2\cos}{d\left(a^8+\right.}$$
$$= \frac{x}{a^8} - \frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7} + \frac{2\cos^5(c+dx)}{5a^3d(a+a\sin(c+dx))^5} - \frac{2\cos^3(c+dx)}{3a^5d(a+a\sin(c+dx))^3} + \frac{}{d\left(a\right.}$$

Mathematica [B] time = 6.07156, size = 275, normalized size = 2.17

$$-\frac{2\sqrt{2}\left(\frac{1}{2}(1-\sin(c+dx))-1\right)^4\left(\frac{\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}}{\sqrt{2}\sqrt{\frac{1}{2}(\sin(c+dx)-1)+1}} + \frac{(1-\sin(c+dx))^4}{112\left(\frac{1}{2}(1-\sin(c+dx))-1\right)^4} + \frac{(1-\sin(c+dx))^3}{40\left(\frac{1}{2}(1-\sin(c+dx))-1\right)^3} + \frac{(1-\sin(c+dx}{12\left(\frac{1}{2}(1-\sin(c+d\right.}\right.}{a^8d\left(\frac{1}{2}(\sin(c+dx)-1)+1\right)^{7/2}(1-\sin(c+dx))^5(\sin(c+dx)+1)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-2*Sqrt[2]*Cos[c + d*x]^9*(-1 + (1 - Sin[c + d*x])/2)^4*((ArcSin[Sqrt[1 -
Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c + d*x]])/(Sqrt[2]*Sqrt[1 + (-1 + Sin[
c + d*x])/2]) + (1 - Sin[c + d*x])/(2*(-1 + (1 - Sin[c + d*x])/2)) + (1 - S
in[c + d*x])^2/(12*(-1 + (1 - Sin[c + d*x])/2)^2) + (1 - Sin[c + d*x])^3/(4
0*(-1 + (1 - Sin[c + d*x])/2)^3) + (1 - Sin[c + d*x])^4/(112*(-1 + (1 - Sin
[c + d*x])/2)^4)))/(a^8*d*(1 + (-1 + Sin[c + d*x])/2)^(7/2)*(1 - Sin[c + d*
x])^5*(1 + Sin[c + d*x])^(9/2))
```

Maple [A] time = 0.125, size = 146, normalized size = 1.2

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^8} - \frac{256}{7\,da^8}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-7} + 128\,\frac{1}{da^8\left(\tan\left(1/2\,dx+c/2\right)+1\right)^6} - \frac{896}{5\,da^8}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*arctan(tan(1/2*d*x+1/2*c))-256/7/d/a^8/(tan(1/2*d*x+1/2*c)+1)^7+128
/d/a^8/(tan(1/2*d*x+1/2*c)+1)^6-896/5/d/a^8/(tan(1/2*d*x+1/2*c)+1)^5+128/d/
a^8/(tan(1/2*d*x+1/2*c)+1)^4-160/3/d/a^8/(tan(1/2*d*x+1/2*c)+1)^3+16/d/a^8/
(tan(1/2*d*x+1/2*c)+1)^2
```

Maxima [B] time = 1.51588, size = 398, normalized size = 3.13

$$\frac{2\left(\frac{8\left(\frac{133\sin(dx+c)}{\cos(dx+c)+1}+\frac{294\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{490\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{175\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{105\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+19\right)}{a^8+\frac{7a^8\sin(dx+c)}{\cos(dx+c)+1}+\frac{21a^8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{35a^8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{35a^8\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{21a^8\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{7a^8\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^8\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^8}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 2/105*(8*(133*sin(d*x + c)/(cos(d*x + c) + 1) + 294*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 490*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 175*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 105*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 19)/(a^8
+ 7*a^8*sin(d*x + c)/(cos(d*x + c) + 1) + 21*a^8*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 35*a^8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 35*a^8*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 21*a^8*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 7*a
^8*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^8*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7) + 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^8)/d
```

Fricas [B] time = 1.71823, size = 655, normalized size = 5.16

$$\frac{(105\,dx-352)\cos(dx+c)^4-(315\,dx+568)\cos(dx+c)^3-24\,(35\,dx-31)\cos(dx+c)^2+840\,dx+60\,(7\,dx+12)\cos(}{105\left(a^8d\cos(dx+c)^4-3\,a^8d\cos(dx+c)^3-8\,a^8d\cos(dx+c)^2+4\,a^8d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*((105*d*x - 352)*cos(d*x + c)^4 - (315*d*x + 568)*cos(d*x + c)^3 - 24
*(35*d*x - 31)*cos(d*x + c)^2 + 840*d*x + 60*(7*d*x + 12)*cos(d*x + c) - ((
105*d*x + 352)*cos(d*x + c)^3 + 12*(35*d*x - 18)*cos(d*x + c)^2 - 840*d*x -
 60*(7*d*x + 16)*cos(d*x + c) - 240)*sin(d*x + c) - 240)/(a^8*d*cos(d*x + c
)^4 - 3*a^8*d*cos(d*x + c)^3 - 8*a^8*d*cos(d*x + c)^2 + 4*a^8*d*cos(d*x + c
) + 8*a^8*d - (a^8*d*cos(d*x + c)^3 + 4*a^8*d*cos(d*x + c)^2 - 4*a^8*d*cos(
d*x + c) - 8*a^8*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20658, size = 134, normalized size = 1.06

$$\frac{\frac{105\,(dx+c)}{a^8} + \frac{16\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+175\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+490\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+294\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+133\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+19\right)}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/105*(105*(d*x + c)/a^8 + 16*(105*tan(1/2*d*x + 1/2*c)^5 + 175*tan(1/2*d*x
 + 1/2*c)^4 + 490*tan(1/2*d*x + 1/2*c)^3 + 294*tan(1/2*d*x + 1/2*c)^2 + 133
*tan(1/2*d*x + 1/2*c) + 19)/(a^8*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

3.89 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=36

$$-\frac{(a-a\sin(c+dx))^4}{8d\left(a^3\sin(c+dx)+a^3\right)^4}$$

```
[Out] -(a - a*Sin[c + d*x])^4/(8*d*(a^3 + a^3*Sin[c + d*x])^4)
```

Rubi [A] time = 0.0461098, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 37}

$$-\frac{(a-a\sin(c+dx))^4}{8d\left(a^3\sin(c+dx)+a^3\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -(a - a*Sin[c + d*x])^4/(8*d*(a^3 + a^3*Sin[c + d*x])^4)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3}{(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= -\frac{(a-a\sin(c+dx))^4}{8d\left(a^3+a^3\sin(c+dx)\right)^4}$$

Mathematica [A] time = 0.102754, size = 28, normalized size = 0.78

$$-\frac{\cos^8(c+dx)}{8a^8 d(\sin(c+dx)+1)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -Cos[c + d*x]^8/(8*a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A] time = 0.105, size = 55, normalized size = 1.5

$$\frac{1}{da^8}\left(-3\,(1+\sin(dx+c))^{-2}+4\,(1+\sin(dx+c))^{-3}+(1+\sin(dx+c))^{-1}-2\,(1+\sin(dx+c))^{-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d/a^8*(-3/(1+sin(d*x+c))^2+4/(1+sin(d*x+c))^3+1/(1+sin(d*x+c))-2/(1+sin(d
*x+c))^4)
```

Maxima [B] time = 0.956296, size = 100, normalized size = 2.78

$$\frac{\sin(dx+c)^3+\sin(dx+c)}{\left(a^8\sin(dx+c)^4+4\,a^8\sin(dx+c)^3+6\,a^8\sin(dx+c)^2+4\,a^8\sin(dx+c)+a^8\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] (sin(d*x + c)^3 + sin(d*x + c))/((a^8*sin(d*x + c)^4 + 4*a^8*sin(d*x + c)^3
 + 6*a^8*sin(d*x + c)^2 + 4*a^8*sin(d*x + c) + a^8)*d)
```

Fricas [B] time = 1.65309, size = 194, normalized size = 5.39

$$-\frac{\left(\cos\left(dx+c\right)^{2}-2\right)\sin\left(dx+c\right)}{a^{8}d\cos\left(dx+c\right)^{4}-8\,a^{8}d\cos\left(dx+c\right)^{2}+8\,a^{8}d-4\left(a^{8}d\cos\left(dx+c\right)^{2}-2\,a^{8}d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c)^2 - 2)*sin(d*x + c)/(a^8*d*cos(d*x + c)^4 - 8*a^8*d*cos(d*x
+ c)^2 + 8*a^8*d - 4*(a^8*d*cos(d*x + c)^2 - 2*a^8*d)*sin(d*x + c))
```

Sympy [A] time = 54.3133, size = 2032, normalized size = 56.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((sin(c + d*x)**9/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c +
d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a
**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x
) + 35*a**8*d) + 7*sin(c + d*x)**8/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*
sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4
+ 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin
(c + d*x) + 35*a**8*d) + sin(c + d*x)**7*cos(c + d*x)**2/(35*a**8*d*sin(c +
 d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a
**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*
x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) + 16*sin(c + d*x)**7/(35*a**8*
d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5
 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*s
in(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) + 7*sin(c + d*x)**6*c
```

```
os(c + d*x)**2/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 73
5*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c
 + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*
d) + 16*sin(c + d*x)**6/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x
)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8
*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) +
 35*a**8*d) + 21*sin(c + d*x)**5*cos(c + d*x)**2/(35*a**8*d*sin(c + d*x)**7
 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*si
n(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 +
245*a**8*d*sin(c + d*x) + 35*a**8*d) + 7*sin(c + d*x)**5/(35*a**8*d*sin(c +
 d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a
**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*
x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) + 27*sin(c + d*x)**4*cos(c + d
*x)**2/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d
*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**
3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) + sin
(c + d*x)**4/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*
a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c +
d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d)
 + 14*sin(c + d*x)**3*cos(c + d*x)**2/(35*a**8*d*sin(c + d*x)**7 + 245*a**8
*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)*
*4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*
sin(c + d*x) + 35*a**8*d) + 6*sin(c + d*x)**2*cos(c + d*x)**4/(35*a**8*d*si
n(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1
225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c
 + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) + 2*sin(c + d*x)**2*cos(c
 + d*x)**2/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a*
*8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*
x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) +
 7*sin(c + d*x)*cos(c + d*x)**4/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin
(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1
225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c
+ d*x) + 35*a**8*d) - 5*cos(c + d*x)**6/(35*a**8*d*sin(c + d*x)**7 + 245*a*
*8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x
)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*
d*sin(c + d*x) + 35*a**8*d) + cos(c + d*x)**4/(35*a**8*d*sin(c + d*x)**7 +
245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c
 + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245
*a**8*d*sin(c + d*x) + 35*a**8*d), Ne(d, 0)), (x*cos(c)**7/(a*sin(c) + a)**
8, True))
```

Giac [A] time = 1.18639, size = 92, normalized size = 2.56

$$\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{8}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2*(tan(1/2*d*x + 1/2*c)^7 + 7*tan(1/2*d*x + 1/2*c)^5 + 7*tan(1/2*d*x + 1/2*
c)^3 + tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^8)
```

3.90 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=58

$$-\frac{\cos^7(c+dx)}{63ad(a\sin(c+dx)+a)^7}-\frac{\cos^7(c+dx)}{9d(a\sin(c+dx)+a)^8}$$

```
[Out] -Cos[c + d*x]^7/(9*d*(a + a*Sin[c + d*x])^8) - Cos[c + d*x]^7/(63*a*d*(a +
a*Sin[c + d*x])^7)
```

Rubi [A] time = 0.0803006, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2672, 2671}

$$-\frac{\cos^7(c+dx)}{63ad(a\sin(c+dx)+a)^7}-\frac{\cos^7(c+dx)}{9d(a\sin(c+dx)+a)^8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -Cos[c + d*x]^7/(9*d*(a + a*Sin[c + d*x])^8) - Cos[c + d*x]^7/(63*a*d*(a +
a*Sin[c + d*x])^7)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{\cos^7(c+dx)}{9d(a+a\sin(c+dx))^8} + \frac{\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{9a}$$
$$= -\frac{\cos^7(c+dx)}{9d(a+a\sin(c+dx))^8} - \frac{\cos^7(c+dx)}{63ad(a+a\sin(c+dx))^7}$$

Mathematica [A] time = 0.0830468, size = 36, normalized size = 0.62

$$-\frac{(\sin(c+dx)+8)\cos^7(c+dx)}{63a^8d(\sin(c+dx)+1)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -(Cos[c + d*x]^7*(8 + Sin[c + d*x]))/(63*a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [B] time = 0.131, size = 145, normalized size = 2.5

$$2\frac{1}{da^8}\Big(7\left(\tan(1/2\,dx+c/2)+1\right)^{-2} + \frac{496}{3\left(\tan(1/2\,dx+c/2)+1\right)^6} - \frac{928}{7\left(\tan(1/2\,dx+c/2)+1\right)^7} - \frac{86}{3\left(\tan(1/2\,dx+c/2)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(7/(tan(1/2*d*x+1/2*c)+1)^2+496/3/(tan(1/2*d*x+1/2*c)+1)^6-928/7/(t
an(1/2*d*x+1/2*c)+1)^7-86/3/(tan(1/2*d*x+1/2*c)+1)^3-128/9/(tan(1/2*d*x+1/2
*c)+1)^9-1/(tan(1/2*d*x+1/2*c)+1)-136/(tan(1/2*d*x+1/2*c)+1)^5+76/(tan(1/2*
d*x+1/2*c)+1)^4+64/(tan(1/2*d*x+1/2*c)+1)^8)
```

Maxima [B] time = 1.02298, size = 506, normalized size = 8.72

$$-\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1} + \frac{225\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{189\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{693\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{315\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{483\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{63\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \right.}{63\left(a^8 + \frac{9a^8\sin(dx+c)}{\cos(dx+c)+1} + \frac{36a^8\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{84a^8\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{126a^8\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{126a^8\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{84a^8\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{36a^8\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/63*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 225*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 189*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 693*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 315*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 483*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 63*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 63*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 + 8)/((a^8 + 9*a^8*sin(d*x + c)/(cos(d*x +
 c) + 1) + 36*a^8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 84*a^8*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 126*a^8*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 126
*a^8*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 84*a^8*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + 36*a^8*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9*a^8*sin(d*x + c
)^8/(cos(d*x + c) + 1)^8 + a^8*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*d)
```

Fricas [B] time = 1.54647, size = 614, normalized size = 10.59

$$\frac{\cos\left(dx+c\right)^{5}-4\,\cos\left(dx+c\right)^{4}+19\,\cos\left(dx+c\right)^{3}+52\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{4}+5\,\cos\left(dx+\right.\right.}{63\left(a^{8}d\cos\left(dx+c\right)^{5}+5\,a^{8}d\cos\left(dx+c\right)^{4}-8\,a^{8}d\cos\left(dx+c\right)^{3}-20\,a^{8}d\cos\left(dx+c\right)^{2}+8\,a^{8}d\cos\left(dx+c\right)+16\,a^{8}d+\left(a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/63*(cos(d*x + c)^5 - 4*cos(d*x + c)^4 + 19*cos(d*x + c)^3 + 52*cos(d*x +
c)^2 - (cos(d*x + c)^4 + 5*cos(d*x + c)^3 + 24*cos(d*x + c)^2 - 28*cos(d*x
+ c) - 56)*sin(d*x + c) - 28*cos(d*x + c) - 56)/(a^8*d*cos(d*x + c)^5 + 5*a
^8*d*cos(d*x + c)^4 - 8*a^8*d*cos(d*x + c)^3 - 20*a^8*d*cos(d*x + c)^2 + 8*
a^8*d*cos(d*x + c) + 16*a^8*d + (a^8*d*cos(d*x + c)^4 - 4*a^8*d*cos(d*x + c
)^3 - 12*a^8*d*cos(d*x + c)^2 + 8*a^8*d*cos(d*x + c) + 16*a^8*d)*sin(d*x +
c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20341, size = 169, normalized size = 2.91

$$-\frac{2\left(63\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+63\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+483\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+315\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+693\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{63\,a^{8}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/63*(63*tan(1/2*d*x + 1/2*c)^8 + 63*tan(1/2*d*x + 1/2*c)^7 + 483*tan(1/2*
d*x + 1/2*c)^6 + 315*tan(1/2*d*x + 1/2*c)^5 + 693*tan(1/2*d*x + 1/2*c)^4 +
189*tan(1/2*d*x + 1/2*c)^3 + 225*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1
/2*c) + 8)/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^9)
```

3.91 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{3a^5d(a\sin(c+dx)+a)^3}+\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)^4}-\frac{4}{5a^3d(a\sin(c+dx)+a)^5}$$

```
[Out] -4/(5*a^3*d*(a + a*Sin[c + d*x])^5) - 1/(3*a^5*d*(a + a*Sin[c + d*x])^3) + 
1/(d*(a^2 + a^2*Sin[c + d*x])^4)
```

Rubi [A] time = 0.0586843, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$-\frac{1}{3a^5d(a\sin(c+dx)+a)^3}+\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)^4}-\frac{4}{5a^3d(a\sin(c+dx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -4/(5*a^3*d*(a + a*Sin[c + d*x])^5) - 1/(3*a^5*d*(a + a*Sin[c + d*x])^3) + 
1/(d*(a^2 + a^2*Sin[c + d*x])^4)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{4a^2}{(a+x)^6} - \frac{4a}{(a+x)^5} + \frac{1}{(a+x)^4}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= -\frac{4}{5a^3 d(a+a\sin(c+dx))^5} - \frac{1}{3a^5 d(a+a\sin(c+dx))^3} + \frac{1}{d\left(a^2+a^2\sin(c+dx)\right)^4}$$

Mathematica [A] time = 0.123634, size = 58, normalized size = 0.89

$$\frac{\left(5\sin^2(c+dx) - 5\sin(c+dx) + 2\right)\cos^6(c+dx)}{15a^8 d(\sin(c+dx)-1)^3(\sin(c+dx)+1)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (Cos[c + d*x]^6*(2 - 5*Sin[c + d*x] + 5*Sin[c + d*x]^2))/(15*a^8*d*(-1 + Si
n[c + d*x])^3*(1 + Sin[c + d*x])^8)
```

Maple [A] time = 0.115, size = 43, normalized size = 0.7

$$\frac{1}{da^8}\left(-\frac{1}{3\,(1+\sin(dx+c))^3} - \frac{4}{5\,(1+\sin(dx+c))^5} + (1+\sin(dx+c))^{-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d/a^8*(-1/3/(1+sin(d*x+c))^3-4/5/(1+sin(d*x+c))^5+1/(1+sin(d*x+c))^4)
```

Maxima [A] time = 0.954115, size = 126, normalized size = 1.94

$$-\frac{5\,\sin(dx+c)^2 - 5\,\sin(dx+c) + 2}{15\left(a^8\sin(dx+c)^5 + 5\,a^8\sin(dx+c)^4 + 10\,a^8\sin(dx+c)^3 + 10\,a^8\sin(dx+c)^2 + 5\,a^8\sin(dx+c) + a^8\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/15*(5*sin(d*x + c)^2 - 5*sin(d*x + c) + 2)/((a^8*sin(d*x + c)^5 + 5*a^8*
sin(d*x + c)^4 + 10*a^8*sin(d*x + c)^3 + 10*a^8*sin(d*x + c)^2 + 5*a^8*sin(
d*x + c) + a^8)*d)
```

Fricas [A] time = 1.68664, size = 247, normalized size = 3.8

$$\frac{5\cos\left(dx+c\right)^2+5\sin\left(dx+c\right)-7}{15\left(5a^8d\cos\left(dx+c\right)^4-20a^8d\cos\left(dx+c\right)^2+16a^8d+\left(a^8d\cos\left(dx+c\right)^4-12a^8d\cos\left(dx+c\right)^2+16a^8d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/15*(5*cos(d*x + c)^2 + 5*sin(d*x + c) - 7)/(5*a^8*d*cos(d*x + c)^4 - 20*a
^8*d*cos(d*x + c)^2 + 16*a^8*d + (a^8*d*cos(d*x + c)^4 - 12*a^8*d*cos(d*x +
 c)^2 + 16*a^8*d)*sin(d*x + c))
```

Sympy [A] time = 53.2147, size = 1658, normalized size = 25.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((-2*sin(c + d*x)**9/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(
c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3
675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c
 + d*x) + 105*a**8*d) - 14*sin(c + d*x)**8/(105*a**8*d*sin(c + d*x)**7 + 73
5*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c
+ d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735
*a**8*d*sin(c + d*x) + 105*a**8*d) - 2*sin(c + d*x)**7*cos(c + d*x)**2/(105
*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c +
d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*
a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) - 41*sin(c +
```

```
 d*x)**7/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a*
*8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*
x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d)
 - 14*sin(c + d*x)**6*cos(c + d*x)**2/(105*a**8*d*sin(c + d*x)**7 + 735*a**
8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x
)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8
*d*sin(c + d*x) + 105*a**8*d) - 63*sin(c + d*x)**6/(105*a**8*d*sin(c + d*x)
**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*
d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)*
*2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) - 42*sin(c + d*x)**5*cos(c + d*x
)**2/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d
*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**
3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) - 4
9*sin(c + d*x)**5/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6
+ 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*s
in(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 10
5*a**8*d) - 70*sin(c + d*x)**4*cos(c + d*x)**2/(105*a**8*d*sin(c + d*x)**7
+ 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*si
n(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 +
 735*a**8*d*sin(c + d*x) + 105*a**8*d) - 15*sin(c + d*x)**4/(105*a**8*d*sin
(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3
675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(
c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) - 70*sin(c + d*x)**3*co
s(c + d*x)**2/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 22
05*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c
 + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a*
*8*d) - 30*sin(c + d*x)**2*cos(c + d*x)**2/(105*a**8*d*sin(c + d*x)**7 + 73
5*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c
+ d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735
*a**8*d*sin(c + d*x) + 105*a**8*d) - 15*cos(c + d*x)**4/(105*a**8*d*sin(c +
 d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*
a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c +
d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d), Ne(d, 0)), (x*cos(c)**5/(a
*sin(c) + a)**8, True))
```

Giac [B] time = 1.16742, size = 185, normalized size = 2.85

$$\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+140\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+170\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+282\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}{15\,a^{8}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/15*(15*tan(1/2*d*x + 1/2*c)^9 + 30*tan(1/2*d*x + 1/2*c)^8 + 140*tan(1/2*d
*x + 1/2*c)^7 + 170*tan(1/2*d*x + 1/2*c)^6 + 282*tan(1/2*d*x + 1/2*c)^5 + 1
70*tan(1/2*d*x + 1/2*c)^4 + 140*tan(1/2*d*x + 1/2*c)^3 + 30*tan(1/2*d*x + 1
/2*c)^2 + 15*tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^10)
```

3.92 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=118

$$-\frac{2\cos^5(c+dx)}{1155a^3d(a\sin(c+dx)+a)^5}-\frac{2\cos^5(c+dx)}{231a^2d(a\sin(c+dx)+a)^6}-\frac{\cos^5(c+dx)}{33ad(a\sin(c+dx)+a)^7}-\frac{\cos^5(c+dx)}{11d(a\sin(c+dx)+a)^8}$$

```
[Out] -Cos[c + d*x]^5/(11*d*(a + a*Sin[c + d*x])^8) - Cos[c + d*x]^5/(33*a*d*(a +
 a*Sin[c + d*x])^7) - (2*Cos[c + d*x]^5)/(231*a^2*d*(a + a*Sin[c + d*x])^6)
 - (2*Cos[c + d*x]^5)/(1155*a^3*d*(a + a*Sin[c + d*x])^5)
```

Rubi [A] time = 0.16757, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2672, 2671}

$$-\frac{2\cos^5(c+dx)}{1155a^3d(a\sin(c+dx)+a)^5}-\frac{2\cos^5(c+dx)}{231a^2d(a\sin(c+dx)+a)^6}-\frac{\cos^5(c+dx)}{33ad(a\sin(c+dx)+a)^7}-\frac{\cos^5(c+dx)}{11d(a\sin(c+dx)+a)^8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -Cos[c + d*x]^5/(11*d*(a + a*Sin[c + d*x])^8) - Cos[c + d*x]^5/(33*a*d*(a +
 a*Sin[c + d*x])^7) - (2*Cos[c + d*x]^5)/(231*a^2*d*(a + a*Sin[c + d*x])^6)
 - (2*Cos[c + d*x]^5)/(1155*a^3*d*(a + a*Sin[c + d*x])^5)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{\cos^5(c+dx)}{11d(a+a\sin(c+dx))^8} + \frac{3\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{11a}$$

$$= -\frac{\cos^5(c+dx)}{11d(a+a\sin(c+dx))^8} - \frac{\cos^5(c+dx)}{33ad(a+a\sin(c+dx))^7} + \frac{2\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{33a^2}$$

$$= -\frac{\cos^5(c+dx)}{11d(a+a\sin(c+dx))^8} - \frac{\cos^5(c+dx)}{33ad(a+a\sin(c+dx))^7} - \frac{2\cos^5(c+dx)}{231a^2d(a+a\sin(c+dx))^6} + \frac{2\int -}{}$$

$$= -\frac{\cos^5(c+dx)}{11d(a+a\sin(c+dx))^8} - \frac{\cos^5(c+dx)}{33ad(a+a\sin(c+dx))^7} - \frac{2\cos^5(c+dx)}{231a^2d(a+a\sin(c+dx))^6} - \frac{}{1155a}$$

Mathematica [A] time = 0.0818712, size = 58, normalized size = 0.49

$$-\frac{\left(2\sin^3(c+dx)+16\sin^2(c+dx)+61\sin(c+dx)+152\right)\cos^5(c+dx)}{1155a^8d(\sin(c+dx)+1)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -(Cos[c + d*x]^5*(152 + 61*Sin[c + d*x] + 16*Sin[c + d*x]^2 + 2*Sin[c + d*x
]^3))/(1155*a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A] time = 0.141, size = 175, normalized size = 1.5

$$2\,\frac{1}{da^8}\left(64\,(\tan(1/2\,dx+c/2)+1)^{-10} - \frac{2376}{7\,(\tan(1/2\,dx+c/2)+1)^7} - 30\,(\tan(1/2\,dx+c/2)+1)^{-3} + 292\,(\tan(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(64/(tan(1/2*d*x+1/2*c)+1)^10-2376/7/(tan(1/2*d*x+1/2*c)+1)^7-30/(t
an(1/2*d*x+1/2*c)+1)^3+292/(tan(1/2*d*x+1/2*c)+1)^6-512/3/(tan(1/2*d*x+1/2*
c)+1)^9-1/(tan(1/2*d*x+1/2*c)+1)-932/5/(tan(1/2*d*x+1/2*c)+1)^5+7/(tan(1/2*
d*x+1/2*c)+1)^2+88/(tan(1/2*d*x+1/2*c)+1)^4-128/11/(tan(1/2*d*x+1/2*c)+1)^1
```

```
1+288/(tan(1/2*d*x+1/2*c)+1)^8)
```

Maxima [B] time = 1.03692, size = 622, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/1155*(517*sin(d*x + c)/(cos(d*x + c) + 1) + 4895*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 11220*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 27060*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 32802*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 374
22*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 23100*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 13860*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 3465*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9 + 1155*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 152)/((a
^8 + 11*a^8*sin(d*x + c)/(cos(d*x + c) + 1) + 55*a^8*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 165*a^8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 330*a^8*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + 462*a^8*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5 + 462*a^8*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 330*a^8*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 + 165*a^8*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 55*a^8*s
in(d*x + c)^9/(cos(d*x + c) + 1)^9 + 11*a^8*sin(d*x + c)^10/(cos(d*x + c) +
 1)^10 + a^8*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*d)
```

Fricas [B] time = 1.62082, size = 760, normalized size = 6.44

$$\frac{2\,\cos(dx+c)^6+12\,\cos(dx+c)^5-25\,\cos(dx+c)^4-70\,\cos(dx+c)^3-245\,\cos(dx+c)^2+\left(2\,\cos\right.}{1155\left(a^8d\cos(dx+c)^6-5\,a^8d\cos(dx+c)^5-18\,a^8d\cos(dx+c)^4+20\,a^8d\cos(dx+c)^3+48\,a^8d\cos(dx+c)^2-16\,a^8d\,\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/1155*(2*cos(d*x + c)^6 + 12*cos(d*x + c)^5 - 25*cos(d*x + c)^4 - 70*cos(d
*x + c)^3 - 245*cos(d*x + c)^2 + (2*cos(d*x + c)^5 - 10*cos(d*x + c)^4 - 35
*cos(d*x + c)^3 + 35*cos(d*x + c)^2 - 210*cos(d*x + c) - 420)*sin(d*x + c)
+ 210*cos(d*x + c) + 420)/(a^8*d*cos(d*x + c)^6 - 5*a^8*d*cos(d*x + c)^5 -
18*a^8*d*cos(d*x + c)^4 + 20*a^8*d*cos(d*x + c)^3 + 48*a^8*d*cos(d*x + c)^2
 - 16*a^8*d*cos(d*x + c) - 32*a^8*d - (a^8*d*cos(d*x + c)^5 + 6*a^8*d*cos(d
```

```
*x + c)^4 - 12*a^8*d*cos(d*x + c)^3 - 32*a^8*d*cos(d*x + c)^2 + 16*a^8*d*co
s(d*x + c) + 32*a^8*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19008, size = 204, normalized size = 1.73

$$-\frac{2\left(1155\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+3465\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+13860\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+23100\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+37422\,\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/1155*(1155*tan(1/2*d*x + 1/2*c)^10 + 3465*tan(1/2*d*x + 1/2*c)^9 + 13860
*tan(1/2*d*x + 1/2*c)^8 + 23100*tan(1/2*d*x + 1/2*c)^7 + 37422*tan(1/2*d*x
+ 1/2*c)^6 + 32802*tan(1/2*d*x + 1/2*c)^5 + 27060*tan(1/2*d*x + 1/2*c)^4 +
11220*tan(1/2*d*x + 1/2*c)^3 + 4895*tan(1/2*d*x + 1/2*c)^2 + 517*tan(1/2*d*
x + 1/2*c) + 152)/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^11)
```

3.93 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=45

$$\frac{1}{5a^3d(a\sin(c+dx)+a)^5}-\frac{1}{3a^2d(a\sin(c+dx)+a)^6}$$

```
[Out] -1/(3*a^2*d*(a + a*Sin[c + d*x])^6) + 1/(5*a^3*d*(a + a*Sin[c + d*x])^5)
```

Rubi [A] time = 0.0525286, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{1}{5a^3d(a\sin(c+dx)+a)^5}-\frac{1}{3a^2d(a\sin(c+dx)+a)^6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/(3*a^2*d*(a + a*Sin[c + d*x])^6) + 1/(5*a^3*d*(a + a*Sin[c + d*x])^5)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{(a+x)^7}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{2a}{(a+x)^7} - \frac{1}{(a+x)^6}\right)dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= -\frac{1}{3a^2 d(a+a\sin(c+dx))^6} + \frac{1}{5a^3 d(a+a\sin(c+dx))^5}$$

Mathematica [A] time = 0.165404, size = 43, normalized size = 0.96

$$\frac{3\sin(c+dx)-2}{15a^8 d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-2 + 3*Sin[c + d*x])/(15*a^8*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^12)
```

Maple [A] time = 0.122, size = 33, normalized size = 0.7

$$\frac{1}{da^8}\left(-\frac{1}{3\,(1+\sin(dx+c))^6}+\frac{1}{5\,(1+\sin(dx+c))^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/d/a^8*(-1/3/(1+sin(d*x+c))^6+1/5/(1+sin(d*x+c))^5)
```

Maxima [B] time = 0.957129, size = 130, normalized size = 2.89

$$\frac{3\,\sin(dx+c)-2}{15\left(a^8\sin(dx+c)^6+6\,a^8\sin(dx+c)^5+15\,a^8\sin(dx+c)^4+20\,a^8\sin(dx+c)^3+15\,a^8\sin(dx+c)^2+6\,a^8\sin(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/15*(3*sin(d*x + c) - 2)/((a^8*sin(d*x + c)^6 + 6*a^8*sin(d*x + c)^5 + 15*
a^8*sin(d*x + c)^4 + 20*a^8*sin(d*x + c)^3 + 15*a^8*sin(d*x + c)^2 + 6*a^8*
sin(d*x + c) + a^8)*d)
```

Fricas [B] time = 1.75978, size = 261, normalized size = 5.8

$$-\frac{3\sin(dx+c)-2}{15\left(a^8d\cos(dx+c)^6-18\,a^8d\cos(dx+c)^4+48\,a^8d\cos(dx+c)^2-32\,a^8d-2\left(3\,a^8d\cos(dx+c)^4-16\,a^8d\cos(dx+c)^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/15*(3*sin(d*x + c) - 2)/(a^8*d*cos(d*x + c)^6 - 18*a^8*d*cos(d*x + c)^4
+ 48*a^8*d*cos(d*x + c)^2 - 32*a^8*d - 2*(3*a^8*d*cos(d*x + c)^4 - 16*a^8*d
*cos(d*x + c)^2 + 16*a^8*d)*sin(d*x + c))
```

Sympy [A] time = 53.88, size = 2020, normalized size = 44.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((15*sin(c + d*x)**9/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(
c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3
675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c
 + d*x) + 105*a**8*d) + 105*sin(c + d*x)**8/(105*a**8*d*sin(c + d*x)**7 + 7
35*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c
 + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 73
5*a**8*d*sin(c + d*x) + 105*a**8*d) + 15*sin(c + d*x)**7*cos(c + d*x)**2/(1
05*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c
+ d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 220
5*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) + 312*sin(
c + d*x)**7/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205
*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c +
```

```
 d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8
*d) + 105*sin(c + d*x)**6*cos(c + d*x)**2/(105*a**8*d*sin(c + d*x)**7 + 735
*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c +
 d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*
a**8*d*sin(c + d*x) + 105*a**8*d) + 504*sin(c + d*x)**6/(105*a**8*d*sin(c +
 d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*
a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c +
d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) + 315*sin(c + d*x)**5*cos(c
 + d*x)**2/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*
a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c +
d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*
d) + 462*sin(c + d*x)**5/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d
*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a
**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*
x) + 105*a**8*d) + 525*sin(c + d*x)**4*cos(c + d*x)**2/(105*a**8*d*sin(c +
d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a
**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d
*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) + 210*sin(c + d*x)**4/(105*a
**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*
x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a*
*8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) + 525*sin(c +
d*x)**3*cos(c + d*x)**2/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*
x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a*
*8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x
) + 105*a**8*d) + 315*sin(c + d*x)**2*cos(c + d*x)**2/(105*a**8*d*sin(c + d
*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a*
*8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*
x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) - 42*sin(c + d*x)**2/(105*a**
8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)
**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8
*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) + 105*sin(c + d*
x)*cos(c + d*x)**2/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6
 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*
sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 1
05*a**8*d) - 14*sin(c + d*x)/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c
 + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 36
75*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c
+ d*x) + 105*a**8*d) - 2/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d
*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a
**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*
x) + 105*a**8*d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) + a)**8, True))
```

Giac [A] time = 1.15983, size = 38, normalized size = 0.84

$$\frac{3 \sin(dx + c) - 2}{15 a^8 d(\sin(dx + c) + 1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(d*x + c) - 2)/(a^8*d*(sin(d*x + c) + 1)^6)
```

3.94 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=183

$$-\frac{8\cos^3(c+dx)}{9009a^2d\left(a^2\sin(c+dx)+a^2\right)^3}-\frac{8\cos^3(c+dx)}{3003d\left(a^2\sin(c+dx)+a^2\right)^4}-\frac{20\cos^3(c+dx)}{3003a^3d(a\sin(c+dx)+a)^5}-\frac{20\cos^3(c+dx)}{1287a^2d(a\sin(c+dx)+a)^6}$$

```
[Out] -Cos[c + d*x]^3/(13*d*(a + a*Sin[c + d*x])^8) - (5*Cos[c + d*x]^3)/(143*a*d
*(a + a*Sin[c + d*x])^7) - (20*Cos[c + d*x]^3)/(1287*a^2*d*(a + a*Sin[c + d
*x])^6) - (20*Cos[c + d*x]^3)/(3003*a^3*d*(a + a*Sin[c + d*x])^5) - (8*Cos[
c + d*x]^3)/(3003*d*(a^2 + a^2*Sin[c + d*x])^4) - (8*Cos[c + d*x]^3)/(9009*
a^2*d*(a^2 + a^2*Sin[c + d*x])^3)
```

Rubi [A] time = 0.271703, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2672, 2671}

$$-\frac{8\cos^3(c+dx)}{9009a^2d\left(a^2\sin(c+dx)+a^2\right)^3}-\frac{8\cos^3(c+dx)}{3003d\left(a^2\sin(c+dx)+a^2\right)^4}-\frac{20\cos^3(c+dx)}{3003a^3d(a\sin(c+dx)+a)^5}-\frac{20\cos^3(c+dx)}{1287a^2d(a\sin(c+dx)+a)^6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -Cos[c + d*x]^3/(13*d*(a + a*Sin[c + d*x])^8) - (5*Cos[c + d*x]^3)/(143*a*d
*(a + a*Sin[c + d*x])^7) - (20*Cos[c + d*x]^3)/(1287*a^2*d*(a + a*Sin[c + d
*x])^6) - (20*Cos[c + d*x]^3)/(3003*a^3*d*(a + a*Sin[c + d*x])^5) - (8*Cos[
c + d*x]^3)/(3003*d*(a^2 + a^2*Sin[c + d*x])^4) - (8*Cos[c + d*x]^3)/(9009*
a^2*d*(a^2 + a^2*Sin[c + d*x])^3)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} + \frac{5\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{13a}$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} + \frac{20\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{143a^2}$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} - \frac{20\cos^3(c+dx)}{1287a^2d(a+a\sin(c+dx))^6} + \frac{20\int}{}$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} - \frac{20\cos^3(c+dx)}{1287a^2d(a+a\sin(c+dx))^6} - \frac{}{3003}$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} - \frac{20\cos^3(c+dx)}{1287a^2d(a+a\sin(c+dx))^6} - \frac{}{3003}$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} - \frac{20\cos^3(c+dx)}{1287a^2d(a+a\sin(c+dx))^6} - \frac{}{3003}$$

Mathematica [A] time = 0.121781, size = 78, normalized size = 0.43

$$-\frac{\left(8\sin^5(c+dx)+64\sin^4(c+dx)+236\sin^3(c+dx)+544\sin^2(c+dx)+911\sin(c+dx)+1240\right)\cos^3(c+dx)}{9009a^8d(\sin(c+dx)+1)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -(Cos[c + d*x]^3*(1240 + 911*Sin[c + d*x] + 544*Sin[c + d*x]^2 + 236*Sin[c
+ d*x]^3 + 64*Sin[c + d*x]^4 + 8*Sin[c + d*x]^5))/(9009*a^8*d*(1 + Sin[c +
d*x])^8)
```

Maple [A] time = 0.148, size = 205, normalized size = 1.1

$$2\frac{1}{da^8}\left(432\,(\tan(1/2\,dx+c/2)+1)^{-10}-\frac{4528}{7\,(\tan(1/2\,dx+c/2)+1)^7}+64\,(\tan(1/2\,dx+c/2)+1)^{-12}+\frac{1336}{3\,(\tan(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(432/(tan(1/2*d*x+1/2*c)+1)^10-4528/7/(tan(1/2*d*x+1/2*c)+1)^7+64/(
tan(1/2*d*x+1/2*c)+1)^12+1336/3/(tan(1/2*d*x+1/2*c)+1)^6-94/3/(tan(1/2*d*x+
1/2*c)+1)^3-5840/9/(tan(1/2*d*x+1/2*c)+1)^9-1/(tan(1/2*d*x+1/2*c)+1)-128/13
/(tan(1/2*d*x+1/2*c)+1)^13-240/(tan(1/2*d*x+1/2*c)+1)^5+7/(tan(1/2*d*x+1/2*
c)+1)^2+100/(tan(1/2*d*x+1/2*c)+1)^4-2272/11/(tan(1/2*d*x+1/2*c)+1)^11+736/
(tan(1/2*d*x+1/2*c)+1)^8)
```

Maxima [B] time = 1.06583, size = 738, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/9009*(7111*sin(d*x + c)/(cos(d*x + c) + 1) + 51675*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 171457*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 451165*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + 785070*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
+ 1076790*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1051050*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 + 810810*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 435435*sin(
d*x + c)^9/(cos(d*x + c) + 1)^9 + 183183*sin(d*x + c)^10/(cos(d*x + c) + 1)
^10 + 45045*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 9009*sin(d*x + c)^12/(c
os(d*x + c) + 1)^12 + 1240)/((a^8 + 13*a^8*sin(d*x + c)/(cos(d*x + c) + 1)
+ 78*a^8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 286*a^8*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 715*a^8*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1287*a^8*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1716*a^8*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6 + 1716*a^8*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1287*a^8*sin(d*x + c
)^8/(cos(d*x + c) + 1)^8 + 715*a^8*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 28
6*a^8*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 78*a^8*sin(d*x + c)^11/(cos(d
*x + c) + 1)^11 + 13*a^8*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + a^8*sin(d*
x + c)^13/(cos(d*x + c) + 1)^13)*d)
```

Fricas [A] time = 1.72716, size = 896, normalized size = 4.9

$$\frac{8\cos(dx+c)^7 - 48\cos(dx+c)^6 - 196\cos(dx+c)^5 + 280\cos(dx+c)^4 + 735\cos(dx+c)^3 - 378}{9009\left(a^8 d\cos(dx+c)^7 + 7a^8 d\cos(dx+c)^6 - 18a^8 d\cos(dx+c)^5 - 56a^8 d\cos(dx+c)^4 + 48a^8 d\cos(dx+c)^3 + 112a^8 c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9009*(8*cos(d*x + c)^7 - 48*cos(d*x + c)^6 - 196*cos(d*x + c)^5 + 280*cos
(d*x + c)^4 + 735*cos(d*x + c)^3 - 378*cos(d*x + c)^2 - (8*cos(d*x + c)^6 +
 56*cos(d*x + c)^5 - 140*cos(d*x + c)^4 - 420*cos(d*x + c)^3 + 315*cos(d*x
+ c)^2 + 693*cos(d*x + c) + 1386)*sin(d*x + c) + 693*cos(d*x + c) + 1386)/(
a^8*d*cos(d*x + c)^7 + 7*a^8*d*cos(d*x + c)^6 - 18*a^8*d*cos(d*x + c)^5 - 5
6*a^8*d*cos(d*x + c)^4 + 48*a^8*d*cos(d*x + c)^3 + 112*a^8*d*cos(d*x + c)^2
 - 32*a^8*d*cos(d*x + c) - 64*a^8*d + (a^8*d*cos(d*x + c)^6 - 6*a^8*d*cos(d
*x + c)^5 - 24*a^8*d*cos(d*x + c)^4 + 32*a^8*d*cos(d*x + c)^3 + 80*a^8*d*co
s(d*x + c)^2 - 32*a^8*d*cos(d*x + c) - 64*a^8*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18117, size = 239, normalized size = 1.31

$$-\frac{2\left(9009\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12} + 45045\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11} + 183183\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10} + 435435\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9} + 810\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/9009*(9009*tan(1/2*d*x + 1/2*c)^12 + 45045*tan(1/2*d*x + 1/2*c)^11 + 183
183*tan(1/2*d*x + 1/2*c)^10 + 435435*tan(1/2*d*x + 1/2*c)^9 + 810810*tan(1/
2*d*x + 1/2*c)^8 + 1051050*tan(1/2*d*x + 1/2*c)^7 + 1076790*tan(1/2*d*x + 1
/2*c)^6 + 785070*tan(1/2*d*x + 1/2*c)^5 + 451165*tan(1/2*d*x + 1/2*c)^4 + 1
71457*tan(1/2*d*x + 1/2*c)^3 + 51675*tan(1/2*d*x + 1/2*c)^2 + 7111*tan(1/2*
d*x + 1/2*c) + 1240)/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^13)
```

3.95 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{7ad(a\sin(c+dx)+a)^7}$$

```
[Out] -1/(7*a*d*(a + a*Sin[c + d*x])^7)
```

Rubi [A] time = 0.0250511, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 32}

$$-\frac{1}{7ad(a\sin(c+dx)+a)^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/(7*a*d*(a + a*Sin[c + d*x])^7)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(a+x)^8}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{1}{7ad(a+a\sin(c+dx))^7}$$

Mathematica [A] time = 0.228168, size = 33, normalized size = 1.5

$$-\frac{1}{7a^8d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/(7*a^8*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^14)
```

Maple [A] time = 0.02, size = 21, normalized size = 1.

$$-\frac{1}{7\,da\left(a+a\sin\left(dx+c\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^8,x)
```

```
[Out] -1/7/a/d/(a+a*sin(d*x+c))^7
```

Maxima [A] time = 0.965244, size = 27, normalized size = 1.23

$$-\frac{1}{7\left(a\sin\left(dx+c\right)+a\right)^{7}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/7/((a*sin(d*x + c) + a)^7*a*d)
```

Fricas [B] time = 1.72512, size = 263, normalized size = 11.95

$$\frac{1}{7\left(7\,a^8d\cos\left(dx+c\right)^6-56\,a^8d\cos\left(dx+c\right)^4+112\,a^8d\cos\left(dx+c\right)^2-64\,a^8d+\left(a^8d\cos\left(dx+c\right)^6-24\,a^8d\cos\left(dx+c\right)^4+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/7/(7*a^8*d*cos(d*x + c)^6 - 56*a^8*d*cos(d*x + c)^4 + 112*a^8*d*cos(d*x +
 c)^2 - 64*a^8*d + (a^8*d*cos(d*x + c)^6 - 24*a^8*d*cos(d*x + c)^4 + 80*a^8
*d*cos(d*x + c)^2 - 64*a^8*d)*sin(d*x + c))
```

Sympy [A] time = 52.7169, size = 128, normalized size = 5.82

$$\begin{cases} -\frac{1}{7a^8d\sin^7\left(c+dx\right)+49a^8d\sin^6\left(c+dx\right)+147a^8d\sin^5\left(c+dx\right)+245a^8d\sin^4\left(c+dx\right)+245a^8d\sin^3\left(c+dx\right)+147a^8d\sin^2\left(c+dx\right)+49a^8d\sin\left(c+dx\right)+7a^8d} & \text{for}\ d\neq \\ \frac{x\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^8} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((-1/(7*a**8*d*sin(c + d*x)**7 + 49*a**8*d*sin(c + d*x)**6 + 147*a
**8*d*sin(c + d*x)**5 + 245*a**8*d*sin(c + d*x)**4 + 245*a**8*d*sin(c + d*x
)**3 + 147*a**8*d*sin(c + d*x)**2 + 49*a**8*d*sin(c + d*x) + 7*a**8*d), Ne(
d, 0)), (x*cos(c)/(a*sin(c) + a)**8, True))
```

Giac [A] time = 1.15364, size = 27, normalized size = 1.23

$$-\frac{1}{7\left(a\sin\left(dx+c\right)+a\right)^7ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="giac")

[Out] -1/7/((a*sin(d*x + c) + a)^7*a*d)
```

3.96 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=194

$$-\frac{1}{256d\left(a^8\sin(c+dx)+a^8\right)}-\frac{1}{256d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{1}{192a^5d(a\sin(c+dx)+a)^3}-\frac{1}{128d\left(a^2\sin(c+dx)+a^2\right)^4}-$$

```
[Out] ArcTanh[Sin[c + d*x]]/(256*a^8*d) - 1/(16*d*(a + a*Sin[c + d*x])^8) - 1/(28
*a*d*(a + a*Sin[c + d*x])^7) - 1/(48*a^2*d*(a + a*Sin[c + d*x])^6) - 1/(80*
a^3*d*(a + a*Sin[c + d*x])^5) - 1/(192*a^5*d*(a + a*Sin[c + d*x])^3) - 1/(1
28*d*(a^2 + a^2*Sin[c + d*x])^4) - 1/(256*d*(a^4 + a^4*Sin[c + d*x])^2) - 1
/(256*d*(a^8 + a^8*Sin[c + d*x]))
```

Rubi [A] time = 0.112255, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2667, 44, 206}

$$-\frac{1}{256d\left(a^8\sin(c+dx)+a^8\right)}-\frac{1}{256d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{1}{192a^5d(a\sin(c+dx)+a)^3}-\frac{1}{128d\left(a^2\sin(c+dx)+a^2\right)^4}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^8,x]
```


```
[Out] ArcTanh[Sin[c + d*x]]/(256*a^8*d) - 1/(16*d*(a + a*Sin[c + d*x])^8) - 1/(28
*a*d*(a + a*Sin[c + d*x])^7) - 1/(48*a^2*d*(a + a*Sin[c + d*x])^6) - 1/(80*
a^3*d*(a + a*Sin[c + d*x])^5) - 1/(192*a^5*d*(a + a*Sin[c + d*x])^3) - 1/(1
28*d*(a^2 + a^2*Sin[c + d*x])^4) - 1/(256*d*(a^4 + a^4*Sin[c + d*x])^2) - 1
/(256*d*(a^8 + a^8*Sin[c + d*x]))
```


Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{a\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^9}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a\,\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^9}+\frac{1}{4a^2(a+x)^8}+\frac{1}{8a^3(a+x)^7}+\frac{1}{16a^4(a+x)^6}+\frac{1}{32a^5(a+x)^5}+\frac{1}{64a^6(a+x)^4}+\frac{1}{128a^7(a+x)^3}\right.\right.}{d}$$

$$= -\frac{1}{16d(a+a\sin(c+dx))^8}-\frac{1}{28ad(a+a\sin(c+dx))^7}-\frac{1}{48a^2d(a+a\sin(c+dx))^6}-\frac{}{80a^3d(}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{256a^8d}-\frac{1}{16d(a+a\sin(c+dx))^8}-\frac{1}{28ad(a+a\sin(c+dx))^7}-\frac{1}{48a^2d(a+a\,\mathrm{s}}$$

Mathematica [A] time = 0.773059, size = 122, normalized size = 0.63

$$-\frac{105\sin^7(c+dx)+840\sin^6(c+dx)+2975\sin^5(c+dx)+6160\sin^4(c+dx)+8351\sin^3(c+dx)+8008\sin^2(c+dx)\cdot}{26880a^8d(\sin(c+dx)+1)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -(4096 - 105*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^16
 + 5993*Sin[c + d*x] + 8008*Sin[c + d*x]^2 + 8351*Sin[c + d*x]^3 + 6160*Sin
[c + d*x]^4 + 2975*Sin[c + d*x]^5 + 840*Sin[c + d*x]^6 + 105*Sin[c + d*x]^7
)/(26880*a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A] time = 0.122, size = 180, normalized size = 0.9

$$-\frac{\ln(\sin(dx+c)-1)}{512\,da^8}-\frac{1}{16\,da^8\,(1+\sin(dx+c))^8}-\frac{1}{28\,da^8\,(1+\sin(dx+c))^7}-\frac{1}{48\,da^8\,(1+\sin(dx+c))^6}-\frac{1}{80\,da^8\,(1+\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^8,x)
```

```
[Out] -1/512/d/a^8*ln(sin(d*x+c)-1)-1/16/d/a^8/(1+sin(d*x+c))^8-1/28/d/a^8/(1+sin
(d*x+c))^7-1/48/d/a^8/(1+sin(d*x+c))^6-1/80/d/a^8/(1+sin(d*x+c))^5-1/128/d/
a^8/(1+sin(d*x+c))^4-1/192/d/a^8/(1+sin(d*x+c))^3-1/256/d/a^8/(1+sin(d*x+c)
)^2-1/256/d/a^8/(1+sin(d*x+c))+1/512/d/a^8*ln(1+sin(d*x+c))
```

Maxima [A] time = 0.968938, size = 288, normalized size = 1.48

$$-\frac{\frac{2\left(105\,\sin(dx+c)^7+840\,\sin(dx+c)^6+2975\,\sin(dx+c)^5+6160\,\sin(dx+c)^4+8351\,\sin(dx+c)^3+8008\,\sin(dx+c)^2+5993\,\sin(dx+c)+4096\right)}{a^8\sin(dx+c)^8+8\,a^8\sin(dx+c)^7+28\,a^8\sin(dx+c)^6+56\,a^8\sin(dx+c)^5+70\,a^8\sin(dx+c)^4+56\,a^8\sin(dx+c)^3+28\,a^8\sin(dx+c)^2+8\,a^8\sin(dx+c)+a^8}-\frac{105\log(\sin(}{a^8}}{53760\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/53760*(2*(105*sin(d*x + c)^7 + 840*sin(d*x + c)^6 + 2975*sin(d*x + c)^5
+ 6160*sin(d*x + c)^4 + 8351*sin(d*x + c)^3 + 8008*sin(d*x + c)^2 + 5993*si
n(d*x + c) + 4096)/(a^8*sin(d*x + c)^8 + 8*a^8*sin(d*x + c)^7 + 28*a^8*sin(
d*x + c)^6 + 56*a^8*sin(d*x + c)^5 + 70*a^8*sin(d*x + c)^4 + 56*a^8*sin(d*x
 + c)^3 + 28*a^8*sin(d*x + c)^2 + 8*a^8*sin(d*x + c) + a^8) - 105*log(sin(d
*x + c) + 1)/a^8 + 105*log(sin(d*x + c) - 1)/a^8)/d
```

Fricas [B] time = 1.96336, size = 1052, normalized size = 5.42

$$\frac{1680\,\cos(dx+c)^6-17360\,\cos(dx+c)^4+45696\,\cos(dx+c)^2+105\left(\cos(dx+c)^8-32\,\cos(dx+c)^6+160\,\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/53760*(1680*cos(d*x + c)^6 - 17360*cos(d*x + c)^4 + 45696*cos(d*x + c)^2
+ 105*(cos(d*x + c)^8 - 32*cos(d*x + c)^6 + 160*cos(d*x + c)^4 - 256*cos(d*
x + c)^2 - 8*(cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 24*cos(d*x + c)^2 - 16)*
sin(d*x + c) + 128)*log(sin(d*x + c) + 1) - 105*(cos(d*x + c)^8 - 32*cos(d*
x + c)^6 + 160*cos(d*x + c)^4 - 256*cos(d*x + c)^2 - 8*(cos(d*x + c)^6 - 10
*cos(d*x + c)^4 + 24*cos(d*x + c)^2 - 16)*sin(d*x + c) + 128)*log(-sin(d*x
+ c) + 1) + 2*(105*cos(d*x + c)^6 - 3290*cos(d*x + c)^4 + 14616*cos(d*x + c
)^2 - 17424)*sin(d*x + c) - 38208)/(a^8*d*cos(d*x + c)^8 - 32*a^8*d*cos(d*x
 + c)^6 + 160*a^8*d*cos(d*x + c)^4 - 256*a^8*d*cos(d*x + c)^2 + 128*a^8*d -
 8*(a^8*d*cos(d*x + c)^6 - 10*a^8*d*cos(d*x + c)^4 + 24*a^8*d*cos(d*x + c)^
2 - 16*a^8*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18536, size = 177, normalized size = 0.91

$$\frac{\frac{840 \log(|\sin(dx+c)+1|)}{a^8} - \frac{840 \log(|\sin(dx+c)-1|)}{a^8} - \frac{2283 \sin(dx+c)^8+19944 \sin(dx+c)^7+77364 \sin(dx+c)^6+175448 \sin(dx+c)^5+258370 \sin(dx+c)^4+2614}{a^8(\sin(dx+c)+1)^8}}{430080\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/430080*(840*log(abs(sin(d*x + c) + 1))/a^8 - 840*log(abs(sin(d*x + c) - 1
))/a^8 - (2283*sin(d*x + c)^8 + 19944*sin(d*x + c)^7 + 77364*sin(d*x + c)^6
 + 175448*sin(d*x + c)^5 + 258370*sin(d*x + c)^4 + 261464*sin(d*x + c)^3 +
192052*sin(d*x + c)^2 + 114152*sin(d*x + c) + 67819)/(a^8*(sin(d*x + c) + 1
)^8))/d
```

3.97 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=245

$$\frac{128\tan(c+dx)}{12155a^8d}-\frac{64\sec(c+dx)}{12155d\left(a^8\sin(c+dx)+a^8\right)}-\frac{64\sec(c+dx)}{12155d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{16\sec(c+dx)}{2431a^2d\left(a^2\sin(c+dx)+a^2\right)^3}-\frac{}{1215}$$

```
[Out] -Sec[c + d*x]/(17*d*(a + a*Sin[c + d*x])^8) - (3*Sec[c + d*x])/(85*a*d*(a +
 a*Sin[c + d*x])^7) - (24*Sec[c + d*x])/(1105*a^2*d*(a + a*Sin[c + d*x])^6)
 - (168*Sec[c + d*x])/(12155*a^3*d*(a + a*Sin[c + d*x])^5) - (112*Sec[c + d
*x])/(12155*d*(a^2 + a^2*Sin[c + d*x])^4) - (16*Sec[c + d*x])/(2431*a^2*d*(
a^2 + a^2*Sin[c + d*x])^3) - (64*Sec[c + d*x])/(12155*d*(a^4 + a^4*Sin[c +
d*x])^2) - (64*Sec[c + d*x])/(12155*d*(a^8 + a^8*Sin[c + d*x])) + (128*Tan[
c + d*x])/(12155*a^8*d)
```

Rubi [A] time = 0.40397, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2672, 3767, 8}

$$\frac{128\tan(c+dx)}{12155a^8d}-\frac{64\sec(c+dx)}{12155d\left(a^8\sin(c+dx)+a^8\right)}-\frac{64\sec(c+dx)}{12155d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{16\sec(c+dx)}{2431a^2d\left(a^2\sin(c+dx)+a^2\right)^3}-\frac{}{1215}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -Sec[c + d*x]/(17*d*(a + a*Sin[c + d*x])^8) - (3*Sec[c + d*x])/(85*a*d*(a +
 a*Sin[c + d*x])^7) - (24*Sec[c + d*x])/(1105*a^2*d*(a + a*Sin[c + d*x])^6)
 - (168*Sec[c + d*x])/(12155*a^3*d*(a + a*Sin[c + d*x])^5) - (112*Sec[c + d
*x])/(12155*d*(a^2 + a^2*Sin[c + d*x])^4) - (16*Sec[c + d*x])/(2431*a^2*d*(
a^2 + a^2*Sin[c + d*x])^3) - (64*Sec[c + d*x])/(12155*d*(a^4 + a^4*Sin[c +
d*x])^2) - (64*Sec[c + d*x])/(12155*d*(a^8 + a^8*Sin[c + d*x])) + (128*Tan[
c + d*x])/(12155*a^8*d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
```

```
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} + \frac{9\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{17a}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} + \frac{24\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{85a^2}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6} + \frac{168\int}{}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6} - \frac{}{12155}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6} - \frac{}{12155}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6} - \frac{}{12155}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6} - \frac{}{12155}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6} - \frac{}{12155}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6} - \frac{}{12155}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6} - \frac{}{12155}$$

Mathematica [A] time = 0.342428, size = 113, normalized size = 0.46

$$\frac{\sec(c+dx)(4862\sin(c+dx) - 6188\sin(3(c+dx)) + 1700\sin(5(c+dx)) - 119\sin(7(c+dx)) + \sin(9(c+dx)) - 7072\,\mathrm{co}}{24310a^8d(\sin(c+dx)+1)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]*(-7072*Cos[2*(c + d*x)] + 3808*Cos[4*(c + d*x)] - 544*Cos[6*(
c + d*x)] + 16*Cos[8*(c + d*x)] + 4862*Sin[c + d*x] - 6188*Sin[3*(c + d*x)]
```

```
 + 1700*Sin[5*(c + d*x)] - 119*Sin[7*(c + d*x)] + Sin[9*(c + d*x)]))/(24310
*a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A] time = 0.098, size = 280, normalized size = 1.1

$$2\,\frac{1}{da^{8}}\left(-\frac{1}{512\,\tan\left(1/2\,dx+c/2\right)-512}-\frac{128}{17\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{17}}+64\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-16}-\frac{1376}{5\,\left(\tan\left(1/2\,dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(-1/512/(tan(1/2*d*x+1/2*c)-1)-128/17/(tan(1/2*d*x+1/2*c)+1)^17+64/
(tan(1/2*d*x+1/2*c)+1)^16-1376/5/(tan(1/2*d*x+1/2*c)+1)^15+784/(tan(1/2*d*x
+1/2*c)+1)^14-21400/13/(tan(1/2*d*x+1/2*c)+1)^13+2692/(tan(1/2*d*x+1/2*c)+1
)^12-38954/11/(tan(1/2*d*x+1/2*c)+1)^11+19109/5/(tan(1/2*d*x+1/2*c)+1)^10-6
847/2/(tan(1/2*d*x+1/2*c)+1)^9+10241/4/(tan(1/2*d*x+1/2*c)+1)^8-12799/8/(ta
n(1/2*d*x+1/2*c)+1)^7+13313/16/(tan(1/2*d*x+1/2*c)+1)^6-57083/160/(tan(1/2*
d*x+1/2*c)+1)^5+7937/64/(tan(1/2*d*x+1/2*c)+1)^4-4351/128/(tan(1/2*d*x+1/2*
c)+1)^3+1793/256/(tan(1/2*d*x+1/2*c)+1)^2-511/512/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.15014, size = 999, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/12155*(18181*sin(d*x + c)/(cos(d*x + c) + 1) + 128384*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 545224*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1667360*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3612364*sin(d*x + c)^5/(cos(d*x + c) +
1)^5 + 5742464*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 6271096*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 3928496*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 85085
0*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 5289856*sin(d*x + c)^10/(cos(d*x +
c) + 1)^10 - 7137416*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 5989984*sin(d*
x + c)^12/(cos(d*x + c) + 1)^12 - 3607604*sin(d*x + c)^13/(cos(d*x + c) + 1
)^13 - 1555840*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 486200*sin(d*x + c)^
15/(cos(d*x + c) + 1)^15 - 97240*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 12
155*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 1896)/((a^8 + 16*a^8*sin(d*x +
```

```
c)/(cos(d*x + c) + 1) + 119*a^8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 544*a
^8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1700*a^8*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 + 3808*a^8*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 6188*a^8*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 7072*a^8*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7 + 4862*a^8*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4862*a^8*sin(d*x + c)^10
/(cos(d*x + c) + 1)^10 - 7072*a^8*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 6
188*a^8*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 3808*a^8*sin(d*x + c)^13/(c
os(d*x + c) + 1)^13 - 1700*a^8*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 544*
a^8*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 119*a^8*sin(d*x + c)^16/(cos(d*
x + c) + 1)^16 - 16*a^8*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - a^8*sin(d*x
 + c)^18/(cos(d*x + c) + 1)^18)*d)
```

Fricas [A] time = 1.87013, size = 624, normalized size = 2.55

$$\frac{1024\,\cos\left(dx+c\right)^{8}-10752\,\cos\left(dx+c\right)^{6}+29568\,\cos\left(dx+c\right)^{4}-27456\,\cos\left(dx+c\right)^{2}+\left(128\,\cos\left(dx+c\right)^{8}-403\right.}{12155\left(a^{8}d\cos\left(dx+c\right)^{9}-32\,a^{8}d\cos\left(dx+c\right)^{7}+160\,a^{8}d\cos\left(dx+c\right)^{5}-256\,a^{8}d\cos\left(dx+c\right)^{3}+128\,a^{8}d\cos\left(dx+c\right)-8\,\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/12155*(1024*cos(d*x + c)^8 - 10752*cos(d*x + c)^6 + 29568*cos(d*x + c)^4
- 27456*cos(d*x + c)^2 + (128*cos(d*x + c)^8 - 4032*cos(d*x + c)^6 + 18480*
cos(d*x + c)^4 - 24024*cos(d*x + c)^2 + 6435)*sin(d*x + c) + 5720)/(a^8*d*c
os(d*x + c)^9 - 32*a^8*d*cos(d*x + c)^7 + 160*a^8*d*cos(d*x + c)^5 - 256*a^
8*d*cos(d*x + c)^3 + 128*a^8*d*cos(d*x + c) - 8*(a^8*d*cos(d*x + c)^7 - 10*
a^8*d*cos(d*x + c)^5 + 24*a^8*d*cos(d*x + c)^3 - 16*a^8*d*cos(d*x + c))*sin
(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21859, size = 336, normalized size = 1.37

$$-\frac{\frac{12155}{a^{8}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{6211205\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16}+55791450\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+303072770\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}+1091397450\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+29095617}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/3111680*(12155/(a^8*(tan(1/2*d*x + 1/2*c) - 1)) + (6211205*tan(1/2*d*x +
 1/2*c)^16 + 55791450*tan(1/2*d*x + 1/2*c)^15 + 303072770*tan(1/2*d*x + 1/2
*c)^14 + 1091397450*tan(1/2*d*x + 1/2*c)^13 + 2909561798*tan(1/2*d*x + 1/2*
c)^12 + 5901218466*tan(1/2*d*x + 1/2*c)^11 + 9405145178*tan(1/2*d*x + 1/2*c
)^10 + 11877161010*tan(1/2*d*x + 1/2*c)^9 + 12017308160*tan(1/2*d*x + 1/2*c
)^8 + 9710430158*tan(1/2*d*x + 1/2*c)^7 + 6263238566*tan(1/2*d*x + 1/2*c)^6
 + 3172666718*tan(1/2*d*x + 1/2*c)^5 + 1247921210*tan(1/2*d*x + 1/2*c)^4 +
365303990*tan(1/2*d*x + 1/2*c)^3 + 77883902*tan(1/2*d*x + 1/2*c)^2 + 104982
14*tan(1/2*d*x + 1/2*c) + 982907)/(a^8*(tan(1/2*d*x + 1/2*c) + 1)^17))/d
```

3.98 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=238

$$\frac{1}{1024d\left(a^8-a^8\sin(c+dx)\right)}-\frac{9}{1024d\left(a^8\sin(c+dx)+a^8\right)}-\frac{1}{128d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{3}{256d\left(a^2\sin(c+dx)+a^2\right)^4}-$$

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(512*a^8*d) - a/(36*d*(a + a*Sin[c + d*x])^9) - 1
/(32*d*(a + a*Sin[c + d*x])^8) - 3/(112*a*d*(a + a*Sin[c + d*x])^7) - 1/(48
*a^2*d*(a + a*Sin[c + d*x])^6) - 1/(64*a^3*d*(a + a*Sin[c + d*x])^5) - 7/(7
68*a^5*d*(a + a*Sin[c + d*x])^3) - 3/(256*d*(a^2 + a^2*Sin[c + d*x])^4) - 1
/(128*d*(a^4 + a^4*Sin[c + d*x])^2) + 1/(1024*d*(a^8 - a^8*Sin[c + d*x])) -
 9/(1024*d*(a^8 + a^8*Sin[c + d*x]))
```

Rubi [A] time = 0.171267, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$\frac{1}{1024d\left(a^8-a^8\sin(c+dx)\right)}-\frac{9}{1024d\left(a^8\sin(c+dx)+a^8\right)}-\frac{1}{128d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{3}{256d\left(a^2\sin(c+dx)+a^2\right)^4}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(512*a^8*d) - a/(36*d*(a + a*Sin[c + d*x])^9) - 1
/(32*d*(a + a*Sin[c + d*x])^8) - 3/(112*a*d*(a + a*Sin[c + d*x])^7) - 1/(48
*a^2*d*(a + a*Sin[c + d*x])^6) - 1/(64*a^3*d*(a + a*Sin[c + d*x])^5) - 7/(7
68*a^5*d*(a + a*Sin[c + d*x])^3) - 3/(256*d*(a^2 + a^2*Sin[c + d*x])^4) - 1
/(128*d*(a^4 + a^4*Sin[c + d*x])^2) + 1/(1024*d*(a^8 - a^8*Sin[c + d*x])) -
 9/(1024*d*(a^8 + a^8*Sin[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{a^3\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{10}}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{1024a^{10}(a-x)^2} + \frac{1}{4a^2(a+x)^{10}} + \frac{1}{4a^3(a+x)^9} + \frac{3}{16a^4(a+x)^8} + \frac{1}{8a^5(a+x)^7} + \frac{5}{64a^6(a+x)^6} + \frac{3}{64a^7(a\right.\right.}{d}$$

$$= -\frac{a}{36d(a+a\sin(c+dx))^9} - \frac{1}{32d(a+a\sin(c+dx))^8} - \frac{3}{112ad(a+a\sin(c+dx))^7} - \frac{}{48a^2d(a}$$

$$= \frac{5\tanh^{-1}(\sin(c+dx))}{512a^8d} - \frac{a}{36d(a+a\sin(c+dx))^9} - \frac{1}{32d(a+a\sin(c+dx))^8} - \frac{3}{112ad(a+a\,s}$$

Mathematica [A] time = 1.76929, size = 175, normalized size = 0.74

$$-\frac{\sec^2(c+dx)\left(-315\sin^9(c+dx) - 2520\sin^8(c+dx) - 8610\sin^7(c+dx) - 15960\sin^6(c+dx) - 16128\sin^5(c+dx) - 5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -(Sec[c + d*x]^2*(5120 - 315*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^18 + 9019*Sin[c + d*x
] + 11736*Sin[c + d*x]^2 + 7074*Sin[c + d*x]^3 - 5544*Sin[c + d*x]^4 - 1612
8*Sin[c + d*x]^5 - 15960*Sin[c + d*x]^6 - 8610*Sin[c + d*x]^7 - 2520*Sin[c
```

```
+ d*x]^8 - 315*Sin[c + d*x]^9))/(32256*a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A] time = 0.135, size = 216, normalized size = 0.9

$$-\frac{1}{1024\,da^{8}\left(\sin\left(dx+c\right)-1\right)}-\frac{5\,\ln\left(\sin\left(dx+c\right)-1\right)}{1024\,da^{8}}-\frac{1}{36\,da^{8}\left(1+\sin\left(dx+c\right)\right)^{9}}-\frac{1}{32\,da^{8}\left(1+\sin\left(dx+c\right)\right)^{8}}-\frac{}{112\,da^{8}\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^8,x)
```

```
[Out] -1/1024/d/a^8/(sin(d*x+c)-1)-5/1024/d/a^8*ln(sin(d*x+c)-1)-1/36/d/a^8/(1+si
n(d*x+c))^9-1/32/d/a^8/(1+sin(d*x+c))^8-3/112/d/a^8/(1+sin(d*x+c))^7-1/48/d
/a^8/(1+sin(d*x+c))^6-1/64/d/a^8/(1+sin(d*x+c))^5-3/256/d/a^8/(1+sin(d*x+c)
)^4-7/768/d/a^8/(1+sin(d*x+c))^3-1/128/d/a^8/(1+sin(d*x+c))^2-9/1024/d/a^8/
(1+sin(d*x+c))+5/1024/d/a^8*ln(1+sin(d*x+c))
```

Maxima [A] time = 0.980043, size = 335, normalized size = 1.41

$$-\frac{\frac{2\left(315\,\sin(dx+c)^{9}+2520\,\sin(dx+c)^{8}+8610\,\sin(dx+c)^{7}+15960\,\sin(dx+c)^{6}+16128\,\sin(dx+c)^{5}+5544\,\sin(dx+c)^{4}-7074\,\sin(dx+c)^{3}-11736\,\sin(dx+c)^{2}-9019\,\mathrm{si}}{a^{8}\sin(dx+c)^{10}+8\,a^{8}\sin(dx+c)^{9}+27\,a^{8}\sin(dx+c)^{8}+48\,a^{8}\sin(dx+c)^{7}+42\,a^{8}\sin(dx+c)^{6}-42\,a^{8}\sin(dx+c)^{4}-48\,a^{8}\sin(dx+c)^{3}-27\,a^{8}\sin(dx+c)^{2}-8\,a^{8}\sin(d}}{64512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/64512*(2*(315*sin(d*x + c)^9 + 2520*sin(d*x + c)^8 + 8610*sin(d*x + c)^7
 + 15960*sin(d*x + c)^6 + 16128*sin(d*x + c)^5 + 5544*sin(d*x + c)^4 - 7074
*sin(d*x + c)^3 - 11736*sin(d*x + c)^2 - 9019*sin(d*x + c) - 5120)/(a^8*sin
(d*x + c)^10 + 8*a^8*sin(d*x + c)^9 + 27*a^8*sin(d*x + c)^8 + 48*a^8*sin(d*
x + c)^7 + 42*a^8*sin(d*x + c)^6 - 42*a^8*sin(d*x + c)^4 - 48*a^8*sin(d*x +
 c)^3 - 27*a^8*sin(d*x + c)^2 - 8*a^8*sin(d*x + c) - a^8) - 315*log(sin(d*x
 + c) + 1)/a^8 + 315*log(sin(d*x + c) - 1)/a^8)/d
```

Fricas [B] time = 2.16685, size = 1241, normalized size = 5.21

$$\frac{5040\,\cos\left(dx+c\right)^{8}-52080\,\cos\left(dx+c\right)^{6}+137088\,\cos\left(dx+c\right)^{4}-114624\,\cos\left(dx+c\right)^{2}+315\left(\cos\left(dx+c\right)^{10}-32\,\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/64512*(5040*cos(d*x + c)^8 - 52080*cos(d*x + c)^6 + 137088*cos(d*x + c)^4
 - 114624*cos(d*x + c)^2 + 315*(cos(d*x + c)^10 - 32*cos(d*x + c)^8 + 160*c
os(d*x + c)^6 - 256*cos(d*x + c)^4 + 128*cos(d*x + c)^2 - 8*(cos(d*x + c)^8
 - 10*cos(d*x + c)^6 + 24*cos(d*x + c)^4 - 16*cos(d*x + c)^2)*sin(d*x + c))
*log(sin(d*x + c) + 1) - 315*(cos(d*x + c)^10 - 32*cos(d*x + c)^8 + 160*cos
(d*x + c)^6 - 256*cos(d*x + c)^4 + 128*cos(d*x + c)^2 - 8*(cos(d*x + c)^8 -
 10*cos(d*x + c)^6 + 24*cos(d*x + c)^4 - 16*cos(d*x + c)^2)*sin(d*x + c))*l
og(-sin(d*x + c) + 1) + 2*(315*cos(d*x + c)^8 - 9870*cos(d*x + c)^6 + 43848
*cos(d*x + c)^4 - 52272*cos(d*x + c)^2 + 8960)*sin(d*x + c) + 14336)/(a^8*d
*cos(d*x + c)^10 - 32*a^8*d*cos(d*x + c)^8 + 160*a^8*d*cos(d*x + c)^6 - 256
*a^8*d*cos(d*x + c)^4 + 128*a^8*d*cos(d*x + c)^2 - 8*(a^8*d*cos(d*x + c)^8
- 10*a^8*d*cos(d*x + c)^6 + 24*a^8*d*cos(d*x + c)^4 - 16*a^8*d*cos(d*x + c)
^2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23705, size = 224, normalized size = 0.94

$$\frac{\frac{2520 \log\left(\left|\sin(dx+c)+1\right|\right)}{a^8} - \frac{2520 \log\left(\left|\sin(dx+c)-1\right|\right)}{a^8} + \frac{504\,(5 \sin(dx+c)-6)}{a^8(\sin(dx+c)-1)} - \frac{7129 \sin(dx+c)^9+68697 \sin(dx+c)^8+296964 \sin(dx+c)^7+758772 \sin(dx+c)}{}}{516096\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/516096*(2520*log(abs(sin(d*x + c) + 1))/a^8 - 2520*log(abs(sin(d*x + c) -
 1))/a^8 + 504*(5*sin(d*x + c) - 6)/(a^8*(sin(d*x + c) - 1)) - (7129*sin(d*
```

```
x + c)^9 + 68697*sin(d*x + c)^8 + 296964*sin(d*x + c)^7 + 758772*sin(d*x +
c)^6 + 1271214*sin(d*x + c)^5 + 1465758*sin(d*x + c)^4 + 1191540*sin(d*x +
c)^3 + 693828*sin(d*x + c)^2 + 295425*sin(d*x + c) + 89553)/(a^8*(sin(d*x +
 c) + 1)^9))/d
```

3.99 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=279

$$\frac{128\tan^3(c+dx)}{12597a^8d}+\frac{128\tan(c+dx)}{4199a^8d}-\frac{32\sec^3(c+dx)}{4199d\left(a^8\sin(c+dx)+a^8\right)}-\frac{32\sec^3(c+dx)}{4199d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{112\sec^3(c}{12597a^2d\left(a^2\sin(c\right.}$$

```
[Out] -Sec[c + d*x]^3/(19*d*(a + a*Sin[c + d*x])^8) - (11*Sec[c + d*x]^3)/(323*a*
d*(a + a*Sin[c + d*x])^7) - (22*Sec[c + d*x]^3)/(969*a^2*d*(a + a*Sin[c + d
*x])^6) - (66*Sec[c + d*x]^3)/(4199*a^3*d*(a + a*Sin[c + d*x])^5) - (48*Sec
[c + d*x]^3)/(4199*d*(a^2 + a^2*Sin[c + d*x])^4) - (112*Sec[c + d*x]^3)/(12
597*a^2*d*(a^2 + a^2*Sin[c + d*x])^3) - (32*Sec[c + d*x]^3)/(4199*d*(a^4 +
a^4*Sin[c + d*x])^2) - (32*Sec[c + d*x]^3)/(4199*d*(a^8 + a^8*Sin[c + d*x])
) + (128*Tan[c + d*x])/(4199*a^8*d) + (128*Tan[c + d*x]^3)/(12597*a^8*d)
```

Rubi [A] time = 0.418573, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2672, 3767}

$$\frac{128\tan^3(c+dx)}{12597a^8d}+\frac{128\tan(c+dx)}{4199a^8d}-\frac{32\sec^3(c+dx)}{4199d\left(a^8\sin(c+dx)+a^8\right)}-\frac{32\sec^3(c+dx)}{4199d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{112\sec^3(c}{12597a^2d\left(a^2\sin(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^8,x]
```


```
[Out] -Sec[c + d*x]^3/(19*d*(a + a*Sin[c + d*x])^8) - (11*Sec[c + d*x]^3)/(323*a*
d*(a + a*Sin[c + d*x])^7) - (22*Sec[c + d*x]^3)/(969*a^2*d*(a + a*Sin[c + d
*x])^6) - (66*Sec[c + d*x]^3)/(4199*a^3*d*(a + a*Sin[c + d*x])^5) - (48*Sec
[c + d*x]^3)/(4199*d*(a^2 + a^2*Sin[c + d*x])^4) - (112*Sec[c + d*x]^3)/(12
597*a^2*d*(a^2 + a^2*Sin[c + d*x])^3) - (32*Sec[c + d*x]^3)/(4199*d*(a^4 +
a^4*Sin[c + d*x])^2) - (32*Sec[c + d*x]^3)/(4199*d*(a^8 + a^8*Sin[c + d*x])
) + (128*Tan[c + d*x])/(4199*a^8*d) + (128*Tan[c + d*x]^3)/(12597*a^8*d)
```


Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
```

```
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} + \frac{11\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{19a}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} + \frac{110\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{323a^2}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6} + \frac{66\int}{}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6} - \frac{}{4199a}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6} - \frac{}{4199a}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6} - \frac{}{4199a}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6} - \frac{}{4199a}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6} - \frac{}{4199a}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6} - \frac{}{4199a}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6} - \frac{}{4199a}$$

Mathematica [A] time = 0.419192, size = 125, normalized size = 0.45

$$\frac{\sec^3(c+dx)(8398\sin(c+dx)-5814\sin(3(c+dx))-2907\sin(5(c+dx))+1463\sin(7(c+dx))-117\sin(9(c+dx))+}{50388a^8d(\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^3*(-10336*Cos[2*(c + d*x)] + 2736*Cos[6*(c + d*x)] - 512*Cos[
8*(c + d*x)] + 16*Cos[10*(c + d*x)] + 8398*Sin[c + d*x] - 5814*Sin[3*(c + d
*x)] - 2907*Sin[5*(c + d*x)] + 1463*Sin[7*(c + d*x)] - 117*Sin[9*(c + d*x)]
 + Sin[11*(c + d*x)]))/(50388*a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A] time = 0.144, size = 340, normalized size = 1.2

$$2\,\frac{1}{da^8}\left(-\frac{1}{1536\,(\tan(1/2\,dx+c/2)-1)^3}-\frac{1}{1024\,(\tan(1/2\,dx+c/2)-1)^2}-\frac{3}{512\,\tan(1/2\,dx+c/2)-512}-\frac{1}{19\,(\tan(1/2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(-1/1536/(tan(1/2*d*x+1/2*c)-1)^3-1/1024/(tan(1/2*d*x+1/2*c)-1)^2-3
/512/(tan(1/2*d*x+1/2*c)-1)-128/19/(tan(1/2*d*x+1/2*c)+1)^19+64/(tan(1/2*d*
x+1/2*c)+1)^18-5248/17/(tan(1/2*d*x+1/2*c)+1)^17+992/(tan(1/2*d*x+1/2*c)+1)
^16-7096/3/(tan(1/2*d*x+1/2*c)+1)^15+4428/(tan(1/2*d*x+1/2*c)+1)^14-87508/1
3/(tan(1/2*d*x+1/2*c)+1)^13+25468/3/(tan(1/2*d*x+1/2*c)+1)^12-18011/2/(tan(
1/2*d*x+1/2*c)+1)^11+32417/4/(tan(1/2*d*x+1/2*c)+1)^10-6215/(tan(1/2*d*x+1/
2*c)+1)^9+32525/8/(tan(1/2*d*x+1/2*c)+1)^8-72425/32/(tan(1/2*d*x+1/2*c)+1)^
7+204605/192/(tan(1/2*d*x+1/2*c)+1)^6-26871/64/(tan(1/2*d*x+1/2*c)+1)^5+217
7/16/(tan(1/2*d*x+1/2*c)+1)^4-54229/1536/(tan(1/2*d*x+1/2*c)+1)^3+7181/1024
/(tan(1/2*d*x+1/2*c)+1)^2-509/512/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.1709, size = 1169, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/12597*(19787*sin(d*x + c)/(cos(d*x + c) + 1) + 136032*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 540806*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1483064*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2552175*sin(d*x + c)^5/(cos(d*x + c) +
1)^5 + 2356608*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1108536*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 - 6930288*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 10934
842*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 7793344*sin(d*x + c)^10/(cos(d*x
+ c) + 1)^10 + 1058148*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 9204208*sin(
d*x + c)^12/(cos(d*x + c) + 1)^12 + 9985222*sin(d*x + c)^13/(cos(d*x + c) +
 1)^13 + 4837248*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 1108536*sin(d*x +
c)^15/(cos(d*x + c) + 1)^15 - 3527160*sin(d*x + c)^16/(cos(d*x + c) + 1)^16
 - 2985489*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 1478048*sin(d*x + c)^18/
(cos(d*x + c) + 1)^18 - 495482*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 - 1007
76*sin(d*x + c)^20/(cos(d*x + c) + 1)^20 - 12597*sin(d*x + c)^21/(cos(d*x +
 c) + 1)^21 + 2024)/((a^8 + 16*a^8*sin(d*x + c)/(cos(d*x + c) + 1) + 117*a^
8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 512*a^8*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 1463*a^8*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2736*a^8*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 + 2907*a^8*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
- 5814*a^8*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 10336*a^8*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9 - 8398*a^8*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 8398
*a^8*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 10336*a^8*sin(d*x + c)^13/(cos
(d*x + c) + 1)^13 + 5814*a^8*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 2907*a
^8*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 2736*a^8*sin(d*x + c)^17/(cos(d*
x + c) + 1)^17 - 1463*a^8*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 - 512*a^8*s
in(d*x + c)^19/(cos(d*x + c) + 1)^19 - 117*a^8*sin(d*x + c)^20/(cos(d*x + c
) + 1)^20 - 16*a^8*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 - a^8*sin(d*x + c)
^22/(cos(d*x + c) + 1)^22)*d)
```

Fricas [A] time = 2.01333, size = 695, normalized size = 2.49

$$\frac{2048\cos(dx+c)^{10}-21504\cos(dx+c)^{8}+59136\cos(dx+c)^{6}-54912\cos(dx+c)^{4}+11440\cos(dx+c)^{2}+\left(256\cos\right.}{12597\left(a^{8}d\cos(dx+c)^{11}-32a^{8}d\cos(dx+c)^{9}+160a^{8}d\cos(dx+c)^{7}-256a^{8}d\cos(dx+c)^{5}+128a^{8}d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/12597*(2048*cos(d*x + c)^10 - 21504*cos(d*x + c)^8 + 59136*cos(d*x + c)^6
 - 54912*cos(d*x + c)^4 + 11440*cos(d*x + c)^2 + (256*cos(d*x + c)^10 - 806
4*cos(d*x + c)^8 + 36960*cos(d*x + c)^6 - 48048*cos(d*x + c)^4 + 12870*cos(
```

```
d*x + c)^2 + 2431)*sin(d*x + c) + 1768)/(a^8*d*cos(d*x + c)^11 - 32*a^8*d*c
os(d*x + c)^9 + 160*a^8*d*cos(d*x + c)^7 - 256*a^8*d*cos(d*x + c)^5 + 128*a
^8*d*cos(d*x + c)^3 - 8*(a^8*d*cos(d*x + c)^9 - 10*a^8*d*cos(d*x + c)^7 + 2
4*a^8*d*cos(d*x + c)^5 - 16*a^8*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19403, size = 406, normalized size = 1.46

$$-\frac{\frac{4199\left(18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-33\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+17\right)}{a^{8}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}+\frac{12823746\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{18}+140368371\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{17}+879644311\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{16}+36932724}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/6449664*(4199*(18*tan(1/2*d*x + 1/2*c)^2 - 33*tan(1/2*d*x + 1/2*c) + 17)
/(a^8*(tan(1/2*d*x + 1/2*c) - 1)^3) + (12823746*tan(1/2*d*x + 1/2*c)^18 + 1
40368371*tan(1/2*d*x + 1/2*c)^17 + 879644311*tan(1/2*d*x + 1/2*c)^16 + 3693
272440*tan(1/2*d*x + 1/2*c)^15 + 11467502592*tan(1/2*d*x + 1/2*c)^14 + 2740
3194676*tan(1/2*d*x + 1/2*c)^13 + 51919375300*tan(1/2*d*x + 1/2*c)^12 + 791
83835016*tan(1/2*d*x + 1/2*c)^11 + 98304418212*tan(1/2*d*x + 1/2*c)^10 + 99
750226290*tan(1/2*d*x + 1/2*c)^9 + 82860874122*tan(1/2*d*x + 1/2*c)^8 + 561
10430792*tan(1/2*d*x + 1/2*c)^7 + 30766700912*tan(1/2*d*x + 1/2*c)^6 + 1346
2452660*tan(1/2*d*x + 1/2*c)^5 + 4616712644*tan(1/2*d*x + 1/2*c)^4 + 119785
1960*tan(1/2*d*x + 1/2*c)^3 + 226248618*tan(1/2*d*x + 1/2*c)^2 + 27911475*t
an(1/2*d*x + 1/2*c) + 2143959)/(a^8*(tan(1/2*d*x + 1/2*c) + 1)^19))/d
```

3.100 $\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=284

$$-\frac{a^2}{80d(a\sin(c+dx)+a)^{10}}+\frac{11}{4096d\left(a^8-a^8\sin(c+dx)\right)}-\frac{55}{4096d\left(a^8\sin(c+dx)+a^8\right)}+\frac{1}{4096d\left(a^4-a^4\sin(c+dx)\right)^2}-$$

```
[Out] (33*ArcTanh[Sin[c + d*x]])/(2048*a^8*d) - a^2/(80*d*(a + a*Sin[c + d*x])^10
) - a/(48*d*(a + a*Sin[c + d*x])^9) - 3/(128*d*(a + a*Sin[c + d*x])^8) - 5/
(224*a*d*(a + a*Sin[c + d*x])^7) - 5/(256*a^2*d*(a + a*Sin[c + d*x])^6) - 2
1/(1280*a^3*d*(a + a*Sin[c + d*x])^5) - 3/(256*a^5*d*(a + a*Sin[c + d*x])^3
) - 7/(512*d*(a^2 + a^2*Sin[c + d*x])^4) + 1/(4096*d*(a^4 - a^4*Sin[c + d*x
])^2) - 45/(4096*d*(a^4 + a^4*Sin[c + d*x])^2) + 11/(4096*d*(a^8 - a^8*Sin[
c + d*x])) - 55/(4096*d*(a^8 + a^8*Sin[c + d*x]))
```

Rubi [A] time = 0.215633, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 44, 206}

$$-\frac{a^2}{80d(a\sin(c+dx)+a)^{10}}+\frac{11}{4096d\left(a^8-a^8\sin(c+dx)\right)}-\frac{55}{4096d\left(a^8\sin(c+dx)+a^8\right)}+\frac{1}{4096d\left(a^4-a^4\sin(c+dx)\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^8,x]
```


```
[Out] (33*ArcTanh[Sin[c + d*x]])/(2048*a^8*d) - a^2/(80*d*(a + a*Sin[c + d*x])^10
) - a/(48*d*(a + a*Sin[c + d*x])^9) - 3/(128*d*(a + a*Sin[c + d*x])^8) - 5/
(224*a*d*(a + a*Sin[c + d*x])^7) - 5/(256*a^2*d*(a + a*Sin[c + d*x])^6) - 2
1/(1280*a^3*d*(a + a*Sin[c + d*x])^5) - 3/(256*a^5*d*(a + a*Sin[c + d*x])^3
) - 7/(512*d*(a^2 + a^2*Sin[c + d*x])^4) + 1/(4096*d*(a^4 - a^4*Sin[c + d*x
])^2) - 45/(4096*d*(a^4 + a^4*Sin[c + d*x])^2) + 11/(4096*d*(a^8 - a^8*Sin[
c + d*x])) - 55/(4096*d*(a^8 + a^8*Sin[c + d*x]))
```


Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{a^5\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{1}{2048a^{11}(a-x)^3} + \frac{11}{4096a^{12}(a-x)^2} + \frac{1}{8a^3(a+x)^{11}} + \frac{3}{16a^4(a+x)^{10}} + \frac{3}{16a^5(a+x)^9} + \frac{5}{32a^6(a+x)^8} + \right.\right.}{}$$

$$= -\frac{a^2}{80d(a+a\sin(c+dx))^{10}} - \frac{a}{48d(a+a\sin(c+dx))^9} - \frac{3}{128d(a+a\sin(c+dx))^8} - \frac{}{224ad(a}$$

$$= \frac{33\tanh^{-1}(\sin(c+dx))}{2048a^8d} - \frac{a^2}{80d(a+a\sin(c+dx))^{10}} - \frac{a}{48d(a+a\sin(c+dx))^9} - \frac{}{128d(a+a}$$

Mathematica [A] time = 2.614, size = 195, normalized size = 0.69

$$\frac{\sec^4(c+dx)\left(-3465\sin^{11}(c+dx) - 27720\sin^{10}(c+dx) - 91245\sin^9(c+dx) - 147840\sin^8(c+dx) - 82698\sin^7(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^4*(-34816 + 3465*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^20 - 66953*Sin[c +
```

```
d*x] - 72776*Sin[c + d*x]^2 + 21395*Sin[c + d*x]^3 + 190080*Sin[c + d*x]^4
+ 255222*Sin[c + d*x]^5 + 114576*Sin[c + d*x]^6 - 82698*Sin[c + d*x]^7 - 14
7840*Sin[c + d*x]^8 - 91245*Sin[c + d*x]^9 - 27720*Sin[c + d*x]^10 - 3465*S
in[c + d*x]^11))/(215040*a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A] time = 0.144, size = 252, normalized size = 0.9

$$\frac{1}{4096\,da^8\,(\sin(dx+c)-1)^2}-\frac{11}{4096\,da^8\,(\sin(dx+c)-1)}-\frac{33\,\ln(\sin(dx+c)-1)}{4096\,da^8}-\frac{1}{80\,da^8\,(1+\sin(dx+c))^{10}}-\frac{}{48\,da^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^8,x)
```

```
[Out] 1/4096/d/a^8/(sin(d*x+c)-1)^2-11/4096/d/a^8/(sin(d*x+c)-1)-33/4096/d/a^8*ln
(sin(d*x+c)-1)-1/80/d/a^8/(1+sin(d*x+c))^10-1/48/d/a^8/(1+sin(d*x+c))^9-3/1
28/d/a^8/(1+sin(d*x+c))^8-5/224/d/a^8/(1+sin(d*x+c))^7-5/256/d/a^8/(1+sin(d
*x+c))^6-21/1280/d/a^8/(1+sin(d*x+c))^5-7/512/d/a^8/(1+sin(d*x+c))^4-3/256/
d/a^8/(1+sin(d*x+c))^3-45/4096/d/a^8/(1+sin(d*x+c))^2-55/4096/d/a^8/(1+sin(
d*x+c))+33/4096/d/a^8*ln(1+sin(d*x+c))
```

Maxima [A] time = 0.989144, size = 412, normalized size = 1.45

$$-\frac{\frac{2\left(3465\,\sin(dx+c)^{11}+27720\,\sin(dx+c)^{10}+91245\,\sin(dx+c)^{9}+147840\,\sin(dx+c)^{8}+82698\,\sin(dx+c)^{7}-114576\,\sin(dx+c)^{6}-255222\,\sin(dx+c)^{5}-190080\,\sin(dx+c)^{4}\cdots\right.}{a^8\sin(dx+c)^{12}+8\,a^8\sin(dx+c)^{11}+26\,a^8\sin(dx+c)^{10}+40\,a^8\sin(dx+c)^{9}+15\,a^8\sin(dx+c)^{8}-48\,a^8\sin(dx+c)^{7}-84\,a^8\sin(dx+c)^{6}-48\,a^8\sin(dx+c)^{5}+15\,a^8\sin\cdots}}{430080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/430080*(2*(3465*sin(d*x + c)^11 + 27720*sin(d*x + c)^10 + 91245*sin(d*x
+ c)^9 + 147840*sin(d*x + c)^8 + 82698*sin(d*x + c)^7 - 114576*sin(d*x + c)
^6 - 255222*sin(d*x + c)^5 - 190080*sin(d*x + c)^4 - 21395*sin(d*x + c)^3 +
 72776*sin(d*x + c)^2 + 66953*sin(d*x + c) + 34816)/(a^8*sin(d*x + c)^12 +
8*a^8*sin(d*x + c)^11 + 26*a^8*sin(d*x + c)^10 + 40*a^8*sin(d*x + c)^9 + 15
*a^8*sin(d*x + c)^8 - 48*a^8*sin(d*x + c)^7 - 84*a^8*sin(d*x + c)^6 - 48*a^
8*sin(d*x + c)^5 + 15*a^8*sin(d*x + c)^4 + 40*a^8*sin(d*x + c)^3 + 26*a^8*s
in(d*x + c)^2 + 8*a^8*sin(d*x + c) + a^8) - 3465*log(sin(d*x + c) + 1)/a^8
```

```
+ 3465*log(sin(d*x + c) - 1)/a^8)/d
```

Fricas [A] time = 2.26353, size = 1332, normalized size = 4.69

$$\frac{55440\,\cos\left(dx+c\right)^{10}-572880\,\cos\left(dx+c\right)^{8}+1507968\,\cos\left(dx+c\right)^{6}-1260864\,\cos\left(dx+c\right)^{4}+157696\,\cos\left(dx+c\right)^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/430080*(55440*cos(d*x + c)^10 - 572880*cos(d*x + c)^8 + 1507968*cos(d*x +
 c)^6 - 1260864*cos(d*x + c)^4 + 157696*cos(d*x + c)^2 + 3465*(cos(d*x + c)
^12 - 32*cos(d*x + c)^10 + 160*cos(d*x + c)^8 - 256*cos(d*x + c)^6 + 128*co
s(d*x + c)^4 - 8*(cos(d*x + c)^10 - 10*cos(d*x + c)^8 + 24*cos(d*x + c)^6 -
 16*cos(d*x + c)^4)*sin(d*x + c))*log(sin(d*x + c) + 1) - 3465*(cos(d*x + c
)^12 - 32*cos(d*x + c)^10 + 160*cos(d*x + c)^8 - 256*cos(d*x + c)^6 + 128*c
os(d*x + c)^4 - 8*(cos(d*x + c)^10 - 10*cos(d*x + c)^8 + 24*cos(d*x + c)^6
- 16*cos(d*x + c)^4)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(3465*cos(d*x
 + c)^10 - 108570*cos(d*x + c)^8 + 482328*cos(d*x + c)^6 - 574992*cos(d*x +
 c)^4 + 98560*cos(d*x + c)^2 + 32256)*sin(d*x + c) + 43008)/(a^8*d*cos(d*x
+ c)^12 - 32*a^8*d*cos(d*x + c)^10 + 160*a^8*d*cos(d*x + c)^8 - 256*a^8*d*c
os(d*x + c)^6 + 128*a^8*d*cos(d*x + c)^4 - 8*(a^8*d*cos(d*x + c)^10 - 10*a^
8*d*cos(d*x + c)^8 + 24*a^8*d*cos(d*x + c)^6 - 16*a^8*d*cos(d*x + c)^4)*sin
(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23477, size = 251, normalized size = 0.88

$$\frac{\frac{27720 \log(|\sin(dx+c)+1|)}{a^8} - \frac{27720 \log(|\sin(dx+c)-1|)}{a^8} + \frac{420\left(99 \sin(dx+c)^2-220 \sin(dx+c)+123\right)}{a^8(\sin(dx+c)-1)^2} - \frac{81191 \sin(dx+c)^{10}+858110 \sin(dx+c)^9+4107195 \sin(}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/3440640*(27720*log(abs(sin(d*x + c) + 1))/a^8 - 27720*log(abs(sin(d*x + c
) - 1))/a^8 + 420*(99*sin(d*x + c)^2 - 220*sin(d*x + c) + 123)/(a^8*(sin(d*
x + c) - 1)^2) - (81191*sin(d*x + c)^10 + 858110*sin(d*x + c)^9 + 4107195*s
in(d*x + c)^8 + 11748840*sin(d*x + c)^7 + 22318590*sin(d*x + c)^6 + 2958354
0*sin(d*x + c)^5 + 27983550*sin(d*x + c)^4 + 19002600*sin(d*x + c)^3 + 9206
235*sin(d*x + c)^2 + 3108990*sin(d*x + c) + 648327)/(a^8*(sin(d*x + c) + 1)
^10))/d
```

3.101 $\int \cos^7(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{2(a\sin(c+dx)+a)^{15/2}}{15a^7d}+\frac{12(a\sin(c+dx)+a)^{13/2}}{13a^6d}-\frac{24(a\sin(c+dx)+a)^{11/2}}{11a^5d}+\frac{16(a\sin(c+dx)+a)^{9/2}}{9a^4d}$$

```
[Out] (16*(a + a*Sin[c + d*x])^(9/2))/(9*a^4*d) - (24*(a + a*Sin[c + d*x])^(11/2)
)/(11*a^5*d) + (12*(a + a*Sin[c + d*x])^(13/2))/(13*a^6*d) - (2*(a + a*Sin[
c + d*x])^(15/2))/(15*a^7*d)
```

Rubi [A] time = 0.0829884, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2(a\sin(c+dx)+a)^{15/2}}{15a^7d}+\frac{12(a\sin(c+dx)+a)^{13/2}}{13a^6d}-\frac{24(a\sin(c+dx)+a)^{11/2}}{11a^5d}+\frac{16(a\sin(c+dx)+a)^{9/2}}{9a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(9/2))/(9*a^4*d) - (24*(a + a*Sin[c + d*x])^(11/2)
)/(11*a^5*d) + (12*(a + a*Sin[c + d*x])^(13/2))/(13*a^6*d) - (2*(a + a*Sin[
c + d*x])^(15/2))/(15*a^7*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^7(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{7/2}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{7/2} - 12a^2(a+x)^{9/2} + 6a(a+x)^{11/2} - (a+x)^{13/2}\right) dx, x, a\sin\right.}{a^7 d}$$
$$= \frac{16(a+a\sin(c+dx))^{9/2}}{9a^4 d} - \frac{24(a+a\sin(c+dx))^{11/2}}{11a^5 d} + \frac{12(a+a\sin(c+dx))^{13/2}}{13a^6 d} -$$

Mathematica [A] time = 4.2383, size = 74, normalized size = 0.76

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^8(-10755\sin(c+dx)+429\sin(3(c+dx))-3366\cos(2(c+dx))}{12870d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Sqrt[a*(1 + Sin[c + d*x])]*(8330 -
 3366*Cos[2*(c + d*x)] - 10755*Sin[c + d*x] + 429*Sin[3*(c + d*x)]))/(12870
*d)
```

Maple [A] time = 0.091, size = 57, normalized size = 0.6

$$\frac{858\ (\cos(dx+c))^2\sin(dx+c) - 3366\ (\cos(dx+c))^2 - 5592\ \sin(dx+c) + 5848}{6435\,a^4 d}(a+a\sin(dx+c))^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/6435/a^4*(a+a*sin(d*x+c))^(9/2)*(429*cos(d*x+c)^2*sin(d*x+c)-1683*cos(d*x
+c)^2-2796*sin(d*x+c)+2924)/d
```

Maxima [A] time = 0.948102, size = 97, normalized size = 1.

$$-\frac{2\left(429\,(a\sin(dx+c)+a)^{\frac{15}{2}} - 2970\,(a\sin(dx+c)+a)^{\frac{13}{2}}a + 7020\,(a\sin(dx+c)+a)^{\frac{11}{2}}a^2 - 5720\,(a\sin(dx+c)+a)^{\frac{9}{2}}a^3\right.}{6435\,a^7 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/6435*(429*(a*sin(d*x + c) + a)^(15/2) - 2970*(a*sin(d*x + c) + a)^(13/2)
*a + 7020*(a*sin(d*x + c) + a)^(11/2)*a^2 - 5720*(a*sin(d*x + c) + a)^(9/2)
*a^3)/(a^7*d)
```

Fricas [A] time = 1.74841, size = 254, normalized size = 2.62

$$\frac{2\left(33\,\cos\left(dx+c\right)^{6}+56\,\cos\left(dx+c\right)^{4}+128\,\cos\left(dx+c\right)^{2}+\left(429\,\cos\left(dx+c\right)^{6}+504\,\cos\left(dx+c\right)^{4}+640\,\cos\left(dx+c\right)^{2}\right.}{6435\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/6435*(33*cos(d*x + c)^6 + 56*cos(d*x + c)^4 + 128*cos(d*x + c)^2 + (429*c
os(d*x + c)^6 + 504*cos(d*x + c)^4 + 640*cos(d*x + c)^2 + 1024)*sin(d*x + c
) + 1024)*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.16911, size = 104, normalized size = 1.07

$$-\frac{2\left(\frac{429\left(a\sin(dx+c)+a\right)^{\frac{15}{2}}}{a^{6}}-\frac{2970\left(a\sin(dx+c)+a\right)^{\frac{13}{2}}}{a^{5}}+\frac{7020\left(a\sin(dx+c)+a\right)^{\frac{11}{2}}}{a^{4}}-\frac{5720\left(a\sin(dx+c)+a\right)^{\frac{9}{2}}}{a^{3}}\right)}{6435\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/6435*(429*(a*sin(d*x + c) + a)^(15/2)/a^6 - 2970*(a*sin(d*x + c) + a)^(1
3/2)/a^5 + 7020*(a*sin(d*x + c) + a)^(11/2)/a^4 - 5720*(a*sin(d*x + c) + a)
^(9/2)/a^3)/(a*d)
```

3.102 $\int \cos^6(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=127

$$-\frac{24a^2\cos^7(c+dx)}{143d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^7(c+dx)}{429d(a\sin(c+dx)+a)^{5/2}}-\frac{256a^4\cos^7(c+dx)}{3003d(a\sin(c+dx)+a)^{7/2}}-\frac{2a\cos^7(c+dx)}{13d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-256*a^4*Cos[c + d*x]^7)/(3003*d*(a + a*Sin[c + d*x])^(7/2)) - (64*a^3*Cos
[c + d*x]^7)/(429*d*(a + a*Sin[c + d*x])^(5/2)) - (24*a^2*Cos[c + d*x]^7)/(
143*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^7)/(13*d*Sqrt[a + a*S
in[c + d*x]])
```

Rubi [A] time = 0.258362, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{24a^2\cos^7(c+dx)}{143d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^7(c+dx)}{429d(a\sin(c+dx)+a)^{5/2}}-\frac{256a^4\cos^7(c+dx)}{3003d(a\sin(c+dx)+a)^{7/2}}-\frac{2a\cos^7(c+dx)}{13d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^7)/(3003*d*(a + a*Sin[c + d*x])^(7/2)) - (64*a^3*Cos
[c + d*x]^7)/(429*d*(a + a*Sin[c + d*x])^(5/2)) - (24*a^2*Cos[c + d*x]^7)/(
143*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^7)/(13*d*Sqrt[a + a*S
in[c + d*x]])
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
```

```
- b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^6(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos^7(c+dx)}{13d\sqrt{a+a\sin(c+dx)}} + \frac{1}{13}(12a)\int \frac{\cos^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= -\frac{24a^2\cos^7(c+dx)}{143d(a+a\sin(c+dx))^{3/2}} - \frac{2a\cos^7(c+dx)}{13d\sqrt{a+a\sin(c+dx)}} + \frac{1}{143}\left(96a^2\right)\int \frac{\cos^6(c}{(a+a\sin(}$$
$$= -\frac{64a^3\cos^7(c+dx)}{429d(a+a\sin(c+dx))^{5/2}} - \frac{24a^2\cos^7(c+dx)}{143d(a+a\sin(c+dx))^{3/2}} - \frac{2a\cos^7(c+dx)}{13d\sqrt{a+a\sin(c+dx}}$$
$$= -\frac{256a^4\cos^7(c+dx)}{3003d(a+a\sin(c+dx))^{7/2}} - \frac{64a^3\cos^7(c+dx)}{429d(a+a\sin(c+dx))^{5/2}} - \frac{24a^2\cos^7(c+d}{143d(a+a\sin(c+d}$$

Mathematica [A] time = 3.90918, size = 99, normalized size = 0.78

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^7(-6377\sin(c+dx)+231\sin(3(c+dx))+1890\cos(2(c+dx))-}{6006d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7*Sqrt[a*(1 + Sin[c + d*x])]*(-5230
+ 1890*Cos[2*(c + d*x)] - 6377*Sin[c + d*x] + 231*Sin[3*(c + d*x)]))/(6006*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.122, size = 75, normalized size = 0.6

$$-\frac{(2+2\sin(dx+c))\,a\,(\sin(dx+c)-1)^4\left(231\,(\sin(dx+c))^3+945\,(\sin(dx+c))^2+1421\,\sin(dx+c)+835\right)}{3003\,d\cos(dx+c)}\,\frac{1}{\sqrt{a+a\sin}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/3003*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^4*(231*sin(d*x+c)^3+945*sin(d*x+c)^
2+1421*sin(d*x+c)+835)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin(dx+c)+a} \cos(dx+c)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^6, x)
```

Fricas [A] time = 1.68498, size = 494, normalized size = 3.89

$$-\frac{2\left(231 \cos(dx+c)^7 - 21 \cos(dx+c)^6 + 28 \cos(dx+c)^5 - 40 \cos(dx+c)^4 + 64 \cos(dx+c)^3 - 128 \cos(dx+c)^2 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3003*(231*cos(d*x + c)^7 - 21*cos(d*x + c)^6 + 28*cos(d*x + c)^5 - 40*co
s(d*x + c)^4 + 64*cos(d*x + c)^3 - 128*cos(d*x + c)^2 - (231*cos(d*x + c)^6
 + 252*cos(d*x + c)^5 + 280*cos(d*x + c)^4 + 320*cos(d*x + c)^3 + 384*cos(d
*x + c)^2 + 512*cos(d*x + c) + 1024)*sin(d*x + c) + 512*cos(d*x + c) + 1024
)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^6, x)
```

3.103 $\int \cos^5(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^5d}-\frac{8(a\sin(c+dx)+a)^{9/2}}{9a^4d}+\frac{8(a\sin(c+dx)+a)^{7/2}}{7a^3d}$$

```
[Out] (8*(a + a*Sin[c + d*x])^(7/2))/(7*a^3*d) - (8*(a + a*Sin[c + d*x])^(9/2))/(
9*a^4*d) + (2*(a + a*Sin[c + d*x])^(11/2))/(11*a^5*d)
```

Rubi [A] time = 0.0726169, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^5d}-\frac{8(a\sin(c+dx)+a)^{9/2}}{9a^4d}+\frac{8(a\sin(c+dx)+a)^{7/2}}{7a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(7/2))/(7*a^3*d) - (8*(a + a*Sin[c + d*x])^(9/2))/(
9*a^4*d) + (2*(a + a*Sin[c + d*x])^(11/2))/(11*a^5*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^5(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{5/2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{5/2}-4a(a+x)^{7/2}+(a+x)^{9/2}\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{8(a+a\sin(c+dx))^{7/2}}{7a^3d} - \frac{8(a+a\sin(c+dx))^{9/2}}{9a^4d} + \frac{2(a+a\sin(c+dx))^{11/2}}{11a^5d}$$

Mathematica [A] time = 0.998606, size = 64, normalized size = 0.88

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6(364\sin(c+dx)+63\cos(2(c+dx))-365)}{693d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*Sqrt[a*(1 + Sin[c + d*x])]*(-365
+ 63*Cos[2*(c + d*x)] + 364*Sin[c + d*x]))/(693*d)
```

Maple [A] time = 0.086, size = 41, normalized size = 0.6

$$-\frac{126\,(\cos(dx+c))^2+364\,\sin(dx+c)-428}{693\,a^3d}(a+a\sin(dx+c))^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/693/a^3*(a+a*sin(d*x+c))^(7/2)*(63*cos(d*x+c)^2+182*sin(d*x+c)-214)/d
```

Maxima [A] time = 0.966128, size = 74, normalized size = 1.01

$$\frac{2\left(63\,(a\sin(dx+c)+a)^{\frac{11}{2}}-308\,(a\sin(dx+c)+a)^{\frac{9}{2}}a+396\,(a\sin(dx+c)+a)^{\frac{7}{2}}a^2\right)}{693\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*(a*sin(d*x + c) + a)^(11/2) - 308*(a*sin(d*x + c) + a)^(9/2)*a +
396*(a*sin(d*x + c) + a)^(7/2)*a^2)/(a^5*d)
```

Fricas [A] time = 1.71955, size = 189, normalized size = 2.59

$$\frac{2\left(7\cos\left(dx+c\right)^{4}+16\cos\left(dx+c\right)^{2}+\left(63\cos\left(dx+c\right)^{4}+80\cos\left(dx+c\right)^{2}+128\right)\sin\left(dx+c\right)+128\right)\sqrt{a\sin\left(dx+c\right)+}}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(7*cos(d*x + c)^4 + 16*cos(d*x + c)^2 + (63*cos(d*x + c)^4 + 80*cos(d
*x + c)^2 + 128)*sin(d*x + c) + 128)*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.7412, size = 81, normalized size = 1.11

$$\frac{2\left(\frac{63\,(a\sin(dx+c)+a)^{\frac{11}{2}}}{a^4}-\frac{308\,(a\sin(dx+c)+a)^{\frac{9}{2}}}{a^3}+\frac{396\,(a\sin(dx+c)+a)^{\frac{7}{2}}}{a^2}\right)}{693\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/693*(63*(a*sin(d*x + c) + a)^(11/2)/a^4 - 308*(a*sin(d*x + c) + a)^(9/2)/
a^3 + 396*(a*sin(d*x + c) + a)^(7/2)/a^2)/(a*d)
```

3.104 $\int \cos^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{16a^2\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}-\frac{2a\cos^5(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-64*a^3*Cos[c + d*x]^5)/(315*d*(a + a*Sin[c + d*x])^(5/2)) - (16*a^2*Cos[c
 + d*x]^5)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^5)/(9*d*Sq
rt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.178518, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{16a^2\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}-\frac{2a\cos^5(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^5)/(315*d*(a + a*Sin[c + d*x])^(5/2)) - (16*a^2*Cos[c
 + d*x]^5)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^5)/(9*d*Sq
rt[a + a*Sin[c + d*x]])
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{1}{9}(8a)\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= -\frac{16a^2\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{1}{63}\left(32a^2\right)\int \frac{\cos^4(c+}{(a+a\sin(c+}$$
$$= -\frac{64a^3\cos^5(c+dx)}{315d(a+a\sin(c+dx))^{5/2}}-\frac{16a^2\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.759997, size = 89, normalized size = 0.94

$$-\frac{\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5(220\sin(c+dx)-35\cos(2(c+dx))+249)}{315d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(249 -
 35*Cos[2*(c + d*x)] + 220*Sin[c + d*x]))/(315*d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))
```

Maple [A] time = 0.109, size = 65, normalized size = 0.7

$$\frac{(2+2\sin(dx+c))\,a\,(\sin(dx+c)-1)^3\left(35\,(\sin(dx+c))^2+110\sin(dx+c)+107\right)}{315\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/315*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^3*(35*sin(d*x+c)^2+110*sin(d*x+c)+107
)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin (dx+c)+a} \cos (dx+c)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4, x)
```

Fricas [A] time = 1.68447, size = 367, normalized size = 3.86

$$-\frac{2\left(35\,\cos(dx+c)^{5}-5\,\cos(dx+c)^{4}+8\,\cos(dx+c)^{3}-16\,\cos(dx+c)^{2}-\left(35\,\cos(dx+c)^{4}+40\,\cos(dx+c)^{3}+48\right.}{315\,(d\cos(dx+c)+d\sin(dx-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*cos(d*x + c)^5 - 5*cos(d*x + c)^4 + 8*cos(d*x + c)^3 - 16*cos(d*
x + c)^2 - (35*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 48*cos(d*x + c)^2 + 64*
cos(d*x + c) + 128)*sin(d*x + c) + 64*cos(d*x + c) + 128)*sqrt(a*sin(d*x +
c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a(\sin (c+dx)+1)} \cos^{4}(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin (dx+c)+a} \cos (dx+c)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4, x)
```

3.105 $\int \cos^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{4(a\sin(c+dx)+a)^{5/2}}{5a^2d}-\frac{2(a\sin(c+dx)+a)^{7/2}}{7a^3d}$$

```
[Out] (4*(a + a*Sin[c + d*x])^(5/2))/(5*a^2*d) - (2*(a + a*Sin[c + d*x])^(7/2))/(
7*a^3*d)
```

Rubi [A] time = 0.0649647, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{4(a\sin(c+dx)+a)^{5/2}}{5a^2d}-\frac{2(a\sin(c+dx)+a)^{7/2}}{7a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(5/2))/(5*a^2*d) - (2*(a + a*Sin[c + d*x])^(7/2))/(
7*a^3*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^{3/2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(a+x)^{3/2}-(a+x)^{5/2}\right)dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{4(a+a\sin(c+dx))^{5/2}}{5a^2 d} - \frac{2(a+a\sin(c+dx))^{7/2}}{7a^3 d}$$

Mathematica [A] time = 0.238839, size = 54, normalized size = 1.1

$$-\frac{2(5\sin(c+dx)-9)\sqrt{a(\sin(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}{35d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Sqrt[a*(1 + Sin[c + d*x])]*(-9
+ 5*Sin[c + d*x]))/(35*d)
```

Maple [A] time = 0.086, size = 31, normalized size = 0.6

$$-\frac{10\sin(dx+c)-18}{35a^2d}(a+a\sin(dx+c))^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/35/a^2*(a+a*sin(d*x+c))^(5/2)*(5*sin(d*x+c)-9)/d
```

Maxima [A] time = 0.960687, size = 51, normalized size = 1.04

$$-\frac{2\left(5\left(a\sin(dx+c)+a\right)^{\frac{7}{2}}-14\left(a\sin(dx+c)+a\right)^{\frac{5}{2}}a\right)}{35a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/35*(5*(a*sin(d*x + c) + a)^(7/2) - 14*(a*sin(d*x + c) + a)^(5/2)*a)/(a^3
*d)
```

Fricas [A] time = 1.59104, size = 124, normalized size = 2.53

$$\frac{2\left(\cos\left(dx+c\right)^{2}+\left(5\,\cos\left(dx+c\right)^{2}+8\right)\sin\left(dx+c\right)+8\right)\sqrt{a\sin\left(dx+c\right)+a}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(cos(d*x + c)^2 + (5*cos(d*x + c)^2 + 8)*sin(d*x + c) + 8)*sqrt(a*sin(
d*x + c) + a)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.02199, size = 58, normalized size = 1.18

$$-\frac{2\left(\frac{5\,(a\sin(dx+c)+a)^{\frac{7}{2}}}{a^{2}}-\frac{14\,(a\sin(dx+c)+a)^{\frac{5}{2}}}{a}\right)}{35\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/35*(5*(a*sin(d*x + c) + a)^(7/2)/a^2 - 14*(a*sin(d*x + c) + a)^(5/2)/a)/
(a*d)
```

3.106 $\int \cos^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=63

$$-\frac{8a^2\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^{3/2}}-\frac{2a\cos^3(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-8*a^2*Cos[c + d*x]^3)/(15*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*
x]^3)/(5*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.110775, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{8a^2\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^{3/2}}-\frac{2a\cos^3(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^3)/(15*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*
x]^3)/(5*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{1}{5}(4a)\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= -\frac{8a^2\cos^3(c+dx)}{15d(a+a\sin(c+dx))^{3/2}} - \frac{2a\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.168565, size = 79, normalized size = 1.25

$$-\frac{2(3\sin(c+dx)+7)\sqrt{a(\sin(c+dx)+1)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}{15d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(7 +
 3*Sin[c + d*x]))/(15*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.102, size = 55, normalized size = 0.9

$$-\frac{(2+2\sin(dx+c))\,a\,(\sin(dx+c)-1)^2\,(3\sin(dx+c)+7)}{15\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/15*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^2*(3*sin(d*x+c)+7)/cos(d*x+c)/(a+a*si
n(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2, x)
```

Fricas [A] time = 1.69323, size = 246, normalized size = 3.9

$$-\frac{2\left(3\cos(dx+c)^{3}-\cos(dx+c)^{2}-\left(3\cos(dx+c)^{2}+4\cos(dx+c)+8\right)\sin(dx+c)+4\cos(dx+c)+8\right)\sqrt{a\sin(dx}}{15\left(d\cos(dx+c)+d\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*cos(d*x + c)^3 - cos(d*x + c)^2 - (3*cos(d*x + c)^2 + 4*cos(d*x +
c) + 8)*sin(d*x + c) + 4*cos(d*x + c) + 8)*sqrt(a*sin(d*x + c) + a)/(d*cos(
d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sin(c+dx)+1\right)}\cos^{2}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin(dx+c)+a}\cos(dx+c)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2, x)
```

3.107 $\int \cos(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=24

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{3ad}$$

```
[Out] (2*(a + a*Sin[c + d*x])^(3/2))/(3*a*d)
```

Rubi [A] time = 0.0327388, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(3/2))/(3*a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{2(a+a\sin(c+dx))^{3/2}}{3ad}\end{aligned}$$

Mathematica [A] time = 0.0809359, size = 44, normalized size = 1.83

$$\frac{2\sqrt{a(\sin(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*Sqrt[a*(1 + Sin[c + d*x])])/(3*d
)
```

Maple [A] time = 0.008, size = 21, normalized size = 0.9

$$\frac{2}{3\,da}\left(a+a\sin\left(dx+c\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3*(a+a*sin(d*x+c))^(3/2)/d/a
```

Maxima [A] time = 0.958411, size = 27, normalized size = 1.12

$$\frac{2\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a*sin(d*x + c) + a)^(3/2)/(a*d)
```

Fricas [A] time = 1.58152, size = 69, normalized size = 2.88

$$\frac{2\sqrt{a\sin\left(dx+c\right)+a}(\sin\left(dx+c\right)+1)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(a*sin(d*x + c) + a)*(sin(d*x + c) + 1)/d
```

Sympy [A] time = 0.484342, size = 58, normalized size = 2.42

$$\begin{cases} \frac{2\sqrt{a\sin(c+dx)+a}\sin(c+dx)}{3d} + \frac{2\sqrt{a\sin(c+dx)+a}}{3d} & \text{for}\ d \neq 0 \\ x\sqrt{a\sin(c)+a}\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)/(3*d) + 2*sqrt(a*sin(c +
 d*x) + a)/(3*d), Ne(d, 0)), (x*sqrt(a*sin(c) + a)*cos(c), True))
```

Giac [A] time = 1.52082, size = 27, normalized size = 1.12

$$\frac{2\,(a\sin(dx+c)+a)^{\frac{3}{2}}}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(a*sin(d*x + c) + a)^(3/2)/(a*d)
```

3.108 $\int \sec(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

```
[Out] (Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rubi [A] time = 0.0604097, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2667, 63, 206}

$$\frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{a\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+a\sin(c+dx)}\right)}{d}\\ &= \frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}\end{aligned}$$

Mathematica [C] time = 0.102338, size = 95, normalized size = 2.38

$$-\frac{(2-2i)\sqrt[4]{-1}\sqrt{a(\sin(c+dx)+1)}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(\sin\left(\frac{1}{4}(2c+dx)\right)+\cos\left(\frac{1}{4}(2c+dx)\right)\right)\right)}{d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-2 + 2*I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*
c + d*x)/4] + Sin[(2*c + d*x)/4])]*Sqrt[a*(1 + Sin[c + d*x])])/(d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.052, size = 32, normalized size = 0.8

$$\frac{\sqrt{2}}{d}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a+a\sin(dx+c)}\frac{1}{\sqrt{a}}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72538, size = 258, normalized size = 6.45

$$\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)}{2d}, -\frac{\sqrt{2}\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}}{\sqrt{a\sin(dx+c)+a}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) +
 a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1))/d, -sqrt(2)*sqrt(-a)*arctan(sqrt(2)*
sqrt(-a)/sqrt(a*sin(d*x + c) + a))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sec(c + d*x), x)
```

Giac [B] time = 1.36625, size = 167, normalized size = 4.18

$$-\frac{2\left(\frac{\sqrt{2}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}-\sqrt{a}\right)}{2\sqrt{-a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}}-\frac{\sqrt{2}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}+2\sqrt{a}\right)}{2\sqrt{-a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*(sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a) - sqrt(a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1
)/sqrt(-a) - sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(a) + 2*sqrt(a))/sqr
t(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/sqrt(-a))/d
```

3.109 $\int \sec^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{2}d}$$

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(Sqrt[2]*d)) + (Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d
```

Rubi [A] time = 0.0793447, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2675, 2649, 206}

$$\frac{\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(Sqrt[2]*d)) + (Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}+\frac{1}{2}a\int\frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx\\ &= \frac{\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}-\frac{a\,\mathrm{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\ &= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{2}d}+\frac{\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}\end{aligned}$$

Mathematica [C] time = 0.221874, size = 106, normalized size = 1.47

$$\frac{\sec(c+dx)\sqrt{a(\sin(c+dx)+1)}\left(1-(1+i)(-1)^{3/4}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(c\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]*(1 - (1 + I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(d
*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])]*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]))*Sqrt[a*(1 + Sin[c + d*x])])/d
```

Maple [A] time = 0.113, size = 83, normalized size = 1.2

$$-\frac{1+\sin(dx+c)}{2\,d\cos(dx+c)}\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin(dx+c)}\frac{1}{\sqrt{a}}\right)a\sqrt{a-a\sin(dx+c)}-2\,a^{3/2}\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/2/a^(1/2)*(1+sin(d*x+c))*(2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(
1/2)/a^(1/2))*a*(a-a*sin(d*x+c))^(1/2)-2*a^(3/2))/cos(d*x+c)/(a+a*sin(d*x+c
))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sec(d*x + c)^2, x)
```

Fricas [B] time = 1.75501, size = 431, normalized size = 5.99

$$\frac{\sqrt{2}\sqrt{a}\cos(dx+c)\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\sqrt{a}}{4d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(a)*cos(d*x + c)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a
*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x
+ c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*
x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*sqrt(a*sin(d*x + c) + a))
/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a(\sin(c+dx)+1)}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sec(c + d*x)**2, x)
```

Giac [B] time = 1.41828, size = 394, normalized size = 5.47

$$\frac{\frac{\sqrt{2}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\sqrt{-a}}-\frac{\left(2\sqrt{2}a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+2a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{-a}\sqrt{a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\cdots\right.}{\sqrt{2}\sqrt{-a}+2\sqrt{-a}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] (sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/s
qrt(-a) - (2*sqrt(2)*a*arctan(sqrt(a)/sqrt(-a)) + 2*a*arctan(sqrt(a)/sqrt(-
a)) - sqrt(-a)*sqrt(a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(2)*sqrt(-a) + 2
*sqrt(-a)) + 2*((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))*a*sgn(tan(1/2*d*x + 1/2*c) + 1) + a^(3/2)*sgn(tan(1/2*d*x + 1/2*c)
 + 1))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^2 - 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*
sqrt(a) - a))/d
```

3.110 $\int \sec^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{3a}{4d\sqrt{a\sin(c+dx)+a}}+\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d}+\frac{\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

```
[Out] (3*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(4*Sqrt[2]*
d) - (3*a)/(4*d*Sqrt[a + a*Sin[c + d*x]]) + (Sec[c + d*x]^2*Sqrt[a + a*Sin[
c + d*x]])/(2*d)
```

Rubi [A] time = 0.123447, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2675, 2667, 51, 63, 206}

$$-\frac{3a}{4d\sqrt{a\sin(c+dx)+a}}+\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d}+\frac{\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (3*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(4*Sqrt[2]*
d) - (3*a)/(4*d*Sqrt[a + a*Sin[c + d*x]]) + (Sec[c + d*x]^2*Sqrt[a + a*Sin[
c + d*x]])/(2*d)
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sec^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}+\frac{1}{4}(3a)\int \frac{\sec(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx \\
&= \frac{\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}+\frac{\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx,x,a\sin(c+dx)\right)}{4d} \\
&= -\frac{3a}{4d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}+\frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)\sqrt{a}\cdots}\right.}{\cdots} \\
&= -\frac{3a}{4d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}+\frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\cdots\right.}{\cdots} \\
&= \frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d}-\frac{3a}{4d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^2(c+dx)\sqrt{a+a\sin(c\cdots}}{2d}
\end{aligned}
$$

Mathematica [C] time = 0.348932, size = 271, normalized size = 2.85

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\frac{\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{2\sin\left(\frac{dx}{2}\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+(-3+3i)\sqrt[4]{-1}\left(\mathrm{si}\right.\right.}{4d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\mathrm{co}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-2 - (3 - 3*I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(d*x)/4]*(Co
s[(2*c + d*x)/4] + Sin[(2*c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]) + (2*Sin[(d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/((Cos[c/2] - Si
n[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + ((Cos[c/2] + Sin[c/2])*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])))*Sqrt[a*(1 + Sin[c + d*x])])/(4*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2)
```

Maple [A] time = 0.145, size = 90, normalized size = 1.

$$2\,\frac{a^3}{d}\left(-1/4\,\frac{1}{a^2}\left(1/2\,\frac{\sqrt{a+a\sin(dx+c)}}{a\sin(dx+c)-a}-3/4\,\frac{\sqrt{2}}{\sqrt{a}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)\right)-1/4\,\frac{1}{a^2\sqrt{a+a\sin(dx+c)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*a^3*(-1/4/a^2*(1/2*(a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c)-a)-3/4*2^(1/2)/a^
(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))-1/4/a^2/(a+a*sin
(d*x+c))^(1/2))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71906, size = 274, normalized size = 2.88

$$\frac{3\sqrt{2}\sqrt{a}\cos(dx+c)^2\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)+4\sqrt{a\sin(dx+c)+a}(3\sin(dx+c)-1)}{16d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*sqrt(a)*cos(d*x + c)^2*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqr
t(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*sqrt(a*sin(d*x
 + c) + a)*(3*sin(d*x + c) - 1))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a(\sin(c+dx)+1)}\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.111 $\int \sec^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$-\frac{5a^2\cos(c+dx)}{8d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}+\frac{5a\sec(c+dx)}{6d\sqrt{a\sin(c+dx)+a}}-\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{2}d}$$

```
[Out] (-5*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(8*Sqrt[2]*d) - (5*a^2*Cos[c + d*x])/(8*d*(a + a*Sin[c + d*x])^(3/2))
+ (5*a*Sec[c + d*x])/(6*d*Sqrt[a + a*Sin[c + d*x]]) + (Sec[c + d*x]^3*Sqrt[
a + a*Sin[c + d*x]])/(3*d)
```

Rubi [A] time = 0.158793, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2675, 2687, 2650, 2649, 206}

$$-\frac{5a^2\cos(c+dx)}{8d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}+\frac{5a\sec(c+dx)}{6d\sqrt{a\sin(c+dx)+a}}-\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-5*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(8*Sqrt[2]*d) - (5*a^2*Cos[c + d*x])/(8*d*(a + a*Sin[c + d*x])^(3/2))
+ (5*a*Sec[c + d*x])/(6*d*Sqrt[a + a*Sin[c + d*x]]) + (Sec[c + d*x]^3*Sqrt[
a + a*Sin[c + d*x]])/(3*d)
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
```

```
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}+\frac{1}{6}(5a)\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= \frac{5a\sec(c+dx)}{6d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}+\frac{1}{4}\left(5a^2\right)\int \frac{1}{(a+a\sin(c$$

$$= -\frac{5a^2\cos(c+dx)}{8d(a+a\sin(c+dx))^{3/2}}+\frac{5a\sec(c+dx)}{6d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^3(c+dx)\sqrt{a+a\sin(c+a}}{3d}$$

$$= -\frac{5a^2\cos(c+dx)}{8d(a+a\sin(c+dx))^{3/2}}+\frac{5a\sec(c+dx)}{6d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^3(c+dx)\sqrt{a+a\sin(c+a}}{3d}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{2}d}-\frac{5a^2\cos(c+dx)}{8d(a+a\sin(c+dx))^{3/2}}+\frac{5a\sec(c+dx)}{6d\sqrt{a+a\sin(c+dx}}$$

Mathematica [C] time = 0.401383, size = 302, normalized size = 2.2

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\frac{12\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{4\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{3\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)}+\right.}{24d\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (((6*Sin[(d*x)/2])/(Cos[c/2] + Sin[c/2]) - (3*(Cos[c/2] - Sin[c/2])*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]))/(Cos[c/2] + Sin[c/2]) - (15 + 15*I)*(-1)^(3
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(
2*c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (12*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
))*Sqrt[a*(1 + Sin[c + d*x])])/(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
3)
```

Maple [A] time = 0.12, size = 153, normalized size = 1.1

$$\frac{1}{(48\sin(dx+c)-48)\cos(dx+c)\,d}\left(\sin(dx+c)\left(15\,(a-a\sin(dx+c))^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)a-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/48/a^(3/2)*(sin(d*x+c)*(15*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-
a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a-20*a^(5/2))-30*a^(5/2)*cos(d*x+c)^2+
15*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2
)/a^(1/2))*a+4*a^(5/2))/(sin(d*x+c)-1)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin(dx+c)+a}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sec(d*x + c)^4, x)
```

Fricas [A] time = 1.76711, size = 517, normalized size = 3.77

$$\frac{15\sqrt{2}\sqrt{a}\cos(dx+c)^3\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(dx+c)-\sqrt{2}\sin(dx+c)+\sqrt{2}\right)\sqrt{a}+3\,a\cos(dx+c)-(a\cos(dx+c)-2\,a)\sin(dx+c)+2\,a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{96\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*sqrt(a)*cos(d*x + c)^3*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*s
in(d*x + c) + a)*(sqrt(2)*cos(d*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))*sq
rt(a) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(
d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(15*c
os(d*x + c)^2 + 10*sin(d*x + c) - 2)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x +
 c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.112 $\int \sec^5(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=149

$$-\frac{35a^2}{96d(a\sin(c+dx)+a)^{3/2}}-\frac{35a}{64d\sqrt{a\sin(c+dx)+a}}+\frac{35\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d}+\frac{\sec^4(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}+$$

```
[Out] (35*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(64*Sqrt[2
]*d) - (35*a^2)/(96*d*(a + a*Sin[c + d*x])^(3/2)) - (35*a)/(64*d*Sqrt[a + a
*Sin[c + d*x]]) + (7*a*Sec[c + d*x]^2)/(16*d*Sqrt[a + a*Sin[c + d*x]]) + (S
ec[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]])/(4*d)
```

Rubi [A] time = 0.203373, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2675, 2687, 2667, 51, 63, 206}

$$-\frac{35a^2}{96d(a\sin(c+dx)+a)^{3/2}}-\frac{35a}{64d\sqrt{a\sin(c+dx)+a}}+\frac{35\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d}+\frac{\sec^4(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (35*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(64*Sqrt[2
]*d) - (35*a^2)/(96*d*(a + a*Sin[c + d*x])^(3/2)) - (35*a)/(64*d*Sqrt[a + a
*Sin[c + d*x]]) + (7*a*Sec[c + d*x]^2)/(16*d*Sqrt[a + a*Sin[c + d*x]]) + (S
ec[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]])/(4*d)
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
```

```
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\sec^4(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} + \frac{1}{8}(7a)\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= \frac{7a\sec^2(c+dx)}{16d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^4(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} + \frac{1}{32}\left(35a^2\right)\int \frac{\mathrm{sec}}{(a+a\,\mathrm{s}}$$

$$= \frac{7a\sec^2(c+dx)}{16d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^4(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} + \frac{\left(35a^3\right)\mathrm{Subst}\left(\int \frac{}{(a-x}\right.}{}$$

$$= -\frac{35a^2}{96d(a+a\sin(c+dx))^{3/2}} + \frac{7a\sec^2(c+dx)}{16d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^4(c+dx)\sqrt{a+a\sin(c}}{4d}$$

$$= -\frac{35a^2}{96d(a+a\sin(c+dx))^{3/2}} - \frac{35a}{64d\sqrt{a+a\sin(c+dx)}} + \frac{7a\sec^2(c+dx)}{16d\sqrt{a+a\sin(c+dx)}} +$$

$$= -\frac{35a^2}{96d(a+a\sin(c+dx))^{3/2}} - \frac{35a}{64d\sqrt{a+a\sin(c+dx)}} + \frac{7a\sec^2(c+dx)}{16d\sqrt{a+a\sin(c+dx)}} +$$

$$= \frac{35\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} - \frac{35a^2}{96d(a+a\sin(c+dx))^{3/2}} - \frac{35a}{64d\sqrt{a+a\sin(c+dx}}$$

Mathematica [C] time = 0.463036, size = 179, normalized size = 1.2

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\frac{329\sin(c+dx)+105\sin(3(c+dx))-70\cos(2(c+dx))-102}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4} - (420-420i)\sqrt[4]{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3 \mathrm{t}\right.}{768d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((-420 + 420*I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*
(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] + Sin[(2*c + d*x)/4])]*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^3 + (-102 - 70*Cos[2*(c + d*x)] + 329*Sin[c +
 d*x] + 105*Sin[3*(c + d*x)])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4))/(76
8*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A] time = 0.211, size = 118, normalized size = 0.8

$$-2\frac{a^5}{d}\left(1/16\,\frac{1}{a^4}\left(1/8\,\frac{\sqrt{a+a\sin(dx+c)}a\,(11\,\sin(dx+c)-15)}{(a\sin(dx+c)-a)^2}-\frac{35\sqrt{2}}{16\sqrt{a}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)\right)+3/16\,\frac{}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2*a^5*(1/16/a^4*(1/8*(a+a*sin(d*x+c))^(1/2)*a*(11*sin(d*x+c)-15)/(a*sin(d*
x+c)-a)^2-35/16*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/
a^(1/2)))+3/16/a^4/(a+a*sin(d*x+c))^(1/2)+1/24/a^3/(a+a*sin(d*x+c))^(3/2))/
d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86235, size = 338, normalized size = 2.27

$$\frac{105\sqrt{2}\sqrt{a}\cos(dx+c)^4\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3\,a}{\sin(dx+c)-1}\right)-4\left(35\,\cos(dx+c)^2-7\left(15\,\cos(dx+c)^2+8\right)\sin(dx\right.}{768\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(105*sqrt(2)*sqrt(a)*cos(d*x + c)^4*log(-(a*sin(d*x + c) + 2*sqrt(2)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*(35*cos(d*x
 + c)^2 - 7*(15*cos(d*x + c)^2 + 8)*sin(d*x + c) + 8)*sqrt(a*sin(d*x + c) +
 a))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.113 $\int \sec^6(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=197

$$-\frac{63a^2\cos(c+dx)}{128d(a\sin(c+dx)+a)^{3/2}}-\frac{21a^2\sec(c+dx)}{80d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^5(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}+\frac{3a\sec^3(c+dx)}{10d\sqrt{a\sin(c+dx)+a}}+$$

```
[Out] (-63*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x
]])])/(128*Sqrt[2]*d) - (63*a^2*Cos[c + d*x])/(128*d*(a + a*Sin[c + d*x])^(
3/2)) - (21*a^2*Sec[c + d*x])/(80*d*(a + a*Sin[c + d*x])^(3/2)) + (21*a*Sec
[c + d*x])/(32*d*Sqrt[a + a*Sin[c + d*x]]) + (3*a*Sec[c + d*x]^3)/(10*d*Sqr
t[a + a*Sin[c + d*x]]) + (Sec[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(5*d)
```

Rubi [A] time = 0.293853, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2675, 2687, 2681, 2650, 2649, 206}

$$-\frac{63a^2\cos(c+dx)}{128d(a\sin(c+dx)+a)^{3/2}}-\frac{21a^2\sec(c+dx)}{80d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^5(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}+\frac{3a\sec^3(c+dx)}{10d\sqrt{a\sin(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-63*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x
]])])/(128*Sqrt[2]*d) - (63*a^2*Cos[c + d*x])/(128*d*(a + a*Sin[c + d*x])^(
3/2)) - (21*a^2*Sec[c + d*x])/(80*d*(a + a*Sin[c + d*x])^(3/2)) + (21*a*Sec
[c + d*x])/(32*d*Sqrt[a + a*Sin[c + d*x]]) + (3*a*Sec[c + d*x]^3)/(10*d*Sqr
t[a + a*Sin[c + d*x]]) + (Sec[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(5*d)
```


Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^6(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\sec^5(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} + \frac{1}{10}(9a)\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= \frac{3a\sec^3(c+dx)}{10d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^5(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} + \frac{1}{20}\left(21a^2\right)\int \frac{\text{se}}{(a+a}$$

$$= -\frac{21a^2\sec(c+dx)}{80d(a+a\sin(c+dx))^{3/2}} + \frac{3a\sec^3(c+dx)}{10d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^5(c+dx)\sqrt{a+a\sin(}}{5d}$$

$$= -\frac{21a^2\sec(c+dx)}{80d(a+a\sin(c+dx))^{3/2}} + \frac{21a\sec(c+dx)}{32d\sqrt{a+a\sin(c+dx)}} + \frac{3a\sec^3(c+dx)}{10d\sqrt{a+a\sin(c+dx)}} +$$

$$= -\frac{63a^2\cos(c+dx)}{128d(a+a\sin(c+dx))^{3/2}} - \frac{21a^2\sec(c+dx)}{80d(a+a\sin(c+dx))^{3/2}} + \frac{21a\sec(c+dx)}{32d\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{63a^2\cos(c+dx)}{128d(a+a\sin(c+dx))^{3/2}} - \frac{21a^2\sec(c+dx)}{80d(a+a\sin(c+dx))^{3/2}} + \frac{21a\sec(c+dx)}{32d\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{63\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{128\sqrt{2}d} - \frac{63a^2\cos(c+dx)}{128d(a+a\sin(c+dx))^{3/2}} - \frac{21a^2\sec(c}{80d(a+a\sin(}$$

Mathematica [C] time = 0.620729, size = 191, normalized size = 0.97

$$\frac{\sqrt{a(\sin(c+dx)+1)}\left(\frac{1572\sin(c+dx)+420\sin(3(c+dx))+1092\cos(2(c+dx))+315\cos(4(c+dx))+649}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5} - (2520+2520i)(-1)^{3/4}\left(\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{5120d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((-2520 - 2520*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2
)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])]*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^4 + (649 + 1092*Cos[2*(c + d*x)] + 315*Cos[
4*(c + d*x)] + 1572*Sin[c + d*x] + 420*Sin[3*(c + d*x)])/(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])^5))/(5120*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A] time = 0.152, size = 244, normalized size = 1.2

$$-\frac{1}{1280\left(\sin(dx+c)-1\right)^2\left(1+\sin(dx+c)\right)\cos(dx+c)\,d}\left(-420\,a^{9/2}\sin(dx+c)\left(\cos(dx+c)\right)^2+\left(630\,\left(a-a\sin(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/1280/a^(7/2)*(-420*a^(9/2)*sin(d*x+c)*cos(d*x+c)^2+(630*(a-a*sin(d*x+c))
^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2-288*
a^(9/2))*sin(d*x+c)-630*a^(9/2)*cos(d*x+c)^4+(-315*(a-a*sin(d*x+c))^(5/2)*2
^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2+84*a^(9/2))*
cos(d*x+c)^2+630*(a-a*sin(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c)
)^(1/2)*2^(1/2)/a^(1/2))*a^2+32*a^(9/2))/(sin(d*x+c)-1)^2/(1+sin(d*x+c))/co
s(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin(dx+c) + a} \sec(dx+c)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sec(d*x + c)^6, x)
```

Fricas [A] time = 1.89484, size = 583, normalized size = 2.96

$$\frac{315\sqrt{2}\sqrt{a}\cos(dx+c)^5 \log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(dx+c) - \sqrt{2}\sin(dx+c) + \sqrt{2}\right)\sqrt{a} + 3a\cos(dx+c) - (a\cos(dx+c) - 2a)\sin(dx+c) + 2a}{\cos(dx+c)^2 - (\cos(dx+c)+2)\sin(dx+c) - \cos(dx+c) - 2}\right.}{2560\, d \cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2560*(315*sqrt(2)*sqrt(a)*cos(d*x + c)^5*log(-(a*cos(d*x + c)^2 - 2*sqrt(
a*sin(d*x + c) + a)*(sqrt(2)*cos(d*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))
*sqrt(a) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(c
os(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(3
15*cos(d*x + c)^4 - 42*cos(d*x + c)^2 + 6*(35*cos(d*x + c)^2 + 24)*sin(d*x
+ c) - 16)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.114 $\int \cos^7(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=97

$$-\frac{2(a\sin(c+dx)+a)^{17/2}}{17a^7d}+\frac{4(a\sin(c+dx)+a)^{15/2}}{5a^6d}-\frac{24(a\sin(c+dx)+a)^{13/2}}{13a^5d}+\frac{16(a\sin(c+dx)+a)^{11/2}}{11a^4d}$$

```
[Out] (16*(a + a*Sin[c + d*x])^(11/2))/(11*a^4*d) - (24*(a + a*Sin[c + d*x])^(13/
2))/(13*a^5*d) + (4*(a + a*Sin[c + d*x])^(15/2))/(5*a^6*d) - (2*(a + a*Sin[
c + d*x])^(17/2))/(17*a^7*d)
```

Rubi [A] time = 0.0861733, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2(a\sin(c+dx)+a)^{17/2}}{17a^7d}+\frac{4(a\sin(c+dx)+a)^{15/2}}{5a^6d}-\frac{24(a\sin(c+dx)+a)^{13/2}}{13a^5d}+\frac{16(a\sin(c+dx)+a)^{11/2}}{11a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(11/2))/(11*a^4*d) - (24*(a + a*Sin[c + d*x])^(13/
2))/(13*a^5*d) + (4*(a + a*Sin[c + d*x])^(15/2))/(5*a^6*d) - (2*(a + a*Sin[
c + d*x])^(17/2))/(17*a^7*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{9/2}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{9/2} - 12a^2(a+x)^{11/2} + 6a(a+x)^{13/2} - (a+x)^{15/2}\right) dx, x,\right.}{a^7d}$$
$$= \frac{16(a+a\sin(c+dx))^{11/2}}{11a^4d} - \frac{24(a+a\sin(c+dx))^{13/2}}{13a^5d} + \frac{4(a+a\sin(c+dx))^{15/}}{5a^6d}$$

Mathematica [A] time = 0.388605, size = 61, normalized size = 0.63

$$-\frac{2(\sin(c+dx)+1)^4\left(715\sin^3(c+dx) - 2717\sin^2(c+dx) + 3641\sin(c+dx) - 1767\right)(a(\sin(c+dx)+1))^{3/2}}{12155d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(1 + Sin[c + d*x])^4*(a*(1 + Sin[c + d*x]))^(3/2)*(-1767 + 3641*Sin[c +
 d*x] - 2717*Sin[c + d*x]^2 + 715*Sin[c + d*x]^3))/(12155*d)
```

Maple [A] time = 0.096, size = 57, normalized size = 0.6

$$\frac{1430\,(\cos(dx+c))^2\sin(dx+c) - 5434\,(\cos(dx+c))^2 - 8712\,\sin(dx+c) + 8968}{12155\,a^4d}(a+a\sin(dx+c))^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/12155/a^4*(a+a*sin(d*x+c))^(11/2)*(715*cos(d*x+c)^2*sin(d*x+c)-2717*cos(d
*x+c)^2-4356*sin(d*x+c)+4484)/d
```

Maxima [A] time = 0.968551, size = 97, normalized size = 1.

$$-\frac{2\left(715\,(a\sin(dx+c)+a)^{\frac{17}{2}} - 4862\,(a\sin(dx+c)+a)^{\frac{15}{2}}a + 11220\,(a\sin(dx+c)+a)^{\frac{13}{2}}a^2 - 8840\,(a\sin(dx+c)+a)\right.}{12155\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/12155*(715*(a*sin(d*x + c) + a)^(17/2) - 4862*(a*sin(d*x + c) + a)^(15/2
)*a + 11220*(a*sin(d*x + c) + a)^(13/2)*a^2 - 8840*(a*sin(d*x + c) + a)^(11
/2)*a^3)/(a^7*d)
```

Fricas [A] time = 1.83001, size = 313, normalized size = 3.23

$$-\frac{2\left(715\,a\cos\left(dx+c\right)^{8}-66\,a\cos\left(dx+c\right)^{6}-112\,a\cos\left(dx+c\right)^{4}-256\,a\cos\left(dx+c\right)^{2}-2\left(429\,a\cos\left(dx+c\right)^{6}+504\,a\cos\right.\right.}{12155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/12155*(715*a*cos(d*x + c)^8 - 66*a*cos(d*x + c)^6 - 112*a*cos(d*x + c)^4
 - 256*a*cos(d*x + c)^2 - 2*(429*a*cos(d*x + c)^6 + 504*a*cos(d*x + c)^4 +
640*a*cos(d*x + c)^2 + 1024*a)*sin(d*x + c) - 2048*a)*sqrt(a*sin(d*x + c) +
 a)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^7, x)
```

3.115 $\int \cos^6(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=159

$$-\frac{32a^2\cos^7(c+dx)}{195d\sqrt{a\sin(c+dx)+a}}-\frac{128a^3\cos^7(c+dx)}{715d(a\sin(c+dx)+a)^{3/2}}-\frac{1024a^4\cos^7(c+dx)}{6435d(a\sin(c+dx)+a)^{5/2}}-\frac{4096a^5\cos^7(c+dx)}{45045d(a\sin(c+dx)+a)^{7/2}}-2a$$

```
[Out] (-4096*a^5*Cos[c + d*x]^7)/(45045*d*(a + a*Sin[c + d*x])^(7/2)) - (1024*a^4
*Cos[c + d*x]^7)/(6435*d*(a + a*Sin[c + d*x])^(5/2)) - (128*a^3*Cos[c + d*x
]^7)/(715*d*(a + a*Sin[c + d*x])^(3/2)) - (32*a^2*Cos[c + d*x]^7)/(195*d*Sq
rt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^7*Sqrt[a + a*Sin[c + d*x]])/(15
*d)
```

Rubi [A] time = 0.302172, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{32a^2\cos^7(c+dx)}{195d\sqrt{a\sin(c+dx)+a}}-\frac{128a^3\cos^7(c+dx)}{715d(a\sin(c+dx)+a)^{3/2}}-\frac{1024a^4\cos^7(c+dx)}{6435d(a\sin(c+dx)+a)^{5/2}}-\frac{4096a^5\cos^7(c+dx)}{45045d(a\sin(c+dx)+a)^{7/2}}-2a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-4096*a^5*Cos[c + d*x]^7)/(45045*d*(a + a*Sin[c + d*x])^(7/2)) - (1024*a^4
*Cos[c + d*x]^7)/(6435*d*(a + a*Sin[c + d*x])^(5/2)) - (128*a^3*Cos[c + d*x
]^7)/(715*d*(a + a*Sin[c + d*x])^(3/2)) - (32*a^2*Cos[c + d*x]^7)/(195*d*Sq
rt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^7*Sqrt[a + a*Sin[c + d*x]])/(15
*d)
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{2a\cos^7(c+dx)\sqrt{a+a\sin(c+dx)}}{15d}+\frac{1}{15}(16a)\int \cos^6(c+dx)\sqrt{a+a\sin(c+\\ &= -\frac{32a^2\cos^7(c+dx)}{195d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^7(c+dx)\sqrt{a+a\sin(c+dx)}}{15d}+\frac{1}{65}\left(64a^2\right)\int\\ &= -\frac{128a^3\cos^7(c+dx)}{715d(a+a\sin(c+dx))^{3/2}}-\frac{32a^2\cos^7(c+dx)}{195d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^7(c+dx)\sqrt{a+}}{15d}\\ &= -\frac{1024a^4\cos^7(c+dx)}{6435d(a+a\sin(c+dx))^{5/2}}-\frac{128a^3\cos^7(c+dx)}{715d(a+a\sin(c+dx))^{3/2}}-\frac{32a^2\cos^7(c+}{195d\sqrt{a+a\sin(}}\\ &= -\frac{4096a^5\cos^7(c+dx)}{45045d(a+a\sin(c+dx))^{7/2}}-\frac{1024a^4\cos^7(c+dx)}{6435d(a+a\sin(c+dx))^{5/2}}-\frac{128a^3\cos^7}{715d(a+a\sin}\end{aligned}$$

Mathematica [A] time = 0.623684, size = 79, normalized size = 0.5

$$-\frac{2\left(3003\sin^4(c+dx)+15708\sin^3(c+dx)+33138\sin^2(c+dx)+34748\sin(c+dx)+16363\right)\cos^7(c+dx)(a(\sin(c+}{45045d(\sin(c+dx)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^7*(a*(1 + Sin[c + d*x]))^(3/2)*(16363 + 34748*Sin[c + d*x]
 + 33138*Sin[c + d*x]^2 + 15708*Sin[c + d*x]^3 + 3003*Sin[c + d*x]^4))/(450
45*d*(1 + Sin[c + d*x])^5)
```

Maple [A] time = 0.112, size = 87, normalized size = 0.6

$$-\frac{(2+2\sin(dx+c))\,a^2(\sin(dx+c)-1)^4\left(3003\,(\sin(dx+c))^4+15708\,(\sin(dx+c))^3+33138\,(\sin(dx+c))^2+3474}{45045\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/45045*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^4*(3003*sin(d*x+c)^4+15708*sin(d
*x+c)^3+33138*sin(d*x+c)^2+34748*sin(d*x+c)+16363)/cos(d*x+c)/(a+a*sin(d*x+
c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^6, x)
```

Fricas [A] time = 1.79285, size = 622, normalized size = 3.91

$$-\frac{2\left(3003\,a\cos(dx+c)^8+6699\,a\cos(dx+c)^7-336\,a\cos(dx+c)^6+448\,a\cos(dx+c)^5-640\,a\cos(dx+c)^4+1024\,a\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45045*(3003*a*cos(d*x + c)^8 + 6699*a*cos(d*x + c)^7 - 336*a*cos(d*x + c
)^6 + 448*a*cos(d*x + c)^5 - 640*a*cos(d*x + c)^4 + 1024*a*cos(d*x + c)^3 -
 2048*a*cos(d*x + c)^2 + 8192*a*cos(d*x + c) + (3003*a*cos(d*x + c)^7 - 369
6*a*cos(d*x + c)^6 - 4032*a*cos(d*x + c)^5 - 4480*a*cos(d*x + c)^4 - 5120*a
*cos(d*x + c)^3 - 6144*a*cos(d*x + c)^2 - 8192*a*cos(d*x + c) - 16384*a)*si
n(d*x + c) + 16384*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x
+ c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^6, x)
```

3.116 $\int \cos^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=73

$$\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^5d}-\frac{8(a\sin(c+dx)+a)^{11/2}}{11a^4d}+\frac{8(a\sin(c+dx)+a)^{9/2}}{9a^3d}$$

```
[Out] (8*(a + a*Sin[c + d*x])^(9/2))/(9*a^3*d) - (8*(a + a*Sin[c + d*x])^(11/2))/
(11*a^4*d) + (2*(a + a*Sin[c + d*x])^(13/2))/(13*a^5*d)
```

Rubi [A] time = 0.0764606, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^5d}-\frac{8(a\sin(c+dx)+a)^{11/2}}{11a^4d}+\frac{8(a\sin(c+dx)+a)^{9/2}}{9a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(9/2))/(9*a^3*d) - (8*(a + a*Sin[c + d*x])^(11/2))/
(11*a^4*d) + (2*(a + a*Sin[c + d*x])^(13/2))/(13*a^5*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= \frac{\operatorname{Subst}\left(\int (a-x)^2(a+x)^{7/2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}\\
&= \frac{\operatorname{Subst}\left(\int \left(4a^2(a+x)^{7/2}-4a(a+x)^{9/2}+(a+x)^{11/2}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}\\
&= \frac{8(a+a\sin(c+dx))^{9/2}}{9a^3 d}-\frac{8(a+a\sin(c+dx))^{11/2}}{11a^4 d}+\frac{2(a+a\sin(c+dx))^{13/2}}{13a^5 d}
\end{aligned}
$$

Mathematica [A] time = 0.158593, size = 51, normalized size = 0.7

$$
\frac{2(\sin(c+dx)+1)^3\left(99\sin^2(c+dx)-270\sin(c+dx)+203\right)(a(\sin(c+dx)+1))^{3/2}}{1287d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(1 + Sin[c + d*x])^3*(a*(1 + Sin[c + d*x]))^(3/2)*(203 - 270*Sin[c + d*x
] + 99*Sin[c + d*x]^2))/(1287*d)
```

Maple [A] time = 0.082, size = 41, normalized size = 0.6

$$
-\frac{198\,(\cos(dx+c))^2+540\,\sin(dx+c)-604}{1287\,a^3 d}(a+a\sin(dx+c))^{\frac{9}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/1287/a^3*(a+a*sin(d*x+c))^(9/2)*(99*cos(d*x+c)^2+270*sin(d*x+c)-302)/d
```

Maxima [A] time = 0.96499, size = 74, normalized size = 1.01

$$
\frac{2\left(99\,(a\sin(dx+c)+a)^{\frac{13}{2}}-468\,(a\sin(dx+c)+a)^{\frac{11}{2}}a+572\,(a\sin(dx+c)+a)^{\frac{9}{2}}a^2\right)}{1287\,a^5 d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1287*(99*(a*sin(d*x + c) + a)^(13/2) - 468*(a*sin(d*x + c) + a)^(11/2)*a
+ 572*(a*sin(d*x + c) + a)^(9/2)*a^2)/(a^5*d)
```

Fricas [A] time = 1.70304, size = 242, normalized size = 3.32

$$-\frac{2\left(99\,a\cos\left(dx+c\right)^{6}-14\,a\cos\left(dx+c\right)^{4}-32\,a\cos\left(dx+c\right)^{2}-2\left(63\,a\cos\left(dx+c\right)^{4}+80\,a\cos\left(dx+c\right)^{2}+128\,a\right)\sin\left(dx\right.}{1287\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/1287*(99*a*cos(d*x + c)^6 - 14*a*cos(d*x + c)^4 - 32*a*cos(d*x + c)^2 -
2*(63*a*cos(d*x + c)^4 + 80*a*cos(d*x + c)^2 + 128*a)*sin(d*x + c) - 256*a)
*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^5, x)
```

3.117 $\int \cos^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{8a^2\cos^5(c+dx)}{33d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^5(c+dx)}{231d(a\sin(c+dx)+a)^{3/2}}-\frac{256a^4\cos^5(c+dx)}{1155d(a\sin(c+dx)+a)^{5/2}}-\frac{2a\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}$$

```
[Out] (-256*a^4*Cos[c + d*x]^5)/(1155*d*(a + a*Sin[c + d*x])^(5/2)) - (64*a^3*Cos
[c + d*x]^5)/(231*d*(a + a*Sin[c + d*x])^(3/2)) - (8*a^2*Cos[c + d*x]^5)/(3
3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]
])/(11*d)
```

Rubi [A] time = 0.234739, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{8a^2\cos^5(c+dx)}{33d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^5(c+dx)}{231d(a\sin(c+dx)+a)^{3/2}}-\frac{256a^4\cos^5(c+dx)}{1155d(a\sin(c+dx)+a)^{5/2}}-\frac{2a\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^5)/(1155*d*(a + a*Sin[c + d*x])^(5/2)) - (64*a^3*Cos
[c + d*x]^5)/(231*d*(a + a*Sin[c + d*x])^(3/2)) - (8*a^2*Cos[c + d*x]^5)/(3
3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]
])/(11*d)
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
```

```
- b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11d}+\frac{1}{11}(12a)\int \cos^4(c+dx)\sqrt{a+a\sin(c+a$$
$$= -\frac{8a^2\cos^5(c+dx)}{33d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11d}+\frac{1}{33}\left(32a^2\right)\int$$
$$= -\frac{64a^3\cos^5(c+dx)}{231d(a+a\sin(c+dx))^{3/2}}-\frac{8a^2\cos^5(c+dx)}{33d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^5(c+dx)\sqrt{a+a}}{11d}$$
$$= -\frac{256a^4\cos^5(c+dx)}{1155d(a+a\sin(c+dx))^{5/2}}-\frac{64a^3\cos^5(c+dx)}{231d(a+a\sin(c+dx))^{3/2}}-\frac{8a^2\cos^5(c+dx}{33d\sqrt{a+a\sin(c+}}$$

Mathematica [A] time = 0.154282, size = 69, normalized size = 0.54

$$-\frac{2\left(105\sin^3(c+dx)+455\sin^2(c+dx)+755\sin(c+dx)+533\right)\cos^5(c+dx)(a(\sin(c+dx)+1))^{3/2}}{1155d(\sin(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5*(a*(1 + Sin[c + d*x]))^(3/2)*(533 + 755*Sin[c + d*x] + 4
55*Sin[c + d*x]^2 + 105*Sin[c + d*x]^3))/(1155*d*(1 + Sin[c + d*x])^4)
```

Maple [A] time = 0.113, size = 77, normalized size = 0.6

$$\frac{(2+2\sin(dx+c))\,a^2(\sin(dx+c)-1)^3\left(105(\sin(dx+c))^3+455(\sin(dx+c))^2+755\sin(dx+c)+533\right)}{1155\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/1155*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^3*(105*sin(d*x+c)^3+455*sin(d*x+c)
^2+755*sin(d*x+c)+533)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4, x)
```

Fricas [A] time = 1.68689, size = 471, normalized size = 3.71

$$-\frac{2\left(105\,a\cos(dx+c)^6+245\,a\cos(dx+c)^5-20\,a\cos(dx+c)^4+32\,a\cos(dx+c)^3-64\,a\cos(dx+c)^2+256\,a\cos(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(105*a*cos(d*x + c)^6 + 245*a*cos(d*x + c)^5 - 20*a*cos(d*x + c)^4
+ 32*a*cos(d*x + c)^3 - 64*a*cos(d*x + c)^2 + 256*a*cos(d*x + c) + (105*a*c
os(d*x + c)^5 - 140*a*cos(d*x + c)^4 - 160*a*cos(d*x + c)^3 - 192*a*cos(d*x
 + c)^2 - 256*a*cos(d*x + c) - 512*a)*sin(d*x + c) + 512*a)*sqrt(a*sin(d*x
+ c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4, x)
```

3.118 $\int \cos^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=49

$$\frac{4(a\sin(c+dx)+a)^{7/2}}{7a^2d}-\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^3d}$$

```
[Out] (4*(a + a*Sin[c + d*x])^(7/2))/(7*a^2*d) - (2*(a + a*Sin[c + d*x])^(9/2))/(
9*a^3*d)
```

Rubi [A] time = 0.0677168, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{4(a\sin(c+dx)+a)^{7/2}}{7a^2d}-\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(7/2))/(7*a^2*d) - (2*(a + a*Sin[c + d*x])^(9/2))/(
9*a^3*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^{5/2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(a+x)^{5/2}-(a+x)^{7/2}\right)\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{4(a+a\sin(c+dx))^{7/2}}{7a^2 d} - \frac{2(a+a\sin(c+dx))^{9/2}}{9a^3 d}$$

Mathematica [A] time = 0.084138, size = 41, normalized size = 0.84

$$-\frac{2(\sin(c+dx)+1)^2(7\sin(c+dx)-11)(a(\sin(c+dx)+1))^{3/2}}{63d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(1 + Sin[c + d*x])^2*(a*(1 + Sin[c + d*x]))^(3/2)*(-11 + 7*Sin[c + d*x]
))/(63*d)
```

Maple [A] time = 0.079, size = 31, normalized size = 0.6

$$-\frac{14\sin(dx+c)-22}{63\,a^2 d}(a+a\sin(dx+c))^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/63/a^2*(a+a*sin(d*x+c))^(7/2)*(7*sin(d*x+c)-11)/d
```

Maxima [A] time = 0.948232, size = 51, normalized size = 1.04

$$-\frac{2\left(7\,(a\sin(dx+c)+a)^{\frac{9}{2}}-18\,(a\sin(dx+c)+a)^{\frac{7}{2}}a\right)}{63\,a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/63*(7*(a*sin(d*x + c) + a)^(9/2) - 18*(a*sin(d*x + c) + a)^(7/2)*a)/(a^3
*d)
```

Fricas [A] time = 1.67518, size = 171, normalized size = 3.49

$$-\frac{2\left(7\,a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}-2\left(5\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)-16\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(7*a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - 2*(5*a*cos(d*x + c)^2 + 8*
a)*sin(d*x + c) - 16*a)*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [A] time = 131.176, size = 252, normalized size = 5.14

$$\begin{cases}\frac{8a\sqrt{a\sin(c+dx)+a}\sin^{4}(c+dx)}{45d}+\frac{152a\sqrt{a\sin(c+dx)+a}\sin^{3}(c+dx)}{315d}+\frac{2a\sqrt{a\sin(c+dx)+a}\sin^{2}(c+dx)\cos^{2}(c+dx)}{5d}+\frac{8a\sqrt{a\sin(c+dx)+a}\sin^{2}(c+dx)}{21d}+\cdots\\ x\left(a\sin(c)+a\right)^{\frac{3}{2}}\cos^{3}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((8*a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)**4/(45*d) + 152*a*sqrt
(a*sin(c + d*x) + a)*sin(c + d*x)**3/(315*d) + 2*a*sqrt(a*sin(c + d*x) + a)
*sin(c + d*x)**2*cos(c + d*x)**2/(5*d) + 8*a*sqrt(a*sin(c + d*x) + a)*sin(c
 + d*x)**2/(21*d) + 4*a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)*cos(c + d*x)*
*2/(5*d) + 8*a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)/(315*d) + 2*a*sqrt(a*s
in(c + d*x) + a)*cos(c + d*x)**2/(5*d) - 16*a*sqrt(a*sin(c + d*x) + a)/(315
*d), Ne(d, 0)), (x*(a*sin(c) + a)**(3/2)*cos(c)**3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

3.119 $\int \cos^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=95

$$-\frac{16a^2\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}-\frac{2a\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}$$

```
[Out] (-64*a^3*Cos[c + d*x]^3)/(105*d*(a + a*Sin[c + d*x])^(3/2)) - (16*a^2*Cos[c
 + d*x]^3)/(35*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^3*Sqrt[a + a
*Sin[c + d*x]])/(7*d)
```

Rubi [A] time = 0.167231, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{16a^2\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}-\frac{2a\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^3)/(105*d*(a + a*Sin[c + d*x])^(3/2)) - (16*a^2*Cos[c
 + d*x]^3)/(35*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^3*Sqrt[a + a
*Sin[c + d*x]])/(7*d)
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7d}+\frac{1}{7}(8a)\int \cos^2(c+dx)\sqrt{a+a\sin(c+dx)}$$
$$= -\frac{16a^2\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7d}+\frac{1}{35}\left(32a^2\right)\int$$
$$= -\frac{64a^3\cos^3(c+dx)}{105d(a+a\sin(c+dx))^{3/2}}-\frac{16a^2\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^3(c+dx)\sqrt{a+a}}{7d}$$

Mathematica [A] time = 0.140159, size = 59, normalized size = 0.62

$$-\frac{2\left(15\sin^2(c+dx)+54\sin(c+dx)+71\right)\cos^3(c+dx)(a(\sin(c+dx)+1))^{3/2}}{105d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^3*(a*(1 + Sin[c + d*x]))^(3/2)*(71 + 54*Sin[c + d*x] + 15*
Sin[c + d*x]^2))/(105*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.113, size = 67, normalized size = 0.7

$$-\frac{(2+2\sin(dx+c))\,a^2(\sin(dx+c)-1)^2\left(15(\sin(dx+c))^2+54\sin(dx+c)+71\right)}{105\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/105*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*(15*sin(d*x+c)^2+54*sin(d*x+c)+7
1)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

Fricas [A] time = 1.62546, size = 336, normalized size = 3.54

$$-\frac{2\left(15\,a\cos\left(dx+c\right)^{4}+39\,a\cos\left(dx+c\right)^{3}-8\,a\cos\left(dx+c\right)^{2}+32\,a\cos\left(dx+c\right)+\left(15\,a\cos\left(dx+c\right)^{3}-24\,a\cos\left(dx+c\right.\right.}{105\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/105*(15*a*cos(d*x + c)^4 + 39*a*cos(d*x + c)^3 - 8*a*cos(d*x + c)^2 + 32
*a*cos(d*x + c) + (15*a*cos(d*x + c)^3 - 24*a*cos(d*x + c)^2 - 32*a*cos(d*x
 + c) - 64*a)*sin(d*x + c) + 64*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c)
 + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

3.120 $\int \cos(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5ad}$$

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(5*a*d)
```

Rubi [A] time = 0.0342689, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(5*a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= \frac{\operatorname{Subst}\left(\int (a+x)^{3/2}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{2(a+a\sin(c+dx))^{5/2}}{5ad}\end{aligned}$$

Mathematica [A] time = 0.0412181, size = 24, normalized size = 1.

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(5*a*d)
```

Maple [A] time = 0.009, size = 21, normalized size = 0.9

$$\frac{2}{5\,da}\left(a+a\sin\left(dx+c\right)\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5*(a+a*sin(d*x+c))^(5/2)/d/a
```

Maxima [A] time = 0.941551, size = 27, normalized size = 1.12

$$\frac{2\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}{5\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(a*sin(d*x + c) + a)^(5/2)/(a*d)
```

Fricas [A] time = 1.57513, size = 104, normalized size = 4.33

$$-\frac{2\left(a\cos\left(dx+c\right)^{2}-2\,a\sin\left(dx+c\right)-2\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(a*cos(d*x + c)^2 - 2*a*sin(d*x + c) - 2*a)*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [A] time = 26.2348, size = 90, normalized size = 3.75

$$\begin{cases} \frac{2a\sqrt{a\sin(c+dx)+a}\sin^2(c+dx)}{5d} + \frac{4a\sqrt{a\sin(c+dx)+a}\sin(c+dx)}{5d} + \frac{2a\sqrt{a\sin(c+dx)+a}}{5d} & \text{for } d \neq 0 \\ x\left(a\sin(c)+a\right)^{\frac{3}{2}}\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((2*a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)**2/(5*d) + 4*a*sqrt(a*
sin(c + d*x) + a)*sin(c + d*x)/(5*d) + 2*a*sqrt(a*sin(c + d*x) + a)/(5*d),
Ne(d, 0)), (x*(a*sin(c) + a)**(3/2)*cos(c), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

3.121 $\int \sec(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a\sqrt{a\sin(c+dx)+a}}{d}$$

```
[Out] (2*Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d -
 (2*a*Sqrt[a + a*Sin[c + d*x]])/d
```

Rubi [A] time = 0.0682787, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2667, 50, 63, 206}

$$\frac{2\sqrt{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a\sqrt{a\sin(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d -
 (2*a*Sqrt[a + a*Sin[c + d*x]])/d
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{2a\sqrt{a+a\sin(c+dx)}}{d}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{2a\sqrt{a+a\sin(c+dx)}}{d}+\frac{\left(4a^2\right)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+a\sin(c+dx)}\right)}{d}\\ &= \frac{2\sqrt{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a\sqrt{a+a\sin(c+dx)}}{d}\end{aligned}$$

Mathematica [A] time = 0.100852, size = 60, normalized size = 0.97

$$\frac{2a\left(\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)-\sqrt{a\sin(c+dx)+a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]

[Out] (2*a*(Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])] -
 Sqrt[a + a*Sin[c + d*x]]))/d
```

Maple [A] time = 0.075, size = 49, normalized size = 0.8

$$-2\,{\frac {a}{d}\left(\sqrt {a+a\sin \left(dx+c \right) }-\sqrt {a}\sqrt {2}{\rm Artanh} \left(1/2\,{\frac {\sqrt {a+a\sin \left(dx+c \right) }\sqrt {2}}{\sqrt {a}}} \right) \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2*a*((a+a*sin(d*x+c))^(1/2)-a^(1/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(
1/2)*2^(1/2)/a^(1/2)))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69702, size = 196, normalized size = 3.16

$$\frac{\sqrt{2} a^{\frac{3}{2}} \log\left(-\frac{a \sin\left(dx + c\right) + 2 \, \sqrt{2} \sqrt{a \sin\left(dx + c\right) + a} \sqrt{a} + 3 \, a}{\sin\left(dx + c\right) - 1}\right) - 2 \, \sqrt{a \sin\left(dx + c\right) + a} a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*a^(3/2)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*
sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 2*sqrt(a*sin(d*x + c) + a)*a)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 8.41583, size = 910, normalized size = 14.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2*(2*sqrt(2)*(a^2*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
 1)*tan(1/2*c)^2 + a^2*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2
*c) - 1))*arctan(1/2*sqrt(2)*(sqrt((a*tan(1/2*c)^2 + a)*tan(1/2*d*x)^2 + a*
tan(1/2*c)^2 + a)*a*tan(1/2*c)^3 + sqrt((a*tan(1/2*c)^2 + a)*tan(1/2*d*x)^2
 + a*tan(1/2*c)^2 + a)*a*tan(1/2*c)^2 + 2*sqrt((a*tan(1/2*c)^2 + a)*tan(1/2
*d*x)^2 + a*tan(1/2*c)^2 + a)*a*tan(1/2*c) + 2*sqrt((a*tan(1/2*c)^2 + a)*ta
n(1/2*d*x)^2 + a*tan(1/2*c)^2 + a)*a + sqrt(a^3*tan(1/2*c)^8 + 2*a^3*tan(1/
2*c)^7 + 6*a^3*tan(1/2*c)^6 + 10*a^3*tan(1/2*c)^5 + 13*a^3*tan(1/2*c)^4 + 1
6*a^3*tan(1/2*c)^3 + 12*a^3*tan(1/2*c)^2 + 8*a^3*tan(1/2*c) + 4*a^3)*tan(1/
2*d*x) + sqrt(a^3*tan(1/2*c)^8 - 2*a^3*tan(1/2*c)^7 + 6*a^3*tan(1/2*c)^6 -
10*a^3*tan(1/2*c)^5 + 13*a^3*tan(1/2*c)^4 - 16*a^3*tan(1/2*c)^3 + 12*a^3*ta
n(1/2*c)^2 - 8*a^3*tan(1/2*c) + 4*a^3))/sqrt(-a^3*tan(1/2*c)^8 - 6*a^3*tan(
1/2*c)^6 - 13*a^3*tan(1/2*c)^4 - 12*a^3*tan(1/2*c)^2 - 4*a^3))/sqrt(-a*tan(
1/2*c)^4 - 2*a*tan(1/2*c)^2 - a) + (a^2*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c) + a^2*sgn(tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) - tan(1/2*c) - 1) - (a^2*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) - tan(1/2*c) - 1)*tan(1/2*c) - a^2*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x))/sqrt(a*tan(1/2*d*x)^2*tan(1/2*c)^2
+ a*tan(1/2*d*x)^2 + a*tan(1/2*c)^2 + a))/d
```

3.122 $\int \sec^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=26

$$\frac{2a\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}$$

```
[Out] (2*a*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d
```

Rubi [A] time = 0.0575865, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2673}

$$\frac{2a\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{2a\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}$$

Mathematica [B] time = 0.157883, size = 67, normalized size = 2.58

$$\frac{2(a(\sin(c+dx)+1))^{3/2}}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(3/2))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A] time = 0.083, size = 37, normalized size = 1.4

$$2\,\frac{a^{2}\left(1+\sin\left(dx+c\right)\right)}{\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2*a^2*(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [B] time = 1.63246, size = 132, normalized size = 5.08

$$-\frac{2\left(a^{\frac{3}{2}}+\frac{2\,a^{\frac{3}{2}}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{\frac{3}{2}}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(a^(3/2) + 2*a^(3/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^(3/2)*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^(3/2))
```

Fricas [A] time = 1.61122, size = 63, normalized size = 2.42

$$\frac{2\sqrt{a\sin\left(dx+c\right)+a}a}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*sin(d*x + c) + a)*a/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.123 $\int \sec^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=73

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{2d}$$

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*d)
 + (Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(2*d)
```

Rubi [A] time = 0.11219, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2675, 2667, 63, 206}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*d)
 + (Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(2*d)
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= \frac{\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{2d}+\frac{1}{4}a\int \sec(c+dx)\sqrt{a+a\sin(c+dx)}\,dx\\ &= \frac{\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{2d}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx,x,a\sin(c+dx)\right)}{4d}\\ &= \frac{\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{2d}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\sqrt{a+a\sin(c+dx)}\right)}{2d}\\ &= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d}+\frac{\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{2d}\end{aligned}$$

Mathematica [A] time = 0.249573, size = 72, normalized size = 0.99

$$\frac{a\left(\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{2}\sqrt{a}}\right)-\frac{2\sqrt{a(\sin(c+dx)+1)}}{\sin(c+dx)-1}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*(Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])] -
 (2*Sqrt[a*(1 + Sin[c + d*x])])/(-1 + Sin[c + d*x])))/(4*d)
```

Maple [A] time = 0.102, size = 70, normalized size = 1.

$$2\,{\frac {{a}^{3}}{d} \left(-1/4\,{\frac {\sqrt {a+a\sin \left(dx+c \right) }}{a \left(a\sin \left(dx+c \right) -a \right) }}+1/8\,{\frac {\sqrt {2}}{{a}^{3/2}}{\rm Artanh} \left(1/2\,{\frac {\sqrt {a+a\sin \left(dx+c \right) }\sqrt {2}}{\sqrt {a}}}\right)} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2*a^3*(-1/4*(a+a*sin(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)+1/8/a^(3/2)*2^(1/2)*a
rctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69733, size = 266, normalized size = 3.64

$$\frac{\left(\sqrt{2} a \sin\left(dx+c\right) - \sqrt{2} a\right) \sqrt{a} \log\left(-\frac{a \sin\left(dx+c\right) + 2 \, \sqrt{2} \sqrt{a \sin\left(dx+c\right) + a} \sqrt{a} + 3 \, a}{\sin\left(dx+c\right) - 1}\right) - 4 \, \sqrt{a \sin\left(dx+c\right) + a} a}{8 \, \left(d \sin\left(dx+c\right) - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*((sqrt(2)*a*sin(d*x + c) - sqrt(2)*a)*sqrt(a)*log(-(a*sin(d*x + c) + 2*
sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*sqr
t(a*sin(d*x + c) + a)*a)/(d*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.124 $\int \sec^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=107

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}+\frac{a\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

```
[Out] -(a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])
])/(2*Sqrt[2]*d) + (a*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d) + (Sec[c
 + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rubi [A] time = 0.134985, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2675, 2649, 206}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}+\frac{a\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])
])/(2*Sqrt[2]*d) + (a*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d) + (Sec[c
 + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= \frac{\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d}+\frac{1}{2}a\int \sec^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx\\ &= \frac{a\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}+\frac{\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d}+\frac{1}{4}a^2\int\\ &= \frac{a\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}+\frac{\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d}-\frac{a^2\,\mathrm{Subs}}{}\\ &= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}d}+\frac{a\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}+\frac{\sec^3(c+d}{}\end{aligned}$$

Mathematica [C] time = 0.382373, size = 130, normalized size = 1.21

$$\frac{\left(\frac{1}{12}+\frac{i}{12}\right)a\sec^3(c+dx)\sqrt{a(\sin(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\left(6(-1)^{3/4}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((1/12 + I/12)*a*Sec[c + d*x]^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*Sqr
t[a*(1 + Sin[c + d*x])]*(6*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 +
Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 - (1 - I)*(-5 +
3*Sin[c + d*x])))/d
```

Maple [A] time = 0.135, size = 107, normalized size = 1.

$$\frac{1+\sin(dx+c)}{(12\sin(dx+c)-12)\cos(dx+c)d}\left(6a^{7/2}\sin(dx+c)-10a^{7/2}+3\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)a^2(a-a\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/12/a^(3/2)*(1+sin(d*x+c))/(sin(d*x+c)-1)*(6*a^(7/2)*sin(d*x+c)-10*a^(7/2)
+3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*(a-a*sin
(d*x+c))^(3/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.76247, size = 585, normalized size = 5.47

$$\frac{3\left(\sqrt{2}a\cos(dx+c)\sin(dx+c)-\sqrt{2}a\cos(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(dx+c)-\sqrt{2}\sin(dx+c)+\sqrt{2}\right)\sqrt{a}+3ac}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos}\right.}{24\left(d\cos(dx+c)\sin(dx+c)-d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(sqrt(2)*a*cos(d*x + c)*sin(d*x + c) - sqrt(2)*a*cos(d*x + c))*sqrt
(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*sin(d*x + c) + a)*(sqrt(2)*cos(d*x +
c) - sqrt(2)*sin(d*x + c) + sqrt(2))*sqrt(a) + 3*a*cos(d*x + c) - (a*cos(d*
x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)) + 4*(3*a*sin(d*x + c) - 5*a)*sqrt(a*sin(d*x +
 c) + a))/(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.125 $\int \sec^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{15a^2}{32d\sqrt{a\sin(c+dx)+a}}+\frac{15a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+a)^{3/2}}{4d}+\frac{5a\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{16d}$$

```
[Out] (15*a^(3/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2
]*d) - (15*a^2)/(32*d*Sqrt[a + a*Sin[c + d*x]]) + (5*a*Sec[c + d*x]^2*Sqrt[
a + a*Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2))/(
4*d)
```

Rubi [A] time = 0.181212, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2675, 2667, 51, 63, 206}

$$-\frac{15a^2}{32d\sqrt{a\sin(c+dx)+a}}+\frac{15a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+a)^{3/2}}{4d}+\frac{5a\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```


```
[Out] (15*a^(3/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2
]*d) - (15*a^2)/(32*d*Sqrt[a + a*Sin[c + d*x]]) + (5*a*Sec[c + d*x]^2*Sqrt[
a + a*Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2))/(
4*d)
```


Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{4d} + \frac{1}{8}(5a)\int \sec^3(c+dx)\sqrt{a+a\sin(c+dx)}$$
$$= \frac{5a\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{16d} + \frac{\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{4d} + \frac{1}{32}$$
$$= \frac{5a\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{16d} + \frac{\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{4d} + \frac{(15}{}$$
$$= -\frac{15a^2}{32d\sqrt{a+a\sin(c+dx)}} + \frac{5a\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{16d} + \frac{\sec^4(c+dx)(}{}$$
$$= -\frac{15a^2}{32d\sqrt{a+a\sin(c+dx)}} + \frac{5a\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{16d} + \frac{\sec^4(c+dx)(}{}$$
$$= \frac{15a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d} - \frac{15a^2}{32d\sqrt{a+a\sin(c+dx)}} + \frac{5a\sec^2(c+dx)\sqrt{a}}{16d}$$

Mathematica [C] time = 0.0714919, size = 44, normalized size = 0.35

$$-\frac{a^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{4d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a^2*Hypergeometric2F1[-1/2, 3, 1/2, (1 + Sin[c + d*x])/2])/(4*d*Sqrt[a +
a*Sin[c + d*x]])
```

Maple [A] time = 0.188, size = 101, normalized size = 0.8

$$-2\,\frac{a^5}{d}\left(1/8\,\frac{1}{a^3}\left(1/8\,\frac{\sqrt{a+a\sin(dx+c)}a\,(7\,\sin(dx+c)-11)}{(a\sin(dx+c)-a)^2}-\frac{15\,\sqrt{2}}{16\,\sqrt{a}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)\right)+1/8\,\frac{}{a^3\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2*a^5*(1/8/a^3*(1/8*(a+a*sin(d*x+c))^(1/2)*a*(7*sin(d*x+c)-11)/(a*sin(d*x+
c)-a)^2-15/16*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^
(1/2)))+1/8/a^3/(a+a*sin(d*x+c))^(1/2))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76802, size = 417, normalized size = 3.28

$$\frac{15\left(\sqrt{2}a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-\sqrt{2}a\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\left(15a\cos\left(dx+c\right)^{2}\right.}{128\left(d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(15*(sqrt(2)*a*cos(d*x + c)^2*sin(d*x + c) - sqrt(2)*a*cos(d*x + c)^2
)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 + 3*a)/(sin(d*x + c) - 1)) - 4*(15*a*cos(d*x + c)^2 + 20*a*sin(d*x + c) -
12*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x
+ c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**(3/2),x)
```

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.126 $\int \sec^6(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=169

$$-\frac{7a^3\cos(c+dx)}{16d(a\sin(c+dx)+a)^{3/2}}+\frac{7a^2\sec(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}-\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

```
[Out] (-7*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(16*Sqrt[2]*d) - (7*a^3*Cos[c + d*x])/(16*d*(a + a*Sin[c + d*x])^(3/2)
) + (7*a^2*Sec[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) + (7*a*Sec[c + d*x
]^3*Sqrt[a + a*Sin[c + d*x]])/(30*d) + (Sec[c + d*x]^5*(a + a*Sin[c + d*x])
^(3/2))/(5*d)
```

Rubi [A] time = 0.221389, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2675, 2687, 2650, 2649, 206}

$$-\frac{7a^3\cos(c+dx)}{16d(a\sin(c+dx)+a)^{3/2}}+\frac{7a^2\sec(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}-\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-7*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(16*Sqrt[2]*d) - (7*a^3*Cos[c + d*x])/(16*d*(a + a*Sin[c + d*x])^(3/2)
) + (7*a^2*Sec[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) + (7*a*Sec[c + d*x
]^3*Sqrt[a + a*Sin[c + d*x]])/(30*d) + (Sec[c + d*x]^5*(a + a*Sin[c + d*x])
^(3/2))/(5*d)
```


Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\sec^5(c+dx)(a+a\sin(c+dx))^{3/2}}{5d} + \frac{1}{10}(7a)\int \sec^4(c+dx)\sqrt{a+a\sin(c+dx)}$$

$$= \frac{7a\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)(a+a\sin(c+dx))^{3/2}}{5d} + \frac{1}{12}$$

$$= \frac{7a^2\sec(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} + \frac{7a\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)(a}{}$$

$$= -\frac{7a^3\cos(c+dx)}{16d(a+a\sin(c+dx))^{3/2}} + \frac{7a^2\sec(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} + \frac{7a\sec^3(c+dx)\sqrt{a+a}}{30d}$$

$$= -\frac{7a^3\cos(c+dx)}{16d(a+a\sin(c+dx))^{3/2}} + \frac{7a^2\sec(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} + \frac{7a\sec^3(c+dx)\sqrt{a+a}}{30d}$$

$$= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}d} - \frac{7a^3\cos(c+dx)}{16d(a+a\sin(c+dx))^{3/2}} + \frac{7a^2\sec(c+}{12d\sqrt{a+a\sin}}$$

Mathematica [C] time = 0.421191, size = 288, normalized size = 1.7

$$\frac{(a(\sin(c+dx)+1))^{3/2}\left(30\sin\left(\frac{1}{2}(c+dx)\right)+\frac{90\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{40\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{24\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\right.\right.}\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((30*Sin[(c + d*x)/2] - 15*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + (105 + 1
05*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (24*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5 + (40*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (90*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))*(a
*(1 + Sin[c + d*x]))^(3/2))/(240*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A] time = 0.132, size = 172, normalized size = 1.

$$-\frac{1}{480\,(\sin(dx+c)-1)^2\cos(dx+c)\,d}\left(210\,a^{7/2}\sin(dx+c)\,(\cos(dx+c))^2+\left(105\,(a-a\sin(dx+c))^{5/2}\sqrt{2}\mathrm{Artanh}\left(1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/480/a^(3/2)*(210*a^(7/2)*sin(d*x+c)*cos(d*x+c)^2+(105*(a-a*sin(d*x+c))^(
5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a-168*a^(7
/2))*sin(d*x+c)-350*a^(7/2)*cos(d*x+c)^2+105*(a-a*sin(d*x+c))^(5/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a+72*a^(7/2))/(sin(d*x
+c)-1)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.95671, size = 667, normalized size = 3.95

$$\frac{105\left(\sqrt{2}a\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-\sqrt{2}a\cos\left(dx+c\right)^{3}\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(dx+c)-\sqrt{2}\sin(dx+c)+\sqrt{2}\right)\sqrt{a}}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c}\right.}{960\left(d\cos\left(dx+c\right)^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/960*(105*(sqrt(2)*a*cos(d*x + c)^3*sin(d*x + c) - sqrt(2)*a*cos(d*x + c)^
3)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*sin(d*x + c) + a)*(sqrt(2)*cos
(d*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))*sqrt(a) + 3*a*cos(d*x + c) - (a
*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) +
2)*sin(d*x + c) - cos(d*x + c) - 2)) - 4*(175*a*cos(d*x + c)^2 - 21*(5*a*co
s(d*x + c)^2 - 4*a)*sin(d*x + c) - 36*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d
*x + c)^3*sin(d*x + c) - d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.127 $\int \cos^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=73

$$\frac{2(a\sin(c+dx)+a)^{15/2}}{15a^5d}-\frac{8(a\sin(c+dx)+a)^{13/2}}{13a^4d}+\frac{8(a\sin(c+dx)+a)^{11/2}}{11a^3d}$$

```
[Out] (8*(a + a*Sin[c + d*x])^(11/2))/(11*a^3*d) - (8*(a + a*Sin[c + d*x])^(13/2)
)/(13*a^4*d) + (2*(a + a*Sin[c + d*x])^(15/2))/(15*a^5*d)
```

Rubi [A] time = 0.0735895, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{15/2}}{15a^5d}-\frac{8(a\sin(c+dx)+a)^{13/2}}{13a^4d}+\frac{8(a\sin(c+dx)+a)^{11/2}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(11/2))/(11*a^3*d) - (8*(a + a*Sin[c + d*x])^(13/2)
)/(13*a^4*d) + (2*(a + a*Sin[c + d*x])^(15/2))/(15*a^5*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{9/2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{9/2}-4a(a+x)^{11/2}+(a+x)^{13/2}\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{8(a+a\sin(c+dx))^{11/2}}{11a^3d} - \frac{8(a+a\sin(c+dx))^{13/2}}{13a^4d} + \frac{2(a+a\sin(c+dx))^{15/2}}{15a^5d}$$

Mathematica [A] time = 0.201794, size = 51, normalized size = 0.7

$$\frac{2(\sin(c+dx)+1)^3\left(143\sin^2(c+dx)-374\sin(c+dx)+263\right)(a(\sin(c+dx)+1))^{5/2}}{2145d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(1 + Sin[c + d*x])^3*(a*(1 + Sin[c + d*x]))^(5/2)*(263 - 374*Sin[c + d*x
] + 143*Sin[c + d*x]^2))/(2145*d)
```

Maple [A] time = 0.082, size = 41, normalized size = 0.6

$$-\frac{286\ (\cos(dx+c))^2+748\ \sin(dx+c)-812}{2145\,a^3d}(a+a\sin(dx+c))^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/2145/a^3*(a+a*sin(d*x+c))^(11/2)*(143*cos(d*x+c)^2+374*sin(d*x+c)-406)/d
```

Maxima [A] time = 0.955465, size = 74, normalized size = 1.01

$$\frac{2\left(143\,(a\sin(dx+c)+a)^{\frac{15}{2}}-660\,(a\sin(dx+c)+a)^{\frac{13}{2}}a+780\,(a\sin(dx+c)+a)^{\frac{11}{2}}a^2\right)}{2145\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/2145*(143*(a*sin(d*x + c) + a)^(15/2) - 660*(a*sin(d*x + c) + a)^(13/2)*a
 + 780*(a*sin(d*x + c) + a)^(11/2)*a^2)/(a^5*d)
```

Fricas [A] time = 1.68519, size = 296, normalized size = 4.05

$$-\frac{2\left(341\,a^2\cos\left(dx+c\right)^6-28\,a^2\cos\left(dx+c\right)^4-64\,a^2\cos\left(dx+c\right)^2-512\,a^2+\left(143\,a^2\cos\left(dx+c\right)^6-252\,a^2\cos\left(dx+c\right)\right.\right.}{2145\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/2145*(341*a^2*cos(d*x + c)^6 - 28*a^2*cos(d*x + c)^4 - 64*a^2*cos(d*x +
c)^2 - 512*a^2 + (143*a^2*cos(d*x + c)^6 - 252*a^2*cos(d*x + c)^4 - 320*a^2
*cos(d*x + c)^2 - 512*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^5, x)
```

3.128 $\int \cos^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=159

$$-\frac{32a^2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{143d}-\frac{128a^3\cos^5(c+dx)}{429d\sqrt{a\sin(c+dx)+a}}-\frac{1024a^4\cos^5(c+dx)}{3003d(a\sin(c+dx)+a)^{3/2}}-\frac{4096a^5\cos^5(c+dx}{15015d(a\sin(c+dx)+}$$

```
[Out] (-4096*a^5*Cos[c + d*x]^5)/(15015*d*(a + a*Sin[c + d*x])^(5/2)) - (1024*a^4
*Cos[c + d*x]^5)/(3003*d*(a + a*Sin[c + d*x])^(3/2)) - (128*a^3*Cos[c + d*x
]^5)/(429*d*Sqrt[a + a*Sin[c + d*x]]) - (32*a^2*Cos[c + d*x]^5*Sqrt[a + a*S
in[c + d*x]])/(143*d) - (2*a*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(13
*d)
```

Rubi [A] time = 0.293338, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{32a^2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{143d}-\frac{128a^3\cos^5(c+dx)}{429d\sqrt{a\sin(c+dx)+a}}-\frac{1024a^4\cos^5(c+dx)}{3003d(a\sin(c+dx)+a)^{3/2}}-\frac{4096a^5\cos^5(c+dx}{15015d(a\sin(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4096*a^5*Cos[c + d*x]^5)/(15015*d*(a + a*Sin[c + d*x])^(5/2)) - (1024*a^4
*Cos[c + d*x]^5)/(3003*d*(a + a*Sin[c + d*x])^(3/2)) - (128*a^3*Cos[c + d*x
]^5)/(429*d*Sqrt[a + a*Sin[c + d*x]]) - (32*a^2*Cos[c + d*x]^5*Sqrt[a + a*S
in[c + d*x]])/(143*d) - (2*a*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(13
*d)
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= -\frac{2a\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{13d}+\frac{1}{13}(16a)\int \cos^4(c+dx)(a+a\sin(c\\
&= -\frac{32a^2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{143d}-\frac{2a\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{13d}\\
&= -\frac{128a^3\cos^5(c+dx)}{429d\sqrt{a+a\sin(c+dx)}}-\frac{32a^2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{143d}-\frac{2a\cos^5(c\\
&= -\frac{1024a^4\cos^5(c+dx)}{3003d(a+a\sin(c+dx))^{3/2}}-\frac{128a^3\cos^5(c+dx)}{429d\sqrt{a+a\sin(c+dx)}}-\frac{32a^2\cos^5(c+dx)}{143\\
&= -\frac{4096a^5\cos^5(c+dx)}{15015d(a+a\sin(c+dx))^{5/2}}-\frac{1024a^4\cos^5(c+dx)}{3003d(a+a\sin(c+dx))^{3/2}}-\frac{128a^3\cos^5(}{429d\sqrt{a+a\,\mathrm{si}}}
\end{aligned}$$

Mathematica [A] time = 0.309193, size = 79, normalized size = 0.5

$$-\frac{2\left(1155\sin^4(c+dx)+6300\sin^3(c+dx)+14210\sin^2(c+dx)+16700\sin(c+dx)+9683\right)\cos^5(c+dx)(a(\sin(c+dx)}{15015d(\sin(c+dx)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5*(a*(1 + Sin[c + d*x]))^(5/2)*(9683 + 16700*Sin[c + d*x]
+ 14210*Sin[c + d*x]^2 + 6300*Sin[c + d*x]^3 + 1155*Sin[c + d*x]^4))/(15015
*d*(1 + Sin[c + d*x])^5)
```

Maple [A] time = 0.112, size = 87, normalized size = 0.6

$$\frac{(2+2\sin(dx+c))a^3(\sin(dx+c)-1)^3\left(1155(\sin(dx+c))^4+6300(\sin(dx+c))^3+14210(\sin(dx+c))^2+16700\,\mathrm{s}\right.}{15015\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/15015*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)^3*(1155*sin(d*x+c)^4+6300*sin(d*x
+c)^3+14210*sin(d*x+c)^2+16700*sin(d*x+c)+9683)/cos(d*x+c)/(a+a*sin(d*x+c))
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

Fricas [A] time = 1.68916, size = 595, normalized size = 3.74

$$\frac{2\left(1155\,a^2\cos(dx+c)^7-2835\,a^2\cos(dx+c)^6-6230\,a^2\cos(dx+c)^5+320\,a^2\cos(dx+c)^4-512\,a^2\cos(dx+c)^3+102\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*a^2*cos(d*x + c)^7 - 2835*a^2*cos(d*x + c)^6 - 6230*a^2*cos(d
*x + c)^5 + 320*a^2*cos(d*x + c)^4 - 512*a^2*cos(d*x + c)^3 + 1024*a^2*cos(
d*x + c)^2 - 4096*a^2*cos(d*x + c) - 8192*a^2 - (1155*a^2*cos(d*x + c)^6 +
3990*a^2*cos(d*x + c)^5 - 2240*a^2*cos(d*x + c)^4 - 2560*a^2*cos(d*x + c)^3
 - 3072*a^2*cos(d*x + c)^2 - 4096*a^2*cos(d*x + c) - 8192*a^2)*sin(d*x + c)
)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

3.129 $\int \cos^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=49

$$\frac{4(a\sin(c+dx)+a)^{9/2}}{9a^2d}-\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^3d}$$

```
[Out] (4*(a + a*Sin[c + d*x])^(9/2))/(9*a^2*d) - (2*(a + a*Sin[c + d*x])^(11/2))/
(11*a^3*d)
```

Rubi [A] time = 0.0665718, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{4(a\sin(c+dx)+a)^{9/2}}{9a^2d}-\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(9/2))/(9*a^2*d) - (2*(a + a*Sin[c + d*x])^(11/2))/
(11*a^3*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= \frac{\mathrm{Subst}\left(\int (a-x)(a+x)^{7/2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}\\
&= \frac{\mathrm{Subst}\left(\int \left(2a(a+x)^{7/2}-(a+x)^{9/2}\right)\,dx, x, a\sin(c+dx)\right)}{a^3 d}\\
&= \frac{4(a+a\sin(c+dx))^{9/2}}{9a^2 d}-\frac{2(a+a\sin(c+dx))^{11/2}}{11a^3 d}
\end{aligned}
$$

Mathematica [A] time = 0.123333, size = 41, normalized size = 0.84

$$
-\frac{2(\sin(c+dx)+1)^2(9\sin(c+dx)-13)(a(\sin(c+dx)+1))^{5/2}}{99d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(1 + Sin[c + d*x])^2*(a*(1 + Sin[c + d*x]))^(5/2)*(-13 + 9*Sin[c + d*x]
))/(99*d)
```

Maple [A] time = 0.082, size = 31, normalized size = 0.6

$$
-\frac{18\,\sin(dx+c)-26}{99\,a^2 d}\,(a+a\sin(dx+c))^{\frac{9}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/99/a^2*(a+a*sin(d*x+c))^(9/2)*(9*sin(d*x+c)-13)/d
```

Maxima [A] time = 0.953337, size = 51, normalized size = 1.04

$$
-\frac{2\left(9\,(a\sin(dx+c)+a)^{\frac{11}{2}}-22\,(a\sin(dx+c)+a)^{\frac{9}{2}}a\right)}{99\,a^3 d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/99*(9*(a*sin(d*x + c) + a)^(11/2) - 22*(a*sin(d*x + c) + a)^(9/2)*a)/(a^
3*d)
```

Fricas [B] time = 1.77678, size = 217, normalized size = 4.43

$$-\frac{2\left(23\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(dx+c\right)^2-32\,a^2+\left(9\,a^2\cos\left(dx+c\right)^4-20\,a^2\cos\left(dx+c\right)^2-32\,a^2\right)\sin\left(dx+c\right)\right)\sqrt{a\sin}}{99\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/99*(23*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 - 32*a^2 + (9*a^2*cos(d
*x + c)^4 - 20*a^2*cos(d*x + c)^2 - 32*a^2)*sin(d*x + c))*sqrt(a*sin(d*x +
c) + a)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

3.130 $\int \cos^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=127

$$-\frac{256a^4\cos^3(c+dx)}{315d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^3(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{8a^2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{9d}$$

```
[Out] (-256*a^4*Cos[c + d*x]^3)/(315*d*(a + a*Sin[c + d*x])^(3/2)) - (64*a^3*Cos[
c + d*x]^3)/(105*d*Sqrt[a + a*Sin[c + d*x]]) - (8*a^2*Cos[c + d*x]^3*Sqrt[a
 + a*Sin[c + d*x]])/(21*d) - (2*a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2)
)/(9*d)
```

Rubi [A] time = 0.224175, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{256a^4\cos^3(c+dx)}{315d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^3(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{8a^2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^3)/(315*d*(a + a*Sin[c + d*x])^(3/2)) - (64*a^3*Cos[
c + d*x]^3)/(105*d*Sqrt[a + a*Sin[c + d*x]]) - (8*a^2*Cos[c + d*x]^3*Sqrt[a
 + a*Sin[c + d*x]])/(21*d) - (2*a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2)
)/(9*d)
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
```

```
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2a\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{9d}+\frac{1}{3}(4a)\int \cos^2(c+dx)(a+a\sin(c+dx$$
$$= -\frac{8a^2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-\frac{2a\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{9d}+$$
$$= -\frac{64a^3\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+dx)}}-\frac{8a^2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-\frac{2a\cos^3(c+$$
$$= -\frac{256a^4\cos^3(c+dx)}{315d(a+a\sin(c+dx))^{3/2}}-\frac{64a^3\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+dx)}}-\frac{8a^2\cos^3(c+dx)\sqrt{a+}}{21d}$$

Mathematica [A] time = 0.264929, size = 69, normalized size = 0.54

$$-\frac{2\left(35\sin^3(c+dx)+165\sin^2(c+dx)+321\sin(c+dx)+319\right)\cos^3(c+dx)(a(\sin(c+dx)+1))^{5/2}}{315d(\sin(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^3*(a*(1 + Sin[c + d*x]))^(5/2)*(319 + 321*Sin[c + d*x] + 1
65*Sin[c + d*x]^2 + 35*Sin[c + d*x]^3))/(315*d*(1 + Sin[c + d*x])^4)
```

Maple [A] time = 0.106, size = 77, normalized size = 0.6

$$-\frac{(2+2\sin(dx+c))\,a^3\,(\sin(dx+c)-1)^2\left(35\,(\sin(dx+c))^3+165\,(\sin(dx+c))^2+321\,\sin(dx+c)+319\right)}{315\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/315*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)^2*(35*sin(d*x+c)^3+165*sin(d*x+c)^
2+321*sin(d*x+c)+319)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

Fricas [A] time = 1.64077, size = 433, normalized size = 3.41

$$\frac{2\left(35\,a^2\cos(dx+c)^5-95\,a^2\cos(dx+c)^4-226\,a^2\cos(dx+c)^3+32\,a^2\cos(dx+c)^2-128\,a^2\cos(dx+c)-256\,a^2-\left(\right.}{315\,(d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*cos(d*x + c)^5 - 95*a^2*cos(d*x + c)^4 - 226*a^2*cos(d*x + c)
^3 + 32*a^2*cos(d*x + c)^2 - 128*a^2*cos(d*x + c) - 256*a^2 - (35*a^2*cos(d
*x + c)^4 + 130*a^2*cos(d*x + c)^3 - 96*a^2*cos(d*x + c)^2 - 128*a^2*cos(d*
x + c) - 256*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) +
d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

3.131 $\int \cos(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a\sin(c+dx)+a)^{7/2}}{7ad}$$

```
[Out] (2*(a + a*Sin[c + d*x])^(7/2))/(7*a*d)
```

Rubi [A] time = 0.0337825, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$\frac{2(a\sin(c+dx)+a)^{7/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(7/2))/(7*a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{5/2}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{2(a+a\sin(c+dx))^{7/2}}{7ad}\end{aligned}$$

Mathematica [A] time = 0.057443, size = 24, normalized size = 1.

$$\frac{2(a\sin(c+dx)+a)^{7/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(7/2))/(7*a*d)
```

Maple [A] time = 0.007, size = 21, normalized size = 0.9

$$\frac{2}{7\,da}\left(a+a\sin\left(dx+c\right)\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/7*(a+a*sin(d*x+c))^(7/2)/d/a
```

Maxima [A] time = 0.943234, size = 27, normalized size = 1.12

$$\frac{2\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}}{7\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(a*sin(d*x + c) + a)^(7/2)/(a*d)
```

Fricas [B] time = 1.60866, size = 146, normalized size = 6.08

$$-\frac{2\left(3\,a^{2}\cos\left(dx+c\right)^{2}-4\,a^{2}+\left(a^{2}\cos\left(dx+c\right)^{2}-4\,a^{2}\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/7*(3*a^2*cos(d*x + c)^2 - 4*a^2 + (a^2*cos(d*x + c)^2 - 4*a^2)*sin(d*x +
 c))*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

3.132 $\int \sec(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=86

$$-\frac{4a^2\sqrt{a\sin(c+dx)+a}}{d}+\frac{4\sqrt{2}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a(a\sin(c+dx)+a)^{3/2}}{3d}$$

```
[Out] (4*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d -
 (4*a^2*Sqrt[a + a*Sin[c + d*x]])/d - (2*a*(a + a*Sin[c + d*x])^(3/2))/(3*d
)
```

Rubi [A] time = 0.0747303, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2667, 50, 63, 206}

$$-\frac{4a^2\sqrt{a\sin(c+dx)+a}}{d}+\frac{4\sqrt{2}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a(a\sin(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```


```
[Out] (4*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d -
 (4*a^2*Sqrt[a + a*Sin[c + d*x]])/d - (2*a*(a + a*Sin[c + d*x])^(3/2))/(3*d
)
```


Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sec(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= \frac{a\,\text{Subst}\left(\int \frac{(a+x)^{3/2}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= -\frac{2a(a+a\sin(c+dx))^{3/2}}{3d} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{\sqrt{a+x}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= -\frac{4a^2\sqrt{a+a\sin(c+dx)}}{d} - \frac{2a(a+a\sin(c+dx))^{3/2}}{3d} + \frac{\left(4a^3\right)\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4a^2\sqrt{a+a\sin(c+dx)}}{d} - \frac{2a(a+a\sin(c+dx))^{3/2}}{3d} + \frac{\left(8a^3\right)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,\right.}{d} \\
&= \frac{4\sqrt{2}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{4a^2\sqrt{a+a\sin(c+dx)}}{d} - \frac{2a(a+a\sin(c+dx}{3d}
\end{aligned}
$$

Mathematica [A] time = 0.179923, size = 73, normalized size = 0.85

$$
\frac{12\sqrt{2}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{2}\sqrt{a}}\right) - 2a^2(\sin(c+dx)+7)\sqrt{a(\sin(c+dx)+1)}}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (12*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])] -
 2*a^2*Sqrt[a*(1 + Sin[c + d*x])]*(7 + Sin[c + d*x]))/(3*d)
```

Maple [A] time = 0.086, size = 66, normalized size = 0.8

$$-2\,\frac{a}{d}\left(1/3\,\left(a+a\sin\left(dx+c\right)\right)^{3/2}+2\,a\sqrt{a+a\sin\left(dx+c\right)}-2\,a^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a+a\sin\left(dx+c\right)}\sqrt{2}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2*a*(1/3*(a+a*sin(d*x+c))^(3/2)+2*a*(a+a*sin(d*x+c))^(1/2)-2*a^(3/2)*2^(1/
2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72769, size = 235, normalized size = 2.73

$$\frac{2\left(3\,\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{a\sin(dx+c)+2\,\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3\,a}{\sin(dx+c)-1}\right)-\left(a^{2}\sin\left(dx+c\right)+7\,a^{2}\right)\sqrt{a\sin\left(dx+c\right)+a}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(2)*a^(5/2)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c)
 + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - (a^2*sin(d*x + c) + 7*a^2)*sqrt(
```

```
a*sin(d*x + c) + a))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 18.8737, size = 1515, normalized size = 17.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(12*sqrt(2)*(a^3*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c
) - 1)*tan(1/2*c)^2 + a^3*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(
1/2*c) - 1))*arctan(1/2*sqrt(2)*(sqrt((a*tan(1/2*c)^2 + a)*tan(1/2*d*x)^2 +
 a*tan(1/2*c)^2 + a)*a*tan(1/2*c)^3 + sqrt((a*tan(1/2*c)^2 + a)*tan(1/2*d*x
)^2 + a*tan(1/2*c)^2 + a)*a*tan(1/2*c)^2 + 2*sqrt((a*tan(1/2*c)^2 + a)*tan(
1/2*d*x)^2 + a*tan(1/2*c)^2 + a)*a*tan(1/2*c) + 2*sqrt((a*tan(1/2*c)^2 + a)
*tan(1/2*d*x)^2 + a*tan(1/2*c)^2 + a)*a - sqrt(a^3*tan(1/2*c)^8 + 2*a^3*tan
(1/2*c)^7 + 6*a^3*tan(1/2*c)^6 + 10*a^3*tan(1/2*c)^5 + 13*a^3*tan(1/2*c)^4
+ 16*a^3*tan(1/2*c)^3 + 12*a^3*tan(1/2*c)^2 + 8*a^3*tan(1/2*c) + 4*a^3)*tan
(1/2*d*x) + sqrt(a^3*tan(1/2*c)^8 - 2*a^3*tan(1/2*c)^7 + 6*a^3*tan(1/2*c)^6
 - 10*a^3*tan(1/2*c)^5 + 13*a^3*tan(1/2*c)^4 - 16*a^3*tan(1/2*c)^3 + 12*a^3
*tan(1/2*c)^2 - 8*a^3*tan(1/2*c) + 4*a^3))/sqrt(-a^3*tan(1/2*c)^8 - 6*a^3*t
an(1/2*c)^6 - 13*a^3*tan(1/2*c)^4 - 12*a^3*tan(1/2*c)^2 - 4*a^3))/sqrt(-a*t
an(1/2*c)^4 - 2*a*tan(1/2*c)^2 - a) + (7*a^4*sgn(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^3 + 9*a^4*sgn(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^2 + 9*a^4*sgn(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c) + 7*a^4*sgn(tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1) - (9*a^4*sgn(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^3 - 3*a^4*sgn(tan(1
```

```
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^2 + 3*a^4*sg
n(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c) - 9*a
^4*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1) - (9*a^4*sg
n(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^3 + 3
*a^4*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c
)^2 + 3*a^4*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*ta
n(1/2*c) + 9*a^4*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1) - (7*a^4*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*ta
n(1/2*c)^3 - 9*a^4*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
- 1)*tan(1/2*c)^2 + 9*a^4*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(
1/2*c) - 1)*tan(1/2*c) - 7*a^4*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
 tan(1/2*c) - 1))*tan(1/2*d*x))*tan(1/2*d*x))*tan(1/2*d*x))/(a*tan(1/2*d*x)
^2*tan(1/2*c)^2 + a*tan(1/2*d*x)^2 + a*tan(1/2*c)^2 + a)^(3/2))/d
```

3.133 $\int \sec^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=55

$$\frac{8a^2\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{2a\sec(c+dx)(a\sin(c+dx)+a)^{3/2}}{d}$$

```
[Out] (8*a^2*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d - (2*a*Sec[c + d*x]*(a + a*
Sin[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.116714, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$\frac{8a^2\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{2a\sec(c+dx)(a\sin(c+dx)+a)^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (8*a^2*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d - (2*a*Sec[c + d*x]*(a + a*
Sin[c + d*x])^(3/2))/d
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2a\sec(c+dx)(a+a\sin(c+dx))^{3/2}}{d} + (4a)\int \sec^2(c+dx)(a+a\sin(c+dx))^{3/2}$$

$$= \frac{8a^2\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d} - \frac{2a\sec(c+dx)(a+a\sin(c+dx))^{3/2}}{d}$$

Mathematica [A] time = 4.05866, size = 36, normalized size = 0.65

$$-\frac{2a^2(\sin(c+dx)-3)\sec(c+dx)\sqrt{a(\sin(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*Sec[c + d*x]*(-3 + Sin[c + d*x])*Sqrt[a*(1 + Sin[c + d*x])])/d
```

Maple [A] time = 0.089, size = 45, normalized size = 0.8

$$-2\,\frac{a^3\,(1+\sin(dx+c))\,(\sin(dx+c)-3)}{\cos(dx+c)\,\sqrt{a+a\sin(dx+c)}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2*a^3*(1+sin(d*x+c))*(sin(d*x+c)-3)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [B] time = 1.64472, size = 258, normalized size = 4.69

$$-\frac{2\left(3\,a^{\frac{5}{2}} - \frac{2\,a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1} + \frac{9\,a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{4\,a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{9\,a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{2\,a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{3\,a^{\frac{5}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2*(3*a^(5/2) - 2*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 9*a^(5/2)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 4*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 + 9*a^(5/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*a^(5/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 + 3*a^(5/2)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)/(
d*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 1)^(5/2))
```

Fricas [A] time = 1.60578, size = 99, normalized size = 1.8

$$-\frac{2\left(a^{2}\sin\left(dx+c\right)-3\,a^{2}\right)\sqrt{a\sin\left(dx+c\right)+a}}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2*(a^2*sin(d*x + c) - 3*a^2)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.134 $\int \sec^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=69

$$\frac{a\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{d}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}$$

```
[Out] -((a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*d)
) + (a*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.109976, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2676, 2667, 63, 206}

$$\frac{a\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{d}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*d)
) + (a*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/d
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sec^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{d}-\frac{1}{2}a^2\int \sec(c+dx)\sqrt{a+a\sin(c+dx)}\,dx\\
&= \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{d}-\frac{a^3\operatorname{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx,x,a\sin(c+dx)\right)}{2d}\\
&= \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{d}-\frac{a^3\operatorname{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\sqrt{a+a\sin(c+dx)}\right)}{d}\\
&= -\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}+\frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{d}
\end{aligned}
$$

Mathematica [A] time = 0.368171, size = 75, normalized size = 1.09

$$
\frac{a^2\left(-\frac{2\sqrt{a(\sin(c+dx)+1)}}{\sin(c+dx)-1}-\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{2}\sqrt{a}}\right)\right)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(-(Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a]
)]) - (2*Sqrt[a*(1 + Sin[c + d*x])])/(-1 + Sin[c + d*x])))/(2*d)
```

Maple [A] time = 0.107, size = 66, normalized size = 1.

$$-\frac{a^3}{d}\left(\frac{1}{a\sin(dx+c)-a}\sqrt{a+a\sin(dx+c)}+\frac{\sqrt{2}}{2}\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a+a\sin(dx+c)}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -a^3*((a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c)-a)+1/2*2^(1/2)/a^(1/2)*arctanh(1
/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75117, size = 263, normalized size = 3.81

$$\frac{\sqrt{2}\left(a^2\sin(dx+c)-a^2\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\sqrt{a\sin(dx+c)+a}a^2}{4\left(d\sin(dx+c)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(a^2*sin(d*x + c) - a^2)*sqrt(a)*log(-(a*sin(d*x + c) - 2*sqrt
(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*sqrt(a*
sin(d*x + c) + a)*a^2)/(d*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.135 $\int \sec^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=30

$$\frac{2a\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}$$

```
[Out] (2*a*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rubi [A] time = 0.058527, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2673}

$$\frac{2a\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{2a\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d}$$

Mathematica [B] time = 5.14894, size = 69, normalized size = 2.3

$$\frac{2(a(\sin(c+dx)+1))^{5/2}}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(5/2))/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A] time = 0.082, size = 47, normalized size = 1.6

$$-\frac{2\,a^{3}\left(1+\sin\left(dx+c\right)\right)}{\left(3\,\sin\left(dx+c\right)-3\right)\cos\left(dx+c\right)d}\frac{1}{\sqrt{a+a\sin\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3*a^3*(1+sin(d*x+c))/(sin(d*x+c)-1)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [B] time = 1.65006, size = 248, normalized size = 8.27

$$-\frac{2\left(a^{\frac{5}{2}}+\frac{4\,a^{\frac{5}{2}}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,a^{\frac{5}{2}}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4\,a^{\frac{5}{2}}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a^{\frac{5}{2}}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)}{3\,d\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-1\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(a^(5/2) + 4*a^(5/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^(5/2)*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^(5/2)*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 + a^(5/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)/(d*(3*sin(d*x + c)/(
cos(d*x + c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 - 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^(5/2))
```

Fricas [A] time = 1.62536, size = 111, normalized size = 3.7

$$-\frac{2\sqrt{a\sin(dx+c)+a}a^2}{3(d\cos(dx+c)\sin(dx+c)-d\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(a*sin(d*x + c) + a)*a^2/(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x
+ c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.136 $\int \sec^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=103

$$\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+a)^{5/2}}{4d}+\frac{3a\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{16d}$$

```
[Out] (3*a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]
*d) + (3*a*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(16*d) + (Sec[c + d*x
]^4*(a + a*Sin[c + d*x])^(5/2))/(4*d)
```

Rubi [A] time = 0.172129, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2675, 2667, 63, 206}

$$\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+a)^{5/2}}{4d}+\frac{3a\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (3*a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]
*d) + (3*a*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(16*d) + (Sec[c + d*x
]^4*(a + a*Sin[c + d*x])^(5/2))/(4*d)
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= \frac{\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{4d}+\frac{1}{8}(3a)\int \sec^3(c+dx)(a+a\sin(c+dx))^{3}\\
&= \frac{3a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{16d}+\frac{\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{4d}+\\
&= \frac{3a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{16d}+\frac{\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{4d}+\\
&= \frac{3a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{16d}+\frac{\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{4d}+\\
&= \frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d}+\frac{3a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{16d}+\end{aligned}$$

Mathematica [A] time = 0.277936, size = 110, normalized size = 1.07

$$\frac{2a^2(7-3\sin(c+dx))\sqrt{a(\sin(c+dx)+1)}+3\sqrt{2}a^{5/2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\tanh^{-1}\left(\frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{2}\sqrt{a}}\right)}{32d(\sin(c+dx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (3*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])]*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^4 + 2*a^2*(7 - 3*Sin[c + d*x])*Sqrt[a*(
1 + Sin[c + d*x])])/(32*d*(-1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.164, size = 107, normalized size = 1.

$$-2\,{\frac {{a}^{5}}{d} \left(-1/8\,{\frac {\sqrt {a+a\sin \left(dx+c \right) }}{a \left(a\sin \left(dx+c \right) -a \right) ^{2}}}-3/8\,{\frac {1}{a} \left(-1/4\,{\frac {\sqrt {a+a\sin \left(dx+c \right) }}{a \left(a\sin \left(dx+c \right) -a \right) }}+1/8\,{\frac {\sqrt {2}}{{a}^{3/2}}{\rm Artanh} \left(1/2\,{\frac {\sqrt {a+a\sin \left(dx+c \right) }\sqrt {2}}{\sqrt {a}}} \right) } \right) } \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2*a^5*(-1/8*(a+a*sin(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)^2-3/8/a*(-1/4*(a+a*s
in(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)+1/8/a^(3/2)*2^(1/2)*arctanh(1/2*(a+a*si
n(d*x+c))^(1/2)*2^(1/2)/a^(1/2))))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74697, size = 387, normalized size = 3.76

$$\frac{3\left(\sqrt{2}a^{2}\cos\left(dx+c\right)^{2}+2\,\sqrt{2}a^{2}\sin\left(dx+c\right)-2\,\sqrt{2}a^{2}\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\,\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3\,a}{\sin(dx+c)-1}\right)+4\left(3\,a^{2}\sin\left(dx+c\right)-7\right.}{64\left(d\cos\left(dx+c\right)^{2}+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*(sqrt(2)*a^2*cos(d*x + c)^2 + 2*sqrt(2)*a^2*sin(d*x + c) - 2*sqrt(2
)*a^2)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sq
rt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(3*a^2*sin(d*x + c) - 7*a^2)*sqrt(a*si
n(d*x + c) + a))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.137 $\int \sec^6(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=139

$$\frac{a^2\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{2}d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+a)^{5/2}}{5d}+\frac{a\sec^3(c+dx)(a}{}$$

```
[Out] -(a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])
])/(4*Sqrt[2]*d) + (a^2*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(4*d) + (a*S
ec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(6*d) + (Sec[c + d*x]^5*(a + a*Si
n[c + d*x])^(5/2))/(5*d)
```

Rubi [A] time = 0.19727, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2675, 2649, 206}

$$\frac{a^2\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{2}d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+a)^{5/2}}{5d}+\frac{a\sec^3(c+dx)(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])
])/(4*Sqrt[2]*d) + (a^2*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(4*d) + (a*S
ec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(6*d) + (Sec[c + d*x]^5*(a + a*Si
n[c + d*x])^(5/2))/(5*d)
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{\sec^5(c+dx)(a+a\sin(c+dx))^{5/2}}{5d}+\frac{1}{2}a\int \sec^4(c+dx)(a+a\sin(c+dx))^{3/2}\,d$$

$$= \frac{a\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{6d}+\frac{\sec^5(c+dx)(a+a\sin(c+dx))^{5/2}}{5d}+\frac{1}{4}a$$

$$= \frac{a^2\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}+\frac{a\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{6d}+\frac{\sec}{}$$

$$= \frac{a^2\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}+\frac{a\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{6d}+\frac{\sec}{}$$

$$= -\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{2}d}+\frac{a^2\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}+\frac{a\sec^3(c}{}$$

Mathematica [C] time = 5.2959, size = 129, normalized size = 0.93

$$\frac{(a(\sin(c+dx)+1))^{5/2}\left(\frac{-80\sin(c+dx)-15\cos(2(c+dx))+89}{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}+(15+15i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(c+dx)\right)-1\right)\right)\right)}{60d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((15 + 15*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])] + (89 - 15*Cos[2*(c + d*x)] - 80*Sin[c + d*x])/(2*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^5))*(a*(1 + Sin[c + d*x]))^(5/2))/(60*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^5)
```

Maple [A] time = 0.126, size = 120, normalized size = 0.9

$$-\frac{1+\sin(dx+c)}{120\left(\sin(dx+c)-1\right)^{2}\cos(dx+c)\,d}\left(30\,a^{11/2}\left(\cos(dx+c)\right)^{2}+80\,a^{11/2}\sin(dx+c)-104\,a^{11/2}+15\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/120*(1+sin(d*x+c))*(30*a^(11/2)*cos(d*x+c)^2+80*a^(11/2)*sin(d*x+c)-104*
a^(11/2)+15*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3
*(a-a*sin(d*x+c))^(5/2))/a^(5/2)/(sin(d*x+c)-1)^2/cos(d*x+c)/(a+a*sin(d*x+c
))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.80312, size = 709, normalized size = 5.1

$$\frac{15\left(\sqrt{2}a^{2}\cos(dx+c)^{3}+2\,\sqrt{2}a^{2}\cos(dx+c)\sin(dx+c)-2\,\sqrt{2}a^{2}\cos(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(\right.}{\cos(dx+}\right.}{240\left(d\cos(dx+c)^{3}+2\,d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/240*(15*(sqrt(2)*a^2*cos(d*x + c)^3 + 2*sqrt(2)*a^2*cos(d*x + c)*sin(d*x
+ c) - 2*sqrt(2)*a^2*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(
a*sin(d*x + c) + a)*(sqrt(2)*cos(d*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))
*sqrt(a) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(c
```

```
os(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(1
5*a^2*cos(d*x + c)^2 + 40*a^2*sin(d*x + c) - 52*a^2)*sqrt(a*sin(d*x + c) +
a))/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.138 $\int \sec^7(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=159

$$-\frac{35a^3}{128d\sqrt{a\sin(c+dx)+a}}+\frac{35a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}d}+\frac{35a^2\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{192d}+\frac{\sec^6(c+dx)(a\sin(c$$

```
[Out] (35*a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(128*Sqrt[
2]*d) - (35*a^3)/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (35*a^2*Sec[c + d*x]^2*
Sqrt[a + a*Sin[c + d*x]])/(192*d) + (7*a*Sec[c + d*x]^4*(a + a*Sin[c + d*x]
)^(3/2))/(48*d) + (Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2))/(6*d)
```

Rubi [A] time = 0.241828, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2675, 2667, 51, 63, 206}

$$-\frac{35a^3}{128d\sqrt{a\sin(c+dx)+a}}+\frac{35a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}d}+\frac{35a^2\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{192d}+\frac{\sec^6(c+dx)(a\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (35*a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(128*Sqrt[
2]*d) - (35*a^3)/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (35*a^2*Sec[c + d*x]^2*
Sqrt[a + a*Sin[c + d*x]])/(192*d) + (7*a*Sec[c + d*x]^4*(a + a*Sin[c + d*x]
)^(3/2))/(48*d) + (Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2))/(6*d)
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{\sec^6(c+dx)(a+a\sin(c+dx))^{5/2}}{6d} + \frac{1}{12}(7a)\int \sec^5(c+dx)(a+a\sin(c+dx))^{3/}$$
$$= \frac{7a\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{48d} + \frac{\sec^6(c+dx)(a+a\sin(c+dx))^{5/2}}{6d} + \frac{1}{96}$$
$$= \frac{35a^2\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{192d} + \frac{7a\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{48d} +$$
$$= \frac{35a^2\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{192d} + \frac{7a\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{48d} +$$
$$= -\frac{35a^3}{128d\sqrt{a+a\sin(c+dx)}} + \frac{35a^2\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{192d} + 7a\sec^4(c+$$
$$= -\frac{35a^3}{128d\sqrt{a+a\sin(c+dx)}} + \frac{35a^2\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{192d} + 7a\sec^4(c+$$
$$= \frac{35a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}d} - \frac{35a^3}{128d\sqrt{a+a\sin(c+dx)}} + \frac{35a^2\sec^2(c+dx)\sqrt{}}{192}$$

Mathematica [C] time = 0.0922075, size = 44, normalized size = 0.28

$$-\frac{a^3\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{8d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(a^3*Hypergeometric2F1[-1/2, 4, 1/2, (1 + Sin[c + d*x])/2])/(8*d*Sqrt[a +
a*Sin[c + d*x]])
```

Maple [A] time = 0.234, size = 113, normalized size = 0.7

$$2\,\frac{a^7}{d}\left(-1/16\,\frac{1}{a^4}\left(-1/48\,\frac{a^2\sqrt{a+a\sin(dx+c)}\left(57\,(\cos(dx+c))^2+158\,\sin(dx+c)-190\right)}{(a\sin(dx+c)-a)^3} - \frac{35\sqrt{2}}{32\sqrt{a}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a+a\sin}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2*a^7*(-1/16/a^4*(-1/48*a^2*(a+a*sin(d*x+c))^(1/2)*(57*cos(d*x+c)^2+158*sin
(d*x+c)-190)/(a*sin(d*x+c)-a)^3-35/32*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(
d*x+c))^(1/2)*2^(1/2)/a^(1/2)))-1/16/a^4/(a+a*sin(d*x+c))^(1/2))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75733, size = 549, normalized size = 3.45

$$\frac{105\left(\sqrt{2}a^2\cos\left(dx+c\right)^4+2\sqrt{2}a^2\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\sqrt{2}a^2\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{}}{\sin(dx+c)-1}\right.}{1536\left(d\cos\left(dx+c\right)^4+2\,d\cos\left(dx+c\right)^2\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(105*(sqrt(2)*a^2*cos(d*x + c)^4 + 2*sqrt(2)*a^2*cos(d*x + c)^2*sin(
d*x + c) - 2*sqrt(2)*a^2*cos(d*x + c)^2)*sqrt(a)*log(-(a*sin(d*x + c) + 2*s
qrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*(245
*a^2*cos(d*x + c)^2 - 160*a^2 - 7*(15*a^2*cos(d*x + c)^2 - 32*a^2)*sin(d*x
+ c))*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2*sin(
d*x + c) - 2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.139 $\int \cos^7(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=97

$$-\frac{2(a\sin(c+dx)+a)^{21/2}}{21a^7d}+\frac{12(a\sin(c+dx)+a)^{19/2}}{19a^6d}-\frac{24(a\sin(c+dx)+a)^{17/2}}{17a^5d}+\frac{16(a\sin(c+dx)+a)^{15/2}}{15a^4d}$$

```
[Out] (16*(a + a*Sin[c + d*x])^(15/2))/(15*a^4*d) - (24*(a + a*Sin[c + d*x])^(17/
2))/(17*a^5*d) + (12*(a + a*Sin[c + d*x])^(19/2))/(19*a^6*d) - (2*(a + a*Si
n[c + d*x])^(21/2))/(21*a^7*d)
```

Rubi [A] time = 0.079213, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2(a\sin(c+dx)+a)^{21/2}}{21a^7d}+\frac{12(a\sin(c+dx)+a)^{19/2}}{19a^6d}-\frac{24(a\sin(c+dx)+a)^{17/2}}{17a^5d}+\frac{16(a\sin(c+dx)+a)^{15/2}}{15a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(15/2))/(15*a^4*d) - (24*(a + a*Sin[c + d*x])^(17/
2))/(17*a^5*d) + (12*(a + a*Sin[c + d*x])^(19/2))/(19*a^6*d) - (2*(a + a*Si
n[c + d*x])^(21/2))/(21*a^7*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{13/2}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{13/2} - 12a^2(a+x)^{15/2} + 6a(a+x)^{17/2} - (a+x)^{19/2}\right) dx, x, a\right.}{a^7 d}$$
$$= \frac{16(a+a\sin(c+dx))^{15/2}}{15a^4 d} - \frac{24(a+a\sin(c+dx))^{17/2}}{17a^5 d} + \frac{12(a+a\sin(c+dx))^{19/2}}{19a^6 d}$$

Mathematica [A] time = 0.601816, size = 64, normalized size = 0.66

$$-\frac{2a^3(\sin(c+dx)+1)^7\left(1615\sin^3(c+dx) - 5865\sin^2(c+dx) + 7365\sin(c+dx) - 3243\right)\sqrt{a(\sin(c+dx)+1)}}{33915d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[c + d*x])^7*Sqrt[a*(1 + Sin[c + d*x])]*(-3243 + 7365*Sin[c
 + d*x] - 5865*Sin[c + d*x]^2 + 1615*Sin[c + d*x]^3))/(33915*d)
```

Maple [A] time = 0.131, size = 57, normalized size = 0.6

$$\frac{3230\ (\cos(dx+c))^2\sin(dx+c) - 11730\ (\cos(dx+c))^2 - 17960\ \sin(dx+c) + 18216}{33915\,a^4 d}(a+a\sin(dx+c))^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 2/33915/a^4*(a+a*sin(d*x+c))^(15/2)*(1615*cos(d*x+c)^2*sin(d*x+c)-5865*cos(
d*x+c)^2-8980*sin(d*x+c)+9108)/d
```

Maxima [A] time = 0.944939, size = 97, normalized size = 1.

$$-\frac{2\left(1615\,(a\sin(dx+c)+a)^{\frac{21}{2}} - 10710\,(a\sin(dx+c)+a)^{\frac{19}{2}}a + 23940\,(a\sin(dx+c)+a)^{\frac{17}{2}}a^2 - 18088\,(a\sin(dx+c)+a\right.}{33915\,a^7 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/33915*(1615*(a*sin(d*x + c) + a)^(21/2) - 10710*(a*sin(d*x + c) + a)^(19
/2)*a + 23940*(a*sin(d*x + c) + a)^(17/2)*a^2 - 18088*(a*sin(d*x + c) + a)^
(15/2)*a^3)/(a^7*d)
```

Fricas [A] time = 1.9035, size = 410, normalized size = 4.23

$$\frac{2\left(1615\,a^3\cos\left(dx+c\right)^{10} - 8300\,a^3\cos\left(dx+c\right)^8 + 264\,a^3\cos\left(dx+c\right)^6 + 448\,a^3\cos\left(dx+c\right)^4 + 1024\,a^3\cos\left(dx+c\right)^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/33915*(1615*a^3*cos(d*x + c)^10 - 8300*a^3*cos(d*x + c)^8 + 264*a^3*cos(d
*x + c)^6 + 448*a^3*cos(d*x + c)^4 + 1024*a^3*cos(d*x + c)^2 + 8192*a^3 - 8
*(680*a^3*cos(d*x + c)^8 - 429*a^3*cos(d*x + c)^6 - 504*a^3*cos(d*x + c)^4
- 640*a^3*cos(d*x + c)^2 - 1024*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)
/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^7, x)
```

3.140 $\int \cos^6(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=223

$$-\frac{48a^2\cos^7(c+dx)(a\sin(c+dx)+a)^{3/2}}{323d}-\frac{64a^3\cos^7(c+dx)\sqrt{a\sin(c+dx)+a}}{323d}-\frac{1024a^4\cos^7(c+dx)}{4199d\sqrt{a\sin(c+dx)+a}}-\frac{12288}{46189d(}$$

```
[Out] (-131072*a^7*Cos[c + d*x]^7)/(969969*d*(a + a*Sin[c + d*x])^(7/2)) - (32768
*a^6*Cos[c + d*x]^7)/(138567*d*(a + a*Sin[c + d*x])^(5/2)) - (12288*a^5*Cos
[c + d*x]^7)/(46189*d*(a + a*Sin[c + d*x])^(3/2)) - (1024*a^4*Cos[c + d*x]^
7)/(4199*d*Sqrt[a + a*Sin[c + d*x]]) - (64*a^3*Cos[c + d*x]^7*Sqrt[a + a*Si
n[c + d*x]])/(323*d) - (48*a^2*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2))/(
323*d) - (2*a*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(5/2))/(19*d)
```

Rubi [A] time = 0.429509, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{48a^2\cos^7(c+dx)(a\sin(c+dx)+a)^{3/2}}{323d}-\frac{64a^3\cos^7(c+dx)\sqrt{a\sin(c+dx)+a}}{323d}-\frac{1024a^4\cos^7(c+dx)}{4199d\sqrt{a\sin(c+dx)+a}}-\frac{12288}{46189d(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^(7/2),x]
```


```
[Out] (-131072*a^7*Cos[c + d*x]^7)/(969969*d*(a + a*Sin[c + d*x])^(7/2)) - (32768
*a^6*Cos[c + d*x]^7)/(138567*d*(a + a*Sin[c + d*x])^(5/2)) - (12288*a^5*Cos
[c + d*x]^7)/(46189*d*(a + a*Sin[c + d*x])^(3/2)) - (1024*a^4*Cos[c + d*x]^
7)/(4199*d*Sqrt[a + a*Sin[c + d*x]]) - (64*a^3*Cos[c + d*x]^7*Sqrt[a + a*Si
n[c + d*x]])/(323*d) - (48*a^2*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2))/(
323*d) - (2*a*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(5/2))/(19*d)
```


Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = -\frac{2a\cos^7(c+dx)(a+a\sin(c+dx))^{5/2}}{19d}+\frac{1}{19}(24a)\int \cos^6(c+dx)(a+a\sin(c+$$
$$= -\frac{48a^2\cos^7(c+dx)(a+a\sin(c+dx))^{3/2}}{323d}-\frac{2a\cos^7(c+dx)(a+a\sin(c+dx))^{5/}}{19d}$$
$$= -\frac{64a^3\cos^7(c+dx)\sqrt{a+a\sin(c+dx)}}{323d}-\frac{48a^2\cos^7(c+dx)(a+a\sin(c+dx))^{3/}}{323d}$$
$$= -\frac{1024a^4\cos^7(c+dx)}{4199d\sqrt{a+a\sin(c+dx)}}-\frac{64a^3\cos^7(c+dx)\sqrt{a+a\sin(c+dx)}}{323d}-\frac{48a^2\cos^7}{}$$
$$= -\frac{12288a^5\cos^7(c+dx)}{46189d(a+a\sin(c+dx))^{3/2}}-\frac{1024a^4\cos^7(c+dx)}{4199d\sqrt{a+a\sin(c+dx)}}-\frac{64a^3\cos^7(c+dx)}{32}$$
$$= -\frac{32768a^6\cos^7(c+dx)}{138567d(a+a\sin(c+dx))^{5/2}}-\frac{12288a^5\cos^7(c+dx)}{46189d(a+a\sin(c+dx))^{3/2}}-\frac{1024a^4\cos}{4199d\sqrt{a+a}}$$
$$= -\frac{131072a^7\cos^7(c+dx)}{969969d(a+a\sin(c+dx))^{7/2}}-\frac{32768a^6\cos^7(c+dx)}{138567d(a+a\sin(c+dx))^{5/2}}-\frac{12288a^5}{46189d(a+}$$

Mathematica [A] time = 0.548844, size = 102, normalized size = 0.46

$$-\frac{2a^3\left(51051\sin^6(c+dx)+378378\sin^5(c+dx)+1222221\sin^4(c+dx)+2244396\sin^3(c+dx)+2546901\sin^2(c+dx)\right.}{969969d(\sin(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Cos[c + d*x]^7*Sqrt[a*(1 + Sin[c + d*x])]*(646739 + 1778602*Sin[c +
 d*x] + 2546901*Sin[c + d*x]^2 + 2244396*Sin[c + d*x]^3 + 1222221*Sin[c + d
*x]^4 + 378378*Sin[c + d*x]^5 + 51051*Sin[c + d*x]^6))/(969969*d*(1 + Sin[c
 + d*x])^4)
```

Maple [A] time = 0.119, size = 107, normalized size = 0.5

$$-\frac{(2+2\sin(dx+c))\,a^4(\sin(dx+c)-1)^4\left(51051\,(\sin(dx+c))^6+378378\,(\sin(dx+c))^5+1222221\,(\sin(dx+c))^4+2\right.}{969969\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2/969969*(1+sin(d*x+c))*a^4*(sin(d*x+c)-1)^4*(51051*sin(d*x+c)^6+378378*si
n(d*x+c)^5+1222221*sin(d*x+c)^4+2244396*sin(d*x+c)^3+2546901*sin(d*x+c)^2+1
778602*sin(d*x+c)+646739)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^6\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^6, x)
```

Fricas [A] time = 1.76278, size = 859, normalized size = 3.85

$$\frac{2\left(51051\,a^3\cos(dx+c)^{10}+225225\,a^3\cos(dx+c)^9-270270\,a^3\cos(dx+c)^8-562716\,a^3\cos(dx+c)^7+10752\,a^3\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/969969*(51051*a^3*cos(d*x + c)^10 + 225225*a^3*cos(d*x + c)^9 - 270270*a^
3*cos(d*x + c)^8 - 562716*a^3*cos(d*x + c)^7 + 10752*a^3*cos(d*x + c)^6 - 1
4336*a^3*cos(d*x + c)^5 + 20480*a^3*cos(d*x + c)^4 - 32768*a^3*cos(d*x + c)
^3 + 65536*a^3*cos(d*x + c)^2 - 262144*a^3*cos(d*x + c) - 524288*a^3 + (510
```

```
51*a^3*cos(d*x + c)^9 - 174174*a^3*cos(d*x + c)^8 - 444444*a^3*cos(d*x + c)
^7 + 118272*a^3*cos(d*x + c)^6 + 129024*a^3*cos(d*x + c)^5 + 143360*a^3*cos
(d*x + c)^4 + 163840*a^3*cos(d*x + c)^3 + 196608*a^3*cos(d*x + c)^2 + 26214
4*a^3*cos(d*x + c) + 524288*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*
cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^6, x)
```

3.141 $\int \cos^5(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=73

$$\frac{2(a\sin(c+dx)+a)^{17/2}}{17a^5d}-\frac{8(a\sin(c+dx)+a)^{15/2}}{15a^4d}+\frac{8(a\sin(c+dx)+a)^{13/2}}{13a^3d}$$

```
[Out] (8*(a + a*Sin[c + d*x])^(13/2))/(13*a^3*d) - (8*(a + a*Sin[c + d*x])^(15/2)
)/(15*a^4*d) + (2*(a + a*Sin[c + d*x])^(17/2))/(17*a^5*d)
```

Rubi [A] time = 0.0746197, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{17/2}}{17a^5d}-\frac{8(a\sin(c+dx)+a)^{15/2}}{15a^4d}+\frac{8(a\sin(c+dx)+a)^{13/2}}{13a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(13/2))/(13*a^3*d) - (8*(a + a*Sin[c + d*x])^(15/2)
)/(15*a^4*d) + (2*(a + a*Sin[c + d*x])^(17/2))/(17*a^5*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\operatorname{Subst}\left(\int (a-x)^2(a+x)^{11/2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(4a^2(a+x)^{11/2}-4a(a+x)^{13/2}+(a+x)^{15/2}\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{8(a+a\sin(c+dx))^{13/2}}{13a^3d} - \frac{8(a+a\sin(c+dx))^{15/2}}{15a^4d} + \frac{2(a+a\sin(c+dx))^{17/2}}{17a^5d}$$

Mathematica [A] time = 0.269581, size = 54, normalized size = 0.74

$$\frac{2a^3(\sin(c+dx)+1)^6\left(195\sin^2(c+dx)-494\sin(c+dx)+331\right)\sqrt{a(\sin(c+dx)+1)}}{3315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*(1 + Sin[c + d*x])^6*Sqrt[a*(1 + Sin[c + d*x])]*(331 - 494*Sin[c + d
*x] + 195*Sin[c + d*x]^2))/(3315*d)
```

Maple [A] time = 0.09, size = 41, normalized size = 0.6

$$-\frac{390\,(\cos(dx+c))^2+988\,\sin(dx+c)-1052}{3315\,a^3d}(a+a\sin(dx+c))^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2/3315/a^3*(a+a*sin(d*x+c))^(13/2)*(195*cos(d*x+c)^2+494*sin(d*x+c)-526)/d
```

Maxima [A] time = 0.954958, size = 74, normalized size = 1.01

$$\frac{2\left(195\,(a\sin(dx+c)+a)^{\frac{17}{2}}-884\,(a\sin(dx+c)+a)^{\frac{15}{2}}a+1020\,(a\sin(dx+c)+a)^{\frac{13}{2}}a^2\right)}{3315\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3315*(195*(a*sin(d*x + c) + a)^(17/2) - 884*(a*sin(d*x + c) + a)^(15/2)*a
 + 1020*(a*sin(d*x + c) + a)^(13/2)*a^2)/(a^5*d)
```

Fricas [B] time = 1.67033, size = 335, normalized size = 4.59

$$\frac{2\left(195\,a^3\cos\left(dx+c\right)^8-1072\,a^3\cos\left(dx+c\right)^6+56\,a^3\cos\left(dx+c\right)^4+128\,a^3\cos\left(dx+c\right)^2+1024\,a^3-4\left(169\,a^3\cos\left(dx\right.\right.}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*a^3*cos(d*x + c)^8 - 1072*a^3*cos(d*x + c)^6 + 56*a^3*cos(d*x +
 c)^4 + 128*a^3*cos(d*x + c)^2 + 1024*a^3 - 4*(169*a^3*cos(d*x + c)^6 - 126
*a^3*cos(d*x + c)^4 - 160*a^3*cos(d*x + c)^2 - 256*a^3)*sin(d*x + c))*sqrt(
a*sin(d*x + c) + a)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}\cos\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^5, x)
```

3.142 $\int \cos^4(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=191

$$-\frac{16384a^6\cos^5(c+dx)}{45045d(a\sin(c+dx)+a)^{5/2}}-\frac{4096a^5\cos^5(c+dx)}{9009d(a\sin(c+dx)+a)^{3/2}}-\frac{512a^4\cos^5(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{128a^3\cos^5(c+dx)\sqrt{a\sin(c}}{429d}$$

```
[Out] (-16384*a^6*Cos[c + d*x]^5)/(45045*d*(a + a*Sin[c + d*x])^(5/2)) - (4096*a^
5*Cos[c + d*x]^5)/(9009*d*(a + a*Sin[c + d*x])^(3/2)) - (512*a^4*Cos[c + d*
x]^5)/(1287*d*Sqrt[a + a*Sin[c + d*x]]) - (128*a^3*Cos[c + d*x]^5*Sqrt[a +
a*Sin[c + d*x]])/(429*d) - (8*a^2*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2)
)/(39*d) - (2*a*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2))/(15*d)
```

Rubi [A] time = 0.365284, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{16384a^6\cos^5(c+dx)}{45045d(a\sin(c+dx)+a)^{5/2}}-\frac{4096a^5\cos^5(c+dx)}{9009d(a\sin(c+dx)+a)^{3/2}}-\frac{512a^4\cos^5(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{128a^3\cos^5(c+dx)\sqrt{a\sin(c}}{429d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(7/2),x]
```


```
[Out] (-16384*a^6*Cos[c + d*x]^5)/(45045*d*(a + a*Sin[c + d*x])^(5/2)) - (4096*a^
5*Cos[c + d*x]^5)/(9009*d*(a + a*Sin[c + d*x])^(3/2)) - (512*a^4*Cos[c + d*
x]^5)/(1287*d*Sqrt[a + a*Sin[c + d*x]]) - (128*a^3*Cos[c + d*x]^5*Sqrt[a +
a*Sin[c + d*x]])/(429*d) - (8*a^2*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2)
)/(39*d) - (2*a*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2))/(15*d)
```


Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= -\frac{2a\cos^5(c+dx)(a+a\sin(c+dx))^{5/2}}{15d}+\frac{1}{3}(4a)\int \cos^4(c+dx)(a+a\sin(c+dx)\\ &= -\frac{8a^2\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{39d}-\frac{2a\cos^5(c+dx)(a+a\sin(c+dx))^{5/2}}{15d}\\ &= -\frac{128a^3\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{429d}-\frac{8a^2\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{39d}\\ &= -\frac{512a^4\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}}-\frac{128a^3\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{429d}-\frac{8a^2\cos^5}{}\\ &= -\frac{4096a^5\cos^5(c+dx)}{9009d(a+a\sin(c+dx))^{3/2}}-\frac{512a^4\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}}-\frac{128a^3\cos^5(c+dx)}{429}\\ &= -\frac{16384a^6\cos^5(c+dx)}{45045d(a+a\sin(c+dx))^{5/2}}-\frac{4096a^5\cos^5(c+dx)}{9009d(a+a\sin(c+dx))^{3/2}}-\frac{512a^4\cos^5(c}{1287d\sqrt{a+a\sin}}\end{aligned}$$

Mathematica [A] time = 0.23296, size = 92, normalized size = 0.48

$$-\frac{2a^3\left(3003\sin^5(c+dx)+19635\sin^4(c+dx)+55230\sin^3(c+dx)+86870\sin^2(c+dx)+81815\sin(c+dx)+41735\right)\mathrm{c}}{45045d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Cos[c + d*x]^5*Sqrt[a*(1 + Sin[c + d*x])]*(41735 + 81815*Sin[c + d*
x] + 86870*Sin[c + d*x]^2 + 55230*Sin[c + d*x]^3 + 19635*Sin[c + d*x]^4 + 3
003*Sin[c + d*x]^5))/(45045*d*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.116, size = 97, normalized size = 0.5

$$\frac{(2+2\sin(dx+c))\,a^4(\sin(dx+c)-1)^3\left(3003\,(\sin(dx+c))^5+19635\,(\sin(dx+c))^4+55230\,(\sin(dx+c))^3+86870\,(\right.}{45045\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 2/45045*(1+sin(d*x+c))*a^4*(sin(d*x+c)-1)^3*(3003*sin(d*x+c)^5+19635*sin(d*
x+c)^4+55230*sin(d*x+c)^3+86870*sin(d*x+c)^2+81815*sin(d*x+c)+41735)/cos(d*
x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^4, x)
```

Fricas [A] time = 1.69027, size = 683, normalized size = 3.58

$$\frac{2\left(3003\,a^3\cos(dx+c)^8+13629\,a^3\cos(dx+c)^7-17346\,a^3\cos(dx+c)^6-36932\,a^3\cos(dx+c)^5+1280\,a^3\cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*a^3*cos(d*x + c)^8 + 13629*a^3*cos(d*x + c)^7 - 17346*a^3*cos
(d*x + c)^6 - 36932*a^3*cos(d*x + c)^5 + 1280*a^3*cos(d*x + c)^4 - 2048*a^3
*cos(d*x + c)^3 + 4096*a^3*cos(d*x + c)^2 - 16384*a^3*cos(d*x + c) - 32768*
a^3 + (3003*a^3*cos(d*x + c)^7 - 10626*a^3*cos(d*x + c)^6 - 27972*a^3*cos(d
*x + c)^5 + 8960*a^3*cos(d*x + c)^4 + 10240*a^3*cos(d*x + c)^3 + 12288*a^3*
cos(d*x + c)^2 + 16384*a^3*cos(d*x + c) + 32768*a^3)*sin(d*x + c))*sqrt(a*s
in(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^4, x)
```

3.143 $\int \cos^3(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=49

$$\frac{4(a\sin(c+dx)+a)^{11/2}}{11a^2d}-\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^3d}$$

```
[Out] (4*(a + a*Sin[c + d*x])^(11/2))/(11*a^2*d) - (2*(a + a*Sin[c + d*x])^(13/2)
)/(13*a^3*d)
```

Rubi [A] time = 0.0661054, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{4(a\sin(c+dx)+a)^{11/2}}{11a^2d}-\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(11/2))/(11*a^2*d) - (2*(a + a*Sin[c + d*x])^(13/2)
)/(13*a^3*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^{9/2}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(a+x)^{9/2}-(a+x)^{11/2}\right)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{4(a+a\sin(c+dx))^{11/2}}{11a^2d} - \frac{2(a+a\sin(c+dx))^{13/2}}{13a^3d}$$

Mathematica [A] time = 0.133037, size = 44, normalized size = 0.9

$$\frac{2\left(26a(a\sin(c+dx)+a)^{11/2}-11(a\sin(c+dx)+a)^{13/2}\right)}{143a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*(26*a*(a + a*Sin[c + d*x])^(11/2) - 11*(a + a*Sin[c + d*x])^(13/2)))/(14
3*a^3*d)
```

Maple [A] time = 0.082, size = 31, normalized size = 0.6

$$-\frac{22\,\sin(dx+c)-30}{143\,a^2d}\,(a+a\sin(dx+c))^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2/143/a^2*(a+a*sin(d*x+c))^(11/2)*(11*sin(d*x+c)-15)/d
```

Maxima [A] time = 0.9574, size = 51, normalized size = 1.04

$$-\frac{2\left(11\,(a\sin(dx+c)+a)^{\frac{13}{2}}-26\,(a\sin(dx+c)+a)^{\frac{11}{2}}a\right)}{143\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/143*(11*(a*sin(d*x + c) + a)^(13/2) - 26*(a*sin(d*x + c) + a)^(11/2)*a)/
(a^3*d)
```

Fricas [B] time = 1.65446, size = 250, normalized size = 5.1

$$\frac{2\left(11\,a^3\cos\left(dx+c\right)^6-68\,a^3\cos\left(dx+c\right)^4+8\,a^3\cos\left(dx+c\right)^2+64\,a^3-8\left(5\,a^3\cos\left(dx+c\right)^4-5\,a^3\cos\left(dx+c\right)^2-8\,a^3\right)\right.}{143\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/143*(11*a^3*cos(d*x + c)^6 - 68*a^3*cos(d*x + c)^4 + 8*a^3*cos(d*x + c)^2
 + 64*a^3 - 8*(5*a^3*cos(d*x + c)^4 - 5*a^3*cos(d*x + c)^2 - 8*a^3)*sin(d*x
 + c))*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^3, x)
```

3.144 $\int \cos^2(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=159

$$-\frac{4096a^5\cos^3(c+dx)}{3465d(a\sin(c+dx)+a)^{3/2}}-\frac{1024a^4\cos^3(c+dx)}{1155d\sqrt{a\sin(c+dx)+a}}-\frac{128a^3\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{231d}-\frac{32a^2\cos^3(c+dx)(a\,\mathrm{s}}{99d}$$

```
[Out] (-4096*a^5*Cos[c + d*x]^3)/(3465*d*(a + a*Sin[c + d*x])^(3/2)) - (1024*a^4*
Cos[c + d*x]^3)/(1155*d*Sqrt[a + a*Sin[c + d*x]]) - (128*a^3*Cos[c + d*x]^3
*Sqrt[a + a*Sin[c + d*x]])/(231*d) - (32*a^2*Cos[c + d*x]^3*(a + a*Sin[c +
d*x])^(3/2))/(99*d) - (2*a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2))/(11*d
)
```

Rubi [A] time = 0.2921, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{4096a^5\cos^3(c+dx)}{3465d(a\sin(c+dx)+a)^{3/2}}-\frac{1024a^4\cos^3(c+dx)}{1155d\sqrt{a\sin(c+dx)+a}}-\frac{128a^3\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{231d}-\frac{32a^2\cos^3(c+dx)(a\,\mathrm{s}}{99d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(7/2),x]
```


```
[Out] (-4096*a^5*Cos[c + d*x]^3)/(3465*d*(a + a*Sin[c + d*x])^(3/2)) - (1024*a^4*
Cos[c + d*x]^3)/(1155*d*Sqrt[a + a*Sin[c + d*x]]) - (128*a^3*Cos[c + d*x]^3
*Sqrt[a + a*Sin[c + d*x]])/(231*d) - (32*a^2*Cos[c + d*x]^3*(a + a*Sin[c +
d*x])^(3/2))/(99*d) - (2*a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2))/(11*d
)
```


Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= -\frac{2a\cos^3(c+dx)(a+a\sin(c+dx))^{5/2}}{11d}+\frac{1}{11}(16a)\int \cos^2(c+dx)(a+a\sin(c\\ &= -\frac{32a^2\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{99d}-\frac{2a\cos^3(c+dx)(a+a\sin(c+dx))}{11d}\\ &= -\frac{128a^3\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{231d}-\frac{32a^2\cos^3(c+dx)(a+a\sin(c+dx)}{99d}\\ &= -\frac{1024a^4\cos^3(c+dx)}{1155d\sqrt{a+a\sin(c+dx)}}-\frac{128a^3\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{231d}-\frac{32a^2\mathrm{co}}{}\\ &= -\frac{4096a^5\cos^3(c+dx)}{3465d(a+a\sin(c+dx))^{3/2}}-\frac{1024a^4\cos^3(c+dx)}{1155d\sqrt{a+a\sin(c+dx)}}-\frac{128a^3\cos^3(c+dx}{2}\end{aligned}$$

Mathematica [A] time = 0.204262, size = 82, normalized size = 0.52

$$-\frac{2a^3\left(315\sin^4(c+dx)+1820\sin^3(c+dx)+4530\sin^2(c+dx)+6396\sin(c+dx)+5419\right)\cos^3(c+dx)\sqrt{a(\sin(c+dx)}}{3465d(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Cos[c + d*x]^3*Sqrt[a*(1 + Sin[c + d*x])]*(5419 + 6396*Sin[c + d*x]
 + 4530*Sin[c + d*x]^2 + 1820*Sin[c + d*x]^3 + 315*Sin[c + d*x]^4))/(3465*d
*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.11, size = 87, normalized size = 0.6

$$-\frac{(2+2\sin(dx+c))\,a^4\,(\sin(dx+c)-1)^2\left(315\,(\sin(dx+c))^4+1820\,(\sin(dx+c))^3+4530\,(\sin(dx+c))^2+6396\,\sin\right.}{3465\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2/3465*(1+sin(d*x+c))*a^4*(sin(d*x+c)-1)^2*(315*sin(d*x+c)^4+1820*sin(d*x+
c)^3+4530*sin(d*x+c)^2+6396*sin(d*x+c)+5419)/cos(d*x+c)/(a+a*sin(d*x+c))^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^2, x)
```

Fricas [A] time = 1.65499, size = 522, normalized size = 3.28

$$\frac{2\left(315\,a^3\cos(dx+c)^6+1505\,a^3\cos(dx+c)^5-2150\,a^3\cos(dx+c)^4-4876\,a^3\cos(dx+c)^3+512\,a^3\cos(dx+c)^2-204\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*a^3*cos(d*x + c)^6 + 1505*a^3*cos(d*x + c)^5 - 2150*a^3*cos(d*x
 + c)^4 - 4876*a^3*cos(d*x + c)^3 + 512*a^3*cos(d*x + c)^2 - 2048*a^3*cos(d
*x + c) - 4096*a^3 + (315*a^3*cos(d*x + c)^5 - 1190*a^3*cos(d*x + c)^4 - 33
40*a^3*cos(d*x + c)^3 + 1536*a^3*cos(d*x + c)^2 + 2048*a^3*cos(d*x + c) + 4
096*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x
 + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^2, x)
```

3.145 $\int \cos(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a\sin(c+dx)+a)^{9/2}}{9ad}$$

```
[Out] (2*(a + a*Sin[c + d*x])^(9/2))/(9*a*d)
```

Rubi [A] time = 0.0332088, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$\frac{2(a\sin(c+dx)+a)^{9/2}}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(9/2))/(9*a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{7/2}\,dx, x, a\sin(c+dx)\right)}{ad}\\
&= \frac{2(a+a\sin(c+dx))^{9/2}}{9ad}
\end{aligned}$$

Mathematica [A] time = 0.0805537, size = 24, normalized size = 1.

$$\frac{2(a\sin(c+dx)+a)^{9/2}}{9ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(9/2))/(9*a*d)
```

Maple [A] time = 0.007, size = 21, normalized size = 0.9

$$\frac{2}{9\,da}\left(a+a\sin\left(dx+c\right)\right)^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 2/9*(a+a*sin(d*x+c))^(9/2)/d/a
```

Maxima [A] time = 0.950317, size = 27, normalized size = 1.12

$$\frac{2\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/9*(a*sin(d*x + c) + a)^(9/2)/(a*d)
```

Fricas [B] time = 1.63613, size = 176, normalized size = 7.33

$$\frac{2\left(a^{3}\cos\left(dx+c\right)^{4}-8\,a^{3}\cos\left(dx+c\right)^{2}+8\,a^{3}-4\left(a^{3}\cos\left(dx+c\right)^{2}-2\,a^{3}\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/9*(a^3*cos(d*x + c)^4 - 8*a^3*cos(d*x + c)^2 + 8*a^3 - 4*(a^3*cos(d*x + c
)^2 - 2*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c), x)
```

3.146 $\int \sec(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=110

$$-\frac{8a^3\sqrt{a\sin(c+dx)+a}}{d}-\frac{4a^2(a\sin(c+dx)+a)^{3/2}}{3d}+\frac{8\sqrt{2}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a(a\sin(c+dx)+a)^{5/2}}{5d}$$

```
[Out] (8*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d -
 (8*a^3*Sqrt[a + a*Sin[c + d*x]])/d - (4*a^2*(a + a*Sin[c + d*x])^(3/2))/(3
*d) - (2*a*(a + a*Sin[c + d*x])^(5/2))/(5*d)
```

Rubi [A] time = 0.0852477, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2667, 50, 63, 206}

$$-\frac{8a^3\sqrt{a\sin(c+dx)+a}}{d}-\frac{4a^2(a\sin(c+dx)+a)^{3/2}}{3d}+\frac{8\sqrt{2}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a(a\sin(c+dx)+a)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^(7/2),x]
```


```
[Out] (8*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d -
 (8*a^3*Sqrt[a + a*Sin[c + d*x]])/d - (4*a^2*(a + a*Sin[c + d*x])^(3/2))/(3
*d) - (2*a*(a + a*Sin[c + d*x])^(5/2))/(5*d)
```


Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sec(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{a\,\text{Subst}\left(\int \frac{(a+x)^{5/2}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= -\frac{2a(a+a\sin(c+dx))^{5/2}}{5d} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{(a+x)^{3/2}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= -\frac{4a^2(a+a\sin(c+dx))^{3/2}}{3d} - \frac{2a(a+a\sin(c+dx))^{5/2}}{5d} + \frac{\left(4a^3\right)\text{Subst}\left(\int \frac{\sqrt{a+x}}{a-x}\,dx,\right.}{d} \\
&= -\frac{8a^3\sqrt{a+a\sin(c+dx)}}{d} - \frac{4a^2(a+a\sin(c+dx))^{3/2}}{3d} - \frac{2a(a+a\sin(c+dx))^{5/2}}{5d} + \\
&= -\frac{8a^3\sqrt{a+a\sin(c+dx)}}{d} - \frac{4a^2(a+a\sin(c+dx))^{3/2}}{3d} - \frac{2a(a+a\sin(c+dx))^{5/2}}{5d} + \\
&= \frac{8\sqrt{2}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{8a^3\sqrt{a+a\sin(c+dx)}}{d} - \frac{4a^2(a+a\sin(c+dx}{3d}
\end{aligned}
$$

Mathematica [A] time = 0.31554, size = 85, normalized size = 0.77

$$
\frac{120\sqrt{2}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{2}\sqrt{a}}\right) - 2a^3\left(3\sin^2(c+dx)+16\sin(c+dx)+73\right)\sqrt{a(\sin(c+dx)+1)}}{15d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (120*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])]
- 2*a^3*Sqrt[a*(1 + Sin[c + d*x])]*(73 + 16*Sin[c + d*x] + 3*Sin[c + d*x]^2
))/(15*d)
```

Maple [A] time = 0.096, size = 83, normalized size = 0.8

$$-2\,{\frac {a}{d}\left(1/5\, \left(a+a\sin \left(dx+c \right) \right) ^{5/2}+2/3\, \left(a+a\sin \left(dx+c \right) \right) ^{3/2}a+4\,{a}^{2}\sqrt {a+a\sin \left(dx+c \right) }-4\,{a}^{5/2}\sqrt {2}{\rm Artanh} \left(1/2\,{\frac {\sqrt {a+a\sin \left(dx+c \right) }\sqrt {2}}{\sqrt {a}}}\right) \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2*a*(1/5*(a+a*sin(d*x+c))^(5/2)+2/3*(a+a*sin(d*x+c))^(3/2)*a+4*a^2*(a+a*si
n(d*x+c))^(1/2)-4*a^(5/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2
)/a^(1/2)))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66972, size = 274, normalized size = 2.49

$$\frac{2\,{\left(30\,\sqrt{2}a^{\frac{7}{2}}\log\left(-\frac{a\sin\left(dx+c\right)+2\,\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}+3\,a}{\sin\left(dx+c\right)-1}\right)+{\left(3\,a^{3}\cos\left(dx+c\right)^{2}-16\,a^{3}\sin\left(dx+c\right)-76\,a^{3}\right)}\sqrt{a\sin\left(dx+c\right)+a}\right)}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(30*sqrt(2)*a^(7/2)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x +
c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + (3*a^3*cos(d*x + c)^2 - 16*a^3
*sin(d*x + c) - 76*a^3)*sqrt(a*sin(d*x + c) + a))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 42.4063, size = 3368, normalized size = 30.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/60*(960*sqrt(2)*(a^4*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2
*c) - 1)*tan(1/2*c)^2 + a^4*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - ta
n(1/2*c) - 1))*arctan(1/2*sqrt(2)*(sqrt((a*tan(1/2*c)^2 + a)*tan(1/2*d*x)^2
 + a*tan(1/2*c)^2 + a)*a*tan(1/2*c)^3 + sqrt((a*tan(1/2*c)^2 + a)*tan(1/2*d
*x)^2 + a*tan(1/2*c)^2 + a)*a*tan(1/2*c)^2 + 2*sqrt((a*tan(1/2*c)^2 + a)*ta
n(1/2*d*x)^2 + a*tan(1/2*c)^2 + a)*a*tan(1/2*c) + 2*sqrt((a*tan(1/2*c)^2 +
a)*tan(1/2*d*x)^2 + a*tan(1/2*c)^2 + a)*a + sqrt(a^3*tan(1/2*c)^8 + 2*a^3*t
an(1/2*c)^7 + 6*a^3*tan(1/2*c)^6 + 10*a^3*tan(1/2*c)^5 + 13*a^3*tan(1/2*c)^
4 + 16*a^3*tan(1/2*c)^3 + 12*a^3*tan(1/2*c)^2 + 8*a^3*tan(1/2*c) + 4*a^3)*t
an(1/2*d*x) + sqrt(a^3*tan(1/2*c)^8 - 2*a^3*tan(1/2*c)^7 + 6*a^3*tan(1/2*c)
^6 - 10*a^3*tan(1/2*c)^5 + 13*a^3*tan(1/2*c)^4 - 16*a^3*tan(1/2*c)^3 + 12*a
^3*tan(1/2*c)^2 - 8*a^3*tan(1/2*c) + 4*a^3))/sqrt(-a^3*tan(1/2*c)^8 - 6*a^3
*tan(1/2*c)^6 - 13*a^3*tan(1/2*c)^4 - 12*a^3*tan(1/2*c)^2 - 4*a^3))/sqrt(-a
*tan(1/2*c)^4 - 2*a*tan(1/2*c)^2 - a) - ((((((73*a^6*sgn(tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^5 - 105*a^6*sgn(tan(1/2*d
```

*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^4 + 190*a^6*sgn(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^3 - 190
*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c
)^2 + 105*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*
tan(1/2*c) - 73*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
 - 1))*tan(1/2*d*x)/(a^9*tan(1/2*c)^18 + 9*a^9*tan(1/2*c)^16 + 36*a^9*tan(1
/2*c)^14 + 84*a^9*tan(1/2*c)^12 + 126*a^9*tan(1/2*c)^10 + 126*a^9*tan(1/2*c
)^8 + 84*a^9*tan(1/2*c)^6 + 36*a^9*tan(1/2*c)^4 + 9*a^9*tan(1/2*c)^2 + a^9)
 - 15*(7*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*t
an(1/2*c)^5 - a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
 1)*tan(1/2*c)^4 + 10*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(
1/2*c) - 1)*tan(1/2*c)^3 + 10*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) - tan(1/2*c) - 1)*tan(1/2*c)^2 - a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) - tan(1/2*c) - 1)*tan(1/2*c) + 7*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) - tan(1/2*c) - 1))/(a^9*tan(1/2*c)^18 + 9*a^9*tan(1/2*c)^16 + 3
6*a^9*tan(1/2*c)^14 + 84*a^9*tan(1/2*c)^12 + 126*a^9*tan(1/2*c)^10 + 126*a^
9*tan(1/2*c)^8 + 84*a^9*tan(1/2*c)^6 + 36*a^9*tan(1/2*c)^4 + 9*a^9*tan(1/2*
c)^2 + a^9))*tan(1/2*d*x) + 10*(19*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^5 - 15*a^6*sgn(tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^4 + 22*a^6*sgn(tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^3 - 22*a^6*sgn(tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^2 + 15*a^6*sgn(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c) - 19*a^
6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))/(a^9*tan(1/
2*c)^18 + 9*a^9*tan(1/2*c)^16 + 36*a^9*tan(1/2*c)^14 + 84*a^9*tan(1/2*c)^12
 + 126*a^9*tan(1/2*c)^10 + 126*a^9*tan(1/2*c)^8 + 84*a^9*tan(1/2*c)^6 + 36*
a^9*tan(1/2*c)^4 + 9*a^9*tan(1/2*c)^2 + a^9))*tan(1/2*d*x) - 10*(19*a^6*sgn
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^5 + 15
*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c
)^4 + 22*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*t
an(1/2*c)^3 + 22*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c
) - 1)*tan(1/2*c)^2 + 15*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - t
an(1/2*c) - 1)*tan(1/2*c) + 19*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) - tan(1/2*c) - 1))/(a^9*tan(1/2*c)^18 + 9*a^9*tan(1/2*c)^16 + 36*a^9*tan
(1/2*c)^14 + 84*a^9*tan(1/2*c)^12 + 126*a^9*tan(1/2*c)^10 + 126*a^9*tan(1/2
*c)^8 + 84*a^9*tan(1/2*c)^6 + 36*a^9*tan(1/2*c)^4 + 9*a^9*tan(1/2*c)^2 + a^
9))*tan(1/2*d*x) + 15*(7*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - t
an(1/2*c) - 1)*tan(1/2*c)^5 + a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) - tan(1/2*c) - 1)*tan(1/2*c)^4 + 10*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan
(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^3 - 10*a^6*sgn(tan(1/2*d*x)*tan(1/2*
c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^2 - a^6*sgn(tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c) - 7*a^6*sgn(tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))/(a^9*tan(1/2*c)^18 + 9*a^9*t
an(1/2*c)^16 + 36*a^9*tan(1/2*c)^14 + 84*a^9*tan(1/2*c)^12 + 126*a^9*tan(1/
2*c)^10 + 126*a^9*tan(1/2*c)^8 + 84*a^9*tan(1/2*c)^6 + 36*a^9*tan(1/2*c)^4

```
+ 9*a^9*tan(1/2*c)^2 + a^9))*tan(1/2*d*x) - (73*a^6*sgn(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^5 + 105*a^6*sgn(tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^4 + 190*a^6*sgn(t
an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)^3 + 190*
a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*tan(1/2*c)
^2 + 105*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)*t
an(1/2*c) + 73*a^6*sgn(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
- 1))/(a^9*tan(1/2*c)^18 + 9*a^9*tan(1/2*c)^16 + 36*a^9*tan(1/2*c)^14 + 84*
a^9*tan(1/2*c)^12 + 126*a^9*tan(1/2*c)^10 + 126*a^9*tan(1/2*c)^8 + 84*a^9*t
an(1/2*c)^6 + 36*a^9*tan(1/2*c)^4 + 9*a^9*tan(1/2*c)^2 + a^9))/(a*tan(1/2*d
*x)^2*tan(1/2*c)^2 + a*tan(1/2*d*x)^2 + a*tan(1/2*c)^2 + a)^(5/2))/d
```

3.147 $\int \sec^2(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=89

$$\frac{64a^3\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{16a^2\sec(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}-\frac{2a\sec(c+dx)(a\sin(c+dx)+a)^{5/2}}{3d}$$

```
[Out] (64*a^3*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*d) - (16*a^2*Sec[c + d*x]
*(a + a*Sin[c + d*x])^(3/2))/(3*d) - (2*a*Sec[c + d*x]*(a + a*Sin[c + d*x])
^(5/2))/(3*d)
```

Rubi [A] time = 0.173659, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$\frac{64a^3\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{16a^2\sec(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}-\frac{2a\sec(c+dx)(a\sin(c+dx)+a)^{5/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (64*a^3*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*d) - (16*a^2*Sec[c + d*x]
*(a + a*Sin[c + d*x])^(3/2))/(3*d) - (2*a*Sec[c + d*x]*(a + a*Sin[c + d*x])
^(5/2))/(3*d)
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = -\frac{2a\sec(c+dx)(a+a\sin(c+dx))^{5/2}}{3d} + \frac{1}{3}(8a)\int \sec^2(c+dx)(a+a\sin(c+dx)$$
$$= -\frac{16a^2\sec(c+dx)(a+a\sin(c+dx))^{3/2}}{3d} - \frac{2a\sec(c+dx)(a+a\sin(c+dx))^{5/2}}{3d} +$$
$$= \frac{64a^3\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{16a^2\sec(c+dx)(a+a\sin(c+dx))^{3/2}}{3d} -$$

Mathematica [A] time = 5.47737, size = 48, normalized size = 0.54

$$\frac{a^3\sec(c+dx)\sqrt{a(\sin(c+dx)+1)}(-20\sin(c+dx)+\cos(2(c+dx))+45)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (a^3*Sec[c + d*x]*(45 + Cos[2*(c + d*x)] - 20*Sin[c + d*x])*Sqrt[a*(1 + Sin
[c + d*x])])/(3*d)
```

Maple [A] time = 0.101, size = 55, normalized size = 0.6

$$-\frac{2a^4(1+\sin(dx+c))\left((\sin(dx+c))^2+10\sin(dx+c)-23\right)}{3d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2/3*a^4*(1+sin(d*x+c))*(sin(d*x+c)^2+10*sin(d*x+c)-23)/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

Maxima [B] time = 1.67178, size = 320, normalized size = 3.6

$$-\frac{2\left(23a^{\frac{7}{2}} - \frac{20a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1} + \frac{88a^{\frac{7}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{60a^{\frac{7}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{130a^{\frac{7}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{60a^{\frac{7}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{88a^{\frac{7}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{20a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1}\right.}{3d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/3*(23*a^(7/2) - 20*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 88*a^(7/2)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 60*a^(7/2)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 130*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 60*a^(7/2)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + 88*a^(7/2)*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 - 20*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 23*a^(7/2)*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^(7/2))
```

Fricas [A] time = 1.66787, size = 134, normalized size = 1.51

$$\frac{2\left(a^3\cos\left(dx+c\right)^2-10\,a^3\sin\left(dx+c\right)+22\,a^3\right)\sqrt{a\sin\left(dx+c\right)+a}}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^3*cos(d*x + c)^2 - 10*a^3*sin(d*x + c) + 22*a^3)*sqrt(a*sin(d*x + c)
 + a)/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.148 $\int \sec^3(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=91

$$\frac{3a^3\sqrt{a\sin(c+dx)+a}}{d}-\frac{3\sqrt{2}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{a\sec^2(c+dx)(a\sin(c+dx)+a)^{5/2}}{d}$$

```
[Out] (-3*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
+ (3*a^3*Sqrt[a + a*Sin[c + d*x]])/d + (a*Sec[c + d*x]^2*(a + a*Sin[c + d*x
])^(5/2))/d
```

Rubi [A] time = 0.127245, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2676, 2667, 50, 63, 206}

$$\frac{3a^3\sqrt{a\sin(c+dx)+a}}{d}-\frac{3\sqrt{2}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{a\sec^2(c+dx)(a\sin(c+dx)+a)^{5/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-3*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
+ (3*a^3*Sqrt[a + a*Sin[c + d*x]])/d + (a*Sec[c + d*x]^2*(a + a*Sin[c + d*x
])^(5/2))/d
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sec^3(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{5/2}}{d} - \frac{1}{2}\left(3a^2\right)\int \sec(c+dx)(a+a\sin(c+dx))^{3}\\
&= \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{5/2}}{d} - \frac{\left(3a^3\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+x}}{a-x}\,dx, x, a\sin(c+dx)\right)}{2d}\\
&= \frac{3a^3\sqrt{a+a\sin(c+dx)}}{d} + \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{5/2}}{d} - \frac{\left(3a^4\right)\operatorname{Subst}\left(\int\right.}{}\\
&= \frac{3a^3\sqrt{a+a\sin(c+dx)}}{d} + \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{5/2}}{d} - \frac{\left(6a^4\right)\operatorname{Subst}\left(\int\right.}{}\\
&= -\frac{3\sqrt{2}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{3a^3\sqrt{a+a\sin(c+dx)}}{d} + \frac{a\sec^2(c+dx)(a+}{}
\end{aligned}
$$

Mathematica [C] time = 0.089461, size = 42, normalized size = 0.46

$$\frac{a(a\sin(c+dx)+a)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (a*Hypergeometric2F1[2, 5/2, 7/2, (1 + Sin[c + d*x])/2]*(a + a*Sin[c + d*x]
)^(5/2))/(10*d)
```

Maple [A] time = 0.158, size = 83, normalized size = 0.9

$$2\,\frac{a^3}{d}\left(\sqrt{a+a\sin(dx+c)}+4\,a\left(-1/4\,\frac{\sqrt{a+a\sin(dx+c)}}{a\sin(dx+c)-a}-3/8\,\frac{\sqrt{2}}{\sqrt{a}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 2*a^3*((a+a*sin(d*x+c))^(1/2)+4*a*(-1/4*(a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c
)-a)-3/8*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68494, size = 297, normalized size = 3.26

$$\frac{3\sqrt{2}\left(a^3\sin\left(dx+c\right)-a^3\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)+4\left(a^3\sin\left(dx+c\right)-2a^3\right)\sqrt{a\sin\left(dx+c\right)+a}}{2\left(d\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*sqrt(2)*(a^3*sin(d*x + c) - a^3)*sqrt(a)*log(-(a*sin(d*x + c) - 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(a^3*
sin(d*x + c) - 2*a^3)*sqrt(a*sin(d*x + c) + a))/(d*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.149 $\int \sec^4(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=61

$$\frac{2a\sec^3(c+dx)(a\sin(c+dx)+a)^{5/2}}{d}-\frac{8a^2\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}$$

```
[Out] (-8*a^2*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(3*d) + (2*a*Sec[c + d*x
]^3*(a + a*Sin[c + d*x])^(5/2))/d
```

Rubi [A] time = 0.113317, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$\frac{2a\sec^3(c+dx)(a\sin(c+dx)+a)^{5/2}}{d}-\frac{8a^2\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-8*a^2*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(3*d) + (2*a*Sec[c + d*x
]^3*(a + a*Sin[c + d*x])^(5/2))/d
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{2a\sec^3(c+dx)(a+a\sin(c+dx))^{5/2}}{d} - (4a)\int \sec^4(c+dx)(a+a\sin(c+dx))^{5/2}$$

$$= -\frac{8a^2\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d} + \frac{2a\sec^3(c+dx)(a+a\sin(c+dx))^{5/2}}{d}$$

Mathematica [A] time = 5.29493, size = 82, normalized size = 1.34

$$\frac{2a^3(3\sin(c+dx)-1)\sqrt{a(\sin(c+dx)+1)}}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Sqrt[a*(1 + Sin[c + d*x])]*(-1 + 3*Sin[c + d*x]))/(3*d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.102, size = 57, normalized size = 0.9

$$-\frac{2\,a^4\,(1+\sin(dx+c))\,(3\,\sin(dx+c)-1)}{(3\,\sin(dx+c)-3)\cos(dx+c)\,d}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2/3*a^4*(1+sin(d*x+c))/(sin(d*x+c)-1)*(3*sin(d*x+c)-1)/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

Maxima [B] time = 2.47511, size = 432, normalized size = 7.08

$$\frac{2\left(a^{\frac{7}{2}} - \frac{6\,a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1} + \frac{5\,a^{\frac{7}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{24\,a^{\frac{7}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{10\,a^{\frac{7}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{36\,a^{\frac{7}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{10\,a^{\frac{7}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{24\,a^{\frac{7}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{5\,a\ldots}{(\cos\ldots}\right.}{3\,d\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1} - \frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - 1\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^(7/2) - 6*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^(7/2)*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 - 24*a^(7/2)*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 + 10*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^(7/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 + 10*a^(7/2)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 - 24*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5*a^(7/2)*sin(d*x + c)^
8/(cos(d*x + c) + 1)^8 - 6*a^(7/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + a^
(7/2)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)/(d*(3*sin(d*x + c)/(cos(d*x +
c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 - 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^(7/2))
```

Fricas [A] time = 1.62355, size = 142, normalized size = 2.33

$$-\frac{2\left(3\,a^3\sin\left(dx+c\right)-a^3\right)\sqrt{a\sin\left(dx+c\right)+a}}{3\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*a^3*sin(d*x + c) - a^3)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c)*si
n(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.150 $\int \sec^5(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=106

$$-\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d}-\frac{a^2\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{8d}+\frac{a\sec^4(c+dx)(a\sin(c+dx)+a)^{5/2}}{2d}$$

```
[Out] -(a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(8*Sqrt[2]*d
) - (a^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(8*d) + (a*Sec[c + d*x]
^4*(a + a*Sin[c + d*x])^(5/2))/(2*d)
```

Rubi [A] time = 0.171282, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2676, 2675, 2667, 63, 206}

$$-\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d}-\frac{a^2\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{8d}+\frac{a\sec^4(c+dx)(a\sin(c+dx)+a)^{5/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(7/2),x]
```


```
[Out] -(a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(8*Sqrt[2]*d
) - (a^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(8*d) + (a*Sec[c + d*x]
^4*(a + a*Sin[c + d*x])^(5/2))/(2*d)
```


Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
```

```
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{a\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{2d} - \frac{1}{4}a^2\int \sec^3(c+dx)(a+a\sin(c+dx))^{3/2}$$

$$= -\frac{a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{8d} + \frac{a\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{2d} -$$

$$= -\frac{a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{8d} + \frac{a\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{2d} -$$

$$= -\frac{a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{8d} + \frac{a\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{2d} -$$

$$= -\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d} - \frac{a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{8d} + \frac{a\sec^4(c$$

Mathematica [A] time = 0.291257, size = 108, normalized size = 1.02

$$\frac{2a^3(\sin(c+dx)+3)\sqrt{a(\sin(c+dx)+1)}-\sqrt{2}a^{7/2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\tanh^{-1}\left(\frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{2}\sqrt{a}}\right)}{16d(\sin(c+dx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-(Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])]*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^4) + 2*a^3*Sqrt[a*(1 + Sin[c + d*x])]*(
3 + Sin[c + d*x]))/(16*d*(-1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.145, size = 75, normalized size = 0.7

$$-2\,{\frac{{a}^{5}}{d}\left(-1/16\,{\frac{\sqrt{a+a\sin\left(dx+c\right)}\left(3+\sin\left(dx+c\right)\right)}{\left(a\sin\left(dx+c\right)-a\right)^{2}}}+1/32\,{\frac{\sqrt{2}}{{a}^{3/2}}{\rm Artanh}\left(1/2\,{\frac{\sqrt{a+a\sin\left(dx+c\right)}\sqrt{2}}{\sqrt{a}}}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2*a^5*(-1/16*(a+a*sin(d*x+c))^(1/2)*(3+sin(d*x+c))/(a*sin(d*x+c)-a)^2+1/32
/a^(3/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77725, size = 382, normalized size = 3.6

$$\frac{\left(\sqrt{2}a^3\cos(dx+c)^2+2\sqrt{2}a^3\sin(dx+c)-2\sqrt{2}a^3\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\left(a^3\sin(dx+c)+3a^3\right.}{32\left(d\cos(dx+c)^2+2d\sin(dx+c)-2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/32*((sqrt(2)*a^3*cos(d*x + c)^2 + 2*sqrt(2)*a^3*sin(d*x + c) - 2*sqrt(2)*
a^3)*sqrt(a)*log(-(a*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt
(a) + 3*a)/(sin(d*x + c) - 1)) - 4*(a^3*sin(d*x + c) + 3*a^3)*sqrt(a*sin(d*
x + c) + a))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.151 $\int \sec^6(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=30

$$\frac{2a\sec^5(c+dx)(a\sin(c+dx)+a)^{5/2}}{5d}$$

```
[Out] (2*a*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2))/(5*d)
```

Rubi [A] time = 0.056813, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2673}

$$\frac{2a\sec^5(c+dx)(a\sin(c+dx)+a)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2))/(5*d)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^{7/2}\,dx=\frac{2a\sec^5(c+dx)(a+a\sin(c+dx))^{5/2}}{5d}$$

Mathematica [B] time = 5.27195, size = 69, normalized size = 2.3

$$\frac{2(a(\sin(c+dx)+1))^{7/2}}{5d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(7/2))/(5*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

Maple [A] time = 0.106, size = 47, normalized size = 1.6

$$\frac{2\,a^{4}\left(1+\sin\left(dx+c\right)\right)}{5\,\left(\sin\left(dx+c\right)-1\right)^{2}\cos\left(dx+c\right)d}\frac{1}{\sqrt{a+a\sin\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 2/5*a^4*(1+sin(d*x+c))/(sin(d*x+c)-1)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [B] time = 1.67032, size = 365, normalized size = 12.17

$$-\frac{2\left(a^{\frac{7}{2}}+\frac{6\,a^{\frac{7}{2}}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{15\,a^{\frac{7}{2}}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{20\,a^{\frac{7}{2}}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{15\,a^{\frac{7}{2}}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{6\,a^{\frac{7}{2}}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^{\frac{7}{2}}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)}{5\,d\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{5\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-1\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(a^(7/2) + 6*a^(7/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^(7/2)*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^(7/2)*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 15*a^(7/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^(7/2)*sin(
d*x + c)^10/(cos(d*x + c) + 1)^10 + a^(7/2)*sin(d*x + c)^12/(cos(d*x + c) +
 1)^12)/(d*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)*(sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 1)^(7/2))
```

Fricas [B] time = 1.62921, size = 142, normalized size = 4.73

$$-\frac{2\sqrt{a\sin\left(dx+c\right)+a}a^{3}}{5\left(d\cos\left(dx+c\right)^{3}+2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(a*sin(d*x + c) + a)*a^3/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)*sin(
d*x + c) - 2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.152 $\int \sec^7(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=135

$$\frac{5a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d}+\frac{5a^2\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{64d}+\frac{\sec^6(c+dx)(a\sin(c+dx)+a)^{7/2}}{6d}+\frac{5a\sec^4(c+d}{}$$

```
[Out] (5*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(64*Sqrt[2]
*d) + (5*a^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(64*d) + (5*a*Sec[c
 + d*x]^4*(a + a*Sin[c + d*x])^(5/2))/(48*d) + (Sec[c + d*x]^6*(a + a*Sin[c
 + d*x])^(7/2))/(6*d)
```

Rubi [A] time = 0.232092, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2675, 2667, 63, 206}

$$\frac{5a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d}+\frac{5a^2\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{64d}+\frac{\sec^6(c+dx)(a\sin(c+dx)+a)^{7/2}}{6d}+\frac{5a\sec^4(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (5*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(64*Sqrt[2]
*d) + (5*a^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(64*d) + (5*a*Sec[c
 + d*x]^4*(a + a*Sin[c + d*x])^(5/2))/(48*d) + (Sec[c + d*x]^6*(a + a*Sin[c
 + d*x])^(7/2))/(6*d)
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec^7(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{\sec^6(c+dx)(a+a\sin(c+dx))^{7/2}}{6d}+\frac{1}{12}(5a)\int \sec^5(c+dx)(a+a\sin(c+dx)) \\
&= \frac{5a\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{48d}+\frac{\sec^6(c+dx)(a+a\sin(c+dx))^{7/2}}{6d}+\frac{1}{3} \\
&= \frac{5a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{64d}+\frac{5a\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{48d} \\
&= \frac{5a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{64d}+\frac{5a\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{48d} \\
&= \frac{5a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{64d}+\frac{5a\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{48d} \\
&= \frac{5a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d}+\frac{5a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{64d}+\frac{5a\sec}{}
\end{aligned}$$

Mathematica [A] time = 0.540617, size = 120, normalized size = 0.89

$$-\frac{2a^3\left(15\sin^2(c+dx)-50\sin(c+dx)+67\right)\sqrt{a(\sin(c+dx)+1)}+15\sqrt{2}a^{7/2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\tanh}{384d(\sin(c+dx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -(15*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])]*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^6 + 2*a^3*Sqrt[a*(1 + Sin[c + d*x])]*
(67 - 50*Sin[c + d*x] + 15*Sin[c + d*x]^2))/(384*d*(-1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.218, size = 144, normalized size = 1.1

$$2\,{\frac {{a}^{7}}{d} \left(-1/12\,{\frac {\sqrt {a+a\sin \left(dx+c \right) }}{a \left(a\sin \left(dx+c \right) -a \right) ^{3}}}-{\frac {5}{12\,a} \left(-1/8\,{\frac {\sqrt {a+a\sin \left(dx+c \right) }}{a \left(a\sin \left(dx+c \right) -a \right) ^{2}}}-3/8\,{\frac {1}{a} \left(-1/4\,{\frac {\sqrt {a+a\sin \left(dx+c \right) }}{a \left(a\sin \left(dx+c \right) -a \right) }}+1/8\,{\frac {\sqrt {2}}{{a}^{3/2}}}{\rm Artar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 2*a^7*(-1/12*(a+a*sin(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)^3-5/12/a*(-1/8*(a+a*
sin(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)^2-3/8/a*(-1/4*(a+a*sin(d*x+c))^(1/2)/a
/(a*sin(d*x+c)-a)+1/8/a^(3/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^
(1/2)/a^(1/2)))))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80116, size = 501, normalized size = 3.71

$$\frac{15\left(3\sqrt{2}a^3\cos\left(dx+c\right)^2-4\sqrt{2}a^3-\left(\sqrt{2}a^3\cos\left(dx+c\right)^2-4\sqrt{2}a^3\right)\sin\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right.}{768\left(3d\cos\left(dx+c\right)^2-\left(d\cos\left(dx+c\right)^2-4d\right)\sin\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/768*(15*(3*sqrt(2)*a^3*cos(d*x + c)^2 - 4*sqrt(2)*a^3 - (sqrt(2)*a^3*cos(
d*x + c)^2 - 4*sqrt(2)*a^3)*sin(d*x + c))*sqrt(a)*log(-(a*sin(d*x + c) + 2*
sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(15
*a^3*cos(d*x + c)^2 + 50*a^3*sin(d*x + c) - 82*a^3)*sqrt(a*sin(d*x + c) + a
))/(3*d*cos(d*x + c)^2 - (d*cos(d*x + c)^2 - 4*d)*sin(d*x + c) - 4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.153 $\int \sec^8(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=171

$$\frac{a^2\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{12d}+\frac{a^3\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{8d}-\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{2}d}+\frac{\sec^7(c+dx)(\cdots}{}$$

```
[Out] -(a^(7/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])
])/(8*Sqrt[2]*d) + (a^3*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(8*d) + (a^2
*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(12*d) + (a*Sec[c + d*x]^5*(a +
 a*Sin[c + d*x])^(5/2))/(10*d) + (Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2)
)/(7*d)
```

Rubi [A] time = 0.267813, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2675, 2649, 206}

$$\frac{a^2\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{12d}+\frac{a^3\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{8d}-\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{2}d}+\frac{\sec^7(c+dx)(\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -(a^(7/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])
])/(8*Sqrt[2]*d) + (a^3*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(8*d) + (a^2
*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(12*d) + (a*Sec[c + d*x]^5*(a +
 a*Sin[c + d*x])^(5/2))/(10*d) + (Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2)
)/(7*d)
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{\sec^7(c+dx)(a+a\sin(c+dx))^{7/2}}{7d}+\frac{1}{2}a\int \sec^6(c+dx)(a+a\sin(c+dx))^{5/2}\,d\\ &= \frac{a\sec^5(c+dx)(a+a\sin(c+dx))^{5/2}}{10d}+\frac{\sec^7(c+dx)(a+a\sin(c+dx))^{7/2}}{7d}+\frac{1}{4}a\\ &= \frac{a^2\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{12d}+\frac{a\sec^5(c+dx)(a+a\sin(c+dx))^{5/2}}{10d}+\\ &= \frac{a^3\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{8d}+\frac{a^2\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{12d}+\frac{a\,s}{}\\ &= \frac{a^3\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{8d}+\frac{a^2\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{12d}+\frac{a\,s}{}\\ &= -\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{2}d}+\frac{a^3\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{8d}+\frac{a^2\sec^3(}{}\end{aligned}$$

Mathematica [C] time = 5.49601, size = 139, normalized size = 0.81

$$\frac{(a(\sin(c+dx)+1))^{7/2}\left(\frac{-2471\sin(c+dx)+105\sin(3(c+dx))-770\cos(2(c+dx))+2286}{4\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^7}+(105+105i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\text{ta}\right.\right.\right.}{840d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(7/2)*((105 + 105*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)
*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + (2286 - 770*Cos[2*(c + d*x)] - 2471*
```

```
Sin[c + d*x] + 105*Sin[3*(c + d*x)])/(4*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])^7)))/(840*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

Maple [A] time = 0.143, size = 139, normalized size = 0.8

$$\frac{1+\sin(dx+c)}{1680\,(\sin(dx+c)-1)^3\cos(dx+c)\,d}\left(-210\,a^{15/2}\sin(dx+c)(\cos(dx+c))^2+770\,a^{15/2}(\cos(dx+c))^2+1288\,a^{15/2}\sin(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] 1/1680/a^(7/2)*(1+sin(d*x+c))/(sin(d*x+c)-1)^3*(-210*a^(15/2)*sin(d*x+c)*co
s(d*x+c)^2+770*a^(15/2)*cos(d*x+c)^2+1288*a^(15/2)*sin(d*x+c)-1528*a^(15/2)
+105*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^4*(a-a*s
in(d*x+c))^(7/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.21181, size = 828, normalized size = 4.84

$$\frac{105\left(3\sqrt{2}a^3\cos(dx+c)^3-4\sqrt{2}a^3\cos(dx+c)-\left(\sqrt{2}a^3\cos(dx+c)^3-4\sqrt{2}a^3\cos(dx+c)\right)\sin(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(d}{}\right.}{3360\left(3\,d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3360*(105*(3*sqrt(2)*a^3*cos(d*x + c)^3 - 4*sqrt(2)*a^3*cos(d*x + c) - (s
qrt(2)*a^3*cos(d*x + c)^3 - 4*sqrt(2)*a^3*cos(d*x + c))*sin(d*x + c))*sqrt(
a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*sin(d*x + c) + a)*(sqrt(2)*cos(d*x + c
) - sqrt(2)*sin(d*x + c) + sqrt(2))*sqrt(a) + 3*a*cos(d*x + c) - (a*cos(d*x
 + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d
*x + c) - cos(d*x + c) - 2)) + 4*(385*a^3*cos(d*x + c)^2 - 764*a^3 - 7*(15*
a^3*cos(d*x + c)^2 - 92*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(3*d*c
os(d*x + c)^3 - 4*d*cos(d*x + c) - (d*cos(d*x + c)^3 - 4*d*cos(d*x + c))*si
n(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.154 $\int \sec^9(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=191

$$-\frac{315a^4}{2048d\sqrt{a\sin(c+dx)+a}}+\frac{315a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}d}+\frac{21a^2\sec^4(c+dx)(a\sin(c+dx)+a)^{3/2}}{256d}+\frac{105a^3\sec^2(c+a}{1}$$

```
[Out] (315*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2048*Sqr
t[2]*d) - (315*a^4)/(2048*d*Sqrt[a + a*Sin[c + d*x]]) + (105*a^3*Sec[c + d*
x]^2*Sqrt[a + a*Sin[c + d*x]])/(1024*d) + (21*a^2*Sec[c + d*x]^4*(a + a*Sin
[c + d*x])^(3/2))/(256*d) + (3*a*Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2))
/(32*d) + (Sec[c + d*x]^8*(a + a*Sin[c + d*x])^(7/2))/(8*d)
```

Rubi [A] time = 0.312283, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2675, 2667, 51, 63, 206}

$$-\frac{315a^4}{2048d\sqrt{a\sin(c+dx)+a}}+\frac{315a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}d}+\frac{21a^2\sec^4(c+dx)(a\sin(c+dx)+a)^{3/2}}{256d}+\frac{105a^3\sec^2(c+a}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a + a*Sin[c + d*x])^(7/2),x]
```


```
[Out] (315*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2048*Sqr
t[2]*d) - (315*a^4)/(2048*d*Sqrt[a + a*Sin[c + d*x]]) + (105*a^3*Sec[c + d*
x]^2*Sqrt[a + a*Sin[c + d*x]])/(1024*d) + (21*a^2*Sec[c + d*x]^4*(a + a*Sin
[c + d*x])^(3/2))/(256*d) + (3*a*Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2))
/(32*d) + (Sec[c + d*x]^8*(a + a*Sin[c + d*x])^(7/2))/(8*d)
```


Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^9(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\sec^8(c+dx)(a+a\sin(c+dx))^{7/2}}{8d} + \frac{1}{16}(9a)\int \sec^7(c+dx)(a+a\sin(c+dx))^{5/}$$

$$= \frac{3a\sec^6(c+dx)(a+a\sin(c+dx))^{5/2}}{32d} + \frac{\sec^8(c+dx)(a+a\sin(c+dx))^{7/2}}{8d} + \frac{1}{64}$$

$$= \frac{21a^2\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{256d} + \frac{3a\sec^6(c+dx)(a+a\sin(c+dx))^{5/2}}{32d}$$

$$= \frac{105a^3\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{1024d} + \frac{21a^2\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{256d}$$

$$= \frac{105a^3\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{1024d} + \frac{21a^2\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{256d}$$

$$= -\frac{315a^4}{2048d\sqrt{a+a\sin(c+dx)}} + \frac{105a^3\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{1024d} + \frac{21a^2\sec}{}$$

$$= -\frac{315a^4}{2048d\sqrt{a+a\sin(c+dx)}} + \frac{105a^3\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{1024d} + \frac{21a^2\sec}{}$$

$$= \frac{315a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}d} - \frac{315a^4}{2048d\sqrt{a+a\sin(c+dx)}} + \frac{105a^3\sec^2(c+dx}{10}$$

Mathematica [C] time = 0.101034, size = 44, normalized size = 0.23

$$-\frac{a^4\,{}_2F_1\left(-\frac{1}{2},5;\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{16d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -(a^4*Hypergeometric2F1[-1/2, 5, 1/2, (1 + Sin[c + d*x])/2])/(16*d*Sqrt[a +
 a*Sin[c + d*x]])
```

Maple [A] time = 0.26, size = 129, normalized size = 0.7

$$-2\,\frac{a^9}{d}\left(1/32\,\frac{1}{a^5}\left(-\frac{\sqrt{a+a\sin(dx+c)}a^3\left(187\,(\cos(dx+c))^2\sin(dx+c)-725\,(\cos(dx+c))^2-1236\,\sin(dx+c)+1364\right.}{128\,(a\sin(dx+c)-a)^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -2*a^9*(1/32/a^5*(-1/128*(a+a*sin(d*x+c))^(1/2)*a^3*(187*cos(d*x+c)^2*sin(d
*x+c)-725*cos(d*x+c)^2-1236*sin(d*x+c)+1364)/(a*sin(d*x+c)-a)^4-315/256*2^(
1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))+1/32/a^5/
(a+a*sin(d*x+c))^(1/2))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.24515, size = 663, normalized size = 3.47

$$\frac{315\left(3\sqrt{2}a^3\cos\left(dx+c\right)^4-4\sqrt{2}a^3\cos\left(dx+c\right)^2-\left(\sqrt{2}a^3\cos\left(dx+c\right)^4-4\sqrt{2}a^3\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a s}{}\right.}{8192\left(3\,d\cos\left(dx+c\right)^4-4\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/8192*(315*(3*sqrt(2)*a^3*cos(d*x + c)^4 - 4*sqrt(2)*a^3*cos(d*x + c)^2 -
(sqrt(2)*a^3*cos(d*x + c)^4 - 4*sqrt(2)*a^3*cos(d*x + c)^2)*sin(d*x + c))*s
qrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) +
3*a)/(sin(d*x + c) - 1)) + 4*(315*a^3*cos(d*x + c)^4 - 1722*a^3*cos(d*x + c
)^2 + 896*a^3 + 6*(175*a^3*cos(d*x + c)^2 - 192*a^3)*sin(d*x + c))*sqrt(a*s
in(d*x + c) + a))/(3*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 - (d*cos(d*x + c
)^4 - 4*d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.155 $\int \sec^{10}(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=233

$$-\frac{11a^5\cos(c+dx)}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{11a^2\sec^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{140d}+\frac{11a^3\sec^3(c+dx)\sqrt{a\sin(c+dx)+a}}{120d}+\frac{11a^4\,\mathrm{se}}{48d\sqrt{a\,\mathrm{si}}}$$

```
[Out] (-11*a^(7/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x
]])])/(64*Sqrt[2]*d) - (11*a^5*Cos[c + d*x])/(64*d*(a + a*Sin[c + d*x])^(3/
2)) + (11*a^4*Sec[c + d*x])/(48*d*Sqrt[a + a*Sin[c + d*x]]) + (11*a^3*Sec[c
 + d*x]^3*Sqrt[a + a*Sin[c + d*x]])/(120*d) + (11*a^2*Sec[c + d*x]^5*(a + a
*Sin[c + d*x])^(3/2))/(140*d) + (11*a*Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(
5/2))/(126*d) + (Sec[c + d*x]^9*(a + a*Sin[c + d*x])^(7/2))/(9*d)
```

Rubi [A] time = 0.354378, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2675, 2687, 2650, 2649, 206}

$$-\frac{11a^5\cos(c+dx)}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{11a^2\sec^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{140d}+\frac{11a^3\sec^3(c+dx)\sqrt{a\sin(c+dx)+a}}{120d}+\frac{11a^4\,\mathrm{se}}{48d\sqrt{a\,\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^(7/2),x]
```


```
[Out] (-11*a^(7/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x
]])])/(64*Sqrt[2]*d) - (11*a^5*Cos[c + d*x])/(64*d*(a + a*Sin[c + d*x])^(3/
2)) + (11*a^4*Sec[c + d*x])/(48*d*Sqrt[a + a*Sin[c + d*x]]) + (11*a^3*Sec[c
 + d*x]^3*Sqrt[a + a*Sin[c + d*x]])/(120*d) + (11*a^2*Sec[c + d*x]^5*(a + a
*Sin[c + d*x])^(3/2))/(140*d) + (11*a*Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(
5/2))/(126*d) + (Sec[c + d*x]^9*(a + a*Sin[c + d*x])^(7/2))/(9*d)
```


Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\sec^9(c+dx)(a+a\sin(c+dx))^{7/2}}{9d} + \frac{1}{18}(11a)\int \sec^8(c+dx)(a+a\sin(c+dx$$

$$= \frac{11a\sec^7(c+dx)(a+a\sin(c+dx))^{5/2}}{126d} + \frac{\sec^9(c+dx)(a+a\sin(c+dx))^{7/2}}{9d} +$$

$$= \frac{11a^2\sec^5(c+dx)(a+a\sin(c+dx))^{3/2}}{140d} + \frac{11a\sec^7(c+dx)(a+a\sin(c+dx))^{5}}{126d}$$

$$= \frac{11a^3\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{120d} + \frac{11a^2\sec^5(c+dx)(a+a\sin(c+dx))^{3/}}{140d}$$

$$= \frac{11a^4\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}} + \frac{11a^3\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{120d} + \frac{11a^2\sec^5(c}{}$$

$$= -\frac{11a^5\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}} + \frac{11a^4\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}} + \frac{11a^3\sec^3(c+dx)\sqrt{a}}{120d}$$

$$= -\frac{11a^5\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}} + \frac{11a^4\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}} + \frac{11a^3\sec^3(c+dx)\sqrt{a}}{120d}$$

$$= -\frac{11a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{64\sqrt{2}d} - \frac{11a^5\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}} + \frac{11a^4\sec(}{48d\sqrt{a+a\,\mathrm{si}}}$$

Mathematica [C] time = 5.66159, size = 388, normalized size = 1.67

$$\frac{(a(\sin(c+dx)+1))^{7/2}\left(630\sin\left(\frac{1}{2}(c+dx)\right) + \frac{3150\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{1680\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{1512\left(\sin\left(\frac{1}{2}(c\right.\right.}{\left(\cos\left(\frac{1}{2}(c+a\right.\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] ((630*Sin[(c + d*x)/2] - 315*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + (3465
+ 3465*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])
]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (1120*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^9 + (1440*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7 + (15
12*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^5 + (1680*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^3 + (3150*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]))*(a*(1 + Sin[c + d*x]))^(7/2))/(20160*d*
```

```
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^9)
```

Maple [A] time = 0.147, size = 205, normalized size = 0.9

$$-\frac{1}{40320\,\left(\sin\left(dx+c\right)-1\right)^{4}\cos\left(dx+c\right)d}\left(-6930\,a^{11/2}\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{4}+42504\,a^{11/2}\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] -1/40320/a^(3/2)*(-6930*a^(11/2)*sin(d*x+c)*cos(d*x+c)^4+42504*a^(11/2)*cos
(d*x+c)^2*sin(d*x+c)+385*(9*(a-a*sin(d*x+c))^(9/2)*2^(1/2)*arctanh(1/2*(a-a
*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a-32*a^(11/2))*sin(d*x+c)+25410*a^(11/2
)*cos(d*x+c)^4-50424*a^(11/2)*cos(d*x+c)^2+3465*(a-a*sin(d*x+c))^(9/2)*2^(1
/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a+7840*a^(11/2))/(s
in(d*x+c)-1)^4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.40159, size = 917, normalized size = 3.94

$$\frac{3465\left(3\sqrt{2}a^{3}\cos\left(dx+c\right)^{5}-4\sqrt{2}a^{3}\cos\left(dx+c\right)^{3}-\left(\sqrt{2}a^{3}\cos\left(dx+c\right)^{5}-4\sqrt{2}a^{3}\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a\,c}{\ }\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/80640*(3465*(3*sqrt(2)*a^3*cos(d*x + c)^5 - 4*sqrt(2)*a^3*cos(d*x + c)^3
- (sqrt(2)*a^3*cos(d*x + c)^5 - 4*sqrt(2)*a^3*cos(d*x + c)^3)*sin(d*x + c))
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*sin(d*x + c) + a)*(sqrt(2)*cos(d
*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))*sqrt(a) + 3*a*cos(d*x + c) - (a*c
os(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)
*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(12705*a^3*cos(d*x + c)^4 - 25212*a^
3*cos(d*x + c)^2 + 3920*a^3 - 77*(45*a^3*cos(d*x + c)^4 - 276*a^3*cos(d*x +
 c)^2 + 80*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(3*d*cos(d*x + c)^5
 - 4*d*cos(d*x + c)^3 - (d*cos(d*x + c)^5 - 4*d*cos(d*x + c)^3)*sin(d*x + c
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.156 $\int \frac{\cos^7(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=97

$$-\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^7d}+\frac{12(a\sin(c+dx)+a)^{11/2}}{11a^6d}-\frac{8(a\sin(c+dx)+a)^{9/2}}{3a^5d}+\frac{16(a\sin(c+dx)+a)^{7/2}}{7a^4d}$$

```
[Out] (16*(a + a*Sin[c + d*x])^(7/2))/(7*a^4*d) - (8*(a + a*Sin[c + d*x])^(9/2))/
(3*a^5*d) + (12*(a + a*Sin[c + d*x])^(11/2))/(11*a^6*d) - (2*(a + a*Sin[c +
 d*x])^(13/2))/(13*a^7*d)
```

Rubi [A] time = 0.075736, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^7d}+\frac{12(a\sin(c+dx)+a)^{11/2}}{11a^6d}-\frac{8(a\sin(c+dx)+a)^{9/2}}{3a^5d}+\frac{16(a\sin(c+dx)+a)^{7/2}}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(7/2))/(7*a^4*d) - (8*(a + a*Sin[c + d*x])^(9/2))/
(3*a^5*d) + (12*(a + a*Sin[c + d*x])^(11/2))/(11*a^6*d) - (2*(a + a*Sin[c +
 d*x])^(13/2))/(13*a^7*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{5/2}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{5/2}-12a^2(a+x)^{7/2}+6a(a+x)^{9/2}-(a+x)^{11/2}\right)dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{16(a+a\sin(c+dx))^{7/2}}{7a^4 d} - \frac{8(a+a\sin(c+dx))^{9/2}}{3a^5 d} + \frac{12(a+a\sin(c+dx))^{11/2}}{11a^6 d} - \frac{2(a+a\sin(c+dx))^{13/2}}{13a^7 d}$$

Mathematica [A] time = 0.288566, size = 61, normalized size = 0.63

$$-\frac{2(\sin(c+dx)+1)^4\left(231\sin^3(c+dx)-945\sin^2(c+dx)+1421\sin(c+dx)-835\right)}{3003d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*(1 + Sin[c + d*x])^4*(-835 + 1421*Sin[c + d*x] - 945*Sin[c + d*x]^2 + 2
31*Sin[c + d*x]^3))/(3003*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.089, size = 57, normalized size = 0.6

$$\frac{462\,(\cos(dx+c))^2\sin(dx+c)-1890\,(\cos(dx+c))^2-3304\,\sin(dx+c)+3560}{3003\,a^4 d}(a+a\sin(dx+c))^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3003/a^4*(a+a*sin(d*x+c))^(7/2)*(231*cos(d*x+c)^2*sin(d*x+c)-945*cos(d*x+
c)^2-1652*sin(d*x+c)+1780)/d
```

Maxima [B] time = 0.969764, size = 379, normalized size = 3.91

$$2\left(15015\sqrt{a\sin(dx+c)+a} - \frac{3003\left(3(a\sin(dx+c)+a)^{\frac{5}{2}}-10(a\sin(dx+c)+a)^{\frac{3}{2}}a+15\sqrt{a\sin(dx+c)+a}\,a^2\right)}{a^2} + \frac{143\left(35(a\sin(dx+c)+a)^{\frac{9}{2}}-180(a\sin(dx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(15015*sqrt(a*sin(d*x + c) + a) - 3003*(3*(a*sin(d*x + c) + a)^(5/2
) - 10*(a*sin(d*x + c) + a)^(3/2)*a + 15*sqrt(a*sin(d*x + c) + a)*a^2)/a^2
+ 143*(35*(a*sin(d*x + c) + a)^(9/2) - 180*(a*sin(d*x + c) + a)^(7/2)*a + 3
78*(a*sin(d*x + c) + a)^(5/2)*a^2 - 420*(a*sin(d*x + c) + a)^(3/2)*a^3 + 31
5*sqrt(a*sin(d*x + c) + a)*a^4)/a^4 - 5*(231*(a*sin(d*x + c) + a)^(13/2) -
1638*(a*sin(d*x + c) + a)^(11/2)*a + 5005*(a*sin(d*x + c) + a)^(9/2)*a^2 -
8580*(a*sin(d*x + c) + a)^(7/2)*a^3 + 9009*(a*sin(d*x + c) + a)^(5/2)*a^4 -
 6006*(a*sin(d*x + c) + a)^(3/2)*a^5 + 3003*sqrt(a*sin(d*x + c) + a)*a^6)/a
^6)/(a*d)
```

Fricas [A] time = 1.91648, size = 228, normalized size = 2.35

$$\frac{2\left(231\cos\left(dx+c\right)^{6}+28\cos\left(dx+c\right)^{4}+64\cos\left(dx+c\right)^{2}+4\left(63\cos\left(dx+c\right)^{4}+80\cos\left(dx+c\right)^{2}+128\right)\sin\left(dx+c\right)+}{3003\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*cos(d*x + c)^6 + 28*cos(d*x + c)^4 + 64*cos(d*x + c)^2 + 4*(63*
cos(d*x + c)^4 + 80*cos(d*x + c)^2 + 128)*sin(d*x + c) + 512)*sqrt(a*sin(d*
x + c) + a)/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30596, size = 97, normalized size = 1.

$$-\frac{2\left(231\,(a\sin\left(dx+c\right)+a)^{\frac{13}{2}}-1638\,(a\sin\left(dx+c\right)+a)^{\frac{11}{2}}a+4004\,(a\sin\left(dx+c\right)+a)^{\frac{9}{2}}a^{2}-3432\,(a\sin\left(dx+c\right)+a)^{\frac{7}{2}}a\right.}{3003\,a^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/3003*(231*(a*sin(d*x + c) + a)^(13/2) - 1638*(a*sin(d*x + c) + a)^(11/2)
*a + 4004*(a*sin(d*x + c) + a)^(9/2)*a^2 - 3432*(a*sin(d*x + c) + a)^(7/2)*
a^3)/(a^7*d)
```

3.157 $\int \frac{\cos^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{16a^2\cos^7(c+dx)}{99d(a\sin(c+dx)+a)^{5/2}}-\frac{64a^3\cos^7(c+dx)}{693d(a\sin(c+dx)+a)^{7/2}}-\frac{2a\cos^7(c+dx)}{11d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-64*a^3*Cos[c + d*x]^7)/(693*d*(a + a*Sin[c + d*x])^(7/2)) - (16*a^2*Cos[c
 + d*x]^7)/(99*d*(a + a*Sin[c + d*x])^(5/2)) - (2*a*Cos[c + d*x]^7)/(11*d*(
a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.170434, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{16a^2\cos^7(c+dx)}{99d(a\sin(c+dx)+a)^{5/2}}-\frac{64a^3\cos^7(c+dx)}{693d(a\sin(c+dx)+a)^{7/2}}-\frac{2a\cos^7(c+dx)}{11d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^7)/(693*d*(a + a*Sin[c + d*x])^(7/2)) - (16*a^2*Cos[c
 + d*x]^7)/(99*d*(a + a*Sin[c + d*x])^(5/2)) - (2*a*Cos[c + d*x]^7)/(11*d*(
a + a*Sin[c + d*x])^(3/2))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= -\frac{2a\cos^7(c+dx)}{11d(a+a\sin(c+dx))^{3/2}}+\frac{1}{11}(8a)\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx \\
&= -\frac{16a^2\cos^7(c+dx)}{99d(a+a\sin(c+dx))^{5/2}}-\frac{2a\cos^7(c+dx)}{11d(a+a\sin(c+dx))^{3/2}}+\frac{1}{99}\left(32a^2\right)\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{5/2}} \\
&= -\frac{64a^3\cos^7(c+dx)}{693d(a+a\sin(c+dx))^{7/2}}-\frac{16a^2\cos^7(c+dx)}{99d(a+a\sin(c+dx))^{5/2}}-\frac{2a\cos^7(c+dx)}{11d(a+a\sin(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.245876, size = 59, normalized size = 0.62

$$-\frac{2\left(63\sin^2(c+dx)+182\sin(c+dx)+151\right)\cos^7(c+dx)}{693d(\sin(c+dx)+1)^3\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^7*(151 + 182*Sin[c + d*x] + 63*Sin[c + d*x]^2))/(693*d*(1
+ Sin[c + d*x])^3*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.125, size = 64, normalized size = 0.7

$$-\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^4\left(63(\sin(dx+c))^2+182\sin(dx+c)+151\right)}{693\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/693*(1+sin(d*x+c))*(sin(d*x+c)-1)^4*(63*sin(d*x+c)^2+182*sin(d*x+c)+151)
/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^6}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 1.89483, size = 432, normalized size = 4.55

$$\frac{2\left(63\cos\left(dx+c\right)^{6}-7\cos\left(dx+c\right)^{5}+10\cos\left(dx+c\right)^{4}-16\cos\left(dx+c\right)^{3}+32\cos\left(dx+c\right)^{2}+\left(63\cos\left(dx+c\right)^{5}+70\,\mathrm{c}\right.\right.}{693\left(ad\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*cos(d*x + c)^6 - 7*cos(d*x + c)^5 + 10*cos(d*x + c)^4 - 16*cos(d*
x + c)^3 + 32*cos(d*x + c)^2 + (63*cos(d*x + c)^5 + 70*cos(d*x + c)^4 + 80*
cos(d*x + c)^3 + 96*cos(d*x + c)^2 + 128*cos(d*x + c) + 256)*sin(d*x + c) -
 128*cos(d*x + c) - 256)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c) + a*d*s
in(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.1936, size = 497, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/11088*(((((((((((151*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)/
a^13 - 693*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) + 1177*
sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) - 1155*sgn(tan(1/2
*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) + 1782*sgn(tan(1/2*d*x + 1/2*
c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) - 3234*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^1
3)*tan(1/2*d*x + 1/2*c) + 3234*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*
d*x + 1/2*c) - 1782*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c
) + 1155*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) - 1177*sg
n(tan(1/2*d*x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) + 693*sgn(tan(1/2*d*
x + 1/2*c) + 1)/a^13)*tan(1/2*d*x + 1/2*c) - 151*sgn(tan(1/2*d*x + 1/2*c) +
 1)/a^13)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(11/2) + 256*sqrt(2)*sgn(tan(1/2*d
*x + 1/2*c) + 1)/a^(37/2))/d
```

3.158 $\int \frac{\cos^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=73

$$\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^5d}-\frac{8(a\sin(c+dx)+a)^{7/2}}{7a^4d}+\frac{8(a\sin(c+dx)+a)^{5/2}}{5a^3d}$$

```
[Out] (8*(a + a*Sin[c + d*x])^(5/2))/(5*a^3*d) - (8*(a + a*Sin[c + d*x])^(7/2))/(
7*a^4*d) + (2*(a + a*Sin[c + d*x])^(9/2))/(9*a^5*d)
```

Rubi [A] time = 0.0672223, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^5d}-\frac{8(a\sin(c+dx)+a)^{7/2}}{7a^4d}+\frac{8(a\sin(c+dx)+a)^{5/2}}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(5/2))/(5*a^3*d) - (8*(a + a*Sin[c + d*x])^(7/2))/(
7*a^4*d) + (2*(a + a*Sin[c + d*x])^(9/2))/(9*a^5*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{3/2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{3/2}-4a(a+x)^{5/2}+(a+x)^{7/2}\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{8(a+a\sin(c+dx))^{5/2}}{5a^3d} - \frac{8(a+a\sin(c+dx))^{7/2}}{7a^4d} + \frac{2(a+a\sin(c+dx))^{9/2}}{9a^5d}$$

Mathematica [A] time = 0.13584, size = 51, normalized size = 0.7

$$\frac{2(\sin(c+dx)+1)^3\left(35\sin^2(c+dx)-110\sin(c+dx)+107\right)}{315d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*(1 + Sin[c + d*x])^3*(107 - 110*Sin[c + d*x] + 35*Sin[c + d*x]^2))/(315*
d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.079, size = 41, normalized size = 0.6

$$-\frac{70\,(\cos(dx+c))^2+220\,\sin(dx+c)-284}{315\,a^3d}(a+a\sin(dx+c))^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/315/a^3*(a+a*sin(d*x+c))^(5/2)*(35*cos(d*x+c)^2+110*sin(d*x+c)-142)/d
```

Maxima [B] time = 0.953644, size = 216, normalized size = 2.96

$$\frac{2\left(315\sqrt{a\sin(dx+c)+a} - \frac{42\left(3(a\sin(dx+c)+a)^{\frac{5}{2}}-10(a\sin(dx+c)+a)^{\frac{3}{2}}a+15\sqrt{a\sin(dx+c)+a}\,a^2\right)}{a^2} + \frac{35(a\sin(dx+c)+a)^{\frac{9}{2}}-180(a\sin(dx+c)+a)^{\frac{7}{2}}a+\cdots}{}\right.}{315\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(a*sin(d*x + c) + a) - 42*(3*(a*sin(d*x + c) + a)^(5/2) - 10
*(a*sin(d*x + c) + a)^(3/2)*a + 15*sqrt(a*sin(d*x + c) + a)*a^2)/a^2 + (35*
(a*sin(d*x + c) + a)^(9/2) - 180*(a*sin(d*x + c) + a)^(7/2)*a + 378*(a*sin(
d*x + c) + a)^(5/2)*a^2 - 420*(a*sin(d*x + c) + a)^(3/2)*a^3 + 315*sqrt(a*s
in(d*x + c) + a)*a^4)/a^4)/(a*d)
```

Fricas [A] time = 1.84219, size = 165, normalized size = 2.26

$$\frac{2\left(35\cos\left(dx+c\right)^{4}+8\cos\left(dx+c\right)^{2}+8\left(5\cos\left(dx+c\right)^{2}+8\right)\sin\left(dx+c\right)+64\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*cos(d*x + c)^4 + 8*cos(d*x + c)^2 + 8*(5*cos(d*x + c)^2 + 8)*sin(
d*x + c) + 64)*sqrt(a*sin(d*x + c) + a)/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23034, size = 74, normalized size = 1.01

$$\frac{2\left(35\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}-180\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a+252\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a^{2}\right)}{315\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(a*sin(d*x + c) + a)^(9/2) - 180*(a*sin(d*x + c) + a)^(7/2)*a + 2
52*(a*sin(d*x + c) + a)^(5/2)*a^2)/(a^5*d)
```

3.159 $\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=63

$$-\frac{8a^2\cos^5(c+dx)}{35d(a\sin(c+dx)+a)^{5/2}}-\frac{2a\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-8*a^2*Cos[c + d*x]^5)/(35*d*(a + a*Sin[c + d*x])^(5/2)) - (2*a*Cos[c + d*
x]^5)/(7*d*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.11114, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{8a^2\cos^5(c+dx)}{35d(a\sin(c+dx)+a)^{5/2}}-\frac{2a\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^5)/(35*d*(a + a*Sin[c + d*x])^(5/2)) - (2*a*Cos[c + d*
x]^5)/(7*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2a\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{1}{7}(4a)\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$
$$= -\frac{8a^2\cos^5(c+dx)}{35d(a+a\sin(c+dx))^{5/2}} - \frac{2a\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.0604323, size = 49, normalized size = 0.78

$$-\frac{2(5\sin(c+dx)+9)\cos^5(c+dx)}{35d(\sin(c+dx)+1)^2\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^5*(9 + 5*Sin[c + d*x]))/(35*d*(1 + Sin[c + d*x])^2*Sqrt[a*
(1 + Sin[c + d*x])])
```

Maple [A] time = 0.111, size = 54, normalized size = 0.9

$$\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^3(5\sin(dx+c)+9)}{35\,d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/35*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(5*sin(d*x+c)+9)/cos(d*x+c)/(a+a*sin(d
*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.92041, size = 306, normalized size = 4.86

$$\frac{2\left(5\,\cos\left(dx+c\right)^{4}-\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)^{2}+\left(5\,\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)^{2}+8\,\cos\left(dx+c\right)+16\right)\sin\left(dx+c\right.}{35\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(d*x + c)^4 - cos(d*x + c)^3 + 2*cos(d*x + c)^2 + (5*cos(d*x + c
)^3 + 6*cos(d*x + c)^2 + 8*cos(d*x + c) + 16)*sin(d*x + c) - 8*cos(d*x + c)
 - 16)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos^{4}\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**4/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] time = 1.98357, size = 346, normalized size = 5.49

$$\frac{\left(\left(\left(\left(\left(\left(\frac{9\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{9}}-\frac{35\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{9}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{49\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{9}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{35\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{9}}\right)\tan\left(\frac{1}{2}\right.}{\left(a\tan\left(\frac{1}{2}\,dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*((((((((9*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)/a^9 - 35
*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^9)*tan(1/2*d*x + 1/2*c) + 49*sgn(tan(1/2*d
*x + 1/2*c) + 1)/a^9)*tan(1/2*d*x + 1/2*c) - 35*sgn(tan(1/2*d*x + 1/2*c) +
1)/a^9)*tan(1/2*d*x + 1/2*c) + 35*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^9)*tan(1/
2*d*x + 1/2*c) - 49*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^9)*tan(1/2*d*x + 1/2*c)
 + 35*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^9)*tan(1/2*d*x + 1/2*c) - 9*sgn(tan(1
/2*d*x + 1/2*c) + 1)/a^9)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2) + 16*sqrt(2)
*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(25/2))/d
```

3.160 $\int \frac{\cos^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=49

$$\frac{4(a\sin(c+dx)+a)^{3/2}}{3a^2d}-\frac{2(a\sin(c+dx)+a)^{5/2}}{5a^3d}$$

```
[Out] (4*(a + a*Sin[c + d*x])^(3/2))/(3*a^2*d) - (2*(a + a*Sin[c + d*x])^(5/2))/(
5*a^3*d)
```

Rubi [A] time = 0.0625084, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{4(a\sin(c+dx)+a)^{3/2}}{3a^2d}-\frac{2(a\sin(c+dx)+a)^{5/2}}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(3/2))/(3*a^2*d) - (2*(a + a*Sin[c + d*x])^(5/2))/(
5*a^3*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\text{Subst}\left(\int (a-x)\sqrt{a+x}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(2a\sqrt{a+x}-(a+x)^{3/2}\right)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{4(a+a\sin(c+dx))^{3/2}}{3a^2d} - \frac{2(a+a\sin(c+dx))^{5/2}}{5a^3d}$$

Mathematica [A] time = 0.0637287, size = 34, normalized size = 0.69

$$-\frac{2(3\sin(c+dx)-7)(a(\sin(c+dx)+1))^{3/2}}{15a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*(a*(1 + Sin[c + d*x]))^(3/2)*(-7 + 3*Sin[c + d*x]))/(15*a^2*d)
```

Maple [A] time = 0.088, size = 31, normalized size = 0.6

$$-\frac{6\sin(dx+c)-14}{15\,a^2d}\,(a+a\sin(dx+c))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/15/a^2*(a+a*sin(d*x+c))^(3/2)*(3*sin(d*x+c)-7)/d
```

Maxima [A] time = 0.953746, size = 101, normalized size = 2.06

$$\frac{2\left(15\sqrt{a\sin(dx+c)+a} - \frac{3\,(a\sin(dx+c)+a)^{\frac{5}{2}} - 10\,(a\sin(dx+c)+a)^{\frac{3}{2}}a + 15\,\sqrt{a\sin(dx+c)+a}a^2}{a^2}\right)}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(a*sin(d*x + c) + a) - (3*(a*sin(d*x + c) + a)^(5/2) - 10*(a*s
in(d*x + c) + a)^(3/2)*a + 15*sqrt(a*sin(d*x + c) + a)*a^2)/a^2)/(a*d)
```

Fricas [A] time = 1.86179, size = 104, normalized size = 2.12

$$\frac{2\left(3\,\cos\left(dx+c\right)^2+4\,\sin\left(dx+c\right)+4\right)\sqrt{a\sin\left(dx+c\right)+a}}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(d*x + c)^2 + 4*sin(d*x + c) + 4)*sqrt(a*sin(d*x + c) + a)/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15475, size = 51, normalized size = 1.04

$$-\frac{2\left(3\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}-10\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a\right)}{15\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(3*(a*sin(d*x + c) + a)^(5/2) - 10*(a*sin(d*x + c) + a)^(3/2)*a)/(a^3
*d)
```

3.161 $\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=30

$$-\frac{2a\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-2*a*Cos[c + d*x]^3)/(3*d*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.0512926, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2673}

$$-\frac{2a\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*a*Cos[c + d*x]^3)/(3*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2a\cos^3(c+dx)}{3d(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.0653603, size = 30, normalized size = 1.

$$-\frac{2a\cos^3(c+dx)}{3d(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*a*Cos[c + d*x]^3)/(3*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.089, size = 44, normalized size = 1.5

$$-\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^2}{3d\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/3*(1+sin(d*x+c))*(sin(d*x+c)-1)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(dx+c)^2}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 1.822, size = 194, normalized size = 6.47

$$\frac{2\left(\cos(dx+c)^2+(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\sqrt{a\sin(dx+c)+a}}{3\left(ad\cos(dx+c)+ad\sin(dx+c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(cos(d*x + c)^2 + (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)*s
qrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] time = 1.82785, size = 193, normalized size = 6.43

$$\frac{\frac{\left(\left(\frac{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^5}-\frac{3\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^5}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{3\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^5}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^5}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}+\frac{2\sqrt{2}\operatorname{sgn}(\mathrm{t}}{}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((((sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)/a^5 - 3*sgn(tan(
1/2*d*x + 1/2*c) + 1)/a^5)*tan(1/2*d*x + 1/2*c) + 3*sgn(tan(1/2*d*x + 1/2*c
) + 1)/a^5)*tan(1/2*d*x + 1/2*c) - sgn(tan(1/2*d*x + 1/2*c) + 1)/a^5)/(a*ta
n(1/2*d*x + 1/2*c)^2 + a)^(3/2) + 2*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a
^(13/2))/d
```

3.162 $\int \frac{\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=22

$$\frac{2\sqrt{a\sin(c+dx)+a}}{ad}$$

```
[Out] (2*Sqrt[a + a*Sin[c + d*x]])/(a*d)
```

Rubi [A] time = 0.0300435, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$\frac{2\sqrt{a\sin(c+dx)+a}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + a*Sin[c + d*x]])/(a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{2\sqrt{a+a\sin(c+dx)}}{ad}$$

Mathematica [A] time = 0.0249866, size = 22, normalized size = 1.

$$\frac{2\sqrt{a\sin(c+dx)+a}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + a*Sin[c + d*x]])/(a*d)
```

Maple [A] time = 0.008, size = 21, normalized size = 1.

$$2\,\frac{\sqrt{a+a\sin(dx+c)}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*(a+a*sin(d*x+c))^(1/2)/d/a
```

Maxima [A] time = 0.946046, size = 27, normalized size = 1.23

$$\frac{2\,\sqrt{a\sin(dx+c)+a}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a*sin(d*x + c) + a)/(a*d)
```

Fricas [A] time = 1.77978, size = 46, normalized size = 2.09

$$\frac{2\sqrt{a\sin\left(dx+c\right)+a}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*sin(d*x + c) + a)/(a*d)
```

Sympy [A] time = 0.839716, size = 32, normalized size = 1.45

$$\begin{cases} \frac{2\sqrt{a\sin\left(c+dx\right)+a}}{ad} & \text{for}\: d \neq 0 \\ \frac{x\cos\left(c\right)}{\sqrt{a\sin\left(c\right)+a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(a*sin(c + d*x) + a)/(a*d), Ne(d, 0)), (x*cos(c)/sqrt(a*si
n(c) + a), True))
```

Giac [A] time = 1.13645, size = 27, normalized size = 1.23

$$\frac{2\sqrt{a\sin\left(dx+c\right)+a}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a*sin(d*x + c) + a)/(a*d)
```

3.163 $\int \frac{\sec(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}-\frac{1}{d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d) - 1
/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.0621285, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2667, 51, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}-\frac{1}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d) - 1
/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{1}{d\sqrt{a+a\sin(c+dx)}}+\frac{\mathrm{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{2d}\\ &= -\frac{1}{d\sqrt{a+a\sin(c+dx)}}+\frac{\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+a\sin(c+dx)}\right)}{d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}-\frac{1}{d\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.0489939, size = 39, normalized size = 0.65

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sin[c + d*x])/2]/(d*Sqrt[a + a*Sin[c
 + d*x]]))
```

Maple [A] time = 0.102, size = 54, normalized size = 0.9

$$-2\,{\frac {a}{d} \left(1/2\,{\frac {1}{a\sqrt {a+a\sin \left(dx+c \right) }}}-1/4\,{\frac {\sqrt {2}}{{a}^{3/2}}{\rm Artanh} \left(1/2\,{\frac {\sqrt {a+a\sin \left(dx+c \right) }\sqrt {2}}{\sqrt {a}}}\right)} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2*a*(1/2/a/(a+a*sin(d*x+c))^(1/2)-1/4/a^(3/2)*2^(1/2)*arctanh(1/2*(a+a*sin
(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16618, size = 252, normalized size = 4.2

$$\frac{\frac{\sqrt{2}(a\sin(dx+c)+a)\log\left(-\frac{\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}}{\sqrt{a}}+\sin(dx+c)+3}{\sin(dx+c)-1}\right)}{\sqrt{a}} - 4\sqrt{a\sin\left(dx+c\right)+a}}{4\left(ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(a*sin(d*x + c) + a)*log(-(2*sqrt(2)*sqrt(a*sin(d*x + c) + a)/
sqrt(a) + sin(d*x + c) + 3)/(sin(d*x + c) - 1))/sqrt(a) - 4*sqrt(a*sin(d*x
+ c) + a))/(a*d*sin(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (c+dx)}{\sqrt{a(\sin (c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A] time = 1.13295, size = 80, normalized size = 1.33

$$-\frac{a\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\sin(dx+c)+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{2}{\sqrt{a\sin(dx+c)+a}a}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*sin(d*x + c) + a)/sqrt(-a))/(sqrt
(-a)*a) + 2/(sqrt(a*sin(d*x + c) + a)*a))/d
```

3.164 $\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=102

$$-\frac{3a\cos(c+dx)}{4d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{2}\sqrt{a}d}$$

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(4*
Sqrt[2]*Sqrt[a]*d) - (3*a*Cos[c + d*x])/(4*d*(a + a*Sin[c + d*x])^(3/2)) +
Sec[c + d*x]/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.0934787, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2687, 2650, 2649, 206}

$$-\frac{3a\cos(c+dx)}{4d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{2}\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(4*
Sqrt[2]*Sqrt[a]*d) - (3*a*Cos[c + d*x])/(4*d*(a + a*Sin[c + d*x])^(3/2)) +
Sec[c + d*x]/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\sec(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{1}{2}(3a)\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx\\
&= -\frac{3a\cos(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{\sec(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{3}{8}\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx\\
&= -\frac{3a\cos(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{\sec(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d}\\
&= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}d}-\frac{3a\cos(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{\sec(c+dx)}{d\sqrt{a+a\sin(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 0.264695, size = 118, normalized size = 1.16

$$
-\frac{\sec(c+dx)\left(-3\sin(c+dx)+(-3-3i)(-1)^{3/4}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2 t}{4d\sqrt{a(\sin(c+dx)+1)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -(Sec[c + d*x]*(-1 - (3 + 3*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-
1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^2 - 3*Sin[c + d*x]))/(4*d*Sqrt[a*(1 + Sin[c + d*x])
```

```
])
```

Maple [A] time = 0.131, size = 130, normalized size = 1.3

$$-\frac{1}{8\,d\cos\left(dx+c\right)}\left(\sin\left(dx+c\right)\left(3\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin\left(dx+c\right)}\sqrt{2}}{\sqrt{a}}\right)a\sqrt{a-a\sin\left(dx+c\right)}-6\,a^{3/2}\right)+3\sqrt{2}\mathrm{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/8*(sin(d*x+c)*(3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1
/2))*a*(a-a*sin(d*x+c))^(1/2)-6*a^(3/2))+3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x
+c))^(1/2)*2^(1/2)/a^(1/2))*a*(a-a*sin(d*x+c))^(1/2)-2*a^(3/2))/a^(3/2)/cos
(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)^{2}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] time = 2.32562, size = 549, normalized size = 5.38

$$\frac{3\sqrt{2}(\cos\left(dx+c\right)\sin\left(dx+c\right)+\cos\left(dx+c\right))\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3\,a\cos(dx+c)-(a\cos}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right.}{16\left(ad\cos\left(dx+c\right)\sin\left(dx+c\right)+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(cos(d*x + c)*sin(d*x + c) + cos(d*x + c))*sqrt(a)*log(-(a*
cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
 sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c)
 + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) -
2)) + 4*sqrt(a*sin(d*x + c) + a)*(3*sin(d*x + c) + 1))/(a*d*cos(d*x + c)*si
n(d*x + c) + a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{2}(c+dx)}{\sqrt{a\left(\sin(c+dx)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] time = 2.25775, size = 566, normalized size = 5.55

$$\frac{\frac{3\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}+\frac{4\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}+\sqrt{a}\right)}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\cdots}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d*x +
 1/2*c) + 1)) + 4*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a) + sqrt(a))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))*sqrt(a) - a)*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 2*(3*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3 + (sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) - (sqrt(a
```

```
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a + a^(3/2))/(
((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a)
 - a)^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.165 $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=116

$$-\frac{5}{8d\sqrt{a\sin(c+dx)+a}}-\frac{5a}{12d(a\sin(c+dx)+a)^{3/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d}+\frac{\sec^2(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (5*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(8*Sqrt[2]*Sqrt[a]*
d) - (5*a)/(12*d*(a + a*Sin[c + d*x])^(3/2)) - 5/(8*d*Sqrt[a + a*Sin[c + d*
x]]) + Sec[c + d*x]^2/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.133365, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2687, 2667, 51, 63, 206}

$$-\frac{5}{8d\sqrt{a\sin(c+dx)+a}}-\frac{5a}{12d(a\sin(c+dx)+a)^{3/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d}+\frac{\sec^2(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (5*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(8*Sqrt[2]*Sqrt[a]*
d) - (5*a)/(12*d*(a + a*Sin[c + d*x])^(3/2)) - 5/(8*d*Sqrt[a + a*Sin[c + d*
x]]) + Sec[c + d*x]^2/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{1}{4}(5a)\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$

$$= \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{(5a^2)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, a\sin(c+dx)\right)}{4d}$$

$$= -\frac{5a}{12d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{(5a)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, a\sin(c\right.}{8d}$$

$$= -\frac{5a}{12d(a+a\sin(c+dx))^{3/2}} - \frac{5}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{5\,\mathrm{Subst}\left(\int \frac{}{(a-}\right.}{}$$

$$= -\frac{5a}{12d(a+a\sin(c+dx))^{3/2}} - \frac{5}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{2a-}\right.}{}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d} - \frac{5a}{12d(a+a\sin(c+dx))^{3/2}} - \frac{5}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^2(c+}{2d\sqrt{a+a\sin}}$$

Mathematica [C] time = 0.0654945, size = 42, normalized size = 0.36

$$-\frac{a\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{6d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -(a*Hypergeometric2F1[-3/2, 2, -1/2, (1 + Sin[c + d*x])/2])/(6*d*(a + a*Sin
[c + d*x])^(3/2))
```

Maple [A] time = 0.185, size = 107, normalized size = 0.9

$$2\,\frac{a^3}{d}\left(-1/4\,\frac{1}{a^3}\left(1/4\,\frac{\sqrt{a+a\sin(dx+c)}}{a\sin(dx+c)-a} - 5/8\,\frac{\sqrt{2}}{\sqrt{a}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)\right) - 1/4\,\frac{1}{a^3\sqrt{a+a\sin(dx+c)}} - 1/12\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*a^3*(-1/4/a^3*(1/4*(a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c)-a)-5/8*2^(1/2)/a^
(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))-1/4/a^3/(a+a*sin
(d*x+c))^(1/2)-1/12/a^2/(a+a*sin(d*x+c))^(3/2))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27795, size = 396, normalized size = 3.41

$$\frac{15\sqrt{2}\left(\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\left(15\cos\left(dx+c\right)^{2}-10\text{ s}\right.}{96\left(ad\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*sqrt(a)*log
(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(
d*x + c) - 1)) - 4*(15*cos(d*x + c)^2 - 10*sin(d*x + c) - 2)*sqrt(a*sin(d*x
 + c) + a))/(a*d*cos(d*x + c)^2*sin(d*x + c) + a*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{3}\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A] time = 1.10862, size = 143, normalized size = 1.23

$$-\frac{a^3\left(\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\sin(dx+c)+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{8(3a\sin(dx+c)+4a)}{(a\sin(dx+c)+a)^{\frac{3}{2}}a^3}+\frac{6\sqrt{a\sin(dx+c)+a}}{(a\sin(dx+c)-a)a^3}\right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*a^3*(15*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*sin(d*x + c) + a)/sqrt(-a))
/(sqrt(-a)*a^3) + 8*(3*a*sin(d*x + c) + 4*a)/((a*sin(d*x + c) + a)^(3/2)*a^
3) + 6*sqrt(a*sin(d*x + c) + a)/((a*sin(d*x + c) - a)*a^3))/d
```

3.166 $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=162

$$-\frac{35a\cos(c+dx)}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{35\sec(c+dx)}{48d\sqrt{a\sin(c+dx)+a}}-\frac{7a\sec(c+dx)}{24d(a\sin(c+dx)+a)^{3/2}}-\frac{35\tanh^{-1}\Big(}{64}$$

```
[Out] (-35*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(6
4*Sqrt[2]*Sqrt[a]*d) - (35*a*Cos[c + d*x])/(64*d*(a + a*Sin[c + d*x])^(3/2)
) - (7*a*Sec[c + d*x])/(24*d*(a + a*Sin[c + d*x])^(3/2)) + (35*Sec[c + d*x]
)/(48*d*Sqrt[a + a*Sin[c + d*x]]) + Sec[c + d*x]^3/(3*d*Sqrt[a + a*Sin[c +
d*x]])
```

Rubi [A] time = 0.217602, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2687, 2681, 2650, 2649, 206}

$$-\frac{35a\cos(c+dx)}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{35\sec(c+dx)}{48d\sqrt{a\sin(c+dx)+a}}-\frac{7a\sec(c+dx)}{24d(a\sin(c+dx)+a)^{3/2}}-\frac{35\tanh^{-1}\Big(}{64}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-35*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(6
4*Sqrt[2]*Sqrt[a]*d) - (35*a*Cos[c + d*x])/(64*d*(a + a*Sin[c + d*x])^(3/2)
) - (7*a*Sec[c + d*x])/(24*d*(a + a*Sin[c + d*x])^(3/2)) + (35*Sec[c + d*x]
)/(48*d*Sqrt[a + a*Sin[c + d*x]]) + Sec[c + d*x]^3/(3*d*Sqrt[a + a*Sin[c +
d*x]])
```


Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sec^3(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{1}{6}(7a)\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$
$$= -\frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^3(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{35}{48}\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= -\frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}} + \frac{35\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^3(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{1}{32}(35a)\int$$
$$= -\frac{35a\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}} - \frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}} + \frac{35\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^3}{3d\sqrt{a+}}$$
$$= -\frac{35a\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}} - \frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}} + \frac{35\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^3}{3d\sqrt{a+}}$$
$$= -\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}d} - \frac{35a\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}} - \frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}} + \frac{}{48d}$$

Mathematica [C] time = 0.585173, size = 117, normalized size = 0.72

$$\frac{\sec^3(c+dx)(329\sin(c+dx)+105\sin(3(c+dx))+70\cos(2(c+dx))+102)+(420+420i)(-1)^{3/4}\left(\sin\left(\frac{1}{2}(c+dx)\right)+c\right.}{768d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((420 + 420*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x
)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + Sec[c + d*x]^3*(102 + 70*Cos
[2*(c + d*x)] + 329*Sin[c + d*x] + 105*Sin[3*(c + d*x)]))/(768*d*Sqrt[a*(1
+ Sin[c + d*x])])
```

Maple [A] time = 0.181, size = 231, normalized size = 1.4

$$\frac{1}{(384\sin(dx+c)-384)(1+\sin(dx+c))\cos(dx+c)d}\left(-210a^{7/2}\sin(dx+c)(\cos(dx+c))^2+\left(210\sqrt{2}\mathrm{Artanh}\left(1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/384*(-210*a^(7/2)*sin(d*x+c)*cos(d*x+c)^2+(210*2^(1/2)*arctanh(1/2*(a-a*s
in(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*(a-a*sin(d*x+c))^(3/2)-112*a^(7/2))*s
in(d*x+c)+(-105*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))
*a^2*(a-a*sin(d*x+c))^(3/2)-70*a^(7/2))*cos(d*x+c)^2+210*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*(a-a*sin(d*x+c))^(3/2)-16*a^(
7/2))/a^(7/2)/(sin(d*x+c)-1)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{4}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 2.43844, size = 624, normalized size = 3.85

$$\frac{105\sqrt{2}\left(\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{3}\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-\cdots}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-\cdots}\right.}{768\left(ad\cos\left(dx+c\right)^{3}\sin\left(dx+c\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(105*sqrt(2)*(cos(d*x + c)^3*sin(d*x + c) + cos(d*x + c)^3)*sqrt(a)*l
og(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x
 + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d
*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x
+ c) - 2)) + 4*(35*cos(d*x + c)^2 + 7*(15*cos(d*x + c)^2 + 8)*sin(d*x + c)
+ 8)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d
*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] time = 2.6105, size = 1006, normalized size = 6.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(105*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*d
*x + 1/2*c) + 1)) + 16*(15*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^5 - 33*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^4*sqrt(a) - 22*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^3*a + 66*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(3/2) + 51*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^2 + 11*a^(5/2))/(((sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^3*sgn(tan(1/
2*d*x + 1/2*c) + 1)) + 6*(53*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^7 + 179*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*sqrt(a) + 127*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a - 195*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) + 7*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^2 + 121*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) - 67*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^3 + 15*a^(7/2))/((
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 2*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*sqrt(a)
- a)^4*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.167 $\int \frac{\sec^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=175

$$-\frac{63}{128d\sqrt{a\sin(c+dx)+a}}-\frac{21a}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{63\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d}+\frac{\sec^4(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{63\sec^2}{160d\sqrt{a\sin}}$$

```
[Out] (63*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(128*Sqrt[2]*Sqrt[
a]*d) - (21*a)/(64*d*(a + a*Sin[c + d*x])^(3/2)) - (9*a*Sec[c + d*x]^2)/(40
*d*(a + a*Sin[c + d*x])^(3/2)) - 63/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (63*
Sec[c + d*x]^2)/(160*d*Sqrt[a + a*Sin[c + d*x]]) + Sec[c + d*x]^4/(4*d*Sqrt
[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.262906, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2687, 2681, 2667, 51, 63, 206}

$$-\frac{63}{128d\sqrt{a\sin(c+dx)+a}}-\frac{21a}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{63\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d}+\frac{\sec^4(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{63\sec^2}{160d\sqrt{a\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (63*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(128*Sqrt[2]*Sqrt[
a]*d) - (21*a)/(64*d*(a + a*Sin[c + d*x])^(3/2)) - (9*a*Sec[c + d*x]^2)/(40
*d*(a + a*Sin[c + d*x])^(3/2)) - 63/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (63*
Sec[c + d*x]^2)/(160*d*Sqrt[a + a*Sin[c + d*x]]) + Sec[c + d*x]^4/(4*d*Sqrt
[a + a*Sin[c + d*x]])
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sec^4(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{1}{8}(9a)\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$
$$= -\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^4(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{63}{80}\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= -\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}} + \frac{63\sec^2(c+dx)}{160d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^4(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{1}{64}(63a)\int$$
$$= -\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}} + \frac{63\sec^2(c+dx)}{160d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^4(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{(63a^2)\,\mathrm{Subs}}{}$$
$$= -\frac{21a}{64d(a+a\sin(c+dx))^{3/2}} - \frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}} + \frac{63\sec^2(c+dx)}{160d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^4}{4d\sqrt{a+}}$$
$$= -\frac{21a}{64d(a+a\sin(c+dx))^{3/2}} - \frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}} - \frac{63}{128d\sqrt{a+a\sin(c+dx)}} + \frac{63\,\mathrm{se}}{160d\sqrt{a}}$$
$$= -\frac{21a}{64d(a+a\sin(c+dx))^{3/2}} - \frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}} - \frac{63}{128d\sqrt{a+a\sin(c+dx)}} + \frac{63\,\mathrm{se}}{160d\sqrt{a}}$$
$$= \frac{63\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d} - \frac{21a}{64d(a+a\sin(c+dx))^{3/2}} - \frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}} - \frac{}{128d\sqrt{a}}$$

Mathematica [C] time = 0.0796178, size = 44, normalized size = 0.25

$$-\frac{a^2\,{}_2F_1\left(-\frac{5}{2},3;-\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{20d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -(a^2*Hypergeometric2F1[-5/2, 3, -3/2, (1 + Sin[c + d*x])/2])/(20*d*(a + a*
Sin[c + d*x])^(5/2))
```

Maple [A] time = 0.244, size = 135, normalized size = 0.8

$$-2\,\frac{a^5}{d}\left(1/16\,\frac{1}{a^5}\left(1/16\,\frac{a\sqrt{a+a\sin(dx+c)}\,(15\,\sin(dx+c)-19)}{(a\sin(dx+c)-a)^2} - \frac{63\,\sqrt{2}}{32\,\sqrt{a}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)\right) + 3/16\,\frac{}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2*a^5*(1/16/a^5*(1/16*(a+a*sin(d*x+c))^(1/2)*a*(15*sin(d*x+c)-19)/(a*sin(d
*x+c)-a)^2-63/32*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)
/a^(1/2)))+3/16/a^5/(a+a*sin(d*x+c))^(1/2)+1/16/a^4/(a+a*sin(d*x+c))^(3/2)+
1/40/a^3/(a+a*sin(d*x+c))^(5/2))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.56751, size = 462, normalized size = 2.64

$$\frac{315\sqrt{2}\left(\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{4}\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\left(315\cos\left(dx+c\right)^{4}-42\right.}{2560\left(ad\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)+ad\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2560*(315*sqrt(2)*(cos(d*x + c)^4*sin(d*x + c) + cos(d*x + c)^4)*sqrt(a)*
log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(s
in(d*x + c) - 1)) - 4*(315*cos(d*x + c)^4 - 42*cos(d*x + c)^2 - 6*(35*cos(d
*x + c)^2 + 24)*sin(d*x + c) - 16)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x +
 c)^4*sin(d*x + c) + a*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11151, size = 194, normalized size = 1.11

$$-\frac{a^5\left(\frac{315\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\sin(dx+c)+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^5}+\frac{10\left(15\,(a\sin(dx+c)+a)^{\frac{3}{2}}-34\sqrt{a\sin(dx+c)+a}a\right)}{(a\sin(dx+c)-a)^2a^5}+\frac{32\left(15\,(a\sin(dx+c)+a)^2+5\,(a\sin(dx+c)+a)a+2\,a^2\right)}{(a\sin(dx+c)+a)^{\frac{5}{2}}a^5}\right)}{1280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/1280*a^5*(315*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*sin(d*x + c) + a)/sqrt(-
a))/(sqrt(-a)*a^5) + 10*(15*(a*sin(d*x + c) + a)^(3/2) - 34*sqrt(a*sin(d*x
+ c) + a)*a)/((a*sin(d*x + c) - a)^2*a^5) + 32*(15*(a*sin(d*x + c) + a)^2 +
 5*(a*sin(d*x + c) + a)*a + 2*a^2)/((a*sin(d*x + c) + a)^(5/2)*a^5))/d
```

3.168 $\int \frac{\sec^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=221

$$-\frac{231a\cos(c+dx)}{512d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^5(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{11\sec^3(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}-\frac{11a\sec^3(c+dx)}{60d(a\sin(c+dx)+a)^{3/2}}+\frac{77\sec}{128d\sqrt{a\sin}}$$

```
[Out] (-231*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(
512*Sqrt[2]*Sqrt[a]*d) - (231*a*Cos[c + d*x])/(512*d*(a + a*Sin[c + d*x])^(
3/2)) - (77*a*Sec[c + d*x])/(320*d*(a + a*Sin[c + d*x])^(3/2)) - (11*a*Sec[
c + d*x]^3)/(60*d*(a + a*Sin[c + d*x])^(3/2)) + (77*Sec[c + d*x])/(128*d*Sq
rt[a + a*Sin[c + d*x]]) + (11*Sec[c + d*x]^3)/(40*d*Sqrt[a + a*Sin[c + d*x]
]) + Sec[c + d*x]^5/(5*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.35688, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2687, 2681, 2650, 2649, 206}

$$-\frac{231a\cos(c+dx)}{512d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^5(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{11\sec^3(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}-\frac{11a\sec^3(c+dx)}{60d(a\sin(c+dx)+a)^{3/2}}+\frac{77\sec}{128d\sqrt{a\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-231*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(
512*Sqrt[2]*Sqrt[a]*d) - (231*a*Cos[c + d*x])/(512*d*(a + a*Sin[c + d*x])^(
3/2)) - (77*a*Sec[c + d*x])/(320*d*(a + a*Sin[c + d*x])^(3/2)) - (11*a*Sec[
c + d*x]^3)/(60*d*(a + a*Sin[c + d*x])^(3/2)) + (77*Sec[c + d*x])/(128*d*Sq
rt[a + a*Sin[c + d*x]]) + (11*Sec[c + d*x]^3)/(40*d*Sqrt[a + a*Sin[c + d*x]
]) + Sec[c + d*x]^5/(5*d*Sqrt[a + a*Sin[c + d*x]])
```


Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sec^5(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{1}{10}(11a)\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$
$$= -\frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^5(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{33}{40}\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= -\frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^3(c+dx)}{40d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^5(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{1}{80}(77a)\int$$
$$= -\frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}} - \frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^3(c+dx)}{40d\sqrt{a+a\sin(c+dx)}} + \frac{\sec}{5d\sqrt{a+}}$$
$$= -\frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}} - \frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{77\sec(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} + \frac{11}{40d\sqrt{}}$$
$$= -\frac{231a\cos(c+dx)}{512d(a+a\sin(c+dx))^{3/2}} - \frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}} - \frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{}{128}$$
$$= -\frac{231a\cos(c+dx)}{512d(a+a\sin(c+dx))^{3/2}} - \frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}} - \frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{}{128}$$
$$= -\frac{231\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{512\sqrt{2}\sqrt{a}d} - \frac{231a\cos(c+dx)}{512d(a+a\sin(c+dx))^{3/2}} - \frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}} -$$

Mathematica [C] time = 0.661624, size = 140, normalized size = 0.63

$$\frac{\frac{1}{16}\sec^5(c+dx)(36850\sin(c+dx)+17787\sin(3(c+dx))+3465\sin(5(c+dx))+11352\cos(2(c+dx))+2310\cos(4(c}{7680d\sqrt{a(s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((3465 + 3465*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d
*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + (Sec[c + d*x]^5*(11090 + 1
1352*Cos[2*(c + d*x)] + 2310*Cos[4*(c + d*x)] + 36850*Sin[c + d*x] + 17787*
Sin[3*(c + d*x)] + 3465*Sin[5*(c + d*x)]))/16)/(7680*d*Sqrt[a*(1 + Sin[c +
d*x])])
```

Maple [A] time = 0.184, size = 308, normalized size = 1.4

$$-\frac{1}{15360\,(\sin(dx+c)-1)^2\,(1+\sin(dx+c))^2\cos(dx+c)\,d}\Big(-6930\,a^{11/2}\sin(dx+c)\,(\cos(dx+c))^4+\Big(-3696\,a^{11/2}-34$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/15360*(-6930*a^(11/2)*sin(d*x+c)*cos(d*x+c)^4+(-3696*a^(11/2)-3465*2^(1/
2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3*(a-a*sin(d*x+c))
^(5/2))*cos(d*x+c)^2*sin(d*x+c)+(-2816*a^(11/2)+13860*2^(1/2)*arctanh(1/2*(
a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3*(a-a*sin(d*x+c))^(5/2))*sin(d*x+
c)-2310*a^(11/2)*cos(d*x+c)^4+(-528*a^(11/2)-10395*2^(1/2)*arctanh(1/2*(a-a
*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3*(a-a*sin(d*x+c))^(5/2))*cos(d*x+c)^
2-256*a^(11/2)+13860*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(
1/2))*a^3*(a-a*sin(d*x+c))^(5/2))/a^(11/2)/(sin(d*x+c)-1)^2/(1+sin(d*x+c))^
2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^6}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^6/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A] time = 2.72939, size = 695, normalized size = 3.14

$$\frac{3465\sqrt{2}\left(\cos(dx+c)^5\sin(dx+c)+\cos(dx+c)^5\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c}\right.}{30720\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30720*(3465*sqrt(2)*(cos(d*x + c)^5*sin(d*x + c) + cos(d*x + c)^5)*sqrt(a
)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(
d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*si
n(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d
```

```
*x + c) - 2)) + 4*(1155*cos(d*x + c)^4 + 264*cos(d*x + c)^2 + 11*(315*cos(d
*x + c)^4 + 168*cos(d*x + c)^2 + 128)*sin(d*x + c) + 128)*sqrt(a*sin(d*x +
c) + a))/(a*d*cos(d*x + c)^5*sin(d*x + c) + a*d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.42085, size = 1438, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/7680*(3465*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2
*d*x + 1/2*c) + 1)) + 64*(165*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^9 - 915*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^8*sqrt(a) + 1340*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^7*a + 1420*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(3/2) - 3434*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a^2 - 1610*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(5/2) + 2700*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^3 + 262
0*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(
7/2) + 845*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))*a^4 + 101*a^(9/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))*sqrt(a) - a)^5*sgn(tan(1/2*d*x + 1/2*c) + 1)) + 10*(1323*(
sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^11 + 879
3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sq
```

```
rt(a) + 20907*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^9*a + 9237*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^8*a^(3/2) - 26274*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^7*a^2 - 12806*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*
tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(5/2) + 30342*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^5*a^3 - 4182*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(7/2) - 12793*(sqrt(a)*ta
n(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^3*a^4 + 9405*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(9/2) -
2721*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))*a^
5 + 337*a^(11/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))*sqrt(a) - a)^6*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.169 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^7d}+\frac{4(a\sin(c+dx)+a)^{9/2}}{3a^6d}-\frac{24(a\sin(c+dx)+a)^{7/2}}{7a^5d}+\frac{16(a\sin(c+dx)+a)^{5/2}}{5a^4d}$$

```
[Out] (16*(a + a*Sin[c + d*x])^(5/2))/(5*a^4*d) - (24*(a + a*Sin[c + d*x])^(7/2))
/(7*a^5*d) + (4*(a + a*Sin[c + d*x])^(9/2))/(3*a^6*d) - (2*(a + a*Sin[c + d
*x])^(11/2))/(11*a^7*d)
```

Rubi [A] time = 0.0822959, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^7d}+\frac{4(a\sin(c+dx)+a)^{9/2}}{3a^6d}-\frac{24(a\sin(c+dx)+a)^{7/2}}{7a^5d}+\frac{16(a\sin(c+dx)+a)^{5/2}}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(5/2))/(5*a^4*d) - (24*(a + a*Sin[c + d*x])^(7/2))
/(7*a^5*d) + (4*(a + a*Sin[c + d*x])^(9/2))/(3*a^6*d) - (2*(a + a*Sin[c + d
*x])^(11/2))/(11*a^7*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{3/2}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{3/2} - 12a^2(a+x)^{5/2} + 6a(a+x)^{7/2} - (a+x)^{9/2}\right)dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{16(a+a\sin(c+dx))^{5/2}}{5a^4 d} - \frac{24(a+a\sin(c+dx))^{7/2}}{7a^5 d} + \frac{4(a+a\sin(c+dx))^{9/2}}{3a^6 d} - \frac{2(a+a\sin(\ldots}{11a\ldots}$$

Mathematica [A] time = 0.225519, size = 54, normalized size = 0.56

$$-\frac{2\left(105\sin^3(c+dx) - 455\sin^2(c+dx) + 755\sin(c+dx) - 533\right)(a(\sin(c+dx)+1))^{5/2}}{1155a^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a*(1 + Sin[c + d*x]))^(5/2)*(-533 + 755*Sin[c + d*x] - 455*Sin[c + d*x
]^2 + 105*Sin[c + d*x]^3))/(1155*a^4*d)
```

Maple [A] time = 0.089, size = 57, normalized size = 0.6

$$\frac{210\,(\cos(dx+c))^2\sin(dx+c) - 910\,(\cos(dx+c))^2 - 1720\,\sin(dx+c) + 1976}{1155\,a^4 d}(a+a\sin(dx+c))^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/1155/a^4*(a+a*sin(d*x+c))^(5/2)*(105*cos(d*x+c)^2*sin(d*x+c)-455*cos(d*x+
c)^2-860*sin(d*x+c)+988)/d
```

Maxima [A] time = 0.973599, size = 97, normalized size = 1.

$$-\frac{2\left(105\,(a\sin(dx+c)+a)^{\frac{11}{2}} - 770\,(a\sin(dx+c)+a)^{\frac{9}{2}}a + 1980\,(a\sin(dx+c)+a)^{\frac{7}{2}}a^2 - 1848\,(a\sin(dx+c)+a)^{\frac{5}{2}}a^3\right)}{1155\,a^7 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/1155*(105*(a*sin(d*x + c) + a)^(11/2) - 770*(a*sin(d*x + c) + a)^(9/2)*a
 + 1980*(a*sin(d*x + c) + a)^(7/2)*a^2 - 1848*(a*sin(d*x + c) + a)^(5/2)*a^
3)/(a^7*d)
```

Fricas [A] time = 2.29302, size = 204, normalized size = 2.1

$$\frac{2\left(245\,\cos\left(dx+c\right)^{4}+32\,\cos\left(dx+c\right)^{2}-\left(105\,\cos\left(dx+c\right)^{4}-160\,\cos\left(dx+c\right)^{2}-256\right)\sin\left(dx+c\right)+256\right)\sqrt{a\sin\left(dx+\right.}}{1155\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(245*cos(d*x + c)^4 + 32*cos(d*x + c)^2 - (105*cos(d*x + c)^4 - 160*
cos(d*x + c)^2 - 256)*sin(d*x + c) + 256)*sqrt(a*sin(d*x + c) + a)/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25644, size = 97, normalized size = 1.

$$-\frac{2\left(105\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}-770\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a+1980\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a^{2}-1848\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a^{3}\right)}{1155\,a^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2/1155*(105*(a*sin(d*x + c) + a)^(11/2) - 770*(a*sin(d*x + c) + a)^(9/2)*a
 + 1980*(a*sin(d*x + c) + a)^(7/2)*a^2 - 1848*(a*sin(d*x + c) + a)^(5/2)*a^
3)/(a^7*d)
```

3.170 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{8a^2\cos^7(c+dx)}{63d(a\sin(c+dx)+a)^{7/2}}-\frac{2a\cos^7(c+dx)}{9d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] (-8*a^2*Cos[c + d*x]^7)/(63*d*(a + a*Sin[c + d*x])^(7/2)) - (2*a*Cos[c + d*
x]^7)/(9*d*(a + a*Sin[c + d*x])^(5/2))
```

Rubi [A] time = 0.11561, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{8a^2\cos^7(c+dx)}{63d(a\sin(c+dx)+a)^{7/2}}-\frac{2a\cos^7(c+dx)}{9d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^7)/(63*d*(a + a*Sin[c + d*x])^(7/2)) - (2*a*Cos[c + d*
x]^7)/(9*d*(a + a*Sin[c + d*x])^(5/2))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2a\cos^7(c+dx)}{9d(a+a\sin(c+dx))^{5/2}} + \frac{1}{9}(4a)\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$$
$$= -\frac{8a^2\cos^7(c+dx)}{63d(a+a\sin(c+dx))^{7/2}} - \frac{2a\cos^7(c+dx)}{9d(a+a\sin(c+dx))^{5/2}}$$

Mathematica [A] time = 0.183941, size = 49, normalized size = 0.78

$$-\frac{2(7\sin(c+dx)+11)\cos^7(c+dx)}{63d(\sin(c+dx)+1)^2(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^7*(11 + 7*Sin[c + d*x]))/(63*d*(1 + Sin[c + d*x])^2*(a*(1
+ Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.178, size = 57, normalized size = 0.9

$$-\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^4(7\sin(dx+c)+11)}{63\,ad\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/63/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^4*(7*sin(d*x+c)+11)/cos(d*x+c)/(a+a*s
in(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^6}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 2.12587, size = 375, normalized size = 5.95

$$\frac{2\left(7\cos\left(dx+c\right)^{5}+17\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{3}+4\cos\left(dx+c\right)^{2}-\left(7\cos\left(dx+c\right)^{4}-10\cos\left(dx+c\right)^{3}-12\cos\left(\right.\right.\right.}{63\left(a^{2}d\cos\left(dx+c\right)+a^{2}d\sin\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*cos(d*x + c)^5 + 17*cos(d*x + c)^4 - 2*cos(d*x + c)^3 + 4*cos(d*x +
 c)^2 - (7*cos(d*x + c)^4 - 10*cos(d*x + c)^3 - 12*cos(d*x + c)^2 - 16*cos(
d*x + c) - 32)*sin(d*x + c) - 16*cos(d*x + c) - 32)*sqrt(a*sin(d*x + c) + a
)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.15408, size = 421, normalized size = 6.68

$$\frac{\left(\left(\left(\left(\left(\left(\left(\frac{11\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{12}}-\frac{63\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{12}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{144\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{12}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{168\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{12}}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4032*((((((((((11*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)/a^12
 - 63*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^12)*tan(1/2*d*x + 1/2*c) + 144*sgn(ta
n(1/2*d*x + 1/2*c) + 1)/a^12)*tan(1/2*d*x + 1/2*c) - 168*sgn(tan(1/2*d*x +
1/2*c) + 1)/a^12)*tan(1/2*d*x + 1/2*c) + 126*sgn(tan(1/2*d*x + 1/2*c) + 1)/
a^12)*tan(1/2*d*x + 1/2*c) - 126*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^12)*tan(1/
2*d*x + 1/2*c) + 168*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^12)*tan(1/2*d*x + 1/2*
c) - 144*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^12)*tan(1/2*d*x + 1/2*c) + 63*sgn(
tan(1/2*d*x + 1/2*c) + 1)/a^12)*tan(1/2*d*x + 1/2*c) - 11*sgn(tan(1/2*d*x +
 1/2*c) + 1)/a^12)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(9/2) + 32*sqrt(2)*sgn(ta
n(1/2*d*x + 1/2*c) + 1)/a^(33/2))/d
```

3.171 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{2(a\sin(c+dx)+a)^{7/2}}{7a^5d}-\frac{8(a\sin(c+dx)+a)^{5/2}}{5a^4d}+\frac{8(a\sin(c+dx)+a)^{3/2}}{3a^3d}$$

```
[Out] (8*(a + a*Sin[c + d*x])^(3/2))/(3*a^3*d) - (8*(a + a*Sin[c + d*x])^(5/2))/(
5*a^4*d) + (2*(a + a*Sin[c + d*x])^(7/2))/(7*a^5*d)
```

Rubi [A] time = 0.0745764, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{7/2}}{7a^5d}-\frac{8(a\sin(c+dx)+a)^{5/2}}{5a^4d}+\frac{8(a\sin(c+dx)+a)^{3/2}}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(3/2))/(3*a^3*d) - (8*(a + a*Sin[c + d*x])^(5/2))/(
5*a^4*d) + (2*(a + a*Sin[c + d*x])^(7/2))/(7*a^5*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2\sqrt{a+x}\,dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \left(4a^2\sqrt{a+x}-4a(a+x)^{3/2}+(a+x)^{5/2}\right)dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= \frac{8(a+a\sin(c+dx))^{3/2}}{3a^3d}-\frac{8(a+a\sin(c+dx))^{5/2}}{5a^4d}+\frac{2(a+a\sin(c+dx))^{7/2}}{7a^5d}
\end{aligned}
$$

Mathematica [A] time = 0.0964159, size = 44, normalized size = 0.6

$$
\frac{2\left(15\sin^2(c+dx)-54\sin(c+dx)+71\right)(a(\sin(c+dx)+1))^{3/2}}{105a^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(3/2)*(71 - 54*Sin[c + d*x] + 15*Sin[c + d*x]^2))
/(105*a^3*d)
```

Maple [A] time = 0.078, size = 41, normalized size = 0.6

$$
-\frac{30\,(\cos(dx+c))^2+108\,\sin(dx+c)-172}{105\,a^3d}\,(a+a\sin(dx+c))^{\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/105/a^3*(a+a*sin(d*x+c))^(3/2)*(15*cos(d*x+c)^2+54*sin(d*x+c)-86)/d
```

Maxima [A] time = 0.963007, size = 74, normalized size = 1.01

$$
\frac{2\left(15\,(a\sin(dx+c)+a)^{\frac{7}{2}}-84\,(a\sin(dx+c)+a)^{\frac{5}{2}}a+140\,(a\sin(dx+c)+a)^{\frac{3}{2}}a^2\right)}{105\,a^5d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(a*sin(d*x + c) + a)^(7/2) - 84*(a*sin(d*x + c) + a)^(5/2)*a + 14
0*(a*sin(d*x + c) + a)^(3/2)*a^2)/(a^5*d)
```

Fricas [A] time = 2.2489, size = 142, normalized size = 1.95

$$\frac{2\left(39\cos\left(dx+c\right)^{2}-\left(15\cos\left(dx+c\right)^{2}-32\right)\sin\left(dx+c\right)+32\right)\sqrt{a\sin\left(dx+c\right)+a}}{105\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(39*cos(d*x + c)^2 - (15*cos(d*x + c)^2 - 32)*sin(d*x + c) + 32)*sqrt
(a*sin(d*x + c) + a)/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20317, size = 74, normalized size = 1.01

$$\frac{2\left(15\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}-84\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a+140\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a^{2}\right)}{105\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(15*(a*sin(d*x + c) + a)^(7/2) - 84*(a*sin(d*x + c) + a)^(5/2)*a + 14
0*(a*sin(d*x + c) + a)^(3/2)*a^2)/(a^5*d)
```

3.172 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{2a\cos^5(c+dx)}{5d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] (-2*a*Cos[c + d*x]^5)/(5*d*(a + a*Sin[c + d*x])^(5/2))
```

Rubi [A] time = 0.0581363, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2673}

$$-\frac{2a\cos^5(c+dx)}{5d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*Cos[c + d*x]^5)/(5*d*(a + a*Sin[c + d*x])^(5/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx=-\frac{2a\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}}$$

Mathematica [A] time = 0.0539437, size = 42, normalized size = 1.4

$$-\frac{2\cos^5(c+dx)\sqrt{a(\sin(c+dx)+1)}}{5a^2d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5*Sqrt[a*(1 + Sin[c + d*x])])/(5*a^2*d*(1 + Sin[c + d*x])^
3)
```

Maple [A] time = 0.105, size = 47, normalized size = 1.6

$$\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^3}{5\,ad\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 2.20367, size = 257, normalized size = 8.57

$$\frac{2\left(\cos(dx+c)^3+3\cos(dx+c)^2-\left(\cos(dx+c)^2-2\cos(dx+c)-4\right)\sin(dx+c)-2\cos(dx+c)-4\right)\sqrt{a\sin(dx+c)+\cdots}}{5\left(a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 - (cos(d*x + c)^2 - 2*cos(d*x + c) -
 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a*sin(d*x + c) + a)/(a^2*d*cos(
d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.0121, size = 269, normalized size = 8.97

$$\frac{\frac{\left(\left(\left(\left(\frac{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^8}-\frac{5\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^8}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{10\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^8}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{10\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^8}\right)\tan\left(\frac{1}{2}\,d\right.\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{5}{2}}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/20*((((((sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)/a^8 - 5*sgn(t
an(1/2*d*x + 1/2*c) + 1)/a^8)*tan(1/2*d*x + 1/2*c) + 10*sgn(tan(1/2*d*x + 1
/2*c) + 1)/a^8)*tan(1/2*d*x + 1/2*c) - 10*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^8
)*tan(1/2*d*x + 1/2*c) + 5*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^8)*tan(1/2*d*x +
 1/2*c) - sgn(tan(1/2*d*x + 1/2*c) + 1)/a^8)/(a*tan(1/2*d*x + 1/2*c)^2 + a)
^(5/2) + 4*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(21/2))/d
```

3.173 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=47

$$\frac{4\sqrt{a\sin(c+dx)+a}}{a^2d}-\frac{2(a\sin(c+dx)+a)^{3/2}}{3a^3d}$$

```
[Out] (4*Sqrt[a + a*Sin[c + d*x]])/(a^2*d) - (2*(a + a*Sin[c + d*x])^(3/2))/(3*a^
3*d)
```

Rubi [A] time = 0.0665267, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{4\sqrt{a\sin(c+dx)+a}}{a^2d}-\frac{2(a\sin(c+dx)+a)^{3/2}}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (4*Sqrt[a + a*Sin[c + d*x]])/(a^2*d) - (2*(a + a*Sin[c + d*x])^(3/2))/(3*a^
3*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{2a}{\sqrt{a+x}} - \sqrt{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{4\sqrt{a+a\sin(c+dx)}}{a^2 d} - \frac{2(a+a\sin(c+dx))^{3/2}}{3a^3 d}$$

Mathematica [A] time = 0.0500463, size = 32, normalized size = 0.68

$$-\frac{2(\sin(c+dx)-5)\sqrt{a(\sin(c+dx)+1)}}{3a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(-5 + Sin[c + d*x])*Sqrt[a*(1 + Sin[c + d*x])])/(3*a^2*d)
```

Maple [A] time = 0.08, size = 29, normalized size = 0.6

$$-\frac{2\sin(dx+c)-10}{3a^2 d}\sqrt{a+a\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/3/a^2*(a+a*sin(d*x+c))^(1/2)*(sin(d*x+c)-5)/d
```

Maxima [A] time = 0.964622, size = 49, normalized size = 1.04

$$-\frac{2\left((a\sin(dx+c)+a)^{\frac{3}{2}} - 6\sqrt{a\sin(dx+c)+a}\,a\right)}{3a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*((a*sin(d*x + c) + a)^(3/2) - 6*sqrt(a*sin(d*x + c) + a)*a)/(a^3*d)
```

Fricas [A] time = 2.20186, size = 78, normalized size = 1.66

$$-\frac{2\sqrt{a\sin\left(dx+c\right)+a}\left(\sin\left(dx+c\right)-5\right)}{3\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(a*sin(d*x + c) + a)*(sin(d*x + c) - 5)/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15639, size = 49, normalized size = 1.04

$$-\frac{2\left(\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}-6\sqrt{a\sin\left(dx+c\right)+a}a\right)}{3\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*((a*sin(d*x + c) + a)^(3/2) - 6*sqrt(a*sin(d*x + c) + a)*a)/(a^3*d)
```

3.174 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{2\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}$$

```
[Out] (-2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(3/2)*d) + (2*Cos[c + d*x])/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.0806601, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2679, 2649, 206}

$$\frac{2\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(3/2)*d) + (2*Cos[c + d*x])/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}+\frac{2\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a} \\
&= \frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}-\frac{4\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad} \\
&= -\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}+\frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.145741, size = 84, normalized size = 1.11

$$
\frac{2\cos^3(c+dx)\left(\sqrt{1-\sin(c+dx)}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\right)}{d(1-\sin(c+dx))^{3/2}(a(\sin(c+dx)+1))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*Cos[c + d*x]^3*(-(Sqrt[2]*ArcTanh[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]) + Sqr
t[1 - Sin[c + d*x]]))/(d*(1 - Sin[c + d*x])^(3/2)*(a*(1 + Sin[c + d*x]))^(3
/2))
```

Maple [A] time = 0.134, size = 94, normalized size = 1.2

$$
2\,\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}}{a^2\cos(dx+c)\sqrt{a+a\sin(dx+c)}d}\left(\sqrt{a-a\sin(dx+c)}-\sqrt{a}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/a^2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*((a-a*sin(d*x+c))^(1/2)-a^(1
/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/cos(d*x+c)
/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B]   time = 2.26083, size = 536, normalized size = 7.05

$$\frac{\frac{\sqrt{2}(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+2\sqrt{a\sin(dx+c)+a}\cdots}{a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 - (cos(
d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x +
c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos
(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 2*sqrt(a*sin(d*x
 + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a^2*d*cos(d*x + c) + a^2*d*s
in(d*x + c) + a^2*d)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 1.90494, size = 257, normalized size = 3.38

$$-\frac{2\left(\frac{\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{1}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}+\frac{\sqrt{2}\left(2\,\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{-a}a^{\frac{3}{2}}}-\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\right.}\right.}{2\,\sqrt{-a}}\right.}{\sqrt{-a}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2*((tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 1/(a*sgn(tan(
1/2*d*x + 1/2*c) + 1)))/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(2)*(2*sqr
t(a)*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sq
rt(-a)*a^(3/2)) - 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sg
n(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.175 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2}{ad\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -2/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.0335983, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$-\frac{2}{ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -2/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{(a+x)^{3/2}}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{2}{ad\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.0299117, size = 22, normalized size = 1.

$$-\frac{2}{ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -2/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Maple [A] time = 0.004, size = 21, normalized size = 1.

$$-2\,\frac{1}{da\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/a/d/(a+a*sin(d*x+c))^(1/2)
```

Maxima [A] time = 0.948987, size = 27, normalized size = 1.23

$$-\frac{2}{\sqrt{a\sin(dx+c)+a}\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(a*sin(d*x + c) + a)*a*d)
```

Fricas [A] time = 2.23214, size = 78, normalized size = 3.55

$$-\frac{2\sqrt{a\sin\left(dx+c\right)+a}}{a^2d\sin\left(dx+c\right)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*sin(d*x + c) + a)/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [A] time = 2.86371, size = 46, normalized size = 2.09

$$\begin{cases} \mathrm{NaN} & \text{for}\: c = \frac{3\pi}{2} \wedge d = 0 \\ \frac{x\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{\frac{3}{2}}} & \text{for}\: d = 0 \\ \mathrm{NaN} & \text{for}\: c = -dx + \frac{3\pi}{2} \\ -\frac{2}{ad\sqrt{a\sin\left(c+dx\right)+a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((nan, Eq(d, 0) & Eq(c, 3*pi/2)), (x*cos(c)/(a*sin(c) + a)**(3/2),
 Eq(d, 0)), (nan, Eq(c, -d*x + 3*pi/2)), (-2/(a*d*sqrt(a*sin(c + d*x) + a))
, True))
```

Giac [A] time = 1.13427, size = 27, normalized size = 1.23

$$-\frac{2}{\sqrt{a\sin\left(dx+c\right)+a}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(a*sin(d*x + c) + a)*a*d)
```

3.176 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{1}{2ad\sqrt{a\sin(c+dx)+a}}-\frac{1}{3d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*a^(3/2)*d) -
 1/(3*d*(a + a*Sin[c + d*x])^(3/2)) - 1/(2*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.0768569, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2667, 51, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{1}{2ad\sqrt{a\sin(c+dx)+a}}-\frac{1}{3d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```


```
[Out] ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*a^(3/2)*d) -
 1/(3*d*(a + a*Sin[c + d*x])^(3/2)) - 1/(2*a*d*Sqrt[a + a*Sin[c + d*x]])
```


Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{a\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= -\frac{1}{3d(a+a\sin(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, a\sin(c+dx)\right)}{2d}\\
&= -\frac{1}{3d(a+a\sin(c+dx))^{3/2}} - \frac{1}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, a\sin(c+dx\right.}{4ad}\\
&= -\frac{1}{3d(a+a\sin(c+dx))^{3/2}} - \frac{1}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+a\sin(c+}\right.}{2ad}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{1}{3d(a+a\sin(c+dx))^{3/2}} - \frac{1}{2ad\sqrt{a+a\sin(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 0.06767, size = 41, normalized size = 0.46

$$
-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{3d(a\sin(c+dx)+a)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sin[c + d*x])/2]/(3*d*(a + a*Sin[c +
 d*x])^(3/2))
```

Maple [A] time = 0.085, size = 71, normalized size = 0.8

$$-\frac{a}{d}\left(-\frac{\sqrt{2}}{4}\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a+a\sin(dx+c)}\frac{1}{\sqrt{a}}\right)a^{-\frac{5}{2}}+\frac{1}{2\,a^{2}}\frac{1}{\sqrt{a+a\sin(dx+c)}}+\frac{1}{3\,a}\left(a+a\sin(dx+c)\right)^{-\frac{3}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -a*(-1/4/a^(5/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)+1/2/a^2/(a+a*sin(d*x+c))^(1/2)+1/3/a/(a+a*sin(d*x+c))^(3/2))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.34851, size = 355, normalized size = 3.99

$$\frac{3\sqrt{2}\left(\cos(dx+c)^{2}-2\sin(dx+c)-2\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3\,a}{\sin(dx+c)-1}\right)+4\sqrt{a\sin(dx+c)+a}(3\sin(dx+c}{24\left(a^{2}d\cos(dx+c)^{2}-2\,a^{2}d\sin(dx+c)-2\,a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(2)*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*sqrt(a)*log(-(a*sin(d
*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) -
 1)) + 4*sqrt(a*sin(d*x + c) + a)*(3*sin(d*x + c) + 5))/(a^2*d*cos(d*x + c)
^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A] time = 1.11997, size = 103, normalized size = 1.16

$$-\frac{1}{12}\,a\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\sin(dx+c)+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2d}+\frac{2\,(3\,a\sin(dx+c)+5\,a)}{(a\sin(dx+c)+a)^{\frac{3}{2}}a^2d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/12*a*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*sin(d*x + c) + a)/sqrt(-a))/(s
qrt(-a)*a^2*d) + 2*(3*a*sin(d*x + c) + 5*a)/((a*sin(d*x + c) + a)^(3/2)*a^2
*d))
```

3.177 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{32\sqrt{2}a^{3/2}d}-\frac{15\cos(c+dx)}{32d(a\sin(c+dx)+a)^{3/2}}+\frac{5\sec(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec(c+dx)}{4d(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-15*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(3
2*Sqrt[2]*a^(3/2)*d) - (15*Cos[c + d*x])/(32*d*(a + a*Sin[c + d*x])^(3/2))
- Sec[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(3/2)) + (5*Sec[c + d*x])/(8*a*d*S
qrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.161535, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2681, 2687, 2650, 2649, 206}

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{32\sqrt{2}a^{3/2}d}-\frac{15\cos(c+dx)}{32d(a\sin(c+dx)+a)^{3/2}}+\frac{5\sec(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec(c+dx)}{4d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-15*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(3
2*Sqrt[2]*a^(3/2)*d) - (15*Cos[c + d*x])/(32*d*(a + a*Sin[c + d*x])^(3/2))
- Sec[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(3/2)) + (5*Sec[c + d*x])/(8*a*d*S
qrt[a + a*Sin[c + d*x]])
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= -\frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{5\int\frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{8a}\\
&= -\frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{5\sec(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}}+\frac{15}{16}\int\frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx\\
&= -\frac{15\cos(c+dx)}{32d(a+a\sin(c+dx))^{3/2}}-\frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{5\sec(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}}+\frac{15\int\ldots}{6\ldots}\\
&= -\frac{15\cos(c+dx)}{32d(a+a\sin(c+dx))^{3/2}}-\frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{5\sec(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}}-\frac{15\,\mathrm{Subst}\ldots}{\ldots}\\
&= -\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{32\sqrt{2}a^{3/2}d}-\frac{15\cos(c+dx)}{32d(a+a\sin(c+dx))^{3/2}}-\frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{\ldots}{8ad\ldots}
\end{aligned}$$

Mathematica [C] time = 0.264118, size = 224, normalized size = 1.67

$$\frac{\frac{8\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-7\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2+14\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+d\right.\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-4 + (8*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 14*Sin[(
c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 7*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^2 + (15 + 15*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*
(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (8*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/
(32*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.148, size = 202, normalized size = 1.5

$$-\frac{1}{(64+64\sin(dx+c))\cos(dx+c)d}\left(\sin(dx+c)\left(30\sqrt{a-a\sin(dx+c)}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)a^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/64/a^(7/2)*(sin(d*x+c)*(30*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/2*(a
-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2-40*a^(5/2))+(-15*(a-a*sin(d*x+c))
^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2+30*a
^(5/2))*cos(d*x+c)^2+30*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin
(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2-24*a^(5/2))/(1+sin(d*x+c))/cos(d*x+c)/(
a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 2.33137, size = 652, normalized size = 4.87

$$\frac{15\sqrt{2}\left(\cos\left(dx+c\right)^{3} - 2\cos\left(dx+c\right)\sin\left(dx+c\right) - 2\cos\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2} - 2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c) - \sin(dx+c}{\cos(dx+c)^{2} - (\cos(dx+c)+2)\,\mathrm{s}}\right.}{128\left(a^{2}d\cos\left(dx+c\right)^{3} - 2\,a^{2}d\cos\left(dx+c\right)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(15*sqrt(2)*(cos(d*x + c)^3 - 2*cos(d*x + c)*sin(d*x + c) - 2*cos(d*x
 + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*
sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x +
 c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x
 + c) - cos(d*x + c) - 2)) + 4*(15*cos(d*x + c)^2 - 20*sin(d*x + c) - 12)*s
qrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d
*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{2}\left(c + dx\right)}{\left(a\left(\sin\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage_2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.178 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d}-\frac{7}{16ad\sqrt{a\sin(c+dx)+a}}-\frac{7}{24d(a\sin(c+dx)+a)^{3/2}}+\frac{7\sec^2(c+dx)}{20ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^2(c}{5d(a\sin(c+}$$

```
[Out] (7*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*a^(3/2)
*d) - 7/(24*d*(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^2/(5*d*(a + a*Sin[
c + d*x])^(3/2)) - 7/(16*a*d*Sqrt[a + a*Sin[c + d*x]]) + (7*Sec[c + d*x]^2)
/(20*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.202478, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2681, 2687, 2667, 51, 63, 206}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d}-\frac{7}{16ad\sqrt{a\sin(c+dx)+a}}-\frac{7}{24d(a\sin(c+dx)+a)^{3/2}}+\frac{7\sec^2(c+dx)}{20ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^2(c}{5d(a\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (7*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*a^(3/2)
*d) - 7/(24*d*(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^2/(5*d*(a + a*Sin[
c + d*x])^(3/2)) - 7/(16*a*d*Sqrt[a + a*Sin[c + d*x]]) + (7*Sec[c + d*x]^2)
/(20*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} + \frac{7\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{10a}$$
$$= -\frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} + \frac{7\sec^2(c+dx)}{20ad\sqrt{a+a\sin(c+dx)}} + \frac{7}{8}\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$
$$= -\frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} + \frac{7\sec^2(c+dx)}{20ad\sqrt{a+a\sin(c+dx)}} + \frac{(7a)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, a\sin\right.}{8d}$$
$$= -\frac{7}{24d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} + \frac{7\sec^2(c+dx)}{20ad\sqrt{a+a\sin(c+dx)}} + \frac{7\,\mathrm{Subst}}{}$$
$$= -\frac{7}{24d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} - \frac{7}{16ad\sqrt{a+a\sin(c+dx)}} + \frac{7\,\mathrm{se}}{20ad\sqrt{a}}$$
$$= -\frac{7}{24d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} - \frac{7}{16ad\sqrt{a+a\sin(c+dx)}} + \frac{7\,\mathrm{se}}{20ad\sqrt{a}}$$
$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d} - \frac{7}{24d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} - \frac{}{16ad\sqrt{a}}$$

Mathematica [C] time = 0.0678499, size = 42, normalized size = 0.28

$$-\frac{a\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{10d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*Hypergeometric2F1[-5/2, 2, -3/2, (1 + Sin[c + d*x])/2])/(10*d*(a + a*Si
n[c + d*x])^(5/2))
```

Maple [A] time = 0.166, size = 124, normalized size = 0.8

$$2\,\frac{a^3}{d}\left(-1/16\,\frac{1}{a^4}\left(1/2\,\frac{\sqrt{a+a\sin(dx+c)}}{a\sin(dx+c)-a} - 7/4\,\frac{\sqrt{2}}{\sqrt{a}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)\right) - 3/16\,\frac{1}{a^4\sqrt{a+a\sin(dx+c)}} - 1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2*a^3*(-1/16/a^4*(1/2*(a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c)-a)-7/4*2^(1/2)/a
^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))-3/16/a^4/(a+a*s
in(d*x+c))^(1/2)-1/12/a^3/(a+a*sin(d*x+c))^(3/2)-1/20/a^2/(a+a*sin(d*x+c))^
(5/2))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.50465, size = 502, normalized size = 3.35

$$\frac{105\sqrt{2}\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-2\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)+4\left(17\right.}{960\left(a^{2}d\cos\left(dx+c\right)^{4}-2a^{2}d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/960*(105*sqrt(2)*(cos(d*x + c)^4 - 2*cos(d*x + c)^2*sin(d*x + c) - 2*cos(
d*x + c)^2)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) +
a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(175*cos(d*x + c)^2 + 21*(5*cos(d
*x + c)^2 - 4)*sin(d*x + c) - 36)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x
+ c)^4 - 2*a^2*d*cos(d*x + c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{3}\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A] time = 1.14058, size = 178, normalized size = 1.19

$$-\frac{1}{480}a^3\left(\frac{105\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\sin(dx+c)+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^4d}+\frac{30\sqrt{a\sin(dx+c)+a}}{(a\sin(dx+c)-a)a^4d}+\frac{4\left(45(a\sin(dx+c)+a)^2+20(a\sin(dx+c)+a)a+12a^2\right)}{(a\sin(dx+c)+a)^{\frac{5}{2}}a^4d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/480*a^3*(105*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*sin(d*x + c) + a)/sqrt(-a
))/(sqrt(-a)*a^4*d) + 30*sqrt(a*sin(d*x + c) + a)/((a*sin(d*x + c) - a)*a^4
*d) + 4*(45*(a*sin(d*x + c) + a)^2 + 20*(a*sin(d*x + c) + a)*a + 12*a^2)/((
a*sin(d*x + c) + a)^(5/2)*a^4*d))
```

3.179 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{105\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{256\sqrt{2}a^{3/2}d}-\frac{105\cos(c+dx)}{256d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^3(c+dx)}{6d(a\sin(c+dx)+a)^{3/2}}+\frac{}{64ad}$$

```
[Out] (-105*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(
256*Sqrt[2]*a^(3/2)*d) - (105*Cos[c + d*x])/(256*d*(a + a*Sin[c + d*x])^(3/
2)) - (7*Sec[c + d*x])/(32*d*(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^3/(
6*d*(a + a*Sin[c + d*x])^(3/2)) + (35*Sec[c + d*x])/(64*a*d*Sqrt[a + a*Sin[
c + d*x]]) + Sec[c + d*x]^3/(4*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.29053, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2681, 2687, 2650, 2649, 206}

$$-\frac{105\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{256\sqrt{2}a^{3/2}d}-\frac{105\cos(c+dx)}{256d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^3(c+dx)}{6d(a\sin(c+dx)+a)^{3/2}}+\frac{}{64ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-105*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(
256*Sqrt[2]*a^(3/2)*d) - (105*Cos[c + d*x])/(256*d*(a + a*Sin[c + d*x])^(3/
2)) - (7*Sec[c + d*x])/(32*d*(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^3/(
6*d*(a + a*Sin[c + d*x])^(3/2)) + (35*Sec[c + d*x])/(64*a*d*Sqrt[a + a*Sin[
c + d*x]]) + Sec[c + d*x]^3/(4*a*d*Sqrt[a + a*Sin[c + d*x]])
```


Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{3\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^3(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} + \frac{7}{8}\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$

$$= -\frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^3(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} + \frac{35\int \frac{}{\sqrt{a}}$$

$$= -\frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{35\sec(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} + \frac{s}{4ad\sqrt{a}}$$

$$= -\frac{105\cos(c+dx)}{256d(a+a\sin(c+dx))^{3/2}} - \frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{}{64a}$$

$$= -\frac{105\cos(c+dx)}{256d(a+a\sin(c+dx))^{3/2}} - \frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{}{64a}$$

$$= -\frac{105\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d} - \frac{105\cos(c+dx)}{256d(a+a\sin(c+dx))^{3/2}} - \frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} -$$

Mathematica [C] time = 0.325417, size = 334, normalized size = 1.71

$$\frac{\frac{192\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{32\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - 123\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2 + 246\sin\left(\frac{1}{2}(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-68 + (64*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 32/(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (136*Sin[(c + d*x)/2])/(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) + 246*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]) - 123*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (315 + 315*I)*
(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^3 + (32*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (192*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(768*d*(a*(1 + Sin[
c + d*x]))^(3/2))
```

Maple [A] time = 0.184, size = 289, normalized size = 1.5

$$\frac{1}{(1536\sin(dx+c)-1536)(1+\sin(dx+c))^2\cos(dx+c)d}\left(\left(\left(-840a^{9/2}-315(a-a\sin(dx+c))^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{a-}}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/1536/a^(11/2)*((-840*a^(9/2)-315*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(d*x+c)*cos(d*x+c)^2+(-3
84*a^(9/2)+1260*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))
^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(d*x+c)+630*a^(9/2)*cos(d*x+c)^4+(-504*a^(9
/2)-945*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2
^(1/2)/a^(1/2))*a^3)*cos(d*x+c)^2-128*a^(9/2)+1260*(a-a*sin(d*x+c))^(3/2)*2
^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(sin(d*x+c)
-1)/(1+sin(d*x+c))^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.48944, size = 729, normalized size = 3.74

$$\frac{315\sqrt{2}\left(\cos(dx+c)^5-2\cos(dx+c)^3\sin(dx+c)-2\cos(dx+c)^3\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(}{\cos(dx+c)^2-(\cos(dx+c)+}\right.}{3072\left(a^2d\cos(dx+c)^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3072*(315*sqrt(2)*(cos(d*x + c)^5 - 2*cos(d*x + c)^3*sin(d*x + c) - 2*cos
(d*x + c)^3)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c)
 + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos
(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*s
in(d*x + c) - cos(d*x + c) - 2)) + 4*(315*cos(d*x + c)^4 - 252*cos(d*x + c)
^2 - 12*(35*cos(d*x + c)^2 + 16)*sin(d*x + c) - 64)*sqrt(a*sin(d*x + c) + a
))/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x + c)^3*sin(d*x + c) - 2*a^2*d*co
s(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.180 $\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=211

$$\frac{99\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d}-\frac{99}{256ad\sqrt{a\sin(c+dx)+a}}-\frac{33}{128d(a\sin(c+dx)+a)^{3/2}}+\frac{11\sec^4(c+dx)}{56ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec}{7d(a\sin}$$

```
[Out] (99*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(256*Sqrt[2]*a^(3/
2)*d) - 33/(128*d*(a + a*Sin[c + d*x])^(3/2)) - (99*Sec[c + d*x]^2)/(560*d*
(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^4/(7*d*(a + a*Sin[c + d*x])^(3/2
)) - 99/(256*a*d*Sqrt[a + a*Sin[c + d*x]]) + (99*Sec[c + d*x]^2)/(320*a*d*S
qrt[a + a*Sin[c + d*x]]) + (11*Sec[c + d*x]^4)/(56*a*d*Sqrt[a + a*Sin[c + d
*x]])
```

Rubi [A] time = 0.342764, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2681, 2687, 2667, 51, 63, 206}

$$\frac{99\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d}-\frac{99}{256ad\sqrt{a\sin(c+dx)+a}}-\frac{33}{128d(a\sin(c+dx)+a)^{3/2}}+\frac{11\sec^4(c+dx)}{56ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec}{7d(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (99*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(256*Sqrt[2]*a^(3/
2)*d) - 33/(128*d*(a + a*Sin[c + d*x])^(3/2)) - (99*Sec[c + d*x]^2)/(560*d*
(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^4/(7*d*(a + a*Sin[c + d*x])^(3/2
)) - 99/(256*a*d*Sqrt[a + a*Sin[c + d*x]]) + (99*Sec[c + d*x]^2)/(320*a*d*S
qrt[a + a*Sin[c + d*x]]) + (11*Sec[c + d*x]^4)/(56*a*d*Sqrt[a + a*Sin[c + d
*x]])
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
```

```
IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= -\frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{11\int \frac{\sec^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{14a} \\
&= -\frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^4(c+dx)}{56ad\sqrt{a+a\sin(c+dx)}} + \frac{99}{112}\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx \\
&= -\frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^4(c+dx)}{56ad\sqrt{a+a\sin(c+dx)}} + \\
&= -\frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{99\sec^2(c+dx)}{320ad\sqrt{a+a\sin(c+dx)}} + \\
&= -\frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{99\sec^2(c+dx)}{320ad\sqrt{a+a\sin(c+dx)}} + \\
&= -\frac{33}{128d(a+a\sin(c+dx))^{3/2}} - \frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \\
&= -\frac{33}{128d(a+a\sin(c+dx))^{3/2}} - \frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} - \\
&= -\frac{33}{128d(a+a\sin(c+dx))^{3/2}} - \frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} - \\
&= \frac{99\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d} - \frac{33}{128d(a+a\sin(c+dx))^{3/2}} - \frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} -
\end{aligned}
$$

Mathematica [C] time = 0.0905095, size = 44, normalized size = 0.21

$$
-\frac{a^2\, {}_2F_1\left(-\frac{7}{2},3;-\frac{5}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{28d(a\sin(c+dx)+a)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a^2*Hypergeometric2F1[-7/2, 3, -5/2, (1 + Sin[c + d*x])/2])/(28*d*(a + a*
Sin[c + d*x])^(7/2))
```

Maple [A] time = 0.234, size = 152, normalized size = 0.7

$$-2\,\frac{a^{5}}{d}\left(1/32\,\frac{1}{a^{6}}\left(1/16\,\frac{a\sqrt{a+a\sin(dx+c)}\,(19\,\sin(dx+c)-23)}{(a\sin(dx+c)-a)^{2}}-\frac{99\,\sqrt{2}}{32\,\sqrt{a}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)\right)+\frac{}{32\,a^{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2*a^5*(1/32/a^6*(1/16*(a+a*sin(d*x+c))^(1/2)*a*(19*sin(d*x+c)-23)/(a*sin(d
*x+c)-a)^2-99/32*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)
/a^(1/2)))+5/32/a^6/(a+a*sin(d*x+c))^(1/2)+1/16/a^5/(a+a*sin(d*x+c))^(3/2)+
3/80/a^4/(a+a*sin(d*x+c))^(5/2)+1/56/a^3/(a+a*sin(d*x+c))^(7/2))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.57181, size = 572, normalized size = 2.71

$$\frac{3465\,\sqrt{2}\left(\cos(dx+c)^{6}-2\,\cos(dx+c)^{4}\sin(dx+c)-2\,\cos(dx+c)^{4}\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\,\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3\,a}{\sin(dx+c)-1}\right)+4\,(5}{35840\,\left(a^{2}d\cos(dx+c)^{6}-2\,a^{2}d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35840*(3465*sqrt(2)*(cos(d*x + c)^6 - 2*cos(d*x + c)^4*sin(d*x + c) - 2*c
os(d*x + c)^4)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c)
 + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(5775*cos(d*x + c)^4 - 1188*co
s(d*x + c)^2 + 11*(315*cos(d*x + c)^4 - 252*cos(d*x + c)^2 - 160)*sin(d*x +
 c) - 480)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^6 - 2*a^2*d*cos(d*
```

```
x + c)^4*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13994, size = 225, normalized size = 1.07

$$-\frac{1}{17920}a^5\left(\frac{3465\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\sin(dx+c)+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^6d}+\frac{70\left(19\,(a\sin(dx+c)+a)^{\frac{3}{2}}-42\sqrt{a\sin(dx+c)+a}a\right)}{(a\sin(dx+c)-a)^2a^6d}+\frac{32\left(175\,(a\sin(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/17920*a^5*(3465*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*sin(d*x + c) + a)/sqrt
(-a))/(sqrt(-a)*a^6*d) + 70*(19*(a*sin(d*x + c) + a)^(3/2) - 42*sqrt(a*sin(
d*x + c) + a)*a)/((a*sin(d*x + c) - a)^2*a^6*d) + 32*(175*(a*sin(d*x + c) +
 a)^3 + 70*(a*sin(d*x + c) + a)^2*a + 42*(a*sin(d*x + c) + a)*a^2 + 20*a^3)
/((a*sin(d*x + c) + a)^(7/2)*a^6*d))
```

3.181 $\int \frac{\sec^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=256

$$-\frac{3003\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{8192\sqrt{2}a^{3/2}d}-\frac{3003\cos(c+dx)}{8192d(a\sin(c+dx)+a)^{3/2}}+\frac{13\sec^5(c+dx)}{80ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^5(c+dx)}{8d(a\sin(c+dx)+a)^{3/2}}+$$

```
[Out] (-3003*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/
(8192*Sqrt[2]*a^(3/2)*d) - (3003*Cos[c + d*x])/(8192*d*(a + a*Sin[c + d*x])
^(3/2)) - (1001*Sec[c + d*x])/(5120*d*(a + a*Sin[c + d*x])^(3/2)) - (143*Se
c[c + d*x]^3)/(960*d*(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^5/(8*d*(a +
 a*Sin[c + d*x])^(3/2)) + (1001*Sec[c + d*x])/(2048*a*d*Sqrt[a + a*Sin[c +
d*x]]) + (143*Sec[c + d*x]^3)/(640*a*d*Sqrt[a + a*Sin[c + d*x]]) + (13*Sec[
c + d*x]^5)/(80*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.429009, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2681, 2687, 2650, 2649, 206}

$$-\frac{3003\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{8192\sqrt{2}a^{3/2}d}-\frac{3003\cos(c+dx)}{8192d(a\sin(c+dx)+a)^{3/2}}+\frac{13\sec^5(c+dx)}{80ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^5(c+dx)}{8d(a\sin(c+dx)+a)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3003*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/
(8192*Sqrt[2]*a^(3/2)*d) - (3003*Cos[c + d*x])/(8192*d*(a + a*Sin[c + d*x])
^(3/2)) - (1001*Sec[c + d*x])/(5120*d*(a + a*Sin[c + d*x])^(3/2)) - (143*Se
c[c + d*x]^3)/(960*d*(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^5/(8*d*(a +
 a*Sin[c + d*x])^(3/2)) + (1001*Sec[c + d*x])/(2048*a*d*Sqrt[a + a*Sin[c +
d*x]]) + (143*Sec[c + d*x]^3)/(640*a*d*Sqrt[a + a*Sin[c + d*x]]) + (13*Sec[
c + d*x]^5)/(80*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
```

```
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{13\int \frac{\sec^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{16a}$$

$$= -\frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{13\sec^5(c+dx)}{80ad\sqrt{a+a\sin(c+dx)}} + \frac{143}{160}\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$

$$= -\frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{13\sec^5(c+dx)}{80ad\sqrt{a+a\sin(c+dx)}} + \frac{429}{}$$

$$= -\frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{143\sec^3(c+dx)}{640ad\sqrt{a+a\sin(c+dx)}} + \frac{}{80a}$$

$$= -\frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx))^{3/2}} - \frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{}{64}$$

$$= -\frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx))^{3/2}} - \frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{}{20}$$

$$= -\frac{3003\cos(c+dx)}{8192d(a+a\sin(c+dx))^{3/2}} - \frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx))^{3/2}} - \frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}}$$

$$= -\frac{3003\cos(c+dx)}{8192d(a+a\sin(c+dx))^{3/2}} - \frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx))^{3/2}} - \frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}}$$

$$= -\frac{3003\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{8192\sqrt{2}a^{3/2}d} - \frac{3003\cos(c+dx)}{8192d(a+a\sin(c+dx))^{3/2}} - \frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx)}$$

Mathematica [C] time = 1.40622, size = 444, normalized size = 1.73

$$\frac{\frac{28800\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{6400\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{1536\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5} - 16245\left(\sin\left(\frac{1}{2}(c+dx)\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8860 + (3840*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 -
1920/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (9920*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 4960/(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2 + (17720*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) +
```

```
32490*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 16245*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^2 + (45045 + 45045*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^3 + (1536*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^5 + (6400*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (28800*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(122880*d*(a*(1 + Sin[
c + d*x]))^(3/2))
```

Maple [A] time = 0.178, size = 367, normalized size = 1.4

$$-\frac{1}{245760\,(\sin(dx+c)-1)^2\,(1+\sin(dx+c))^3\cos(dx+c)\,d}\left(-120120\,a^{13/2}\sin(dx+c)\,(\cos(dx+c))^4+\left(-54912\,a^{13/2}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/245760/a^(15/2)*(-120120*a^(13/2)*sin(d*x+c)*cos(d*x+c)^4+(-54912*a^(13/
2)-180180*(a-a*sin(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)
*2^(1/2)/a^(1/2))*a^4)*cos(d*x+c)^2*sin(d*x+c)+(-39936*a^(13/2)+360360*(a-a
*sin(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/
2))*a^4)*sin(d*x+c)+90090*a^(13/2)*cos(d*x+c)^6+9009*(-8*a^(13/2)+5*(a-a*si
n(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))
*a^4)*cos(d*x+c)^4+(-18304*a^(13/2)-360360*(a-a*sin(d*x+c))^(5/2)*2^(1/2)*a
rctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^4)*cos(d*x+c)^2-9216*a
^(13/2)+360360*(a-a*sin(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^
(1/2)*2^(1/2)/a^(1/2))*a^4)/(sin(d*x+c)-1)^2/(1+sin(d*x+c))^3/cos(d*x+c)/(a
+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.89251, size = 805, normalized size = 3.14

$$\frac{45045\,\sqrt{2}\left(\cos\left(dx+c\right)^{7}-2\,\cos\left(dx+c\right)^{5}\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)^{5}\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)}{\cos(dx+c)^{2}-(\cos(d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/491520*(45045*sqrt(2)*(cos(d*x + c)^7 - 2*cos(d*x + c)^5*sin(d*x + c) - 2
*cos(d*x + c)^5)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x
+ c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a
*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) +
2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(45045*cos(d*x + c)^6 - 36036*cos(
d*x + c)^4 - 9152*cos(d*x + c)^2 - 156*(385*cos(d*x + c)^4 + 176*cos(d*x +
c)^2 + 128)*sin(d*x + c) - 4608)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x +
 c)^7 - 2*a^2*d*cos(d*x + c)^5*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.182 $\int \frac{\cos^{10}(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{16a^2\cos^{11}(c+dx)}{195d(a\sin(c+dx)+a)^{9/2}}-\frac{64a^3\cos^{11}(c+dx)}{2145d(a\sin(c+dx)+a)^{11/2}}-\frac{2a\cos^{11}(c+dx)}{15d(a\sin(c+dx)+a)^{7/2}}$$

```
[Out] (-64*a^3*Cos[c + d*x]^11)/(2145*d*(a + a*Sin[c + d*x])^(11/2)) - (16*a^2*Co
s[c + d*x]^11)/(195*d*(a + a*Sin[c + d*x])^(9/2)) - (2*a*Cos[c + d*x]^11)/(
15*d*(a + a*Sin[c + d*x])^(7/2))
```

Rubi [A] time = 0.194634, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{16a^2\cos^{11}(c+dx)}{195d(a\sin(c+dx)+a)^{9/2}}-\frac{64a^3\cos^{11}(c+dx)}{2145d(a\sin(c+dx)+a)^{11/2}}-\frac{2a\cos^{11}(c+dx)}{15d(a\sin(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^10/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^11)/(2145*d*(a + a*Sin[c + d*x])^(11/2)) - (16*a^2*Co
s[c + d*x]^11)/(195*d*(a + a*Sin[c + d*x])^(9/2)) - (2*a*Cos[c + d*x]^11)/(
15*d*(a + a*Sin[c + d*x])^(7/2))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^{10}(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2a\cos^{11}(c+dx)}{15d(a+a\sin(c+dx))^{7/2}} + \frac{1}{15}(8a)\int \frac{\cos^{10}(c+dx)}{(a+a\sin(c+dx))^{7/2}}\,dx$$
$$= -\frac{16a^2\cos^{11}(c+dx)}{195d(a+a\sin(c+dx))^{9/2}} - \frac{2a\cos^{11}(c+dx)}{15d(a+a\sin(c+dx))^{7/2}} + \frac{1}{195}\left(32a^2\right)\int \frac{\cos^{10}(c+dx)}{(a+a\sin(c+dx}$$
$$= -\frac{64a^3\cos^{11}(c+dx)}{2145d(a+a\sin(c+dx))^{11/2}} - \frac{16a^2\cos^{11}(c+dx)}{195d(a+a\sin(c+dx))^{9/2}} - \frac{2a\cos^{11}(c+dx)}{15d(a+a\sin(c+dx))^{7/2}}$$

Mathematica [A] time = 0.663395, size = 59, normalized size = 0.62

$$-\frac{2\left(143\sin^2(c+dx)+374\sin(c+dx)+263\right)\cos^{11}(c+dx)}{2145d(\sin(c+dx)+1)^3(a(\sin(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^10/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^11*(263 + 374*Sin[c + d*x] + 143*Sin[c + d*x]^2))/(2145*d*
(1 + Sin[c + d*x])^3*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A] time = 0.16, size = 67, normalized size = 0.7

$$-\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^6\left(143(\sin(dx+c))^2+374\sin(dx+c)+263\right)}{2145a^2\cos(dx+c)d}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^10/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/2145/a^2*(1+sin(d*x+c))*(sin(d*x+c)-1)^6*(143*sin(d*x+c)^2+374*sin(d*x+c
)+263)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{10}}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^10/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 2.27415, size = 568, normalized size = 5.98

$$-\frac{2\left(143\cos(dx+c)^8 - 341\cos(dx+c)^7 - 736\cos(dx+c)^6 + 28\cos(dx+c)^5 - 40\cos(dx+c)^4 + 64\cos(dx+c)^3 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/2145*(143*cos(d*x + c)^8 - 341*cos(d*x + c)^7 - 736*cos(d*x + c)^6 + 28*
cos(d*x + c)^5 - 40*cos(d*x + c)^4 + 64*cos(d*x + c)^3 - 128*cos(d*x + c)^2
 + (143*cos(d*x + c)^7 + 484*cos(d*x + c)^6 - 252*cos(d*x + c)^5 - 280*cos(
d*x + c)^4 - 320*cos(d*x + c)^3 - 384*cos(d*x + c)^2 - 512*cos(d*x + c) - 1
024)*sin(d*x + c) + 512*cos(d*x + c) + 1024)*sqrt(a*sin(d*x + c) + a)/(a^3*
d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**10/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.47535, size = 648, normalized size = 6.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/2145*((((((((((((((((263*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*
c)/a^19 - 2145*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^19)*tan(1/2*d*x + 1/2*c) + 7
335*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^19)*tan(1/2*d*x + 1/2*c) - 13585*sgn(ta
n(1/2*d*x + 1/2*c) + 1)/a^19)*tan(1/2*d*x + 1/2*c) + 15795*sgn(tan(1/2*d*x
+ 1/2*c) + 1)/a^19)*tan(1/2*d*x + 1/2*c) - 17589*sgn(tan(1/2*d*x + 1/2*c) +
 1)/a^19)*tan(1/2*d*x + 1/2*c) + 29315*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^19)*
tan(1/2*d*x + 1/2*c) - 45045*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^19)*tan(1/2*d*
x + 1/2*c) + 45045*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^19)*tan(1/2*d*x + 1/2*c)
 - 29315*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^19)*tan(1/2*d*x + 1/2*c) + 17589*s
gn(tan(1/2*d*x + 1/2*c) + 1)/a^19)*tan(1/2*d*x + 1/2*c) - 15795*sgn(tan(1/2
*d*x + 1/2*c) + 1)/a^19)*tan(1/2*d*x + 1/2*c) + 13585*sgn(tan(1/2*d*x + 1/2
*c) + 1)/a^19)*tan(1/2*d*x + 1/2*c) - 7335*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^
19)*tan(1/2*d*x + 1/2*c) + 2145*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^19)*tan(1/2
*d*x + 1/2*c) - 263*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^19)/(a*tan(1/2*d*x + 1/
2*c)^2 + a)^(15/2) + 1024*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(53/2))/d
```

3.183 $\int \frac{\cos^9(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=121

$$\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^9d}-\frac{16(a\sin(c+dx)+a)^{11/2}}{11a^8d}+\frac{16(a\sin(c+dx)+a)^{9/2}}{3a^7d}-\frac{64(a\sin(c+dx)+a)^{7/2}}{7a^6d}+\frac{32(a\sin(c+d}{5a^5d}$$

```
[Out] (32*(a + a*Sin[c + d*x])^(5/2))/(5*a^5*d) - (64*(a + a*Sin[c + d*x])^(7/2))
/(7*a^6*d) + (16*(a + a*Sin[c + d*x])^(9/2))/(3*a^7*d) - (16*(a + a*Sin[c +
 d*x])^(11/2))/(11*a^8*d) + (2*(a + a*Sin[c + d*x])^(13/2))/(13*a^9*d)
```

Rubi [A] time = 0.0900153, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^9d}-\frac{16(a\sin(c+dx)+a)^{11/2}}{11a^8d}+\frac{16(a\sin(c+dx)+a)^{9/2}}{3a^7d}-\frac{64(a\sin(c+dx)+a)^{7/2}}{7a^6d}+\frac{32(a\sin(c+d}{5a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (32*(a + a*Sin[c + d*x])^(5/2))/(5*a^5*d) - (64*(a + a*Sin[c + d*x])^(7/2))
/(7*a^6*d) + (16*(a + a*Sin[c + d*x])^(9/2))/(3*a^7*d) - (16*(a + a*Sin[c +
 d*x])^(11/2))/(11*a^8*d) + (2*(a + a*Sin[c + d*x])^(13/2))/(13*a^9*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^9(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int (a-x)^4(a+x)^{3/2}\,dx, x, a\sin(c+dx)\right)}{a^9 d}$$
$$= \frac{\text{Subst}\left(\int \left(16a^4(a+x)^{3/2} - 32a^3(a+x)^{5/2} + 24a^2(a+x)^{7/2} - 8a(a+x)^{9/2} + (a+x)^{11/2}\right) d}{a^9 d}$$
$$= \frac{32(a+a\sin(c+dx))^{5/2}}{5a^5 d} - \frac{64(a+a\sin(c+dx))^{7/2}}{7a^6 d} + \frac{16(a+a\sin(c+dx))^{9/2}}{3a^7 d} - \frac{16(a+a\,\text{s}}{}$$

Mathematica [A] time = 0.290182, size = 64, normalized size = 0.53

$$\frac{2\left(1155\sin^4(c+dx) - 6300\sin^3(c+dx) + 14210\sin^2(c+dx) - 16700\sin(c+dx) + 9683\right)(a(\sin(c+dx)+1))^{5/2}}{15015a^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(5/2)*(9683 - 16700*Sin[c + d*x] + 14210*Sin[c +
d*x]^2 - 6300*Sin[c + d*x]^3 + 1155*Sin[c + d*x]^4))/(15015*a^5*d)
```

Maple [A] time = 0.102, size = 67, normalized size = 0.6

$$\frac{2310\,(\cos(dx+c))^4 + 12600\,(\cos(dx+c))^2\sin(dx+c) - 33040\,(\cos(dx+c))^2 - 46000\,\sin(dx+c) + 50096}{15015\,a^5 d}(a + a\,\text{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/15015/a^5*(a+a*sin(d*x+c))^(5/2)*(1155*cos(d*x+c)^4+6300*cos(d*x+c)^2*sin
(d*x+c)-16520*cos(d*x+c)^2-23000*sin(d*x+c)+25048)/d
```

Maxima [A] time = 0.942051, size = 120, normalized size = 0.99

$$\frac{2\left(1155\,(a\sin(dx+c)+a)^{\frac{13}{2}} - 10920\,(a\sin(dx+c)+a)^{\frac{11}{2}}a + 40040\,(a\sin(dx+c)+a)^{\frac{9}{2}}a^2 - 68640\,(a\sin(dx+c)+a\right.}{15015\,a^9 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*(a*sin(d*x + c) + a)^(13/2) - 10920*(a*sin(d*x + c) + a)^(11/
2)*a + 40040*(a*sin(d*x + c) + a)^(9/2)*a^2 - 68640*(a*sin(d*x + c) + a)^(7
/2)*a^3 + 48048*(a*sin(d*x + c) + a)^(5/2)*a^4)/(a^9*d)
```

Fricas [A] time = 2.30215, size = 247, normalized size = 2.04

$$-\frac{2\left(1155\cos\left(dx+c\right)^{6}-6230\cos\left(dx+c\right)^{4}-512\cos\left(dx+c\right)^{2}+2\left(1995\cos\left(dx+c\right)^{4}-1280\cos\left(dx+c\right)^{2}-2048\right)\sin\right.}{15015\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15015*(1155*cos(d*x + c)^6 - 6230*cos(d*x + c)^4 - 512*cos(d*x + c)^2 +
2*(1995*cos(d*x + c)^4 - 1280*cos(d*x + c)^2 - 2048)*sin(d*x + c) - 4096)*s
qrt(a*sin(d*x + c) + a)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31304, size = 120, normalized size = 0.99

$$\frac{2\left(1155\left(a\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}-10920\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}a+40040\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a^{2}-68640\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}\right.}{15015\,a^{9}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2/15015*(1155*(a*sin(d*x + c) + a)^(13/2) - 10920*(a*sin(d*x + c) + a)^(11/
2)*a + 40040*(a*sin(d*x + c) + a)^(9/2)*a^2 - 68640*(a*sin(d*x + c) + a)^(7
/2)*a^3 + 48048*(a*sin(d*x + c) + a)^(5/2)*a^4)/(a^9*d)
```

3.184 $\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{8a^2\cos^9(c+dx)}{99d(a\sin(c+dx)+a)^{9/2}}-\frac{2a\cos^9(c+dx)}{11d(a\sin(c+dx)+a)^{7/2}}$$

```
[Out] (-8*a^2*Cos[c + d*x]^9)/(99*d*(a + a*Sin[c + d*x])^(9/2)) - (2*a*Cos[c + d*
x]^9)/(11*d*(a + a*Sin[c + d*x])^(7/2))
```

Rubi [A] time = 0.11884, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2674, 2673}

$$-\frac{8a^2\cos^9(c+dx)}{99d(a\sin(c+dx)+a)^{9/2}}-\frac{2a\cos^9(c+dx)}{11d(a\sin(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^9)/(99*d*(a + a*Sin[c + d*x])^(9/2)) - (2*a*Cos[c + d*
x]^9)/(11*d*(a + a*Sin[c + d*x])^(7/2))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2a\cos^9(c+dx)}{11d(a+a\sin(c+dx))^{7/2}} + \frac{1}{11}(4a)\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^{7/2}}\,dx$$
$$= -\frac{8a^2\cos^9(c+dx)}{99d(a+a\sin(c+dx))^{9/2}} - \frac{2a\cos^9(c+dx)}{11d(a+a\sin(c+dx))^{7/2}}$$

Mathematica [A] time = 0.335438, size = 49, normalized size = 0.78

$$-\frac{2(9\sin(c+dx)+13)\cos^9(c+dx)}{99d(\sin(c+dx)+1)^2(a(\sin(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^9*(13 + 9*Sin[c + d*x]))/(99*d*(1 + Sin[c + d*x])^2*(a*(1
+ Sin[c + d*x]))^(5/2))
```

Maple [A] time = 0.12, size = 57, normalized size = 0.9

$$\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^5(9\sin(dx+c)+13)}{99a^2\cos(dx+c)d}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/99/a^2*(1+sin(d*x+c))*(sin(d*x+c)-1)^5*(9*sin(d*x+c)+13)/cos(d*x+c)/(a+a*
sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^8}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^8/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 2.23003, size = 431, normalized size = 6.84

$$-\frac{2\left(9\cos(dx+c)^{6} - 23\cos(dx+c)^{5} - 52\cos(dx+c)^{4} + 4\cos(dx+c)^{3} - 8\cos(dx+c)^{2} + \left(9\cos(dx+c)^{5} + 32\cos\right.\right.}{99\left(a^{3}d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/99*(9*cos(d*x + c)^6 - 23*cos(d*x + c)^5 - 52*cos(d*x + c)^4 + 4*cos(d*x
 + c)^3 - 8*cos(d*x + c)^2 + (9*cos(d*x + c)^5 + 32*cos(d*x + c)^4 - 20*cos
(d*x + c)^3 - 24*cos(d*x + c)^2 - 32*cos(d*x + c) - 64)*sin(d*x + c) + 32*c
os(d*x + c) + 64)*sqrt(a*sin(d*x + c) + a)/(a^3*d*cos(d*x + c) + a^3*d*sin(
d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.32693, size = 497, normalized size = 7.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/1584*((((((((((((13*sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)/a^
15 - 99*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^15)*tan(1/2*d*x + 1/2*c) + 319*sgn(
tan(1/2*d*x + 1/2*c) + 1)/a^15)*tan(1/2*d*x + 1/2*c) - 561*sgn(tan(1/2*d*x
+ 1/2*c) + 1)/a^15)*tan(1/2*d*x + 1/2*c) + 594*sgn(tan(1/2*d*x + 1/2*c) + 1
)/a^15)*tan(1/2*d*x + 1/2*c) - 462*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^15)*tan(
1/2*d*x + 1/2*c) + 462*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^15)*tan(1/2*d*x + 1/
2*c) - 594*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^15)*tan(1/2*d*x + 1/2*c) + 561*s
gn(tan(1/2*d*x + 1/2*c) + 1)/a^15)*tan(1/2*d*x + 1/2*c) - 319*sgn(tan(1/2*d
*x + 1/2*c) + 1)/a^15)*tan(1/2*d*x + 1/2*c) + 99*sgn(tan(1/2*d*x + 1/2*c) +
 1)/a^15)*tan(1/2*d*x + 1/2*c) - 13*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^15)/(a*
tan(1/2*d*x + 1/2*c)^2 + a)^(11/2) + 64*sqrt(2)*sgn(tan(1/2*d*x + 1/2*c) +
1)/a^(41/2))/d
```

3.185 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^7d}+\frac{12(a\sin(c+dx)+a)^{7/2}}{7a^6d}-\frac{24(a\sin(c+dx)+a)^{5/2}}{5a^5d}+\frac{16(a\sin(c+dx)+a)^{3/2}}{3a^4d}$$

```
[Out] (16*(a + a*Sin[c + d*x])^(3/2))/(3*a^4*d) - (24*(a + a*Sin[c + d*x])^(5/2))
/(5*a^5*d) + (12*(a + a*Sin[c + d*x])^(7/2))/(7*a^6*d) - (2*(a + a*Sin[c +
d*x])^(9/2))/(9*a^7*d)
```

Rubi [A] time = 0.0831496, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^7d}+\frac{12(a\sin(c+dx)+a)^{7/2}}{7a^6d}-\frac{24(a\sin(c+dx)+a)^{5/2}}{5a^5d}+\frac{16(a\sin(c+dx)+a)^{3/2}}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(3/2))/(3*a^4*d) - (24*(a + a*Sin[c + d*x])^(5/2))
/(5*a^5*d) + (12*(a + a*Sin[c + d*x])^(7/2))/(7*a^6*d) - (2*(a + a*Sin[c +
d*x])^(9/2))/(9*a^7*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3\sqrt{a+x}\,dx, x, a\sin(c+dx)\right)}{a^7 d}\\
&= \frac{\text{Subst}\left(\int \left(8a^3\sqrt{a+x} - 12a^2(a+x)^{3/2} + 6a(a+x)^{5/2} - (a+x)^{7/2}\right)dx, x, a\sin(c+dx)\right)}{a^7 d}\\
&= \frac{16(a+a\sin(c+dx))^{3/2}}{3a^4 d} - \frac{24(a+a\sin(c+dx))^{5/2}}{5a^5 d} + \frac{12(a+a\sin(c+dx))^{7/2}}{7a^6 d} - \frac{2(a+a\,\mathrm{si}}{9}
\end{aligned}
$$

Mathematica [A] time = 0.174714, size = 54, normalized size = 0.56

$$
-\frac{2\left(35\sin^3(c+dx) - 165\sin^2(c+dx) + 321\sin(c+dx) - 319\right)(a(\sin(c+dx)+1))^{3/2}}{315a^4 d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a*(1 + Sin[c + d*x]))^(3/2)*(-319 + 321*Sin[c + d*x] - 165*Sin[c + d*x
]^2 + 35*Sin[c + d*x]^3))/(315*a^4*d)
```

Maple [A] time = 0.161, size = 57, normalized size = 0.6

$$
\frac{70\,(\cos(dx+c))^2\sin(dx+c) - 330\,(\cos(dx+c))^2 - 712\,\sin(dx+c) + 968}{315\,a^4 d}(a+a\sin(dx+c))^{\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/315/a^4*(a+a*sin(d*x+c))^(3/2)*(35*cos(d*x+c)^2*sin(d*x+c)-165*cos(d*x+c)
^2-356*sin(d*x+c)+484)/d
```

Maxima [A] time = 0.936901, size = 97, normalized size = 1.

$$
-\frac{2\left(35\,(a\sin(dx+c)+a)^{\frac{9}{2}} - 270\,(a\sin(dx+c)+a)^{\frac{7}{2}}a + 756\,(a\sin(dx+c)+a)^{\frac{5}{2}}a^2 - 840\,(a\sin(dx+c)+a)^{\frac{3}{2}}a^3\right)}{315\,a^7 d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*(a*sin(d*x + c) + a)^(9/2) - 270*(a*sin(d*x + c) + a)^(7/2)*a +
756*(a*sin(d*x + c) + a)^(5/2)*a^2 - 840*(a*sin(d*x + c) + a)^(3/2)*a^3)/(a
^7*d)
```

Fricas [A] time = 2.2687, size = 176, normalized size = 1.81

$$-\frac{2\left(35\cos\left(dx+c\right)^{4}-226\cos\left(dx+c\right)^{2}+2\left(65\cos\left(dx+c\right)^{2}-64\right)\sin\left(dx+c\right)-128\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*cos(d*x + c)^4 - 226*cos(d*x + c)^2 + 2*(65*cos(d*x + c)^2 - 64)
*sin(d*x + c) - 128)*sqrt(a*sin(d*x + c) + a)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24723, size = 97, normalized size = 1.

$$-\frac{2\left(35\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}-270\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a+756\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a^{2}-840\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a^{3}\right)}{315\,a^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/315*(35*(a*sin(d*x + c) + a)^(9/2) - 270*(a*sin(d*x + c) + a)^(7/2)*a +
756*(a*sin(d*x + c) + a)^(5/2)*a^2 - 840*(a*sin(d*x + c) + a)^(3/2)*a^3)/(a
^7*d)
```

3.186 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{2a\cos^7(c+dx)}{7d(a\sin(c+dx)+a)^{7/2}}$$

```
[Out] (-2*a*Cos[c + d*x]^7)/(7*d*(a + a*Sin[c + d*x])^(7/2))
```

Rubi [A] time = 0.0571351, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2673}

$$-\frac{2a\cos^7(c+dx)}{7d(a\sin(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*a*Cos[c + d*x]^7)/(7*d*(a + a*Sin[c + d*x])^(7/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx=-\frac{2a\cos^7(c+dx)}{7d(a+a\sin(c+dx))^{7/2}}$$

Mathematica [A] time = 0.111325, size = 42, normalized size = 1.4

$$-\frac{2\cos^7(c+dx)\sqrt{a(\sin(c+dx)+1)}}{7a^3d(\sin(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^7*Sqrt[a*(1 + Sin[c + d*x])])/(7*a^3*d*(1 + Sin[c + d*x])^
4)
```

Maple [A] time = 0.104, size = 47, normalized size = 1.6

$$-\frac{(2+2\sin(dx+c))(\sin(dx+c)-1)^4}{7a^2\cos(dx+c)d}\frac{1}{\sqrt{a+a\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/7/a^2*(1+sin(d*x+c))*(sin(d*x+c)-1)^4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(dx+c)^6}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 2.15058, size = 309, normalized size = 10.3

$$-\frac{2\left(\cos(dx+c)^4-3\cos(dx+c)^3-8\cos(dx+c)^2+\left(\cos(dx+c)^3+4\cos(dx+c)^2-4\cos(dx+c)-8\right)\sin(dx+c\right.}{7\left(a^3d\cos(dx+c)+a^3d\sin(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")

[Out] -2/7*(cos(d*x + c)^4 - 3*cos(d*x + c)^3 - 8*cos(d*x + c)^2 + (cos(d*x + c)^
3 + 4*cos(d*x + c)^2 - 4*cos(d*x + c) - 8)*sin(d*x + c) + 4*cos(d*x + c) +
8)*sqrt(a*sin(d*x + c) + a)/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*
d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**(5/2),x)

[Out] Timed out

Giac [B] time = 2.16901, size = 344, normalized size = 11.47

$$\frac{\left(\left(\left(\left(\left(\left(\left(\frac{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{11}}-\frac{7\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{11}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{21\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{11}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{35\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{11}}\right)\tan\left(\frac{1}{2}\,d\right.\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] 1/21*((((((((sgn(tan(1/2*d*x + 1/2*c) + 1)*tan(1/2*d*x + 1/2*c)/a^11 - 7*sg
n(tan(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*c) + 21*sgn(tan(1/2*d*x
 + 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*c) - 35*sgn(tan(1/2*d*x + 1/2*c) + 1
)/a^11)*tan(1/2*d*x + 1/2*c) + 35*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1
/2*d*x + 1/2*c) - 21*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*
c) + 7*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^11)*tan(1/2*d*x + 1/2*c) - sgn(tan(1
/2*d*x + 1/2*c) + 1)/a^11)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2) + 8*sqrt(2)
*sgn(tan(1/2*d*x + 1/2*c) + 1)/a^(29/2))/d

3.187 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5a^5d}-\frac{8(a\sin(c+dx)+a)^{3/2}}{3a^4d}+\frac{8\sqrt{a\sin(c+dx)+a}}{a^3d}$$

```
[Out] (8*Sqrt[a + a*Sin[c + d*x]])/(a^3*d) - (8*(a + a*Sin[c + d*x])^(3/2))/(3*a^
4*d) + (2*(a + a*Sin[c + d*x])^(5/2))/(5*a^5*d)
```

Rubi [A] time = 0.074642, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5a^5d}-\frac{8(a\sin(c+dx)+a)^{3/2}}{3a^4d}+\frac{8\sqrt{a\sin(c+dx)+a}}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (8*Sqrt[a + a*Sin[c + d*x]])/(a^3*d) - (8*(a + a*Sin[c + d*x])^(3/2))/(3*a^
4*d) + (2*(a + a*Sin[c + d*x])^(5/2))/(5*a^5*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{4a^2}{\sqrt{a+x}} - 4a\sqrt{a+x} + (a+x)^{3/2}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{8\sqrt{a+a\sin(c+dx)}}{a^3 d} - \frac{8(a+a\sin(c+dx))^{3/2}}{3a^4 d} + \frac{2(a+a\sin(c+dx))^{5/2}}{5a^5 d}$$

Mathematica [A] time = 0.0712361, size = 44, normalized size = 0.62

$$\frac{2\left(3\sin^2(c+dx) - 14\sin(c+dx) + 43\right)\sqrt{a(\sin(c+dx)+1)}}{15a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Sin[c + d*x])]*(43 - 14*Sin[c + d*x] + 3*Sin[c + d*x]^2))/(1
5*a^3*d)
```

Maple [A] time = 0.081, size = 41, normalized size = 0.6

$$-\frac{6\,(\cos(dx+c))^2 + 28\,\sin(dx+c) - 92}{15\,a^3 d}\sqrt{a+a\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/15/a^3*(a+a*sin(d*x+c))^(1/2)*(3*cos(d*x+c)^2+14*sin(d*x+c)-46)/d
```

Maxima [A] time = 0.951528, size = 74, normalized size = 1.04

$$\frac{2\left(3\,(a\sin(dx+c)+a)^{\frac{5}{2}} - 20\,(a\sin(dx+c)+a)^{\frac{3}{2}}a + 60\,\sqrt{a\sin(dx+c)+a}\,a^2\right)}{15\,a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(a*sin(d*x + c) + a)^(5/2) - 20*(a*sin(d*x + c) + a)^(3/2)*a + 60*s
qrt(a*sin(d*x + c) + a)*a^2)/(a^5*d)
```

Fricas [A] time = 2.1591, size = 111, normalized size = 1.56

$$-\frac{2\left(3\cos\left(dx+c\right)^{2}+14\sin\left(dx+c\right)-46\right)\sqrt{a\sin\left(dx+c\right)+a}}{15\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*cos(d*x + c)^2 + 14*sin(d*x + c) - 46)*sqrt(a*sin(d*x + c) + a)/(a
^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20441, size = 74, normalized size = 1.04

$$\frac{2\left(3\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}-20\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a+60\sqrt{a\sin\left(dx+c\right)+a}a^{2}\right)}{15\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(a*sin(d*x + c) + a)^(5/2) - 20*(a*sin(d*x + c) + a)^(3/2)*a + 60*s
qrt(a*sin(d*x + c) + a)*a^2)/(a^5*d)
```

3.188 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=108

$$\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{2\cos^3(c+dx)}{3ad(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(5/2)*d) + (2*Cos[c + d*x]^3)/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) +
(4*Cos[c + d*x])/(a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.144076, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2679, 2649, 206}

$$\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{2\cos^3(c+dx)}{3ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(5/2)*d) + (2*Cos[c + d*x]^3)/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) +
(4*Cos[c + d*x])/(a^2*d*Sqrt[a + a*Sin[c + d*x]])
```


Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}}+\frac{2\int\frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{a}\\
&= \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}}+\frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}}+\frac{4\int\frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}\\
&= \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}}+\frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}}-\frac{8\,\mathrm{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^2d}\\
&= -\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}+\frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}}+\frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.179209, size = 96, normalized size = 0.89

$$-\frac{2\cos(c+dx)\left(\sqrt{1-\sin(c+dx)}(\sin(c+dx)-7)+6\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\right)}{3a^2d\sqrt{1-\sin(c+dx)}\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]*(6*Sqrt[2]*ArcTanh[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]] + Sqrt[
1 - Sin[c + d*x]]*(-7 + Sin[c + d*x])))/(3*a^2*d*Sqrt[1 - Sin[c + d*x]]*Sqr
t[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.15, size = 112, normalized size = 1.

$$-\frac{2+2\,\sin(dx+c)}{3\,a^4\cos(dx+c)\,d}\sqrt{-a(\sin(dx+c)-1)}\left(6\,a^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)-(a-a\sin(dx+c))^{\frac{3}{2}}-6\,a\sqrt{a-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(6*a^(3/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(3/2)-6*a*(a-a*s
in(d*x+c))^(1/2))/a^4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{4}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 2.35373, size = 593, normalized size = 5.49

$$\frac{2\left(\frac{3\sqrt{2}(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^{2}-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-\left(\cos\left(dx+c\right)^{2}+\right.\right.}{3\left(a^{3}d\cos\left(dx+c\right)+a^{3}d\sin\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 -
 (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(
d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
- (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - (cos(d*x +
 c)^2 + (cos(d*x + c) + 8)*sin(d*x + c) - 7*cos(d*x + c) - 8)*sqrt(a*sin(d*
```

```
x + c) + a))/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.06983, size = 344, normalized size = 3.19

$$-\frac{2\left(\frac{\left(\left(\frac{7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{9}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{9}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{7}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{3}{2}}}+\frac{4\sqrt{2}\left(3\,a\arctan\left(\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\right.\right.}{\cdots}\cdots\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*((((7*tan(1/2*d*x + 1/2*c)/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)) - 9/(a*sg
n(tan(1/2*d*x + 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c) + 9/(a*sgn(tan(1/2*d*x +
 1/2*c) + 1)))*tan(1/2*d*x + 1/2*c) - 7/(a*sgn(tan(1/2*d*x + 1/2*c) + 1)))/
(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) + 4*sqrt(2)*(3*a*arctan(sqrt(a)/sqrt(-
a)) + 2*sqrt(-a)*sqrt(a))*sgn(tan(1/2*d*x + 1/2*c) + 1)/(sqrt(-a)*a^3) - 12
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1
/2*c) + 1)))/d
```

3.189 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2\sqrt{a\sin(c+dx)+a}}{a^3d}-\frac{4}{a^2d\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -4/(a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Sqrt[a + a*Sin[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.0676261, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 43}

$$-\frac{2\sqrt{a\sin(c+dx)+a}}{a^3d}-\frac{4}{a^2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -4/(a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Sqrt[a + a*Sin[c + d*x]])/(a^3*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{(a+x)^{3/2}}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{2a}{(a+x)^{3/2}} - \frac{1}{\sqrt{a+x}}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= -\frac{4}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\sqrt{a+a\sin(c+dx)}}{a^3d}$$

Mathematica [A] time = 0.0544934, size = 30, normalized size = 0.67

$$-\frac{2(\sin(c+dx)+3)}{a^2d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(3 + Sin[c + d*x]))/(a^2*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.087, size = 29, normalized size = 0.6

$$-2\,\frac{3+\sin(dx+c)}{a^2\sqrt{a+a\sin(dx+c)}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/a^2/(a+a*sin(d*x+c))^(1/2)*(3+sin(d*x+c))/d
```

Maxima [A] time = 0.942827, size = 57, normalized size = 1.27

$$-\frac{2\left(\frac{\sqrt{a\sin(dx+c)+a}}{a^2} + \frac{2}{\sqrt{a\sin(dx+c)+a}a}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2*(sqrt(a*sin(d*x + c) + a)/a^2 + 2/(sqrt(a*sin(d*x + c) + a)*a))/(a*d)
```

Fricas [A] time = 2.13332, size = 104, normalized size = 2.31

$$-\frac{2\sqrt{a\sin(dx+c)+a}(\sin(dx+c)+3)}{a^3d\sin(dx+c)+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*sin(d*x + c) + a)*(sin(d*x + c) + 3)/(a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [A] time = 30.4713, size = 267, normalized size = 5.93

$$\begin{cases} \text{NaN} \\ \frac{x\cos^3(c)}{(a\sin(c)+a)^{\frac{5}{2}}} \\ -\frac{8\sqrt{a\sin(c+dx)+a}\sin^2(c+dx)}{3a^3d\sin^2(c+dx)+6a^3d\sin(c+dx)+3a^3d} - \frac{24\sqrt{a\sin(c+dx)+a}\sin(c+dx)}{3a^3d\sin^2(c+dx)+6a^3d\sin(c+dx)+3a^3d} - \frac{2\sqrt{a\sin(c+dx)+a}\cos^2(c+dx)}{3a^3d\sin^2(c+dx)+6a^3d\sin(c+dx)+3a^3d} - \frac{16\sqrt{a\sin(c-}{3a^3d\sin^2(c+dx)+6a^3d} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((nan, (Eq(d, 0) | Eq(c, -d*x + 3*pi/2)) & (Eq(c, 3*pi/2) | Eq(c,
-d*x + 3*pi/2))), (x*cos(c)**3/(a*sin(c) + a)**(5/2), Eq(d, 0)), (-8*sqrt(a
*sin(c + d*x) + a)*sin(c + d*x)**2/(3*a**3*d*sin(c + d*x)**2 + 6*a**3*d*sin
(c + d*x) + 3*a**3*d) - 24*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)/(3*a**3*d*
sin(c + d*x)**2 + 6*a**3*d*sin(c + d*x) + 3*a**3*d) - 2*sqrt(a*sin(c + d*x)
 + a)*cos(c + d*x)**2/(3*a**3*d*sin(c + d*x)**2 + 6*a**3*d*sin(c + d*x) + 3
*a**3*d) - 16*sqrt(a*sin(c + d*x) + a)/(3*a**3*d*sin(c + d*x)**2 + 6*a**3*d
*sin(c + d*x) + 3*a**3*d), True))
```

Giac [A] time = 1.1691, size = 49, normalized size = 1.09

$$-\frac{2\left(\sqrt{a\sin\left(dx+c\right)+a}+\frac{2\,a}{\sqrt{a\sin\left(dx+c\right)+a}}\right)}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2*(sqrt(a*sin(d*x + c) + a) + 2*a/sqrt(a*sin(d*x + c) + a))/(a^3*d)
```

3.190 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{2}a^{5/2}d}-\frac{\cos(c+dx)}{ad(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]/(Sqrt[2]
*a^(5/2)*d) - Cos[c + d*x]/(a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.0815447, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2680, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{2}a^{5/2}d}-\frac{\cos(c+dx)}{ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]/(Sqrt[2]
*a^(5/2)*d) - Cos[c + d*x]/(a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= -\frac{\cos(c+dx)}{ad(a+a\sin(c+dx))^{3/2}}-\frac{\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}\\ &= -\frac{\cos(c+dx)}{ad(a+a\sin(c+dx))^{3/2}}+\frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^2d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}-\frac{\cos(c+dx)}{ad(a+a\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.228569, size = 100, normalized size = 1.33

$$\frac{\sec(c+dx)\left(2(\sin(c+dx)-1)+\sqrt{2-2\sin(c+dx)}\tanh^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2\right)}{2a^2d\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]*(ArcTanh[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^2*Sqrt[2 - 2*Sin[c + d*x]] + 2*(-1 + Sin[c + d*x])))/(2*a
^2*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.119, size = 123, normalized size = 1.6

$$-\frac{1}{2\,d\cos(dx+c)}\left(-\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin(dx+c)}\frac{1}{\sqrt{a}}\right)a\sin(dx+c)-\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin(dx+c)}\frac{1}{\sqrt{a}}\right)a+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/2/a^(7/2)*(-2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*
a*sin(d*x+c)-2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a+
2*(a-a*sin(d*x+c))^(1/2)*a^(1/2))*(-a*(sin(d*x+c)-1))^(1/2)/cos(d*x+c)/(a+a
*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{2}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 2.31028, size = 666, normalized size = 8.88

$$\frac{\frac{\sqrt{2}\left(a\cos(dx+c)^{2}-a\cos(dx+c)-(a\cos(dx+c)+2\,a)\sin(dx+c)-2\,a\right)\log\left(-\frac{\cos(dx+c)^{2}-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right.}{\sqrt{a}}}{4\left(a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d\cos\left(dx+c\right)-2\,a^{3}d-\left(a^{3}d\cos\left(dx+c\right)+2\,a^{3}d\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(a*cos(d*x + c)^2 - a*cos(d*x + c) - (a*cos(d*x + c) + 2*a)*si
n(d*x + c) - 2*a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) +
2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a
) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) -
 cos(d*x + c) - 2))/sqrt(a) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - si
n(d*x + c) + 1))/(a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (a^
3*d*cos(d*x + c) + 2*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.18793, size = 396, normalized size = 5.28

$$-\frac{\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{2\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^3+\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 \cdots}\right.\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\cdots\right.}\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -(sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x +
1/2*c) + 1)) - 2*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^3 + (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*sqrt(a) - (sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a))*a + a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))*sqrt(a) - a)^2*a^2*sgn(tan(1/2*d*x + 1/2*c) + 1)))/d
```

3.191 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=24

$$-\frac{2}{3ad(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -2/(3*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.0348205, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 32}

$$-\frac{2}{3ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -2/(3*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(a+x)^{5/2}}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{2}{3ad(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.0399404, size = 24, normalized size = 1.

$$-\frac{2}{3ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -2/(3*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Maple [A] time = 0.006, size = 21, normalized size = 0.9

$$-\frac{2}{3\,da}\left(a+a\sin\left(dx+c\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3/a/d/(a+a*sin(d*x+c))^(3/2)
```

Maxima [A] time = 0.935473, size = 27, normalized size = 1.12

$$-\frac{2}{3\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((a*sin(d*x + c) + a)^(3/2)*a*d)
```

Fricas [B] time = 2.14845, size = 116, normalized size = 4.83

$$\frac{2\sqrt{a\sin(dx+c)+a}}{3\left(a^3d\cos(dx+c)^2-2a^3d\sin(dx+c)-2a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(a*sin(d*x + c) + a)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) -
 2*a^3*d)
```

Sympy [A] time = 30.179, size = 65, normalized size = 2.71

$$\begin{cases} -\frac{2}{3a^2d\sqrt{a\sin(c+dx)+a}\sin(c+dx)+3a^2d\sqrt{a\sin(c+dx)+a}} & \text{for } d\neq 0 \\ \frac{x\cos(c)}{(a\sin(c)+a)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((-2/(3*a**2*d*sqrt(a*sin(c + d*x) + a)*sin(c + d*x) + 3*a**2*d*sq
rt(a*sin(c + d*x) + a)), Ne(d, 0)), (x*cos(c)/(a*sin(c) + a)**(5/2), True))
```

Giac [A] time = 1.11838, size = 27, normalized size = 1.12

$$-\frac{2}{3\left(a\sin(dx+c)+a\right)^{\frac{3}{2}}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/((a*sin(d*x + c) + a)^(3/2)*a*d)
```

3.192 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{1}{4a^2d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d}-\frac{1}{6ad(a\sin(c+dx)+a)^{3/2}}-\frac{1}{5d(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(4*Sqrt[2]*a^(5/2)*d) -
 1/(5*d*(a + a*Sin[c + d*x])^(5/2)) - 1/(6*a*d*(a + a*Sin[c + d*x])^(3/2))
- 1/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.0886034, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2667, 51, 63, 206}

$$-\frac{1}{4a^2d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d}-\frac{1}{6ad(a\sin(c+dx)+a)^{3/2}}-\frac{1}{5d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(4*Sqrt[2]*a^(5/2)*d) -
 1/(5*d*(a + a*Sin[c + d*x])^(5/2)) - 1/(6*a*d*(a + a*Sin[c + d*x])^(3/2))
- 1/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{a\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{7/2}}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{1}{5d(a+a\sin(c+dx))^{5/2}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, a\sin(c+dx)\right)}{2d}$$

$$= -\frac{1}{5d(a+a\sin(c+dx))^{5/2}} - \frac{1}{6ad(a+a\sin(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, a\sin(\right.}{4ad}$$

$$= -\frac{1}{5d(a+a\sin(c+dx))^{5/2}} - \frac{1}{6ad(a+a\sin(c+dx))^{3/2}} - \frac{1}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}}{}$$

$$= -\frac{1}{5d(a+a\sin(c+dx))^{5/2}} - \frac{1}{6ad(a+a\sin(c+dx))^{3/2}} - \frac{1}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} - \frac{1}{5d(a+a\sin(c+dx))^{5/2}} - \frac{1}{6ad(a+a\sin(c+dx))^{3/2}} - \frac{}{4a^2d\sqrt{a}}$$

Mathematica [C] time = 0.0842239, size = 41, normalized size = 0.36

$$-\frac{{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{5d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sin[c + d*x])/2]/(5*d*(a + a*Sin[c +
 d*x])^(5/2))
```

Maple [A] time = 0.097, size = 88, normalized size = 0.8

$$-2\,\frac{a}{d}\left(-1/16\,\frac{\sqrt{2}}{a^{7/2}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)+1/8\,\frac{1}{a^{3}\sqrt{a+a\sin(dx+c)}}+1/12\,\frac{1}{a^{2}\left(a+a\sin(dx+c)\right)^{3/2}}+1/10$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2*a*(-1/16/a^(7/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1
/2))+1/8/a^3/(a+a*sin(d*x+c))^(1/2)+1/12/a^2/(a+a*sin(d*x+c))^(3/2)+1/10/a/
(a+a*sin(d*x+c))^(5/2))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.47456, size = 452, normalized size = 4.

$$\frac{15\,\sqrt{2}\left(3\,\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-4\right)\sin\left(dx+c\right)-4\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\,\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3\,a}{\sin(dx+c)-1}\right)-4\left(15\,\cos\left(dx+\right.\right.}{240\left(3\,a^{3}d\cos\left(dx+c\right)^{2}-4\,a^{3}d+\left(a^{3}d\cos\left(dx+c\right)^{2}-4\,a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/240*(15*sqrt(2)*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*x + c) - 4
)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 + 3*a)/(sin(d*x + c) - 1)) - 4*(15*cos(d*x + c)^2 - 40*sin(d*x + c) - 52)*
sqrt(a*sin(d*x + c) + a))/(3*a^3*d*cos(d*x + c)^2 - 4*a^3*d + (a^3*d*cos(d*
x + c)^2 - 4*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12648, size = 130, normalized size = 1.15

$$-\frac{1}{120}\,a\left(\frac{15\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\sin(dx+c)+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}a^3d}+\frac{2\left(15\,(a\sin\left(dx+c\right)+a)^2+10\,(a\sin\left(dx+c\right)+a)a+12\,a^2\right)}{(a\sin\left(dx+c\right)+a)^{\frac{5}{2}}a^3d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/120*a*(15*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*sin(d*x + c) + a)/sqrt(-a))/
(sqrt(-a)*a^3*d) + 2*(15*(a*sin(d*x + c) + a)^2 + 10*(a*sin(d*x + c) + a)*a
 + 12*a^2)/((a*sin(d*x + c) + a)^(5/2)*a^3*d))
```

3.193 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=167

$$\frac{35\sec(c+dx)}{96a^2d\sqrt{a\sin(c+dx)+a}}-\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{128\sqrt{2}a^{5/2}d}-\frac{35\cos(c+dx)}{128ad(a\sin(c+dx)+a)^{3/2}}-\frac{7\sec(c+dx)}{48ad(a\sin(c+dx)+a)^{3/2}}-\frac{}{6d(}$$

```
[Out] (-35*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
28*Sqrt[2]*a^(5/2)*d) - Sec[c + d*x]/(6*d*(a + a*Sin[c + d*x])^(5/2)) - (35
*Cos[c + d*x])/(128*a*d*(a + a*Sin[c + d*x])^(3/2)) - (7*Sec[c + d*x])/(48*
a*d*(a + a*Sin[c + d*x])^(3/2)) + (35*Sec[c + d*x])/(96*a^2*d*Sqrt[a + a*Si
n[c + d*x]])
```

Rubi [A] time = 0.230418, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2681, 2687, 2650, 2649, 206}

$$\frac{35\sec(c+dx)}{96a^2d\sqrt{a\sin(c+dx)+a}}-\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{128\sqrt{2}a^{5/2}d}-\frac{35\cos(c+dx)}{128ad(a\sin(c+dx)+a)^{3/2}}-\frac{7\sec(c+dx)}{48ad(a\sin(c+dx)+a)^{3/2}}-\frac{}{6d(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-35*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
28*Sqrt[2]*a^(5/2)*d) - Sec[c + d*x]/(6*d*(a + a*Sin[c + d*x])^(5/2)) - (35
*Cos[c + d*x])/(128*a*d*(a + a*Sin[c + d*x])^(3/2)) - (7*Sec[c + d*x])/(48*
a*d*(a + a*Sin[c + d*x])^(3/2)) + (35*Sec[c + d*x])/(96*a^2*d*Sqrt[a + a*Si
n[c + d*x]])
```


Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}} + \frac{7\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{12a}$$

$$= -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}} - \frac{7\sec(c+dx)}{48ad(a+a\sin(c+dx))^{3/2}} + \frac{35\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{96a^2}$$

$$= -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}} - \frac{7\sec(c+dx)}{48ad(a+a\sin(c+dx))^{3/2}} + \frac{35\sec(c+dx)}{96a^2d\sqrt{a+a\sin(c+dx)}} + \frac{35\int}{}$$

$$= -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}} - \frac{35\cos(c+dx)}{128ad(a+a\sin(c+dx))^{3/2}} - \frac{7\sec(c+dx)}{48ad(a+a\sin(c+dx))^{3/2}} + \frac{}{96a}$$

$$= -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}} - \frac{35\cos(c+dx)}{128ad(a+a\sin(c+dx))^{3/2}} - \frac{7\sec(c+dx)}{48ad(a+a\sin(c+dx))^{3/2}} + \frac{}{96a}$$

$$= -\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d} - \frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}} - \frac{35\cos(c+dx)}{128ad(a+a\sin(c+dx))^{3/2}} - \frac{}{48}$$

Mathematica [C] time = 0.395521, size = 284, normalized size = 1.7

$$\frac{\frac{48\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} - 57\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4 + 114\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-32 + (64*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 88*Sin
[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 44*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^2 + 114*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^3 - 57*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (105 + 105*I)*(-1)
^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^5 + (48*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(384*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A] time = 0.187, size = 266, normalized size = 1.6

$$-\frac{1}{768\,(1+\sin(dx+c))^2\cos(dx+c)\,d}\left(\left(210\,a^{7/2} - 105\sqrt{a-a\sin(dx+c)}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)a^3\right)\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/768/a^(11/2)*((210*a^(7/2)-105*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(d*x+c)*cos(d*x+c)^2+(-44
8*a^(7/2)+420*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(
1/2)*2^(1/2)/a^(1/2))*a^3)*sin(d*x+c)+(490*a^(7/2)-315*(a-a*sin(d*x+c))^(1/
2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*cos(d*x
+c)^2-320*a^(7/2)+420*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d
*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(1+sin(d*x+c))^2/cos(d*x+c)/(a+a*sin(d*x
+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 2.44893, size = 755, normalized size = 4.52

$$\frac{105\sqrt{2}\Big(3\cos(dx+c)^3+\big(\cos(dx+c)^3-4\cos(dx+c)\big)\sin(dx+c)-4\cos(dx+c)\Big)\sqrt{a}\log\Big(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx}}{}}{1536\,\big(3a^3d\cos(dx+c)^3-4a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(105*sqrt(2)*(3*cos(d*x + c)^3 + (cos(d*x + c)^3 - 4*cos(d*x + c))*s
in(d*x + c) - 4*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sq
rt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(
d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (co
s(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(245*cos(d*x + c)^2 +
```

```
 7*(15*cos(d*x + c)^2 - 32)*sin(d*x + c) - 160)*sqrt(a*sin(d*x + c) + a))/(
3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x + c)^3 - 4*a
^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] sage2
```

3.194 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{9}{32a^2d\sqrt{a\sin(c+dx)+a}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d}+\frac{9\sec^2(c+dx)}{40a^2d\sqrt{a\sin(c+dx)+a}}-\frac{3}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{}{70ad(}$$

```
[Out] (9*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*a^(5/2)
*d) - Sec[c + d*x]^2/(7*d*(a + a*Sin[c + d*x])^(5/2)) - 3/(16*a*d*(a + a*Si
n[c + d*x])^(3/2)) - (9*Sec[c + d*x]^2)/(70*a*d*(a + a*Sin[c + d*x])^(3/2))
 - 9/(32*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (9*Sec[c + d*x]^2)/(40*a^2*d*Sqr
t[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.27491, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2681, 2687, 2667, 51, 63, 206}

$$-\frac{9}{32a^2d\sqrt{a\sin(c+dx)+a}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d}+\frac{9\sec^2(c+dx)}{40a^2d\sqrt{a\sin(c+dx)+a}}-\frac{3}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{}{70ad(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (9*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*a^(5/2)
*d) - Sec[c + d*x]^2/(7*d*(a + a*Sin[c + d*x])^(5/2)) - 3/(16*a*d*(a + a*Si
n[c + d*x])^(3/2)) - (9*Sec[c + d*x]^2)/(70*a*d*(a + a*Sin[c + d*x])^(3/2))
 - 9/(32*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (9*Sec[c + d*x]^2)/(40*a^2*d*Sqr
t[a + a*Sin[c + d*x]])
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{9\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{14a}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx))^{3/2}} + \frac{9\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{20a^2}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx))^{3/2}} + \frac{9\sec^2(c+dx)}{40a^2d\sqrt{a+a\sin(c+dx)}} + \frac{9\int}{}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx))^{3/2}} + \frac{9\sec^2(c+dx)}{40a^2d\sqrt{a+a\sin(c+dx)}} + \frac{9\,\mathrm{Su}}{}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{3}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx))^{3/2}} + \frac{}{40a}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{3}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx))^{3/2}} - \frac{}{32a}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{3}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx))^{3/2}} - \frac{}{32a}$$

$$= \frac{9\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d} - \frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{3}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9}{70ad(}$$

Mathematica [C] time = 0.0895356, size = 42, normalized size = 0.23

$$-\frac{a\,{}_2F_1\left(-\frac{7}{2},2;-\frac{5}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{14d(a\sin(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(a*Hypergeometric2F1[-7/2, 2, -5/2, (1 + Sin[c + d*x])/2])/(14*d*(a + a*Si
n[c + d*x])^(7/2))
```

Maple [A] time = 0.194, size = 141, normalized size = 0.8

$$2\,\frac{a^3}{d}\left(-1/16\,\frac{1}{a^5}\left(1/4\,\frac{\sqrt{a+a\sin(dx+c)}}{a\sin(dx+c)-a} - \frac{9\sqrt{2}}{8\sqrt{a}}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{\sqrt{a}}\right)\right) - 1/8\,\frac{1}{a^5\sqrt{a+a\sin(dx+c)}} - 1/16\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 2*a^3*(-1/16/a^5*(1/4*(a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c)-a)-9/8*2^(1/2)/a
^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))-1/8/a^5/(a+a*si
n(d*x+c))^(1/2)-1/16/a^4/(a+a*sin(d*x+c))^(3/2)-1/20/a^3/(a+a*sin(d*x+c))^(
5/2)-1/28/a^2/(a+a*sin(d*x+c))^(7/2))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.53116, size = 601, normalized size = 3.25

$$\frac{315\,\sqrt{2}\left(3\,\cos\left(dx+c\right)^{4}-4\,\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{4}-4\,\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\,\sqrt{2}\sqrt{a\sin(dx+}}{\sin(dx+c)-1}\right.}{4480\left(3\,a^{3}d\cos\left(dx+c\right)^{4}-4\,a^{3}d\cos\left(dx+c\right)^{2}+\left(a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(315*sqrt(2)*(3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + (cos(d*x + c)^4
- 4*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*(315*cos(d*
x + c)^4 - 1092*cos(d*x + c)^2 - 120*(7*cos(d*x + c)^2 - 3)*sin(d*x + c) +
200)*sqrt(a*sin(d*x + c) + a))/(3*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x +
c)^2 + (a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14311, size = 201, normalized size = 1.09

$$-\frac{1}{2240}a^3\left(\frac{315\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\sin(dx+c)+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^5d}+\frac{70\sqrt{a\sin(dx+c)+a}}{(a\sin(dx+c)-a)a^5d}+\frac{8\left(70\left(a\sin(dx+c)+a\right)^3+35\left(a\sin(dx+c)+\right.\right.}{(a\sin(dx+c)-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/2240*a^3*(315*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*sin(d*x + c) + a)/sqrt(-
a))/(sqrt(-a)*a^5*d) + 70*sqrt(a*sin(d*x + c) + a)/((a*sin(d*x + c) - a)*a^
5*d) + 8*(70*(a*sin(d*x + c) + a)^3 + 35*(a*sin(d*x + c) + a)^2*a + 28*(a*s
in(d*x + c) + a)*a^2 + 20*a^3)/((a*sin(d*x + c) + a)^(7/2)*a^5*d))
```

3.195 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=233

$$\frac{11\sec^3(c+dx)}{64a^2d\sqrt{a\sin(c+dx)+a}}+\frac{385\sec(c+dx)}{1024a^2d\sqrt{a\sin(c+dx)+a}}-\frac{1155\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4096\sqrt{2}a^{5/2}d}-\frac{1155\cos(c+dx)}{4096ad(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-1155*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/
(4096*Sqrt[2]*a^(5/2)*d) - Sec[c + d*x]^3/(8*d*(a + a*Sin[c + d*x])^(5/2))
- (1155*Cos[c + d*x])/(4096*a*d*(a + a*Sin[c + d*x])^(3/2)) - (77*Sec[c + d
*x])/(512*a*d*(a + a*Sin[c + d*x])^(3/2)) - (11*Sec[c + d*x]^3)/(96*a*d*(a
+ a*Sin[c + d*x])^(3/2)) + (385*Sec[c + d*x])/(1024*a^2*d*Sqrt[a + a*Sin[c
+ d*x]]) + (11*Sec[c + d*x]^3)/(64*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.363838, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2681, 2687, 2650, 2649, 206}

$$\frac{11\sec^3(c+dx)}{64a^2d\sqrt{a\sin(c+dx)+a}}+\frac{385\sec(c+dx)}{1024a^2d\sqrt{a\sin(c+dx)+a}}-\frac{1155\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4096\sqrt{2}a^{5/2}d}-\frac{1155\cos(c+dx)}{4096ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-1155*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/
(4096*Sqrt[2]*a^(5/2)*d) - Sec[c + d*x]^3/(8*d*(a + a*Sin[c + d*x])^(5/2))
- (1155*Cos[c + d*x])/(4096*a*d*(a + a*Sin[c + d*x])^(3/2)) - (77*Sec[c + d
*x])/(512*a*d*(a + a*Sin[c + d*x])^(3/2)) - (11*Sec[c + d*x]^3)/(96*a*d*(a
+ a*Sin[c + d*x])^(3/2)) + (385*Sec[c + d*x])/(1024*a^2*d*Sqrt[a + a*Sin[c
+ d*x]]) + (11*Sec[c + d*x]^3)/(64*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
```

```
IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} + \frac{11\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{16a}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{11\sec^3(c+dx)}{96ad(a+a\sin(c+dx))^{3/2}} + \frac{33\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{64a^2}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{11\sec^3(c+dx)}{96ad(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^3(c+dx)}{64a^2d\sqrt{a+a\sin(c+dx)}} + \frac{77\int}{}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{77\sec(c+dx)}{512ad(a+a\sin(c+dx))^{3/2}} - \frac{11\sec^3(c+dx)}{96ad(a+a\sin(c+dx))^{3/2}} + \frac{}{64a}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{77\sec(c+dx)}{512ad(a+a\sin(c+dx))^{3/2}} - \frac{11\sec^3(c+dx)}{96ad(a+a\sin(c+dx))^{3/2}} + \frac{}{102}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{1155\cos(c+dx)}{4096ad(a+a\sin(c+dx))^{3/2}} - \frac{77\sec(c+dx)}{512ad(a+a\sin(c+dx))^{3/2}} - \frac{}{9}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{1155\cos(c+dx)}{4096ad(a+a\sin(c+dx))^{3/2}} - \frac{77\sec(c+dx)}{512ad(a+a\sin(c+dx))^{3/2}} - \frac{}{9}$$

$$= -\frac{1155\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d} - \frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{1155\cos(c+dx)}{4096ad(a+a\sin(c+dx))^{3/2}}$$

Mathematica [C] time = 0.515932, size = 394, normalized size = 1.69

$$\frac{\frac{1920\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{256\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - 1545\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4 + 3090\sin\left(\frac{1}{2}(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-736 + (768*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 38
4/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (1472*Sin[(c + d*x)/2])/(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]) + 2072*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]) - 1036*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 3090*Sin
[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 1545*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^4 + (3465 + 3465*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*
```

```
(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5
 + (256*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^3 + (1920*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]))/(12288*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A] time = 0.192, size = 355, normalized size = 1.5

$$\frac{1}{(24576 \sin(dx+c) - 24576)(1+\sin(dx+c))^3 \cos(dx+c)\, d}\left(6930\, a^{11/2} \sin(dx+c)(\cos(dx+c))^4 - 924\left(16\, a^{11/2} + 1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/24576/a^(15/2)*(6930*a^(11/2)*sin(d*x+c)*cos(d*x+c)^4-924*(16*a^(11/2)+15
*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(d*x+c))^(3/2)
*2^(1/2)*a^4)*cos(d*x+c)^2*sin(d*x+c)+(-5632*a^(11/2)+27720*arctanh(1/2*(a-
a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*a^4)*si
n(d*x+c)+(16170*a^(11/2)+3465*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^
(1/2))*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*a^4)*cos(d*x+c)^4-1320*(8*a^(11/2)+21
*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(d*x+c))^(3/2)
*2^(1/2)*a^4)*cos(d*x+c)^2-2560*a^(11/2)+27720*arctanh(1/2*(a-a*sin(d*x+c))
^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*a^4)/(sin(d*x+c)-1)/
(1+sin(d*x+c))^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.61488, size = 833, normalized size = 3.58

$$\frac{3465\sqrt{2}\Big(3\cos(dx+c)^5-4\cos(dx+c)^3+\big(\cos(dx+c)^5-4\cos(dx+c)^3\big)\sin(dx+c)\Big)\sqrt{a}\log\Big(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(}{}}{49152\,\big(3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/49152*(3465*sqrt(2)*(3*cos(d*x + c)^5 - 4*cos(d*x + c)^3 + (cos(d*x + c)^
5 - 4*cos(d*x + c)^3)*sin(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt
(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*
a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2
 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(8085*cos(d*x +
 c)^4 - 5280*cos(d*x + c)^2 + 11*(315*cos(d*x + c)^4 - 672*cos(d*x + c)^2 -
 256)*sin(d*x + c) - 1280)*sqrt(a*sin(d*x + c) + a))/(3*a^3*d*cos(d*x + c)^
5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3
)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.196 $\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=124

$$\frac{10ae^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}+\frac{10ae^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{9/2}}{9de}+\frac{2ae\sin(c+dx)(e\cos(c}{7d}$$

```
[Out] (-2*a*(e*Cos[c + d*x])^(9/2))/(9*d*e) + (10*a*e^4*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (10*a*e^3*Sqrt[e*Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*a*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(
7*d)
```

Rubi [A] time = 0.0842286, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2635, 2642, 2641}

$$\frac{10ae^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}+\frac{10ae^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{9/2}}{9de}+\frac{2ae\sin(c+dx)(e\cos(c}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x]),x]
```


```
[Out] (-2*a*(e*Cos[c + d*x])^(9/2))/(9*d*e) + (10*a*e^4*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (10*a*e^3*Sqrt[e*Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*a*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(
7*d)
```


Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))\,dx &= -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + a\int (e\cos(c+dx))^{7/2}\,dx \\
&= -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + \frac{2ae(e\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}\left(5ae^2\right)\int (e\cos(\\
&= -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2ae(e\cos(c+dx))}{7d} \\
&= -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2ae(e\cos(c+dx))}{7d} \\
&= -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}} + \frac{10ae^3\sqrt{e\cos(c}{2}
\end{aligned}$$

Mathematica [A] time = 0.583878, size = 98, normalized size = 0.79

$$-\frac{ae^3\sqrt{e\cos(c+dx)}\left(\sqrt{\cos(c+dx)}(-138\sin(c+dx)-18\sin(3(c+dx))+28\cos(2(c+dx))+7\cos(4(c+dx))+21)-12\right.}{252d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*e^3*Sqrt[e*Cos[c + d*x]]*(-120*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c +
 d*x]]*(21 + 28*Cos[2*(c + d*x)] + 7*Cos[4*(c + d*x)] - 138*Sin[c + d*x] -
18*Sin[3*(c + d*x)])))/(252*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.454, size = 249, normalized size = 2.

$$-\frac{2\,a e^{4}}{63\,d}\left(-224\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{11}+144\,\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}+560\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{9}-216\,\left(\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] -2/63/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^4*(-224*si
n(1/2*d*x+1/2*c)^11+144*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+560*sin(1/2
*d*x+1/2*c)^9-216*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-560*sin(1/2*d*x+1
/2*c)^7+168*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+280*sin(1/2*d*x+1/2*c)^
5+15*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-70
*sin(1/2*d*x+1/2*c)^3+7*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\left(a\sin\left(dx+c\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ae^{3}\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+ae^{3}\cos\left(dx+c\right)^{3}\right)\sqrt{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*e^3*cos(d*x + c)^3*sin(d*x + c) + a*e^3*cos(d*x + c)^3)*sqrt(e*
cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{7}{2}}(a\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a), x)
```

3.197 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=95

$$\frac{6ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{7/2}}{7de}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{3/2}}{5d}$$

```
[Out] (-2*a*(e*Cos[c + d*x])^(7/2))/(7*d*e) + (6*a*e^2*Sqrt[e*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a*e*(e*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0671417, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2635, 2640, 2639}

$$\frac{6ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{7/2}}{7de}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*(e*Cos[c + d*x])^(7/2))/(7*d*e) + (6*a*e^2*Sqrt[e*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a*e*(e*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(5*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))\,dx &= -\frac{2a(e\cos(c+dx))^{7/2}}{7de} + a\int (e\cos(c+dx))^{5/2}\,dx\\ &= -\frac{2a(e\cos(c+dx))^{7/2}}{7de} + \frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\left(3ae^2\right)\int \sqrt{e\cos\\ &= -\frac{2a(e\cos(c+dx))^{7/2}}{7de} + \frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\left(3ae^2\sqrt{e\cos(c+\\ &= -\frac{2a(e\cos(c+dx))^{7/2}}{7de} + \frac{6ae^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2ae(e\cos(c+\end{aligned}$$

Mathematica [C] time = 2.60873, size = 264, normalized size = 2.78

$$ae^3\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(168(\cos(dx)-i\sin(dx))\sqrt{i\sin(2(c+dx))+\cos(2(c+dx))+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*e^3*Csc[c/2]*Sec[c/2]*(-154*Cos[d*x] - 182*Cos[2*c + d*x] + 14*Cos[2*c +
 3*d*x] - 14*Cos[4*c + 3*d*x] - 30*Sin[c] + 168*Hypergeometric2F1[-1/4, 1/2
, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] - I*Sin[d*x])*Sqrt
[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] + 56*Hypergeometric2F1[1/2, 3/4
, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] + I*Sin[d*x])*Sqrt
[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] + 20*Sin[c + 2*d*x] - 20*Sin[3*
c + 2*d*x] + 5*Sin[3*c + 4*d*x] - 5*Sin[5*c + 4*d*x]))/(560*d*Sqrt[e*Cos[c
+ d*x]])
```

Maple [A] time = 0.411, size = 214, normalized size = 2.3

$$\frac{2\,ae^3}{35\,d}\left(-80\,\left(\sin\left(1/2\,dx+c/2\right)\right)^9+56\,\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)+160\,\left(\sin\left(1/2\,dx+c/2\right)\right)^7-56\,\left(\sin\left(1/2\,dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] 2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^3*(-80*sin(
1/2*d*x+1/2*c)^9+56*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+160*sin(1/2*d*x
+1/2*c)^7-56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-120*sin(1/2*d*x+1/2*c)
^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+14*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+4
0*sin(1/2*d*x+1/2*c)^3-5*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sin\left(dx+c\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ae^2\cos\left(dx+c\right)^2\sin\left(dx+c\right)+ae^2\cos\left(dx+c\right)^2\right)\sqrt{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*cos(d*x + c)^2*sin(d*x + c) + a*e^2*cos(d*x + c)^2)*sqrt(e*
cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a), x)
```

3.198 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=95

$$\frac{2ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{5/2}}{5de}+\frac{2ae\sin(c+dx)\sqrt{e\cos(c+dx)}}{3d}$$

```
[Out] (-2*a*(e*Cos[c + d*x])^(5/2))/(5*d*e) + (2*a*e^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) + (2*a*e*Sqrt[e*Cos[c + d*x]
]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0649123, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2635, 2642, 2641}

$$\frac{2ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{5/2}}{5de}+\frac{2ae\sin(c+dx)\sqrt{e\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*(e*Cos[c + d*x])^(5/2))/(5*d*e) + (2*a*e^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) + (2*a*e*Sqrt[e*Cos[c + d*x]
]*Sin[c + d*x])/(3*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))\,dx &= -\frac{2a(e\cos(c+dx))^{5/2}}{5de}+a\int (e\cos(c+dx))^{3/2}\,dx\\ &= -\frac{2a(e\cos(c+dx))^{5/2}}{5de}+\frac{2ae\sqrt{e\cos(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}\left(ae^2\right)\int \frac{1}{\sqrt{e\cos(c\\ &= -\frac{2a(e\cos(c+dx))^{5/2}}{5de}+\frac{2ae\sqrt{e\cos(c+dx)}\sin(c+dx)}{3d}+\frac{\left(ae^2\sqrt{\cos(c+dx)}\right)}{3\sqrt{e\cos(c}\\ &= -\frac{2a(e\cos(c+dx))^{5/2}}{5de}+\frac{2ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}+\frac{2ae\sqrt{e\cos(c+a}{3a}\end{aligned}$$

Mathematica [A] time = 0.363893, size = 75, normalized size = 0.79

$$-\frac{a(e\cos(c+dx))^{3/2}\left(\sqrt{\cos(c+dx)}(-10\sin(c+dx)+3\cos(2(c+dx))+3)-10F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*(e*Cos[c + d*x])^(3/2)*(-10*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*
x]]*(3 + 3*Cos[2*(c + d*x)] - 10*Sin[c + d*x])))/(15*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.352, size = 179, normalized size = 1.9

$$-\frac{2\,ae^2}{15\,d}\left(-24\,(\sin(1/2\,dx+c/2))^7+20\,(\sin(1/2\,dx+c/2))^4\cos(1/2\,dx+c/2)+36\,(\sin(1/2\,dx+c/2))^5+5\sqrt{2\,(\sin(1/2\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] -2/15/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^2*(-24*sin
(1/2*d*x+1/2*c)^7+20*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+36*sin(1/2*d*x
+1/2*c)^5+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)-18*sin(1/2*d*x+1/2*c)^3+3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}} (a\sin(dx+c)+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ae\cos(dx+c)\sin(dx+c)+ae\cos(dx+c))\sqrt{e\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*e*cos(d*x + c)*sin(d*x + c) + a*e*cos(d*x + c))*sqrt(e*cos(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a), x)
```

3.199 $\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=63

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{3/2}}{3de}$$

```
[Out] (-2*a*(e*Cos[c + d*x])^(3/2))/(3*d*e) + (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0460988, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2669, 2640, 2639}

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{3/2}}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*(e*Cos[c + d*x])^(3/2))/(3*d*e) + (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))\,dx &= -\frac{2a(e\cos(c+dx))^{3/2}}{3de}+a\int \sqrt{e\cos(c+dx)}\,dx\\
&= -\frac{2a(e\cos(c+dx))^{3/2}}{3de}+\frac{\left(a\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{2a(e\cos(c+dx))^{3/2}}{3de}+\frac{2a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 0.958106, size = 260, normalized size = 4.13

$$\frac{a\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)(\cos(dx)+i\sin(dx))\sqrt{e\cos(c+dx)}\left(6(\cos(dx)-i\sin(dx))\sqrt{i\sin(2(c+dx))+\cos(2(c+dx))+1}\,{}_2F_1\left(-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sqrt[e*Cos[c + d*x]]*Csc[c/2]*Sec[c/2]*(Cos[d*x] + I*Sin[d*x])*(-6*Cos[d
*x] - 6*Cos[2*c + d*x] - 2*Sin[c] + 6*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E
^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] - I*Sin[d*x])*Sqrt[1 + Cos[2
*(c + d*x)] + I*Sin[2*(c + d*x)]] + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^
((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] + I*Sin[d*x])*Sqrt[1 + Cos[2*
(c + d*x)] + I*Sin[2*(c + d*x)]] + Sin[c + 2*d*x] - Sin[3*c + 2*d*x]))/(6*d
*((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c]))
```

Maple [A] time = 0.303, size = 120, normalized size = 1.9

$$\frac{2\,ae}{3\,d}\left(-4\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{5}+3\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\mathrm{EllipticE}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e*(-4*sin(1/2*
d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+4*sin(1/2*d*x+1/2*c)^3-sin(1/2*d*x+
1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e \cos (dx + c)}(a \sin (dx + c) + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a), x)
```

3.200 $\int \frac{a+a\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}}{de}$$

```
[Out] (-2*a*Sqrt[e*Cos[c + d*x]])/(d*e) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.0463531, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2669, 2642, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```


```
[Out] (-2*a*Sqrt[e*Cos[c + d*x]])/(d*e) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]])
```


Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx &= -\frac{2a\sqrt{e\cos(c+dx)}}{de}+a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx\\
&= -\frac{2a\sqrt{e\cos(c+dx)}}{de}+\frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{e\cos(c+dx)}}\\
&= -\frac{2a\sqrt{e\cos(c+dx)}}{de}+\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 20.9496, size = 48, normalized size = 0.79

$$-\frac{2a\left(\cos(c+dx)-\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*a*(Cos[c + d*x] - Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]))/(d*Sqr
t[e*Cos[c + d*x]])
```

Maple [A] time = 0.22, size = 103, normalized size = 1.7

$$-2\,\frac{a\left(\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2)\,,\sqrt{2}\right)-2\,(\sin(1/2\,dx+c/2))^3+\sin}{\sin(1/2\,dx+c/2)\sqrt{-2\,(\sin(1/2\,dx+c/2))^2\,e+e}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*((2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
```

```
),2^(1/2))-2*sin(1/2*d*x+1/2*c)^3+sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a \sin(dx + c) + a}{\sqrt{e \cos(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e \cos(dx + c)}(a \sin(dx + c) + a)}{e \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)/(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin(dx+c)+a}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

3.201 $\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2a}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] (2*a)/(d*e*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(d*e*Sqrt[e*Co
s[c + d*x]])
```

Rubi [A] time = 0.0676036, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2636, 2640, 2639}

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2a}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*a)/(d*e*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(d*e*Sqrt[e*Co
s[c + d*x]])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{2a}{de\sqrt{e\cos(c+dx)}}+a\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2a}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}-\frac{a\int \sqrt{e\cos(c+dx)}\,dx}{e^2}\\ &= \frac{2a}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}-\frac{\left(a\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{e^2\sqrt{\cos(c+dx)}}\\ &= \frac{2a}{de\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.914775, size = 188, normalized size = 2.07

$$-\frac{a\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(3(\cos(dx)-i\sin(dx))\sqrt{i\sin(2(c+dx))+\cos(2(c+dx))+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)+\cdots}{6de\sqrt{e\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] -(a*Csc[c/2]*Sec[c/2]*(-6*(Cos[d*x] + Sin[c]) + 3*Hypergeometric2F1[-1/4, 1
/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] - I*Sin[d*x])*Sq
rt[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] + Hypergeometric2F1[1/2, 3/4,
 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] + I*Sin[d*x])*Sqrt[
1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]]))/(6*d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 0.488, size = 117, normalized size = 1.3

$$-2\,\frac{\left(\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2)\,,\sqrt{2}\right)-2\,(\sin(1/2\,dx+c/2))^2\cos(1/2\,dx+c/2)-\sin(1/2\,dx+c/2)\right)a}{e\sqrt{-2\,(\sin(1/2\,dx+c/2))^2e+e}\sin(1/2\,dx+c/2)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*((sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))*a/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin(dx+c)+a}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)}{e^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)/(e^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a \sin (dx + c) + a}{(e \cos (dx + c))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

3.202 $\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a}{3de(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] (2*a)/(3*d*e*(e*Cos[c + d*x])^(3/2)) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(3*d*e*(
e*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0652218, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2636, 2642, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a}{3de(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a)/(3*d*e*(e*Cos[c + d*x])^(3/2)) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(3*d*e*(
e*Cos[c + d*x])^(3/2))
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx &= \frac{2a}{3de(e\cos(c+dx))^{3/2}}+a\int \frac{1}{(e\cos(c+dx))^{5/2}}\,dx\\
&= \frac{2a}{3de(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}+\frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}\\
&= \frac{2a}{3de(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}+\frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3e^2\sqrt{e\cos(c+dx)}}\\
&= \frac{2a}{3de(e\cos(c+dx))^{3/2}}+\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.429061, size = 86, normalized size = 0.89

$$\frac{2a\left(\cos(c+dx)-(\sin(c+dx)-1)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3de^2\sqrt{e\cos(c+dx)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a*(Cos[c + d*x] - Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(-1 + Sin
[c + d*x])))/(3*d*e^2*Sqrt[e*Cos[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])^2)
```

Maple [A] time = 0.609, size = 189, normalized size = 2.

$$-\frac{2a}{3\,de^{2}}\left(2\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c)
)*a/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a\sin\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}\left(a\sin\left(dx+c\right)+a\right)}{e^{3}\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)/(e^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin(dx+c)+a}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

3.203 $\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=126

$$\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}$$

```
[Out] (2*a)/(5*d*e*(e*Cos[c + d*x])^(5/2)) - (6*a*Sqrt[e*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(5*d*e*(
e*Cos[c + d*x])^(5/2)) + (6*a*Sin[c + d*x])/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.0863084, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2636, 2640, 2639}

$$\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```


```
[Out] (2*a)/(5*d*e*(e*Cos[c + d*x])^(5/2)) - (6*a*Sqrt[e*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(5*d*e*(
e*Cos[c + d*x])^(5/2)) + (6*a*Sin[c + d*x])/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```


Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx &= \frac{2a}{5de(e\cos(c+dx))^{5/2}}+a\int \frac{1}{(e\cos(c+dx))^{7/2}}\,dx\\
&= \frac{2a}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{(3a)\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx}{5e^2}\\
&= \frac{2a}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{(3a)\int \sqrt{e\cos(c+dx)}\,dx}{5e^4}\\
&= \frac{2a}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{\left(3a\sqrt{e\cos(c+dx)}\right)\int \sqrt{c}}{5e^4\sqrt{\cos(c+a}}\\
&= \frac{2a}{5de(e\cos(c+dx))^{5/2}}-\frac{6a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{6a\sin(c}{5de^3\sqrt{e\cos}}
\end{aligned}$$

Mathematica [C] time = 1.22272, size = 144, normalized size = 1.14

$$\frac{2ae^{i(c+dx)}\left(i\sqrt{1+e^{2i(c+dx)}}\left(e^{i(c+dx)}-i\right)^2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-6e^{i(c+dx)}-3ie^{2i(c+dx)}+i\right)}{5de^3\left(e^{i(c+dx)}-i\right)^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*a*E^(I*(c + d*x))*(I - 6*E^(I*(c + d*x)) - (3*I)*E^((2*I)*(c + d*x)) + I
*(-I + E^(I*(c + d*x)))^2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1
/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(5*d*e^3*(-I + E^(I*(c + d*x)))^2*Sqr
t[e*Cos[c + d*x]])
```

Maple [B] time = 0.962, size = 304, normalized size = 2.4

$$-\frac{2a}{5de^3}\left(12\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2}\,(\sin(1/2\,dx+c/2))^4-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2
*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-sin(1/2
*d*x+1/2*c))*a/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin(dx+c)+a}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)}{e^4\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)/(e^4*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sin\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

3.204 $\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=168

$$\frac{130a^2e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{130a^2e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}-\frac{2\left(a^2\sin(c+dx)\right.}{}$$

```
[Out] (-26*a^2*(e*Cos[c + d*x])^(9/2))/(99*d*e) + (130*a^2*e^4*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (130*a^2*e^3*Sqr
t[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (26*a^2*e*(e*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(77*d) - (2*(e*Cos[c + d*x])^(9/2)*(a^2 + a^2*Sin[c + d*x]))/
(11*d*e)
```

Rubi [A] time = 0.144141, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2678, 2669, 2635, 2642, 2641}

$$\frac{130a^2e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{130a^2e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}-\frac{2\left(a^2\sin(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-26*a^2*(e*Cos[c + d*x])^(9/2))/(99*d*e) + (130*a^2*e^4*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (130*a^2*e^3*Sqr
t[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (26*a^2*e*(e*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(77*d) - (2*(e*Cos[c + d*x])^(9/2)*(a^2 + a^2*Sin[c + d*x]))/
(11*d*e)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2\,dx &= -\frac{2(e\cos(c+dx))^{9/2}\left(a^2+a^2\sin(c+dx)\right)}{11de}+\frac{1}{11}(13a)\int (e\cos(c+dx))^{7/2}(a+ \\
&= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}-\frac{2(e\cos(c+dx))^{9/2}\left(a^2+a^2\sin(c+dx)\right)}{11de}+\frac{1}{11}\Big(13a \\
&= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}+\frac{26a^2e(e\cos(c+dx))^{5/2}\sin(c+dx)}{77d}-\frac{2(e\cos(c+ \\
&= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}+\frac{130a^2e^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{231d}+\frac{26a^2e(e\cos(\\
&= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}+\frac{130a^2e^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{231d}+\frac{26a^2e(e\cos(\\
&= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}+\frac{130a^2e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{130a^2e^3\surd
\end{aligned}
$$

Mathematica [C] time = 0.101918, size = 66, normalized size = 0.39

$$-\frac{32\sqrt[4]{2}a^2(e\cos(c+dx))^{9/2}\,{}_2F_1\left(-\frac{13}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-32*2^(1/4)*a^2*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[-13/4, 9/4, 13/4,
 (1 - Sin[c + d*x])/2])/(9*d*e*(1 + Sin[c + d*x])^(9/4))
```

Maple [A] time = 0.452, size = 295, normalized size = 1.8

$$-\frac{2\,a^2e^4}{693\,d}\left(-4032\,\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^{12}+10080\,(\sin(1/2\,dx+c/2))^{10}\cos(1/2\,dx+c/2)-4928\,(\sin(1/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/693/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*e^4*(-403
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+10080*sin(1/2*d*x+1/2*c)^10*cos(
1/2*d*x+1/2*c)-4928*sin(1/2*d*x+1/2*c)^11-8208*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^8+12320*sin(1/2*d*x+1/2*c)^9+2232*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)-12320*sin(1/2*d*x+1/2*c)^7+924*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+6160*sin(1/2*d*x+1/2*c)^5+195*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-498*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-1540*sin(1/2*d*x+1/2*c)^3+154*sin(1/2*d*x+1/2*
c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(e\cos(dx+c))^{\frac{7}{2}}(a\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\left(a^2e^3\cos\left(dx+c\right)^5-2\,a^2e^3\cos\left(dx+c\right)^3\sin\left(dx+c\right)-2\,a^2e^3\cos\left(dx+c\right)^3\right)\sqrt{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*e^3*cos(d*x + c)^5 - 2*a^2*e^3*cos(d*x + c)^3*sin(d*x + c) -
 2*a^2*e^3*cos(d*x + c)^3)*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\left(a\sin\left(dx+c\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)^2, x)
```

3.205 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=137

$$\frac{22a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}-\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{7/2}}{9de}+\frac{22a^2e\sin(}{}$$

```
[Out] (-22*a^2*(e*Cos[c + d*x])^(7/2))/(63*d*e) + (22*a^2*e^2*Sqrt[e*Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (22*a^2*e*(e*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(45*d) - (2*(e*Cos[c + d*x])^(7/2)*(a^2 + a^2*
Sin[c + d*x]))/(9*d*e)
```

Rubi [A] time = 0.122008, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2678, 2669, 2635, 2640, 2639}

$$\frac{22a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}-\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{7/2}}{9de}+\frac{22a^2e\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (-22*a^2*(e*Cos[c + d*x])^(7/2))/(63*d*e) + (22*a^2*e^2*Sqrt[e*Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (22*a^2*e*(e*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(45*d) - (2*(e*Cos[c + d*x])^(7/2)*(a^2 + a^2*
Sin[c + d*x]))/(9*d*e)
```


Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2\,dx &= -\frac{2(e\cos(c+dx))^{7/2}\left(a^2+a^2\sin(c+dx)\right)}{9de}+\frac{1}{9}(11a)\int (e\cos(c+dx))^{5/2}(a+a\\ &= -\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}-\frac{2(e\cos(c+dx))^{7/2}\left(a^2+a^2\sin(c+dx)\right)}{9de}+\frac{1}{9}\left(11a^2\right.\\ &= -\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}+\frac{22a^2e(e\cos(c+dx))^{3/2}\sin(c+dx)}{45d}-\frac{2(e\cos(c+\\ &= -\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}+\frac{22a^2e(e\cos(c+dx))^{3/2}\sin(c+dx)}{45d}-\frac{2(e\cos(c+\\ &= -\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}+\frac{22a^2e^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{22a^2e(e\end{aligned}$$

Mathematica [C] time = 0.113194, size = 66, normalized size = 0.48

$$-\frac{16\ 2^{3/4}a^2(e\cos(c+dx))^{7/2}\,{}_2F_1\left(-\frac{11}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-16*2^(3/4)*a^2*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[-11/4, 7/4, 11/4,
 (1 - Sin[c + d*x])/2])/(7*d*e*(1 + Sin[c + d*x])^(7/4))
```

Maple [A] time = 0.433, size = 260, normalized size = 1.9

$$\frac{2a^2e^3}{315d}\left(-1120\left(\sin\left(1/2\,dx+c/2\right)\right)^{10}\cos\left(1/2\,dx+c/2\right)+2240\,\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}-1440\,\left(\sin\left(1/2\,dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*e^3*(-1120
*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+2240*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^8-1440*sin(1/2*d*x+1/2*c)^9-1064*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+2880*sin(1/2*d*x+1/2*c)^7-56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)-2160*sin(1/2*d*x+1/2*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+84*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+720*sin(1/2*d*x+1/2*c)^3-90*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sin\left(dx+c\right)+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2e^2\cos\left(dx+c\right)^4-2\,a^2e^2\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\,a^2e^2\cos\left(dx+c\right)^2\right)\sqrt{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*e^2*cos(d*x + c)^4 - 2*a^2*e^2*cos(d*x + c)^2*sin(d*x + c) -
 2*a^2*e^2*cos(d*x + c)^2)*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}\,(a\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^2, x)
```

3.206 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=137

$$\frac{6a^2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}-\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{5/2}}{7de}+\frac{6a^2e\sin(c+d}{}$$

```
[Out] (-18*a^2*(e*Cos[c + d*x])^(5/2))/(35*d*e) + (6*a^2*e^2*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2])/(7*d*Sqrt[e*Cos[c + d*x]]) + (6*a^2*e*Sqrt[e*Cos[
c + d*x]]*Sin[c + d*x])/(7*d) - (2*(e*Cos[c + d*x])^(5/2)*(a^2 + a^2*Sin[c
+ d*x]))/(7*d*e)
```

Rubi [A] time = 0.124766, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2678, 2669, 2635, 2642, 2641}

$$\frac{6a^2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}-\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{5/2}}{7de}+\frac{6a^2e\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-18*a^2*(e*Cos[c + d*x])^(5/2))/(35*d*e) + (6*a^2*e^2*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2])/(7*d*Sqrt[e*Cos[c + d*x]]) + (6*a^2*e*Sqrt[e*Cos[
c + d*x]]*Sin[c + d*x])/(7*d) - (2*(e*Cos[c + d*x])^(5/2)*(a^2 + a^2*Sin[c
+ d*x]))/(7*d*e)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2\,dx &= -\frac{2(e\cos(c+dx))^{5/2}\left(a^2+a^2\sin(c+dx)\right)}{7de}+\frac{1}{7}(9a)\int (e\cos(c+dx))^{3/2}(a+a\\ &= -\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}-\frac{2(e\cos(c+dx))^{5/2}\left(a^2+a^2\sin(c+dx)\right)}{7de}+\frac{1}{7}\left(9a^2\right)\\ &= -\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}+\frac{6a^2e\sqrt{e\cos(c+dx)}\sin(c+dx)}{7d}-\frac{2(e\cos(c+dx))}{}\\ &= -\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}+\frac{6a^2e\sqrt{e\cos(c+dx)}\sin(c+dx)}{7d}-\frac{2(e\cos(c+dx))}{}\\ &= -\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}+\frac{6a^2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}+\frac{6a^2e\sqrt{e\cos}}{}\end{aligned}$$

Mathematica [C] time = 0.0744977, size = 66, normalized size = 0.48

$$-\frac{16\sqrt[4]{2}a^2(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{9}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-16*2^(1/4)*a^2*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-9/4, 5/4, 9/4, (
1 - Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [A] time = 0.46, size = 203, normalized size = 1.5

$$-\frac{2\,a^2e^2}{35\,d}\left(-80\,\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+120\,\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)-112\,\left(\sin\left(1/2\,dx+c/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*e^2*(-80*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+120*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)-112*sin(1/2*d*x+1/2*c)^7+168*sin(1/2*d*x+1/2*c)^5+15*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-20*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-84*sin(1/2*d*x+1/2*c)
^3+14*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a\sin\left(dx+c\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2e\cos\left(dx+c\right)^3-2\,a^2e\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^2e\cos\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*e*cos(d*x + c)^3 - 2*a^2*e*cos(d*x + c)*sin(d*x + c) - 2*a^2
*e*cos(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^2, x)
```

3.207 $\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=105

$$-\frac{14a^2(e\cos(c+dx))^{3/2}}{15de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{3/2}}{5de}+\frac{14a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-14*a^2*(e*Cos[c + d*x])^(3/2))/(15*d*e) + (14*a^2*Sqrt[e*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) - (2*(e*Cos[c + d*x])^(3/
2)*(a^2 + a^2*Sin[c + d*x]))/(5*d*e)
```

Rubi [A] time = 0.091795, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2678, 2669, 2640, 2639}

$$-\frac{14a^2(e\cos(c+dx))^{3/2}}{15de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{3/2}}{5de}+\frac{14a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-14*a^2*(e*Cos[c + d*x])^(3/2))/(15*d*e) + (14*a^2*Sqrt[e*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) - (2*(e*Cos[c + d*x])^(3/
2)*(a^2 + a^2*Sin[c + d*x]))/(5*d*e)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2\,dx &= -\frac{2(e\cos(c+dx))^{3/2}\left(a^2+a^2\sin(c+dx)\right)}{5de}+\frac{1}{5}(7a)\int \sqrt{e\cos(c+dx)}(a+a\sin\\ &= -\frac{14a^2(e\cos(c+dx))^{3/2}}{15de}-\frac{2(e\cos(c+dx))^{3/2}\left(a^2+a^2\sin(c+dx)\right)}{5de}+\frac{1}{5}\left(7a^2\right)\\ &= -\frac{14a^2(e\cos(c+dx))^{3/2}}{15de}-\frac{2(e\cos(c+dx))^{3/2}\left(a^2+a^2\sin(c+dx)\right)}{5de}+\frac{\left(7a^2\sqrt{e\,c}\right.}{}\\ &= -\frac{14a^2(e\cos(c+dx))^{3/2}}{15de}+\frac{14a^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+d}{}\end{aligned}$$

Mathematica [C] time = 0.0441518, size = 66, normalized size = 0.63

$$-\frac{8\ 2^{3/4}a^2(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(\sin(c+dx)+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-8*2^(3/4)*a^2*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-7/4, 3/4, 7/4, (1
 - Sin[c + d*x])/2])/(3*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [A] time = 0.388, size = 188, normalized size = 1.8

$$\frac{2a^2e}{15d}\left(-24\left(\sin\left(1/2dx+c/2\right)\right)^6\cos\left(1/2dx+c/2\right)+24\left(\sin\left(1/2dx+c/2\right)\right)^4\cos\left(1/2dx+c/2\right)-40\left(\sin\left(1/2dx+c/2\right)\right)^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*e*(-24*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)-40*sin(1/2*d*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)+40*sin(1/2*d*x+1/2*c)^3-10*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos\left(dx+c\right)}\left(a\sin\left(dx+c\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2\cos\left(dx+c\right)^2-2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\sqrt{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^2, x)
```

3.208 $\int \frac{(a+a\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=105

$$-\frac{10a^2\sqrt{e\cos(c+dx)}}{3de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)\sqrt{e\cos(c+dx)}}{3de}+\frac{10a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}$$

```
[Out] (-10*a^2*Sqrt[e*Cos[c + d*x]])/(3*d*e) + (10*a^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) - (2*Sqrt[e*Cos[c + d*x]]*(a
^2 + a^2*Sin[c + d*x]))/(3*d*e)
```

Rubi [A] time = 0.0951575, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2678, 2669, 2642, 2641}

$$-\frac{10a^2\sqrt{e\cos(c+dx)}}{3de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)\sqrt{e\cos(c+dx)}}{3de}+\frac{10a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/Sqrt[e*Cos[c + d*x]],x]
```


```
[Out] (-10*a^2*Sqrt[e*Cos[c + d*x]])/(3*d*e) + (10*a^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) - (2*Sqrt[e*Cos[c + d*x]]*(a
^2 + a^2*Sin[c + d*x]))/(3*d*e)
```


Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}\,dx &= -\frac{2\sqrt{e\cos(c+dx)}\left(a^2+a^2\sin(c+dx)\right)}{3de}+\frac{1}{3}(5a)\int \frac{a+a\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx\\ &= -\frac{10a^2\sqrt{e\cos(c+dx)}}{3de}-\frac{2\sqrt{e\cos(c+dx)}\left(a^2+a^2\sin(c+dx)\right)}{3de}+\frac{1}{3}\left(5a^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,d\\ &= -\frac{10a^2\sqrt{e\cos(c+dx)}}{3de}-\frac{2\sqrt{e\cos(c+dx)}\left(a^2+a^2\sin(c+dx)\right)}{3de}+\frac{\left(5a^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c}}}{3\sqrt{e\cos(c+dx)}}\\ &= -\frac{10a^2\sqrt{e\cos(c+dx)}}{3de}+\frac{10a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}\left(a^2+a^2\sin(c\right.}{3de}\end{aligned}$$

Mathematica [C] time = 0.0333864, size = 64, normalized size = 0.61

$$-\frac{8\sqrt[4]{2}a^2\sqrt{e\cos(c+dx)}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt[4]{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-8*2^(1/4)*a^2*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-5/4, 1/4, 5/4, (1 -
 Sin[c + d*x])/2])/(d*e*(1 + Sin[c + d*x])^(1/4))
```

Maple [A] time = 0.328, size = 152, normalized size = 1.5

$$-\frac{2a^2}{3d}\left(-4\left(\sin\left(1/2\,dx+c/2\right)\right)^4\cos\left(1/2\,dx+c/2\right)+5\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\mathrm{EllipticF}\left(\cos(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*(-4*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^3+6*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^2}{\sqrt{e\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/sqrt(e*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a^2\cos\left(dx+c\right)^2-2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\sqrt{e\cos\left(dx+c\right)}}{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c))/(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^2}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/sqrt(e*cos(d*x + c)), x)
```

3.209 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}$$

```
[Out] (-6*a^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c +
 d*x]]) + (4*a^4*(e*Cos[c + d*x])^(3/2))/(d*e^3*(a^2 - a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.132392, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2670, 2680, 2640, 2639}

$$\frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-6*a^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c +
 d*x]]) + (4*a^4*(e*Cos[c + d*x])^(3/2))/(d*e^3*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{a^4\int \frac{(e\cos(c+dx))^{5/2}}{(a-a\sin(c+dx))^2}\,dx}{e^4}\\
&= \frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{\left(3a^2\right)\int\sqrt{e\cos(c+dx)}\,dx}{e^2}\\
&= \frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{\left(3a^2\sqrt{e\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{e^2\sqrt{\cos(c+dx)}}\\
&= -\frac{6a^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)}
\end{aligned}
$$

Mathematica [C] time = 0.061673, size = 64, normalized size = 0.75

$$
\frac{4\ 2^{3/4}a^2\sqrt[4]{\sin(c+dx)+1}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt{e\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (4*2^(3/4)*a^2*Hypergeometric2F1[-3/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(1/4))/(d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 0.483, size = 120, normalized size = 1.4

$$-2\,\frac{\left(3\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-4\,(\sin(1/2\,dx+c/2))^2\cos(\right.}{e\sqrt{-2\,(\sin(1/2\,dx+c/2))^2e+e}\sin(1/2\,dx+c/2)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)
)*a^2/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2\right)\sqrt{e\cos(dx+c)}}{e^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c))/(e^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(3/2), x)
```

3.210 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{2a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}$$

```
[Out] (-2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c
 + d*x]]) + (4*a^4*Sqrt[e*Cos[c + d*x]])/(3*d*e^3*(a^2 - a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.129371, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2670, 2680, 2642, 2641}

$$\frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{2a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c
 + d*x]]) + (4*a^4*Sqrt[e*Cos[c + d*x]])/(3*d*e^3*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{5/2}}\,dx &= \frac{a^4\int \frac{(e\cos(c+dx))^{3/2}}{(a-a\sin(c+dx))^2}\,dx}{e^4}\\
&= \frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)} - \frac{a^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}\\
&= \frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)} - \frac{\left(a^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3e^2\sqrt{e\cos(c+dx)}}\\
&= -\frac{2a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)}
\end{aligned}
$$

Mathematica [C] time = 0.0602419, size = 66, normalized size = 0.74

$$
\frac{4\sqrt[4]{2}a^2(\sin(c+dx)+1)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(e\cos(c+dx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*2^(1/4)*a^2*Hypergeometric2F1[-3/4, -1/4, 1/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(3/4))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.619, size = 193, normalized size = 2.2

$$\frac{2a^2}{3de^2}\left(2\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\left(\sin\left(1/2\,dx+c/2\right)\right)^2-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*
e+e)^(1/2)/e^2*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c
))*a^2/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2\right)\sqrt{e\cos(dx+c)}}{e^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c))/(e^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(5/2), x)
```

3.211 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=127

$$\frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5\left(a^2-a^2\sin(c+dx)\right)}+\frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}-\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*a^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c
 + d*x]]) + (2*a^4*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a - a*Sin[c + d*x])^2)
 + (2*a^4*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a^2 - a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.180943, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2670, 2681, 2683, 2640, 2639}

$$\frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5\left(a^2-a^2\sin(c+dx)\right)}+\frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}-\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(7/2),x]
```


```
[Out] (-2*a^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c
 + d*x]]) + (2*a^4*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a - a*Sin[c + d*x])^2)
 + (2*a^4*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a^2 - a^2*Sin[c + d*x]))
```


Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
```

```
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{7/2}}\,dx &= \frac{a^4\int \frac{\sqrt{e\cos(c+dx)}}{(a-a\sin(c+dx))^2}\,dx}{e^4}\\
&= \frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}+\frac{a^3\int \frac{\sqrt{e\cos(c+dx)}}{a-a\sin(c+dx)}\,dx}{5e^4}\\
&= \frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}+\frac{2a^3(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))}-\frac{a^2\int \sqrt{e\cos(c+dx)}\,dx}{5e^4}\\
&= \frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}+\frac{2a^3(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))}-\frac{\left(a^2\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5e^4\sqrt{\cos(c+dx)}}\\
&= -\frac{2a^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}+\frac{2a^3(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))}
\end{aligned}
$$

Mathematica [C] time = 0.0769828, size = 66, normalized size = 0.52

$$
\frac{2\,2^{3/4}a^2(\sin(c+dx)+1)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(e\cos(c+dx))^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*2^(3/4)*a^2*Hypergeometric2F1[-5/4, 1/4, -1/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(5/4))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [B] time = 1.003, size = 305, normalized size = 2.4

$$-\frac{2\,a^2}{5\,de^3}\left(4\,\mathrm{EllipticE}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\left(\sin\left(1/2\,dx+c/2\right)\right)^4-8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(4*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^4-8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-4*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x
+1/2*c))*a^2/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(dx+c\right)+a\right)^2}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a^{2}\cos\left(dx+c\right)^{2}-2\,a^{2}\sin\left(dx+c\right)-2\,a^{2}\right)\sqrt{e\cos\left(dx+c\right)}}{e^{4}\cos\left(dx+c\right)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c))/(e^4*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^{2}}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(7/2), x)
```

3.212 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}}+\frac{2a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7de^4\sqrt{e\cos(c+dx)}}+\frac{4\left(a^2\sin(c+dx)+a^2\right)}{7de(e\cos(c+dx))^{7/2}}$$

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*d*e^4*Sqrt[e*Cos[c
+ d*x]]) + (2*a^2*Sin[c + d*x])/(7*d*e^3*(e*Cos[c + d*x])^(3/2)) + (4*(a^2
+ a^2*Sin[c + d*x]))/(7*d*e*(e*Cos[c + d*x])^(7/2))
```

Rubi [A] time = 0.0914094, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2676, 2636, 2642, 2641}

$$\frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}}+\frac{2a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7de^4\sqrt{e\cos(c+dx)}}+\frac{4\left(a^2\sin(c+dx)+a^2\right)}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*d*e^4*Sqrt[e*Cos[c
+ d*x]]) + (2*a^2*Sin[c + d*x])/(7*d*e^3*(e*Cos[c + d*x])^(3/2)) + (4*(a^2
+ a^2*Sin[c + d*x]))/(7*d*e*(e*Cos[c + d*x])^(7/2))
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{9/2}}\,dx &= \frac{4\left(a^2+a^2\sin(c+dx)\right)}{7de(e\cos(c+dx))^{7/2}}+\frac{\left(3a^2\right)\int \frac{1}{(e\cos(c+dx))^{5/2}}\,dx}{7e^2}\\ &= \frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}}+\frac{4\left(a^2+a^2\sin(c+dx)\right)}{7de(e\cos(c+dx))^{7/2}}+\frac{a^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{7e^4}\\ &= \frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}}+\frac{4\left(a^2+a^2\sin(c+dx)\right)}{7de(e\cos(c+dx))^{7/2}}+\frac{\left(a^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{7e^4\sqrt{e\cos(c+dx)}}\\ &= \frac{2a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7de^4\sqrt{e\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}}+\frac{4\left(a^2+a^2\sin(c+dx)\right)}{7de(e\cos(c+dx))^{7/2}}\end{aligned}$$

Mathematica [C] time = 0.112598, size = 66, normalized size = 0.58

$$\frac{2\sqrt[4]{2}a^2(\sin(c+dx)+1)^{7/4}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*2^(1/4)*a^2*Hypergeometric2F1[-7/4, 3/4, -3/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(7/4))/(7*d*e*(e*Cos[c + d*x])^(7/2))
```

Maple [B] time = 1.218, size = 375, normalized size = 3.3

$$-\frac{2a^2}{7e^4d}\left(8\sqrt{(\sin(1/2\,dx+c/2))^2}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,(\sin(1/2\,dx+c/2))^6-12\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] -2/7/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(8*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^4+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+6*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1
/2*c))*a^2/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2\right)\sqrt{e\cos(dx+c)}}{e^5\cos(dx+c)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c))/(e^5*cos(d*x + c)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(9/2), x)
```

3.213 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=145

$$\frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}}-\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3de^6\sqrt{\cos(c+dx)}}+\frac{4\left(a^2\sin(c+dx)+a^2\right)}{9de(e\cos(c+dx))^{9/2}}$$

```
[Out] (-2*a^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(3*d*e^6*Sqrt[Cos[c
 + d*x]]) + (2*a^2*Sin[c + d*x])/(9*d*e^3*(e*Cos[c + d*x])^(5/2)) + (2*a^2*
Sin[c + d*x])/(3*d*e^5*Sqrt[e*Cos[c + d*x]]) + (4*(a^2 + a^2*Sin[c + d*x]))
/(9*d*e*(e*Cos[c + d*x])^(9/2))
```

Rubi [A] time = 0.112072, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2676, 2636, 2640, 2639}

$$\frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}}-\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3de^6\sqrt{\cos(c+dx)}}+\frac{4\left(a^2\sin(c+dx)+a^2\right)}{9de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (-2*a^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(3*d*e^6*Sqrt[Cos[c
 + d*x]]) + (2*a^2*Sin[c + d*x])/(9*d*e^3*(e*Cos[c + d*x])^(5/2)) + (2*a^2*
Sin[c + d*x])/(3*d*e^5*Sqrt[e*Cos[c + d*x]]) + (4*(a^2 + a^2*Sin[c + d*x]))
/(9*d*e*(e*Cos[c + d*x])^(9/2))
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{11/2}}\,dx &= \frac{4\left(a^2+a^2\sin(c+dx)\right)}{9de(e\cos(c+dx))^{9/2}} + \frac{\left(5a^2\right)\int \frac{1}{(e\cos(c+dx))^{7/2}}\,dx}{9e^2} \\
&= \frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}} + \frac{4\left(a^2+a^2\sin(c+dx)\right)}{9de(e\cos(c+dx))^{9/2}} + \frac{a^2\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx}{3e^4} \\
&= \frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}} + \frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}} + \frac{4\left(a^2+a^2\sin(c+dx)\right)}{9de(e\cos(c+dx))^{9/2}} - \frac{a^2\int \sqrt{e\cos(c+dx}}{3e^6} \\
&= \frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}} + \frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}} + \frac{4\left(a^2+a^2\sin(c+dx)\right)}{9de(e\cos(c+dx))^{9/2}} - \frac{\left(a^2\sqrt{e\cos(c+dx)}\right)}{3e^6\sqrt{\cos}} \\
&= -\frac{2a^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^6\sqrt{\cos(c+dx)}} + \frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}} + \frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}} + \frac{4\left(a^2\right.}{9de(}
\end{aligned}
$$

Mathematica [C] time = 0.160718, size = 66, normalized size = 0.46

$$
\frac{2^{3/4}a^2(\sin(c+dx)+1)^{9/4}\,{}_2F_1\left(-\frac{9}{4},\frac{5}{4};-\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9de(e\cos(c+dx))^{9/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2^(3/4)*a^2*Hypergeometric2F1[-9/4, 5/4, -5/4, (1 - Sin[c + d*x])/2]*(1 +
Sin[c + d*x])^(9/4))/(9*d*e*(e*Cos[c + d*x])^(9/2))
```

Maple [B] time = 1.622, size = 488, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(11/2),x)
```

```
[Out] -2/9/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c)
^4-8*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+
e)^(1/2)/e^5*(48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*d
*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^4-152*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)-2*sin(1/2*d*x+1/2*c))*a^2/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^{2}\cos\left(dx+c\right)^{2}-2\,a^{2}\sin\left(dx+c\right)-2\,a^{2}\right)\sqrt{e\cos\left(dx+c\right)}}{e^{6}\cos\left(dx+c\right)^{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(e*cos(d*x
+ c))/(e^6*cos(d*x + c)^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(dx+c\right)+a\right)^{2}}{\left(e\cos\left(dx+c\right)\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(11/2), x)
```

3.214 $\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=203

$$\frac{170a^3e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{170a^3e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}-\frac{34\left(a^3\sin(c+dx\right.}{}$$

```
[Out] (-34*a^3*(e*Cos[c + d*x])^(9/2))/(99*d*e) + (170*a^3*e^4*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (170*a^3*e^3*Sqr
t[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (34*a^3*e*(e*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(77*d) - (2*a*(e*Cos[c + d*x])^(9/2)*(a + a*Sin[c + d*x])^2)/
(13*d*e) - (34*(e*Cos[c + d*x])^(9/2)*(a^3 + a^3*Sin[c + d*x]))/(143*d*e)
```

Rubi [A] time = 0.211256, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2678, 2669, 2635, 2642, 2641}

$$\frac{170a^3e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{170a^3e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}-\frac{34\left(a^3\sin(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-34*a^3*(e*Cos[c + d*x])^(9/2))/(99*d*e) + (170*a^3*e^4*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (170*a^3*e^3*Sqr
t[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (34*a^3*e*(e*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(77*d) - (2*a*(e*Cos[c + d*x])^(9/2)*(a + a*Sin[c + d*x])^2)/
(13*d*e) - (34*(e*Cos[c + d*x])^(9/2)*(a^3 + a^3*Sin[c + d*x]))/(143*d*e)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^3\,dx &= -\frac{2a(e\cos(c+dx))^{9/2}(a+a\sin(c+dx))^2}{13de}+\frac{1}{13}(17a)\int (e\cos(c+dx))^{7/2}(a+ \\
&= -\frac{2a(e\cos(c+dx))^{9/2}(a+a\sin(c+dx))^2}{13de}-\frac{34(e\cos(c+dx))^{9/2}\left(a^3+a^3\sin(\right.}{143de} \\
&= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}-\frac{2a(e\cos(c+dx))^{9/2}(a+a\sin(c+dx))^2}{13de}-\frac{34(e\cos}{} \\
&= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}+\frac{34a^3e(e\cos(c+dx))^{5/2}\sin(c+dx)}{77d}-\frac{2a(e\cos(c+}{} \\
&= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}+\frac{170a^3e^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{231d}+\frac{34a^3e(e\cos(}{} \\
&= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}+\frac{170a^3e^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{231d}+\frac{34a^3e(e\cos(}{} \\
&= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}+\frac{170a^3e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{170a^3e^3\sqrt{}}{}
\end{aligned}$$

Mathematica [C] time = 0.0742641, size = 66, normalized size = 0.33

$$-\frac{64\sqrt[4]{2}a^3(e\cos(c+dx))^{9/2}\,{}_2F_1\left(-\frac{17}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-64*2^(1/4)*a^3*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[-17/4, 9/4, 13/4,
 (1 - Sin[c + d*x])/2])/(9*d*e*(1 + Sin[c + d*x])^(9/4))
```

Maple [A] time = 0.595, size = 321, normalized size = 1.6

$$-\frac{2\,a^3e^4}{9009\,d}\left(88704\,(\sin(1/2\,dx+c/2))^{15}-157248\,\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^{12}-310464\,(\sin(1/2\,dx+c/2))^{13}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/9009/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*e^4*(887
04*sin(1/2*d*x+1/2*c)^15-157248*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-31
0464*sin(1/2*d*x+1/2*c)^13+393120*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+
337568*sin(1/2*d*x+1/2*c)^11-361296*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8
-67760*sin(1/2*d*x+1/2*c)^9+148824*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-
126280*sin(1/2*d*x+1/2*c)^7-12012*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+1
01948*sin(1/2*d*x+1/2*c)^5+3315*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-5694*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)-30338*sin(1/2*d*x+1/2*c)^3+3311*sin(1/2*d*x+1/2*
c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{7}{2}}\,(a\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,a^3e^3\cos\left(dx+c\right)^5-4\,a^3e^3\cos\left(dx+c\right)^3+\left(a^3e^3\cos\left(dx+c\right)^5-4\,a^3e^3\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*e^3*cos(d*x + c)^5 - 4*a^3*e^3*cos(d*x + c)^3 + (a^3*e^3*c
os(d*x + c)^5 - 4*a^3*e^3*cos(d*x + c)^3)*sin(d*x + c))*sqrt(e*cos(d*x + c)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\left(a\sin\left(dx+c\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)^3, x)
```

3.215 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=170

$$\frac{2a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{10a^3(e\cos(c+dx))^{7/2}}{21de}-\frac{10\left(a^3\sin(c+dx)+a^3\right)(e\cos(c+dx))^{7/2}}{33de}+\frac{2a^3e\sin(c$$

```
[Out] (-10*a^3*(e*Cos[c + d*x])^(7/2))/(21*d*e) + (2*a^3*e^2*Sqrt[e*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*a^3*e*(e*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(3*d) - (2*a*(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c +
d*x])^2)/(11*d*e) - (10*(e*Cos[c + d*x])^(7/2)*(a^3 + a^3*Sin[c + d*x]))/(3
3*d*e)
```

Rubi [A] time = 0.189465, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2678, 2669, 2635, 2640, 2639}

$$\frac{2a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{10a^3(e\cos(c+dx))^{7/2}}{21de}-\frac{10\left(a^3\sin(c+dx)+a^3\right)(e\cos(c+dx))^{7/2}}{33de}+\frac{2a^3e\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-10*a^3*(e*Cos[c + d*x])^(7/2))/(21*d*e) + (2*a^3*e^2*Sqrt[e*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*a^3*e*(e*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(3*d) - (2*a*(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c +
d*x])^2)/(11*d*e) - (10*(e*Cos[c + d*x])^(7/2)*(a^3 + a^3*Sin[c + d*x]))/(3
3*d*e)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3\,dx = -\frac{2a(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}{11de} + \frac{1}{11}(15a)\int (e\cos(c+dx))^{5/2}(a+$$

$$= -\frac{2a(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}{11de} - \frac{10(e\cos(c+dx))^{7/2}\left(a^3+a^3\sin(\right.}{33de}$$

$$= -\frac{10a^3(e\cos(c+dx))^{7/2}}{21de} - \frac{2a(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}{11de} - \frac{10(e\cos}{}$$

$$= -\frac{10a^3(e\cos(c+dx))^{7/2}}{21de} + \frac{2a^3e(e\cos(c+dx))^{3/2}\sin(c+dx)}{3d} - \frac{2a(e\cos(c+}{}$$

$$= -\frac{10a^3(e\cos(c+dx))^{7/2}}{21de} + \frac{2a^3e(e\cos(c+dx))^{3/2}\sin(c+dx)}{3d} - \frac{2a(e\cos(c+}{}$$

$$= -\frac{10a^3(e\cos(c+dx))^{7/2}}{21de} + \frac{2a^3e^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2a^3e(e\cos}{}$$

Mathematica [C] time = 0.113621, size = 66, normalized size = 0.39

$$-\frac{32\ 2^{3/4}a^3(e\cos(c+dx))^{7/2}\,{}_2F_1\left(-\frac{15}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-32*2^(3/4)*a^3*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[-15/4, 7/4, 11/4,
 (1 - Sin[c + d*x])/2])/(7*d*e*(1 + Sin[c + d*x])^(7/4))
```

Maple [A] time = 0.494, size = 264, normalized size = 1.6

$$\frac{2\,a^3e^3}{231\,d}\left(1344\,(\sin(1/2\,dx+c/2))^{13}-2464\,(\sin(1/2\,dx+c/2))^{10}\cos(1/2\,dx+c/2)-4032\,(\sin(1/2\,dx+c/2))^{11}+4928\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/231/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*e^3*(1344*
sin(1/2*d*x+1/2*c)^13-2464*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-4032*si
n(1/2*d*x+1/2*c)^11+4928*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+2928*sin(1
/2*d*x+1/2*c)^9-3080*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+864*sin(1/2*d*
x+1/2*c)^7+616*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-1908*sin(1/2*d*x+1/2
*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))+804*sin(1/2*d*x+1/2*c)^3-111*sin(1/2*d*x
+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(e\cos(dx+c))^{\frac{5}{2}}\,(a\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,a^3e^2\cos\left(dx+c\right)^4-4\,a^3e^2\cos\left(dx+c\right)^2+\left(a^3e^2\cos\left(dx+c\right)^4-4\,a^3e^2\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*e^2*cos(d*x + c)^4 - 4*a^3*e^2*cos(d*x + c)^2 + (a^3*e^2*c
os(d*x + c)^4 - 4*a^3*e^2*cos(d*x + c)^2)*sin(d*x + c))*sqrt(e*cos(d*x + c)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sin\left(dx+c\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^3, x)
```

3.216 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=172

$$\frac{26a^3e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}-\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}-\frac{26\left(a^3\sin(c+dx)+\right.}{63}$$

```
[Out] (-26*a^3*(e*Cos[c + d*x])^(5/2))/(35*d*e) + (26*a^3*e^2*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (26*a^3*e*Sqrt[e*C
os[c + d*x]]*Sin[c + d*x])/(21*d) - (2*a*(e*Cos[c + d*x])^(5/2)*(a + a*Sin[
c + d*x])^2)/(9*d*e) - (26*(e*Cos[c + d*x])^(5/2)*(a^3 + a^3*Sin[c + d*x]))
/(63*d*e)
```

Rubi [A] time = 0.188479, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2678, 2669, 2635, 2642, 2641}

$$\frac{26a^3e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}-\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}-\frac{26\left(a^3\sin(c+dx)+\right.}{63}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-26*a^3*(e*Cos[c + d*x])^(5/2))/(35*d*e) + (26*a^3*e^2*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (26*a^3*e*Sqrt[e*C
os[c + d*x]]*Sin[c + d*x])/(21*d) - (2*a*(e*Cos[c + d*x])^(5/2)*(a + a*Sin[
c + d*x])^2)/(9*d*e) - (26*(e*Cos[c + d*x])^(5/2)*(a^3 + a^3*Sin[c + d*x]))
/(63*d*e)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3\,dx &= -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}{9de}+\frac{1}{9}(13a)\int (e\cos(c+dx))^{3/2}(a+a \\
&= -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}{9de}-\frac{26(e\cos(c+dx))^{5/2}\left(a^3+a^3\sin(\right.}{63de} \\
&= -\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}-\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}{9de}-\frac{26(e\cos}{} \\
&= -\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e\sqrt{e\cos(c+dx)}\sin(c+dx)}{21d}-\frac{2a(e\cos(c+d}{} \\
&= -\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e\sqrt{e\cos(c+dx)}\sin(c+dx)}{21d}-\frac{2a(e\cos(c+d}{} \\
&= -\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{26a^3e\sqrt{e\,c}}{}
\end{aligned}$$

Mathematica [C] time = 0.0734213, size = 66, normalized size = 0.38

$$-\frac{32\sqrt[4]{2}a^3(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{13}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-32*2^(1/4)*a^3*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-13/4, 5/4, 9/4,
(1 - Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [A] time = 0.449, size = 251, normalized size = 1.5

$$-\frac{2\,a^3e^2}{315\,d}\Bigg(1120\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{11}-2160\,\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}-2800\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{9}+3240\,(\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*e^2*(1120
*sin(1/2*d*x+1/2*c)^11-2160*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-2800*si
n(1/2*d*x+1/2*c)^9+3240*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+784*sin(1/2
*d*x+1/2*c)^7-840*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+1624*sin(1/2*d*x+
1/2*c)^5+195*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-120*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)-1162*sin(1/2*d*x+1/2*c)^3+217*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}\,(a\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,a^3e\cos\left(dx+c\right)^3-4\,a^3e\cos\left(dx+c\right)+\left(a^3e\cos\left(dx+c\right)^3-4\,a^3e\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)},$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*e*cos(d*x + c)^3 - 4*a^3*e*cos(d*x + c) + (a^3*e*cos(d*x +
 c)^3 - 4*a^3*e*cos(d*x + c))*sin(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a\sin\left(dx+c\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^3, x)
```

3.217 $\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=140

$$-\frac{22a^3(e\cos(c+dx))^{3/2}}{15de}-\frac{22\left(a^3\sin(c+dx)+a^3\right)(e\cos(c+dx))^{3/2}}{35de}+\frac{22a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2a(a\sin(c$$

```
[Out] (-22*a^3*(e*Cos[c + d*x])^(3/2))/(15*d*e) + (22*a^3*Sqrt[e*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) - (2*a*(e*Cos[c + d*x])^(
3/2)*(a + a*Sin[c + d*x])^2)/(7*d*e) - (22*(e*Cos[c + d*x])^(3/2)*(a^3 + a^
3*Sin[c + d*x]))/(35*d*e)
```

Rubi [A] time = 0.146904, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2678, 2669, 2640, 2639}

$$-\frac{22a^3(e\cos(c+dx))^{3/2}}{15de}-\frac{22\left(a^3\sin(c+dx)+a^3\right)(e\cos(c+dx))^{3/2}}{35de}+\frac{22a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2a(a\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-22*a^3*(e*Cos[c + d*x])^(3/2))/(15*d*e) + (22*a^3*Sqrt[e*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) - (2*a*(e*Cos[c + d*x])^(
3/2)*(a + a*Sin[c + d*x])^2)/(7*d*e) - (22*(e*Cos[c + d*x])^(3/2)*(a^3 + a^
3*Sin[c + d*x]))/(35*d*e)
```


Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3\,dx &= -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}{7de}+\frac{1}{7}(11a)\int \sqrt{e\cos(c+dx)}(a+a\sin\\
&= -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}{7de}-\frac{22(e\cos(c+dx))^{3/2}\left(a^3+a^3\sin(c\right.}{35de}\\
&= -\frac{22a^3(e\cos(c+dx))^{3/2}}{15de}-\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}{7de}-\frac{22(e\cos(}{}\\
&= -\frac{22a^3(e\cos(c+dx))^{3/2}}{15de}-\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}{7de}-\frac{22(e\cos(}{}\\
&= -\frac{22a^3(e\cos(c+dx))^{3/2}}{15de}+\frac{22a^3\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c+}{}\end{aligned}$$

Mathematica [C] time = 0.0444513, size = 66, normalized size = 0.47

$$-\frac{16\ 2^{3/4}a^3(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{11}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(\sin(c+dx)+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-16*2^(3/4)*a^3*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-11/4, 3/4, 7/4,
(1 - Sin[c + d*x])/2])/(3*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [A] time = 0.452, size = 214, normalized size = 1.5

$$\frac{2a^3e}{105d}\left(240\,(\sin(1/2\,dx+c/2))^9-504\,(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)-480\,(\sin(1/2\,dx+c/2))^7+504\,(\sin(1/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/105/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*e*(240*sin
(1/2*d*x+1/2*c)^9-504*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-480*sin(1/2*d
*x+1/2*c)^7+504*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-200*sin(1/2*d*x+1/2
*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-126*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)+440*sin(1/2*d*x+1/2*c)^3-125*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^3, x)
```

3.218 $\int \frac{(a+a\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=136

$$-\frac{6a^3\sqrt{e\cos(c+dx)}}{de}-\frac{6\left(a^3\sin(c+dx)+a^3\right)\sqrt{e\cos(c+dx)}}{5de}+\frac{6a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2a(a\sin(c+dx)+a}{5de}$$

```
[Out] (-6*a^3*Sqrt[e*Cos[c + d*x]])/(d*e) + (6*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*(a + a
*Sin[c + d*x])^2)/(5*d*e) - (6*Sqrt[e*Cos[c + d*x]]*(a^3 + a^3*Sin[c + d*x]
))/(5*d*e)
```

Rubi [A] time = 0.147357, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2678, 2669, 2642, 2641}

$$-\frac{6a^3\sqrt{e\cos(c+dx)}}{de}-\frac{6\left(a^3\sin(c+dx)+a^3\right)\sqrt{e\cos(c+dx)}}{5de}+\frac{6a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2a(a\sin(c+dx)+a}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/Sqrt[e*Cos[c + d*x]],x]
```


```
[Out] (-6*a^3*Sqrt[e*Cos[c + d*x]])/(d*e) + (6*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*(a + a
*Sin[c + d*x])^2)/(5*d*e) - (6*Sqrt[e*Cos[c + d*x]]*(a^3 + a^3*Sin[c + d*x]
))/(5*d*e)
```


Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}\,dx &= -\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2}{5de}+\frac{1}{5}(9a)\int \frac{(a+a\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}\,dx\\
&= -\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2}{5de}-\frac{6\sqrt{e\cos(c+dx)}\left(a^3+a^3\sin(c+dx)\right)}{5de}+\left(3a^2\right)\int \frac{a}{\cdots}\\
&= -\frac{6a^3\sqrt{e\cos(c+dx)}}{de}-\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2}{5de}-\frac{6\sqrt{e\cos(c+dx)}\left(a^3+a^3\sin(\cdots\right.}{5de}\\
&= -\frac{6a^3\sqrt{e\cos(c+dx)}}{de}-\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2}{5de}-\frac{6\sqrt{e\cos(c+dx)}\left(a^3+a^3\sin(\cdots\right.}{5de}\\
&= -\frac{6a^3\sqrt{e\cos(c+dx)}}{de}+\frac{6a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+d\cdots}{5de}
\end{aligned}
$$

Mathematica [C] time = 0.0305894, size = 64, normalized size = 0.47

$$
-\frac{16\sqrt[4]{2}a^3\sqrt{e\cos(c+dx)}\,{}_2F_1\left(-\frac{9}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt[4]{\sin(c+dx)+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-16*2^(1/4)*a^3*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-9/4, 1/4, 5/4, (1
- Sin[c + d*x])/2])/(d*e*(1 + Sin[c + d*x])^(1/4))
```

Maple [A] time = 0.447, size = 178, normalized size = 1.3

$$-\frac{2a^3}{5d}\left(8\,(\sin(1/2\,dx+c/2))^7-20\,(\sin(1/2\,dx+c/2))^4\cos(1/2\,dx+c/2)-12\,(\sin(1/2\,dx+c/2))^5+15\sqrt{2\,(\sin(1/2\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/5/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*(8*sin(1/2*
d*x+1/2*c)^7-20*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*
c)^5+15*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
-34*sin(1/2*d*x+1/2*c)^3+19*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a\sin(dx+c)+a)^3}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/sqrt(e*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c))/(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^3}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/sqrt(e*cos(d*x + c)), x)
```

3.219 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3}+\frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2}-\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}$$

```
[Out] (14*a^3*(e*Cos[c + d*x])^(3/2))/(3*d*e^3) - (14*a^3*Sqrt[e*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (4*a^5*(e*Cos[c + d*x
])^(7/2))/(d*e^5*(a - a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.197596, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2670, 2680, 2682, 2640, 2639}

$$\frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3}+\frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2}-\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (14*a^3*(e*Cos[c + d*x])^(3/2))/(3*d*e^3) - (14*a^3*Sqrt[e*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (4*a^5*(e*Cos[c + d*x
])^(7/2))/(d*e^5*(a - a*Sin[c + d*x])^2)
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{a^6\int \frac{(e\cos(c+dx))^{9/2}}{(a-a\sin(c+dx))^3}\,dx}{e^6}\\
&= \frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2}-\frac{\left(7a^4\right)\int \frac{(e\cos(c+dx))^{5/2}}{a-a\sin(c+dx)}\,dx}{e^4}\\
&= \frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3}+\frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2}-\frac{\left(7a^3\right)\int \sqrt{e\cos(c+dx)}\,dx}{e^2}\\
&= \frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3}+\frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2}-\frac{\left(7a^3\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{e^2\sqrt{\cos(c+dx)}}\\
&= \frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3}-\frac{14a^3\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2}
\end{aligned}$$

Mathematica [C] time = 0.0536187, size = 64, normalized size = 0.6

$$\frac{8\ 2^{3/4}a^3\sqrt[4]{\sin(c+dx)+1}\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*2^(3/4)*a^3*Hypergeometric2F1[-7/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(1/4))/(d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 0.667, size = 146, normalized size = 1.4

$$-\frac{2a^3}{3de}\left(-4\,(\sin(1/2\,dx+c/2))^5+21\,\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2)\,,\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-4*sin(1/2*d
*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+4*sin(1/2*d*x+1/2*c)^3-13*sin(1/2*d*x+1/2*c))*a^3/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c))/(e^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(3/2), x)
```

3.220 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=110

$$\frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3}+\frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2}-\frac{10a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}$$

```
[Out] (10*a^3*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) - (10*a^3*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (4*a^5*(e*Cos[c + d*x
])^(5/2))/(3*d*e^5*(a - a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.202305, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2670, 2680, 2682, 2642, 2641}

$$\frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3}+\frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2}-\frac{10a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(5/2),x]
```



```
[Out] (10*a^3*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) - (10*a^3*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (4*a^5*(e*Cos[c + d*x
])^(5/2))/(3*d*e^5*(a - a*Sin[c + d*x])^2)
```


Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{5/2}}\,dx &= \frac{a^6\int \frac{(e\cos(c+dx))^{7/2}}{(a-a\sin(c+dx))^3}\,dx}{e^6}\\
&= \frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2}-\frac{\left(5a^4\right)\int \frac{(e\cos(c+dx))^{3/2}}{a-a\sin(c+dx)}\,dx}{3e^4}\\
&= \frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3}+\frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2}-\frac{\left(5a^3\right)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}\\
&= \frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3}+\frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2}-\frac{\left(5a^3\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3e^2\sqrt{e\cos(c+dx)}}\\
&= \frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3}-\frac{10a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2}
\end{aligned}
$$

Mathematica [C] time = 0.0527323, size = 66, normalized size = 0.6

$$
\frac{8\sqrt[4]{2}a^3(\sin(c+dx)+1)^{3/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{3}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(e\cos(c+dx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (8*2^(1/4)*a^3*Hypergeometric2F1[-5/4, -3/4, 1/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(3/4))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.744, size = 219, normalized size = 2.

$$\frac{2a^3}{3de^2}\left(10\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\left(\sin\left(1/2\,dx+c/2\right)\right)^2-12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*
e+e)^(1/2)/e^2*(10*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*sin(1/2
*d*x+1/2*c)^5-5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+12*sin(1/2*d*x+1/2*c)^3-7*sin(1/2*d*x+1/2*c))*a^3/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^3}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,a^{3}\cos\left(dx+c\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(dx+c\right)^{2}-4\,a^{3}\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^{3}\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c))/(e^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^{3}}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(5/2), x)
```

3.221 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{6a^6(e\cos(c+dx))^{3/2}}{5de^5\left(a^3-a^3\sin(c+dx)\right)}+\frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}+\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}$$

```
[Out] (6*a^3*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c
+ d*x]]) + (4*a^5*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a - a*Sin[c + d*x])^2)
- (6*a^6*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a^3 - a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.202382, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2670, 2680, 2683, 2640, 2639}

$$-\frac{6a^6(e\cos(c+dx))^{3/2}}{5de^5\left(a^3-a^3\sin(c+dx)\right)}+\frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}+\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (6*a^3*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c
+ d*x]]) + (4*a^5*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a - a*Sin[c + d*x])^2)
- (6*a^6*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a^3 - a^3*Sin[c + d*x]))
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
```

```
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{7/2}}\,dx &= \frac{a^6\int \frac{(e\cos(c+dx))^{5/2}}{(a-a\sin(c+dx))^3}\,dx}{e^6}\\
&= \frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}-\frac{\left(3a^4\right)\int \frac{\sqrt{e\cos(c+dx)}}{a-a\sin(c+dx)}\,dx}{5e^4}\\
&= \frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}-\frac{6a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))}+\frac{\left(3a^3\right)\int \sqrt{e\cos(c+dx)}\,dx}{5e^4}\\
&= \frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}-\frac{6a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))}+\frac{\left(3a^3\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5e^4\sqrt{\cos(c+dx)}}\\
&= \frac{6a^3\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}-\frac{6a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))}
\end{aligned}$$

Mathematica [C] time = 0.0896046, size = 66, normalized size = 0.52

$$\frac{4\ 2^{3/4}a^3(\sin(c+dx)+1)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{3}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (4*2^(3/4)*a^3*Hypergeometric2F1[-5/4, -3/4, -1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(5/4))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [B] time = 1.153, size = 332, normalized size = 2.6

$$\frac{2a^3}{5de^3}\left(12\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2}\,(\sin(1/2\,dx+c/2))^4-24$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] 2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-
2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-20
*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)+20*sin(1/2*d*x+1/2*c)^3-sin(1/2*d*x+1/2*c))*a^3/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(3\,a^{3}\cos\left(dx+c\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(dx+c\right)^{2}-4\,a^{3}\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^{4}\cos\left(dx+c\right)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c))/(e^4*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^{3}}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(7/2), x)
```

3.222 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2a^6\sqrt{e\cos(c+dx)}}{21de^5\left(a^3-a^3\sin(c+dx)\right)}+\frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2}-\frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}$$

```
[Out] (-2*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt[e*Cos[
c + d*x]]) + (4*a^5*Sqrt[e*Cos[c + d*x]])/(7*d*e^5*(a - a*Sin[c + d*x])^2)
- (2*a^6*Sqrt[e*Cos[c + d*x]])/(21*d*e^5*(a^3 - a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.197608, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2670, 2680, 2683, 2642, 2641}

$$-\frac{2a^6\sqrt{e\cos(c+dx)}}{21de^5\left(a^3-a^3\sin(c+dx)\right)}+\frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2}-\frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(9/2),x]
```


```
[Out] (-2*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt[e*Cos[
c + d*x]]) + (4*a^5*Sqrt[e*Cos[c + d*x]])/(7*d*e^5*(a - a*Sin[c + d*x])^2)
- (2*a^6*Sqrt[e*Cos[c + d*x]])/(21*d*e^5*(a^3 - a^3*Sin[c + d*x]))
```


Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
```

```
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{9/2}}\,dx &= \frac{a^6\int \frac{(e\cos(c+dx))^{3/2}}{(a-a\sin(c+dx))^3}\,dx}{e^6}\\
&= \frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2}-\frac{a^4\int \frac{1}{\sqrt{e\cos(c+dx)}(a-a\sin(c+dx))}\,dx}{7e^4}\\
&= \frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2}-\frac{2a^4\sqrt{e\cos(c+dx)}}{21de^5(a-a\sin(c+dx))}-\frac{a^3\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{21e^4}\\
&= \frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2}-\frac{2a^4\sqrt{e\cos(c+dx)}}{21de^5(a-a\sin(c+dx))}-\frac{\left(a^3\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21e^4\sqrt{e\cos(c+dx)}}\\
&= -\frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}+\frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2}-\frac{2a^4\sqrt{e\cos(c+dx)}}{21de^5(a-a\sin(c+dx))}
\end{aligned}
$$

Mathematica [C] time = 0.0829027, size = 66, normalized size = 0.52

$$\frac{4\sqrt[4]{2}a^3(\sin(c+dx)+1)^{7/4}\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (4*2^(1/4)*a^3*Hypergeometric2F1[-7/4, -1/4, -3/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(7/4))/(7*d*e*(e*Cos[c + d*x])^(7/2))
```

Maple [B] time = 1.273, size = 401, normalized size = 3.2

$$\frac{2a^3}{21e^4d}\left(8\sqrt{(\sin(1/2dx+c/2))^2}\text{EllipticF}\left(\cos(1/2dx+c/2),\sqrt{2}\right)\sqrt{2(\sin(1/2dx+c/2))^2-1}(\sin(1/2dx+c/2))^6-12\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] 2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(8*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^4+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+6*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+28*sin(
1/2*d*x+1/2*c)^5-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-22*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)-28*sin(1/2*d*x+1/2*c)^3-5*sin(1/2*d*x+1/2*c))*a^3/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3+\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^5\cos\left(dx+c\right)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c))/(e^5*cos(d*x + c)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^3}{\left(e\cos\left(dx+c\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(9/2), x)
```

3.223 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=165

$$\frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7\left(a^3-a^3\sin(c+dx)\right)}+\frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3}+\frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2}-\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15de^6\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*a^3*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*e^6*Sqrt[Cos[
c + d*x]]) + (2*a^6*(e*Cos[c + d*x])^(3/2))/(9*d*e^7*(a - a*Sin[c + d*x])^3
) + (2*a^5*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a - a*Sin[c + d*x])^2) + (2*a
^6*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a^3 - a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.243165, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2670, 2681, 2683, 2640, 2639}

$$\frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7\left(a^3-a^3\sin(c+dx)\right)}+\frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3}+\frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2}-\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15de^6\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (-2*a^3*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*e^6*Sqrt[Cos[
c + d*x]]) + (2*a^6*(e*Cos[c + d*x])^(3/2))/(9*d*e^7*(a - a*Sin[c + d*x])^3
) + (2*a^5*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a - a*Sin[c + d*x])^2) + (2*a
^6*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a^3 - a^3*Sin[c + d*x]))
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
```

```
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{11/2}}\,dx = \frac{a^6\int \frac{\sqrt{e\cos(c+dx)}}{(a-a\sin(c+dx))^3}\,dx}{e^6}$$

$$= \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{a^5\int \frac{\sqrt{e\cos(c+dx)}}{(a-a\sin(c+dx))^2}\,dx}{3e^6}$$

$$= \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} + \frac{a^4\int \frac{\sqrt{e\cos(c+dx)}}{a-a\sin(c+dx)}\,dx}{15e^6}$$

$$= \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} + \frac{2a^4(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))} - \frac{a^3\int \sqrt{e}}{}$$

$$= \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} + \frac{2a^4(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))} - \frac{\left(a^3\sqrt{e\,c}\right.}{}$$

$$= -\frac{2a^3\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^6\sqrt{\cos(c+dx)}} + \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2}$$

Mathematica [C] time = 0.139316, size = 66, normalized size = 0.4

$$\frac{2\,2^{3/4}a^3(\sin(c+dx)+1)^{9/4}\,{}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*2^(3/4)*a^3*Hypergeometric2F1[-9/4, 1/4, -5/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(9/4))/(9*d*e*(e*Cos[c + d*x])^(9/2))
```

Maple [B] time = 1.977, size = 514, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(11/2),x)
```

```
[Out] -2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c
)^4-8*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e
+e)^(1/2)/e^5*(48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/
2*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*sin(1/2*d*x+1/2*c)^4-152*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+36*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-36*sin(1/2*d*x+1/2*c)^3-11*sin(1/2*d*x+1/2*c)
)*a^3/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3+\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^6\cos\left(dx+c\right)^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(e*cos(d*x + c))/(e^6*cos(d*x + c)^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^3}{\left(e\cos\left(dx+c\right)\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(11/2), x)
```

3.224 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=210

$$\frac{442a^4e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{442a^4(e\cos(c+dx))^{5/2}}{385de}+\frac{442a^4e\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}-\frac{34\left(a^2\sin(c+dx\right.}{}$$

```
[Out] (-442*a^4*(e*Cos[c + d*x])^(5/2))/(385*d*e) + (442*a^4*e^2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (442*a^4*e*Sqr
t[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) - (2*a*(e*Cos[c + d*x])^(5/2)*(a +
a*Sin[c + d*x])^3)/(11*d*e) - (34*(e*Cos[c + d*x])^(5/2)*(a^2 + a^2*Sin[c +
 d*x])^2)/(99*d*e) - (442*(e*Cos[c + d*x])^(5/2)*(a^4 + a^4*Sin[c + d*x]))/
(693*d*e)
```

Rubi [A] time = 0.247785, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2678, 2669, 2635, 2642, 2641}

$$\frac{442a^4e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{442a^4(e\cos(c+dx))^{5/2}}{385de}+\frac{442a^4e\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}-\frac{34\left(a^2\sin(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^4,x]
```


```
[Out] (-442*a^4*(e*Cos[c + d*x])^(5/2))/(385*d*e) + (442*a^4*e^2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (442*a^4*e*Sqr
t[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) - (2*a*(e*Cos[c + d*x])^(5/2)*(a +
a*Sin[c + d*x])^3)/(11*d*e) - (34*(e*Cos[c + d*x])^(5/2)*(a^2 + a^2*Sin[c +
 d*x])^2)/(99*d*e) - (442*(e*Cos[c + d*x])^(5/2)*(a^4 + a^4*Sin[c + d*x]))/
(693*d*e)
```


Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^4\,dx = -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3}{11de} + \frac{1}{11}(17a)\int (e\cos(c+dx))^{3/2}(a$$

$$= -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3}{11de} - \frac{34(e\cos(c+dx))^{5/2}\left(a^2+a^2\sin\right.}{99de}$$

$$= -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3}{11de} - \frac{34(e\cos(c+dx))^{5/2}\left(a^2+a^2\sin\right.}{99de}$$

$$= -\frac{442a^4(e\cos(c+dx))^{5/2}}{385de} - \frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3}{11de} - \frac{34(e}{}$$

$$= -\frac{442a^4(e\cos(c+dx))^{5/2}}{385de} + \frac{442a^4e\sqrt{e\cos(c+dx)}\sin(c+dx)}{231d} - \frac{2a(e\cos(c}{}$$

$$= -\frac{442a^4(e\cos(c+dx))^{5/2}}{385de} + \frac{442a^4e\sqrt{e\cos(c+dx)}\sin(c+dx)}{231d} - \frac{2a(e\cos(c}{}$$

$$= -\frac{442a^4(e\cos(c+dx))^{5/2}}{385de} + \frac{442a^4e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}} + \frac{442a^4}{}$$

Mathematica [C] time = 0.103337, size = 66, normalized size = 0.31

$$-\frac{64\sqrt[4]{2}a^4(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{17}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-64*2^(1/4)*a^4*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-17/4, 5/4, 9/4,
(1 - Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [A] time = 0.504, size = 295, normalized size = 1.4

$$-\frac{2a^4e^2}{3465d}\left(20160\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^{12}-50400(\sin(1/2dx+c/2))^{10}\cos(1/2dx+c/2)+49280(\sin(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -2/3465/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^4*e^2*(201
60*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-50400*sin(1/2*d*x+1/2*c)^10*cos
(1/2*d*x+1/2*c)+49280*sin(1/2*d*x+1/2*c)^11-6480*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^8-123200*sin(1/2*d*x+1/2*c)^9+60120*sin(1/2*d*x+1/2*c)^6*cos(1/
2*d*x+1/2*c)+78848*sin(1/2*d*x+1/2*c)^7-23100*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+4928*sin(1/2*d*x+1/2*c)^5+3315*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-150*sin(
1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-17864*sin(1/2*d*x+1/2*c)^3+4004*sin(1/2
*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}} (a\sin(dx+c)+a)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(a^4e\cos(dx+c)^5-8\,a^4e\cos(dx+c)^3+8\,a^4e\cos(dx+c)-4\left(a^4e\cos(dx+c)^3-2\,a^4e\cos(dx+c)\right)\sin(dx+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*e*cos(d*x + c)^5 - 8*a^4*e*cos(d*x + c)^3 + 8*a^4*e*cos(d*x +
 c) - 4*(a^4*e*cos(d*x + c)^3 - 2*a^4*e*cos(d*x + c))*sin(d*x + c))*sqrt(e*
cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^4, x)
```

3.225 $\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=178

$$-\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}-\frac{10\left(a^2\sin(c+dx)+a^2\right)^2(e\cos(c+dx))^{3/2}}{21de}-\frac{22\left(a^4\sin(c+dx)+a^4\right)(e\cos(c+dx))^{3/2}}{21de}+\frac{22a^4}{}$$

```
[Out] (-22*a^4*(e*Cos[c + d*x])^(3/2))/(9*d*e) + (22*a^4*Sqrt[e*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2])/(3*d*Sqrt[Cos[c + d*x]]) - (2*a*(e*Cos[c + d*x])^(3
/2)*(a + a*Sin[c + d*x])^3)/(9*d*e) - (10*(e*Cos[c + d*x])^(3/2)*(a^2 + a^2
*Sin[c + d*x])^2)/(21*d*e) - (22*(e*Cos[c + d*x])^(3/2)*(a^4 + a^4*Sin[c +
d*x]))/(21*d*e)
```

Rubi [A] time = 0.200701, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2678, 2669, 2640, 2639}

$$-\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}-\frac{10\left(a^2\sin(c+dx)+a^2\right)^2(e\cos(c+dx))^{3/2}}{21de}-\frac{22\left(a^4\sin(c+dx)+a^4\right)(e\cos(c+dx))^{3/2}}{21de}+\frac{22a^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4,x]
```


```
[Out] (-22*a^4*(e*Cos[c + d*x])^(3/2))/(9*d*e) + (22*a^4*Sqrt[e*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2])/(3*d*Sqrt[Cos[c + d*x]]) - (2*a*(e*Cos[c + d*x])^(3
/2)*(a + a*Sin[c + d*x])^3)/(9*d*e) - (10*(e*Cos[c + d*x])^(3/2)*(a^2 + a^2
*Sin[c + d*x])^2)/(21*d*e) - (22*(e*Cos[c + d*x])^(3/2)*(a^4 + a^4*Sin[c +
d*x]))/(21*d*e)
```


Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^4\,dx = -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}+\frac{1}{3}(5a)\int \sqrt{e\cos(c+dx)}(a+a\sin$$

$$= -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}-\frac{10(e\cos(c+dx))^{3/2}\left(a^2+a^2\sin(\right.}{21de}$$

$$= -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}-\frac{10(e\cos(c+dx))^{3/2}\left(a^2+a^2\sin(\right.}{21de}$$

$$= -\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}-\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}-\frac{10(e\cos}{}$$

$$= -\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}-\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}-\frac{10(e\cos}{}$$

$$= -\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}+\frac{22a^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c}{}$$

Mathematica [C] time = 0.0668708, size = 66, normalized size = 0.37

$$-\frac{32\ 2^{3/4}a^4(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{15}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(\sin(c+dx)+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-32*2^(3/4)*a^4*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-15/4, 3/4, 7/4,
(1 - Sin[c + d*x])/2])/(3*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [A] time = 0.519, size = 258, normalized size = 1.5

$$\frac{2\,a^4e}{63\,d}\Bigg(224\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{10}\cos\left(1/2\,dx+c/2\right)-448\,\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}+576\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/63/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^4*e*(224*sin(
1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-448*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^8+576*sin(1/2*d*x+1/2*c)^9-392*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-
1152*sin(1/2*d*x+1/2*c)^7+616*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+192*s
in(1/2*d*x+1/2*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-168*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)+384*sin(1/2*d*x+1/2*c)^3-132*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{e\cos\left(dx+c\right)}\left(a\sin\left(dx+c\right)+a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^4\cos\left(dx+c\right)^4-8\,a^4\cos\left(dx+c\right)^2+8\,a^4-4\left(a^4\cos\left(dx+c\right)^2-2\,a^4\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^4, x)
```

3.226 $\int \frac{(a+a\sin(c+dx))^4}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=178

$$-\frac{78a^4\sqrt{e\cos(c+dx)}}{7de}-\frac{26\left(a^2\sin(c+dx)+a^2\right)^2\sqrt{e\cos(c+dx)}}{35de}-\frac{78\left(a^4\sin(c+dx)+a^4\right)\sqrt{e\cos(c+dx)}}{35de}+\frac{78a^4\sqrt{\cos(c}}{7d}$$

```
[Out] (-78*a^4*Sqrt[e*Cos[c + d*x]])/(7*d*e) + (78*a^4*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(7*d*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*
(a + a*Sin[c + d*x])^3)/(7*d*e) - (26*Sqrt[e*Cos[c + d*x]]*(a^2 + a^2*Sin[c
 + d*x])^2)/(35*d*e) - (78*Sqrt[e*Cos[c + d*x]]*(a^4 + a^4*Sin[c + d*x]))/(
35*d*e)
```

Rubi [A] time = 0.21279, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2678, 2669, 2642, 2641}

$$-\frac{78a^4\sqrt{e\cos(c+dx)}}{7de}-\frac{26\left(a^2\sin(c+dx)+a^2\right)^2\sqrt{e\cos(c+dx)}}{35de}-\frac{78\left(a^4\sin(c+dx)+a^4\right)\sqrt{e\cos(c+dx)}}{35de}+\frac{78a^4\sqrt{\cos(c}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-78*a^4*Sqrt[e*Cos[c + d*x]])/(7*d*e) + (78*a^4*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(7*d*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*
(a + a*Sin[c + d*x])^3)/(7*d*e) - (26*Sqrt[e*Cos[c + d*x]]*(a^2 + a^2*Sin[c
 + d*x])^2)/(35*d*e) - (78*Sqrt[e*Cos[c + d*x]]*(a^4 + a^4*Sin[c + d*x]))/(
35*d*e)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^4}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3}{7de}+\frac{1}{7}(13a)\int \frac{(a+a\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}\,dx$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3}{7de}-\frac{26\sqrt{e\cos(c+dx)}\left(a^2+a^2\sin(c+dx)\right)^2}{35de}+\frac{1}{35}\big(11$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3}{7de}-\frac{26\sqrt{e\cos(c+dx)}\left(a^2+a^2\sin(c+dx)\right)^2}{35de}-\frac{78\sqrt{e\,c}}{}$$

$$= -\frac{78a^4\sqrt{e\cos(c+dx)}}{7de}-\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3}{7de}-\frac{26\sqrt{e\cos(c+dx)}\left(a^2+a^2\right.}{35de}$$

$$= -\frac{78a^4\sqrt{e\cos(c+dx)}}{7de}-\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3}{7de}-\frac{26\sqrt{e\cos(c+dx)}\left(a^2+a^2\right.}{35de}$$

$$= -\frac{78a^4\sqrt{e\cos(c+dx)}}{7de}+\frac{78a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}(a+a\sin(c}{7de}$$

Mathematica [C] time = 0.0598027, size = 64, normalized size = 0.36

$$-\frac{32\sqrt[4]{2}a^4\sqrt{e\cos(c+dx)}\,{}_2F_1\left(-\frac{13}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt[4]{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-32*2^(1/4)*a^4*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-13/4, 1/4, 5/4, (1
 - Sin[c + d*x])/2])/(d*e*(1 + Sin[c + d*x])^(1/4))
```

Maple [A] time = 0.615, size = 222, normalized size = 1.3

$$-\frac{2a^4}{35d}\left(80\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^8-120\left(\sin(1/2\,dx+c/2)\right)^6\cos(1/2\,dx+c/2)+224\left(\sin(1/2\,dx+c/2)\right)^7\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^4*(80*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-120*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+224*sin(1/2*d*x+1/2*c)^7-280*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-336
*sin(1/2*d*x+1/2*c)^5+195*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+160*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-392*sin(1/2*d*x+1/2*c)^3+252*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^4}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/sqrt(e*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4\cos(dx+c)^4-8\,a^4\cos(dx+c)^2+8\,a^4-4\left(a^4\cos(dx+c)^2-2\,a^4\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^4}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/sqrt(e*cos(d*x + c)), x)
```

3.227 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=156

$$\frac{44a^8(e\cos(c+dx))^{7/2}}{3de^5\left(a^4-a^4\sin(c+dx)\right)}+\frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3}-\frac{154a^4\sin(c+dx)(e\cos(c+dx))^{3/2}}{15de^3}-\frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos}}{5de^2\sqrt{\cos(c+dx)}}$$

```
[Out] (-154*a^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos
[c + d*x]]) - (154*a^4*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*d*e^3) + (4
*a^7*(e*Cos[c + d*x])^(11/2))/(d*e^7*(a - a*Sin[c + d*x])^3) + (44*a^8*(e*C
os[c + d*x])^(7/2))/(3*d*e^5*(a^4 - a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.234053, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2670, 2680, 2635, 2640, 2639}

$$\frac{44a^8(e\cos(c+dx))^{7/2}}{3de^5\left(a^4-a^4\sin(c+dx)\right)}+\frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3}-\frac{154a^4\sin(c+dx)(e\cos(c+dx))^{3/2}}{15de^3}-\frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos}}{5de^2\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-154*a^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos
[c + d*x]]) - (154*a^4*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*d*e^3) + (4
*a^7*(e*Cos[c + d*x])^(11/2))/(d*e^7*(a - a*Sin[c + d*x])^3) + (44*a^8*(e*C
os[c + d*x])^(7/2))/(3*d*e^5*(a^4 - a^4*Sin[c + d*x]))
```


Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{3/2}}\,dx = \frac{a^8\int \frac{(e\cos(c+dx))^{13/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}$$

$$= \frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3} - \frac{\left(11a^6\right)\int \frac{(e\cos(c+dx))^{9/2}}{(a-a\sin(c+dx))^2}\,dx}{e^6}$$

$$= \frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3} + \frac{44a^6(e\cos(c+dx))^{7/2}}{3de^5\left(a^2-a^2\sin(c+dx)\right)} - \frac{\left(77a^4\right)\int (e\cos(c+dx))^{5/2}\,dx}{3e^4}$$

$$= -\frac{154a^4(e\cos(c+dx))^{3/2}\sin(c+dx)}{15de^3} + \frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3} + \frac{44a^6(e\cos(c+dx))^{7/2}}{3de^5\left(a^2-a^2\sin(c+dx)\right)}$$

$$= -\frac{154a^4(e\cos(c+dx))^{3/2}\sin(c+dx)}{15de^3} + \frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3} + \frac{44a^6(e\cos(c+dx))^{7/2}}{3de^5\left(a^2-a^2\sin(c+dx)\right)}$$

$$= -\frac{154a^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}} - \frac{154a^4(e\cos(c+dx))^{3/2}\sin(c+dx)}{15de^3} + \frac{4a^7(e\cos(c}{de^7(a-a\sin}$$

Mathematica [C] time = 0.08358, size = 64, normalized size = 0.41

$$\frac{16\ 2^{3/4}a^4\sqrt[4]{\sin(c+dx)+1}\,{}_2F_1\left(-\frac{11}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (16*2^(3/4)*a^4*Hypergeometric2F1[-11/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(
1 + Sin[c + d*x])^(1/4))/(d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 0.767, size = 190, normalized size = 1.2

$$-\frac{2a^4}{15de}\left(-24\ (\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)+24\ (\sin(1/2\,dx+c/2))^4\cos(1/2\,dx+c/2)-80\ (\sin(1/2\,dx+c/2))^5\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/15/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-24*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
-80*sin(1/2*d*x+1/2*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-246*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)+80*sin(1/2*d*x+1/2*c)^3-140*sin(1/2*d*x+1/2*c))*a^4/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(a^{4}\cos\left(dx+c\right)^{4}-8\,a^{4}\cos\left(dx+c\right)^{2}+8\,a^{4}-4\,\left(a^{4}\cos\left(dx+c\right)^{2}-2\,a^{4}\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^{2}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^2*cos(d*x + c)^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^{4}}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(3/2), x)
```

3.228 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=152

$$\frac{12a^8(e\cos(c+dx))^{5/2}}{de^5\left(a^4-a^4\sin(c+dx)\right)}+\frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3}-\frac{10a^4\sin(c+dx)\sqrt{e\cos(c+dx)}}{de^3}-\frac{10a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{e\cos(c+dx)}}$$

```
[Out] (-10*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*e^2*Sqrt[e*Cos[c
+ d*x]]) - (10*a^4*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(d*e^3) + (4*a^7*(e*C
os[c + d*x])^(9/2))/(3*d*e^7*(a - a*Sin[c + d*x])^3) + (12*a^8*(e*Cos[c + d
*x])^(5/2))/(d*e^5*(a^4 - a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.223586, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2670, 2680, 2635, 2642, 2641}

$$\frac{12a^8(e\cos(c+dx))^{5/2}}{de^5\left(a^4-a^4\sin(c+dx)\right)}+\frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3}-\frac{10a^4\sin(c+dx)\sqrt{e\cos(c+dx)}}{de^3}-\frac{10a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-10*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*e^2*Sqrt[e*Cos[c
+ d*x]]) - (10*a^4*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(d*e^3) + (4*a^7*(e*C
os[c + d*x])^(9/2))/(3*d*e^7*(a - a*Sin[c + d*x])^3) + (12*a^8*(e*Cos[c + d
*x])^(5/2))/(d*e^5*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{5/2}}\,dx &= \frac{a^8\int \frac{(e\cos(c+dx))^{11/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}\\
&= \frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3}-\frac{\left(3a^6\right)\int \frac{(e\cos(c+dx))^{7/2}}{(a-a\sin(c+dx))^2}\,dx}{e^6}\\
&= \frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3}+\frac{12a^6(e\cos(c+dx))^{5/2}}{de^5\left(a^2-a^2\sin(c+dx)\right)}-\frac{\left(15a^4\right)\int (e\cos(c+dx))^{3/2}\,dx}{e^4}\\
&= -\frac{10a^4\sqrt{e\cos(c+dx)}\sin(c+dx)}{de^3}+\frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3}+\frac{12a^6(e\cos(c+dx))^{5/2}}{de^5\left(a^2-a^2\sin(c+dx)\right)}-\\
&= -\frac{10a^4\sqrt{e\cos(c+dx)}\sin(c+dx)}{de^3}+\frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3}+\frac{12a^6(e\cos(c+dx))^{5/2}}{de^5\left(a^2-a^2\sin(c+dx)\right)}-\\
&= -\frac{10a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{e\cos(c+dx)}}-\frac{10a^4\sqrt{e\cos(c+dx)}\sin(c+dx)}{de^3}+\frac{4a^7(e\cos(c+dx}{3de^7(a-a\sin(c+}
\end{aligned}
$$

Mathematica [C] time = 0.0749828, size = 66, normalized size = 0.43

$$\frac{16\sqrt[4]{2}a^4(\sin(c+dx)+1)^{3/4}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (16*2^(1/4)*a^4*Hypergeometric2F1[-9/4, -3/4, 1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(3/4))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.776, size = 263, normalized size = 1.7

$$\frac{2\,a^4}{3\,de^2}\left(-8\,(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)+30\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2)\,,\sqrt{2}\right)\sqrt{(\mathrm{si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*
e+e)^(1/2)/e^2*(-8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-
48*sin(1/2*d*x+1/2*c)^5-15*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-18*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+48*sin(1/2*d*x+1/2*c)^3-20*sin(1/2*d*x+1/2*c))*a^4/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4\cos\left(dx+c\right)^4-8\,a^4\cos\left(dx+c\right)^2+8\,a^4-4\,\left(a^4\cos\left(dx+c\right)^2-2\,a^4\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^3\cos\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^3*cos(d*x + c)^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^4}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(5/2), x)
```

3.229 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{28a^8(e\cos(c+dx))^{3/2}}{5de^5\left(a^4-a^4\sin(c+dx)\right)}+\frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3}+\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}$$

```
[Out] (42*a^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c
 + d*x]]) + (4*a^7*(e*Cos[c + d*x])^(7/2))/(5*d*e^7*(a - a*Sin[c + d*x])^3)
 - (28*a^8*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a^4 - a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.199432, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2670, 2680, 2640, 2639}

$$-\frac{28a^8(e\cos(c+dx))^{3/2}}{5de^5\left(a^4-a^4\sin(c+dx)\right)}+\frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3}+\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(7/2),x]
```


```
[Out] (42*a^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c
 + d*x]]) + (4*a^7*(e*Cos[c + d*x])^(7/2))/(5*d*e^7*(a - a*Sin[c + d*x])^3)
 - (28*a^8*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a^4 - a^4*Sin[c + d*x]))
```


Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
```

```
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{7/2}}\,dx &= \frac{a^8\int \frac{(e\cos(c+dx))^{9/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8} \\
&= \frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3} - \frac{\left(7a^6\right)\int \frac{(e\cos(c+dx))^{5/2}}{(a-a\sin(c+dx))^2}\,dx}{5e^6} \\
&= \frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3} - \frac{28a^6(e\cos(c+dx))^{3/2}}{5de^5\left(a^2-a^2\sin(c+dx)\right)} + \frac{\left(21a^4\right)\int \sqrt{e\cos(c+dx)}\,dx}{5e^4} \\
&= \frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3} - \frac{28a^6(e\cos(c+dx))^{3/2}}{5de^5\left(a^2-a^2\sin(c+dx)\right)} + \frac{\left(21a^4\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+d}}{5e^4\sqrt{\cos(c+dx)}} \\
&= \frac{42a^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3} - \frac{28a^6(e\cos(c+dx))^{3/2}}{5de^5\left(a^2-a^2\sin(c+dx)\right)}
\end{aligned}
$$

Mathematica [C] time = 0.0989687, size = 66, normalized size = 0.52

$$
\frac{8\ 2^{3/4}a^4(\sin(c+dx)+1)^{5/4}\,{}_2F_1\left(-\frac{7}{4},-\frac{5}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(e\cos(c+dx))^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (8*2^(3/4)*a^4*Hypergeometric2F1[-7/4, -5/4, -1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(5/4))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [B] time = 1.299, size = 332, normalized size = 2.6

$$\frac{2a^4}{5de^3}\left(84\,\mathrm{EllipticE}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\left(\sin\left(1/2\,dx+c/2\right)\right)^4-128\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] 2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-
2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(84*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^4-128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-84*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-
80*sin(1/2*d*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+80*sin(1/2*d*x+1/2*c)^3-12*sin(1/2*d*x+1/2*c))*a^4/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^4}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4\cos\left(dx+c\right)^4-8\,a^4\cos\left(dx+c\right)^2+8\,a^4-4\left(a^4\cos\left(dx+c\right)^2-2\,a^4\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^4\cos\left(dx+c\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^4*cos(d*x + c)^4),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(7/2), x)
```

3.230 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{20a^8\sqrt{e\cos(c+dx)}}{21de^5\left(a^4-a^4\sin(c+dx)\right)}+\frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3}+\frac{10a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}$$

```
[Out] (10*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt[e*Cos[
c + d*x]]) + (4*a^7*(e*Cos[c + d*x])^(5/2))/(7*d*e^7*(a - a*Sin[c + d*x])^3
) - (20*a^8*Sqrt[e*Cos[c + d*x]])/(21*d*e^5*(a^4 - a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.198481, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2670, 2680, 2642, 2641}

$$-\frac{20a^8\sqrt{e\cos(c+dx)}}{21de^5\left(a^4-a^4\sin(c+dx)\right)}+\frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3}+\frac{10a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(9/2),x]
```


```
[Out] (10*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt[e*Cos[
c + d*x]]) + (4*a^7*(e*Cos[c + d*x])^(5/2))/(7*d*e^7*(a - a*Sin[c + d*x])^3
) - (20*a^8*Sqrt[e*Cos[c + d*x]])/(21*d*e^5*(a^4 - a^4*Sin[c + d*x]))
```


Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
```

```
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{9/2}}\,dx &= \frac{a^8\int \frac{(e\cos(c+dx))^{7/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8} \\
&= \frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3} - \frac{\left(5a^6\right)\int \frac{(e\cos(c+dx))^{3/2}}{(a-a\sin(c+dx))^2}\,dx}{7e^6} \\
&= \frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3} - \frac{20a^6\sqrt{e\cos(c+dx)}}{21de^5\left(a^2-a^2\sin(c+dx)\right)} + \frac{\left(5a^4\right)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{21e^4} \\
&= \frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3} - \frac{20a^6\sqrt{e\cos(c+dx)}}{21de^5\left(a^2-a^2\sin(c+dx)\right)} + \frac{\left(5a^4\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21e^4\sqrt{e\cos(c+dx)}} \\
&= \frac{10a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}} + \frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3} - \frac{20a^6\sqrt{e\cos(c+dx)}}{21de^5\left(a^2-a^2\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [C] time = 0.117539, size = 66, normalized size = 0.52

$$\frac{8\sqrt[4]{2}a^4(\sin(c+dx)+1)^{7/4}\,{}_2F_1\left(-\frac{7}{4},-\frac{5}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (8*2^(1/4)*a^4*Hypergeometric2F1[-7/4, -5/4, -3/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(7/4))/(7*d*e*(e*Cos[c + d*x])^(7/2))
```

Maple [B] time = 1.585, size = 401, normalized size = 3.2

$$-\frac{2a^4}{21e^4d}\left(40\sqrt{(\sin(1/2\,dx+c/2))^2}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}(\sin(1/2\,dx+c/2))^6-6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(40*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6-60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4-128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-
112*sin(1/2*d*x+1/2*c)^5-5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+112*sin(1/2*d*x+1/2*c)^3-4*sin(1/2*d*x+1/2*c))*a^4/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4\cos\left(dx+c\right)^4-8\,a^4\cos\left(dx+c\right)^2+8\,a^4-4\left(a^4\cos\left(dx+c\right)^2-2\,a^4\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^5\cos\left(dx+c\right)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^5*cos(d*x + c)^5),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sin\left(dx+c\right)+a\right)^4}{\left(e\cos\left(dx+c\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(9/2), x)
```

3.231 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{2a^8(e\cos(c+dx))^{3/2}}{15de^7\left(a^4-a^4\sin(c+dx)\right)}-\frac{2a^8(e\cos(c+dx))^{3/2}}{15de^7\left(a^2-a^2\sin(c+dx)\right)^2}+\frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3}+\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15de^6\sqrt{\cos(c+dx)}}$$

```
[Out] (2*a^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*e^6*Sqrt[Cos[c
 + d*x]]) + (4*a^7*(e*Cos[c + d*x])^(3/2))/(9*d*e^7*(a - a*Sin[c + d*x])^3)
 - (2*a^8*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a^2 - a^2*Sin[c + d*x])^2) - (
2*a^8*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a^4 - a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.249945, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2670, 2680, 2681, 2683, 2640, 2639}

$$-\frac{2a^8(e\cos(c+dx))^{3/2}}{15de^7\left(a^4-a^4\sin(c+dx)\right)}-\frac{2a^8(e\cos(c+dx))^{3/2}}{15de^7\left(a^2-a^2\sin(c+dx)\right)^2}+\frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3}+\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15de^6\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*a^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*e^6*Sqrt[Cos[c
 + d*x]]) + (4*a^7*(e*Cos[c + d*x])^(3/2))/(9*d*e^7*(a - a*Sin[c + d*x])^3)
 - (2*a^8*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a^2 - a^2*Sin[c + d*x])^2) - (
2*a^8*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{11/2}}\,dx = \frac{a^8\int \frac{(e\cos(c+dx))^{5/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}$$

$$= \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{a^6\int \frac{\sqrt{e\cos(c+dx)}}{(a-a\sin(c+dx))^2}\,dx}{3e^6}$$

$$= \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} - \frac{a^5\int \frac{\sqrt{e\cos(c+dx)}}{a-a\sin(c+dx)}\,dx}{15e^6}$$

$$= \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} - \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))} + \frac{a^4\int \sqrt{e\cos}}{1}$$

$$= \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} - \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))} + \frac{\left(a^4\sqrt{e\cos}\right.}{}$$

$$= \frac{2a^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^6\sqrt{\cos(c+dx)}} + \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} -$$

Mathematica [C] time = 0.146502, size = 66, normalized size = 0.39

$$\frac{4\,2^{3/4}a^4(\sin(c+dx)+1)^{9/4}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{4};-\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (4*2^(3/4)*a^4*Hypergeometric2F1[-9/4, -3/4, -5/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(9/4))/(9*d*e*(e*Cos[c + d*x])^(9/2))
```

Maple [B] time = 2.043, size = 514, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x)
```

```
[Out] 2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c)
^4-8*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+
e)^(1/2)/e^5*(48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*d
*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^4-272*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+176*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)-144*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+42*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+144*sin(1/2*d*x+1/2*c)^3+4*sin(1/2*d*x+1/2*c
))*a^4/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4\cos(dx+c)^4-8\,a^4\cos(dx+c)^2+8\,a^4-4\left(a^4\cos(dx+c)^2-2\,a^4\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^6\cos(dx+c)^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^6*cos(d*x + c)^6),
```

x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(11/2), x)
```

3.232 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{13/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{2a^8\sqrt{e\cos(c+dx)}}{77de^7\left(a^4-a^4\sin(c+dx)\right)}-\frac{2a^8\sqrt{e\cos(c+dx)}}{77de^7\left(a^2-a^2\sin(c+dx)\right)^2}+\frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3}-\frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{77de^6\sqrt{e\cos(c+dx)}}$$

```
[Out] (-2*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*e^6*Sqrt[e*Cos[
c + d*x]]) + (4*a^7*Sqrt[e*Cos[c + d*x]])/(11*d*e^7*(a - a*Sin[c + d*x])^3)
 - (2*a^8*Sqrt[e*Cos[c + d*x]])/(77*d*e^7*(a^2 - a^2*Sin[c + d*x])^2) - (2*
a^8*Sqrt[e*Cos[c + d*x]])/(77*d*e^7*(a^4 - a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.259435, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2670, 2680, 2681, 2683, 2642, 2641}

$$-\frac{2a^8\sqrt{e\cos(c+dx)}}{77de^7\left(a^4-a^4\sin(c+dx)\right)}-\frac{2a^8\sqrt{e\cos(c+dx)}}{77de^7\left(a^2-a^2\sin(c+dx)\right)^2}+\frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3}-\frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{77de^6\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(13/2),x]
```

```
[Out] (-2*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*e^6*Sqrt[e*Cos[
c + d*x]]) + (4*a^7*Sqrt[e*Cos[c + d*x]])/(11*d*e^7*(a - a*Sin[c + d*x])^3)
 - (2*a^8*Sqrt[e*Cos[c + d*x]])/(77*d*e^7*(a^2 - a^2*Sin[c + d*x])^2) - (2*
a^8*Sqrt[e*Cos[c + d*x]])/(77*d*e^7*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{13/2}}\,dx &= \frac{a^8\int \frac{(e\cos(c+dx))^{3/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8} \\
&= \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3} - \frac{a^6\int \frac{1}{\sqrt{e\cos(c+dx)}(a-a\sin(c+dx))^2}\,dx}{11e^6} \\
&= \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3} - \frac{2a^6\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))^2} - \frac{\left(3a^5\right)\int \frac{1}{\sqrt{e\cos(c+dx)}(a-a\sin(c+dx))}\,dx}{77e^6} \\
&= \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3} - \frac{2a^6\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))^2} - \frac{2a^5\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))} - \\
&= \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3} - \frac{2a^6\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))^2} - \frac{2a^5\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))} - \\
&= -\frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77de^6\sqrt{e\cos(c+dx)}} + \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3} - \frac{2a^6\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))^2}
\end{aligned}
$$

Mathematica [C] time = 0.211174, size = 66, normalized size = 0.39

$$
\frac{4\sqrt[4]{2}a^4(\sin(c+dx)+1)^{11/4}\,{}_2F_1\left(-\frac{11}{4},-\frac{1}{4};-\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11de(e\cos(c+dx))^{11/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(13/2),x]
```

```
[Out] (4*2^(1/4)*a^4*Hypergeometric2F1[-11/4, -1/4, -7/4, (1 - Sin[c + d*x])/2]*(
1 + Sin[c + d*x])^(11/4))/(11*d*e*(e*Cos[c + d*x])^(11/2))
```

Maple [B] time = 2.272, size = 583, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(13/2),x)
```

```
[Out] 2/77/(32*sin(1/2*d*x+1/2*c)^10-80*sin(1/2*d*x+1/2*c)^8+80*sin(1/2*d*x+1/2*c
)^6-40*sin(1/2*d*x+1/2*c)^4+10*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^6*(32*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2
*d*x+1/2*c)^10-80*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8+32*sin(1/2*
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+80*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/
2*c)^6-64*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-40*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*sin(1/2*d*x+1/2*c)^4+176*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+10*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-144*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+176*sin(1/2*d*x+1/2*c)^5-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-78*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-176*sin(1/2*d*x+1/2*c)^3-12*sin(1/2*d*x+1/2*c)
)*a^4/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4\cos(dx+c)^4 - 8\,a^4\cos(dx+c)^2 + 8\,a^4 - 4\left(a^4\cos(dx+c)^2 - 2\,a^4\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^7\cos(dx+c)^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(13/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*
x + c)^2 - 2*a^4)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^7*cos(d*x + c)^7),
```

```
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(13/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(13/2), x)
```

3.233 $\int \frac{(e\cos(c+dx))^{11/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=132

$$\frac{10e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{21ad}+\frac{2e^3\sin(c+dx)(e\cos(c+dx))^{5/2}}{7ad}+\frac{10e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad\sqrt{e\cos(c+dx)}}+\frac{2e(e\cos(c+dx}{9ad}$$

```
[Out] (2*e*(e*Cos[c + d*x])^(9/2))/(9*a*d) + (10*e^6*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2])/(21*a*d*Sqrt[e*Cos[c + d*x]]) + (10*e^5*Sqrt[e*Cos[c + d*
x]]*Sin[c + d*x])/(21*a*d) + (2*e^3*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7
*a*d)
```

Rubi [A] time = 0.117489, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2682, 2635, 2642, 2641}

$$\frac{10e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{21ad}+\frac{2e^3\sin(c+dx)(e\cos(c+dx))^{5/2}}{7ad}+\frac{10e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad\sqrt{e\cos(c+dx)}}+\frac{2e(e\cos(c+dx}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (2*e*(e*Cos[c + d*x])^(9/2))/(9*a*d) + (10*e^6*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2])/(21*a*d*Sqrt[e*Cos[c + d*x]]) + (10*e^5*Sqrt[e*Cos[c + d*
x]]*Sin[c + d*x])/(21*a*d) + (2*e^3*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7
*a*d)
```


Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\cos(c+dx))^{11/2}}{a+a\sin(c+dx)}\,dx &= \frac{2e(e\cos(c+dx))^{9/2}}{9ad}+\frac{e^2\int (e\cos(c+dx))^{7/2}\,dx}{a}\\
&= \frac{2e(e\cos(c+dx))^{9/2}}{9ad}+\frac{2e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{7ad}+\frac{\left(5e^4\right)\int (e\cos(c+dx))^{3/2}\,dx}{7a}\\
&= \frac{2e(e\cos(c+dx))^{9/2}}{9ad}+\frac{10e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{21ad}+\frac{2e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{7ad}+\\
&= \frac{2e(e\cos(c+dx))^{9/2}}{9ad}+\frac{10e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{21ad}+\frac{2e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{7ad}+\\
&= \frac{2e(e\cos(c+dx))^{9/2}}{9ad}+\frac{10e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad\sqrt{e\cos(c+dx)}}+\frac{10e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{21ad}+
\end{aligned}$$

Mathematica [C] time = 0.168022, size = 66, normalized size = 0.5

$$-\frac{8\sqrt[4]{2}(e\cos(c+dx))^{13/2}\,{}_2F_1\left(-\frac{5}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{13ade(\sin(c+dx)+1)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-8*2^(1/4)*(e*Cos[c + d*x])^(13/2)*Hypergeometric2F1[-5/4, 13/4, 17/4, (1
- Sin[c + d*x])/2])/(13*a*d*e*(1 + Sin[c + d*x])^(13/4))
```

Maple [A] time = 0.563, size = 251, normalized size = 1.9

$$-\frac{2e^6}{63da}\left(224\left(\sin\left(1/2\,dx+c/2\right)\right)^{11}+144\,\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}-560\left(\sin\left(1/2\,dx+c/2\right)\right)^{9}-216\left(\sin\left(1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/63/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^6*(224*sin
(1/2*d*x+1/2*c)^11+144*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-560*sin(1/2*
d*x+1/2*c)^9-216*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+560*sin(1/2*d*x+1/
2*c)^7+168*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-280*sin(1/2*d*x+1/2*c)^5
+15*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*
sin(1/2*d*x+1/2*c)^3-7*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(e\cos\left(dx+c\right)\right)^{\frac{11}{2}}}{a\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(11/2)/(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}e^{5}\cos\left(dx+c\right)^{5}}{a\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^5*cos(d*x + c)^5/(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{11}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(11/2)/(a*sin(d*x + c) + a), x)
```

3.234 $\int \frac{(e\cos(c+dx))^{9/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2e^3\sin(c+dx)(e\cos(c+dx))^{3/2}}{5ad}+\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5ad\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{7/2}}{7ad}$$

```
[Out] (2*e*(e*Cos[c + d*x])^(7/2))/(7*a*d) + (6*e^4*Sqrt[e*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]) + (2*e^3*(e*Cos[c + d*x])^(3/
2)*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.0954738, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2682, 2635, 2640, 2639}

$$\frac{2e^3\sin(c+dx)(e\cos(c+dx))^{3/2}}{5ad}+\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5ad\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{7/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (2*e*(e*Cos[c + d*x])^(7/2))/(7*a*d) + (6*e^4*Sqrt[e*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]) + (2*e^3*(e*Cos[c + d*x])^(3/
2)*Sin[c + d*x])/(5*a*d)
```


Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{9/2}}{a+a\sin(c+dx)}\,dx &= \frac{2e(e\cos(c+dx))^{7/2}}{7ad}+\frac{e^2\int(e\cos(c+dx))^{5/2}\,dx}{a}\\ &= \frac{2e(e\cos(c+dx))^{7/2}}{7ad}+\frac{2e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{5ad}+\frac{\left(3e^4\right)\int\sqrt{e\cos(c+dx)}\,dx}{5a}\\ &= \frac{2e(e\cos(c+dx))^{7/2}}{7ad}+\frac{2e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{5ad}+\frac{\left(3e^4\sqrt{e\cos(c+dx)}\right)\int\sqrt{\cos(c+dx}}{5a\sqrt{\cos(c+dx)}}\\ &= \frac{2e(e\cos(c+dx))^{7/2}}{7ad}+\frac{6e^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}}+\frac{2e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{5ad}\end{aligned}$$

Mathematica [C] time = 0.0919839, size = 66, normalized size = 0.65

$$-\frac{4\,2^{3/4}(e\cos(c+dx))^{11/2}\,{}_2F_1\left(-\frac{3}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11ade(\sin(c+dx)+1)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-4*2^(3/4)*(e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[-3/4, 11/4, 15/4, (1
 - Sin[c + d*x])/2])/(11*a*d*e*(1 + Sin[c + d*x])^(11/4))
```

Maple [A] time = 0.598, size = 216, normalized size = 2.1

$$\frac{2\,e^5}{35\,da}\left(80\,(\sin(1/2\,dx+c/2))^9+56\,(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)-160\,(\sin(1/2\,dx+c/2))^7-56\,(\sin(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/35/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^5*(80*sin(1
/2*d*x+1/2*c)^9+56*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-160*sin(1/2*d*x+
1/2*c)^7-56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+120*sin(1/2*d*x+1/2*c)^
5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))+14*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-40
*sin(1/2*d*x+1/2*c)^3+5*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}e^4\cos(dx+c)^4}{a\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^4*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(a*sin(d*x + c) + a), x)
```

3.235 $\int \frac{(e\cos(c+dx))^{7/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{3ad}+\frac{2e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{5/2}}{5ad}$$

```
[Out] (2*e*(e*Cos[c + d*x])^(5/2))/(5*a*d) + (2*e^4*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2])/(3*a*d*Sqrt[e*Cos[c + d*x]]) + (2*e^3*Sqrt[e*Cos[c + d*x]]
*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.0949842, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2682, 2635, 2642, 2641}

$$\frac{2e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{3ad}+\frac{2e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x]),x]
```


```
[Out] (2*e*(e*Cos[c + d*x])^(5/2))/(5*a*d) + (2*e^4*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2])/(3*a*d*Sqrt[e*Cos[c + d*x]]) + (2*e^3*Sqrt[e*Cos[c + d*x]]
*Sin[c + d*x])/(3*a*d)
```


Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{7/2}}{a+a\sin(c+dx)}\,dx &= \frac{2e(e\cos(c+dx))^{5/2}}{5ad}+\frac{e^2\int(e\cos(c+dx))^{3/2}\,dx}{a}\\ &= \frac{2e(e\cos(c+dx))^{5/2}}{5ad}+\frac{2e^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{3ad}+\frac{e^4\int\frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3a}\\ &= \frac{2e(e\cos(c+dx))^{5/2}}{5ad}+\frac{2e^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{3ad}+\frac{\left(e^4\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a\sqrt{e\cos(c+dx)}}\\ &= \frac{2e(e\cos(c+dx))^{5/2}}{5ad}+\frac{2e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}+\frac{2e^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{3ad}\end{aligned}$$

Mathematica [C] time = 0.0788619, size = 66, normalized size = 0.65

$$-\frac{4\sqrt[4]{2}(e\cos(c+dx))^{9/2}\,{}_2F_1\left(-\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9ade(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-4*2^(1/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[-1/4, 9/4, 13/4, (1 -
Sin[c + d*x])/2])/(9*a*d*e*(1 + Sin[c + d*x])^(9/4))
```

Maple [A] time = 0.516, size = 181, normalized size = 1.8

$$-\frac{2e^4}{15da}\left(24\left(\sin(1/2dx+c/2)\right)^7+20\left(\sin(1/2dx+c/2)\right)^4\cos(1/2dx+c/2)-36\left(\sin(1/2dx+c/2)\right)^5+5\sqrt{2\left(\sin(1/2d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/15/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(24*sin(
1/2*d*x+1/2*c)^7+20*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-36*sin(1/2*d*x+
1/2*c)^5+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)+18*sin(1/2*d*x+1/2*c)^3-3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}e^3\cos(dx+c)^3}{a\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^3*cos(d*x + c)^3/(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a), x)
```

3.236 $\int \frac{(e\cos(c+dx))^{5/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=68

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{ad\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{3/2}}{3ad}$$

```
[Out] (2*e*(e*Cos[c + d*x])^(3/2))/(3*a*d) + (2*e^2*Sqrt[e*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0754318, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2682, 2640, 2639}

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{ad\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{3/2}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*e*(e*Cos[c + d*x])^(3/2))/(3*a*d) + (2*e^2*Sqrt[e*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]])
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{5/2}}{a+a\sin(c+dx)}\,dx &= \frac{2e(e\cos(c+dx))^{3/2}}{3ad}+\frac{e^2\int \sqrt{e\cos(c+dx)}\,dx}{a}\\ &= \frac{2e(e\cos(c+dx))^{3/2}}{3ad}+\frac{\left(e^2\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{a\sqrt{\cos(c+dx)}}\\ &= \frac{2e(e\cos(c+dx))^{3/2}}{3ad}+\frac{2e^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.098988, size = 66, normalized size = 0.97

$$-\frac{2\ 2^{3/4}(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7ade(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[1/4, 7/4, 11/4, (1 - S
in[c + d*x])/2])/(7*a*d*e*(1 + Sin[c + d*x])^(7/4))
```

Maple [A] time = 0.591, size = 122, normalized size = 1.8

$$\frac{2e^3}{3da}\left(4\left(\sin(1/2\,dx+c/2)\right)^5+3\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] 2/3/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(4*sin(1/2
*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
```

```
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^3+sin(1/2*d*x
+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}e^{2}\cos(dx+c)^{2}}{a\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^2*cos(d*x + c)^2/(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \cos(dx + c))^{\frac{5}{2}}}{a \sin(dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a), x)
```

3.237 $\int \frac{(e\cos(c+dx))^{3/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=66

$$\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{ad}$$

```
[Out] (2*e*Sqrt[e*Cos[c + d*x]])/(a*d) + (2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2])/(a*d*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.0756069, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2682, 2642, 2641}

$$\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*e*Sqrt[e*Cos[c + d*x]])/(a*d) + (2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2])/(a*d*Sqrt[e*Cos[c + d*x]])
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\cos(c+dx))^{3/2}}{a+a\sin(c+dx)}\,dx &= \frac{2e\sqrt{e\cos(c+dx)}}{ad}+\frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{a}\\
&= \frac{2e\sqrt{e\cos(c+dx)}}{ad}+\frac{\left(e^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a\sqrt{e\cos(c+dx)}}\\
&= \frac{2e\sqrt{e\cos(c+dx)}}{ad}+\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{e\cos(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 0.0747973, size = 66, normalized size = 1.

$$
-\frac{2\sqrt[4]{2}(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5ade(\sin(c+dx)+1)^{5/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[3/4, 5/4, 9/4, (1 - Si
n[c + d*x])/2])/(5*a*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [A] time = 0.345, size = 110, normalized size = 1.7

$$
-2\,\frac{e^2\left(\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2)\,,\sqrt{2}\right)+2\,(\sin(1/2\,dx+c/2))^3-\mathrm{si}\right.}{a\sin(1/2\,dx+c/2)\sqrt{-2\,(\sin(1/2\,dx+c/2))^2e+e}d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*((2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+2*sin(1/2*d*x+1/2*c)^3-sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}e\cos(dx+c)}{a\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos{(dx+c)})^{\frac{3}{2}}}{a\sin{(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a), x)
```

3.238 $\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=74

$$-\frac{2(e\cos(c+dx))^{3/2}}{de(a\sin(c+dx)+a)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]]
) - (2*(e*Cos[c + d*x])^(3/2))/(d*e*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0679212, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2683, 2640, 2639}

$$-\frac{2(e\cos(c+dx))^{3/2}}{de(a\sin(c+dx)+a)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]]
) - (2*(e*Cos[c + d*x])^(3/2))/(d*e*(a + a*Sin[c + d*x]))
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx &= -\frac{2(e\cos(c+dx))^{3/2}}{de(a+a\sin(c+dx))}-\frac{\int\sqrt{e\cos(c+dx)}\,dx}{a}\\
&= -\frac{2(e\cos(c+dx))^{3/2}}{de(a+a\sin(c+dx))}-\frac{\sqrt{e\cos(c+dx)}\int\sqrt{\cos(c+dx)}\,dx}{a\sqrt{\cos(c+dx)}}\\
&= -\frac{2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{de(a+a\sin(c+dx))}
\end{aligned}
$$

Mathematica [C] time = 0.0419868, size = 66, normalized size = 0.89

$$
-\frac{2^{3/4}(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3ade(\sin(c+dx)+1)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(2^(3/4)*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 5/4, 7/4, (1 - Sin[
c + d*x])/2])/(3*a*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [A] time = 0.831, size = 115, normalized size = 1.6

$$
-2\,\frac{\left(\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-2\,(\sin(1/2\,dx+c/2))^2\cos(1/\cdots\right.}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^2e+e}\sin(1/2\,dx+c/2)\,ad}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)/a*((sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
```

```
),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))*e/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{a\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}}{a\sin\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos(dx+c)}}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a), x)
```

3.239 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=78

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{3de(a\sin(c+dx)+a)}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*a*d*Sqrt[e*Cos[c + d*x]
]) - (2*Sqrt[e*Cos[c + d*x]])/(3*d*e*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0724614, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2683, 2642, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{3de(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*a*d*Sqrt[e*Cos[c + d*x]
]) - (2*Sqrt[e*Cos[c + d*x]])/(3*d*e*(a + a*Sin[c + d*x]))
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))}\,dx &= -\frac{2\sqrt{e\cos(c+dx)}}{3de(a+a\sin(c+dx))}+\frac{\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3a}\\ &= -\frac{2\sqrt{e\cos(c+dx)}}{3de(a+a\sin(c+dx))}+\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a\sqrt{e\cos(c+dx)}}\\ &= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{3de(a+a\sin(c+dx))}\end{aligned}$$

Mathematica [C] time = 0.039761, size = 64, normalized size = 0.82

$$-\frac{\sqrt[4]{2}\sqrt{e\cos(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{ade\sqrt[4]{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])),x]
```

```
[Out] -((2^(1/4)*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 7/4, 5/4, (1 - Sin[c
 + d*x])/2])/(a*d*e*(1 + Sin[c + d*x])^(1/4)))
```

Maple [B] time = 0.914, size = 190, normalized size = 2.4

$$-\frac{2}{3\,da}\left(2\,\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)
^2*e+e)^(1/2)*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
```

```
*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}}{ae\cos(dx+c)\sin(dx+c)+ae\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a*e*cos(d*x + c)*sin(d*x + c) + a*e*cos(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)), x)
```

3.240 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=112

$$-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5ade^2\sqrt{\cos(c+dx)}}+\frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}}-\frac{2}{5de(a\sin(c+dx)+a)\sqrt{e\cos(c+dx)}}$$

```
[Out] (-6*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a*d*e^2*Sqrt[Cos[c +
 d*x]]) + (6*Sin[c + d*x])/(5*a*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(5*d*e*Sqrt[e
*Cos[c + d*x]]*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0968475, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2683, 2636, 2640, 2639}

$$-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5ade^2\sqrt{\cos(c+dx)}}+\frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}}-\frac{2}{5de(a\sin(c+dx)+a)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])),x]
```


```
[Out] (-6*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a*d*e^2*Sqrt[Cos[c +
 d*x]]) + (6*Sin[c + d*x])/(5*a*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(5*d*e*Sqrt[e
*Cos[c + d*x]]*(a + a*Sin[c + d*x]))
```


Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))}\,dx &= -\frac{2}{5de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))}+\frac{3\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx}{5a}\\ &= \frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}}-\frac{2}{5de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))}-\frac{3\int \sqrt{e\cos(c}{5ae}\\ &= \frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}}-\frac{2}{5de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))}-\frac{\left(3\sqrt{e\cos(c}{5a}\\ &= -\frac{6\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ade^2\sqrt{\cos(c+dx)}}+\frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}}-\frac{}{5de\sqrt{e\cos(c+}}\end{aligned}$$

Mathematica [C] time = 0.0563637, size = 63, normalized size = 0.56

$$\frac{\sqrt[4]{\sin(c+dx)+1}\,{}_2F_1\left(-\frac{1}{4},\frac{9}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{\sqrt[4]{2}ade\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (Hypergeometric2F1[-1/4, 9/4, 3/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])
^(1/4))/(2^(1/4)*a*d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [B] time = 1.286, size = 304, normalized size = 2.7

$$-\frac{2}{5ade}\left(12\,\mathrm{EllipticE}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}-24\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a/sin(1/2*d*x+1/2*c)
/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2
*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+sin(1/2
*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a\sin\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}}{ae^{2}\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+ae^{2}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a*e^2*cos(d*x + c)^2*sin(d*x + c) + a*e^2*co
s(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)), x)
```

3.241 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=112

$$\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ade^2\sqrt{e\cos(c+dx)}}+\frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}}-\frac{2}{7de(a\sin(c+dx)+a)(e\cos(c+dx))^{3/2}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*a*d*e^2*Sqrt[e*Cos[c
+ d*x]]) + (10*Sin[c + d*x])/(21*a*d*e*(e*Cos[c + d*x])^(3/2)) - 2/(7*d*e*(
e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0943606, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2683, 2636, 2642, 2641}

$$\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ade^2\sqrt{e\cos(c+dx)}}+\frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}}-\frac{2}{7de(a\sin(c+dx)+a)(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*a*d*e^2*Sqrt[e*Cos[c
+ d*x]]) + (10*Sin[c + d*x])/(21*a*d*e*(e*Cos[c + d*x])^(3/2)) - 2/(7*d*e*(
e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x]))
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))}\,dx = -\frac{2}{7de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))} + \frac{5\int \frac{1}{(e\cos(c+dx))^{5/2}}\,dx}{7a}$$

$$= \frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}} - \frac{2}{7de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))} + \frac{5\int \frac{1}{\sqrt{e}}$$

$$= \frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}} - \frac{2}{7de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))} + \frac{\left(5\sqrt{\mathrm{co}}\right.}{}$$

$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ade^2\sqrt{e\cos(c+dx)}} + \frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}} - \frac{}{7de(e\cos(c+}$$

Mathematica [C] time = 0.0609495, size = 66, normalized size = 0.59

$$\frac{(\sin(c+dx)+1)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{11}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3\ 2^{3/4}ade(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (Hypergeometric2F1[-3/4, 11/4, 1/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(3/4))/(3*2^(3/4)*a*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [B] time = 1.511, size = 375, normalized size = 3.4

$$-\frac{2}{21\,e^{2}ad}\left(40\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^2*(40*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6-60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/
2*d*x+1/2*c)^4+40*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-
5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*sin
(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sin\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}}{ae^{3}\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+ae^{3}\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a*e^3*cos(d*x + c)^3*sin(d*x + c) + a*e^3*co
s(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)), x)
```

3.242 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=143

$$\frac{14\sin(c+dx)}{15ade^3\sqrt{e\cos(c+dx)}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15ade^4\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}}-\frac{2}{9de(a\sin(c+dx)+a)(e\cos(c+dx))^{5/2}}$$

```
[Out] (-14*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*a*d*e^4*Sqrt[Cos[c
 + d*x]]) + (14*Sin[c + d*x])/(45*a*d*e*(e*Cos[c + d*x])^(5/2)) + (14*Sin[c
 + d*x])/(15*a*d*e^3*Sqrt[e*Cos[c + d*x]]) - 2/(9*d*e*(e*Cos[c + d*x])^(5/2
)*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.114257, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2683, 2636, 2640, 2639}

$$\frac{14\sin(c+dx)}{15ade^3\sqrt{e\cos(c+dx)}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15ade^4\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}}-\frac{2}{9de(a\sin(c+dx)+a)(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])),x]
```



```
[Out] (-14*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*a*d*e^4*Sqrt[Cos[c
 + d*x]]) + (14*Sin[c + d*x])/(45*a*d*e*(e*Cos[c + d*x])^(5/2)) + (14*Sin[c
 + d*x])/(15*a*d*e^3*Sqrt[e*Cos[c + d*x]]) - 2/(9*d*e*(e*Cos[c + d*x])^(5/2
)*(a + a*Sin[c + d*x]))
```


Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))}\,dx = -\frac{2}{9de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))} + \frac{7\int \frac{1}{(e\cos(c+dx))^{7/2}}\,dx}{9a}$$

$$= \frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}} - \frac{2}{9de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))} + \frac{7\int \frac{}{(e\,c}}{}$$

$$= \frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15ade^3\sqrt{e\cos(c+dx)}} - \frac{2}{9de(e\cos(c+dx))^{5/2}(a\,-}$$

$$= \frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15ade^3\sqrt{e\cos(c+dx)}} - \frac{2}{9de(e\cos(c+dx))^{5/2}(a\,-}$$

$$= -\frac{14\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^4\sqrt{\cos(c+dx)}} + \frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}} + \frac{14\sin(}{15ade^3\sqrt{e}}$$

Mathematica [C] time = 0.082537, size = 66, normalized size = 0.46

$$\frac{(\sin(c+dx)+1)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{13}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{10\sqrt[4]{2}ade(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (Hypergeometric2F1[-5/4, 13/4, -1/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^(5/4))/(10*2^(1/4)*a*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [B] time = 2.212, size = 488, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c
)^4-8*sin(1/2*d*x+1/2*c)^2+1)/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^3*(336*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-672*sin(
1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-672*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d
*x+1/2*c)^6+1344*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+504*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-1064*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c
)-168*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+392*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+5*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}}{ae^4\cos(dx+c)^4\sin(dx+c)+ae^4\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a*e^4*cos(d*x + c)^4*sin(d*x + c) + a*e^4*co
s(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}(a\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)), x)
```

3.243 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=145

$$\frac{6e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{7a^2d}+\frac{18e^3\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^2d}+\frac{6e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}+\frac{4e(e\cos(c+d}{5d\left(a^2\sin(c+d\right.}$$

```
[Out] (6*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Sqrt[e*Cos[c
+ d*x]]) + (6*e^5*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(7*a^2*d) + (18*e^3*(e
*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(35*a^2*d) + (4*e*(e*Cos[c + d*x])^(9/2)
)/(5*d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.109562, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2680, 2635, 2642, 2641}

$$\frac{6e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{7a^2d}+\frac{18e^3\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^2d}+\frac{6e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}+\frac{4e(e\cos(c+d}{5d\left(a^2\sin(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] (6*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Sqrt[e*Cos[c
+ d*x]]) + (6*e^5*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(7*a^2*d) + (18*e^3*(e
*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(35*a^2*d) + (4*e*(e*Cos[c + d*x])^(9/2)
)/(5*d*(a^2 + a^2*Sin[c + d*x]))
```


Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^2}\,dx = \frac{4e(e\cos(c+dx))^{9/2}}{5d\left(a^2+a^2\sin(c+dx)\right)} + \frac{\left(9e^2\right)\int (e\cos(c+dx))^{7/2}\,dx}{5a^2}$$

$$= \frac{18e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^2d} + \frac{4e(e\cos(c+dx))^{9/2}}{5d\left(a^2+a^2\sin(c+dx)\right)} + \frac{\left(9e^4\right)\int (e\cos(c+dx))^{3/2}\,dx}{7a^2}$$

$$= \frac{6e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{7a^2d} + \frac{18e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^2d} + \frac{4e(e\cos(c+dx))^{9/2}}{5d\left(a^2+a^2\sin(c+dx)\right)}$$

$$= \frac{6e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{7a^2d} + \frac{18e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^2d} + \frac{4e(e\cos(c+dx))^{9/2}}{5d\left(a^2+a^2\sin(c+dx)\right)}$$

$$= \frac{6e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{6e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{7a^2d} + \frac{18e^3(e\cos(c+dx))^{5/2}}{35a^2d}$$

Mathematica [C] time = 0.188916, size = 66, normalized size = 0.46

$$-\frac{4\sqrt[4]{2}(e\cos(c+dx))^{13/2}\,{}_2F_1\left(-\frac{1}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{13a^2de(\sin(c+dx)+1)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-4*2^(1/4)*(e*Cos[c + d*x])^(13/2)*Hypergeometric2F1[-1/4, 13/4, 17/4, (1
- Sin[c + d*x])/2])/(13*a^2*d*e*(1 + Sin[c + d*x])^(13/4))
```

Maple [A] time = 0.56, size = 203, normalized size = 1.4

$$-\frac{2\,e^6}{35\,a^2d}\left(-80\,\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+120\,\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)+112\,\left(\sin\left(1/2\,dx+c/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/35/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^6*(-80*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+120*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+112*sin(1/2*d*x+1/2*c)^7-168*sin(1/2*d*x+1/2*c)^5+15*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-20*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+84*sin(1/2*d*x+1/2*c)
^3-14*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e\cos\left(dx+c\right)\right)^{\frac{11}{2}}}{\left(a\sin\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(11/2)/(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}e^5\cos\left(dx+c\right)^5}{a^2\cos\left(dx+c\right)^2-2\,a^2\sin\left(dx+c\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^5*cos(d*x + c)^5/(a^2*cos(d*x + c)^2 - 2*a
^2*sin(d*x + c) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{11}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(11/2)/(a*sin(d*x + c) + a)^2, x)
```

3.244 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=114

$$\frac{14e^3\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^2d}+\frac{14e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^2d\sqrt{\cos(c+dx)}}+\frac{4e(e\cos(c+dx))^{7/2}}{3d\left(a^2\sin(c+dx)+a^2\right)}$$

```
[Out] (14*e^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c
 + d*x]]) + (14*e^3*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*a^2*d) + (4*e*
(e*Cos[c + d*x])^(7/2))/(3*d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0922453, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2680, 2635, 2640, 2639}

$$\frac{14e^3\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^2d}+\frac{14e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^2d\sqrt{\cos(c+dx)}}+\frac{4e(e\cos(c+dx))^{7/2}}{3d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (14*e^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c
 + d*x]]) + (14*e^3*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*a^2*d) + (4*e*
(e*Cos[c + d*x])^(7/2))/(3*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^2}\,dx &= \frac{4e(e\cos(c+dx))^{7/2}}{3d\left(a^2+a^2\sin(c+dx)\right)}+\frac{\left(7e^2\right)\int(e\cos(c+dx))^{5/2}\,dx}{3a^2}\\
&= \frac{14e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^2d}+\frac{4e(e\cos(c+dx))^{7/2}}{3d\left(a^2+a^2\sin(c+dx)\right)}+\frac{\left(7e^4\right)\int\sqrt{e\cos(c+dx)}\,dx}{5a^2}\\
&= \frac{14e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^2d}+\frac{4e(e\cos(c+dx))^{7/2}}{3d\left(a^2+a^2\sin(c+dx)\right)}+\frac{\left(7e^4\sqrt{e\cos(c+dx)}\right)\int\sqrt{\cos}\ldots}{5a^2\sqrt{\cos(c+dx}\ldots}\\
&= \frac{14e^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}}+\frac{14e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^2d}+\frac{4e(e\cos(c+\ldots}{3d\left(a^2+a^2\sin(\ldots\right.}
\end{aligned}
$$

Mathematica [C] time = 0.090007, size = 66, normalized size = 0.58

$$
-\frac{2\,2^{3/4}(e\cos(c+dx))^{11/2}\,{}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11a^2de(\sin(c+dx)+1)^{11/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[1/4, 11/4, 15/4, (1 -
 Sin[c + d*x])/2])/(11*a^2*d*e*(1 + Sin[c + d*x])^(11/4))
```

Maple [A] time = 0.658, size = 190, normalized size = 1.7

$$\frac{2e^5}{15a^2d}\left(-24\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)+24\left(\sin\left(1/2\,dx+c/2\right)\right)^4\cos\left(1/2\,dx+c/2\right)+40\left(\sin\left(1/2\,dx+c/2\right)\right)^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/15/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^5*(-24*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+40*sin(1/2*d*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)-40*sin(1/2*d*x+1/2*c)^3+10*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}e^4\cos(dx+c)^4}{a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^4*cos(d*x + c)^4/(a^2*cos(d*x + c)^2 - 2*a
^2*sin(d*x + c) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(a*sin(d*x + c) + a)^2, x)
```

3.245 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=112

$$\frac{10e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{3a^2d}+\frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}+\frac{4e(e\cos(c+dx))^{5/2}}{d\left(a^2\sin(c+dx)+a^2\right)}$$

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[e*Cos[c
 + d*x]]) + (10*e^3*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) + (4*e*(e*
Cos[c + d*x])^(5/2))/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0970945, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2680, 2635, 2642, 2641}

$$\frac{10e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{3a^2d}+\frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}+\frac{4e(e\cos(c+dx))^{5/2}}{d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[e*Cos[c
 + d*x]]) + (10*e^3*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) + (4*e*(e*
Cos[c + d*x])^(5/2))/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^2}\,dx &= \frac{4e(e\cos(c+dx))^{5/2}}{d\left(a^2+a^2\sin(c+dx)\right)}+\frac{\left(5e^2\right)\int (e\cos(c+dx))^{3/2}\,dx}{a^2}\\
&= \frac{10e^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{3a^2d}+\frac{4e(e\cos(c+dx))^{5/2}}{d\left(a^2+a^2\sin(c+dx)\right)}+\frac{\left(5e^4\right)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3a^2}\\
&= \frac{10e^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{3a^2d}+\frac{4e(e\cos(c+dx))^{5/2}}{d\left(a^2+a^2\sin(c+dx)\right)}+\frac{\left(5e^4\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}}{3a^2\sqrt{e\cos(c+dx)}}\\
&= \frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}+\frac{10e^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{3a^2d}+\frac{4e(e\cos(c+dx))^{5/2}}{d\left(a^2+a^2\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [C] time = 0.0765677, size = 66, normalized size = 0.59

$$-\frac{2\sqrt[4]{2}(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9a^2de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[3/4, 9/4, 13/4, (1 - S
in[c + d*x])/2])/(9*a^2*d*e*(1 + Sin[c + d*x])^(9/4))
```

Maple [A] time = 0.434, size = 155, normalized size = 1.4

$$-\frac{2e^4}{3a^2d}\left(-4(\sin(1/2dx+c/2))^4\cos(1/2dx+c/2)+5\sqrt{2(\sin(1/2dx+c/2))^2-1}\sqrt{(\sin(1/2dx+c/2))^2}\mathrm{EllipticF}\left(\cos(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/3/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(-4*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+12*sin(1/2*d*x+1/2*c)^3-6*sin(1/2*d*x+1/2*c)
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos(dx+c)}e^3\cos(dx+c)^3}{a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^3*cos(d*x + c)^3/(a^2*cos(d*x + c)^2 - 2*a
^2*sin(d*x + c) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a)^2, x)
```

3.246 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{6e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{a^2d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{3/2}}{d\left(a^2\sin(c+dx)+a^2\right)}$$

```
[Out] (-6*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(a^2*d*Sqrt[Cos[c +
 d*x]]) - (4*e*(e*Cos[c + d*x])^(3/2))/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0779328, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2680, 2640, 2639}

$$-\frac{6e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{a^2d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{3/2}}{d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-6*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(a^2*d*Sqrt[Cos[c +
 d*x]]) - (4*e*(e*Cos[c + d*x])^(3/2))/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^2}\,dx &= -\frac{4e(e\cos(c+dx))^{3/2}}{d\left(a^2+a^2\sin(c+dx)\right)}-\frac{\left(3e^2\right)\int\sqrt{e\cos(c+dx)}\,dx}{a^2}\\ &= -\frac{4e(e\cos(c+dx))^{3/2}}{d\left(a^2+a^2\sin(c+dx)\right)}-\frac{\left(3e^2\sqrt{e\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{a^2\sqrt{\cos(c+dx)}}\\ &= -\frac{6e^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{3/2}}{d\left(a^2+a^2\sin(c+dx)\right)}\end{aligned}$$

Mathematica [C] time = 0.0938018, size = 66, normalized size = 0.84

$$-\frac{2^{3/4}(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7a^2de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(2^(3/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[5/4, 7/4, 11/4, (1 - Sin
[c + d*x])/2])/(7*a^2*d*e*(1 + Sin[c + d*x])^(7/4))
```

Maple [A] time = 0.717, size = 120, normalized size = 1.5

$$-2\,\frac{\left(3\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-4\,(\sin(1/2\,dx+c/2))^2\cos(\right.}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^2\,e+e}\sin(1/2\,dx+c/2)\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)/a^2*(3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1/2*
c))*e^3/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}e^2\cos(dx+c)^2}{a^2\cos(dx+c)^2-2a^2\sin(dx+c)-2a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^2*cos(d*x + c)^2/(a^2*cos(d*x + c)^2 - 2*a
^2*sin(d*x + c) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^2, x)
```

3.247 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=83

$$-\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}-\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2\sin(c+dx)+a^2\right)}$$

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[e*Cos[c
 + d*x]]) - (4*e*Sqrt[e*Cos[c + d*x]])/(3*d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.074943, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2680, 2642, 2641}

$$-\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}-\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[e*Cos[c
 + d*x]]) - (4*e*Sqrt[e*Cos[c + d*x]])/(3*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^2}\,dx &= -\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2+a^2\sin(c+dx)\right)}-\frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3a^2}\\
&= -\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2+a^2\sin(c+dx)\right)}-\frac{\left(e^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^2\sqrt{e\cos(c+dx)}}\\
&= -\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}-\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2+a^2\sin(c+dx)\right)}
\end{aligned}
$$

Mathematica [C] time = 0.076772, size = 66, normalized size = 0.8

$$
-\frac{\sqrt[4]{2}(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5a^2de(\sin(c+dx)+1)^{5/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(2^(1/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, 7/4, 9/4, (1 - Sin[
c + d*x])/2])/(5*a^2*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [A] time = 1.057, size = 193, normalized size = 2.3

$$
\frac{2\,e^2}{3\,a^2d}\left(2\,\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{(\sin(1/2\,dx+c/2))^2}\,(\sin(1/2\,dx+c/2))^2-\sqrt{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*e+e)^(1/2)*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1/2*c
))*e^2/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos{(dx+c)})^{\frac{3}{2}}}{(a\sin{(dx+c)}+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos{(dx+c)}}e\cos{(dx+c)}}{a^2\cos{(dx+c)}^2-2\,a^2\sin{(dx+c)}-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e*cos(d*x + c)/(a^2*cos(d*x + c)^2 - 2*a^2*s
in(d*x + c) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a)^2, x)
```

3.248 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$-\frac{2(e\cos(c+dx))^{3/2}}{5de\left(a^2\sin(c+dx)+a^2\right)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^2d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{5de(a\sin(c+dx)+a)^2}$$

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d
*x]]) - (2*(e*Cos[c + d*x])^(3/2))/(5*d*e*(a + a*Sin[c + d*x])^2) - (2*(e*C
os[c + d*x])^(3/2))/(5*d*e*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.119396, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2681, 2683, 2640, 2639}

$$-\frac{2(e\cos(c+dx))^{3/2}}{5de\left(a^2\sin(c+dx)+a^2\right)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^2d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{5de(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d
*x]]) - (2*(e*Cos[c + d*x])^(3/2))/(5*d*e*(a + a*Sin[c + d*x])^2) - (2*(e*C
os[c + d*x])^(3/2))/(5*d*e*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
```

```
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}\,dx &= -\frac{2(e\cos(c+dx))^{3/2}}{5de(a+a\sin(c+dx))^2}+\frac{\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx}{5a}\\
&= -\frac{2(e\cos(c+dx))^{3/2}}{5de(a+a\sin(c+dx))^2}-\frac{2(e\cos(c+dx))^{3/2}}{5de\left(a^2+a^2\sin(c+dx)\right)}-\frac{\int \sqrt{e\cos(c+dx)}\,dx}{5a^2}\\
&= -\frac{2(e\cos(c+dx))^{3/2}}{5de(a+a\sin(c+dx))^2}-\frac{2(e\cos(c+dx))^{3/2}}{5de\left(a^2+a^2\sin(c+dx)\right)}-\frac{\sqrt{e\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{5a^2\sqrt{\cos(c+dx)}}\\
&= -\frac{2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{5de(a+a\sin(c+dx))^2}-\frac{2(e\cos(c+dx))^{3/2}}{5de\left(a^2+a^2\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [C] time = 0.0416356, size = 66, normalized size = 0.57

$$-\frac{(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3\sqrt[4]{2}a^2de(\sin(c+dx)+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 9/4, 7/4, (1 - Sin[c + d*x]
)/2])/(3*2^(1/4)*a^2*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [B] time = 1.456, size = 303, normalized size = 2.6

$$-\frac{2e}{5a^2d}\left(4\,\mathrm{EllipticE}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\left(\sin\left(1/2\,dx+c/2\right)\right)^4-8\,\left(\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^2/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(4*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^4-8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-4*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x
+1/2*c))*e/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{e\cos\left(dx+c\right)}}{\left(a\sin\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^2-2\,a^2\sin\left(dx+c\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2
*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e \cos (dx + c)}}{(a \sin (dx + c) + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^2, x)
```

3.249 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$-\frac{2\sqrt{e\cos(c+dx)}}{7de\left(a^2\sin(c+dx)+a^2\right)}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{7de(a\sin(c+dx)+a)^2}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Sqrt[e*Cos[c + d*
x]]) - (2*Sqrt[e*Cos[c + d*x]])/(7*d*e*(a + a*Sin[c + d*x])^2) - (2*Sqrt[e*
Cos[c + d*x]])/(7*d*e*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.127073, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2681, 2683, 2642, 2641}

$$-\frac{2\sqrt{e\cos(c+dx)}}{7de\left(a^2\sin(c+dx)+a^2\right)}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{7de(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Sqrt[e*Cos[c + d*
x]]) - (2*Sqrt[e*Cos[c + d*x]])/(7*d*e*(a + a*Sin[c + d*x])^2) - (2*Sqrt[e*
Cos[c + d*x]])/(7*d*e*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
```

```
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2}\,dx &= -\frac{2\sqrt{e\cos(c+dx)}}{7de(a+a\sin(c+dx))^2}+\frac{3\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))}\,dx}{7a}\\
&= -\frac{2\sqrt{e\cos(c+dx)}}{7de(a+a\sin(c+dx))^2}-\frac{2\sqrt{e\cos(c+dx)}}{7de\left(a^2+a^2\sin(c+dx)\right)}+\frac{\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{7a^2}\\
&= -\frac{2\sqrt{e\cos(c+dx)}}{7de(a+a\sin(c+dx))^2}-\frac{2\sqrt{e\cos(c+dx)}}{7de\left(a^2+a^2\sin(c+dx)\right)}+\frac{\sqrt{\cos(c+dx)}\int \frac{}{\sqrt{\cos}}}{7a^2\sqrt{e\cos(c+}}\\
&= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{7de(a+a\sin(c+dx))^2}-\frac{2\sqrt{e\cos(c+}}{7de\left(a^2+a^2\sin\right.}
\end{aligned}$$

Mathematica [C] time = 0.0448084, size = 64, normalized size = 0.55

$$-\frac{\sqrt{e\cos(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{3/4}a^2de\sqrt[4]{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] -((Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 11/4, 5/4, (1 - Sin[c + d*x]
)/2])/(2^(3/4)*a^2*d*e*(1 + Sin[c + d*x])^(1/4)))
```

Maple [B] time = 1.764, size = 372, normalized size = 3.2

$$-\frac{2}{7\,a^{2}d}\left(8\sqrt{(\sin(1/2\,dx+c/2))^{2}}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2)\,,\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^{2}-1}\,(\sin(1/2\,dx+c/2))^{6}-12\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/7/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(8*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^4+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+6*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1
/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos(dx+c)}}{a^{2}e\cos(dx+c)^{3}-2\,a^{2}e\cos(dx+c)\sin(dx+c)-2\,a^{2}e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(a^2*e*cos(d*x + c)^3 - 2*a^2*e*cos(d*x + c)
*sin(d*x + c) - 2*a^2*e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos\left(dx+c\right)}\left(a\sin\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^2), x)
```

3.250 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3a^2de^2\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}}-\frac{2}{9de\left(a^2\sin(c+dx)+a^2\right)\sqrt{e\cos(c+dx)}}-\frac{}{9de(a\sin(c+dx)}$$

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(3*a^2*d*e^2*Sqrt[Cos[c
 + d*x]]) + (2*Sin[c + d*x])/(3*a^2*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(9*d*e*Sq
rt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2) - 2/(9*d*e*Sqrt[e*Cos[c + d*x]]*
(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.1574, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2681, 2683, 2636, 2640, 2639}

$$-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3a^2de^2\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}}-\frac{2}{9de\left(a^2\sin(c+dx)+a^2\right)\sqrt{e\cos(c+dx)}}-\frac{}{9de(a\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(3*a^2*d*e^2*Sqrt[Cos[c
 + d*x]]) + (2*Sin[c + d*x])/(3*a^2*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(9*d*e*Sq
rt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2) - 2/(9*d*e*Sqrt[e*Cos[c + d*x]]*
(a^2 + a^2*Sin[c + d*x]))
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}\,dx = -\frac{2}{9de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2}+\frac{5\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))}\,dx}{9a}$$

$$= -\frac{2}{9de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2}-\frac{2}{9de\sqrt{e\cos(c+dx)}\left(a^2+a^2\sin(\right.}$$

$$= \frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}}-\frac{2}{9de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2}-\frac{}{9de\sqrt{e\,\mathrm{co}}}$$

$$= \frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}}-\frac{2}{9de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2}-\frac{}{9de\sqrt{e\,\mathrm{co}}}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2de^2\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}}-\frac{}{9de\sqrt{e\cos(c}}$$

Mathematica [C] time = 0.0682538, size = 66, normalized size = 0.44

$$\frac{\sqrt[4]{\sin(c+dx)+1}\,{}_2F_1\left(-\frac{1}{4},\frac{13}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{2\sqrt[4]{2}a^2de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (Hypergeometric2F1[-1/4, 13/4, 3/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(1/4))/(2*2^(1/4)*a^2*d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [B] time = 2.269, size = 488, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/9/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c)
^4-8*sin(1/2*d*x+1/2*c)^2+1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
2*e+e)^(1/2)/e*(48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2
*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1
/2*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^4-152*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+56*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)+2*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a\sin\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}}{a^{2}e^{2}\cos\left(dx+c\right)^{4}-2\,a^{2}e^{2}\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-2\,a^{2}e^{2}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(a^2*e^2*cos(d*x + c)^4 - 2*a^2*e^2*cos(d*x
+ c)^2*sin(d*x + c) - 2*a^2*e^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a\sin\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^2), x)
```

3.251 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^2de^2\sqrt{e\cos(c+dx)}}+\frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}}-\frac{2}{11de\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{3/2}}-\frac{}{11de(a\sin(}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(33*a^2*d*e^2*Sqrt[e*Cos[
c + d*x]]) + (10*Sin[c + d*x])/(33*a^2*d*e*(e*Cos[c + d*x])^(3/2)) - 2/(11*
d*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2) - 2/(11*d*e*(e*Cos[c + d
*x])^(3/2)*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.161927, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2681, 2683, 2636, 2642, 2641}

$$\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^2de^2\sqrt{e\cos(c+dx)}}+\frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}}-\frac{2}{11de\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{3/2}}-\frac{}{11de(a\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(33*a^2*d*e^2*Sqrt[e*Cos[
c + d*x]]) + (10*Sin[c + d*x])/(33*a^2*d*e*(e*Cos[c + d*x])^(3/2)) - 2/(11*
d*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2) - 2/(11*d*e*(e*Cos[c + d
*x])^(3/2)*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}\,dx = -\frac{2}{11de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2} + \frac{7\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))}\,d}{11a}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2} - \frac{2}{11de(e\cos(c+dx))^{3/2}\left(a^2+a^2\right.}$$

$$= \frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}} - \frac{2}{11de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2} - \frac{}{11de}$$

$$= \frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}} - \frac{2}{11de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2} - \frac{}{11de}$$

$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^2de^2\sqrt{e\cos(c+dx)}} + \frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}} - \frac{}{11de(e\cos(c+}$$

Mathematica [C] time = 0.0725248, size = 66, normalized size = 0.44

$$\frac{(\sin(c+dx)+1)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{15}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{6\,2^{3/4}a^2de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (Hypergeometric2F1[-3/4, 15/4, 1/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(3/4))/(6*2^(3/4)*a^2*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [B] time = 2.612, size = 557, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/33/(32*sin(1/2*d*x+1/2*c)^10-80*sin(1/2*d*x+1/2*c)^8+80*sin(1/2*d*x+1/2*
c)^6-40*sin(1/2*d*x+1/2*c)^4+10*sin(1/2*d*x+1/2*c)^2-1)/a^2/sin(1/2*d*x+1/2
*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^2*(160*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^10-400*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8+160*
sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+400*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1
/2*d*x+1/2*c)^6-320*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-200*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+264*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+50*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-104*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)-5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+28*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)-6*sin(1/2*d*x+1/2*c))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}}{a^{2}e^{3}\cos\left(dx+c\right)^{5}-2\,a^{2}e^{3}\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-2\,a^{2}e^{3}\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(a^2*e^3*cos(d*x + c)^5 - 2*a^2*e^3*cos(d*x
+ c)^3*sin(d*x + c) - 2*a^2*e^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sin\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^2), x)
```

3.252 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=181

$$\frac{42\sin(c+dx)}{65a^2de^3\sqrt{e\cos(c+dx)}}-\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{65a^2de^4\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}}-\frac{2}{13de\left(a^2\sin(c+dx)+a^2\right)(e\,\mathrm{co}}$$

```
[Out] (-42*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(65*a^2*d*e^4*Sqrt[Cos
[c + d*x]]) + (14*Sin[c + d*x])/(65*a^2*d*e*(e*Cos[c + d*x])^(5/2)) + (42*S
in[c + d*x])/(65*a^2*d*e^3*Sqrt[e*Cos[c + d*x]]) - 2/(13*d*e*(e*Cos[c + d*x
])^(5/2)*(a + a*Sin[c + d*x])^2) - 2/(13*d*e*(e*Cos[c + d*x])^(5/2)*(a^2 +
a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.183031, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2681, 2683, 2636, 2640, 2639}

$$\frac{42\sin(c+dx)}{65a^2de^3\sqrt{e\cos(c+dx)}}-\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{65a^2de^4\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}}-\frac{2}{13de\left(a^2\sin(c+dx)+a^2\right)(e\,\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (-42*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(65*a^2*d*e^4*Sqrt[Cos
[c + d*x]]) + (14*Sin[c + d*x])/(65*a^2*d*e*(e*Cos[c + d*x])^(5/2)) + (42*S
in[c + d*x])/(65*a^2*d*e^3*Sqrt[e*Cos[c + d*x]]) - 2/(13*d*e*(e*Cos[c + d*x
])^(5/2)*(a + a*Sin[c + d*x])^2) - 2/(13*d*e*(e*Cos[c + d*x])^(5/2)*(a^2 +
a^2*Sin[c + d*x]))
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}\,dx = -\frac{2}{13de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2} + \frac{9\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))} \ldots}{13a}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2} - \frac{2}{13de(e\cos(c+dx))^{5/2}\left(a^2+a \ldots\right.}$$

$$= \frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}} - \frac{2}{13de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2} - \frac{\ldots}{13d \ldots}$$

$$= \frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}} + \frac{42\sin(c+dx)}{65a^2de^3\sqrt{e\cos(c+dx)}} - \frac{2}{13de(e\cos(c+dx))^{5/2} \ldots}$$

$$= \frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}} + \frac{42\sin(c+dx)}{65a^2de^3\sqrt{e\cos(c+dx)}} - \frac{2}{13de(e\cos(c+dx))^{5/2} \ldots}$$

$$= -\frac{42\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^4\sqrt{\cos(c+dx)}} + \frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}} + \frac{42\sin \ldots}{65a^2de^3\sqrt{\ldots}}$$

Mathematica [C] time = 0.105818, size = 66, normalized size = 0.36

$$\frac{(\sin(c+dx)+1)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{17}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{20\sqrt[4]{2}a^2de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (Hypergeometric2F1[-5/4, 17/4, -1/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^(5/4))/(20*2^(1/4)*a^2*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [B] time = 3.316, size = 670, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2/65/(64*sin(1/2*d*x+1/2*c)^12-192*sin(1/2*d*x+1/2*c)^10+240*sin(1/2*d*x+1
/2*c)^8-160*sin(1/2*d*x+1/2*c)^6+60*sin(1/2*d*x+1/2*c)^4-12*sin(1/2*d*x+1/2
*c)^2+1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(13
44*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^12-2688*sin(1/2*d*x+1/2*c)^1
4*cos(1/2*d*x+1/2*c)-4032*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^10+80
64*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+5040*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*s
in(1/2*d*x+1/2*c)^8-10304*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-3360*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+7168*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^8+1260*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-2896*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-252*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2+656*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+21*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-86*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*sin(1/2*d*x+1/2*c))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}}{a^2e^4\cos\left(dx+c\right)^6-2\,a^2e^4\cos\left(dx+c\right)^4\sin\left(dx+c\right)-2\,a^2e^4\cos\left(dx+c\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(a^2*e^4*cos(d*x + c)^6 - 2*a^2*e^4*cos(d*x
+ c)^4*sin(d*x + c) - 2*a^2*e^4*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\left(a\sin\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)^2), x)
```

3.253 $\int \frac{(e\cos(c+dx))^{15/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=169

$$\frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d}+\frac{26e^7\sin(c+dx)\sqrt{e\cos(c+dx)}}{21a^3d}+\frac{26e^5\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^3d}+\frac{26e^8\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{21a^3d\sqrt{e\cos(c+}$$

```
[Out] (26*e^3*(e*Cos[c + d*x])^(9/2))/(45*a^3*d) + (26*e^8*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2])/(21*a^3*d*Sqrt[e*Cos[c + d*x]]) + (26*e^7*Sqrt[e*Co
s[c + d*x]]*Sin[c + d*x])/(21*a^3*d) + (26*e^5*(e*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(35*a^3*d) + (4*e*(e*Cos[c + d*x])^(13/2))/(5*a*d*(a + a*Sin[c + d
*x])^2)
```

Rubi [A] time = 0.179051, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2680, 2682, 2635, 2642, 2641}

$$\frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d}+\frac{26e^7\sin(c+dx)\sqrt{e\cos(c+dx)}}{21a^3d}+\frac{26e^5\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^3d}+\frac{26e^8\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{21a^3d\sqrt{e\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(15/2)/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (26*e^3*(e*Cos[c + d*x])^(9/2))/(45*a^3*d) + (26*e^8*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2])/(21*a^3*d*Sqrt[e*Cos[c + d*x]]) + (26*e^7*Sqrt[e*Co
s[c + d*x]]*Sin[c + d*x])/(21*a^3*d) + (26*e^5*(e*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(35*a^3*d) + (4*e*(e*Cos[c + d*x])^(13/2))/(5*a*d*(a + a*Sin[c + d
*x])^2)
```


Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\cos(c+dx))^{15/2}}{(a+a\sin(c+dx))^3}\,dx &= \frac{4e(e\cos(c+dx))^{13/2}}{5ad(a+a\sin(c+dx))^2} + \frac{\left(13e^2\right)\int \frac{(e\cos(c+dx))^{11/2}}{a+a\sin(c+dx)}\,dx}{5a^2} \\
&= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d} + \frac{4e(e\cos(c+dx))^{13/2}}{5ad(a+a\sin(c+dx))^2} + \frac{\left(13e^4\right)\int (e\cos(c+dx))^{7/2}\,dx}{5a^3} \\
&= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d} + \frac{26e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^3d} + \frac{4e(e\cos(c+dx))^{13/2}}{5ad(a+a\sin(c+dx))^2} + \frac{(13e}{} \\
&= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d} + \frac{26e^7\sqrt{e\cos(c+dx)}\sin(c+dx)}{21a^3d} + \frac{26e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^3d} \\
&= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d} + \frac{26e^7\sqrt{e\cos(c+dx)}\sin(c+dx)}{21a^3d} + \frac{26e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^3d} \\
&= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d} + \frac{26e^8\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}} + \frac{26e^7\sqrt{e\cos(c+dx)}\sin(c+dx}{21a^3d}
\end{aligned}
$$

Mathematica [C] time = 0.369593, size = 66, normalized size = 0.39

$$-\frac{4\sqrt[4]{2}(e\cos(c+dx))^{17/2}\,{}_2F_1\left(-\frac{1}{4},\frac{17}{4};\frac{21}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{17a^3de(\sin(c+dx)+1)^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(15/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*2^(1/4)*(e*Cos[c + d*x])^(17/2)*Hypergeometric2F1[-1/4, 17/4, 21/4, (1
- Sin[c + d*x])/2])/(17*a^3*d*e*(1 + Sin[c + d*x])^(17/4))
```

Maple [A] time = 0.658, size = 251, normalized size = 1.5

$$-\frac{2\,e^8}{315\,a^3d}\left(-1120\,(\sin(1/2\,dx+c/2))^{11}-2160\,\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^8+2800\,(\sin(1/2\,dx+c/2))^9+324\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/315/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^8*(-112
0*sin(1/2*d*x+1/2*c)^11-2160*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+2800*s
in(1/2*d*x+1/2*c)^9+3240*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-784*sin(1/
2*d*x+1/2*c)^7-840*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-1624*sin(1/2*d*x
+1/2*c)^5+195*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-120*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+1162*sin(1/2*d*x+1/2*c)^3-217*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(e\cos(dx+c))^{\frac{15}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(15/2)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}e^{7}\cos\left(dx+c\right)^{7}}{3\,a^{3}\cos\left(dx+c\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(dx+c\right)^{2}-4\,a^{3}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^7*cos(d*x + c)^7/(3*a^3*cos(d*x + c)^2 - 4
*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(15/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.254 $\int \frac{(e\cos(c+dx))^{13/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=138

$$\frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d}+\frac{22e^5\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^3d}+\frac{22e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^3d\sqrt{\cos(c+dx)}}+\frac{4e(e\cos(c+dx))^{1}}{3ad(a\sin(c+dx)+}$$

```
[Out] (22*e^3*(e*Cos[c + d*x])^(7/2))/(21*a^3*d) + (22*e^6*Sqrt[e*Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2])/(5*a^3*d*Sqrt[Cos[c + d*x]]) + (22*e^5*(e*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(15*a^3*d) + (4*e*(e*Cos[c + d*x])^(11/2))/(3*a*
d*(a + a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.152068, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2680, 2682, 2635, 2640, 2639}

$$\frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d}+\frac{22e^5\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^3d}+\frac{22e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^3d\sqrt{\cos(c+dx)}}+\frac{4e(e\cos(c+dx))^{1}}{3ad(a\sin(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(13/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (22*e^3*(e*Cos[c + d*x])^(7/2))/(21*a^3*d) + (22*e^6*Sqrt[e*Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2])/(5*a^3*d*Sqrt[Cos[c + d*x]]) + (22*e^5*(e*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(15*a^3*d) + (4*e*(e*Cos[c + d*x])^(11/2))/(3*a*
d*(a + a*Sin[c + d*x])^2)
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{13/2}}{(a+a\sin(c+dx))^3}\,dx &= \frac{4e(e\cos(c+dx))^{11/2}}{3ad(a+a\sin(c+dx))^2}+\frac{\left(11e^2\right)\int \frac{(e\cos(c+dx))^{9/2}}{a+a\sin(c+dx)}\,dx}{3a^2}\\ &= \frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d}+\frac{4e(e\cos(c+dx))^{11/2}}{3ad(a+a\sin(c+dx))^2}+\frac{\left(11e^4\right)\int (e\cos(c+dx))^{5/2}\,dx}{3a^3}\\ &= \frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d}+\frac{22e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^3d}+\frac{4e(e\cos(c+dx))^{11/2}}{3ad(a+a\sin(c+dx))^2}+\frac{\left(11e^{6}\right.}{}\\ &= \frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d}+\frac{22e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^3d}+\frac{4e(e\cos(c+dx))^{11/2}}{3ad(a+a\sin(c+dx))^2}+\frac{\left(11e^{6}\right.}{}\\ &= \frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d}+\frac{22e^6\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}}+\frac{22e^5(e\cos(c+dx))^{3/2}\sin(c+}{15a^3d}\end{aligned}$$

Mathematica [C] time = 0.228969, size = 66, normalized size = 0.48

$$-\frac{2\,2^{3/4}(e\cos(c+dx))^{15/2}\,{}_2F_1\left(\frac{1}{4},\frac{15}{4};\frac{19}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{15a^3de(\sin(c+dx)+1)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(13/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(15/2)*Hypergeometric2F1[1/4, 15/4, 19/4, (1 -
 Sin[c + d*x])/2])/(15*a^3*d*e*(1 + Sin[c + d*x])^(15/4))
```

Maple [A] time = 0.662, size = 216, normalized size = 1.6

$$\frac{2\,e^7}{105\,a^3d}\left(-240\,\left(\sin\left(1/2\,dx+c/2\right)\right)^9-504\,\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)+480\,\left(\sin\left(1/2\,dx+c/2\right)\right)^7+504\,(\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/105/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^7*(-240*
sin(1/2*d*x+1/2*c)^9-504*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+480*sin(1/
2*d*x+1/2*c)^7+504*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+200*sin(1/2*d*x+
1/2*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-126*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)-440*sin(1/2*d*x+1/2*c)^3+125*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{13}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(13/2)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}e^{6}\cos\left(dx+c\right)^{6}}{3\,a^{3}\cos\left(dx+c\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(dx+c\right)^{2}-4\,a^{3}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^6*cos(d*x + c)^6/(3*a^3*cos(d*x + c)^2 - 4
*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(13/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.255 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=132

$$\frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d}+\frac{6e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{a^3d}+\frac{6e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{e\cos(c+dx)}}+\frac{4e(e\cos(c+dx))^{9/2}}{ad(a\sin(c+dx)+a)^2}$$

```
[Out] (18*e^3*(e*Cos[c + d*x])^(5/2))/(5*a^3*d) + (6*e^6*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2])/(a^3*d*Sqrt[e*Cos[c + d*x]]) + (6*e^5*Sqrt[e*Cos[c +
d*x]]*Sin[c + d*x])/(a^3*d) + (4*e*(e*Cos[c + d*x])^(9/2))/(a*d*(a + a*Sin[
c + d*x])^2)
```

Rubi [A] time = 0.150978, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2680, 2682, 2635, 2642, 2641}

$$\frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d}+\frac{6e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{a^3d}+\frac{6e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{e\cos(c+dx)}}+\frac{4e(e\cos(c+dx))^{9/2}}{ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (18*e^3*(e*Cos[c + d*x])^(5/2))/(5*a^3*d) + (6*e^6*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2])/(a^3*d*Sqrt[e*Cos[c + d*x]]) + (6*e^5*Sqrt[e*Cos[c +
d*x]]*Sin[c + d*x])/(a^3*d) + (4*e*(e*Cos[c + d*x])^(9/2))/(a*d*(a + a*Sin[
c + d*x])^2)
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^3}\,dx &= \frac{4e(e\cos(c+dx))^{9/2}}{ad(a+a\sin(c+dx))^2}+\frac{\left(9e^2\right)\int \frac{(e\cos(c+dx))^{7/2}}{a+a\sin(c+dx)}\,dx}{a^2}\\
&= \frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d}+\frac{4e(e\cos(c+dx))^{9/2}}{ad(a+a\sin(c+dx))^2}+\frac{\left(9e^4\right)\int (e\cos(c+dx))^{3/2}\,dx}{a^3}\\
&= \frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d}+\frac{6e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{a^3d}+\frac{4e(e\cos(c+dx))^{9/2}}{ad(a+a\sin(c+dx))^2}+\frac{\left(3e^6\right)\int \frac{}{\sqrt{}}}{}\\
&= \frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d}+\frac{6e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{a^3d}+\frac{4e(e\cos(c+dx))^{9/2}}{ad(a+a\sin(c+dx))^2}+\frac{\left(3e^6\sqrt{\cos}\right.}{a}\\
&= \frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d}+\frac{6e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{e\cos(c+dx)}}+\frac{6e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{a^3d}+
\end{aligned}$$

Mathematica [C] time = 0.17769, size = 66, normalized size = 0.5

$$-\frac{2\sqrt[4]{2}(e\cos(c+dx))^{13/2}\,{}_2F_1\left(\frac{3}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{13a^3de(\sin(c+dx)+1)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(13/2)*Hypergeometric2F1[3/4, 13/4, 17/4, (1 -
 Sin[c + d*x])/2])/(13*a^3*d*e*(1 + Sin[c + d*x])^(13/4))
```

Maple [A] time = 0.763, size = 181, normalized size = 1.4

$$-\frac{2\,e^6}{5\,a^3d}\left(-8\,\left(\sin\left(1/2\,dx+c/2\right)\right)^7-20\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4\cos\left(1/2\,dx+c/2\right)+12\,\left(\sin\left(1/2\,dx+c/2\right)\right)^5+15\,\sqrt{2\,\left(\sin(1/2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/5/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^6*(-8*sin
(1/2*d*x+1/2*c)^7-20*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*sin(1/2*d*x
+1/2*c)^5+15*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)+34*sin(1/2*d*x+1/2*c)^3-19*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(e\cos(dx+c))^{\frac{11}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(11/2)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}e^5\cos(dx+c)^5}{3a^3\cos(dx+c)^2-4a^3+\left(a^3\cos(dx+c)^2-4a^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^5*cos(d*x + c)^5/(3*a^3*cos(d*x + c)^2 - 4
*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.256 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d}-\frac{14e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{a^3d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{7/2}}{ad(a\sin(c+dx)+a)^2}$$

```
[Out] (-14*e^3*(e*Cos[c + d*x])^(3/2))/(3*a^3*d) - (14*e^4*Sqrt[e*Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2])/(a^3*d*Sqrt[Cos[c + d*x]]) - (4*e*(e*Cos[c + d*x]
)^(7/2))/(a*d*(a + a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.154847, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2680, 2682, 2640, 2639}

$$-\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d}-\frac{14e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{a^3d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{7/2}}{ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-14*e^3*(e*Cos[c + d*x])^(3/2))/(3*a^3*d) - (14*e^4*Sqrt[e*Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2])/(a^3*d*Sqrt[Cos[c + d*x]]) - (4*e*(e*Cos[c + d*x]
)^(7/2))/(a*d*(a + a*Sin[c + d*x])^2)
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
```

```
&& EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^3}\,dx &= -\frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^2}-\frac{\left(7e^2\right)\int \frac{(e\cos(c+dx))^{5/2}}{a+a\sin(c+dx)}\,dx}{a^2}\\
&= -\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d}-\frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^2}-\frac{\left(7e^4\right)\int \sqrt{e\cos(c+dx)}\,dx}{a^3}\\
&= -\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d}-\frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^2}-\frac{\left(7e^4\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{a^3\sqrt{\cos(c+dx)}}\\
&= -\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d}-\frac{14e^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^2}
\end{aligned}
$$

Mathematica [C] time = 0.0975292, size = 66, normalized size = 0.64

$$
-\frac{2^{3/4}(e\cos(c+dx))^{11/2}\,{}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11a^3de(\sin(c+dx)+1)^{11/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(2^(3/4)*(e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[5/4, 11/4, 15/4, (1 - S
in[c + d*x])/2])/(11*a^3*d*e*(1 + Sin[c + d*x])^(11/4))
```

Maple [A] time = 0.987, size = 146, normalized size = 1.4

$$-\frac{2e^5}{3a^3d}\left(4\left(\sin\left(1/2\,dx+c/2\right)\right)^5+21\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\mathrm{EllipticE}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/3/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)/a^3*(4*sin(1/2*
d*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)-4*sin(1/2*d*x+1/2*c)^3+13*sin(1/2*d*x+1/2*c))*e^5/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e\cos\left(dx+c\right)\right)^{\frac{9}{2}}}{\left(a\sin\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}e^4\cos\left(dx+c\right)^4}{3\,a^3\cos\left(dx+c\right)^2-4\,a^3+\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^4*cos(d*x + c)^4/(3*a^3*cos(d*x + c)^2 - 4
*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(a*sin(d*x + c) + a)^3, x)
```

3.257 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=107

$$-\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d}-\frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d\sqrt{e\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{5/2}}{3ad(a\sin(c+dx)+a)^2}$$

```
[Out] (-10*e^3*Sqrt[e*Cos[c + d*x]])/(3*a^3*d) - (10*e^4*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2])/(3*a^3*d*Sqrt[e*Cos[c + d*x]]) - (4*e*(e*Cos[c + d*x]
)^(5/2))/(3*a*d*(a + a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.136039, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2680, 2682, 2642, 2641}

$$-\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d}-\frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d\sqrt{e\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{5/2}}{3ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-10*e^3*Sqrt[e*Cos[c + d*x]])/(3*a^3*d) - (10*e^4*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2])/(3*a^3*d*Sqrt[e*Cos[c + d*x]]) - (4*e*(e*Cos[c + d*x]
)^(5/2))/(3*a*d*(a + a*Sin[c + d*x])^2)
```


Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
```

```
&& EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^3}\,dx &= -\frac{4e(e\cos(c+dx))^{5/2}}{3ad(a+a\sin(c+dx))^2}-\frac{\left(5e^2\right)\int \frac{(e\cos(c+dx))^{3/2}}{a+a\sin(c+dx)}\,dx}{3a^2}\\ &= -\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d}-\frac{4e(e\cos(c+dx))^{5/2}}{3ad(a+a\sin(c+dx))^2}-\frac{\left(5e^4\right)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3a^3}\\ &= -\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d}-\frac{4e(e\cos(c+dx))^{5/2}}{3ad(a+a\sin(c+dx))^2}-\frac{\left(5e^4\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^3\sqrt{e\cos(c+dx)}}\\ &= -\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d}-\frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d\sqrt{e\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{5/2}}{3ad(a+a\sin(c+dx))^2}\end{aligned}$$

Mathematica [C] time = 0.0799999, size = 66, normalized size = 0.62

$$-\frac{\sqrt[4]{2}(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9a^3de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(2^(1/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[7/4, 9/4, 13/4, (1 - Sin
[c + d*x])/2])/(9*a^3*d*e*(1 + Sin[c + d*x])^(9/4))
```

Maple [A] time = 1.101, size = 219, normalized size = 2.1

$$\frac{2e^4}{3a^3d}\left(10\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\text{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\left(\sin\left(1/2\,dx+c/2\right)\right)^2+12\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*e+e)^(1/2)*(10*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+12*sin(1/2
*d*x+1/2*c)^5-5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)-12*sin(1/2*d*x+1/2*c)^3+7*sin(1/2*d*x+1/2*c))*e^4/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}{\left(a\sin\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}e^3\cos\left(dx+c\right)^3}{3\,a^3\cos\left(dx+c\right)^2-4\,a^3+\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^3*cos(d*x + c)^3/(3*a^3*cos(d*x + c)^2 - 4
*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a)^3, x)
```

3.258 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=118

$$\frac{6e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^3d\sqrt{\cos(c+dx)}}+\frac{6e(e\cos(c+dx))^{3/2}}{5d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a\sin(c+dx)+a)^2}$$

```
[Out] (6*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^3*d*Sqrt[Cos[c
+ d*x]]) - (4*e*(e*Cos[c + d*x])^(3/2))/(5*a*d*(a + a*Sin[c + d*x])^2) + (6
*e*(e*Cos[c + d*x])^(3/2))/(5*d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.130675, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2680, 2683, 2640, 2639}

$$\frac{6e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^3d\sqrt{\cos(c+dx)}}+\frac{6e(e\cos(c+dx))^{3/2}}{5d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^3*d*Sqrt[Cos[c
+ d*x]]) - (4*e*(e*Cos[c + d*x])^(3/2))/(5*a*d*(a + a*Sin[c + d*x])^2) + (6
*e*(e*Cos[c + d*x])^(3/2))/(5*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
```

```
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^3}\,dx &= -\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a+a\sin(c+dx))^2}-\frac{\left(3e^2\right)\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx}{5a^2}\\
&= -\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a+a\sin(c+dx))^2}+\frac{6e(e\cos(c+dx))^{3/2}}{5d\left(a^3+a^3\sin(c+dx)\right)}+\frac{\left(3e^2\right)\int \sqrt{e\cos(c+dx)}\,dx}{5a^3}\\
&= -\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a+a\sin(c+dx))^2}+\frac{6e(e\cos(c+dx))^{3/2}}{5d\left(a^3+a^3\sin(c+dx)\right)}+\frac{\left(3e^2\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,d}{5a^3\sqrt{\cos(c+dx)}}\\
&= \frac{6e^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a+a\sin(c+dx))^2}+\frac{6e(e\cos(c+dx))^{3/2}}{5d\left(a^3+a^3\sin(c+dx)\right)}
\end{aligned}
$$

Mathematica [C] time = 0.0840857, size = 66, normalized size = 0.56

$$
-\frac{(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7\sqrt[4]{2}a^3de(\sin(c+dx)+1)^{7/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, 9/4, 11/4, (1 - Sin[c + d*x
])/2])/(7*2^(1/4)*a^3*d*e*(1 + Sin[c + d*x])^(7/4))
```

Maple [B] time = 1.941, size = 330, normalized size = 2.8

$$\frac{2e^3}{5a^3d}\left(12\,\mathrm{EllipticE}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\left(\sin\left(1/2\,dx+c/2\right)\right)^4-24$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^3/sin(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+20
*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)-20*sin(1/2*d*x+1/2*c)^3+sin(1/2*d*x+1/2*c))*e^3/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\left(a\sin\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}e^2\cos\left(dx+c\right)^2}{3\,a^3\cos\left(dx+c\right)^2-4\,a^3+\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e^2*cos(d*x + c)^2/(3*a^3*cos(d*x + c)^2 - 4
*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^3, x)
```

3.259 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=118

$$-\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a\sin(c+dx)+a)^2}$$

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*a^3*d*Sqrt[e*Cos[
c + d*x]]) - (4*e*Sqrt[e*Cos[c + d*x]])/(7*a*d*(a + a*Sin[c + d*x])^2) + (2
*e*Sqrt[e*Cos[c + d*x]])/(21*d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.133818, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2680, 2683, 2642, 2641}

$$-\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*a^3*d*Sqrt[e*Cos[
c + d*x]]) - (4*e*Sqrt[e*Cos[c + d*x]])/(7*a*d*(a + a*Sin[c + d*x])^2) + (2
*e*Sqrt[e*Cos[c + d*x]])/(21*d*(a^3 + a^3*Sin[c + d*x]))
```


Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
```

```
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^3}\,dx &= -\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a+a\sin(c+dx))^2} - \frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))}\,dx}{7a^2} \\
&= -\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a+a\sin(c+dx))^2} + \frac{2e\sqrt{e\cos(c+dx)}}{21d\left(a^3+a^3\sin(c+dx)\right)} - \frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{21a^3} \\
&= -\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a+a\sin(c+dx))^2} + \frac{2e\sqrt{e\cos(c+dx)}}{21d\left(a^3+a^3\sin(c+dx)\right)} - \frac{\left(e^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21a^3\sqrt{e\cos(c+dx)}} \\
&= -\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}} - \frac{4e\sqrt{e\cos(c+dx)}}{7ad(a+a\sin(c+dx))^2} + \frac{2e\sqrt{e\cos(c+dx)}}{21d\left(a^3+a^3\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [C] time = 0.0659085, size = 66, normalized size = 0.56

$$-\frac{(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5\ 2^{3/4}a^3de(\sin(c+dx)+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, 11/4, 9/4, (1 - Sin[c + d*x
])/2])/(5*2^(3/4)*a^3*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [B] time = 1.905, size = 401, normalized size = 3.4

$$\frac{2e^2}{21a^3d}\left(8\sqrt{(\sin(1/2dx+c/2))^2}\mathrm{EllipticF}\left(\cos(1/2dx+c/2),\sqrt{2}\right)\sqrt{2(\sin(1/2dx+c/2))^2-1}(\sin(1/2dx+c/2))^6-12\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(8*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^4+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+6*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-28*sin(
1/2*d*x+1/2*c)^5-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-22*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)+28*sin(1/2*d*x+1/2*c)^3+5*sin(1/2*d*x+1/2*c))*e^2/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{e\cos(dx+c)}e\cos(dx+c)}{3a^3\cos(dx+c)^2-4a^3+\left(a^3\cos(dx+c)^2-4a^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))*e*cos(d*x + c)/(3*a^3*cos(d*x + c)^2 - 4*a^3
 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a)^3, x)
```

3.260 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=153

$$-\frac{2(e\cos(c+dx))^{3/2}}{15de\left(a^3\sin(c+dx)+a^3\right)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15a^3d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{15ade(a\sin(c+dx)+a)^2}-\frac{2(e\cos(c+dx))^{3/2}}{9de(a\sin(c+dx)+a)^3}$$

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*a^3*d*Sqrt[Cos[c +
d*x]]) - (2*(e*Cos[c + d*x])^(3/2))/(9*d*e*(a + a*Sin[c + d*x])^3) - (2*(e*
Cos[c + d*x])^(3/2))/(15*a*d*e*(a + a*Sin[c + d*x])^2) - (2*(e*Cos[c + d*x]
)^(3/2))/(15*d*e*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.173375, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2681, 2683, 2640, 2639}

$$-\frac{2(e\cos(c+dx))^{3/2}}{15de\left(a^3\sin(c+dx)+a^3\right)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15a^3d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{15ade(a\sin(c+dx)+a)^2}-\frac{2(e\cos(c+dx))^{3/2}}{9de(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*a^3*d*Sqrt[Cos[c +
d*x]]) - (2*(e*Cos[c + d*x])^(3/2))/(9*d*e*(a + a*Sin[c + d*x])^3) - (2*(e*
Cos[c + d*x])^(3/2))/(15*a*d*e*(a + a*Sin[c + d*x])^2) - (2*(e*Cos[c + d*x]
)^(3/2))/(15*d*e*(a^3 + a^3*Sin[c + d*x]))
```


Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^3}\,dx &= -\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3}+\frac{\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}\,dx}{3a}\\ &= -\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{15ade(a+a\sin(c+dx))^2}+\frac{\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx}{15a^2}\\ &= -\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{15ade(a+a\sin(c+dx))^2}-\frac{2(e\cos(c+dx))^{3/2}}{15de\left(a^3+a^3\sin(c+dx)\right)}-\frac{\int \sqrt{e\cos}\\ &= -\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{15ade(a+a\sin(c+dx))^2}-\frac{2(e\cos(c+dx))^{3/2}}{15de\left(a^3+a^3\sin(c+dx)\right)}-\frac{\sqrt{e\cos}\\ &= -\frac{2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{15ade(a+a\sin(c+dx))^2}-\frac{}{1}\end{aligned}$$

Mathematica [C] time = 0.0426006, size = 66, normalized size = 0.43

$$-\frac{(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{13}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{6\sqrt[4]{2}a^3de(\sin(c+dx)+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 13/4, 7/4, (1 - Sin[c + d*x
])/2])/(6*2^(1/4)*a^3*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [B] time = 2.585, size = 512, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c
)^4-8*sin(1/2*d*x+1/2*c)^2+1)/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)
^2*e+e)^(1/2)*(48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/
2*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*sin(1/2*d*x+1/2*c)^4-152*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)-36*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)+36*sin(1/2*d*x+1/2*c)^3+11*sin(1/2*d*x+1/2*c)
)*e/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{\left(a\sin\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}}{3\,a^{3}\cos\left(dx+c\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(dx+c\right)^{2}-4\,a^{3}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x
 + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{\left(a\sin\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^3, x)
```

3.261 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=153

$$-\frac{10\sqrt{e\cos(c+dx)}}{77de\left(a^3\sin(c+dx)+a^3\right)}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^3d\sqrt{e\cos(c+dx)}}-\frac{10\sqrt{e\cos(c+dx)}}{77ade(a\sin(c+dx)+a)^2}-\frac{2\sqrt{e\cos(c+dx)}}{11de(a\sin(c+dx)+a)^3}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*a^3*d*Sqrt[e*Cos[c +
d*x]]) - (2*Sqrt[e*Cos[c + d*x]])/(11*d*e*(a + a*Sin[c + d*x])^3) - (10*Sqr
t[e*Cos[c + d*x]])/(77*a*d*e*(a + a*Sin[c + d*x])^2) - (10*Sqrt[e*Cos[c + d
*x]])/(77*d*e*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.184146, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2681, 2683, 2642, 2641}

$$-\frac{10\sqrt{e\cos(c+dx)}}{77de\left(a^3\sin(c+dx)+a^3\right)}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^3d\sqrt{e\cos(c+dx)}}-\frac{10\sqrt{e\cos(c+dx)}}{77ade(a\sin(c+dx)+a)^2}-\frac{2\sqrt{e\cos(c+dx)}}{11de(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*a^3*d*Sqrt[e*Cos[c +
d*x]]) - (2*Sqrt[e*Cos[c + d*x]])/(11*d*e*(a + a*Sin[c + d*x])^3) - (10*Sqr
t[e*Cos[c + d*x]])/(77*a*d*e*(a + a*Sin[c + d*x])^2) - (10*Sqrt[e*Cos[c + d
*x]])/(77*d*e*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3}\,dx &= -\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3}+\frac{5\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2}\,dx}{11a}\\ &= -\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3}-\frac{10\sqrt{e\cos(c+dx)}}{77ade(a+a\sin(c+dx))^2}+\frac{15\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\,\mathrm{s}}}{77a^2}\\ &= -\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3}-\frac{10\sqrt{e\cos(c+dx)}}{77ade(a+a\sin(c+dx))^2}-\frac{10\sqrt{e\cos(c+dx}}{77de\left(a^3+a^3\sin(c+\right.}\\ &= -\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3}-\frac{10\sqrt{e\cos(c+dx)}}{77ade(a+a\sin(c+dx))^2}-\frac{10\sqrt{e\cos(c+dx}}{77de\left(a^3+a^3\sin(c+\right.}\\ &= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^3d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3}-\frac{10\sqrt{e\cos(c}}{77ade(a+a\sin}\end{aligned}$$

Mathematica [C] time = 0.0491033, size = 66, normalized size = 0.43

$$-\frac{\sqrt{e\cos(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{15}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{2\,2^{3/4}a^3de\sqrt[4]{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3),x]
```

```
[Out] -(Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 15/4, 5/4, (1 - Sin[c + d*x])
/2])/(2*2^(3/4)*a^3*d*e*(1 + Sin[c + d*x])^(1/4))
```

Maple [B] time = 3.264, size = 580, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/77/(32*sin(1/2*d*x+1/2*c)^10-80*sin(1/2*d*x+1/2*c)^8+80*sin(1/2*d*x+1/2*
c)^6-40*sin(1/2*d*x+1/2*c)^4+10*sin(1/2*d*x+1/2*c)^2-1)/a^3/sin(1/2*d*x+1/2
*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(160*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1
/2*d*x+1/2*c)^10-400*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8+160*sin(
1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+400*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d
*x+1/2*c)^6-320*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-200*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+264*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+
50*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-104*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+44*sin(1/2*d*x+1/2*c)^5-5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+72*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-44*sin(1/2*d*x+1/2*c)^3-17*sin(1/2*d*x
+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e \cos(dx + c)}(a \sin(dx + c) + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{e\cos(dx+c)}}{3\,a^3e\cos(dx+c)^3-4\,a^3e\cos(dx+c)+\left(a^3e\cos(dx+c)^3-4\,a^3e\cos(dx+c)\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(3*a^3*e*cos(d*x + c)^3 - 4*a^3*e*cos(d*x +
c) + (a^3*e*cos(d*x + c)^3 - 4*a^3*e*cos(d*x + c))*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^3), x)
```

3.262 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=187

$$-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{39a^3de^2\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}}-\frac{14}{117de\left(a^3\sin(c+dx)+a^3\right)\sqrt{e\cos(c+dx)}}-\frac{}{117ade(a\,\mathrm{si}}$$

```
[Out] (-14*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(39*a^3*d*e^2*Sqrt[Cos
[c + d*x]]) + (14*Sin[c + d*x])/(39*a^3*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(13*d
*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3) - 14/(117*a*d*e*Sqrt[e*Cos[
c + d*x]]*(a + a*Sin[c + d*x])^2) - 14/(117*d*e*Sqrt[e*Cos[c + d*x]]*(a^3 +
 a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.224771, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2681, 2683, 2636, 2640, 2639}

$$-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{39a^3de^2\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}}-\frac{14}{117de\left(a^3\sin(c+dx)+a^3\right)\sqrt{e\cos(c+dx)}}-\frac{}{117ade(a\,\mathrm{si}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^3),x]
```


```
[Out] (-14*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(39*a^3*d*e^2*Sqrt[Cos
[c + d*x]]) + (14*Sin[c + d*x])/(39*a^3*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(13*d
*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3) - 14/(117*a*d*e*Sqrt[e*Cos[
c + d*x]]*(a + a*Sin[c + d*x])^2) - 14/(117*d*e*Sqrt[e*Cos[c + d*x]]*(a^3 +
 a^3*Sin[c + d*x]))
```


Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}\,dx = -\frac{2}{13de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3} + \frac{7\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}}{13a}$$

$$= -\frac{2}{13de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3} - \frac{14}{117ade\sqrt{e\cos(c+dx)}(a+a\,\mathrm{si}}$$

$$= -\frac{2}{13de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3} - \frac{14}{117ade\sqrt{e\cos(c+dx)}(a+a\,\mathrm{si}}$$

$$= \frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}} - \frac{2}{13de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3} - \frac{}{117ade}$$

$$= \frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}} - \frac{2}{13de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3} - \frac{}{117ade}$$

$$= -\frac{14\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3de^2\sqrt{\cos(c+dx)}} + \frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}} - \frac{}{13de\sqrt{e\,\mathrm{co}}}$$

Mathematica [C] time = 0.061958, size = 66, normalized size = 0.35

$$\frac{\sqrt[4]{\sin(c+dx)+1}\,{}_2F_1\left(-\frac{1}{4},\frac{17}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{4\sqrt[4]{2}a^3de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^3),x]
```

```
[Out] (Hypergeometric2F1[-1/4, 17/4, 3/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(1/4))/(4*2^(1/4)*a^3*d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [B] time = 4.014, size = 696, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/117/(64*sin(1/2*d*x+1/2*c)^12-192*sin(1/2*d*x+1/2*c)^10+240*sin(1/2*d*x+
1/2*c)^8-160*sin(1/2*d*x+1/2*c)^6+60*sin(1/2*d*x+1/2*c)^4-12*sin(1/2*d*x+1/
2*c)^2+1)/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e*(134
4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^12-2688*sin(1/2*d*x+1/2*c)^14
*cos(1/2*d*x+1/2*c)-4032*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^10+806
4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+5040*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*si
n(1/2*d*x+1/2*c)^8-10304*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-3360*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+7168*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^8+1260*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-2896*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-252*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^2+656*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-52*sin(1/2*d*x+1/2*c)^5
+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-138*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+52
*sin(1/2*d*x+1/2*c)^3+23*sin(1/2*d*x+1/2*c))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{e\cos\left(dx+c\right)}}{3\,a^{3}e^{2}\cos\left(dx+c\right)^{4}-4\,a^{3}e^{2}\cos\left(dx+c\right)^{2}+\left(a^{3}e^{2}\cos\left(dx+c\right)^{4}-4\,a^{3}e^{2}\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*cos(d*x + c))/(3*a^3*e^2*cos(d*x + c)^4 - 4*a^3*e^2*cos(d*
x + c)^2 + (a^3*e^2*cos(d*x + c)^4 - 4*a^3*e^2*cos(d*x + c)^2)*sin(d*x + c)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^3), x)
```

3.263 $\int \frac{(e\cos(c+dx))^{15/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=180

$$\frac{78e^7\sin(c+dx)\sqrt{e\cos(c+dx)}}{7a^4d}+\frac{234e^5\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^4d}+\frac{52e^3(e\cos(c+dx))^{9/2}}{5d\left(a^4\sin(c+dx)+a^4\right)}+\frac{78e^8\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.}{7a^4d\sqrt{e\cos(c+}}$$

```
[Out] (78*e^8*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^4*d*Sqrt[e*Cos[c
 + d*x]]) + (78*e^7*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(7*a^4*d) + (234*e^5
*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(35*a^4*d) + (4*e*(e*Cos[c + d*x])^(1
3/2))/(a*d*(a + a*Sin[c + d*x])^3) + (52*e^3*(e*Cos[c + d*x])^(9/2))/(5*d*(
a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.175337, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2680, 2635, 2642, 2641}

$$\frac{78e^7\sin(c+dx)\sqrt{e\cos(c+dx)}}{7a^4d}+\frac{234e^5\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^4d}+\frac{52e^3(e\cos(c+dx))^{9/2}}{5d\left(a^4\sin(c+dx)+a^4\right)}+\frac{78e^8\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.}{7a^4d\sqrt{e\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(15/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (78*e^8*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^4*d*Sqrt[e*Cos[c
 + d*x]]) + (78*e^7*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(7*a^4*d) + (234*e^5
*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(35*a^4*d) + (4*e*(e*Cos[c + d*x])^(1
3/2))/(a*d*(a + a*Sin[c + d*x])^3) + (52*e^3*(e*Cos[c + d*x])^(9/2))/(5*d*(
a^4 + a^4*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\cos(c+dx))^{15/2}}{(a+a\sin(c+dx))^4}\,dx &= \frac{4e(e\cos(c+dx))^{13/2}}{ad(a+a\sin(c+dx))^3} + \frac{\left(13e^2\right)\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^2}\,dx}{a^2}\\
&= \frac{4e(e\cos(c+dx))^{13/2}}{ad(a+a\sin(c+dx))^3} + \frac{52e^3(e\cos(c+dx))^{9/2}}{5d\left(a^4+a^4\sin(c+dx)\right)} + \frac{\left(117e^4\right)\int (e\cos(c+dx))^{7/2}\,dx}{5a^4}\\
&= \frac{234e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^4d} + \frac{4e(e\cos(c+dx))^{13/2}}{ad(a+a\sin(c+dx))^3} + \frac{52e^3(e\cos(c+dx))^{9/2}}{5d\left(a^4+a^4\sin(c+dx)\right)} + \cdots\\
&= \frac{78e^7\sqrt{e\cos(c+dx)}\sin(c+dx)}{7a^4d} + \frac{234e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^4d} + \frac{4e(e\cos(c+dx))^{1\cdots}}{ad(a+a\sin(c+d\cdots}\\
&= \frac{78e^7\sqrt{e\cos(c+dx)}\sin(c+dx)}{7a^4d} + \frac{234e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^4d} + \frac{4e(e\cos(c+dx))^{1\cdots}}{ad(a+a\sin(c+d\cdots}\\
&= \frac{78e^8\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^4d\sqrt{e\cos(c+dx)}} + \frac{78e^7\sqrt{e\cos(c+dx)}\sin(c+dx)}{7a^4d} + \frac{234e^5(e\cos(c+dx\cdots}{35a^4\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.38013, size = 66, normalized size = 0.37

$$
-\frac{2\sqrt[4]{2}(e\cos(c+dx))^{17/2}\,{}_2F_1\left(\frac{3}{4},\frac{17}{4};\frac{21}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{17a^4de(\sin(c+dx)+1)^{17/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(15/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(17/2)*Hypergeometric2F1[3/4, 17/4, 21/4, (1 -
 Sin[c + d*x])/2])/(17*a^4*d*e*(1 + Sin[c + d*x])^(17/4))
```

Maple [A] time = 0.931, size = 225, normalized size = 1.3

$$-\frac{2e^8}{35a^4d}\Bigl(80\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^8-120(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)-224(\sin(1/2\,dx+c/2))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -2/35/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^8*(80*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-120*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)-224*sin(1/2*d*x+1/2*c)^7-280*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+336*sin(1/2*d*x+1/2*c)^5+195*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+160*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+392*sin(1/2*d*x+1/2*c)^3-252*sin(1/2*d*x+1/2*c))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}e^7\cos(dx+c)^7}{a^4\cos(dx+c)^4-8a^4\cos(dx+c)^2+8a^4-4\left(a^4\cos(dx+c)^2-2a^4\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^7*cos(d*x + c)^7/(a^4*cos(d*x + c)^4 - 8*a^
4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(15/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.264 $\int \frac{(e\cos(c+dx))^{13/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=149

$$-\frac{154e^5\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^4d}-\frac{44e^3(e\cos(c+dx))^{7/2}}{3d\left(a^4\sin(c+dx)+a^4\right)}-\frac{154e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^4d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+d}{ad(a\sin(c+dx}$$

```
[Out] (-154*e^6*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos
[c + d*x]]) - (154*e^5*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*a^4*d) - (4
*e*(e*Cos[c + d*x])^(11/2))/(a*d*(a + a*Sin[c + d*x])^3) - (44*e^3*(e*Cos[c
 + d*x])^(7/2))/(3*d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.153125, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2680, 2635, 2640, 2639}

$$-\frac{154e^5\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^4d}-\frac{44e^3(e\cos(c+dx))^{7/2}}{3d\left(a^4\sin(c+dx)+a^4\right)}-\frac{154e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^4d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+d}{ad(a\sin(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(13/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-154*e^6*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos
[c + d*x]]) - (154*e^5*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*a^4*d) - (4
*e*(e*Cos[c + d*x])^(11/2))/(a*d*(a + a*Sin[c + d*x])^3) - (44*e^3*(e*Cos[c
 + d*x])^(7/2))/(3*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{13/2}}{(a+a\sin(c+dx))^4}\,dx &= -\frac{4e(e\cos(c+dx))^{11/2}}{ad(a+a\sin(c+dx))^3}-\frac{\left(11e^2\right)\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^2}\,dx}{a^2}\\ &= -\frac{4e(e\cos(c+dx))^{11/2}}{ad(a+a\sin(c+dx))^3}-\frac{44e^3(e\cos(c+dx))^{7/2}}{3d\left(a^4+a^4\sin(c+dx)\right)}-\frac{\left(77e^4\right)\int (e\cos(c+dx))^{5/2}\,dx}{3a^4}\\ &= -\frac{154e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^4d}-\frac{4e(e\cos(c+dx))^{11/2}}{ad(a+a\sin(c+dx))^3}-\frac{44e^3(e\cos(c+dx))^{7/2}}{3d\left(a^4+a^4\sin(c+dx)\right)}-\\ &= -\frac{154e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^4d}-\frac{4e(e\cos(c+dx))^{11/2}}{ad(a+a\sin(c+dx))^3}-\frac{44e^3(e\cos(c+dx))^{7/2}}{3d\left(a^4+a^4\sin(c+dx)\right)}-\\ &= -\frac{154e^6\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}}-\frac{154e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^4d}-\frac{4e(e\cos(c-}{ad(a+a\sin}\end{aligned}$$

Mathematica [C] time = 0.219982, size = 66, normalized size = 0.44

$$-\frac{2^{3/4}(e\cos(c+dx))^{15/2}\,{}_2F_1\left(\frac{5}{4},\frac{15}{4};\frac{19}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{15a^4de(\sin(c+dx)+1)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(13/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(2^(3/4)*(e*Cos[c + d*x])^(15/2)*Hypergeometric2F1[5/4, 15/4, 19/4, (1 - S
in[c + d*x])/2])/(15*a^4*d*e*(1 + Sin[c + d*x])^(15/4))
```

Maple [A] time = 1.056, size = 190, normalized size = 1.3

$$-\frac{2e^7}{15a^4d}\left(-24\left(\sin(1/2\,dx+c/2)\right)^6\cos(1/2\,dx+c/2)+24\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2)+80\left(\sin(1/2\,dx+c/2)\right)^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -2/15/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)/a^4*(-24*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+80*sin(1/2*d*x+1/2*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-246*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)-80*sin(1/2*d*x+1/2*c)^3+140*sin(1/2*d*x+1/2*c))*e^
7/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}e^6\cos(dx+c)^6}{a^4\cos(dx+c)^4-8\,a^4\cos(dx+c)^2+8\,a^4-4\left(a^4\cos(dx+c)^2-2\,a^4\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^6*cos(d*x + c)^6/(a^4*cos(d*x + c)^4 - 8*a^
4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(13/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.265 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=145

$$-\frac{10e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{a^4d}-\frac{12e^3(e\cos(c+dx))^{5/2}}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{10e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^4d\sqrt{e\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{9/2}}{3ad(a\sin(c+dx)+a)^3}$$

```
[Out] (-10*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(a^4*d*Sqrt[e*Cos[c
+ d*x]]) - (10*e^5*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(a^4*d) - (4*e*(e*Cos
[c + d*x])^(9/2))/(3*a*d*(a + a*Sin[c + d*x])^3) - (12*e^3*(e*Cos[c + d*x])
^(5/2))/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.152142, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2680, 2635, 2642, 2641}

$$-\frac{10e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{a^4d}-\frac{12e^3(e\cos(c+dx))^{5/2}}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{10e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^4d\sqrt{e\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{9/2}}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-10*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(a^4*d*Sqrt[e*Cos[c
+ d*x]]) - (10*e^5*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(a^4*d) - (4*e*(e*Cos
[c + d*x])^(9/2))/(3*a*d*(a + a*Sin[c + d*x])^3) - (12*e^3*(e*Cos[c + d*x])
^(5/2))/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^4}\,dx &= -\frac{4e(e\cos(c+dx))^{9/2}}{3ad(a+a\sin(c+dx))^3}-\frac{\left(3e^2\right)\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^2}\,dx}{a^2}\\
&= -\frac{4e(e\cos(c+dx))^{9/2}}{3ad(a+a\sin(c+dx))^3}-\frac{12e^3(e\cos(c+dx))^{5/2}}{d\left(a^4+a^4\sin(c+dx)\right)}-\frac{\left(15e^4\right)\int (e\cos(c+dx))^{3/2}\,dx}{a^4}\\
&= -\frac{10e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{a^4d}-\frac{4e(e\cos(c+dx))^{9/2}}{3ad(a+a\sin(c+dx))^3}-\frac{12e^3(e\cos(c+dx))^{5/2}}{d\left(a^4+a^4\sin(c+dx)\right)}-\frac{\left(5e\right.}{}\\
&= -\frac{10e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{a^4d}-\frac{4e(e\cos(c+dx))^{9/2}}{3ad(a+a\sin(c+dx))^3}-\frac{12e^3(e\cos(c+dx))^{5/2}}{d\left(a^4+a^4\sin(c+dx)\right)}-\frac{\left(5e\right.}{}\\
&= -\frac{10e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^4d\sqrt{e\cos(c+dx)}}-\frac{10e^5\sqrt{e\cos(c+dx)}\sin(c+dx)}{a^4d}-\frac{4e(e\cos(c+dx)}{3ad(a+a\sin(c+}\end{aligned}$$

Mathematica [C] time = 0.171778, size = 66, normalized size = 0.46

$$-\frac{\sqrt[4]{2}(e\cos(c+dx))^{13/2}\,{}_2F_1\left(\frac{7}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{13a^4de(\sin(c+dx)+1)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(2^(1/4)*(e*Cos[c + d*x])^(13/2)*Hypergeometric2F1[7/4, 13/4, 17/4, (1 - S
in[c + d*x])/2])/(13*a^4*d*e*(1 + Sin[c + d*x])^(13/4))
```

Maple [A] time = 1.204, size = 263, normalized size = 1.8

$$\frac{2e^6}{3a^4d}\left(-8\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)+30\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{(\mathrm{si}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*e+e)^(1/2)*(-8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
48*sin(1/2*d*x+1/2*c)^5-15*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-18*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-48*sin(1/2*d*x+1/2*c)^3+20*sin(1/2*d*x+1/2*c))*e^6/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e\cos\left(dx+c\right)\right)^{\frac{11}{2}}}{\left(a\sin\left(dx+c\right)+a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(11/2)/(a*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}e^5\cos\left(dx+c\right)^5}{a^4\cos\left(dx+c\right)^4-8\,a^4\cos\left(dx+c\right)^2+8\,a^4-4\left(a^4\cos\left(dx+c\right)^2-2\,a^4\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^5*cos(d*x + c)^5/(a^4*cos(d*x + c)^4 - 8*a^
4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.266 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{28e^3(e\cos(c+dx))^{3/2}}{5d\left(a^4\sin(c+dx)+a^4\right)}+\frac{42e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^4d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a\sin(c+dx)+a)^3}$$

```
[Out] (42*e^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos[c
 + d*x]]) - (4*e*(e*Cos[c + d*x])^(7/2))/(5*a*d*(a + a*Sin[c + d*x])^3) + (
28*e^3*(e*Cos[c + d*x])^(3/2))/(5*d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.133817, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2680, 2640, 2639}

$$\frac{28e^3(e\cos(c+dx))^{3/2}}{5d\left(a^4\sin(c+dx)+a^4\right)}+\frac{42e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^4d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (42*e^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos[c
 + d*x]]) - (4*e*(e*Cos[c + d*x])^(7/2))/(5*a*d*(a + a*Sin[c + d*x])^3) + (
28*e^3*(e*Cos[c + d*x])^(3/2))/(5*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^4}\,dx &= -\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a+a\sin(c+dx))^3} - \frac{\left(7e^2\right)\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^2}\,dx}{5a^2} \\
&= -\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a+a\sin(c+dx))^3} + \frac{28e^3(e\cos(c+dx))^{3/2}}{5d\left(a^4+a^4\sin(c+dx)\right)} + \frac{\left(21e^4\right)\int \sqrt{e\cos(c+dx)}\,dx}{5a^4} \\
&= -\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a+a\sin(c+dx))^3} + \frac{28e^3(e\cos(c+dx))^{3/2}}{5d\left(a^4+a^4\sin(c+dx)\right)} + \frac{\left(21e^4\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+d}}{5a^4\sqrt{\cos(c+dx)}} \\
&= \frac{42e^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}} - \frac{4e(e\cos(c+dx))^{7/2}}{5ad(a+a\sin(c+dx))^3} + \frac{28e^3(e\cos(c+dx))^{3/2}}{5d\left(a^4+a^4\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [C] time = 0.0905803, size = 66, normalized size = 0.55

$$-\frac{(e\cos(c+dx))^{11/2}\,{}_2F_1\left(\frac{9}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11\sqrt[4]{2}a^4de(\sin(c+dx)+1)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -((e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[9/4, 11/4, 15/4, (1 - Sin[c + d
*x])/2])/(11*2^(1/4)*a^4*d*e*(1 + Sin[c + d*x])^(11/4))
```

Maple [B] time = 2.181, size = 332, normalized size = 2.8

$$\frac{2e^5}{5a^4d}\left(84\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2\,(\sin(1/2\,dx+c/2))^4-12}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^4/sin(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(84*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^4-128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-84*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
80*sin(1/2*d*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-80*sin(1/2*d*x+1/2*c)^3+12*sin(1/2*d*x+1/2*c))*e^5/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(a*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}e^4\cos(dx+c)^4}{a^4\cos(dx+c)^4-8\,a^4\cos(dx+c)^2+8\,a^4-4\left(a^4\cos(dx+c)^2-2\,a^4\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^4*cos(d*x + c)^4/(a^4*cos(d*x + c)^4 - 8*a^
4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.267 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{20e^3\sqrt{e\cos(c+dx)}}{21d\left(a^4\sin(c+dx)+a^4\right)}+\frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^4d\sqrt{e\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a\sin(c+dx)+a)^3}$$

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*a^4*d*Sqrt[e*Cos[
c + d*x]]) - (4*e*(e*Cos[c + d*x])^(5/2))/(7*a*d*(a + a*Sin[c + d*x])^3) +
(20*e^3*Sqrt[e*Cos[c + d*x]])/(21*d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.133545, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2680, 2642, 2641}

$$\frac{20e^3\sqrt{e\cos(c+dx)}}{21d\left(a^4\sin(c+dx)+a^4\right)}+\frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^4d\sqrt{e\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*a^4*d*Sqrt[e*Cos[
c + d*x]]) - (4*e*(e*Cos[c + d*x])^(5/2))/(7*a*d*(a + a*Sin[c + d*x])^3) +
(20*e^3*Sqrt[e*Cos[c + d*x]])/(21*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^4}\,dx &= -\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a+a\sin(c+dx))^3}-\frac{\left(5e^2\right)\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^2}\,dx}{7a^2}\\
&= -\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a+a\sin(c+dx))^3}+\frac{20e^3\sqrt{e\cos(c+dx)}}{21d\left(a^4+a^4\sin(c+dx)\right)}+\frac{\left(5e^4\right)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{21a^4}\\
&= -\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a+a\sin(c+dx))^3}+\frac{20e^3\sqrt{e\cos(c+dx)}}{21d\left(a^4+a^4\sin(c+dx)\right)}+\frac{\left(5e^4\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21a^4\sqrt{e\cos(c+dx)}}\\
&= \frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^4d\sqrt{e\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a+a\sin(c+dx))^3}+\frac{20e^3\sqrt{e\cos(c+dx)}}{21d\left(a^4+a^4\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [C] time = 0.0783708, size = 66, normalized size = 0.55

$$-\frac{(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{9}{4},\frac{11}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9\;2^{3/4}a^4de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -((e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[9/4, 11/4, 13/4, (1 - Sin[c + d*
x])/2])/(9*2^(3/4)*a^4*d*e*(1 + Sin[c + d*x])^(9/4))
```

Maple [B] time = 2.124, size = 401, normalized size = 3.3

$$-\frac{2\,e^4}{21\,a^4d}\left(40\,\sqrt{(\sin(1/2\,dx+c/2))^2}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2)\,,\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\,(\sin(1/2\,dx+c/2))^6-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(40*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6-60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4-128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
112*sin(1/2*d*x+1/2*c)^5-5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-112*sin(1/2*d*x+1/2*c)^3+4*sin(1/2*d*x+1/2*c))*e^4/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}e^3\cos(dx+c)^3}{a^4\cos(dx+c)^4-8a^4\cos(dx+c)^2+8a^4-4\left(a^4\cos(dx+c)^2-2a^4\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^3*cos(d*x + c)^3/(a^4*cos(d*x + c)^4 - 8*a^
4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a)^4, x)
```

3.268 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=154

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15a^4d\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^4\sin(c+dx)+a^4\right)}+\frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a\sin(c+dx)+a)^3}$$

```
[Out] (2*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*a^4*d*Sqrt[Cos[c
 + d*x]]) - (4*e*(e*Cos[c + d*x])^(3/2))/(9*a*d*(a + a*Sin[c + d*x])^3) + (
2*e*(e*Cos[c + d*x])^(3/2))/(15*d*(a^2 + a^2*Sin[c + d*x])^2) + (2*e*(e*Cos
[c + d*x])^(3/2))/(15*d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.182129, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2680, 2681, 2683, 2640, 2639}

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15a^4d\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^4\sin(c+dx)+a^4\right)}+\frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^4,x]
```


```
[Out] (2*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*a^4*d*Sqrt[Cos[c
 + d*x]]) - (4*e*(e*Cos[c + d*x])^(3/2))/(9*a*d*(a + a*Sin[c + d*x])^3) + (
2*e*(e*Cos[c + d*x])^(3/2))/(15*d*(a^2 + a^2*Sin[c + d*x])^2) + (2*e*(e*Cos
[c + d*x])^(3/2))/(15*d*(a^4 + a^4*Sin[c + d*x]))
```


Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^4}\,dx = -\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} - \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}\,dx}{3a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2+a^2\sin(c+dx)\right)^2} - \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx}{15a^3}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2+a^2\sin(c+dx)\right)^2} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^4+a^4\sin(c+dx)\right)} + \frac{e^2\int\sqrt{e}\ldots}{}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2+a^2\sin(c+dx)\right)^2} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^4+a^4\sin(c+dx)\right)} + \frac{\left(e^2\sqrt{e\ldots}\right.}{}$$

$$= \frac{2e^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}} - \frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2+a^2\sin(c+dx)\right)^2} + \ldots$$

Mathematica [C] time = 0.087732, size = 66, normalized size = 0.43

$$-\frac{(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},\frac{13}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{14\sqrt[4]{2}a^4de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -((e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, 13/4, 11/4, (1 - Sin[c + d*
x])/2])/(14*2^(1/4)*a^4*d*e*(1 + Sin[c + d*x])^(7/4))
```

Maple [B] time = 2.808, size = 514, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c)
^4-8*sin(1/2*d*x+1/2*c)^2+1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
```

```
2*e+e)^(1/2)*(48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*d
*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2
*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^4-272*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+176*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+144*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+42*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-144*sin(1/2*d*x+1/2*c)^3-4*sin(1/2*d*x+1/2*c
))*e^3/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}e^2\cos(dx+c)^2}{a^4\cos(dx+c)^4-8\,a^4\cos(dx+c)^2+8\,a^4-4\left(a^4\cos(dx+c)^2-2\,a^4\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e^2*cos(d*x + c)^2/(a^4*cos(d*x + c)^4 - 8*a^
4*cos(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^4, x)
```

3.269 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=154

$$-\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^4d\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^4\sin(c+dx)+a^4\right)}+\frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a\sin(c+dx)+a)^3}$$

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*a^4*d*Sqrt[e*Cos[
c + d*x]]) - (4*e*Sqrt[e*Cos[c + d*x]])/(11*a*d*(a + a*Sin[c + d*x])^3) + (
2*e*Sqrt[e*Cos[c + d*x]])/(77*d*(a^2 + a^2*Sin[c + d*x])^2) + (2*e*Sqrt[e*C
os[c + d*x]])/(77*d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.18481, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2680, 2681, 2683, 2642, 2641}

$$-\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^4d\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^4\sin(c+dx)+a^4\right)}+\frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*a^4*d*Sqrt[e*Cos[
c + d*x]]) - (4*e*Sqrt[e*Cos[c + d*x]])/(11*a*d*(a + a*Sin[c + d*x])^3) + (
2*e*Sqrt[e*Cos[c + d*x]])/(77*d*(a^2 + a^2*Sin[c + d*x])^2) + (2*e*Sqrt[e*C
os[c + d*x]])/(77*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^4}\,dx = -\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} - \frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^2}\,dx}{11a^2}$$

$$= -\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2+a^2\sin(c+dx)\right)^2} - \frac{\left(3e^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))}\,d}{77a^3}$$

$$= -\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2+a^2\sin(c+dx)\right)^2} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^4+a^4\sin(c+dx)\right)} - \frac{e^2\int}{}$$

$$= -\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2+a^2\sin(c+dx)\right)^2} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^4+a^4\sin(c+dx)\right)} - \frac{(e^2\sqrt{}}{}$$

$$= -\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^4d\sqrt{e\cos(c+dx)}} - \frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2+a^2\sin(c+dx)\right)^2}$$

Mathematica [C] time = 0.0742038, size = 66, normalized size = 0.43

$$-\frac{(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{15}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{10\ 2^{3/4}a^4de(\sin(c+dx)+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -((e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, 15/4, 9/4, (1 - Sin[c + d*x
])/2])/(10*2^(3/4)*a^4*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [B] time = 3.526, size = 583, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 2/77/(32*sin(1/2*d*x+1/2*c)^10-80*sin(1/2*d*x+1/2*c)^8+80*sin(1/2*d*x+1/2*c
)^6-40*sin(1/2*d*x+1/2*c)^4+10*sin(1/2*d*x+1/2*c)^2-1)/a^4/sin(1/2*d*x+1/2*
```

```
c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(32*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2
*d*x+1/2*c)^10-80*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8+32*sin(1/2*
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+80*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/
2*c)^6-64*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-40*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*sin(1/2*d*x+1/2*c)^4+176*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+10*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-144*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)-176*sin(1/2*d*x+1/2*c)^5-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-78*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+176*sin(1/2*d*x+1/2*c)^3+12*sin(1/2*d*x+1/2*c)
)*e^2/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}\,e\cos(dx+c)}{a^4\cos(dx+c)^4-8\,a^4\cos(dx+c)^2+8\,a^4-4\left(a^4\cos(dx+c)^2-2\,a^4\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*e*cos(d*x + c)/(a^4*cos(d*x + c)^4 - 8*a^4*co
s(d*x + c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a)^4, x)
```

3.270 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=191

$$-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^4\sin(c+dx)+a^4\right)}-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{39a^4d\sqrt{\cos(c+dx)}}-\frac{10(e\cos(c+dx))^{3/2}}{117ade(a\sin(c+dx)+a}$$

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(39*a^4*d*Sqrt[Cos[c + 
d*x]]) - (2*(e*Cos[c + d*x])^(3/2))/(13*d*e*(a + a*Sin[c + d*x])^4) - (10*(
e*Cos[c + d*x])^(3/2))/(117*a*d*e*(a + a*Sin[c + d*x])^3) - (2*(e*Cos[c + d
*x])^(3/2))/(39*d*e*(a^2 + a^2*Sin[c + d*x])^2) - (2*(e*Cos[c + d*x])^(3/2)
)/(39*d*e*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.228498, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2681, 2683, 2640, 2639}

$$-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^4\sin(c+dx)+a^4\right)}-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{39a^4d\sqrt{\cos(c+dx)}}-\frac{10(e\cos(c+dx))^{3/2}}{117ade(a\sin(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^4,x]
```


```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(39*a^4*d*Sqrt[Cos[c + 
d*x]]) - (2*(e*Cos[c + d*x])^(3/2))/(13*d*e*(a + a*Sin[c + d*x])^4) - (10*(
e*Cos[c + d*x])^(3/2))/(117*a*d*e*(a + a*Sin[c + d*x])^3) - (2*(e*Cos[c + d
*x])^(3/2))/(39*d*e*(a^2 + a^2*Sin[c + d*x])^2) - (2*(e*Cos[c + d*x])^(3/2)
)/(39*d*e*(a^4 + a^4*Sin[c + d*x]))
```


Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^4}\,dx &= -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4}+\frac{5\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^3}\,dx}{13a}\\
&= -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4}-\frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3}+\frac{5\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}\,dx}{39a^2}\\
&= -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4}-\frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^2+a^2\sin(c+dx)\right)^2}+\int\\
&= -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4}-\frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^2+a^2\sin(c+dx)\right)^2}-39\\
&= -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4}-\frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{39de\left(a^2+a^2\sin(c+dx)\right)^2}-39\\
&= -\frac{2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^4d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4}-\frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3}
\end{aligned}$$

Mathematica [C] time = 0.0461394, size = 66, normalized size = 0.35

$$-\frac{(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{17}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{12\sqrt[4]{2}a^4de(\sin(c+dx)+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -((e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 17/4, 7/4, (1 - Sin[c + d*x
])/2])/(12*2^(1/4)*a^4*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [B] time = 3.932, size = 694, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -2/117/(64*sin(1/2*d*x+1/2*c)^12-192*sin(1/2*d*x+1/2*c)^10+240*sin(1/2*d*x+
1/2*c)^8-160*sin(1/2*d*x+1/2*c)^6+60*sin(1/2*d*x+1/2*c)^4-12*sin(1/2*d*x+1/
2*c)^2+1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(192*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^12-384*sin(1/2*d*x+1/2*c)^14*cos
(1/2*d*x+1/2*c)-576*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^10+1152*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+720*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^8-1472*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-480*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6+1024*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^8+180*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-280*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)-36*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-40*
sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-208*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-120*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+208*sin(1/2*d*
x+1/2*c)^3+20*sin(1/2*d*x+1/2*c))*e/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos(dx+c)}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{e\cos(dx+c)}}{a^4\cos(dx+c)^4-8\,a^4\cos(dx+c)^2+8\,a^4-4\left(a^4\cos(dx+c)^2-2\,a^4\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x + c)^2 +
8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos(dx+c)}}{(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^4, x)
```

3.271 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=191

$$-\frac{2\sqrt{e\cos(c+dx)}}{33de\left(a^4\sin(c+dx)+a^4\right)}-\frac{2\sqrt{e\cos(c+dx)}}{33de\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^4d\sqrt{e\cos(c+dx)}}-\frac{14\sqrt{e\cos(c+dx)}}{165ade(a\sin(c+dx)+a}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(33*a^4*d*Sqrt[e*Cos[c + d
*x]]) - (2*Sqrt[e*Cos[c + d*x]])/(15*d*e*(a + a*Sin[c + d*x])^4) - (14*Sqrt
[e*Cos[c + d*x]])/(165*a*d*e*(a + a*Sin[c + d*x])^3) - (2*Sqrt[e*Cos[c + d*
x]])/(33*d*e*(a^2 + a^2*Sin[c + d*x])^2) - (2*Sqrt[e*Cos[c + d*x]])/(33*d*e
*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.241544, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2681, 2683, 2642, 2641}

$$-\frac{2\sqrt{e\cos(c+dx)}}{33de\left(a^4\sin(c+dx)+a^4\right)}-\frac{2\sqrt{e\cos(c+dx)}}{33de\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^4d\sqrt{e\cos(c+dx)}}-\frac{14\sqrt{e\cos(c+dx)}}{165ade(a\sin(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(33*a^4*d*Sqrt[e*Cos[c + d
*x]]) - (2*Sqrt[e*Cos[c + d*x]])/(15*d*e*(a + a*Sin[c + d*x])^4) - (14*Sqrt
[e*Cos[c + d*x]])/(165*a*d*e*(a + a*Sin[c + d*x])^3) - (2*Sqrt[e*Cos[c + d*
x]])/(33*d*e*(a^2 + a^2*Sin[c + d*x])^2) - (2*Sqrt[e*Cos[c + d*x]])/(33*d*e
*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^4}\,dx = -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4}+\frac{7\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^3}\,dx}{15a}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4}-\frac{14\sqrt{e\cos(c+dx)}}{165ade(a+a\sin(c+dx))^3}+\frac{7\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\,\mathrm{s}}{33a^2}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4}-\frac{14\sqrt{e\cos(c+dx)}}{165ade(a+a\sin(c+dx))^3}-\frac{2\sqrt{e\cos(c+a}}{33de\left(a^2+a^2\sin(c\right.}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4}-\frac{14\sqrt{e\cos(c+dx)}}{165ade(a+a\sin(c+dx))^3}-\frac{2\sqrt{e\cos(c+a}}{33de\left(a^2+a^2\sin(c\right.}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4}-\frac{14\sqrt{e\cos(c+dx)}}{165ade(a+a\sin(c+dx))^3}-\frac{2\sqrt{e\cos(c+a}}{33de\left(a^2+a^2\sin(c\right.}$$

$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^4d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4}-\frac{14\sqrt{e\cos(c}}{165ade(a+a\sin}$$

Mathematica [C] time = 0.0584454, size = 66, normalized size = 0.35

$$-\frac{\sqrt{e\cos(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{19}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{4\;2^{3/4}a^4de\sqrt[4]{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4),x]
```

```
[Out] -(Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 19/4, 5/4, (1 - Sin[c + d*x])
/2])/(4*2^(3/4)*a^4*d*e*(1 + Sin[c + d*x])^(1/4))
```

Maple [B] time = 4.301, size = 762, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/165/(128*sin(1/2*d*x+1/2*c)^14-448*sin(1/2*d*x+1/2*c)^12+672*sin(1/2*d*x
+1/2*c)^10-560*sin(1/2*d*x+1/2*c)^8+280*sin(1/2*d*x+1/2*c)^6-84*sin(1/2*d*x
+1/2*c)^4+14*sin(1/2*d*x+1/2*c)^2-1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*e+e)^(1/2)*(640*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^14-22
40*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^12+640*sin(1/2*d*x+1/2*c)^14
*cos(1/2*d*x+1/2*c)+3360*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^10-192
0*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-2800*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^8+2496*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+1400*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^6-1792*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^8-420*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+616*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)+70*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^2-40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+240*sin(1/2*d*x+1/2*c)^5-5*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+160*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-240*sin
```

```
(1/2*d*x+1/2*c)^3-28*sin(1/2*d*x+1/2*c))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}}{a^4e\cos(dx+c)^5 - 8\,a^4e\cos(dx+c)^3 + 8\,a^4e\cos(dx+c) - 4\left(a^4e\cos(dx+c)^3 - 2\,a^4e\cos(dx+c)\right)\sin(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a^4*e*cos(d*x + c)^5 - 8*a^4*e*cos(d*x + c)^
3 + 8*a^4*e*cos(d*x + c) - 4*(a^4*e*cos(d*x + c)^3 - 2*a^4*e*cos(d*x + c))*
sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^4), x)
```

3.272 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=225

$$-\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{221a^4de^2\sqrt{\cos(c+dx)}}+\frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}}-\frac{14}{221de\left(a^4\sin(c+dx)+a^4\right)\sqrt{e\cos(c+dx)}}-\frac{}{221de\left(a^2\mathrm{s}\right.}$$

```
[Out] (-42*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(221*a^4*d*e^2*Sqrt[Co
s[c + d*x]]) + (42*Sin[c + d*x])/(221*a^4*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(17
*d*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4) - 18/(221*a*d*e*Sqrt[e*Co
s[c + d*x]]*(a + a*Sin[c + d*x])^3) - 14/(221*d*e*Sqrt[e*Cos[c + d*x]]*(a^2
 + a^2*Sin[c + d*x])^2) - 14/(221*d*e*Sqrt[e*Cos[c + d*x]]*(a^4 + a^4*Sin[c
 + d*x]))
```

Rubi [A] time = 0.299081, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2681, 2683, 2636, 2640, 2639}

$$-\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{221a^4de^2\sqrt{\cos(c+dx)}}+\frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}}-\frac{14}{221de\left(a^4\sin(c+dx)+a^4\right)\sqrt{e\cos(c+dx)}}-\frac{}{221de\left(a^2\mathrm{s}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^4),x]
```



```
[Out] (-42*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(221*a^4*d*e^2*Sqrt[Co
s[c + d*x]]) + (42*Sin[c + d*x])/(221*a^4*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(17
*d*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4) - 18/(221*a*d*e*Sqrt[e*Co
s[c + d*x]]*(a + a*Sin[c + d*x])^3) - 14/(221*d*e*Sqrt[e*Cos[c + d*x]]*(a^2
 + a^2*Sin[c + d*x])^2) - 14/(221*d*e*Sqrt[e*Cos[c + d*x]]*(a^4 + a^4*Sin[c
 + d*x]))
```


Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
```

```
IntegersQ[2*m, 2*p]
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p - 1)*(a + b*
Sin[e + f*x])), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^4}\,dx = -\frac{2}{17de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^4} + \frac{9\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}\,dx}{17a}$$

$$= -\frac{2}{17de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^4} - \frac{18}{221ade\sqrt{e\cos(c+dx)}(a+a\sin}$$

$$= -\frac{2}{17de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^4} - \frac{18}{221ade\sqrt{e\cos(c+dx)}(a+a\sin}$$

$$= -\frac{2}{17de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^4} - \frac{18}{221ade\sqrt{e\cos(c+dx)}(a+a\sin}$$

$$= \frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}} - \frac{2}{17de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^4} - \frac{}{221ade}$$

$$= \frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}} - \frac{2}{17de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^4} - \frac{}{221ade}$$

$$= -\frac{42\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{221a^4de^2\sqrt{\cos(c+dx)}} + \frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}} - \frac{}{17de\sqrt{e\cos}}$$

Mathematica [C] time = 0.0876748, size = 66, normalized size = 0.29

$$\frac{\sqrt[4]{\sin(c+dx)+1}\,{}_2F_1\left(-\frac{1}{4},\frac{21}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{8\sqrt[4]{2}a^4de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^4),x]
```

```
[Out] (Hypergeometric2F1[-1/4, 21/4, 3/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(1/4))/(8*2^(1/4)*a^4*d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [B] time = 5.447, size = 878, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -2/221/(256*sin(1/2*d*x+1/2*c)^16-1024*sin(1/2*d*x+1/2*c)^14+1792*sin(1/2*d
*x+1/2*c)^12-1792*sin(1/2*d*x+1/2*c)^10+1120*sin(1/2*d*x+1/2*c)^8-448*sin(1
/2*d*x+1/2*c)^6+112*sin(1/2*d*x+1/2*c)^4-16*sin(1/2*d*x+1/2*c)^2+1)/a^4/sin
(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e*(5376*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^16-10752*sin(1/2*d*x+1/2*c)^18*cos(1/2*d*x+1/2
*c)-21504*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^14+43008*sin(1/2*d*x+
1/2*c)^16*cos(1/2*d*x+1/2*c)+37632*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^12-76160*sin(1/2*d*x+1/2*c)^14*cos(1/2*d*x+1/2*c)-37632*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^10+77952*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^12+23520*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^8-50560*sin(1/2*d*x+1
/2*c)^10*cos(1/2*d*x+1/2*c)-9408*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c
)^6+21376*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+2352*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^4-5656*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-336*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+792*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)-272*sin(1/2*d*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-242*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+272*sin(1/2*d*x+1/2*c)^3+36*sin(1/2*d*
x+1/2*c))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos\left(dx+c\right)}}{a^4e^2\cos\left(dx+c\right)^6-8\,a^4e^2\cos\left(dx+c\right)^4+8\,a^4e^2\cos\left(dx+c\right)^2-4\left(a^4e^2\cos\left(dx+c\right)^4-2\,a^4e^2\cos\left(dx+c\right)^2\right)\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))/(a^4*e^2*cos(d*x + c)^6 - 8*a^4*e^2*cos(d*x +
 c)^4 + 8*a^4*e^2*cos(d*x + c)^2 - 4*(a^4*e^2*cos(d*x + c)^4 - 2*a^4*e^2*co
s(d*x + c)^2)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a\sin\left(dx+c\right)+a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^4), x)
```

3.273 $\int (e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=236

$$\frac{3e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{4d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{3e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\mathrm{sinh}}{4d(\sin(c+dx)+\cos(c+dx)+}$$

```
[Out] -(a*(e*Cos[c + d*x])^(5/2))/(2*d*e*Sqrt[a + a*Sin[c + d*x]]) + (3*e*Sqrt[e*
Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d) - (3*e^(3/2)*ArcSinh[Sqrt[e*C
os[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d
*(1 + Cos[c + d*x] + Sin[c + d*x])) + (3*e^(3/2)*ArcTan[(Sqrt[e]*Sin[c + d*
x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x]]*S
qrt[a + a*Sin[c + d*x]])/(4*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rubi [A] time = 0.357076, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2678, 2685, 2677, 2775, 203, 2833, 63, 215}

$$\frac{3e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{4d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{3e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\mathrm{sinh}}{4d(\sin(c+dx)+\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -(a*(e*Cos[c + d*x])^(5/2))/(2*d*e*Sqrt[a + a*Sin[c + d*x]]) + (3*e*Sqrt[e*
Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d) - (3*e^(3/2)*ArcSinh[Sqrt[e*C
os[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d
*(1 + Cos[c + d*x] + Sin[c + d*x])) + (3*e^(3/2)*ArcTan[(Sqrt[e]*Sin[c + d*
x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x]]*S
qrt[a + a*Sin[c + d*x]])/(4*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}} + \frac{1}{4}(3a)\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}} + \frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4d} + \frac{1}{8}\left(3e^2\right)\int$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}} + \frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4d} + \frac{\left(3ae^2\sqrt{1}\right.}{}$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}} + \frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4d} - \frac{\left(3ae^2\sqrt{1}\right.}{}$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}} + \frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4d} + \frac{3e^{3/2}\tan}{}$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}} + \frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4d} - \frac{3e^{3/2}\sinh}{}$$

Mathematica [C] time = 0.921097, size = 269, normalized size = 1.14

$$-\frac{iee^{-i(c+dx)}\sqrt{a(\sin(c+dx)+1)}\sqrt{e\cos(c+dx)}\left(-3dxe^{2i(c+dx)}-2e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}+2ie^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}+e^{3i(c+dx)}\right.}{4d\left(e^{i(c+dx)}+i\right)\sqrt{1+e^{2i(c+}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-I/4)*e*Sqrt[e*Cos[c + d*x]]*((-I)*Sqrt[1 + E^((2*I)*(c + d*x))] - 2*E^(I
*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))] + (2*I)*E^((2*I)*(c + d*x))*Sqrt[
1 + E^((2*I)*(c + d*x))] + E^((3*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))
```

```
] - 3*d*E^((2*I)*(c + d*x))*x + 3*E^((2*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x
))] - (3*I)*E^((2*I)*(c + d*x))*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqr
t[a*(1 + Sin[c + d*x])])/(d*E^(I*(c + d*x))*(I + E^(I*(c + d*x)))*Sqrt[1 +
E^((2*I)*(c + d*x))])
```

Maple [A] time = 0.236, size = 241, normalized size = 1.

$$\frac{1}{8\,d\left(-1+\cos\left(dx+c\right)-\sin\left(dx+c\right)\right)\left(\cos\left(dx+c\right)\right)^{2}}\left(-3\sqrt{-2\,\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\arctan\left(1/2\,\sqrt{2}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(-3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)+3*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+4*cos(d*x+c)^3+4*cos(d*x+c)^2*sin(d*x+c
)+2*cos(d*x+c)^2-6*cos(d*x+c)*sin(d*x+c)-6*cos(d*x+c))*(e*cos(d*x+c))^(3/2)
*(a*(1+sin(d*x+c)))^(1/2)/(-1+cos(d*x+c)-sin(d*x+c))/cos(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}\sqrt{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*sqrt(a*sin(d*x + c) + a), x)
```

3.274 $\int \sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$-\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a\sin(c+dx)+a}}+\frac{\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{\sqrt{e}\sqrt{\cos(c+dx)+1}\cdots}{d(\sin(c+\cdots}$$

```
[Out] -((a*(e*Cos[c + d*x])^(3/2))/(d*e*Sqrt[a + a*Sin[c + d*x]])) + (Sqrt[e]*Arc
Sinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c
+ d*x]])/(d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (Sqrt[e]*ArcTan[(Sqrt[e]*S
in[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c
+ d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rubi [A] time = 0.269584, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2678, 2684, 2775, 203, 2833, 63, 215}

$$-\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a\sin(c+dx)+a}}+\frac{\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{\sqrt{e}\sqrt{\cos(c+dx)+1}\cdots}{d(\sin(c+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((a*(e*Cos[c + d*x])^(3/2))/(d*e*Sqrt[a + a*Sin[c + d*x]])) + (Sqrt[e]*Arc
Sinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c
+ d*x]])/(d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (Sqrt[e]*ArcTan[(Sqrt[e]*S
in[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c
+ d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{1}{2}a\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{\left(ae\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+d}}{\sqrt{e\cos(c+dx}}}{2(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{\left(ae\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{e}}\right.}{2d(a+a\cos(c+dx)+a\sin(c+}$$

$$= -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\sqrt{}}{d(1+\cos(c+dx)+\sin(c+dx))}$$

$$= -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{\sqrt{e}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(}}{d(1+\cos(c+dx)+\sin(c+dx))}$$

Mathematica [C] time = 0.855958, size = 195, normalized size = 1.01

$$-\frac{i\sqrt{a(\sin(c+dx)+1)}\sqrt{e\cos(c+dx)}\left(idxe^{i(c+dx)}+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}-i\sqrt{1+e^{2i(c+dx)}}-e^{i(c+dx)}\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)\right.}{d\left(e^{i(c+dx)}+i\right)\sqrt{1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[e*Cos[c + d*x]]*((-I)*Sqrt[1 + E^((2*I)*(c + d*x))] + E^(I*(c +
d*x))*Sqrt[1 + E^((2*I)*(c + d*x))] + I*d*E^(I*(c + d*x))*x + I*E^(I*(c + d
*x))*ArcSinh[E^(I*(c + d*x))] - E^(I*(c + d*x))*Log[1 + Sqrt[1 + E^((2*I)*(
c + d*x))]])*Sqrt[a*(1 + Sin[c + d*x])])/(d*(I + E^(I*(c + d*x)))*Sqrt[1 +
E^((2*I)*(c + d*x))])
```

Maple [A] time = 0.155, size = 213, normalized size = 1.1

$$-\frac{1}{2d(1-\cos(dx+c)+\sin(dx+c))\cos(dx+c)}\left(\sqrt{-2\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sqrt{2}\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)+(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
/cos(d*x+c))*2^(1/2)*sin(d*x+c)+2*cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-2*co
s(d*x+c))*(e*cos(d*x+c))^(1/2)*(a*(1+sin(d*x+c)))^(1/2)/(1-cos(d*x+c)+sin(d
*x+c))/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a(\sin(c+dx)+1)}\sqrt{e\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sqrt(e*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a), x)
```

3.275 $\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=161

$$\frac{2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d\sqrt{e}(\sin(c+dx)+\cos(c+dx)+1)}-\frac{2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}\Big(\sqrt{e}}{d\sqrt{e}(\sin(c+dx)+\cos(c+dx)+1)}$$

```
[Out] (-2*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a
*Sin[c + d*x]])/(d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x])) + (2*ArcTan[(
Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1
 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*Sqrt[e]*(1 + Cos[c + d*x] + S
in[c + d*x]))
```

Rubi [A] time = 0.209145, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2677, 2775, 203, 2833, 63, 215}

$$\frac{2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d\sqrt{e}(\sin(c+dx)+\cos(c+dx)+1)}-\frac{2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}\Big(\sqrt{e}}{d\sqrt{e}(\sin(c+dx)+\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a
*Sin[c + d*x]])/(d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x])) + (2*ArcTan[(
Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1
 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*Sqrt[e]*(1 + Cos[c + d*x] + S
in[c + d*x]))
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \frac{\left(a\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{a+a\cos(c+dx)+a\sin(c+dx)} + \frac{\left(a\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c\right.}{a+a\cos(c+a}$$

$$= -\frac{\left(a\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{d(a+a\cos(c+dx)+a\sin(c+dx))} - \frac{\left(2a\sqrt{1+c\right.}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{d\sqrt{e}(1+\cos(c+dx)+\sin(c+dx))} - \frac{\left(2a\sqrt{1+\cos(c+dx\right.}{}$$

$$= -\frac{2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{d\sqrt{e}(1+\cos(c+dx)+\sin(c+dx))} + \frac{2\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+}}\right.}{d\sqrt{e}(1+\cos}$$

Mathematica [C] time = 0.463393, size = 108, normalized size = 0.67

$$\frac{\sqrt{1+e^{2i(c+dx)}}\sqrt{a(\sin(c+dx)+1)}\left(i\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-\sinh^{-1}\left(e^{i(c+dx)}\right)+dx\right)}{d\left(1-ie^{i(c+dx)}\right)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[1 + E^((2*I)*(c + d*x))]*(d*x - ArcSinh[E^(I*(c + d*x))] + I*Log[1 +
Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a*(1 + Sin[c + d*x])])/(d*(1 - I*E^(I*
(c + d*x)))*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 0.109, size = 142, normalized size = 0.9

$$-\frac{\sqrt{2}\left(-1+\cos(dx+c)+\sin(dx+c)\right)}{d\sin(dx+c)}\left(\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)-\text{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx}{1+\cos(d}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*2^(1/2)*(arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-arct
anh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))
)*(a*(1+sin(d*x+c)))^(1/2)*(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)/(e*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin(dx+c)+a}}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a(\sin(c+dx)+1)}}{\sqrt{e\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/sqrt(e*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \sin (dx + c) + a}}{\sqrt{e \cos (dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

3.276 $\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{2\sqrt{a\sin(c+dx)+a}}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] (2*Sqrt[a + a*Sin[c + d*x]])/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.0681025, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2671}

$$\frac{2\sqrt{a\sin(c+dx)+a}}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[a + a*Sin[c + d*x]])/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx=\frac{2\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 0.116662, size = 34, normalized size = 1.

$$\frac{2\sqrt{a(\sin(c+dx)+1)}}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Sin[c + d*x])])/(d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 0.123, size = 34, normalized size = 1.

$$2\,\frac{\cos(dx+c)\sqrt{a(1+\sin(dx+c))}}{d(e\cos(dx+c))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/d*cos(d*x+c)*(a*(1+sin(d*x+c)))^(1/2)/(e*cos(d*x+c))^(3/2)
```

Maxima [B] time = 1.5534, size = 177, normalized size = 5.21

$$\frac{2\left(\sqrt{a}\sqrt{e}-\frac{\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{\left(e^2+\frac{e^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] 2*(sqrt(a)*sqrt(e) - sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/((e^2 + e^2*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2)*d*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*(-sin(d*x + c)/(
cos(d*x + c) + 1) + 1)^(3/2))
```

Fricas [A] time = 2.56197, size = 95, normalized size = 2.79

$$\frac{2\sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+c)+a}}{de^2\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 2*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*e^2*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}{\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/(e*cos(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin\left(dx+c\right)+a}}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

3.277 $\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{4(a\sin(c+dx)+a)^{3/2}}{3ade(e\cos(c+dx))^{3/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{de(e\cos(c+dx))^{3/2}}$$

```
[Out] (-2*Sqrt[a + a*Sin[c + d*x]])/(d*e*(e*Cos[c + d*x])^(3/2)) + (4*(a + a*Sin[
c + d*x])^(3/2))/(3*a*d*e*(e*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.145952, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{4(a\sin(c+dx)+a)^{3/2}}{3ade(e\cos(c+dx))^{3/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[a + a*Sin[c + d*x]])/(d*e*(e*Cos[c + d*x])^(3/2)) + (4*(a + a*Sin[
c + d*x])^(3/2))/(3*a*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{a+a\sin(c+dx)}}{de(e\cos(c+dx))^{3/2}} + \frac{2\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{5/2}}\,dx}{a}$$
$$= -\frac{2\sqrt{a+a\sin(c+dx)}}{de(e\cos(c+dx))^{3/2}} + \frac{4(a+a\sin(c+dx))^{3/2}}{3ade(e\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.232978, size = 46, normalized size = 0.62

$$\frac{2(2\sin(c+dx)-1)\sqrt{a(\sin(c+dx)+1)}}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Sin[c + d*x])]*(-1 + 2*Sin[c + d*x]))/(3*d*e*(e*Cos[c + d*x]
)^(3/2))
```

Maple [A] time = 0.119, size = 44, normalized size = 0.6

$$\frac{(4\,\sin(dx+c)-2)\cos(dx+c)}{3\,d}\sqrt{a\,(1+\sin(dx+c))}\,(e\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*(2*sin(d*x+c)-1)*(a*(1+sin(d*x+c)))^(1/2)*cos(d*x+c)/(e*cos(d*x+c))^(
5/2)
```

Maxima [B] time = 1.58294, size = 278, normalized size = 3.76

$$-\frac{2\left(\sqrt{a}\sqrt{e}-\frac{4\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{4\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\sqrt{a}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{3\left(e^3+\frac{2e^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{e^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] -2/3*(sqrt(a)*sqrt(e) - 4*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1) +
 4*sqrt(a)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - sqrt(a)*sqrt(e)*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
)^2/((e^3 + 2*e^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + e^3*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(5/2))
```

Fricas [A] time = 2.57561, size = 128, normalized size = 1.73

$$\frac{2\sqrt{e\cos\left(dx+c\right)}\sqrt{a\sin\left(dx+c\right)+a}\left(2\,\sin\left(dx+c\right)-1\right)}{3\,de^{3}\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)*(2*sin(d*x + c) - 1)/(d*e
^3*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \sin(dx+c)+a}}{(e \cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

3.278 $\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{16(a\sin(c+dx)+a)^{5/2}}{15a^2de(e\cos(c+dx))^{5/2}}+\frac{8(a\sin(c+dx)+a)^{3/2}}{3ade(e\cos(c+dx))^{5/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{3de(e\cos(c+dx))^{5/2}}$$

```
[Out] (-2*Sqrt[a + a*Sin[c + d*x]])/(3*d*e*(e*Cos[c + d*x])^(5/2)) + (8*(a + a*Si
n[c + d*x])^(3/2))/(3*a*d*e*(e*Cos[c + d*x])^(5/2)) - (16*(a + a*Sin[c + d*
x])^(5/2))/(15*a^2*d*e*(e*Cos[c + d*x])^(5/2))
```

Rubi [A] time = 0.223558, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{16(a\sin(c+dx)+a)^{5/2}}{15a^2de(e\cos(c+dx))^{5/2}}+\frac{8(a\sin(c+dx)+a)^{3/2}}{3ade(e\cos(c+dx))^{5/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{3de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*Sqrt[a + a*Sin[c + d*x]])/(3*d*e*(e*Cos[c + d*x])^(5/2)) + (8*(a + a*Si
n[c + d*x])^(3/2))/(3*a*d*e*(e*Cos[c + d*x])^(5/2)) - (16*(a + a*Sin[c + d*
x])^(5/2))/(15*a^2*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
```

```
&& EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx &= -\frac{2\sqrt{a+a\sin(c+dx)}}{3de(e\cos(c+dx))^{5/2}}+\frac{4\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{7/2}}\,dx}{3a}\\
&= -\frac{2\sqrt{a+a\sin(c+dx)}}{3de(e\cos(c+dx))^{5/2}}+\frac{8(a+a\sin(c+dx))^{3/2}}{3ade(e\cos(c+dx))^{5/2}}-\frac{8\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{7/2}}\,dx}{3a^2}\\
&= -\frac{2\sqrt{a+a\sin(c+dx)}}{3de(e\cos(c+dx))^{5/2}}+\frac{8(a+a\sin(c+dx))^{3/2}}{3ade(e\cos(c+dx))^{5/2}}-\frac{16(a+a\sin(c+dx))^{5/2}}{15a^2de(e\cos(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.328401, size = 56, normalized size = 0.49

$$\frac{2\sqrt{a(\sin(c+dx)+1)}(4\sin(c+dx)+4\cos(2(c+dx))+3)}{15de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Sin[c + d*x])]*(3 + 4*Cos[2*(c + d*x)] + 4*Sin[c + d*x]))/(1
5*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [A] time = 0.124, size = 54, normalized size = 0.5

$$\frac{\left(16\,(\cos(dx+c))^2+8\,\sin(dx+c)-2\right)\cos(dx+c)}{15\,d}\sqrt{a\,(1+\sin(dx+c))}\,(e\cos(dx+c))^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] 2/15/d*(8*cos(d*x+c)^2+4*sin(d*x+c)-1)*(a*(1+sin(d*x+c)))^(1/2)*cos(d*x+c)/
(e*cos(d*x+c))^(7/2)
```

Maxima [B] time = 1.59846, size = 381, normalized size = 3.31

$$\frac{2\left(7\sqrt{a}\sqrt{e}+\frac{8\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{a}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{25\sqrt{a}\sqrt{e}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{8\sqrt{a}\sqrt{e}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{7\sqrt{a}\sqrt{e}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{3}}{15\left(e^{4}+\frac{3e^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3e^{4}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{e^{4}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxima"
)
```

```
[Out] 2/15*(7*sqrt(a)*sqrt(e) + 8*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1)
 - 25*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 25*sqrt(a)*sqrt
(e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 8*sqrt(a)*sqrt(e)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 7*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((e^4 + 3*e^4*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 3*e^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + e^4*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2))
```

Fricas [A] time = 2.4318, size = 155, normalized size = 1.35

$$\frac{2\sqrt{e\cos(dx+c)}\left(8\cos(dx+c)^{2}+4\sin(dx+c)-1\right)\sqrt{a\sin(dx+c)+a}}{15\,de^{4}\cos(dx+c)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="fricas"
)
```

```
[Out] 2/15*sqrt(e*cos(d*x + c))*(8*cos(d*x + c)^2 + 4*sin(d*x + c) - 1)*sqrt(a*si
n(d*x + c) + a)/(d*e^4*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin(dx+c)+a}}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

3.279 $\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{32(a\sin(c+dx)+a)^{7/2}}{35a^3de(e\cos(c+dx))^{7/2}}+\frac{16(a\sin(c+dx)+a)^{5/2}}{5a^2de(e\cos(c+dx))^{7/2}}-\frac{12(a\sin(c+dx)+a)^{3/2}}{5ade(e\cos(c+dx))^{7/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{5de(e\cos(c+dx))^{7/2}}$$

```
[Out] (-2*Sqrt[a + a*Sin[c + d*x]])/(5*d*e*(e*Cos[c + d*x])^(7/2)) - (12*(a + a*S
in[c + d*x])^(3/2))/(5*a*d*e*(e*Cos[c + d*x])^(7/2)) + (16*(a + a*Sin[c + d
*x])^(5/2))/(5*a^2*d*e*(e*Cos[c + d*x])^(7/2)) - (32*(a + a*Sin[c + d*x])^(
7/2))/(35*a^3*d*e*(e*Cos[c + d*x])^(7/2))
```

Rubi [A] time = 0.307486, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{32(a\sin(c+dx)+a)^{7/2}}{35a^3de(e\cos(c+dx))^{7/2}}+\frac{16(a\sin(c+dx)+a)^{5/2}}{5a^2de(e\cos(c+dx))^{7/2}}-\frac{12(a\sin(c+dx)+a)^{3/2}}{5ade(e\cos(c+dx))^{7/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{5de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (-2*Sqrt[a + a*Sin[c + d*x]])/(5*d*e*(e*Cos[c + d*x])^(7/2)) - (12*(a + a*S
in[c + d*x])^(3/2))/(5*a*d*e*(e*Cos[c + d*x])^(7/2)) + (16*(a + a*Sin[c + d
*x])^(5/2))/(5*a^2*d*e*(e*Cos[c + d*x])^(7/2)) - (32*(a + a*Sin[c + d*x])^(
7/2))/(35*a^3*d*e*(e*Cos[c + d*x])^(7/2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
```

```
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{9/2}}\,dx &= -\frac{2\sqrt{a+a\sin(c+dx)}}{5de(e\cos(c+dx))^{7/2}}+\frac{6\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{9/2}}\,dx}{5a}\\
&= -\frac{2\sqrt{a+a\sin(c+dx)}}{5de(e\cos(c+dx))^{7/2}}-\frac{12(a+a\sin(c+dx))^{3/2}}{5ade(e\cos(c+dx))^{7/2}}+\frac{24\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{9/2}}\,dx}{5a^2}\\
&= -\frac{2\sqrt{a+a\sin(c+dx)}}{5de(e\cos(c+dx))^{7/2}}-\frac{12(a+a\sin(c+dx))^{3/2}}{5ade(e\cos(c+dx))^{7/2}}+\frac{16(a+a\sin(c+dx))^{5/2}}{5a^2de(e\cos(c+dx))^{7/2}}-\frac{16\int \frac{(a+a\sin(c+a}{(e\cos(c+dx}}{5a^3}\\
&= -\frac{2\sqrt{a+a\sin(c+dx)}}{5de(e\cos(c+dx))^{7/2}}-\frac{12(a+a\sin(c+dx))^{3/2}}{5ade(e\cos(c+dx))^{7/2}}+\frac{16(a+a\sin(c+dx))^{5/2}}{5a^2de(e\cos(c+dx))^{7/2}}-\frac{32(a+a\sin(c}{35a^3de(e\cos(c}
\end{aligned}$$

Mathematica [A] time = 0.757956, size = 74, normalized size = 0.48

$$\frac{2\sec^4(c+dx)\sqrt{a(\sin(c+dx)+1)}\sqrt{e\cos(c+dx)}(10\sin(c+dx)+4\sin(3(c+dx))-4\cos(2(c+dx))-5)}{35de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^4*Sqrt[a*(1 + Sin[c + d*x])]*(-5 - 4*C
os[2*(c + d*x)] + 10*Sin[c + d*x] + 4*Sin[3*(c + d*x)]))/(35*d*e^5)
```

Maple [A] time = 0.138, size = 70, normalized size = 0.5

$$\frac{\left(32\,(\cos(dx+c))^2\sin(dx+c)-16\,(\cos(dx+c))^2+12\,\sin(dx+c)-2\right)\cos(dx+c)}{35\,d}\sqrt{a\,(1+\sin(dx+c))}\,(e\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] 2/35/d*(16*cos(d*x+c)^2*sin(d*x+c)-8*cos(d*x+c)^2+6*sin(d*x+c)-1)*(a*(1+sin
(d*x+c)))^(1/2)*cos(d*x+c)/(e*cos(d*x+c))^(9/2)
```

Maxima [B] time = 1.6216, size = 482, normalized size = 3.13

$$-\frac{2\left(9\sqrt{a}\sqrt{e}-\frac{44\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{14\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{84\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{84\sqrt{a}\sqrt{e}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{14\sqrt{a}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{44\sqrt{a}\sqrt{e}\sin(dx+c}{(\cos(dx+c)+1)^7}\right.}{35\left(e^5+\frac{4e^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6e^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4e^5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{e^5\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\mathrm{si}}{\mathrm{cos}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(9/2),x, algorithm="maxima"
)
```

```
[Out] -2/35*(9*sqrt(a)*sqrt(e) - 44*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) +
1) - 14*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 84*sqrt(a)*sq
rt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 84*sqrt(a)*sqrt(e)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 + 14*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6 + 44*sqrt(a)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 9*sqrt(a)*
sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)^4/((e^5 + 4*e^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*e^5*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*e^5*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + e^5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2))
```

Fricas [A] time = 2.16577, size = 188, normalized size = 1.22

$$-\frac{2\sqrt{e\cos(dx+c)}\left(8\cos(dx+c)^2-2\left(8\cos(dx+c)^2+3\right)\sin(dx+c)+1\right)\sqrt{a\sin(dx+c)+a}}{35\,de^5\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(9/2),x, algorithm="fricas"
)
```

```
[Out] -2/35*sqrt(e*cos(d*x + c))*(8*cos(d*x + c)^2 - 2*(8*cos(d*x + c)^2 + 3)*sin
(d*x + c) + 1)*sqrt(a*sin(d*x + c) + a)/(d*e^5*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sin(dx+c)+a}}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)/(e*cos(d*x + c))^(9/2), x)
```

3.280 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=319

$$-\frac{3a^2(e\cos(c+dx))^{7/2}}{8de\sqrt{a\sin(c+dx)+a}}-\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a\sin(c+dx)+a)^{3/2}}+\frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a\sin(c+dx)+a}}+\frac{45ae^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx}}{64d(\sin(c+dx)+}$$

```
[Out] (-15*a^3*(e*Cos[c + d*x])^(7/2))/(32*d*e*(a + a*Sin[c + d*x])^(3/2)) + (15*
a^2*e*(e*Cos[c + d*x])^(3/2))/(64*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a^2*(e*C
os[c + d*x])^(7/2))/(8*d*e*Sqrt[a + a*Sin[c + d*x]]) - (a*(e*Cos[c + d*x])^
(7/2)*Sqrt[a + a*Sin[c + d*x]])/(4*d*e) + (45*a*e^(5/2)*ArcSinh[Sqrt[e*Cos[
c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(64*d*(
1 + Cos[c + d*x] + Sin[c + d*x])) + (45*a*e^(5/2)*ArcTan[(Sqrt[e]*Sin[c + d
*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x]]*
Sqrt[a + a*Sin[c + d*x]])/(64*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rubi [A] time = 0.564244, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2678, 2686, 2679, 2684, 2775, 203, 2833, 63, 215}

$$-\frac{3a^2(e\cos(c+dx))^{7/2}}{8de\sqrt{a\sin(c+dx)+a}}-\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a\sin(c+dx)+a)^{3/2}}+\frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a\sin(c+dx)+a}}+\frac{45ae^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx}}{64d(\sin(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-15*a^3*(e*Cos[c + d*x])^(7/2))/(32*d*e*(a + a*Sin[c + d*x])^(3/2)) + (15*
a^2*e*(e*Cos[c + d*x])^(3/2))/(64*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a^2*(e*C
os[c + d*x])^(7/2))/(8*d*e*Sqrt[a + a*Sin[c + d*x]]) - (a*(e*Cos[c + d*x])^
(7/2)*Sqrt[a + a*Sin[c + d*x]])/(4*d*e) + (45*a*e^(5/2)*ArcSinh[Sqrt[e*Cos[
c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(64*d*(
1 + Cos[c + d*x] + Sin[c + d*x])) + (45*a*e^(5/2)*ArcTan[(Sqrt[e]*Sin[c + d
*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x]]*
Sqrt[a + a*Sin[c + d*x]])/(64*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2686

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1))/(f*g*(2*p - 1)*
(a + b*Sin[e + f*x])^(3/2)), x] + Dist[(2*a*(p - 2))/(2*p - 1), Int[(g*Cos[
e + f*x])^p/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 2] && IntegerQ[2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}\,dx = -\frac{a(e\cos(c+dx))^{7/2}\sqrt{a+a\sin(c+dx)}}{4de} + \frac{1}{8}(9a)\int (e\cos(c+dx))^{5/2}\sqrt{a+a$$

$$= -\frac{3a^2(e\cos(c+dx))^{7/2}}{8de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{7/2}\sqrt{a+a\sin(c+dx)}}{4de} + \frac{1}{16}\big(15a$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} - \frac{3a^2(e\cos(c+dx))^{7/2}}{8de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{7/2}}{4d$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos(c+dx))}{8de\sqrt{a+a\sin(c+}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos(c+dx))}{8de\sqrt{a+a\sin(c+}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos(c+dx))}{8de\sqrt{a+a\sin(c+}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos(c+dx))}{8de\sqrt{a+a\sin(c+}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos(c+dx))}{8de\sqrt{a+a\sin(c+}$$

Mathematica [C] time = 0.166952, size = 78, normalized size = 0.24

$$-\frac{16\sqrt[4]{2}a\sqrt{a(\sin(c+dx)+1)}(e\cos(c+dx))^{7/2}\,{}_2F_1\left(-\frac{9}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-16*2^(1/4)*a*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[-9/4, 7/4, 11/4, (1
 - Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(7*d*e*(1 + Sin[c + d*x])^(
9/4))
```

Maple [A] time = 0.201, size = 314, normalized size = 1.

$$\frac{1}{128\,d\left(\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(\cos\left(dx+c\right)\right)^{2}-2\,\sin\left(dx+c\right)+\cos\left(dx+c\right)-2\right)\left(\cos\left(dx+c\right)\right)^{3}}\left(32\,\sin\left(dx+c\right)\left(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/128/d*(32*sin(d*x+c)*cos(d*x+c)^4-32*cos(d*x+c)^5+48*cos(d*x+c)^3*sin(d*x
+c)+45*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)+45*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+80*cos(d*x+c)^4-60*cos(d*x+c)^2*sin(d*x+c
)+12*cos(d*x+c)^3+90*cos(d*x+c)*sin(d*x+c)+30*cos(d*x+c)^2-90*cos(d*x+c))*(
e*cos(d*x+c))^(5/2)*(a*(1+sin(d*x+c)))^(3/2)/(cos(d*x+c)*sin(d*x+c)+cos(d*x
+c)^2-2*sin(d*x+c)+cos(d*x+c)-2)/cos(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^(3/2), x)
```

3.281 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=278

$$-\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a\sin(c+dx)+a}}+\frac{7ae^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{8d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{7ae^{3/2}\sqrt{\cos(c+}{8}$$

```
[Out] (-7*a^2*(e*Cos[c + d*x])^(5/2))/(12*d*e*Sqrt[a + a*Sin[c + d*x]]) + (7*a*e*
Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(8*d) - (a*(e*Cos[c + d*x])^
(5/2)*Sqrt[a + a*Sin[c + d*x]])/(3*d*e) - (7*a*e^(3/2)*ArcSinh[Sqrt[e*Cos[c
 + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(8*d*(1
+ Cos[c + d*x] + Sin[c + d*x])) + (7*a*e^(3/2)*ArcTan[(Sqrt[e]*Sin[c + d*x]
)/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x]]*Sqr
t[a + a*Sin[c + d*x]])/(8*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rubi [A] time = 0.474727, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2678, 2685, 2677, 2775, 203, 2833, 63, 215}

$$-\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a\sin(c+dx)+a}}+\frac{7ae^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{8d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{7ae^{3/2}\sqrt{\cos(c+}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2),x]
```



```
[Out] (-7*a^2*(e*Cos[c + d*x])^(5/2))/(12*d*e*Sqrt[a + a*Sin[c + d*x]]) + (7*a*e*
Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(8*d) - (a*(e*Cos[c + d*x])^
(5/2)*Sqrt[a + a*Sin[c + d*x]])/(3*d*e) - (7*a*e^(3/2)*ArcSinh[Sqrt[e*Cos[c
 + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(8*d*(1
+ Cos[c + d*x] + Sin[c + d*x])) + (7*a*e^(3/2)*ArcTan[(Sqrt[e]*Sin[c + d*x]
)/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x]]*Sqr
t[a + a*Sin[c + d*x]])/(8*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```


Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
```

```
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}\,dx = -\frac{a(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}{3de}+\frac{1}{6}(7a)\int (e\cos(c+dx))^{3/2}\sqrt{a+$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}}-\frac{a(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}{3de}+\frac{1}{8}\big(7a$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}}+\frac{7ae\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{8d}-\frac{a(e\,c$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}}+\frac{7ae\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{8d}-\frac{a(e\,c$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}}+\frac{7ae\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{8d}-\frac{a(e\,c$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}}+\frac{7ae\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{8d}-\frac{a(e\,c$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}}+\frac{7ae\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{8d}-\frac{a(e\,c$$

Mathematica [C] time = 0.131691, size = 78, normalized size = 0.28

$$-\frac{8\,2^{3/4}a\sqrt{a(\sin(c+dx)+1)}(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{7}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*2^(3/4)*a*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-7/4, 5/4, 9/4, (1 -
 Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(5*d*e*(1 + Sin[c + d*x])^(7/
4))
```

Maple [A] time = 0.17, size = 288, normalized size = 1.

$$\frac{1}{48\,d\left(\cos(dx+c)\sin(dx+c)+(\cos(dx+c))^{2}-2\sin(dx+c)+\cos(dx+c)-2\right)(\cos(dx+c))^{2}}\left(16\,(\cos(dx+c))^{3}\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/48/d*(16*cos(d*x+c)^3*sin(d*x+c)-21*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)
+21*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-16*cos(d*x
+c)^4+28*cos(d*x+c)^2*sin(d*x+c)+44*cos(d*x+c)^3-42*cos(d*x+c)*sin(d*x+c)+1
4*cos(d*x+c)^2-42*cos(d*x+c))*(e*cos(d*x+c))^(3/2)*(a*(1+sin(d*x+c)))^(3/2)
/(cos(d*x+c)*sin(d*x+c)+cos(d*x+c)^2-2*sin(d*x+c)+cos(d*x+c)-2)/cos(d*x+c)^
2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}\,(a\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}\,(a\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^(3/2), x)
```

3.282 $\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=243

$$-\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a\sin(c+dx)+a}}-\frac{a\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{3/2}}{2de}+\frac{5a\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\Big(\frac{}{\sqrt{\cos(}}}{4d(\sin(c+dx)+\cos(c+dx)+1)}$$

```
[Out] (-5*a^2*(e*Cos[c + d*x])^(3/2))/(4*d*e*Sqrt[a + a*Sin[c + d*x]]) - (a*(e*Co
s[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]])/(2*d*e) + (5*a*Sqrt[e]*ArcSinh[
Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x
]])/(4*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (5*a*Sqrt[e]*ArcTan[(Sqrt[e]*
Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c
 + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rubi [A] time = 0.358337, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2678, 2684, 2775, 203, 2833, 63, 215}

$$-\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a\sin(c+dx)+a}}-\frac{a\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{3/2}}{2de}+\frac{5a\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\Big(\frac{}{\sqrt{\cos(}}}{4d(\sin(c+dx)+\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2),x]
```



```
[Out] (-5*a^2*(e*Cos[c + d*x])^(3/2))/(4*d*e*Sqrt[a + a*Sin[c + d*x]]) - (a*(e*Co
s[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]])/(2*d*e) + (5*a*Sqrt[e]*ArcSinh[
Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x
]])/(4*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (5*a*Sqrt[e]*ArcTan[(Sqrt[e]*
Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c
 + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```


Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}\,dx = -\frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de} + \frac{1}{4}(5a)\int \sqrt{e\cos(c+dx)}\sqrt{a+a\sin$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de} + \frac{1}{8}\left(5a^2\right)\int$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de} + \frac{\left(5a^2e\sqrt{1}\right.}{}$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de} + \frac{\left(5a^2e\sqrt{1}\right.}{}$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de} + \frac{5a^2\sqrt{e}\tan}{}$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de} + \frac{5a^2\sqrt{e}\sin}{}$$

Mathematica [C] time = 0.117161, size = 77, normalized size = 0.32

$$-\frac{8\sqrt[4]{2}(a(\sin(c+dx)+1))^{3/2}(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*2^(1/4)*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-5/4, 3/4, 7/4, (1 - S
in[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(3/2))/(3*d*e*(1 + Sin[c + d*x])^(9/
4))
```

Maple [A] time = 0.166, size = 262, normalized size = 1.1

$$\frac{1}{8\,d\left(\cos(dx+c)\sin(dx+c)+(\cos(dx+c))^2-2\,\sin(dx+c)+\cos(dx+c)-2\right)\cos(dx+c)}\left(5\sqrt{-2\,\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)+5*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+4*cos(d*x+c)^2*sin(d*x+c)-4*cos(d*x+c)^3
+10*cos(d*x+c)*sin(d*x+c)+14*cos(d*x+c)^2-10*cos(d*x+c))*(a*(1+sin(d*x+c)))
^(3/2)*(e*cos(d*x+c))^(1/2)/(cos(d*x+c)*sin(d*x+c)+cos(d*x+c)^2-2*sin(d*x+c
)+cos(d*x+c)-2)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos\left(dx+c\right)}\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^(3/2), x)
```

3.283 $\int \frac{(a+a\sin(c+dx))^{3/2}}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=198

$$-\frac{a\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}{de}+\frac{3a\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d\sqrt{e}(\sin(c+dx)+\cos(c+dx)+1)}-\frac{3a\sqrt{\cos}}{}$$

```
[Out] -((a*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e)) - (3*a*ArcSinh[S
qrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]
])/(d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x])) + (3*a*ArcTan[(Sqrt[e]*Sin
[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c +
d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x]
))
```

Rubi [A] time = 0.284202, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2678, 2677, 2775, 203, 2833, 63, 215}

$$-\frac{a\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}{de}+\frac{3a\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d\sqrt{e}(\sin(c+dx)+\cos(c+dx)+1)}-\frac{3a\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] -((a*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e)) - (3*a*ArcSinh[S
qrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]
])/(d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x])) + (3*a*ArcTan[(Sqrt[e]*Sin
[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c +
d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x]
))
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
```

```
*m, 2*p]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{a\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{de}+\frac{1}{2}(3a)\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$$

$$= -\frac{a\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{de}+\frac{\left(3a^2\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\mathrm{co}}}{\sqrt{e\cos}}}{2(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= -\frac{a\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{de}-\frac{\left(3a^2\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\mathrm{Subst}\Big(}{2d(a+a\cos(c+dx)+a\sin(}$$

$$= -\frac{a\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{de}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+d}}{d\sqrt{e}(a+a\cos(c+dx)+a\sin(c+}$$

$$= -\frac{a\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{de}-\frac{3a^2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\,\mathrm{s}}}{d\sqrt{e}(a+a\cos(c+dx)+a\sin(c+dx))}$$

Mathematica [C] time = 0.10468, size = 75, normalized size = 0.38

$$-\frac{4\,2^{3/4}(a(\sin(c+dx)+1))^{3/2}\sqrt{e\cos(c+dx)}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-4*2^(3/4)*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-3/4, 1/4, 5/4, (1 - Sin
[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(3/2))/(d*e*(1 + Sin[c + d*x])^(7/4))
```

Maple [A] time = 0.145, size = 228, normalized size = 1.2

$$\frac{1}{2\,d\left(\cos(dx+c)\sin(dx+c)+(\cos(dx+c))^2-2\,\sin(dx+c)+\cos(dx+c)-2\right)}\left(-3\sqrt{-2\,\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\Big(1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(-3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)+3*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+2*cos(d*x+c)*sin(d*x+c)-2*cos(d*x+c)^2+
2*cos(d*x+c))*(a*(1+sin(d*x+c)))^(3/2)/(cos(d*x+c)*sin(d*x+c)+cos(d*x+c)^2-
2*sin(d*x+c)+cos(d*x+c)-2)/(e*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^{\frac{3}{2}}}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)/sqrt(e*cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^{\frac{3}{2}}}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)/sqrt(e*cos(d*x + c)), x)
```

3.284 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=210

$$-\frac{2a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{de^{3/2}(a\sin(c+dx)+a\cos(c+dx)+a)}-\frac{2a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}}{de^{3/2}(a\sin(c+dx)+a\cos(c+dx)+}$$

```
[Out] (4*a*Sqrt[a + a*Sin[c + d*x]])/(d*e*Sqrt[e*Cos[c + d*x]]) - (2*a^2*ArcSinh[
Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x
]])/(d*e^(3/2)*(a + a*Cos[c + d*x] + a*Sin[c + d*x])) - (2*a^2*ArcTan[(Sqrt
[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + C
os[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e^(3/2)*(a + a*Cos[c + d*x] + a*S
in[c + d*x]))
```

Rubi [A] time = 0.293432, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2676, 2684, 2775, 203, 2833, 63, 215}

$$-\frac{2a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{de^{3/2}(a\sin(c+dx)+a\cos(c+dx)+a)}-\frac{2a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}}{de^{3/2}(a\sin(c+dx)+a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (4*a*Sqrt[a + a*Sin[c + d*x]])/(d*e*Sqrt[e*Cos[c + d*x]]) - (2*a^2*ArcSinh[
Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x
]])/(d*e^(3/2)*(a + a*Cos[c + d*x] + a*Sin[c + d*x])) - (2*a^2*ArcTan[(Sqrt
[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + C
os[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e^(3/2)*(a + a*Cos[c + d*x] + a*S
in[c + d*x]))
```

Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
```

```
gersQ[2*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{a^2\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx}{e^2}$$

$$= \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{\left(a^2\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{e(a+a\cos(c+dx)+a\sin(c+dx))} + \frac{\left(a^2\sqrt{1\right.}{}$$

$$= \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{\left(a^2\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x}}\,dx, x, \mathrm{c}\right.}{de(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{2a^2\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+}}{de^{3/2}(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{2a^2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{de^{3/2}(a+a\cos(c+dx)+a\sin(c+dx))} - \frac{2a^2}{}$$

Mathematica [C] time = 0.115476, size = 75, normalized size = 0.36

$$\frac{4\sqrt[4]{2}(a(\sin(c+dx)+1))^{3/2}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de(\sin(c+dx)+1)^{5/4}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (4*2^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(a*(1 +
 Sin[c + d*x]))^(3/2))/(d*e*Sqrt[e*Cos[c + d*x]]*(1 + Sin[c + d*x])^(5/4))
```

Maple [A] time = 0.12, size = 323, normalized size = 1.5

$$-2\,\frac{(a(1+\sin(dx+c)))^{3/2}(-1+\cos(dx+c))}{d\sin(dx+c)(1-\cos(dx+c)+\sin(dx+c))(e\cos(dx+c))^{3/2}}\left(\sqrt{2}\arctan\left(1/2\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sin(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d*(a*(1+sin(d*x+c)))^(3/2)*(-1+cos(d*x+c))*(2^(1/2)*arctan(1/2*2^(1/2)*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+2^(1/2)*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*cos(d*x+c)*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)-2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2))-2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)/cos(d*x+c))+2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)
/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(1-cos(d*x+c)+sin(d*x+c))/(e*cos(d*x+
c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^{\frac{3}{2}}}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)/(e*cos(d*x + c))^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.285 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{5/2}} dx$

Optimal. Leaf size=36

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] (2*(a + a*Sin[c + d*x])^(3/2))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0725385, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2671}

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(3/2))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{5/2}} dx = \frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.106918, size = 36, normalized size = 1.

$$\frac{2(a(\sin(c+dx)+1))^{3/2}}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(3/2))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.089, size = 34, normalized size = 0.9

$$\frac{2\cos\left(dx+c\right)}{3d}\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{3}{2}}\left(e\cos\left(dx+c\right)\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*cos(d*x+c)*(a*(1+sin(d*x+c)))^(3/2)/(e*cos(d*x+c))^(5/2)
```

Maxima [B] time = 1.57786, size = 177, normalized size = 4.92

$$\frac{2\left(a^{\frac{3}{2}}\sqrt{e}-\frac{a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)}{3\left(e^{3}+\frac{e^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] 2/3*(a^(3/2)*sqrt(e) - a^(3/2)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*(sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 1)/((e^3 + e^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d*(-sin(d*x + c)
/(cos(d*x + c) + 1) + 1)^(5/2))
```

Fricas [A] time = 2.78553, size = 112, normalized size = 3.11

$$-\frac{2\sqrt{e\cos\left(dx+c\right)}\sqrt{a\sin\left(dx+c\right)+aa}}{3\left(de^{3}\sin\left(dx+c\right)-de^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/3*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)*a/(d*e^3*sin(d*x + c) -
d*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.286 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=74

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{de(e\cos(c+dx))^{5/2}}-\frac{4(a\sin(c+dx)+a)^{5/2}}{5ade(e\cos(c+dx))^{5/2}}$$

```
[Out] (2*(a + a*Sin[c + d*x])^(3/2))/(d*e*(e*Cos[c + d*x])^(5/2)) - (4*(a + a*Sin
[c + d*x])^(5/2))/(5*a*d*e*(e*Cos[c + d*x])^(5/2))
```

Rubi [A] time = 0.148083, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{de(e\cos(c+dx))^{5/2}}-\frac{4(a\sin(c+dx)+a)^{5/2}}{5ade(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(3/2))/(d*e*(e*Cos[c + d*x])^(5/2)) - (4*(a + a*Sin
[c + d*x])^(5/2))/(5*a*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{7/2}}\,dx}{a}$$
$$= \frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{5/2}} - \frac{4(a+a\sin(c+dx))^{5/2}}{5ade(e\cos(c+dx))^{5/2}}$$

Mathematica [A] time = 0.128922, size = 72, normalized size = 0.97

$$-\frac{2a(2\sin(c+dx)-3)\sqrt{a(\sin(c+dx)+1)}}{5de^3\sqrt{e\cos(c+dx)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*a*Sqrt[a*(1 + Sin[c + d*x])]*(-3 + 2*Sin[c + d*x]))/(5*d*e^3*Sqrt[e*Cos
[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2)
```

Maple [A] time = 0.096, size = 44, normalized size = 0.6

$$-\frac{(4\sin(dx+c)-6)\cos(dx+c)}{5d}(a(1+\sin(dx+c)))^{\frac{3}{2}}(e\cos(dx+c))^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/d*(2*sin(d*x+c)-3)*(a*(1+sin(d*x+c)))^(3/2)*cos(d*x+c)/(e*cos(d*x+c))^
(7/2)
```

Maxima [B] time = 1.5794, size = 279, normalized size = 3.77

$$\frac{2\left(3a^{\frac{3}{2}}\sqrt{e}-\frac{4a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{5\left(e^4+\frac{2e^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{e^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxima"
)
```

```
[Out] 2/5*(3*a^(3/2)*sqrt(e) - 4*a^(3/2)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1)
+ 4*a^(3/2)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*a^(3/2)*sqrt(e)
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 1)^2/((e^4 + 2*e^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + e^4*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4)*d*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*(-sin(
d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2))
```

Fricas [A] time = 2.54234, size = 177, normalized size = 2.39

$$\frac{2\sqrt{e\cos(dx+c)}(2a\sin(dx+c)-3a)\sqrt{a\sin(dx+c)+a}}{5\left(de^4\cos(dx+c)\sin(dx+c)-de^4\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(7/2),x, algorithm="fricas"
)
```

```
[Out] 2/5*sqrt(e*cos(d*x + c))*(2*a*sin(d*x + c) - 3*a)*sqrt(a*sin(d*x + c) + a)/
(d*e^4*cos(d*x + c)*sin(d*x + c) - d*e^4*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.287 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{16(a\sin(c+dx)+a)^{7/2}}{21a^2de(e\cos(c+dx))^{7/2}}+\frac{8(a\sin(c+dx)+a)^{5/2}}{3ade(e\cos(c+dx))^{7/2}}-\frac{2(a\sin(c+dx)+a)^{3/2}}{de(e\cos(c+dx))^{7/2}}$$

```
[Out] (-2*(a + a*Sin[c + d*x])^(3/2))/(d*e*(e*Cos[c + d*x])^(7/2)) + (8*(a + a*Si
n[c + d*x])^(5/2))/(3*a*d*e*(e*Cos[c + d*x])^(7/2)) - (16*(a + a*Sin[c + d*
x])^(7/2))/(21*a^2*d*e*(e*Cos[c + d*x])^(7/2))
```

Rubi [A] time = 0.229943, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{16(a\sin(c+dx)+a)^{7/2}}{21a^2de(e\cos(c+dx))^{7/2}}+\frac{8(a\sin(c+dx)+a)^{5/2}}{3ade(e\cos(c+dx))^{7/2}}-\frac{2(a\sin(c+dx)+a)^{3/2}}{de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (-2*(a + a*Sin[c + d*x])^(3/2))/(d*e*(e*Cos[c + d*x])^(7/2)) + (8*(a + a*Si
n[c + d*x])^(5/2))/(3*a*d*e*(e*Cos[c + d*x])^(7/2)) - (16*(a + a*Sin[c + d*
x])^(7/2))/(21*a^2*d*e*(e*Cos[c + d*x])^(7/2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{9/2}}\,dx &= -\frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{7/2}}+\frac{4\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{9/2}}\,dx}{a}\\
&= -\frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{7/2}}+\frac{8(a+a\sin(c+dx))^{5/2}}{3ade(e\cos(c+dx))^{7/2}}-\frac{8\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{9/2}}\,dx}{3a^2}\\
&= -\frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{7/2}}+\frac{8(a+a\sin(c+dx))^{5/2}}{3ade(e\cos(c+dx))^{7/2}}-\frac{16(a+a\sin(c+dx))^{7/2}}{21a^2de(e\cos(c+dx))^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.196198, size = 105, normalized size = 0.93

$$
\frac{2a\sqrt{a(\sin(c+dx)+1)}(12\sin(c+dx)+4\cos(2(c+dx))-5)}{21de^4\sqrt{e\cos(c+dx)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Sin[c + d*x])]*(-5 + 4*Cos[2*(c + d*x)] + 12*Sin[c + d*x])
)/(21*d*e^4*Sqrt[e*Cos[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.102, size = 54, normalized size = 0.5

$$
\frac{\left(16\,(\cos(dx+c))^2+24\,\sin(dx+c)-18\right)\cos(dx+c)}{21\,d}\,(a\,(1+\sin(dx+c)))^{\frac{3}{2}}\,(e\cos(dx+c))^{-\frac{9}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] 2/21/d*(8*cos(d*x+c)^2+12*sin(d*x+c)-9)*(a*(1+sin(d*x+c)))^(3/2)*cos(d*x+c)
/(e*cos(d*x+c))^(9/2)
```

MaximA [B] time = 1.60803, size = 379, normalized size = 3.35

$$-\frac{2\left(a^{\frac{3}{2}}\sqrt{e}-\frac{24\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{33\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{33\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{24\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{3}}{21\left(e^{5}+\frac{3\,e^{5}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\,e^{5}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{e^{5}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(9/2),x, algorithm="maxima"
)
```

```
[Out] -2/21*(a^(3/2)*sqrt(e) - 24*a^(3/2)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1)
 + 33*a^(3/2)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 33*a^(3/2)*sqrt
(e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 24*a^(3/2)*sqrt(e)*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - a^(3/2)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((e^5 + 3*e^5*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*e^5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + e^5*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3
/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2))
```

Fricas [A] time = 2.2969, size = 215, normalized size = 1.9

$$-\frac{2\left(8\,a\cos\left(dx+c\right)^{2}+12\,a\sin\left(dx+c\right)-9\,a\right)\sqrt{e\cos\left(dx+c\right)}\sqrt{a\sin\left(dx+c\right)+a}}{21\left(de^{5}\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-de^{5}\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(9/2),x, algorithm="fricas"
)
```

```
[Out] -2/21*(8*a*cos(d*x + c)^2 + 12*a*sin(d*x + c) - 9*a)*sqrt(e*cos(d*x + c))*s
qrt(a*sin(d*x + c) + a)/(d*e^5*cos(d*x + c)^2*sin(d*x + c) - d*e^5*cos(d*x
+ c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.288 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=152

$$\frac{32(a\sin(c+dx)+a)^{9/2}}{45a^3de(e\cos(c+dx))^{9/2}}-\frac{16(a\sin(c+dx)+a)^{7/2}}{5a^2de(e\cos(c+dx))^{9/2}}+\frac{4(a\sin(c+dx)+a)^{5/2}}{ade(e\cos(c+dx))^{9/2}}-\frac{2(a\sin(c+dx)+a)^{3/2}}{3de(e\cos(c+dx))^{9/2}}$$

```
[Out] (-2*(a + a*Sin[c + d*x])^(3/2))/(3*d*e*(e*Cos[c + d*x])^(9/2)) + (4*(a + a*
Sin[c + d*x])^(5/2))/(a*d*e*(e*Cos[c + d*x])^(9/2)) - (16*(a + a*Sin[c + d*
x])^(7/2))/(5*a^2*d*e*(e*Cos[c + d*x])^(9/2)) + (32*(a + a*Sin[c + d*x])^(9
/2))/(45*a^3*d*e*(e*Cos[c + d*x])^(9/2))
```

Rubi [A] time = 0.308801, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{32(a\sin(c+dx)+a)^{9/2}}{45a^3de(e\cos(c+dx))^{9/2}}-\frac{16(a\sin(c+dx)+a)^{7/2}}{5a^2de(e\cos(c+dx))^{9/2}}+\frac{4(a\sin(c+dx)+a)^{5/2}}{ade(e\cos(c+dx))^{9/2}}-\frac{2(a\sin(c+dx)+a)^{3/2}}{3de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(11/2),x]
```


```
[Out] (-2*(a + a*Sin[c + d*x])^(3/2))/(3*d*e*(e*Cos[c + d*x])^(9/2)) + (4*(a + a*
Sin[c + d*x])^(5/2))/(a*d*e*(e*Cos[c + d*x])^(9/2)) - (16*(a + a*Sin[c + d*
x])^(7/2))/(5*a^2*d*e*(e*Cos[c + d*x])^(9/2)) + (32*(a + a*Sin[c + d*x])^(9
/2))/(45*a^3*d*e*(e*Cos[c + d*x])^(9/2))
```


Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
```

```
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{11/2}}\,dx = -\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{9/2}} + \frac{2\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{11/2}}\,dx}{a}$$

$$= -\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{9/2}} + \frac{4(a+a\sin(c+dx))^{5/2}}{ade(e\cos(c+dx))^{9/2}} - \frac{8\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{11/2}}\,dx}{a^2}$$

$$= -\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{9/2}} + \frac{4(a+a\sin(c+dx))^{5/2}}{ade(e\cos(c+dx))^{9/2}} - \frac{16(a+a\sin(c+dx))^{7/2}}{5a^2de(e\cos(c+dx))^{9/2}} + \frac{16\int \frac{(a+a\si}{(e\cos}}{5}$$

$$= -\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{9/2}} + \frac{4(a+a\sin(c+dx))^{5/2}}{ade(e\cos(c+dx))^{9/2}} - \frac{16(a+a\sin(c+dx))^{7/2}}{5a^2de(e\cos(c+dx))^{9/2}} + \frac{32(a+a\,s}{45a^3de(e\,c}$$

Mathematica [A] time = 0.248208, size = 74, normalized size = 0.49

$$\frac{2\sec^5(c+dx)(a(\sin(c+dx)+1))^{3/2}\sqrt{e\cos(c+dx)}(6\sin(c+dx)-4\sin(3(c+dx))+12\cos(2(c+dx))+7)}{45de^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^5*(a*(1 + Sin[c + d*x]))^(3/2)*(7 + 12
*Cos[2*(c + d*x)] + 6*Sin[c + d*x] - 4*Sin[3*(c + d*x)]))/(45*d*e^6)
```

Maple [A] time = 0.112, size = 70, normalized size = 0.5

$$-\frac{\left(32\,(\cos(dx+c))^2\sin(dx+c)-48\,(\cos(dx+c))^2-20\,\sin(dx+c)+10\right)\cos(dx+c)}{45\,d}(a\,(1+\sin(dx+c)))^{\frac{3}{2}}\,(e\cos(\,$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(11/2),x)
```

```
[Out] -2/45/d*(16*cos(d*x+c)^2*sin(d*x+c)-24*cos(d*x+c)^2-10*sin(d*x+c)+5)*(a*(1+
sin(d*x+c)))^(3/2)*cos(d*x+c)/(e*cos(d*x+c))^(11/2)
```

Maxima [B] time = 1.66688, size = 482, normalized size = 3.17

$$\frac{2\left(19\,a^{\frac{3}{2}}\sqrt{e}-\frac{12\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{58\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{116\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{116\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{58\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{12\,a^{\frac{3}{2}}\sqrt{e}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right.}{45\left(e^{6}+\frac{4\,e^{6}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,e^{6}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4\,e^{6}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{e^{6}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(11/2),x, algorithm="maxima
")
```

```
[Out] 2/45*(19*a^(3/2)*sqrt(e) - 12*a^(3/2)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) +
1) - 58*a^(3/2)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 116*a^(3/2)*s
qrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 116*a^(3/2)*sqrt(e)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 + 58*a^(3/2)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 + 12*a^(3/2)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 19*a^(3/
2)*sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^4/((e^6 + 4*e^6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*e^6*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*e^6*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + e^6*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2))
```

Fricas [A] time = 2.38017, size = 248, normalized size = 1.63

$$-\frac{2\left(24\,a\cos\left(dx+c\right)^{2}-2\left(8\,a\cos\left(dx+c\right)^{2}-5\,a\right)\sin\left(dx+c\right)-5\,a\right)\sqrt{e\cos\left(dx+c\right)}\sqrt{a\sin\left(dx+c\right)+a}}{45\left(de^{6}\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-de^{6}\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(11/2),x, algorithm="fricas
")
```

```
[Out] -2/45*(24*a*cos(d*x + c)^2 - 2*(8*a*cos(d*x + c)^2 - 5*a)*sin(d*x + c) - 5*
a)*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*e^6*cos(d*x + c)^3*sin(
```

```
d*x + c) - d*e^6*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(11/2),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(11/2),x, algorithm="giac")

[Out] Timed out
```

3.289 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=323

$$\frac{77a^2e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{64d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{77a^2e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+}}{64d(\sin(c+dx)+\cos(c+d}$$

```
[Out] (-77*a^3*(e*Cos[c + d*x])^(5/2))/(96*d*e*Sqrt[a + a*Sin[c + d*x]]) + (77*a^
2*e*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(64*d) - (11*a^2*(e*Cos[
c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]])/(24*d*e) - (77*a^2*e^(3/2)*ArcSin
h[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d
*x]])/(64*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (77*a^2*e^(3/2)*ArcTan[(Sq
rt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 +
 Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(64*d*(1 + Cos[c + d*x] + Sin[c +
d*x])) - (a*(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2))/(4*d*e)
```

Rubi [A] time = 0.543058, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2678, 2685, 2677, 2775, 203, 2833, 63, 215}

$$\frac{77a^2e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{64d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{77a^2e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+}}{64d(\sin(c+dx)+\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-77*a^3*(e*Cos[c + d*x])^(5/2))/(96*d*e*Sqrt[a + a*Sin[c + d*x]]) + (77*a^
2*e*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(64*d) - (11*a^2*(e*Cos[
c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]])/(24*d*e) - (77*a^2*e^(3/2)*ArcSin
h[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d
*x]])/(64*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (77*a^2*e^(3/2)*ArcTan[(Sq
rt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 +
 Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(64*d*(1 + Cos[c + d*x] + Sin[c +
d*x])) - (a*(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2))/(4*d*e)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}\,dx &= -\frac{a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}}{4de}+\frac{1}{8}(11a)\int (e\cos(c+dx))^{3/2}(a+ \\
&= -\frac{11a^2(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}{24de}-\frac{a(e\cos(c+dx))^{5/2}(a+a\sin(c}{4de} \\
&= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}}-\frac{11a^2(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}{24de}-\frac{a(e}{} \\
&= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}}+\frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{64d}-\frac{11a^2}{} \\
&= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}}+\frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{64d}-\frac{11a^2}{} \\
&= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}}+\frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{64d}-\frac{11a^2}{} \\
&= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}}+\frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{64d}-\frac{11a^2}{} \\
&= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}}+\frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{64d}-\frac{11a^2}{}
\end{aligned}$$

Mathematica [C] time = 0.283164, size = 77, normalized size = 0.24

$$-\frac{16\ 2^{3/4}(a(\sin(c+dx)+1))^{5/2}(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{11}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-16*2^(3/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-11/4, 5/4, 9/4, (1 -
 Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(5/2))/(5*d*e*(1 + Sin[c + d*x])^(
15/4))
```

Maple [A] time = 0.208, size = 344, normalized size = 1.1

$$-\frac{1}{384\,d\left((\cos(dx+c))^2\sin(dx+c)-(\cos(dx+c))^3+2\,\cos(dx+c)\sin(dx+c)+3\,(\cos(dx+c))^2-4\,\sin(dx+c)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/384/d*(96*sin(d*x+c)*cos(d*x+c)^4+96*cos(d*x+c)^5-368*cos(d*x+c)^3*sin(d
*x+c)+231*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)-231*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+272*cos(d*x+c)^4-308*cos(d*x+c)^2*sin
(d*x+c)-676*cos(d*x+c)^3+462*cos(d*x+c)*sin(d*x+c)-154*cos(d*x+c)^2+462*cos
(d*x+c))*(e*cos(d*x+c))^(3/2)*(a*(1+sin(d*x+c)))^(5/2)/(cos(d*x+c)^2*sin(d*
x+c)-cos(d*x+c)^3+2*cos(d*x+c)*sin(d*x+c)+3*cos(d*x+c)^2-4*sin(d*x+c)+2*cos
(d*x+c)-4)/cos(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}\,(a\sin(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^(5/2), x)
```

3.290 $\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=286

$$-\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a\sin(c+dx)+a}}-\frac{3a^2\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{3/2}}{4de}+\frac{15a^2\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}}{8d(\sin(c+dx)+\cos(c+dx)}$$

```
[Out] (-15*a^3*(e*Cos[c + d*x])^(3/2))/(8*d*e*Sqrt[a + a*Sin[c + d*x]]) - (3*a^2*
(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]])/(4*d*e) + (15*a^2*Sqrt[e]*
ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin
[c + d*x]])/(8*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (15*a^2*Sqrt[e]*ArcTa
n[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqr
t[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(8*d*(1 + Cos[c + d*x] + Sin[
c + d*x])) - (a*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2))/(3*d*e)
```

Rubi [A] time = 0.436506, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2678, 2684, 2775, 203, 2833, 63, 215}

$$-\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a\sin(c+dx)+a}}-\frac{3a^2\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{3/2}}{4de}+\frac{15a^2\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}}{8d(\sin(c+dx)+\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-15*a^3*(e*Cos[c + d*x])^(3/2))/(8*d*e*Sqrt[a + a*Sin[c + d*x]]) - (3*a^2*
(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]])/(4*d*e) + (15*a^2*Sqrt[e]*
ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin
[c + d*x]])/(8*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (15*a^2*Sqrt[e]*ArcTa
n[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqr
t[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(8*d*(1 + Cos[c + d*x] + Sin[
c + d*x])) - (a*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2))/(3*d*e)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
```

```
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{5/2}\,dx &= -\frac{a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}{3de}+\frac{1}{2}(3a)\int \sqrt{e\cos(c+dx)}(a+a\,\mathrm{s}\\
&= -\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a(e\cos(c+dx))^{3/2}(a+a\sin(c-}{3de}\\
&= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}}-\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a(e\,\mathrm{co}}{}\\
&= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}}-\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a(e\,\mathrm{co}}{}\\
&= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}}-\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a(e\,\mathrm{co}}{}\\
&= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}}-\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a(e\,\mathrm{co}}{}\\
&= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}}-\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a(e\,\mathrm{co}}{}
\end{aligned}
$$

Mathematica [C] time = 0.121617, size = 78, normalized size = 0.27

$$
-\frac{16\sqrt[4]{2}a(a(\sin(c+dx)+1))^{3/2}(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{9}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(\sin(c+dx)+1)^{9/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-16*2^(1/4)*a*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-9/4, 3/4, 7/4, (1
- Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(3/2))/(3*d*e*(1 + Sin[c + d*x])^
(9/4))
```

Maple [A] time = 0.183, size = 317, normalized size = 1.1

$$\frac{1}{48\,d\left((\cos(dx+c))^{3}-(\cos(dx+c))^{2}\sin(dx+c)-3\,(\cos(dx+c))^{2}-2\,\cos(dx+c)\sin(dx+c)-2\,\cos(dx+c)+4\,\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/48/d*(16*cos(d*x+c)^4+16*cos(d*x+c)^3*sin(d*x+c)-45*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^
(1/2)*sin(d*x+c)-45*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d
*x+c)+52*cos(d*x+c)^3-68*cos(d*x+c)^2*sin(d*x+c)-158*cos(d*x+c)^2-90*cos(d*
x+c)*sin(d*x+c)+90*cos(d*x+c))*(a*(1+sin(d*x+c)))^(5/2)*(e*cos(d*x+c))^(1/2
)/(cos(d*x+c)^3-cos(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)^2-2*cos(d*x+c)*sin(d*x
+c)-2*cos(d*x+c)+4*sin(d*x+c)+4)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)*(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)*(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^(5/2), x)
```

3.291 $\int \frac{(a+a\sin(c+dx))^{5/2}}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=247

$$-\frac{7a^2\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}{4de}+\frac{21a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{4d\sqrt{e}(\sin(c+dx)+\cos(c+dx)+1)}-\frac{21a^2}{}$$

```
[Out] (-7*a^2*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*e) - (21*a^2*Ar
cSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c
 + d*x]])/(4*d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x])) + (21*a^2*ArcTan[
(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[
1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*Sqrt[e]*(1 + Cos[c + d*x]
+ Sin[c + d*x])) - (a*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2))/(2*d
*e)
```

Rubi [A] time = 0.358691, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2678, 2677, 2775, 203, 2833, 63, 215}

$$-\frac{7a^2\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}{4de}+\frac{21a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{4d\sqrt{e}(\sin(c+dx)+\cos(c+dx)+1)}-\frac{21a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-7*a^2*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*e) - (21*a^2*Ar
cSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c
 + d*x]])/(4*d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x])) + (21*a^2*ArcTan[
(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[
1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*Sqrt[e]*(1 + Cos[c + d*x]
+ Sin[c + d*x])) - (a*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2))/(2*d
*e)
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
```

```
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}}{2de}+\frac{1}{4}(7a)\int \frac{(a+a\sin(c+dx))^{3/2}}{\sqrt{e\cos(c+dx)}}\,dx$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}}{2de}+\frac{1}{8}\left(21a^2\right)$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}}{2de}+\frac{\left(21a^3\sqrt{1}\right.}{}$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}}{2de}-\frac{\left(21a^3\sqrt{1}\right.}{}$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}}{2de}+\frac{21a^3\tan}{}$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4de}-\frac{a\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}}{2de}-\frac{21a^3\sinh}{}$$

Mathematica [C] time = 0.103092, size = 76, normalized size = 0.31

$$-\frac{8\ 2^{3/4}a(a(\sin(c+dx)+1))^{3/2}\sqrt{e\cos(c+dx)}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-8*2^(3/4)*a*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-7/4, 1/4, 5/4, (1 - S
in[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(3/2))/(d*e*(1 + Sin[c + d*x])^(7/4)
)
```

Maple [A] time = 0.169, size = 284, normalized size = 1.2

$$-\frac{1}{8d\left((\cos(dx+c))^2\sin(dx+c)-(\cos(dx+c))^3+2\cos(dx+c)\sin(dx+c)+3(\cos(dx+c))^2-4\sin(dx+c)+2\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/8/d*(a*(1+sin(d*x+c)))^(5/2)*(21*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ar
ctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)-2
1*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+4*cos(d*x+c)
^2*sin(d*x+c)+4*cos(d*x+c)^3-22*cos(d*x+c)*sin(d*x+c)+18*cos(d*x+c)^2-22*co
s(d*x+c))/(cos(d*x+c)^2*sin(d*x+c)-cos(d*x+c)^3+2*cos(d*x+c)*sin(d*x+c)+3*c
os(d*x+c)^2-4*sin(d*x+c)+2*cos(d*x+c)-4)/(e*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^{\frac{5}{2}}}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)/sqrt(e*cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^{\frac{5}{2}}}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)/sqrt(e*cos(d*x + c)), x)
```

3.292 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=239

$$\frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a\sin(c+dx)+a}}-\frac{5a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{de^{3/2}(\sin(c+dx)+\cos(c+dx)+1)}-\frac{5a^2\sqrt{\cos(c+dx)+1}}{de^{3/2}(\sin}$$

```
[Out] (5*a^3*(e*Cos[c + d*x])^(3/2))/(d*e^3*Sqrt[a + a*Sin[c + d*x]]) - (5*a^2*Ar
cSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c
 + d*x]])/(d*e^(3/2)*(1 + Cos[c + d*x] + Sin[c + d*x])) - (5*a^2*ArcTan[(Sq
rt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 +
 Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e^(3/2)*(1 + Cos[c + d*x] + Sin
[c + d*x])) + (4*a*(a + a*Sin[c + d*x])^(3/2))/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.364332, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2676, 2678, 2684, 2775, 203, 2833, 63, 215}

$$\frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a\sin(c+dx)+a}}-\frac{5a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{de^{3/2}(\sin(c+dx)+\cos(c+dx)+1)}-\frac{5a^2\sqrt{\cos(c+dx)+1}}{de^{3/2}(\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(3/2),x]
```


```
[Out] (5*a^3*(e*Cos[c + d*x])^(3/2))/(d*e^3*Sqrt[a + a*Sin[c + d*x]]) - (5*a^2*Ar
cSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c
 + d*x]])/(d*e^(3/2)*(1 + Cos[c + d*x] + Sin[c + d*x])) - (5*a^2*ArcTan[(Sq
rt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 +
 Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e^(3/2)*(1 + Cos[c + d*x] + Sin
[c + d*x])) + (4*a*(a + a*Sin[c + d*x])^(3/2))/(d*e*Sqrt[e*Cos[c + d*x]])
```


Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
```

```
gersQ[2*m, 2*p]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{\left(5a^2\right)\int \sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\,dx}{e^2}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{\left(5a^3\right)\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx}{2e^2}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{\left(5a^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+}\right.}{2e(a+a\cos(c+dx)+a\,\mathrm{si}}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{\left(5a^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+}\right.}{2de(a+a\cos(c+}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{5a^3\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1}}{de^{3/2}(a+a\cos(c+dx}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{5a^3\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+}}{de^{3/2}(a+a\cos(c+dx)+a}$$

Mathematica [C] time = 0.153979, size = 75, normalized size = 0.31

$$\frac{8\sqrt[4]{2}(a(\sin(c+dx)+1))^{5/2}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de(\sin(c+dx)+1)^{9/4}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*2^(1/4)*Hypergeometric2F1[-5/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(a*(1 +
 Sin[c + d*x]))^(5/2))/(d*e*Sqrt[e*Cos[c + d*x]]*(1 + Sin[c + d*x])^(9/4))
```

Maple [B] time = 0.128, size = 445, normalized size = 1.9

$$-\frac{1}{4\,d\left(-\left(\cos\left(dx+c\right)\right)^{2}+2\,\sin\left(dx+c\right)+2\right)}\left(5\,\sqrt{-2\,\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\arctan\left(1/2\,\sqrt{2}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right)\sqrt{2}\sin\left(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)+5*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)-5*cos(d*x+c)*2^(1/2)*arctan(1/2*2^(1/2)
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
-5*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*2^(1
/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))-5*2^(1/2)*a
rctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)-5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))+4*cos
(d*x+c)*sin(d*x+c)-36*cos(d*x+c))*(a*(1+sin(d*x+c)))^(5/2)/(-cos(d*x+c)^2+2
*sin(d*x+c)+2)/(e*cos(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)/(e*cos(d*x + c))^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.293 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{2a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{de^{5/2}(\sin(c+dx)+\cos(c+dx)+1)}+\frac{2a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}}{de^{5/2}(\sin(c+dx)+\cos(c+dx)+1}$$

```
[Out] (2*a^2*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a
+ a*Sin[c + d*x]])/(d*e^(5/2)*(1 + Cos[c + d*x] + Sin[c + d*x])) - (2*a^2*A
rcTan[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]
*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e^(5/2)*(1 + Cos[c + d
*x] + Sin[c + d*x])) + (4*a*(a + a*Sin[c + d*x])^(3/2))/(3*d*e*(e*Cos[c + d
*x])^(3/2))
```

Rubi [A] time = 0.29602, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2676, 2677, 2775, 203, 2833, 63, 215}

$$-\frac{2a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{de^{5/2}(\sin(c+dx)+\cos(c+dx)+1)}+\frac{2a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}}{de^{5/2}(\sin(c+dx)+\cos(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(5/2),x]
```


```
[Out] (2*a^2*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a
+ a*Sin[c + d*x]])/(d*e^(5/2)*(1 + Cos[c + d*x] + Sin[c + d*x])) - (2*a^2*A
rcTan[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]
*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e^(5/2)*(1 + Cos[c + d
*x] + Sin[c + d*x])) + (4*a*(a + a*Sin[c + d*x])^(3/2))/(3*d*e*(e*Cos[c + d
*x])^(3/2))
```


Rule 2676

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e +
f*x])^(m - 1))/(f*g*(p + 1)), x] + Dist[(b^2*(2*m + p - 1))/(g^2*(p + 1)),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
```

```
gersQ[2*m, 2*p]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} - \frac{a^2\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{e^2}$$

$$= \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} - \frac{\left(a^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{e^2(a+a\cos(c+dx)+a\sin(c+dx))} - \Big(a^3$$

$$= \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} + \frac{\left(a^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x}}\,dx, x\right.}{de^2(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} - \frac{2a^3\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c}{de^{5/2}(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} + \frac{2a^3\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{de^{5/2}(a+a\cos(c+dx)+a\sin(c+dx))} - \frac{2}{}$$

Mathematica [C] time = 0.188837, size = 77, normalized size = 0.38

$$\frac{4\, 2^{3/4}(a(\sin(c+dx)+1))^{5/2}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(\sin(c+dx)+1)^{7/4}(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*2^(3/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, (1 - Sin[c + d*x])/2]*(a*(1 +
 Sin[c + d*x]))^(5/2))/(3*d*e*(e*Cos[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(7/
4))
```

Maple [B] time = 0.137, size = 545, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(5/2),x)

[Out] -1/3/d*(3*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)-3*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)-3*cos(d*x+c)^2*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+3*cos(d*x+c)^2*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))-4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)-6*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+6*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-3*cos(d*x+c)*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+3*cos(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))+6*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-6*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)))*(a*(1+sin(d*x+c)))^(5/2)/(1+sin(d*x+c))/sin(d*x+c)/(e*cos(d*x+c))^(5/2)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^{\frac{5}{2}}}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((a*sin(d*x + c) + a)^(5/2)/(e*cos(d*x + c))^(5/2), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.294 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5de(e\cos(c+dx))^{5/2}}$$

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Rubi [A] time = 0.0746067, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2671}

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{7/2}}\,dx=\frac{2(a+a\sin(c+dx))^{5/2}}{5de(e\cos(c+dx))^{5/2}}$$

Mathematica [A] time = 0.173051, size = 36, normalized size = 1.

$$\frac{2(a(\sin(c+dx)+1))^{5/2}}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(5/2))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [A] time = 0.092, size = 34, normalized size = 0.9

$$\frac{2\cos\left(dx+c\right)}{5d}\left(a\left(1+\sin\left(dx+c\right)\right)\right)^{\frac{5}{2}}\left(e\cos\left(dx+c\right)\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] 2/5/d*cos(d*x+c)*(a*(1+sin(d*x+c)))^(5/2)/(e*cos(d*x+c))^(7/2)
```

Maxima [B] time = 1.5902, size = 177, normalized size = 4.92

$$\frac{2\left(a^{\frac{5}{2}}\sqrt{e}-\frac{a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)}{5\left(e^{4}+\frac{e^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxima"
)
```

```
[Out] 2/5*(a^(5/2)*sqrt(e) - a^(5/2)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)/((e^4 + e^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(7/2))
```

Fricas [B] time = 2.66705, size = 265, normalized size = 7.36

$$-\frac{2\left(a^{2}\cos\left(dx+c\right)+a^{2}\sin\left(dx+c\right)+a^{2}\right)\sqrt{e\cos\left(dx+c\right)}\sqrt{a\sin\left(dx+c\right)+a}}{5\left(de^{4}\cos\left(dx+c\right)^{2}-de^{4}\cos\left(dx+c\right)-2\,de^{4}+\left(de^{4}\cos\left(dx+c\right)+2\,de^{4}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(7/2),x, algorithm="fricas"
)
```

```
[Out] -2/5*(a^2*cos(d*x + c) + a^2*sin(d*x + c) + a^2)*sqrt(e*cos(d*x + c))*sqrt(
a*sin(d*x + c) + a)/(d*e^4*cos(d*x + c)^2 - d*e^4*cos(d*x + c) - 2*d*e^4 +
(d*e^4*cos(d*x + c) + 2*d*e^4)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.295 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=76

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{3de(e\cos(c+dx))^{7/2}}-\frac{4(a\sin(c+dx)+a)^{7/2}}{21ade(e\cos(c+dx))^{7/2}}$$

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(3*d*e*(e*Cos[c + d*x])^(7/2)) - (4*(a + a*S
in[c + d*x])^(7/2))/(21*a*d*e*(e*Cos[c + d*x])^(7/2))
```

Rubi [A] time = 0.148437, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{3de(e\cos(c+dx))^{7/2}}-\frac{4(a\sin(c+dx)+a)^{7/2}}{21ade(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(3*d*e*(e*Cos[c + d*x])^(7/2)) - (4*(a + a*S
in[c + d*x])^(7/2))/(21*a*d*e*(e*Cos[c + d*x])^(7/2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{9/2}}\,dx = \frac{2(a+a\sin(c+dx))^{5/2}}{3de(e\cos(c+dx))^{7/2}} - \frac{2\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{9/2}}\,dx}{3a}$$
$$= \frac{2(a+a\sin(c+dx))^{5/2}}{3de(e\cos(c+dx))^{7/2}} - \frac{4(a+a\sin(c+dx))^{7/2}}{21ade(e\cos(c+dx))^{7/2}}$$

Mathematica [A] time = 0.181336, size = 54, normalized size = 0.71

$$-\frac{2(2\sin(c+dx)-5)\sec^4(c+dx)(a(\sin(c+dx)+1))^{5/2}\sqrt{e\cos(c+dx)}}{21de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^4*(a*(1 + Sin[c + d*x]))^(5/2)*(-5 +
2*Sin[c + d*x]))/(21*d*e^5)
```

Maple [A] time = 0.095, size = 44, normalized size = 0.6

$$-\frac{(4\sin(dx+c)-10)\cos(dx+c)}{21d}(a(1+\sin(dx+c)))^{\frac{5}{2}}(e\cos(dx+c))^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] -2/21/d*(2*sin(d*x+c)-5)*(a*(1+sin(d*x+c)))^(5/2)*cos(d*x+c)/(e*cos(d*x+c))
^(9/2)
```

Maxima [B] time = 1.58881, size = 279, normalized size = 3.67

$$\frac{2\left(5a^{\frac{5}{2}}\sqrt{e}-\frac{4a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{21\left(e^5+\frac{2e^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{e^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(9/2),x, algorithm="maxima"
)
```

```
[Out] 2/21*(5*a^(5/2)*sqrt(e) - 4*a^(5/2)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1)
 + 4*a^(5/2)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*a^(5/2)*sqrt(e
)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((e^5 + 2*e^5*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + e^5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d*(-sin
(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2))
```

Fricas [A] time = 2.59937, size = 185, normalized size = 2.43

$$\frac{2\left(2a^2\sin\left(dx+c\right)-5a^2\right)\sqrt{e\cos\left(dx+c\right)}\sqrt{a\sin\left(dx+c\right)+a}}{21\left(de^5\cos\left(dx+c\right)^2+2de^5\sin\left(dx+c\right)-2de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(9/2),x, algorithm="fricas"
)
```

```
[Out] 2/21*(2*a^2*sin(d*x + c) - 5*a^2)*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c)
+ a)/(d*e^5*cos(d*x + c)^2 + 2*d*e^5*sin(d*x + c) - 2*d*e^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.296 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=113

$$\frac{16(a\sin(c+dx)+a)^{9/2}}{45a^2de(e\cos(c+dx))^{9/2}}-\frac{8(a\sin(c+dx)+a)^{7/2}}{5ade(e\cos(c+dx))^{9/2}}+\frac{2(a\sin(c+dx)+a)^{5/2}}{de(e\cos(c+dx))^{9/2}}$$

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(d*e*(e*Cos[c + d*x])^(9/2)) - (8*(a + a*Sin
[c + d*x])^(7/2))/(5*a*d*e*(e*Cos[c + d*x])^(9/2)) + (16*(a + a*Sin[c + d*x
])^(9/2))/(45*a^2*d*e*(e*Cos[c + d*x])^(9/2))
```

Rubi [A] time = 0.223381, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{16(a\sin(c+dx)+a)^{9/2}}{45a^2de(e\cos(c+dx))^{9/2}}-\frac{8(a\sin(c+dx)+a)^{7/2}}{5ade(e\cos(c+dx))^{9/2}}+\frac{2(a\sin(c+dx)+a)^{5/2}}{de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(d*e*(e*Cos[c + d*x])^(9/2)) - (8*(a + a*Sin
[c + d*x])^(7/2))/(5*a*d*e*(e*Cos[c + d*x])^(9/2)) + (16*(a + a*Sin[c + d*x
])^(9/2))/(45*a^2*d*e*(e*Cos[c + d*x])^(9/2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{11/2}}\,dx = \frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{9/2}} - \frac{4\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{11/2}}\,dx}{a}$$

$$= \frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{9/2}} - \frac{8(a+a\sin(c+dx))^{7/2}}{5ade(e\cos(c+dx))^{9/2}} + \frac{8\int \frac{(a+a\sin(c+dx))^{9/2}}{(e\cos(c+dx))^{11/2}}\,dx}{5a^2}$$

$$= \frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{9/2}} - \frac{8(a+a\sin(c+dx))^{7/2}}{5ade(e\cos(c+dx))^{9/2}} + \frac{16(a+a\sin(c+dx))^{9/2}}{45a^2de(e\cos(c+dx))^{9/2}}$$

Mathematica [A] time = 0.217391, size = 64, normalized size = 0.57

$$\frac{2\left(8\sin^2(c+dx)-20\sin(c+dx)+17\right)\sec^5(c+dx)(a(\sin(c+dx)+1))^{5/2}\sqrt{e\cos(c+dx)}}{45de^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^5*(a*(1 + Sin[c + d*x]))^(5/2)*(17 - 2
0*Sin[c + d*x] + 8*Sin[c + d*x]^2))/(45*d*e^6)
```

Maple [A] time = 0.105, size = 54, normalized size = 0.5

$$-\frac{\left(16\,(\cos(dx+c))^2+40\,\sin(dx+c)-50\right)\cos(dx+c)}{45\,d}\,(a\,(1+\sin(dx+c)))^{\frac{5}{2}}\,(e\cos(dx+c))^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(11/2),x)
```

```
[Out] -2/45/d*(8*cos(d*x+c)^2+20*sin(d*x+c)-25)*(a*(1+sin(d*x+c)))^(5/2)*cos(d*x+
c)/(e*cos(d*x+c))^(11/2)
```

Maxima [B] time = 1.59627, size = 381, normalized size = 3.37

$$\frac{2\left(17\,a^{\frac{5}{2}}\sqrt{e}-\frac{40\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{49\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{49\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{40\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{17\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)}{45\left(e^{6}+\frac{3\,e^{6}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\,e^{6}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{e^{6}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)d\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(11/2),x, algorithm="maxima
")
```

```
[Out] 2/45*(17*a^(5/2)*sqrt(e) - 40*a^(5/2)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) +
1) + 49*a^(5/2)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 49*a^(5/2)*sq
rt(e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 40*a^(5/2)*sqrt(e)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 17*a^(5/2)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((e^6 + 3*e^6*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 3*e^6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + e^6
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d*sqrt(sin(d*x + c)/(cos(d*x + c) + 1
) + 1)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2))
```

Fricas [A] time = 2.49685, size = 254, normalized size = 2.25

$$\frac{2\left(8\,a^{2}\cos\left(dx+c\right)^{2}+20\,a^{2}\sin\left(dx+c\right)-25\,a^{2}\right)\sqrt{e\cos\left(dx+c\right)}\sqrt{a\sin\left(dx+c\right)+a}}{45\left(de^{6}\cos\left(dx+c\right)^{3}+2\,de^{6}\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,de^{6}\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(11/2),x, algorithm="fricas
")
```

```
[Out] 2/45*(8*a^2*cos(d*x + c)^2 + 20*a^2*sin(d*x + c) - 25*a^2)*sqrt(e*cos(d*x +
 c))*sqrt(a*sin(d*x + c) + a)/(d*e^6*cos(d*x + c)^3 + 2*d*e^6*cos(d*x + c)*
sin(d*x + c) - 2*d*e^6*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.297 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{13/2}}\,dx$

Optimal. Leaf size=150

$$\frac{32(a\sin(c+dx)+a)^{11/2}}{77a^3de(e\cos(c+dx))^{11/2}}-\frac{16(a\sin(c+dx)+a)^{9/2}}{7a^2de(e\cos(c+dx))^{11/2}}+\frac{4(a\sin(c+dx)+a)^{7/2}}{ade(e\cos(c+dx))^{11/2}}-\frac{2(a\sin(c+dx)+a)^{5/2}}{de(e\cos(c+dx))^{11/2}}$$

```
[Out] (-2*(a + a*Sin[c + d*x])^(5/2))/(d*e*(e*Cos[c + d*x])^(11/2)) + (4*(a + a*S
in[c + d*x])^(7/2))/(a*d*e*(e*Cos[c + d*x])^(11/2)) - (16*(a + a*Sin[c + d*
x])^(9/2))/(7*a^2*d*e*(e*Cos[c + d*x])^(11/2)) + (32*(a + a*Sin[c + d*x])^(
11/2))/(77*a^3*d*e*(e*Cos[c + d*x])^(11/2))
```

Rubi [A] time = 0.305198, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{32(a\sin(c+dx)+a)^{11/2}}{77a^3de(e\cos(c+dx))^{11/2}}-\frac{16(a\sin(c+dx)+a)^{9/2}}{7a^2de(e\cos(c+dx))^{11/2}}+\frac{4(a\sin(c+dx)+a)^{7/2}}{ade(e\cos(c+dx))^{11/2}}-\frac{2(a\sin(c+dx)+a)^{5/2}}{de(e\cos(c+dx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(13/2),x]
```

```
[Out] (-2*(a + a*Sin[c + d*x])^(5/2))/(d*e*(e*Cos[c + d*x])^(11/2)) + (4*(a + a*S
in[c + d*x])^(7/2))/(a*d*e*(e*Cos[c + d*x])^(11/2)) - (16*(a + a*Sin[c + d*
x])^(9/2))/(7*a^2*d*e*(e*Cos[c + d*x])^(11/2)) + (32*(a + a*Sin[c + d*x])^(
11/2))/(77*a^3*d*e*(e*Cos[c + d*x])^(11/2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
```

```
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{13/2}}\,dx = -\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{11/2}} + \frac{6\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{13/2}}\,dx}{a}$$

$$= -\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{11/2}} + \frac{4(a+a\sin(c+dx))^{7/2}}{ade(e\cos(c+dx))^{11/2}} - \frac{8\int \frac{(a+a\sin(c+dx))^{9/2}}{(e\cos(c+dx))^{13/2}}\,dx}{a^2}$$

$$= -\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{11/2}} + \frac{4(a+a\sin(c+dx))^{7/2}}{ade(e\cos(c+dx))^{11/2}} - \frac{16(a+a\sin(c+dx))^{9/2}}{7a^2de(e\cos(c+dx))^{11/2}} + \frac{16\int \frac{(a+a\,\mathrm{s}}{(e\,\mathrm{co}}}{}$$

$$= -\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{11/2}} + \frac{4(a+a\sin(c+dx))^{7/2}}{ade(e\cos(c+dx))^{11/2}} - \frac{16(a+a\sin(c+dx))^{9/2}}{7a^2de(e\cos(c+dx))^{11/2}} + \frac{32(a+a\,\mathrm{s}}{77a^3de(e\,\mathrm{c}}$$

Mathematica [A] time = 0.282116, size = 74, normalized size = 0.49

$$\frac{2\left(16\sin^3(c+dx)-40\sin^2(c+dx)+26\sin(c+dx)+5\right)\sec^6(c+dx)(a(\sin(c+dx)+1))^{5/2}\sqrt{e\cos(c+dx)}}{77de^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(13/2),x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^6*(a*(1 + Sin[c + d*x]))^(5/2)*(5 + 26
*Sin[c + d*x] - 40*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3))/(77*d*e^7)
```

Maple [A] time = 0.106, size = 70, normalized size = 0.5

$$-\frac{\left(32\,(\cos(dx+c))^2\sin(dx+c)-80\,(\cos(dx+c))^2-84\,\sin(dx+c)+70\right)\cos(dx+c)}{77\,d}(a\,(1+\sin(dx+c)))^{\frac{5}{2}}\,(e\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(13/2),x)
```

```
[Out] -2/77/d*(16*cos(d*x+c)^2*sin(d*x+c)-40*cos(d*x+c)^2-42*sin(d*x+c)+35)*(a*(1
+sin(d*x+c)))^(5/2)*cos(d*x+c)/(e*cos(d*x+c))^(13/2)
```

Maxima [B] time = 1.65589, size = 482, normalized size = 3.21

$$\frac{2\left(5\,a^{\frac{5}{2}}\sqrt{e}+\frac{52\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{150\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{180\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{180\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{150\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{52\,a^{\frac{5}{2}}\sqrt{e}\sin(dx+c)}{(\cos(dx+c)+1)^{7}}\right.}{77\left(e^{7}+\frac{4\,e^{7}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,e^{7}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4\,e^{7}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{e^{7}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(}{\cos(d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(13/2),x, algorithm="maxima
")
```

```
[Out] 2/77*(5*a^(5/2)*sqrt(e) + 52*a^(5/2)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1
) - 150*a^(5/2)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 180*a^(5/2)*s
qrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 180*a^(5/2)*sqrt(e)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 + 150*a^(5/2)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 - 52*a^(5/2)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5*a^(5/
2)*sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^4/((e^7 + 4*e^7*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*e^7*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*e^7*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + e^7*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2))
```

Fricas [A] time = 2.1243, size = 298, normalized size = 1.99

$$-\frac{2\left(40\,a^{2}\cos\left(dx+c\right)^{2}-35\,a^{2}-2\left(8\,a^{2}\cos\left(dx+c\right)^{2}-21\,a^{2}\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}\sqrt{a\sin\left(dx+c\right)+a}}{77\left(de^{7}\cos\left(dx+c\right)^{4}+2\,de^{7}\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-2\,de^{7}\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(13/2),x, algorithm="fricas
")
```

```
[Out] -2/77*(40*a^2*cos(d*x + c)^2 - 35*a^2 - 2*(8*a^2*cos(d*x + c)^2 - 21*a^2)*s
in(d*x + c))*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*e^7*cos(d*x +
```

```
c)^4 + 2*d*e^7*cos(d*x + c)^2*sin(d*x + c) - 2*d*e^7*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(13/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.298 $\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=244

$$\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{4d(a\sin(c+dx)+a\cos(c+dx)+a)}+\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}}{4d(a\sin(c+dx)+a\cos(c+dx)+}$$

```
[Out] -(a*(e*Cos[c + d*x])^(7/2))/(2*d*e*(a + a*Sin[c + d*x])^(3/2)) + (e*(e*Cos[
c + d*x])^(3/2))/(4*d*Sqrt[a + a*Sin[c + d*x]]) + (3*e^(5/2)*ArcSinh[Sqrt[e
*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4
*d*(a + a*Cos[c + d*x] + a*Sin[c + d*x])) + (3*e^(5/2)*ArcTan[(Sqrt[e]*Sin[
c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d
*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*(a + a*Cos[c + d*x] + a*Sin[c + d*x]))
```

Rubi [A] time = 0.366696, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2686, 2679, 2684, 2775, 203, 2833, 63, 215}

$$\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{4d(a\sin(c+dx)+a\cos(c+dx)+a)}+\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}}{4d(a\sin(c+dx)+a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] -(a*(e*Cos[c + d*x])^(7/2))/(2*d*e*(a + a*Sin[c + d*x])^(3/2)) + (e*(e*Cos[
c + d*x])^(3/2))/(4*d*Sqrt[a + a*Sin[c + d*x]]) + (3*e^(5/2)*ArcSinh[Sqrt[e
*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4
*d*(a + a*Cos[c + d*x] + a*Sin[c + d*x])) + (3*e^(5/2)*ArcTan[(Sqrt[e]*Sin[
c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d
*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*(a + a*Cos[c + d*x] + a*Sin[c + d*x]))
```


Rule 2686

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1))/(f*g*(2*p - 1)*
(a + b*Sin[e + f*x])^(3/2)), x] + Dist[(2*a*(p - 2))/(2*p - 1), Int[(g*Cos[
e + f*x])^p/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 2] && IntegerQ[2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}} + \frac{1}{4}a\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{3/2}}\,dx$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}} + \frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}} + \frac{1}{8}\left(3e^2\right)\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}} + \frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}} + \frac{\left(3e^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx}\right.}{8(a+a\cos(c+dx)+a\sin(}$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}} + \frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}} + \frac{\left(3e^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx}\right.}{8d(a+a\cos(c+d}$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}} + \frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}} + \frac{3e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1}}{4d(a+a\cos(c+dx)}$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}} + \frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}} + \frac{3e^{5/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+}}{4d(a+a\cos(c+dx)+a\sin}$$

Mathematica [C] time = 0.189415, size = 77, normalized size = 0.32

$$-\frac{4\sqrt[4]{2}(e\cos(c+dx))^{7/2}\,{}_2F_1\left(-\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(\sin(c+dx)+1)^{5/4}\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-4*2^(1/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[-1/4, 7/4, 11/4, (1 -
Sin[c + d*x])/2])/(7*d*e*(1 + Sin[c + d*x])^(5/4)*Sqrt[a*(1 + Sin[c + d*x])
```

```
])
```

Maple [A] time = 0.149, size = 239, normalized size = 1.

$$-\frac{1}{8\,d\left(-1+\cos\left(dx+c\right)+\sin\left(dx+c\right)\right)\left(\cos\left(dx+c\right)\right)^{2}}\left(3\,\sqrt{-2\,\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\arctan\left(1/2\,\sqrt{2}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right)\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/8/d*(3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)+3*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+4*cos(d*x+c)^3-4*cos(d*x+c)^2*sin(d*x+c
)+2*cos(d*x+c)^2+6*cos(d*x+c)*sin(d*x+c)-6*cos(d*x+c))*(e*cos(d*x+c))^(5/2)
/(-1+cos(d*x+c)+sin(d*x+c))/cos(d*x+c)^2/(a*(1+sin(d*x+c)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/sqrt(a*sin(d*x + c) + a), x)
```

3.299 $\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=200

$$\frac{e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{ad(\sin(c+dx)+\cos(c+dx)+1)}-\frac{e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}}{ad(\sin(c+dx)+\cos(c+dx)+1)}$$

```
[Out] (e*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a*d) - (e^(3/2)*ArcSinh[
Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x
]])/(a*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (e^(3/2)*ArcTan[(Sqrt[e]*Sin[
c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d
*x]]*Sqrt[a + a*Sin[c + d*x]])/(a*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rubi [A] time = 0.278785, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2685, 2677, 2775, 203, 2833, 63, 215}

$$\frac{e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{ad(\sin(c+dx)+\cos(c+dx)+1)}-\frac{e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}}{ad(\sin(c+dx)+\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (e*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a*d) - (e^(3/2)*ArcSinh[
Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x
]])/(a*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (e^(3/2)*ArcTan[(Sqrt[e]*Sin[
c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d
*x]]*Sqrt[a + a*Sin[c + d*x]])/(a*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{ad} + \frac{e^2\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{2a}$$

$$= \frac{e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{ad} + \frac{\left(e^2\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+dx}}{\sqrt{e\cos(c+dx)}}}{2(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{ad} - \frac{\left(e^2\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{}{\sqrt{ex}}\right.}{2d(a+a\cos(c+dx)+a\sin(c+a}$$

$$= \frac{e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{ad} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\sqrt{}}{d(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{ad} - \frac{e^{3/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c}}{d(a+a\cos(c+dx)+a\sin(c+dx))}$$

Mathematica [C] time = 0.116269, size = 77, normalized size = 0.38

$$-\frac{2\,2^{3/4}(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{3/4}\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[1/4, 5/4, 9/4, (1 - Si
n[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(3/4)*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.125, size = 212, normalized size = 1.1

$$\frac{1}{2\,d\,(-1+\cos(dx+c)+\sin(dx+c))\cos(dx+c)}\left(\sqrt{-2\,\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\,\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sqrt{2}\sin(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)-(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c))*2^(1/2)*sin(d*x+c)+2*cos(d*x+c)*sin(d*x+c)-2*cos(d*x+c)^2+2*cos
(d*x+c))*(e*cos(d*x+c))^(3/2)/(-1+cos(d*x+c)+sin(d*x+c))/(a*(1+sin(d*x+c)))
^(1/2)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/sqrt(a*sin(d*x + c) + a), x)
```

3.300 $\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=169

$$\frac{2\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d(a\sin(c+dx)+a\cos(c+dx)+a)}+\frac{2\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}}{d(a\sin(c+dx)+a\cos(c+dx)+a}$$

```
[Out] (2*Sqrt[e]*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqr
t[a + a*Sin[c + d*x]])/(d*(a + a*Cos[c + d*x] + a*Sin[c + d*x])) + (2*Sqrt[
e]*ArcTan[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x
]])]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(a + a*Cos[c + d*x
] + a*Sin[c + d*x]))
```

Rubi [A] time = 0.196235, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2684, 2775, 203, 2833, 63, 215}

$$\frac{2\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d(a\sin(c+dx)+a\cos(c+dx)+a)}+\frac{2\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}}{d(a\sin(c+dx)+a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[e]*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqr
t[a + a*Sin[c + d*x]])/(d*(a + a*Cos[c + d*x] + a*Sin[c + d*x])) + (2*Sqrt[
e]*ArcTan[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x
]])]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(a + a*Cos[c + d*x
] + a*Sin[c + d*x]))
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(e\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{a+a\cos(c+dx)+a\sin(c+dx)} - \frac{\left(e\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+\right.}{a+a\cos(c+dx)}$$

$$= \frac{\left(e\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{d(a+a\cos(c+dx)+a\sin(c+dx))} - \frac{\left(2e\sqrt{1+\cos(\right.}{}$$

$$= \frac{2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{d(a+a\cos(c+dx)+a\sin(c+dx))} + \frac{\left(2\sqrt{1+\cos(c+dx\right.}{}$$

$$= \frac{2\sqrt{e}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{d(a+a\cos(c+dx)+a\sin(c+dx))} + \frac{2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos}}\right.}{d(a+a\cos}$$

Mathematica [C] time = 0.0829778, size = 77, normalized size = 0.46

$$-\frac{2\sqrt[4]{2}(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de\sqrt[4]{\sin(c+dx)+1}\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 3/4, 7/4, (1 - Si
n[c + d*x])/2])/(3*d*e*(1 + Sin[c + d*x])^(1/4)*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.095, size = 141, normalized size = 0.8

$$\frac{\sqrt{2}\,(1-\cos(dx+c)+\sin(dx+c))}{d\sin(dx+c)}\sqrt{e\cos(dx+c)}\left(\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*(e*cos(d*x+c))^(1/2)*(1-cos(d*x+c)+sin(d*x+c))*(arctan(1/2*2^(1
/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+arctanh(1/2*2^(1/2)*(-2*cos(d*x+c
```

```
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)))/(a*(1+sin(d*x+c)))^(1/2)/si
n(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos(dx+c)}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos(dx+c)}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/sqrt(a*sin(d*x + c) + a), x)
```

3.301 $\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=34

$$-\frac{2\sqrt{e\cos(c+dx)}}{de\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.0604187, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2671}

$$-\frac{2\sqrt{e\cos(c+dx)}}{de\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\sqrt{e\cos(c+dx)}}{de\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.0641767, size = 34, normalized size = 1.

$$-\frac{2\sqrt{e\cos(c+dx)}}{de\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(d*e*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.102, size = 34, normalized size = 1.

$$-2\,\frac{\cos\left(dx+c\right)}{d\sqrt{e\cos\left(dx+c\right)}\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/d*cos(d*x+c)/(e*cos(d*x+c))^(1/2)/(a*(1+sin(d*x+c)))^(1/2)
```

Maxima [B] time = 1.58804, size = 176, normalized size = 5.18

$$-\frac{2\left(\sqrt{a}\sqrt{e}-\frac{\sqrt{a}\sqrt{e}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)}{\left(ae+\frac{ae\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2*(sqrt(a)*sqrt(e) - sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/((a*e + a*e*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*sqrt(-sin(d*x
 + c)/(cos(d*x + c) + 1) + 1))
```

Fricas [A] time = 2.43498, size = 107, normalized size = 3.15

$$-\frac{2\sqrt{e\cos\left(dx+c\right)}\sqrt{a\sin\left(dx+c\right)+a}}{ade\sin\left(dx+c\right)+ade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)/(a*d*e*sin(d*x + c) + a*d*
e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\sqrt{e\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(c + d*x) + 1))*sqrt(e*cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos\left(dx+c\right)}\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)), x)
```

3.302 $\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{4\sqrt{a\sin(c+dx)+a}}{3ade\sqrt{e\cos(c+dx)}}-\frac{2}{3de\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}$$

```
[Out] -2/(3*d*e*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]) + (4*Sqrt[a + a*Si
n[c + d*x]])/(3*a*d*e*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.132634, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{4\sqrt{a\sin(c+dx)+a}}{3ade\sqrt{e\cos(c+dx)}}-\frac{2}{3de\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] -2/(3*d*e*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]) + (4*Sqrt[a + a*Si
n[c + d*x]])/(3*a*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2}{3de\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx}{3a}$$
$$= -\frac{2}{3de\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}} + \frac{4\sqrt{a+a\sin(c+dx)}}{3ade\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 0.102878, size = 46, normalized size = 0.61

$$\frac{2(2\sin(c+dx)+1)}{3de\sqrt{a(\sin(c+dx)+1)}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (2*(1 + 2*Sin[c + d*x]))/(3*d*e*Sqrt[e*Cos[c + d*x]]*Sqrt[a*(1 + Sin[c + d*
x])])
```

Maple [A] time = 0.107, size = 44, normalized size = 0.6

$$\frac{(4\sin(dx+c)+2)\cos(dx+c)}{3d}(e\cos(dx+c))^{-\frac{3}{2}}\frac{1}{\sqrt{a(1+\sin(dx+c))}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d*(2*sin(d*x+c)+1)*cos(d*x+c)/(e*cos(d*x+c))^(3/2)/(a*(1+sin(d*x+c)))^(
1/2)
```

Maxima [B] time = 1.6021, size = 284, normalized size = 3.74

$$\frac{2\left(\sqrt{a}\sqrt{e}+\frac{4\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\sqrt{a}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{3\left(ae^2+\frac{2ae^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{ae^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2/3*(sqrt(a)*sqrt(e) + 4*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1) -
4*sqrt(a)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - sqrt(a)*sqrt(e)*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)
^2/((a*e^2 + 2*a*e^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*e^2*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-
sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))
```

Fricas [A] time = 2.62174, size = 177, normalized size = 2.33

$$\frac{2\sqrt{e\cos\left(dx+c\right)}\sqrt{a\sin\left(dx+c\right)+a}(2\,\sin\left(dx+c\right)+1)}{3\left(ade^2\cos\left(dx+c\right)\sin\left(dx+c\right)+ade^2\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/3*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)*(2*sin(d*x + c) + 1)/(a*d
*e^2*cos(d*x + c)*sin(d*x + c) + a*d*e^2*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*sqrt(a*sin(d*x + c) + a)), x)
```

3.303 $\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=115

$$\frac{16(a\sin(c+dx)+a)^{3/2}}{15a^2de(e\cos(c+dx))^{3/2}}-\frac{8\sqrt{a\sin(c+dx)+a}}{5ade(e\cos(c+dx))^{3/2}}-\frac{2}{5de\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{3/2}}$$

```
[Out] -2/(5*d*e*(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]) - (8*Sqrt[a + a*
Sin[c + d*x]])/(5*a*d*e*(e*Cos[c + d*x])^(3/2)) + (16*(a + a*Sin[c + d*x])^
(3/2))/(15*a^2*d*e*(e*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.208582, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{16(a\sin(c+dx)+a)^{3/2}}{15a^2de(e\cos(c+dx))^{3/2}}-\frac{8\sqrt{a\sin(c+dx)+a}}{5ade(e\cos(c+dx))^{3/2}}-\frac{2}{5de\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] -2/(5*d*e*(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]) - (8*Sqrt[a + a*
Sin[c + d*x]])/(5*a*d*e*(e*Cos[c + d*x])^(3/2)) + (16*(a + a*Sin[c + d*x])^
(3/2))/(15*a^2*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
```

```
&& EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2}{5de(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}} + \frac{4\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx}{5a}$$

$$= -\frac{2}{5de(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}} - \frac{8\sqrt{a+a\sin(c+dx)}}{5ade(e\cos(c+dx))^{3/2}} + \frac{8\int \ldots}{\ldots}$$

$$= -\frac{2}{5de(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}} - \frac{8\sqrt{a+a\sin(c+dx)}}{5ade(e\cos(c+dx))^{3/2}} + \frac{16(a \ldots}{15a^2 \ldots}$$

Mathematica [A] time = 0.09899, size = 56, normalized size = 0.49

$$\frac{2\left(8\sin^2(c+dx)+4\sin(c+dx)-7\right)}{15de\sqrt{a(\sin(c+dx)+1)}(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (2*(-7 + 4*Sin[c + d*x] + 8*Sin[c + d*x]^2))/(15*d*e*(e*Cos[c + d*x])^(3/2)
*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.112, size = 54, normalized size = 0.5

$$\frac{\left(-16\,(\cos(dx+c))^2+8\,\sin(dx+c)+2\right)\cos(dx+c)}{15\,d}(e\cos(dx+c))^{-\frac{5}{2}}\frac{1}{\sqrt{a\,(1+\sin(dx+c))}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d*(-8*cos(d*x+c)^2+4*sin(d*x+c)+1)*cos(d*x+c)/(e*cos(d*x+c))^(5/2)/(a*
(1+sin(d*x+c)))^(1/2)
```

Maxima [B] time = 1.62497, size = 387, normalized size = 3.37

$$-\frac{2\left(7\sqrt{a}\sqrt{e}-\frac{8\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{25\sqrt{a}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{8\sqrt{a}\sqrt{e}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{7\sqrt{a}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{15\left(ae^3+\frac{3ae^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3ae^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{ae^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*(7*sqrt(a)*sqrt(e) - 8*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1
) - 25*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 25*sqrt(a)*sqr
t(e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 8*sqrt(a)*sqrt(e)*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 7*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((a*e^3 + 3*a*e^3*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a*e^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
a*e^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d*(sin(d*x + c)/(cos(d*x + c) +
1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2))
```

Fricas [A] time = 2.26388, size = 211, normalized size = 1.83

$$-\frac{2\sqrt{e\cos(dx+c)}\left(8\cos(dx+c)^2-4\sin(dx+c)-1\right)\sqrt{a\sin(dx+c)+a}}{15\left(ade^3\cos(dx+c)^2\sin(dx+c)+ade^3\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2/15*sqrt(e*cos(d*x + c))*(8*cos(d*x + c)^2 - 4*sin(d*x + c) - 1)*sqrt(a*s
in(d*x + c) + a)/(a*d*e^3*cos(d*x + c)^2*sin(d*x + c) + a*d*e^3*cos(d*x + c
)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*sqrt(a*sin(d*x + c) + a)), x)
```

3.304 $\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=154

$$-\frac{32(a\sin(c+dx)+a)^{5/2}}{35a^3de(e\cos(c+dx))^{5/2}}+\frac{16(a\sin(c+dx)+a)^{3/2}}{7a^2de(e\cos(c+dx))^{5/2}}-\frac{4\sqrt{a\sin(c+dx)+a}}{7ade(e\cos(c+dx))^{5/2}}-\frac{2}{7de\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{5/2}}$$

```
[Out] -2/(7*d*e*(e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]) - (4*Sqrt[a + a*
Sin[c + d*x]])/(7*a*d*e*(e*Cos[c + d*x])^(5/2)) + (16*(a + a*Sin[c + d*x])^
(3/2))/(7*a^2*d*e*(e*Cos[c + d*x])^(5/2)) - (32*(a + a*Sin[c + d*x])^(5/2))
/(35*a^3*d*e*(e*Cos[c + d*x])^(5/2))
```

Rubi [A] time = 0.287667, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{32(a\sin(c+dx)+a)^{5/2}}{35a^3de(e\cos(c+dx))^{5/2}}+\frac{16(a\sin(c+dx)+a)^{3/2}}{7a^2de(e\cos(c+dx))^{5/2}}-\frac{4\sqrt{a\sin(c+dx)+a}}{7ade(e\cos(c+dx))^{5/2}}-\frac{2}{7de\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] -2/(7*d*e*(e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]) - (4*Sqrt[a + a*
Sin[c + d*x]])/(7*a*d*e*(e*Cos[c + d*x])^(5/2)) + (16*(a + a*Sin[c + d*x])^
(3/2))/(7*a^2*d*e*(e*Cos[c + d*x])^(5/2)) - (32*(a + a*Sin[c + d*x])^(5/2))
/(35*a^3*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
```

```
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2}{7de(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}} + \frac{6\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx}{7a}$$

$$= -\frac{2}{7de(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}} - \frac{4\sqrt{a+a\sin(c+dx)}}{7ade(e\cos(c+dx))^{5/2}} + \frac{8\int \frac{(a}{(}}{}$$

$$= -\frac{2}{7de(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}} - \frac{4\sqrt{a+a\sin(c+dx)}}{7ade(e\cos(c+dx))^{5/2}} + \frac{16(a}{7a^2de}$$

$$= -\frac{2}{7de(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}} - \frac{4\sqrt{a+a\sin(c+dx)}}{7ade(e\cos(c+dx))^{5/2}} + \frac{16(a}{7a^2de}$$

Mathematica [A] time = 0.178332, size = 66, normalized size = 0.43

$$\frac{2(10\sin(c+dx)+4\sin(3(c+dx))+4\cos(2(c+dx))+5)}{35de\sqrt{a(\sin(c+dx)+1)}(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (2*(5 + 4*Cos[2*(c + d*x)] + 10*Sin[c + d*x] + 4*Sin[3*(c + d*x)]))/(35*d*e
*(e*Cos[c + d*x])^(5/2)*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A] time = 0.121, size = 70, normalized size = 0.5

$$\frac{\left(32\,(\cos(dx+c))^2\sin(dx+c)+16\,(\cos(dx+c))^2+12\,\sin(dx+c)+2\right)\cos(dx+c)}{35\,d}(e\cos(dx+c))^{-\frac{7}{2}}\frac{1}{\sqrt{a\,(1+\sin(d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/35/d*(16*cos(d*x+c)^2*sin(d*x+c)+8*cos(d*x+c)^2+6*sin(d*x+c)+1)*cos(d*x+c
)/(e*cos(d*x+c))^(7/2)/(a*(1+sin(d*x+c)))^(1/2)
```

Maxima [B] time = 1.65325, size = 490, normalized size = 3.18

$$\frac{2\left(9\sqrt{a}\sqrt{e}+\frac{44\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{14\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{84\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sqrt{a}\sqrt{e}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{14\sqrt{a}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{44\sqrt{a}\sqrt{e}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\cdots}{35\left(ae^4+\frac{4ae^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6ae^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4ae^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{ae^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(\cdots}{\cos(d\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2/35*(9*sqrt(a)*sqrt(e) + 44*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1
) - 14*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 84*sqrt(a)*sqr
t(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(a)*sqrt(e)*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 + 14*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 - 44*sqrt(a)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 9*sqrt(a)*s
qrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 1)^4/((a*e^4 + 4*a*e^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*e^
4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*e^4*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + a*e^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2))
```

Fricas [A] time = 2.06038, size = 240, normalized size = 1.56

$$\frac{2\sqrt{e\cos(dx+c)}\left(8\cos(dx+c)^2+2\left(8\cos(dx+c)^2+3\right)\sin(dx+c)+1\right)\sqrt{a\sin(dx+c)+a}}{35\left(ade^4\cos(dx+c)^3\sin(dx+c)+ade^4\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/35*sqrt(e*cos(d*x + c))*(8*cos(d*x + c)^2 + 2*(8*cos(d*x + c)^2 + 3)*sin(
d*x + c) + 1)*sqrt(a*sin(d*x + c) + a)/(a*d*e^4*cos(d*x + c)^3*sin(d*x + c)
```

```
+ a*d*e^4*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*sqrt(a*sin(d*x + c) + a)), x)
```

3.305 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=247

$$\frac{5e^3\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}{4a^2d}+\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{4a^2d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{5e^{7/2}\sqrt{}}{}$$

```
[Out] (e*(e*Cos[c + d*x])^(5/2))/(2*a*d*Sqrt[a + a*Sin[c + d*x]]) + (5*e^3*Sqrt[e
*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*a^2*d) - (5*e^(7/2)*ArcSinh[Sqr
t[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])
/(4*a^2*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (5*e^(7/2)*ArcTan[(Sqrt[e]*S
in[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c
+ d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*a^2*d*(1 + Cos[c + d*x] + Sin[c + d*x]
))
```

Rubi [A] time = 0.371736, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2679, 2685, 2677, 2775, 203, 2833, 63, 215}

$$\frac{5e^3\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}{4a^2d}+\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{4a^2d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{5e^{7/2}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (e*(e*Cos[c + d*x])^(5/2))/(2*a*d*Sqrt[a + a*Sin[c + d*x]]) + (5*e^3*Sqrt[e
*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*a^2*d) - (5*e^(7/2)*ArcSinh[Sqr
t[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])
/(4*a^2*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (5*e^(7/2)*ArcTan[(Sqrt[e]*S
in[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c
+ d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*a^2*d*(1 + Cos[c + d*x] + Sin[c + d*x]
))
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
```

```
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{\left(5e^2\right)\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} + \frac{\left(5e^4\right)\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{8a^2}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} + \frac{\left(5e^4\sqrt{1+\cos(c+dx)}\sqrt{a\right.}{8a(a+a\cos(c}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} - \frac{\left(5e^4\sqrt{1+\cos(c+dx)}\sqrt{a\right.}{8ad(}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} + \frac{5e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(c}{\sqrt{e\cos(c+dx)}\sqrt{1}\right.}{4d\left(a^2+\right.}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} - \frac{5e^{7/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{4d\left(a^2+a^2\cos\right.}$$

Mathematica [C] time = 0.124953, size = 80, normalized size = 0.32

$$-\frac{2\,2^{3/4}\sqrt{a(\sin(c+dx)+1)}(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9a^2de(\sin(c+dx)+1)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[1/4, 9/4, 13/4, (1 - S
in[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(9*a^2*d*e*(1 + Sin[c + d*x])^(
11/4))
```

Maple [A] time = 0.148, size = 266, normalized size = 1.1

$$-\frac{1}{8\,d\left(\cos(dx+c)\sin(dx+c)-(\cos(dx+c))^2-2\,\sin(dx+c)-\cos(dx+c)+2\right)\cos(dx+c)}(e\cos(dx+c))^{\frac{7}{2}}\left(5\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/8/d*(e*cos(d*x+c))^(7/2)*(5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(
1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)-5*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+4*cos(d*x+c)^2*sin
(d*x+c)+4*cos(d*x+c)^3+10*cos(d*x+c)*sin(d*x+c)-14*cos(d*x+c)^2+10*cos(d*x+
c))/(cos(d*x+c)*sin(d*x+c)-cos(d*x+c)^2-2*sin(d*x+c)-cos(d*x+c)+2)/(a*(1+si
n(d*x+c)))^(3/2)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.306 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=215

$$\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d\left(a^2\sin(c+dx)+a^2\cos(c+dx)+a^2\right)}+\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\mathrm{sinh}}{d\left(a^2\sin(c+dx)+a^2\cos(c+dx)\right.}$$

```
[Out] (e*(e*Cos[c + d*x])^(3/2))/(a*d*Sqrt[a + a*Sin[c + d*x]]) + (3*e^(5/2)*ArcS
inh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c +
 d*x]])/(d*(a^2 + a^2*Cos[c + d*x] + a^2*Sin[c + d*x])) + (3*e^(5/2)*ArcTan
[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt
[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(a^2 + a^2*Cos[c + d*x] + a
^2*Sin[c + d*x]))
```

Rubi [A] time = 0.286914, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2679, 2684, 2775, 203, 2833, 63, 215}

$$\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d\left(a^2\sin(c+dx)+a^2\cos(c+dx)+a^2\right)}+\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\mathrm{sinh}}{d\left(a^2\sin(c+dx)+a^2\cos(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^(3/2),x]
```


```
[Out] (e*(e*Cos[c + d*x])^(3/2))/(a*d*Sqrt[a + a*Sin[c + d*x]]) + (3*e^(5/2)*ArcS
inh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c +
 d*x]])/(d*(a^2 + a^2*Cos[c + d*x] + a^2*Sin[c + d*x])) + (3*e^(5/2)*ArcTan
[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt
[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(a^2 + a^2*Cos[c + d*x] + a
^2*Sin[c + d*x]))
```


Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
```

```
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\left(3e^2\right)\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a}$$

$$= \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\left(3e^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{2a(a+a\cos(c+dx)+a\sin(c+dx))} - \frac{(3e$$

$$= \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\left(3e^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x}}\,dx, x\right.}{2ad(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{3e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(}{d\left(a^2+a^2\cos(c+dx)+a^2\sin(c+dx)\right)}$$

$$= \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{3e^{5/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{d\left(a^2+a^2\cos(c+dx)+a^2\sin(c+dx)\right)} +$$

Mathematica [C] time = 0.183856, size = 80, normalized size = 0.37

$$-\frac{2\sqrt[4]{2}\sqrt{a(\sin(c+dx)+1)}(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7a^2de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[3/4, 7/4, 11/4, (1 - S
in[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(7*a^2*d*e*(1 + Sin[c + d*x])^(
9/4))
```

Maple [A] time = 0.113, size = 232, normalized size = 1.1

$$-\frac{1}{2\,d\left(\cos(dx+c)\sin(dx+c)-(\cos(dx+c))^2-2\,\sin(dx+c)-\cos(dx+c)+2\right)}(e\cos(dx+c))^{\frac{5}{2}}\left(-3\sqrt{-2\,\frac{\cos(dx}{1+\cos(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d*(e*cos(d*x+c))^(5/2)*(-3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan
(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)-3*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+2*cos(d*x+c)*sin(
d*x+c)+2*cos(d*x+c)^2-2*cos(d*x+c))/(cos(d*x+c)*sin(d*x+c)-cos(d*x+c)^2-2*s
in(d*x+c)-cos(d*x+c)+2)/(a*(1+sin(d*x+c)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.307 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{2e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{a^2d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{2e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh}{a^2d(\sin(c+dx)+\cos(c+dx)+}$$

```
[Out] (-2*(e*Cos[c + d*x])^(5/2))/(d*e*(a + a*Sin[c + d*x])^(3/2)) - (2*e*Sqrt[e*
Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a^2*d) + (2*e^(3/2)*ArcSinh[Sqrt[e
*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a
^2*d*(1 + Cos[c + d*x] + Sin[c + d*x])) - (2*e^(3/2)*ArcTan[(Sqrt[e]*Sin[c
+ d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x
]]*Sqrt[a + a*Sin[c + d*x]])/(a^2*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rubi [A] time = 0.362183, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2681, 2685, 2677, 2775, 203, 2833, 63, 215}

$$-\frac{2e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{a^2d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{2e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh}{a^2d(\sin(c+dx)+\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^(3/2),x]
```



```
[Out] (-2*(e*Cos[c + d*x])^(5/2))/(d*e*(a + a*Sin[c + d*x])^(3/2)) - (2*e*Sqrt[e*
Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a^2*d) + (2*e^(3/2)*ArcSinh[Sqrt[e
*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a
^2*d*(1 + Cos[c + d*x] + Sin[c + d*x])) - (2*e^(3/2)*ArcTan[(Sqrt[e]*Sin[c
+ d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x
]]*Sqrt[a + a*Sin[c + d*x]])/(a^2*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```


Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
```

```
IntegersQ[2*m, 2*p]
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx}{a}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^2d} - \frac{e^2\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{a^2}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^2d} - \frac{\left(e^2\sqrt{1+\cos(c+dx)}\sqrt{a}\right.}{a(a+a\cos(c}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^2d} + \frac{\left(e^2\sqrt{1+\cos(c+dx)}\sqrt{a}\right.}{ad(}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^2d} - \frac{2e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(}{\sqrt{e\cos(c+dx)}\sqrt{}}\right.}{d\left(a^2+\right.}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^2d} + \frac{2e^{3/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right.}{d\left(a^2+a^2\,\mathrm{co}\right.}$$

Mathematica [C] time = 0.122615, size = 80, normalized size = 0.34

$$-\frac{2^{3/4}\sqrt{a(\sin(c+dx)+1)}(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5a^2de(\sin(c+dx)+1)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(2^(3/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, 5/4, 9/4, (1 - Sin[
c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(5*a^2*d*e*(1 + Sin[c + d*x])^(7/4
```

```
))
```

Maple [A] time = 0.101, size = 321, normalized size = 1.4

$$-2\,\frac{(e\cos(dx+c))^{3/2}\,(-1+\cos(dx+c))}{d\sin(dx+c)\,(a\,(1+\sin(dx+c)))^{3/2}\,(-1+\cos(dx+c)+\sin(dx+c))}\left(\sqrt{2}\arctan\left(1/2\,\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\mathrm{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/d*(e*cos(d*x+c))^(3/2)*(-1+cos(d*x+c))*(2^(1/2)*arctan(1/2*2^(1/2)*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-2^(1/2)*arctanh(1/2*2^(1/2)*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*cos(d*x+c)*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2))-2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)/cos(d*x+c))-2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)/(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(a*(1+sin(d*x+c)))^(3/2)/(-1+cos(d*x+c)+s
in(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.308 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=36

$$-\frac{2(e\cos(c+dx))^{3/2}}{3de(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-2*(e*Cos[c + d*x])^(3/2))/(3*d*e*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.06364, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2671}

$$-\frac{2(e\cos(c+dx))^{3/2}}{3de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(3/2))/(3*d*e*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{3/2}}\,dx=-\frac{2(e\cos(c+dx))^{3/2}}{3de(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.0717418, size = 49, normalized size = 1.36

$$-\frac{2\sqrt{a(\sin(c+dx)+1)}(e\cos(c+dx))^{3/2}}{3a^2de(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(3/2)*Sqrt[a*(1 + Sin[c + d*x])])/(3*a^2*d*e*(1 + Sin[
c + d*x])^2)
```

Maple [A] time = 0.105, size = 34, normalized size = 0.9

$$-\frac{2\cos(dx+c)}{3d}\sqrt{e\cos(dx+c)}\,(a(1+\sin(dx+c)))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(e*cos(d*x+c))^(1/2)*cos(d*x+c)/(a*(1+sin(d*x+c)))^(3/2)
```

Maxima [B] time = 1.56429, size = 177, normalized size = 4.92

$$-\frac{2\left(\sqrt{a}\sqrt{e}-\frac{\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{3\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] -2/3*(sqrt(a)*sqrt(e) - sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*sqrt(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d*(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)^(5/2))
```

Fricas [B] time = 2.87672, size = 250, normalized size = 6.94

$$\frac{2\sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{3\left(a^2d\cos(dx+c)^2-a^2d\cos(dx+c)-2a^2d-\left(a^2d\cos(dx+c)+2a^2d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x +
 c) + 1)/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (a^2*d*cos(
d*x + c) + 2*a^2*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{e\cos(c+dx)}}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{e\cos(dx+c)}}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^(3/2), x)
```

3.309 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{4\sqrt{e\cos(c+dx)}}{5ade\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{e\cos(c+dx)}}{5de(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(5*d*e*(a + a*Sin[c + d*x])^(3/2)) - (4*Sqrt[e*Co
s[c + d*x]])/(5*a*d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.129558, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{4\sqrt{e\cos(c+dx)}}{5ade\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{e\cos(c+dx)}}{5de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(5*d*e*(a + a*Sin[c + d*x])^(3/2)) - (4*Sqrt[e*Co
s[c + d*x]])/(5*a*d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{e\cos(c+dx)}}{5de(a+a\sin(c+dx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}\,dx}{5a}$$
$$= -\frac{2\sqrt{e\cos(c+dx)}}{5de(a+a\sin(c+dx))^{3/2}} - \frac{4\sqrt{e\cos(c+dx)}}{5ade\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.110112, size = 59, normalized size = 0.78

$$-\frac{2(2\sin(c+dx)+3)\sqrt{a(\sin(c+dx)+1)}\sqrt{e\cos(c+dx)}}{5a^2de(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*Sqrt[a*(1 + Sin[c + d*x])]*(3 + 2*Sin[c + d*x]))/(
5*a^2*d*e*(1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.113, size = 44, normalized size = 0.6

$$-\frac{(4\sin(dx+c)+6)\cos(dx+c)}{5d}(a(1+\sin(dx+c)))^{-\frac{3}{2}}\frac{1}{\sqrt{e\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/5/d*(2*sin(d*x+c)+3)*cos(d*x+c)/(a*(1+sin(d*x+c)))^(3/2)/(e*cos(d*x+c))^
(1/2)
```

Maxima [B] time = 1.57702, size = 285, normalized size = 3.75

$$-\frac{2\left(3\sqrt{a}\sqrt{e} + \frac{4\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1} - \frac{4\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{3\sqrt{a}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{5\left(a^2e + \frac{2a^2e\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^2e\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2/5*(3*sqrt(a)*sqrt(e) + 4*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1)
 - 4*sqrt(a)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sqrt(a)*sqrt(e
)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 1)^2/((a^2*e + 2*a^2*e*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*e*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/
2)*sqrt(-sin(d*x + c)/(cos(d*x + c) + 1) + 1))
```

Fricas [A] time = 2.81083, size = 181, normalized size = 2.38

$$\frac{2 \sqrt{e \cos(dx+c)} \sqrt{a \sin(dx+c)+a}(2 \sin(dx+c)+3)}{5 \left(a^2 de \cos(dx+c)^2 - 2 a^2 de \sin(dx+c) - 2 a^2 de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/5*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)*(2*sin(d*x + c) + 3)/(a^2
*d*e*cos(d*x + c)^2 - 2*a^2*d*e*sin(d*x + c) - 2*a^2*d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a(\sin(c+dx)+1))^{\frac{3}{2}} \sqrt{e \cos(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/((a*(sin(c + d*x) + 1))**(3/2)*sqrt(e*cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos\left(dx+c\right)}\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^(3/2)), x)
```

3.310 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$\frac{16\sqrt{a\sin(c+dx)+a}}{21a^2de\sqrt{e\cos(c+dx)}}-\frac{8}{21ade\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}-\frac{2}{7de(a\sin(c+dx)+a)^{3/2}\sqrt{e\cos(c+dx)}}$$

```
[Out] -2/(7*d*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)) - 8/(21*a*d*e*Sq
rt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]) + (16*Sqrt[a + a*Sin[c + d*x]]
)/(21*a^2*d*e*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.209761, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{16\sqrt{a\sin(c+dx)+a}}{21a^2de\sqrt{e\cos(c+dx)}}-\frac{8}{21ade\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}-\frac{2}{7de(a\sin(c+dx)+a)^{3/2}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -2/(7*d*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)) - 8/(21*a*d*e*Sq
rt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]) + (16*Sqrt[a + a*Sin[c + d*x]]
)/(21*a^2*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
```

```
&& EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2}{7de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}} + \frac{4\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}}{7a}$$

$$= -\frac{2}{7de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}} - \frac{8}{21ade\sqrt{e\cos(c+dx)}\sqrt{a+a}}$$

$$= -\frac{2}{7de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}} - \frac{8}{21ade\sqrt{e\cos(c+dx)}\sqrt{a+a}}$$

Mathematica [A] time = 0.103559, size = 56, normalized size = 0.49

$$\frac{16\sin^2(c+dx)+24\sin(c+dx)+2}{21de(a(\sin(c+dx)+1))^{3/2}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (2 + 24*Sin[c + d*x] + 16*Sin[c + d*x]^2)/(21*d*e*Sqrt[e*Cos[c + d*x]]*(a*(
1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.093, size = 54, normalized size = 0.5

$$\frac{\left(-16\,(\cos(dx+c))^2+24\,\sin(dx+c)+18\right)\cos(dx+c)}{21\,d}(e\cos(dx+c))^{-\frac{3}{2}}(a(1+\sin(dx+c)))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/21/d*(-8*cos(d*x+c)^2+12*sin(d*x+c)+9)*cos(d*x+c)/(e*cos(d*x+c))^(3/2)/(a
*(1+sin(d*x+c)))^(3/2)
```

Maxima [B] time = 1.6065, size = 397, normalized size = 3.45

$$\frac{2\left(\sqrt{a}\sqrt{e}+\frac{24\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{33\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{33\sqrt{a}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{24\sqrt{a}\sqrt{e}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{\sqrt{a}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{21\left(a^2e^2+\frac{3a^2e^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2e^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2e^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/21*(sqrt(a)*sqrt(e) + 24*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1)
+ 33*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 33*sqrt(a)*sqrt(
e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 24*sqrt(a)*sqrt(e)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)
*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((a^2*e^2 + 3*a^2*e^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*e^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 + a^2*e^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d*(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))
```

Fricas [A] time = 2.82499, size = 252, normalized size = 2.19

$$\frac{2\sqrt{e\cos(dx+c)}\left(8\cos(dx+c)^2-12\sin(dx+c)-9\right)\sqrt{a\sin(dx+c)+a}}{21\left(a^2de^2\cos(dx+c)^3-2a^2de^2\cos(dx+c)\sin(dx+c)-2a^2de^2\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/21*sqrt(e*cos(d*x + c))*(8*cos(d*x + c)^2 - 12*sin(d*x + c) - 9)*sqrt(a*s
in(d*x + c) + a)/(a^2*d*e^2*cos(d*x + c)^3 - 2*a^2*d*e^2*cos(d*x + c)*sin(d
*x + c) - 2*a^2*d*e^2*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^(3/2)), x)
```

3.311 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=154

$$\frac{32(a\sin(c+dx)+a)^{3/2}}{45a^3de(e\cos(c+dx))^{3/2}}-\frac{16\sqrt{a\sin(c+dx)+a}}{15a^2de(e\cos(c+dx))^{3/2}}-\frac{4}{15ade\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{3/2}}-\frac{2}{9de(a\sin(c+dx)+a)^{3}}$$

```
[Out] -2/(9*d*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)) - 4/(15*a*d*e*
(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]) - (16*Sqrt[a + a*Sin[c + d
*x]])/(15*a^2*d*e*(e*Cos[c + d*x])^(3/2)) + (32*(a + a*Sin[c + d*x])^(3/2))
/(45*a^3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.288734, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{32(a\sin(c+dx)+a)^{3/2}}{45a^3de(e\cos(c+dx))^{3/2}}-\frac{16\sqrt{a\sin(c+dx)+a}}{15a^2de(e\cos(c+dx))^{3/2}}-\frac{4}{15ade\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{3/2}}-\frac{2}{9de(a\sin(c+dx)+a)^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -2/(9*d*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)) - 4/(15*a*d*e*
(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]) - (16*Sqrt[a + a*Sin[c + d
*x]])/(15*a^2*d*e*(e*Cos[c + d*x])^(3/2)) + (32*(a + a*Sin[c + d*x])^(3/2))
/(45*a^3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
```

```
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2}{9de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}} + \frac{2\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+}}}{3a}$$

$$= -\frac{2}{9de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}} - \frac{4}{15ade(e\cos(c+dx))^{3/2}\sqrt{a}}$$

$$= -\frac{2}{9de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}} - \frac{4}{15ade(e\cos(c+dx))^{3/2}\sqrt{a}}$$

$$= -\frac{2}{9de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}} - \frac{4}{15ade(e\cos(c+dx))^{3/2}\sqrt{a}}$$

Mathematica [A] time = 0.171883, size = 66, normalized size = 0.43

$$-\frac{2(-6\sin(c+dx)+4\sin(3(c+dx))+12\cos(2(c+dx))+7)}{45de(a(\sin(c+dx)+1))^{3/2}(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-2*(7 + 12*Cos[2*(c + d*x)] - 6*Sin[c + d*x] + 4*Sin[3*(c + d*x)]))/(45*d*
e*(e*Cos[c + d*x])^(3/2)*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A] time = 0.104, size = 70, normalized size = 0.5

$$-\frac{\left(32\,(\cos(dx+c))^2\sin(dx+c)+48\,(\cos(dx+c))^2-20\,\sin(dx+c)-10\right)\cos(dx+c)}{45\,d}(e\cos(dx+c))^{-\frac{5}{2}}(a(1+\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x)
```

[Out] -2/45/d*(16*cos(d*x+c)^2*sin(d*x+c)+24*cos(d*x+c)^2-10*sin(d*x+c)-5)*cos(d*x+c)/(e*cos(d*x+c))^(5/2)/(a*(1+sin(d*x+c)))^(3/2)

Maxima [B] time = 1.65257, size = 504, normalized size = 3.27

$$-\frac{2\left(19\sqrt{a}\sqrt{e}+\frac{12\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{58\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{116\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{116\sqrt{a}\sqrt{e}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{58\sqrt{a}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{12\sqrt{a}\sqrt{e}\sin(dx+}{(\cos(dx+c)+1)}\right.}{45\left(a^2e^3+\frac{4a^2e^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2e^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^2e^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2e^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")

[Out] -2/45*(19*sqrt(a)*sqrt(e) + 12*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1) - 58*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 116*sqrt(a)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 116*sqrt(a)*sqrt(e)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 58*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 12*sqrt(a)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 19*sqrt(a)*sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((a^2*e^3 + 4*a^2*e^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*e^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^2*e^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*e^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2))

Fricas [A] time = 2.84959, size = 289, normalized size = 1.88

$$\frac{2\sqrt{e\cos(dx+c)}\left(24\cos(dx+c)^2+2\left(8\cos(dx+c)^2-5\right)\sin(dx+c)-5\right)\sqrt{a\sin(dx+c)+a}}{45\left(a^2de^3\cos(dx+c)^4-2a^2de^3\cos(dx+c)^2\sin(dx+c)-2a^2de^3\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 2/45*sqrt(e*cos(d*x + c))*(24*cos(d*x + c)^2 + 2*(8*cos(d*x + c)^2 - 5)*sin(d*x + c) - 5)*sqrt(a*sin(d*x + c) + a)/(a^2*d*e^3*cos(d*x + c)^4 - 2*a^2*d

```
*e^3*cos(d*x + c)^2*sin(d*x + c) - 2*a^2*d*e^3*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^(3/2)), x)
```

3.312 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{256(a\sin(c+dx)+a)^{5/2}}{385a^4de(e\cos(c+dx))^{5/2}}+\frac{128(a\sin(c+dx)+a)^{3/2}}{77a^3de(e\cos(c+dx))^{5/2}}-\frac{32\sqrt{a\sin(c+dx)+a}}{77a^2de(e\cos(c+dx))^{5/2}}-\frac{16}{77ade\sqrt{a\sin(c+dx)+a}(e\cos(c+d}$$

```
[Out] -2/(11*d*e*(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2)) - 16/(77*a*d*
e*(e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]) - (32*Sqrt[a + a*Sin[c +
 d*x]])/(77*a^2*d*e*(e*Cos[c + d*x])^(5/2)) + (128*(a + a*Sin[c + d*x])^(3/
2))/(77*a^3*d*e*(e*Cos[c + d*x])^(5/2)) - (256*(a + a*Sin[c + d*x])^(5/2))/
(385*a^4*d*e*(e*Cos[c + d*x])^(5/2))
```

Rubi [A] time = 0.371601, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{256(a\sin(c+dx)+a)^{5/2}}{385a^4de(e\cos(c+dx))^{5/2}}+\frac{128(a\sin(c+dx)+a)^{3/2}}{77a^3de(e\cos(c+dx))^{5/2}}-\frac{32\sqrt{a\sin(c+dx)+a}}{77a^2de(e\cos(c+dx))^{5/2}}-\frac{16}{77ade\sqrt{a\sin(c+dx)+a}(e\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```


```
[Out] -2/(11*d*e*(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2)) - 16/(77*a*d*
e*(e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]) - (32*Sqrt[a + a*Sin[c +
 d*x]])/(77*a^2*d*e*(e*Cos[c + d*x])^(5/2)) + (128*(a + a*Sin[c + d*x])^(3/
2))/(77*a^3*d*e*(e*Cos[c + d*x])^(5/2)) - (256*(a + a*Sin[c + d*x])^(5/2))/
(385*a^4*d*e*(e*Cos[c + d*x])^(5/2))
```


Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} + \frac{8\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+a\sin(c}}}{11a}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} - \frac{16}{77ade(e\cos(c+dx))^{5/2}\sqrt{}}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} - \frac{16}{77ade(e\cos(c+dx))^{5/2}\sqrt{}}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} - \frac{16}{77ade(e\cos(c+dx))^{5/2}\sqrt{}}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} - \frac{16}{77ade(e\cos(c+dx))^{5/2}\sqrt{}}$$

Mathematica [A] time = 0.26944, size = 76, normalized size = 0.39

$$\frac{2(104\sin(c+dx)+48\sin(3(c+dx))+8\cos(2(c+dx))-16\cos(4(c+dx))+45)}{385de(a(\sin(c+dx)+1))^{3/2}(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (2*(45 + 8*Cos[2*(c + d*x)] - 16*Cos[4*(c + d*x)] + 104*Sin[c + d*x] + 48*S
in[3*(c + d*x)]))/(385*d*e*(e*Cos[c + d*x])^(5/2)*(a*(1 + Sin[c + d*x]))^(3
/2))
```

Maple [A] time = 0.114, size = 80, normalized size = 0.4

$$\frac{\left(-256\,(\cos(dx+c))^4+384\,(\cos(dx+c))^2\sin(dx+c)+288\,(\cos(dx+c))^2+112\,\sin(dx+c)+42\right)\cos(dx+c)}{385\,d}(e\,\mathrm{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/385/d*(-128*cos(d*x+c)^4+192*cos(d*x+c)^2*sin(d*x+c)+144*cos(d*x+c)^2+56*
sin(d*x+c)+21)*cos(d*x+c)/(e*cos(d*x+c))^(7/2)/(a*(1+sin(d*x+c)))^(3/2)
```

Maxima [B] time = 1.69086, size = 609, normalized size = 3.16

$$\frac{2\left(37\sqrt{a}\sqrt{e} + \frac{496\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1} + \frac{559\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{544\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{1526\sqrt{a}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{1526\sqrt{a}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{544\sqrt{a}\sqrt{e}\sin}{(\cos(dx+}\right.}{385\left(a^2e^4 + \frac{5a^2e^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10a^2e^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{10a^2e^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{5a^2e^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{a^2e^4\sin}{(\cos(d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/385*(37*sqrt(a)*sqrt(e) + 496*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c)
+ 1) + 559*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 544*sqrt(a
)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1526*sqrt(a)*sqrt(e)*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + 1526*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 544*sqrt(a)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5
59*sqrt(a)*sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 496*sqrt(a)*sqrt(e
)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 37*sqrt(a)*sqrt(e)*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((a^2*e^
4 + 5*a^2*e^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^2*e^4*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 10*a^2*e^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
5*a^2*e^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^2*e^4*sin(d*x + c)^10/(co
s(d*x + c) + 1)^10)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*
x + c)/(cos(d*x + c) + 1) + 1)^(7/2))
```

Fricas [A] time = 2.66741, size = 323, normalized size = 1.67

$$\frac{2\left(128\cos(dx+c)^4 - 144\cos(dx+c)^2 - 8\left(24\cos(dx+c)^2 + 7\right)\sin(dx+c) - 21\right)\sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+c)+a}}{385\left(a^2de^4\cos(dx+c)^5 - 2a^2de^4\cos(dx+c)^3\sin(dx+c) - 2a^2de^4\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/385*(128*cos(d*x + c)^4 - 144*cos(d*x + c)^2 - 8*(24*cos(d*x + c)^2 + 7)*
sin(d*x + c) - 21)*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)/(a^2*d*e^4
*cos(d*x + c)^5 - 2*a^2*d*e^4*cos(d*x + c)^3*sin(d*x + c) - 2*a^2*d*e^4*cos
(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(a*sin(d*x + c) + a)^(3/2)), x)
```

3.313 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=261

$$\frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a\sin(c+dx)+a}}+\frac{21e^{9/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{4d\left(a^3\sin(c+dx)+a^3\cos(c+dx)+a^3\right)}+\frac{21e^{9/2}\sqrt{\cos(c+dx}}{4d\left(a^3\,\mathrm{s}\right.}$$

```
[Out] (e*(e*Cos[c + d*x])^(7/2))/(2*a*d*(a + a*Sin[c + d*x])^(3/2)) + (7*e^3*(e*C
os[c + d*x])^(3/2))/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (21*e^(9/2)*ArcSin
h[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d
*x]])/(4*d*(a^3 + a^3*Cos[c + d*x] + a^3*Sin[c + d*x])) + (21*e^(9/2)*ArcTa
n[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqr
t[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*(a^3 + a^3*Cos[c + d*x]
+ a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.46874, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2680, 2686, 2679, 2684, 2775, 203, 2833, 63, 215}

$$\frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a\sin(c+dx)+a}}+\frac{21e^{9/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{4d\left(a^3\sin(c+dx)+a^3\cos(c+dx)+a^3\right)}+\frac{21e^{9/2}\sqrt{\cos(c+dx}}{4d\left(a^3\,\mathrm{s}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (e*(e*Cos[c + d*x])^(7/2))/(2*a*d*(a + a*Sin[c + d*x])^(3/2)) + (7*e^3*(e*C
os[c + d*x])^(3/2))/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (21*e^(9/2)*ArcSin
h[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d
*x]])/(4*d*(a^3 + a^3*Cos[c + d*x] + a^3*Sin[c + d*x])) + (21*e^(9/2)*ArcTa
n[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqr
t[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*(a^3 + a^3*Cos[c + d*x]
+ a^3*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
```

```
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2686

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-2*b*(g*Cos[e + f*x])^(p + 1))/(f*g*(2*p - 1)*
(a + b*Sin[e + f*x])^(3/2)), x] + Dist[(2*a*(p - 2))/(2*p - 1), Int[(g*Cos[
e + f*x])^p/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 2] && IntegerQ[2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^{3/2}} + \frac{\left(7e^2\right)\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{\left(7e^2\right)\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\left(21e^4\right)\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx}{8a^2}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\left(21e^5\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(\right.}{8a^2(a+a\cos(c+dx)+}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\left(21e^5\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(\right.}{8a^2d(a+a\cos}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{21e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx}}\right.}{4d\left(a^3+a^3\cos(c+\right.}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{21e^{9/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\text{co}}}{4d\left(a^3+a^3\cos(c+dx)\right.}$$

Mathematica [C] time = 0.16791, size = 80, normalized size = 0.31

$$-\frac{2\sqrt[4]{2}\sqrt{a(\sin(c+dx)+1)}(e\cos(c+dx))^{11/2}\,{}_2F_1\left(\frac{3}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11a^3de(\sin(c+dx)+1)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[3/4, 11/4, 15/4, (1 -
 Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(11*a^3*d*e*(1 + Sin[c + d*x]
)^(13/4))
```

Maple [A] time = 0.146, size = 282, normalized size = 1.1

$$\frac{1}{8\,d\left(\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right)+\left(\cos\left(dx+c\right)\right)^{3}+2\,\cos\left(dx+c\right)\sin\left(dx+c\right)-3\,\left(\cos\left(dx+c\right)\right)^{2}-4\,\sin\left(dx+c\right)-2\,\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/8/d*(e*cos(d*x+c))^(9/2)*(21*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(
1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)+21*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+4*cos(d*x+c)^2*si
n(d*x+c)-4*cos(d*x+c)^3-22*cos(d*x+c)*sin(d*x+c)-18*cos(d*x+c)^2+22*cos(d*x
+c))/(cos(d*x+c)^2*sin(d*x+c)+cos(d*x+c)^3+2*cos(d*x+c)*sin(d*x+c)-3*cos(d*
x+c)^2-4*sin(d*x+c)-2*cos(d*x+c)+4)/(a*(1+sin(d*x+c)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e\cos\left(dx+c\right)\right)^{\frac{9}{2}}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.314 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{5e^3\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}{a^3d}-\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{a^3d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{5e^{7/2}}{}$$

```
[Out] (-4*e*(e*Cos[c + d*x])^(5/2))/(a*d*(a + a*Sin[c + d*x])^(3/2)) - (5*e^3*Sqr
t[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a^3*d) + (5*e^(7/2)*ArcSinh[Sq
rt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]
)/(a^3*d*(1 + Cos[c + d*x] + Sin[c + d*x])) - (5*e^(7/2)*ArcTan[(Sqrt[e]*Si
n[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c +
 d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a^3*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rubi [A] time = 0.366482, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2680, 2685, 2677, 2775, 203, 2833, 63, 215}

$$-\frac{5e^3\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}{a^3d}-\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{a^3d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{5e^{7/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-4*e*(e*Cos[c + d*x])^(5/2))/(a*d*(a + a*Sin[c + d*x])^(3/2)) - (5*e^3*Sqr
t[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a^3*d) + (5*e^(7/2)*ArcSinh[Sq
rt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]
)/(a^3*d*(1 + Cos[c + d*x] + Sin[c + d*x])) - (5*e^(7/2)*ArcTan[(Sqrt[e]*Si
n[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c +
 d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a^3*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```


Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2685

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[(g*Sqrt[g*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(b*f), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2677

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[(a*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[(b*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{\left(5e^2\right)\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^3d} - \frac{\left(5e^4\right)\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{2a^3}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^3d} - \frac{\left(5e^4\sqrt{1+\cos(c+dx)}\right.}{2a^2(a+a\cos}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^3d} + \frac{\left(5e^4\sqrt{1+\cos(c+dx)}\right.}{2a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^3d} - \frac{5e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\sin}{\sqrt{e\cos(c+dx)}}\right.}{d\left(a^3+\right.}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{a^3d} + \frac{5e^{7/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+d}}{\sqrt{e}}\right.}{d\left(a^3+a^3\,\mathrm{c}\right.}$$

Mathematica [C] time = 0.11486, size = 80, normalized size = 0.33

$$-\frac{2^{3/4}\sqrt{a(\sin(c+dx)+1)}(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9a^3de(\sin(c+dx)+1)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(2^(3/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[5/4, 9/4, 13/4, (1 - Sin
[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(9*a^3*d*e*(1 + Sin[c + d*x])^(11
```

```
/4))
```

Maple [B] time = 0.117, size = 443, normalized size = 1.9

$$\frac{1}{4\,d\left(2\,\sin\left(dx+c\right)+\left(\cos\left(dx+c\right)\right)^{2}-2\right)}\left(5\,\sqrt{-2\,\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\,\arctan\left(1/2\,\sqrt{2}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right)\sqrt{2}\sin\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/4/d*(5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*sin(d*x+c)-5*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+5*cos(d*x+c)*2^(1/2)*arctan(1/2*2^(1/2)*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-
5*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*2^(1/
2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))+5*2^(1/2)*ar
ctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)-5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2
*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))+4*cos(
d*x+c)*sin(d*x+c)+36*cos(d*x+c))*(e*cos(d*x+c))^(7/2)/(2*sin(d*x+c)+cos(d*x
+c)^2-2)/(a*(1+sin(d*x+c)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.315 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=218

$$-\frac{2e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d\left(a^3\sin(c+dx)+a^3\cos(c+dx)+a^3\right)}-\frac{2e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\mathrm{si}}{d\left(a^3\sin(c+dx)+a^3\cos(c+dx\right.}$$

```
[Out] (-4*e*(e*Cos[c + d*x])^(3/2))/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) - (2*e^(5/
2)*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*
Sin[c + d*x]])/(d*(a^3 + a^3*Cos[c + d*x] + a^3*Sin[c + d*x])) - (2*e^(5/2)
*ArcTan[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]
)]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(a^3 + a^3*Cos[c + d
*x] + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.295897, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2680, 2684, 2775, 203, 2833, 63, 215}

$$-\frac{2e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d\left(a^3\sin(c+dx)+a^3\cos(c+dx)+a^3\right)}-\frac{2e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\mathrm{si}}{d\left(a^3\sin(c+dx)+a^3\cos(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*e*(e*Cos[c + d*x])^(3/2))/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) - (2*e^(5/
2)*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*
Sin[c + d*x]])/(d*(a^3 + a^3*Cos[c + d*x] + a^3*Sin[c + d*x])) - (2*e^(5/2)
*ArcTan[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]
)]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(a^3 + a^3*Cos[c + d
*x] + a^3*Sin[c + d*x]))
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2684

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x
]])/(a + a*Cos[e + f*x] + b*Sin[e + f*x]), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[(g*Sqrt[1 + Cos[e + f*x]]*Sqrt[a + b*Sin[e +
 f*x]])/(b + b*Cos[e + f*x] + a*Sin[e + f*x]), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{\left(e^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{a^2(a+a\cos(c+dx)+a\sin(c+dx))} +$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{\left(e^3\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x}}\,d\right.}{a^2d(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a}}{d\left(a^3+a^3\cos(c+dx)+a^3\sin(c+dx)\right)}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2e^{5/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+d}}{d\left(a^3+a^3\cos(c+dx)+a^3\sin(c+dx)\right)}$$

Mathematica [C] time = 0.12835, size = 80, normalized size = 0.37

$$-\frac{\sqrt[4]{2}\sqrt{a(\sin(c+dx)+1)}(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7a^3de(\sin(c+dx)+1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(2^(1/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, 7/4, 11/4, (1 - Sin
[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(7*a^3*d*e*(1 + Sin[c + d*x])^(9/
4))
```

Maple [B] time = 0.115, size = 545, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d*(3*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*
sin(d*x+c)*cos(d*x+c)+3*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)+3*cos(d*x+c)^2*2
^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+3*cos(d*x+c
)^2*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c))-4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+
c)-6*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*sin(d
*x+c)-6*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)/cos(d*x+c))*sin(d*x+c)+3*cos(d*x+c)*2^(1/2)*arctan(1/2*2^(1/2)*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+3*cos(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c))-6*2^(1/2)*arct
an(1/2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-6*2^(1/2)*arctanh(1/2*
2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)))*(e*cos
(d*x+c))^(5/2)/(sin(d*x+c)-1)/(a*(1+sin(d*x+c)))^(5/2)/(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.316 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=36

$$-\frac{2(e\cos(c+dx))^{5/2}}{5de(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] (-2*(e*Cos[c + d*x])^(5/2))/(5*d*e*(a + a*Sin[c + d*x])^(5/2))
```

Rubi [A] time = 0.0705864, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2671}

$$-\frac{2(e\cos(c+dx))^{5/2}}{5de(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(5/2))/(5*d*e*(a + a*Sin[c + d*x])^(5/2))
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^{5/2}}\,dx=-\frac{2(e\cos(c+dx))^{5/2}}{5de(a+a\sin(c+dx))^{5/2}}$$

Mathematica [A] time = 0.107092, size = 49, normalized size = 1.36

$$-\frac{2\sqrt{a(\sin(c+dx)+1)}(e\cos(c+dx))^{5/2}}{5a^3de(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(5/2)*Sqrt[a*(1 + Sin[c + d*x])])/(5*a^3*d*e*(1 + Sin[
c + d*x])^3)
```

Maple [A] time = 0.087, size = 34, normalized size = 0.9

$$-\frac{2\cos(dx+c)}{5d}(e\cos(dx+c))^{\frac{3}{2}}(a(1+\sin(dx+c)))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/5/d*(e*cos(d*x+c))^(3/2)*cos(d*x+c)/(a*(1+sin(d*x+c)))^(5/2)
```

Maxima [B] time = 1.58983, size = 177, normalized size = 4.92

$$-\frac{2\left(\sqrt{a}e^{\frac{3}{2}}-\frac{\sqrt{a}e^{\frac{3}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{5\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] -2/5*(sqrt(a)*e^(3/2) - sqrt(a)*e^(3/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 1)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d*(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)^(7/2))
```

Fricas [B] time = 3.42259, size = 174, normalized size = 4.83

$$-\frac{2\sqrt{e\cos\left(dx+c\right)}\sqrt{a\sin\left(dx+c\right)+a}(e\sin\left(dx+c\right)-e)}{5\left(a^3d\cos\left(dx+c\right)^2-2\,a^3d\sin\left(dx+c\right)-2\,a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/5*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)*(e*sin(d*x + c) - e)/(a^
3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.317 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{4(e\cos(c+dx))^{3/2}}{21ade(a\sin(c+dx)+a)^{3/2}}-\frac{2(e\cos(c+dx))^{3/2}}{7de(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] (-2*(e*Cos[c + d*x])^(3/2))/(7*d*e*(a + a*Sin[c + d*x])^(5/2)) - (4*(e*Cos[
c + d*x])^(3/2))/(21*a*d*e*(a + a*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.13138, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{4(e\cos(c+dx))^{3/2}}{21ade(a\sin(c+dx)+a)^{3/2}}-\frac{2(e\cos(c+dx))^{3/2}}{7de(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(3/2))/(7*d*e*(a + a*Sin[c + d*x])^(5/2)) - (4*(e*Cos[
c + d*x])^(3/2))/(21*a*d*e*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2(e\cos(c+dx))^{3/2}}{7de(a+a\sin(c+dx))^{5/2}} + \frac{2\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{3/2}}\,dx}{7a}$$
$$= -\frac{2(e\cos(c+dx))^{3/2}}{7de(a+a\sin(c+dx))^{5/2}} - \frac{4(e\cos(c+dx))^{3/2}}{21ade(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.102302, size = 59, normalized size = 0.78

$$-\frac{2(2\sin(c+dx)+5)\sqrt{a(\sin(c+dx)+1)}(e\cos(c+dx))^{3/2}}{21a^3de(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(3/2)*Sqrt[a*(1 + Sin[c + d*x])]*(5 + 2*Sin[c + d*x]))
/(21*a^3*d*e*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.11, size = 44, normalized size = 0.6

$$-\frac{(4\sin(dx+c)+10)\cos(dx+c)}{21\,d}\sqrt{e\cos(dx+c)}\,(a(1+\sin(dx+c)))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/21/d*(2*sin(d*x+c)+5)*cos(d*x+c)*(e*cos(d*x+c))^(1/2)/(a*(1+sin(d*x+c)))
^(5/2)
```

Maxima [B] time = 1.58982, size = 279, normalized size = 3.67

$$-\frac{2\left(5\sqrt{a}\sqrt{e} + \frac{4\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1} - \frac{4\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{5\sqrt{a}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{21\left(a^3 + \frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] -2/21*(5*sqrt(a)*sqrt(e) + 4*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) + 1
) - 4*sqrt(a)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sqrt(a)*sqrt(
e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*sqrt(-sin(d*x + c)/(cos(d*x + c) +
1) + 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((a^3 + 2*a^3*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d*(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2))
```

Fricas [B] time = 3.31139, size = 378, normalized size = 4.97

$$\frac{2\sqrt{e\cos(dx+c)}\left(2\cos(dx+c)^2+(2\cos(dx+c)-3)\sin(dx+c)+5\cos(dx+c)+3\right)\sqrt{a\sin(dx+c)+a}}{21\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)-4a^3d+\left(a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)-4a^3d\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 2/21*sqrt(e*cos(d*x + c))*(2*cos(d*x + c)^2 + (2*cos(d*x + c) - 3)*sin(d*x
+ c) + 5*cos(d*x + c) + 3)*sqrt(a*sin(d*x + c) + a)/(a^3*d*cos(d*x + c)^3 +
 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x +
 c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos(dx+c)}}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(a*sin(d*x + c) + a)^(5/2), x)
```

3.318 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{16\sqrt{e\cos(c+dx)}}{45a^2de\sqrt{a\sin(c+dx)+a}}-\frac{8\sqrt{e\cos(c+dx)}}{45ade(a\sin(c+dx)+a)^{3/2}}-\frac{2\sqrt{e\cos(c+dx)}}{9de(a\sin(c+dx)+a)^{5/2}}$$

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(9*d*e*(a + a*Sin[c + d*x])^(5/2)) - (8*Sqrt[e*Co
s[c + d*x]])/(45*a*d*e*(a + a*Sin[c + d*x])^(3/2)) - (16*Sqrt[e*Cos[c + d*x
]])/(45*a^2*d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.20212, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{16\sqrt{e\cos(c+dx)}}{45a^2de\sqrt{a\sin(c+dx)+a}}-\frac{8\sqrt{e\cos(c+dx)}}{45ade(a\sin(c+dx)+a)^{3/2}}-\frac{2\sqrt{e\cos(c+dx)}}{9de(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(9*d*e*(a + a*Sin[c + d*x])^(5/2)) - (8*Sqrt[e*Co
s[c + d*x]])/(45*a*d*e*(a + a*Sin[c + d*x])^(3/2)) - (16*Sqrt[e*Cos[c + d*x
]])/(45*a^2*d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
```

```
&& EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{e\cos(c+dx)}}{9de(a+a\sin(c+dx))^{5/2}} + \frac{4\int \frac{1}{\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{3/2}}\,dx}{9a}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{9de(a+a\sin(c+dx))^{5/2}} - \frac{8\sqrt{e\cos(c+dx)}}{45ade(a+a\sin(c+dx))^{3/2}} + \frac{8\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a\cdots}}}{45a^2\cdots}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{9de(a+a\sin(c+dx))^{5/2}} - \frac{8\sqrt{e\cos(c+dx)}}{45ade(a+a\sin(c+dx))^{3/2}} - \frac{16\sqrt{e\cos(c+\cdots}}{45a^2de\sqrt{a+a\sin\cdots}}$$

Mathematica [A] time = 0.142075, size = 69, normalized size = 0.6

$$-\frac{2\left(8\sin^2(c+dx)+20\sin(c+dx)+17\right)\sqrt{a(\sin(c+dx)+1)}\sqrt{e\cos(c+dx)}}{45a^3de(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*Sqrt[a*(1 + Sin[c + d*x])]*(17 + 20*Sin[c + d*x] +
 8*Sin[c + d*x]^2))/(45*a^3*d*e*(1 + Sin[c + d*x])^3)
```

Maple [A] time = 0.115, size = 54, normalized size = 0.5

$$-\frac{\left(-16\,(\cos(dx+c))^2+40\,\sin(dx+c)+50\right)\cos(dx+c)}{45\,d}\,(a\,(1+\sin(dx+c)))^{-\frac{5}{2}}\,\frac{1}{\sqrt{e\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/45/d*(-8*cos(d*x+c)^2+20*sin(d*x+c)+25)*cos(d*x+c)/(a*(1+sin(d*x+c)))^(5
/2)/(e*cos(d*x+c))^(1/2)
```

Maxima [B] time = 1.61961, size = 387, normalized size = 3.37

$$-\frac{2\left(17\sqrt{a}\sqrt{e}+\frac{40\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{49\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{49\sqrt{a}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{40\sqrt{a}\sqrt{e}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{17\sqrt{a}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right.}{45\left(a^3e+\frac{3a^3e\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^3e\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3e\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2/45*(17*sqrt(a)*sqrt(e) + 40*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) +
 1) + 49*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 49*sqrt(a)*s
qrt(e)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 40*sqrt(a)*sqrt(e)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 - 17*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x + c)
+ 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((a^3*e + 3*a^3*e*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*e*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + a^3*e*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d*(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)^(11/2)*sqrt(-sin(d*x + c)/(cos(d*x + c) + 1) + 1))
```

Fricas [A] time = 3.10313, size = 251, normalized size = 2.18

$$-\frac{2\sqrt{e\cos(dx+c)}\left(8\cos(dx+c)^2-20\sin(dx+c)-25\right)\sqrt{a\sin(dx+c)+a}}{45\left(3a^3de\cos(dx+c)^2-4a^3de+\left(a^3de\cos(dx+c)^2-4a^3de\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2/45*sqrt(e*cos(d*x + c))*(8*cos(d*x + c)^2 - 20*sin(d*x + c) - 25)*sqrt(a
*sin(d*x + c) + a)/(3*a^3*d*e*cos(d*x + c)^2 - 4*a^3*d*e + (a^3*d*e*cos(d*x
 + c)^2 - 4*a^3*d*e)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos\left(dx+c\right)}(a\sin\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^(5/2)), x)
```

3.319 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{32\sqrt{a\sin(c+dx)+a}}{77a^3de\sqrt{e\cos(c+dx)}}-\frac{16}{77a^2de\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}-\frac{12}{77ade(a\sin(c+dx)+a)^{3/2}\sqrt{e\cos(c+dx)}}-\frac{}{11de(a}$$

```
[Out] -2/(11*d*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2)) - 12/(77*a*d*e*
Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)) - 16/(77*a^2*d*e*Sqrt[e*Co
s[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]) + (32*Sqrt[a + a*Sin[c + d*x]])/(77*a
^3*d*e*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.296674, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{32\sqrt{a\sin(c+dx)+a}}{77a^3de\sqrt{e\cos(c+dx)}}-\frac{16}{77a^2de\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}-\frac{12}{77ade(a\sin(c+dx)+a)^{3/2}\sqrt{e\cos(c+dx)}}-\frac{}{11de(a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2)),x]
```


```
[Out] -2/(11*d*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2)) - 12/(77*a*d*e*
Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)) - 16/(77*a^2*d*e*Sqrt[e*Co
s[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]) + (32*Sqrt[a + a*Sin[c + d*x]])/(77*a
^3*d*e*Sqrt[e*Cos[c + d*x]])
```


Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
```

```
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2}{11de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{5/2}} + \frac{6\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3}}}{11a}$$

$$= -\frac{2}{11de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{5/2}} - \frac{12}{77ade\sqrt{e\cos(c+dx)}(a+a\sin}$$

$$= -\frac{2}{11de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{5/2}} - \frac{12}{77ade\sqrt{e\cos(c+dx)}(a+a\sin}$$

$$= -\frac{2}{11de\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{5/2}} - \frac{12}{77ade\sqrt{e\cos(c+dx)}(a+a\sin}$$

Mathematica [A] time = 0.137724, size = 66, normalized size = 0.43

$$\frac{32\sin^3(c+dx)+80\sin^2(c+dx)+52\sin(c+dx)-10}{77de(a(\sin(c+dx)+1))^{5/2}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-10 + 52*Sin[c + d*x] + 80*Sin[c + d*x]^2 + 32*Sin[c + d*x]^3)/(77*d*e*Sqr
t[e*Cos[c + d*x]]*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A] time = 0.112, size = 70, normalized size = 0.5

$$-\frac{\left(32\,(\cos(dx+c))^2\sin(dx+c)+80\,(\cos(dx+c))^2-84\,\sin(dx+c)-70\right)\cos(dx+c)}{77\,d}(e\cos(dx+c))^{-\frac{3}{2}}(a(1+\sin(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/77/d*(16*cos(d*x+c)^2*sin(d*x+c)+40*cos(d*x+c)^2-42*sin(d*x+c)-35)*cos(d
*x+c)/(e*cos(d*x+c))^(3/2)/(a*(1+sin(d*x+c)))^(5/2)
```

Maxima [B] time = 1.66377, size = 504, normalized size = 3.27

$$-\frac{2\left(5\sqrt{a}\sqrt{e}-\frac{52\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}-\frac{150\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{180\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{180\sqrt{a}\sqrt{e}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{150\sqrt{a}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{52\sqrt{a}\sqrt{e}\sin(}{(\cos(dx+c)}\right.}{77\left(a^3e^2+\frac{4a^3e^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3e^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^3e^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3e^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/77*(5*sqrt(a)*sqrt(e) - 52*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c) +
1) - 150*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 180*sqrt(a)*
sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 180*sqrt(a)*sqrt(e)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 + 150*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 52*sqrt(a)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5*sqrt
(a)*sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)^4/((a^3*e^2 + 4*a^3*e^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 6*a^3*e^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^3*e^2*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + a^3*e^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin
(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)^(3/2))
```

Fricas [A] time = 2.89052, size = 328, normalized size = 2.13

$$\frac{2\sqrt{e\cos(dx+c)}\left(40\cos(dx+c)^2+2\left(8\cos(dx+c)^2-21\right)\sin(dx+c)-35\right)\sqrt{a\sin(dx+c)+a}}{77\left(3a^3de^2\cos(dx+c)^3-4a^3de^2\cos(dx+c)+\left(a^3de^2\cos(dx+c)^3-4a^3de^2\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/77*sqrt(e*cos(d*x + c))*(40*cos(d*x + c)^2 + 2*(8*cos(d*x + c)^2 - 21)*si
n(d*x + c) - 35)*sqrt(a*sin(d*x + c) + a)/(3*a^3*d*e^2*cos(d*x + c)^3 - 4*a
```

```
^3*d*e^2*cos(d*x + c) + (a^3*d*e^2*cos(d*x + c)^3 - 4*a^3*d*e^2*cos(d*x + c
))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^(5/2)), x)
```

3.320 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=193

$$\frac{256(a\sin(c+dx)+a)^{3/2}}{585a^4de(e\cos(c+dx))^{3/2}}-\frac{128\sqrt{a\sin(c+dx)+a}}{195a^3de(e\cos(c+dx))^{3/2}}-\frac{32}{195a^2de\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{3/2}}-\frac{}{117ade(a\sin(c+}$$

```
[Out] -2/(13*d*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2)) - 16/(117*a*d
*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)) - 32/(195*a^2*d*e*(e*
Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]) - (128*Sqrt[a + a*Sin[c + d*x
]])/(195*a^3*d*e*(e*Cos[c + d*x])^(3/2)) + (256*(a + a*Sin[c + d*x])^(3/2))
/(585*a^4*d*e*(e*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.375388, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{256(a\sin(c+dx)+a)^{3/2}}{585a^4de(e\cos(c+dx))^{3/2}}-\frac{128\sqrt{a\sin(c+dx)+a}}{195a^3de(e\cos(c+dx))^{3/2}}-\frac{32}{195a^2de\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{3/2}}-\frac{}{117ade(a\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(5/2)),x]
```


```
[Out] -2/(13*d*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2)) - 16/(117*a*d
*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)) - 32/(195*a^2*d*e*(e*
Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]) - (128*Sqrt[a + a*Sin[c + d*x
]])/(195*a^3*d*e*(e*Cos[c + d*x])^(3/2)) + (256*(a + a*Sin[c + d*x])^(3/2))
/(585*a^4*d*e*(e*Cos[c + d*x])^(3/2))
```


Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} + \frac{8\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx}}{13a}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} - \frac{16}{117ade(e\cos(c+dx))^{3/2}(a}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} - \frac{16}{117ade(e\cos(c+dx))^{3/2}(a}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} - \frac{16}{117ade(e\cos(c+dx))^{3/2}(a}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} - \frac{16}{117ade(e\cos(c+dx))^{3/2}(a}$$

Mathematica [A] time = 0.330774, size = 76, normalized size = 0.39

$$-\frac{2(-40\sin(c+dx)+80\sin(3(c+dx))+136\cos(2(c+dx))-16\cos(4(c+dx))+77)}{585de(a(\sin(c+dx)+1))^{5/2}(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*(77 + 136*Cos[2*(c + d*x)] - 16*Cos[4*(c + d*x)] - 40*Sin[c + d*x] + 80
*Sin[3*(c + d*x)]))/(585*d*e*(e*Cos[c + d*x])^(3/2)*(a*(1 + Sin[c + d*x]))^
(5/2))
```

Maple [A] time = 0.115, size = 80, normalized size = 0.4

$$-\frac{\left(-256\,(\cos(dx+c))^4+640\,(\cos(dx+c))^2\sin(dx+c)+800\,(\cos(dx+c))^2-240\,\sin(dx+c)-150\right)\cos(dx+c)}{585\,d}(e\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/585/d*(-128*cos(d*x+c)^4+320*cos(d*x+c)^2*sin(d*x+c)+400*cos(d*x+c)^2-12
0*sin(d*x+c)-75)*cos(d*x+c)/(e*cos(d*x+c))^(5/2)/(a*(1+sin(d*x+c)))^(5/2)
```

Maxima [B] time = 1.69006, size = 609, normalized size = 3.16

$$-\frac{2\left(197\,\sqrt{a}\sqrt{e}+\frac{400\,\sqrt{a}\sqrt{e}\sin(dx+c)}{\cos(dx+c)+1}+\frac{15\,\sqrt{a}\sqrt{e}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1760\,\sqrt{a}\sqrt{e}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2230\,\sqrt{a}\sqrt{e}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2230\,\sqrt{a}\sqrt{e}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\cdots\right.}{585\left(a^3e^3+\frac{5\,a^3e^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,a^3e^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\,a^3e^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5\,a^3e^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/585*(197*sqrt(a)*sqrt(e) + 400*sqrt(a)*sqrt(e)*sin(d*x + c)/(cos(d*x + c
) + 1) + 15*sqrt(a)*sqrt(e)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1760*sqrt
(a)*sqrt(e)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2230*sqrt(a)*sqrt(e)*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4 + 2230*sqrt(a)*sqrt(e)*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 1760*sqrt(a)*sqrt(e)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
- 15*sqrt(a)*sqrt(e)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 400*sqrt(a)*sqrt
(e)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 197*sqrt(a)*sqrt(e)*sin(d*x + c)^
10/(cos(d*x + c) + 1)^10)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((a^3
*e^3 + 5*a^3*e^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*e^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*e^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + 5*a^3*e^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^3*e^3*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin
(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2))
```

Fricas [A] time = 2.9877, size = 366, normalized size = 1.9

$$-\frac{2\left(128\,\cos(dx+c)^4-400\,\cos(dx+c)^2-40\left(8\,\cos(dx+c)^2-3\right)\sin(dx+c)+75\right)\sqrt{e\cos(dx+c)}\sqrt{a\sin(dx+c)+\cdots}}{585\left(3\,a^3de^3\cos(dx+c)^4-4\,a^3de^3\cos(dx+c)^2+\left(a^3de^3\cos(dx+c)^4-4\,a^3de^3\cos(dx+c)^2\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] -2/585*(128*cos(d*x + c)^4 - 400*cos(d*x + c)^2 - 40*(8*cos(d*x + c)^2 - 3)
*sin(d*x + c) + 75)*sqrt(e*cos(d*x + c))*sqrt(a*sin(d*x + c) + a)/(3*a^3*d*
e^3*cos(d*x + c)^4 - 4*a^3*d*e^3*cos(d*x + c)^2 + (a^3*d*e^3*cos(d*x + c)^4
 - 4*a^3*d*e^3*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^(5/2)), x)
```

3.321 $\int \frac{(e\cos(c+dx))^{7/3}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3\sqrt[6]{2}a(e\cos(c+dx))^{10/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{8}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{5de\sqrt[6]{\sin(c+dx)+1}(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-3*2^(1/6)*a*(e*Cos[c + d*x])^(10/3)*Hypergeometric2F1[-1/6, 5/3, 8/3, (1
- Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(3
/2))
```

Rubi [A] time = 0.0954889, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 70, 69}

$$-\frac{3\sqrt[6]{2}a(e\cos(c+dx))^{10/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{8}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{5de\sqrt[6]{\sin(c+dx)+1}(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/3)/Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-3*2^(1/6)*a*(e*Cos[c + d*x])^(10/3)*Hypergeometric2F1[-1/6, 5/3, 8/3, (1
- Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(3
/2))
```


Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
```

```
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\cos(c+dx))^{7/3}}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\left(a^2(e\cos(c+dx))^{10/3}\right)\operatorname{Subst}\left(\int (a-ax)^{2/3}\sqrt[6]{a+ax}\,dx, x, \sin(c+dx)\right)}{de(a-a\sin(c+dx))^{5/3}(a+a\sin(c+dx))^{5/3}}\\
&= \frac{\left(\sqrt[6]{2}a^2(e\cos(c+dx))^{10/3}\right)\operatorname{Subst}\left(\int \sqrt[6]{\frac{1}{2}+\frac{x}{2}}(a-ax)^{2/3}\,dx, x, \sin(c+dx)\right)}{de(a-a\sin(c+dx))^{5/3}(a+a\sin(c+dx))^{3/2}\sqrt[6]{\frac{a+a\sin(c+dx)}{a}}}\\
&= -\frac{3\sqrt[6]{2}a(e\cos(c+dx))^{10/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{8}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{5de\sqrt[6]{1+\sin(c+dx)}(a+a\sin(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.153058, size = 77, normalized size = 0.99

$$-\frac{3\sqrt[6]{2}(e\cos(c+dx))^{10/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{8}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(\sin(c+dx)+1)^{7/6}\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*2^(1/6)*(e*Cos[c + d*x])^(10/3)*Hypergeometric2F1[-1/6, 5/3, 8/3, (1 -
Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(7/6)*Sqrt[a*(1 + Sin[c + d*x])
])
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{7}{3}}\frac{1}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{3}}}{\sqrt{a\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(7/3)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e\cos\left(dx+c\right)\right)^{\frac{1}{3}}e^{2}\cos\left(dx+c\right)^{2}}{\sqrt{a\sin\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*cos(d*x + c))^(1/3)*e^2*cos(d*x + c)^2/sqrt(a*sin(d*x + c) + a)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{3}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/3)/sqrt(a*sin(d*x + c) + a), x)
```

3.322 $\int \frac{(e\cos(c+dx))^{5/3}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3a\sqrt[6]{\sin(c+dx)+1}(e\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{4\sqrt[6]{2}de(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-3*a*(e*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/6, 4/3, 7/3, (1 - Sin[c + 
d*x])/2]*(1 + Sin[c + d*x])^(1/6))/(4*2^(1/6)*d*e*(a + a*Sin[c + d*x])^(3/2
))
```

Rubi [A] time = 0.0973489, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 70, 69}

$$-\frac{3a\sqrt[6]{\sin(c+dx)+1}(e\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{4\sqrt[6]{2}de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/3)/Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-3*a*(e*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/6, 4/3, 7/3, (1 - Sin[c + 
d*x])/2]*(1 + Sin[c + d*x])^(1/6))/(4*2^(1/6)*d*e*(a + a*Sin[c + d*x])^(3/2
))
```


Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
```

```
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{5/3}}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\left(a^2(e\cos(c+dx))^{8/3}\right)\mathrm{Subst}\left(\int \frac{\sqrt[3]{a-ax}}{\sqrt[6]{a+ax}}\,dx, x, \sin(c+dx)\right)}{de(a-a\sin(c+dx))^{4/3}(a+a\sin(c+dx))^{4/3}}\\ &= \frac{\left(a^2(e\cos(c+dx))^{8/3}\sqrt[6]{\frac{a+a\sin(c+dx)}{a}}\right)\mathrm{Subst}\left(\int \frac{\sqrt[3]{a-ax}}{\sqrt[6]{\frac{1}{2}+\frac{x}{2}}}\,dx, x, \sin(c+dx)\right)}{\sqrt[6]{2}de(a-a\sin(c+dx))^{4/3}(a+a\sin(c+dx))^{3/2}}\\ &= -\frac{3a(e\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{3};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[6]{1+\sin(c+dx)}}{4\sqrt[6]{2}de(a+a\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.108071, size = 77, normalized size = 0.99

$$-\frac{3(e\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{4\sqrt[6]{2}de(\sin(c+dx)+1)^{5/6}\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*(e*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/6, 4/3, 7/3, (1 - Sin[c + d*
x])/2])/(4*2^(1/6)*d*e*(1 + Sin[c + d*x])^(5/6)*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{3}}\frac{1}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{3}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(5/3)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(e\cos(dx+c))^{\frac{2}{3}}\,e\cos(dx+c)}{\sqrt{a\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*cos(d*x + c))^(2/3)*e*cos(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{3}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/3)/sqrt(a*sin(d*x + c) + a), x)
```

3.323 $\int \frac{(e\cos(c+dx))^{2/3}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3\sqrt[3]{2}a(\sin(c+dx)+1)^{2/3}(e\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-3*2^(1/3)*a*(e*Cos[c + d*x])^(5/3)*Hypergeometric2F1[2/3, 5/6, 11/6, (1 -
 Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(2/3))/(5*d*e*(a + a*Sin[c + d*x])^(3/
2))
```

Rubi [A] time = 0.0970896, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 70, 69}

$$-\frac{3\sqrt[3]{2}a(\sin(c+dx)+1)^{2/3}(e\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(2/3)/Sqrt[a + a*Sin[c + d*x]],x]
```


```
[Out] (-3*2^(1/3)*a*(e*Cos[c + d*x])^(5/3)*Hypergeometric2F1[2/3, 5/6, 11/6, (1 -
 Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(2/3))/(5*d*e*(a + a*Sin[c + d*x])^(3/
2))
```


Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\cos(c+dx))^{2/3}}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\left(a^2(e\cos(c+dx))^{5/3}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{a-ax}(a+ax)^{2/3}}\,dx,x,\sin(c+dx)\right)}{de(a-a\sin(c+dx))^{5/6}(a+a\sin(c+dx))^{5/6}}\\
&= \frac{\left(a^2(e\cos(c+dx))^{5/3}\left(\frac{a+a\sin(c+dx)}{a}\right)^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{x}{2}\right)^{2/3}\sqrt[6]{a-ax}}\,dx,x,\sin(c+dx)\right)}{2^{2/3}de(a-a\sin(c+dx))^{5/6}(a+a\sin(c+dx))^{3/2}}\\
&= -\frac{3\sqrt[3]{2}a(e\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{2/3}}{5de(a+a\sin(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0766419, size = 77, normalized size = 0.99

$$-\frac{3\sqrt[3]{2}(e\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{5de\sqrt[3]{\sin(c+dx)+1}\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(2/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*2^(1/3)*(e*Cos[c + d*x])^(5/3)*Hypergeometric2F1[2/3, 5/6, 11/6, (1 - S
in[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(1/3)*Sqrt[a*(1 + Sin[c + d*x])]
)
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{2}{3}}\frac{1}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{2}{3}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(2/3)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(e\cos(dx+c))^{\frac{2}{3}}}{\sqrt{a\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*cos(d*x + c))^(2/3)/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(c+dx))^{\frac{2}{3}}}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(2/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**(2/3)/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{2}{3}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(2/3)/sqrt(a*sin(d*x + c) + a), x)
```

3.324 $\int \frac{\sqrt[3]{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3a(\sin(c+dx)+1)^{5/6}(e\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\,2^{5/6}de(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] (-3*a*(e*Cos[c + d*x])^(4/3)*Hypergeometric2F1[2/3, 5/6, 5/3, (1 - Sin[c +
d*x])/2]*(1 + Sin[c + d*x])^(5/6))/(2*2^(5/6)*d*e*(a + a*Sin[c + d*x])^(3/2
))
```

Rubi [A] time = 0.0855781, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 70, 69}

$$-\frac{3a(\sin(c+dx)+1)^{5/6}(e\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\,2^{5/6}de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(1/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*a*(e*Cos[c + d*x])^(4/3)*Hypergeometric2F1[2/3, 5/6, 5/3, (1 - Sin[c +
d*x])/2]*(1 + Sin[c + d*x])^(5/6))/(2*2^(5/6)*d*e*(a + a*Sin[c + d*x])^(3/2
))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeqQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\left(a^2(e\cos(c+dx))^{4/3}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a-ax}(a+ax)^{5/6}}\,dx, x, \sin(c+dx)\right)}{de(a-a\sin(c+dx))^{2/3}(a+a\sin(c+dx))^{2/3}}\\
&= \frac{\left(a^2(e\cos(c+dx))^{4/3}\left(\frac{a+a\sin(c+dx)}{a}\right)^{5/6}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{x}{2}\right)^{5/6}\sqrt[3]{a-ax}}\,dx, x, \sin(c+dx)\right)}{2^{5/6}de(a-a\sin(c+dx))^{2/3}(a+a\sin(c+dx))^{3/2}}\\
&= -\frac{3a(e\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{5/6}}{2\,2^{5/6}de(a+a\sin(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.074867, size = 77, normalized size = 0.99

$$-\frac{3(e\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\,2^{5/6}de\sqrt[6]{\sin(c+dx)+1}\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(1/3)/Sqrt[a + a*Sin[c + d*x]],x]

[Out] (-3*(e*Cos[c + d*x])^(4/3)*Hypergeometric2F1[2/3, 5/6, 5/3, (1 - Sin[c + d*
x])/2])/(2*2^(5/6)*d*e*(1 + Sin[c + d*x])^(1/6)*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int \sqrt[3]{e\cos(dx+c)}\frac{1}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{1}{3}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*cos(d*x + c))^(1/3)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(e\cos(dx+c))^{\frac{1}{3}}}{\sqrt{a\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*cos(d*x + c))^(1/3)/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{e\cos(c+dx)}}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**(1/3)/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{1}{3}}}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(1/3)/sqrt(a*sin(d*x + c) + a), x)
```

3.325 $\int \frac{1}{\sqrt[3]{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{3\sqrt[6]{\sin(c+dx)+1}(e\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\sqrt[6]{2}de\sqrt{a\sin(c+dx)+a}}$$

```
[Out] (-3*(e*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 7/6, 4/3, (1 - Sin[c + d*
x])/2]*(1 + Sin[c + d*x])^(1/6))/(2*2^(1/6)*d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.0872455, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 70, 69}

$$-\frac{3\sqrt[6]{\sin(c+dx)+1}(e\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\sqrt[6]{2}de\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(1/3)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (-3*(e*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 7/6, 4/3, (1 - Sin[c + d*
x])/2]*(1 + Sin[c + d*x])^(1/6))/(2*2^(1/6)*d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt[3]{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\left(a^2(e\cos(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{1}{(a-ax)^{2/3}(a+ax)^{7/6}}\,dx, x, \sin(c+dx)\right)}{de\sqrt[3]{a-a\sin(c+dx)}\sqrt[3]{a+a\sin(c+dx)}}\\
&= \frac{\left(a(e\cos(c+dx))^{2/3}\sqrt[6]{\frac{a+a\sin(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{x}{2}\right)^{7/6}(a-ax)^{2/3}}\,dx, x, \sin(c+dx)\right)}{2\sqrt[6]{2}de\sqrt[3]{a-a\sin(c+dx)}\sqrt{a+a\sin(c+dx)}}\\
&= -\frac{3(e\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[6]{1+\sin(c+dx)}}{2\sqrt[6]{2}de\sqrt{a+a\sin(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.0771017, size = 77, normalized size = 1.

$$
-\frac{3\sqrt[6]{\sin(c+dx)+1}(e\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\sqrt[6]{2}de\sqrt{a(\sin(c+dx)+1)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(1/3)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (-3*(e*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 7/6, 4/3, (1 - Sin[c + d*
x])/2]*(1 + Sin[c + d*x])^(1/6))/(2*2^(1/6)*d*e*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$
\int \frac{1}{\sqrt[3]{e\cos(dx+c)}}\frac{1}{\sqrt{a+a\sin(dx+c)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos{(dx+c)})^{\frac{1}{3}}\sqrt{a\sin{(dx+c)}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(1/3)*sqrt(a*sin(d*x + c) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\cos{(dx+c)})^{\frac{2}{3}}\sqrt{a\sin{(dx+c)}+a}}{ae\cos{(dx+c)}\sin{(dx+c)}+ae\cos{(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((e*cos(d*x + c))^(2/3)*sqrt(a*sin(d*x + c) + a)/(a*e*cos(d*x + c)*
sin(d*x + c) + a*e*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a(\sin{(c+dx)}+1)}\sqrt[3]{e\cos{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(1/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{1}{3}}\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(1/3)*sqrt(a*sin(d*x + c) + a)), x)
```

3.326 $\int \frac{1}{(e\cos(c+dx))^{4/3}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{3(\sin(c+dx)+1)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{2/3}de\sqrt{a\sin(c+dx)+a}\sqrt[3]{e\cos(c+dx)}}$$

```
[Out] (3*Hypergeometric2F1[-1/6, 5/3, 5/6, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^(2/3))/(2^(2/3)*d*e*(e*Cos[c + d*x])^(1/3)*Sqrt[a + a*Sin[c + d*x]])
```

Rubi [A] time = 0.0945724, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 70, 69}

$$\frac{3(\sin(c+dx)+1)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{2/3}de\sqrt{a\sin(c+dx)+a}\sqrt[3]{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(4/3)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 5/3, 5/6, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^(2/3))/(2^(2/3)*d*e*(e*Cos[c + d*x])^(1/3)*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{4/3}\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2\sqrt[6]{a-a\sin(c+dx)}\sqrt[6]{a+a\sin(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{(a-ax)^{7/6}(a+ax)^{5/3}}\,dx,x,\sin\right.}{de\sqrt[3]{e\cos(c+dx)}}$$

$$= \frac{\left(a\sqrt[6]{a-a\sin(c+dx)}\left(\frac{a+a\sin(c+dx)}{a}\right)^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{x}{2}\right)^{5/3}(a-ax)^{7/6}}\,dx,x,\sin(c\right.}{2\,2^{2/3}de\sqrt[3]{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{2/3}}{2^{2/3}de\sqrt[3]{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.0869293, size = 75, normalized size = 1.

$$\frac{3(\sin(c+dx)+1)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{2/3}de\sqrt{a(\sin(c+dx)+1)}\sqrt[3]{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(4/3)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 5/3, 5/6, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^(2/3))/(2^(2/3)*d*e*(e*Cos[c + d*x])^(1/3)*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-\frac{4}{3}}\frac{1}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{4}{3}}\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(4/3)*sqrt(a*sin(d*x + c) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(e\cos(dx+c))^{\frac{2}{3}}\sqrt{a\sin(dx+c)+a}}{ae^2\cos(dx+c)^2\sin(dx+c)+ae^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((e*cos(d*x + c))^(2/3)*sqrt(a*sin(d*x + c) + a)/(a*e^2*cos(d*x + c
)^2*sin(d*x + c) + a*e^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(4/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{4}{3}}\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((e*cos(d*x + c))^(4/3)*sqrt(a*sin(d*x + c) + a)), x)
```

3.327 $\int (e\cos(c+dx))^p (a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=95

$$-\frac{a^8 2^{\frac{p}{2}+\frac{17}{2}} (\sin(c+dx)+1)^{\frac{1}{2}(-p-1)} (e\cos(c+dx))^{p+1} {}_2F_1\left(\frac{1}{2}(-p-15),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

```
[Out] -((2^(17/2 + p/2)*a^8*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-15 - p)/
2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)
/2))/(d*e*(1 + p)))
```

Rubi [A] time = 0.0809274, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2688, 69}

$$-\frac{a^8 2^{\frac{p}{2}+\frac{17}{2}} (\sin(c+dx)+1)^{\frac{1}{2}(-p-1)} (e\cos(c+dx))^{p+1} {}_2F_1\left(\frac{1}{2}(-p-15),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -((2^(17/2 + p/2)*a^8*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-15 - p)/
2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)
/2))/(d*e*(1 + p)))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^8\,dx = \frac{\left(a^8(e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\Big(}{de}$$
$$= -\frac{2^{\frac{17}{2}+\frac{p}{2}}a^8(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-15-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+}{de(1+p)}$$

Mathematica [A] time = 0.198832, size = 94, normalized size = 0.99

$$-\frac{a^8 2^{\frac{p+17}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{1}{2}(-p-15),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -((2^((17 + p)/2)*a^8*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(-1
5 - p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((
-1 - p)/2))/(d*(1 + p)))
```

Maple [F] time = 8.375, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p(a+a\sin(dx+c))^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^8(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^8*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(\left(a^8\cos(dx+c)^8 - 32a^8\cos(dx+c)^6 + 160a^8\cos(dx+c)^4 - 256a^8\cos(dx+c)^2 + 128a^8 - 8\left(a^8\cos(dx+\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] integral((a^8*cos(d*x + c)^8 - 32*a^8*cos(d*x + c)^6 + 160*a^8*cos(d*x + c)
^4 - 256*a^8*cos(d*x + c)^2 + 128*a^8 - 8*(a^8*cos(d*x + c)^6 - 10*a^8*cos(
d*x + c)^4 + 24*a^8*cos(d*x + c)^2 - 16*a^8)*sin(d*x + c))*(e*cos(d*x + c))
^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^8(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^8*(e*cos(d*x + c))^p, x)
```

3.328 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=95

$$-\frac{a^3 2^{\frac{p}{2}+\frac{7}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-5),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

```
[Out] -((2^(7/2 + p/2)*a^3*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-5 - p)/2,
 (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2
))/(d*e*(1 + p)))
```

Rubi [A] time = 0.0801343, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2688, 69}

$$-\frac{a^3 2^{\frac{p}{2}+\frac{7}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-5),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((2^(7/2 + p/2)*a^3*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-5 - p)/2,
 (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2
))/(d*e*(1 + p)))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^3\,dx = \frac{\left(a^3(e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\operatorname{Subst}\left(\right.}{de}$$
$$= -\frac{2^{\frac{7}{2}+\frac{p}{2}}a^3(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-5-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin}{de(1+p)}$$

Mathematica [A] time = 0.105313, size = 94, normalized size = 0.99

$$-\frac{a^3 2^{\frac{p+7}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{1}{2}(-p-5),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((2^((7 + p)/2)*a^3*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(-5
- p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1
 - p)/2))/(d*(1 + p)))
```

Maple [F] time = 3.378, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p(a+a\sin(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^3(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3+\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)\right)\left(e\cos\left(dx+c\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*(e*cos(d*x + c))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(dx+c\right)+a\right)^3\left(e\cos\left(dx+c\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3*(e*cos(d*x + c))^p, x)
```

3.329 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=95

$$-\frac{a^2 2^{\frac{p}{2}+\frac{5}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-3),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

```
[Out] -((2^(5/2 + p/2)*a^2*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-3 - p)/2,
 (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2
))/(d*e*(1 + p)))
```

Rubi [A] time = 0.0790328, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2688, 69}

$$-\frac{a^2 2^{\frac{p}{2}+\frac{5}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-3),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2^(5/2 + p/2)*a^2*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-3 - p)/2,
 (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2
))/(d*e*(1 + p)))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^2\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}}{de}$$
$$= -\frac{2^{\frac{5}{2}+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-3-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+}{de(1+p)}$$

Mathematica [A] time = 0.103447, size = 94, normalized size = 0.99

$$-\frac{a^2 2^{\frac{p+5}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{1}{2}(-p-3),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2^((5 + p)/2)*a^2*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(-3
- p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1
 - p)/2))/(d*(1 + p)))
```

Maple [F] time = 2.616, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p(a+a\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^2(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2\cos\left(dx+c\right)^2-2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\left(e\cos\left(dx+c\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*(e*cos(d*x + c)
)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(dx+c\right)+a\right)^2\left(e\cos\left(dx+c\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*(e*cos(d*x + c))^p, x)
```

3.330 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=93

$$-\frac{a2^{\frac{p}{2}+\frac{3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-1),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

```
[Out] -((2^(3/2 + p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-1 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))
/(d*e*(1 + p)))
```

Rubi [A] time = 0.0568303, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2688, 69}

$$-\frac{a2^{\frac{p}{2}+\frac{3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-1),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((2^(3/2 + p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-1 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))
/(d*e*(1 + p)))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))\,dx = \frac{\left(a(e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\left(\int(\right.}{de}$$

$$= -\frac{2^{\frac{3}{2}+\frac{p}{2}}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-1-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin}{de(1+p)}$$

Mathematica [C] time = 1.34029, size = 245, normalized size = 2.63

$$-\frac{ia2^{-p-1}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{p+1}(\sin(c+dx)+1)\left((p+1)e^{i(c+dx)}\left(ipe^{i(c+dx)}\,{}_2F_1\left(1,\frac{p+3}{2};\frac{3-p}{2};-e^{2i(c+dx)}\right)-2(p-1)\,{}_2F_1\left(1\right.\right.\right.}{d(p-1)p(p+1)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x]),x]
```

```
[Out] ((-I)*2^(-1 - p)*a*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^(1 + p)*(e*C
os[c + d*x])^p*((-I)*(-1 + p)*p*Hypergeometric2F1[1, (1 + p)/2, (1 - p)/2,
-E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(1 + p)*(-2*(-1 + p)*Hypergeometric
2F1[1, (2 + p)/2, 1 - p/2, -E^((2*I)*(c + d*x))] + I*E^(I*(c + d*x))*p*Hype
rgeometric2F1[1, (3 + p)/2, (3 - p)/2, -E^((2*I)*(c + d*x))]))*(1 + Sin[c +
 d*x]))/(d*(-1 + p)*p*(1 + p)*Cos[c + d*x]^p*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2)
```

Maple [F] time = 0.907, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p\,(a+a\sin(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)\,(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sin(dx+c)+a)\,(e\cos(dx+c))^p\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int (e\cos(c+dx))^p\,dx+\int (e\cos(c+dx))^p\sin(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral((e*cos(c + d*x))**p, x) + Integral((e*cos(c + d*x))**p*sin(c +
d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)\,(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

3.331 $\int \frac{(e\cos(c+dx))^p}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{2^{\frac{p}{2}-\frac{1}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{3-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ade(p+1)}$$

```
[Out] -((2^(-1/2 + p/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(3 - p)/2, (1
+ p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))/(
a*d*e*(1 + p)))
```

Rubi [A] time = 0.0990199, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2688, 69}

$$-\frac{2^{\frac{p}{2}-\frac{1}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{3-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ade(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((2^(-1/2 + p/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(3 - p)/2, (1
+ p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))/(
a*d*e*(1 + p)))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{a+a\sin(c+dx)}\,dx = \frac{\left((e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\left(\int(1-x)^{\frac{1}{2}(-1+p)}(1+\right.}{ade}$$
$$= -\frac{2^{-\frac{1}{2}+\frac{p}{2}}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{3-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{ade(1+p)}$$

Mathematica [A] time = 0.165018, size = 94, normalized size = 0.99

$$-\frac{2^{\frac{p-1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{3-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((2^((-1 + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(3 - p)
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p
)/2))/(a*d*(1 + p)))
```

Maple [F] time = 0.133, size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\cos(dx+c))^p}{a\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^p/(a*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(e\cos(c+dx))^p}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral((e*cos(c + d*x))**p/(sin(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{a\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a), x)
```

3.332 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=93

$$-\frac{2^{\frac{p-3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{5-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^2de(p+1)}$$

```
[Out] -((2^((-3 + p)/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(5 - p)/2, (1
+ p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))/(
a^2*d*e*(1 + p)))
```

Rubi [A] time = 0.0901636, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2688, 69}

$$-\frac{2^{\frac{p-3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{5-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^2de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2^((-3 + p)/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(5 - p)/2, (1
+ p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))/(
a^2*d*e*(1 + p)))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^2}\,dx = \frac{\left((e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\left(\int (1-x)^{\frac{1}{2}(-1+p)}(1\right.}{a^2de}$$

$$= -\frac{2^{\frac{1}{2}(-3+p)}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{5-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{a^2de(1+p)}$$

Mathematica [A] time = 0.159679, size = 94, normalized size = 1.01

$$-\frac{2^{\frac{p-3}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{5-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^2d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2^((-3 + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(5 - p)
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p
)/2))/(a^2*d*(1 + p)))
```

Maple [F] time = 0.26, size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(a+a\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(e\cos(dx+c))^p}{a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(e*cos(d*x + c))^p/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a
^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^2, x)
```

3.333 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=93

$$-\frac{2^{\frac{p-5}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{7-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^3de(p+1)}$$

```
[Out] -((2^((-5 + p)/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(7 - p)/2, (1
+ p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))/(
a^3*d*e*(1 + p)))
```

Rubi [A] time = 0.0921136, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2688, 69}

$$-\frac{2^{\frac{p-5}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{7-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^3de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] -((2^((-5 + p)/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(7 - p)/2, (1
+ p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))/(
a^3*d*e*(1 + p)))
```


Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^3}\,dx = \frac{\left((e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\left(\int(1-x)^{\frac{1}{2}(-1+p)}(1\right.}{a^3de}$$
$$= -\frac{2^{\frac{1}{2}(-5+p)}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{7-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{a^3de(1+p)}$$

Mathematica [A] time = 0.152979, size = 94, normalized size = 1.01

$$-\frac{2^{\frac{p-5}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{7-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^3d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((2^((-5 + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(7 - p)
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p
)/2))/(a^3*d*(1 + p)))
```

Maple [F] time = 0.32, size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(a+a\sin(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(e\cos(dx+c))^p}{3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(e*cos(d*x + c))^p/(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x +
 c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^3, x)
```

3.334 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=93

$$-\frac{2^{\frac{p-15}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{17-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^8de(p+1)}$$

```
[Out] -((2^((-15 + p)/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(17 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))
/(a^8*d*e*(1 + p)))
```

Rubi [A] time = 0.0913771, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2688, 69}

$$-\frac{2^{\frac{p-15}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{17-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^8de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -((2^((-15 + p)/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(17 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))
/(a^8*d*e*(1 + p)))
```

Rule 2688

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^m*(g*Cos[e + f*x])^(p + 1))/(f*g*(1 + Sin[
e + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2)), Subst[Int[(1 + (b*x)
/a)^(m + (p - 1)/2)*(1 - (b*x)/a)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^8}\,dx = \frac{\left((e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\left(\int(1-x)^{\frac{1}{2}(-1+p)}(1\right.}{a^8de}$$
$$= -\frac{2^{\frac{1}{2}(-15+p)}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{17-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{a^8de(1+p)}$$

Mathematica [A] time = 0.174044, size = 94, normalized size = 1.01

$$-\frac{2^{\frac{p-15}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^p\,{}_2F_1\left(\frac{17-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^8d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -((2^((-15 + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(17 -
p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 -
 p)/2))/(a^8*d*(1 + p)))
```

Maple [F] time = 1.833, size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(a+a\sin(dx+c))^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\cos(dx+c))^p}{a^8\cos(dx+c)^8 - 32\,a^8\cos(dx+c)^6 + 160\,a^8\cos(dx+c)^4 - 256\,a^8\cos(dx+c)^2 + 128\,a^8 - 8\left(a^8\cos(dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^p/(a^8*cos(d*x + c)^8 - 32*a^8*cos(d*x + c)^6 + 1
60*a^8*cos(d*x + c)^4 - 256*a^8*cos(d*x + c)^2 + 128*a^8 - 8*(a^8*cos(d*x +
 c)^6 - 10*a^8*cos(d*x + c)^4 + 24*a^8*cos(d*x + c)^2 - 16*a^8)*sin(d*x + c
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^8, x)
```

3.335 $\int (e\cos(c+dx))^p (a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=103

$$-\frac{a^4 2^{\frac{p}{2}+4}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-6),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -((2^(4 + p/2)*a^4*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-6 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])
^(p/2)*Sqrt[a + a*Sin[c + d*x]]))
```

Rubi [A] time = 0.123197, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$-\frac{a^4 2^{\frac{p}{2}+4}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-6),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -((2^(4 + p/2)*a^4*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-6 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])
^(p/2)*Sqrt[a + a*Sin[c + d*x]]))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{7/2}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Sub}}{de}$$

$$= \frac{\left(2^{3+\frac{p}{2}}a^5(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right.}{}$$

$$= -\frac{2^{4+\frac{p}{2}}a^4(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-6-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.223415, size = 102, normalized size = 0.99

$$-\frac{a^4 2^{\frac{p}{2}+4}\cos(c+dx)(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^p\,{}_2F_1\left(-\frac{p}{2}-3,\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -((2^(4 + p/2)*a^4*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[-3 - p
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*(1 + p)*(1 + Sin[c + d*
x])^(p/2)*Sqrt[a*(1 + Sin[c + d*x])]))
```

Maple [F] time = 0.107, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p\,(a+a\sin(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{7}{2}}(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)\right)\sqrt{a\sin(dx+c)+a}\,(e\cos(dx+c))^p\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(a*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin (dx+c)+a)^{\frac{7}{2}} (e\cos (dx+c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*(e*cos(d*x + c))^p, x)
```

3.336 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=103

$$-\frac{a^3 2^{\frac{p}{2}+3}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-4),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -((2^(3 + p/2)*a^3*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-4 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])
^(p/2)*Sqrt[a + a*Sin[c + d*x]]))
```

Rubi [A] time = 0.116803, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$-\frac{a^3 2^{\frac{p}{2}+3}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-4),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((2^(3 + p/2)*a^3*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-4 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])
^(p/2)*Sqrt[a + a*Sin[c + d*x]]))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeqQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{5/2}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Sub}}{de}$$
$$= \frac{\left(2^{2+\frac{p}{2}}a^4(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right.}{}$$
$$= -\frac{2^{3+\frac{p}{2}}a^3(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-4-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.185934, size = 102, normalized size = 0.99

$$-\frac{a^3 2^{\frac{p}{2}+3}\cos(c+dx)(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^p\,{}_2F_1\left(-\frac{p}{2}-2,\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((2^(3 + p/2)*a^3*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[-2 - p
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*(1 + p)*(1 + Sin[c + d*
x])^(p/2)*Sqrt[a*(1 + Sin[c + d*x])]))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p(a+a\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}} (e\cos(dx+c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2\right)\sqrt{a\sin(dx+c)+a}\,(e\cos(dx+c))^p\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(a*sin(d*x
+ c) + a)*(e*cos(d*x + c))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{5}{2}}(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*(e*cos(d*x + c))^p, x)
```

3.337 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=103

$$-\frac{a^2 2^{\frac{p}{2}+2}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-2),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -((2^(2 + p/2)*a^2*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-2 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])
^(p/2)*Sqrt[a + a*Sin[c + d*x]]))
```

Rubi [A] time = 0.113855, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$-\frac{a^2 2^{\frac{p}{2}+2}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-2),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((2^(2 + p/2)*a^2*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-2 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])
^(p/2)*Sqrt[a + a*Sin[c + d*x]]))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{3/2}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Sub}}{de}$$

$$= \frac{\left(2^{1+\frac{p}{2}}a^3(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right.}{}$$

$$= -\frac{2^{2+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-2-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A] time = 0.184172, size = 101, normalized size = 0.98

$$-\frac{2^{\frac{p}{2}+2}\cos(c+dx)(a(\sin(c+dx)+1))^{3/2}(\sin(c+dx)+1)^{-\frac{p}{2}-2}(e\cos(c+dx))^p\,{}_2F_1\left(-\frac{p}{2}-1,\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((2^(2 + p/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[-1 - p/2,
(1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-2 - p/2)*(
a*(1 + Sin[c + d*x]))^(3/2))/(d*(1 + p)))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p\,(a+a\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sin(dx+c)+a)^{\frac{3}{2}}(e\cos(dx+c))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(3/2)*(e*cos(d*x + c))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^{\frac{3}{2}}(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*(e*cos(d*x + c))^p, x)
```

3.338 $\int (e\cos(c+dx))^p\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{a2^{\frac{p}{2}+1}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(-\frac{p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

```
[Out] -((2^(1 + p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[-p/2, (1 + p)/2
, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])^(p/2)*S
qrt[a + a*Sin[c + d*x]]))
```

Rubi [A] time = 0.104876, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$-\frac{a2^{\frac{p}{2}+1}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(-\frac{p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((2^(1 + p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[-p/2, (1 + p)/2
, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])^(p/2)*S
qrt[a + a*Sin[c + d*x]]))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^p\sqrt{a+a\sin(c+dx)}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}}{de}$$

$$= \frac{\left(2^{p/2}a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)+\frac{p}{2}}\right.}{d}$$

$$= -\frac{2^{1+\frac{p}{2}}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(-\frac{p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] time = 3.71412, size = 310, normalized size = 3.2

$$\frac{(1+i)2^{-p}e^{-\frac{1}{2}idx}\sqrt{a(\sin(c+dx)+1)}\cos^{-p}(c+dx)(e\cos(c+dx))^p\left(e^{-idx}\left(i\sin(c)\left(-1+e^{2idx}\right)+\cos(c)\left(1+e^{2idx}\right)\right)\right)^p\left(i\sin(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((1 + I)*(e*Cos[c + d*x])^p*(E^(I*d*x)*(1 + 2*p)*Hypergeometric2F1[(1 - 2*p
)/4, -p, (5 - 2*p)/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[c/2] + I
*Sin[c/2]) + (-1 + 2*p)*Hypergeometric2F1[(-1 - 2*p)/4, -p, (3 - 2*p)/4, -(
E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(I*Cos[c/2] + Sin[c/2]))*(((1 + E^((2
*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x))^p*Sqrt[a*(1 +
Sin[c + d*x]])]/(2^p*d*E^((I/2)*d*x)*(-1 + 2*p)*(1 + 2*p)*Cos[c + d*x]^p*(1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c])^p*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]))
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int (e\cos (dx+c))^{p} \sqrt{a+a\sin (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin (dx+c)+a}\,(e\cos (dx+c))^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a\sin (dx+c)+a}\,(e\cos (dx+c))^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\,(\sin (c+dx)+1)}\,(e\cos (c+dx))^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(dx+c)+a}\,(e\cos(dx+c))^p\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

3.339 $\int \frac{(e\cos(c+dx))^p}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=101

$$-\frac{a2^{p/2}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{2-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -((2^(p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(2 - p)/2, (1 + p)/
2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(d*e*(1 +
 p)*(a + a*Sin[c + d*x])^(3/2)))
```

Rubi [A] time = 0.108896, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$-\frac{a2^{p/2}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{2-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((2^(p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(2 - p)/2, (1 + p)/
2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(d*e*(1 +
 p)*(a + a*Sin[c + d*x])^(3/2)))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\mathrm{Subst}\left(\int(a-ax)^{\frac{1}{2}(-\right.}{de}$$

$$= \frac{\left(2^{-1+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{-1+\frac{1}{2}(-1-p)+\frac{p}{2}}\left(\frac{a+a\sin(c+}{a}\right.\right.}{de}$$

$$= -\frac{2^{p/2}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{2-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{1-\frac{p}{2}}}{de(1+p)(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.110652, size = 97, normalized size = 0.96

$$-\frac{2^{p/2}\cos(c+dx)(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^p\,{}_2F_1\left(1-\frac{p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((2^(p/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[1 - p/2, (1 +
p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*(1 + p)*(1 + Sin[c + d*x])^(p/2)
*Sqrt[a*(1 + Sin[c + d*x])]))
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p \frac{1}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\cos(dx+c))^p}{\sqrt{a\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^p/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(c+dx))^p}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**p/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{\sqrt{a\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/sqrt(a*sin(d*x + c) + a), x)
```

3.340 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{2^{\frac{p}{2}-1}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{4-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -((2^(-1 + p/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(4 - p)/2, (1 +
p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(d*e*(
1 + p)*(a + a*Sin[c + d*x])^(3/2)))
```

Rubi [A] time = 0.121318, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$-\frac{2^{\frac{p}{2}-1}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{4-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((2^(-1 + p/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(4 - p)/2, (1 +
p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(d*e*(
1 + p)*(a + a*Sin[c + d*x])^(3/2)))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\left(\int(a-ax)^{\frac{1}{2}}\right.}{de}$$

$$= \frac{\left(2^{-2+\frac{p}{2}}a(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{-1+\frac{1}{2}(-1-p)+\frac{p}{2}}\left(\frac{a+a\sin(c}{a}\right.\right.}{de}$$

$$= -\frac{2^{-1+\frac{p}{2}}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{4-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{1-\frac{p}{2}}}{de(1+p)(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.167749, size = 101, normalized size = 0.99

$$-\frac{2^{\frac{p}{2}-1}\cos(c+dx)(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^p\,{}_2F_1\left(2-\frac{p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(p+1)(a(\sin(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((2^(-1 + p/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[2 - p/2,
(1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(
d*(1 + p)*(a*(1 + Sin[c + d*x]))^(3/2)))
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p\,(a+a\sin(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sin(dx+c)+a}\,(e\cos(dx+c))^p}{a^2\cos(dx+c)^2-2\,a^2\sin(dx+c)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(d*x + c) + a)*(e*cos(d*x + c))^p/(a^2*cos(d*x + c)^2 -
 2*a^2*sin(d*x + c) - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(c+dx))^p}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**p/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \cos(dx+c))^p}{(a \sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^(3/2), x)
```

3.341 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{2^{\frac{p}{2}-2}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{6-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ade(p+1)(a\sin(c+dx)+a)^{3/2}}$$

```
[Out] -((2^(-2 + p/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(6 - p)/2, (1 +
p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(a*d*e
*(1 + p)*(a + a*Sin[c + d*x])^(3/2)))
```

Rubi [A] time = 0.123095, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$-\frac{2^{\frac{p}{2}-2}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{6-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ade(p+1)(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((2^(-2 + p/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(6 - p)/2, (1 +
p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(a*d*e
*(1 + p)*(a + a*Sin[c + d*x])^(3/2)))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\left(\int(a-ax)^{\frac{1}{2}}\right.}{de}$$

$$= \frac{\left(2^{-3+\frac{p}{2}}(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{-1+\frac{1}{2}(-1-p)+\frac{p}{2}}\left(\frac{a+a\sin(c+}{a}\right.\right.}{de}$$

$$= -\frac{2^{-2+\frac{p}{2}}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{6-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{1-\frac{p}{2}}}{ade(1+p)(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.132011, size = 102, normalized size = 0.97

$$-\frac{2^{\frac{p}{2}-2}\cos(c+dx)(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^p\,{}_2F_1\left(3-\frac{p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^2d(p+1)\sqrt{a(\sin(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((2^(-2 + p/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[3 - p/2,
(1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(a^2*d*(1 + p)*(1 + Sin[c + d*
x])^(p/2)*Sqrt[a*(1 + Sin[c + d*x])]))
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int(e\cos(dx+c))^p\,(a+a\sin(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{a\sin(dx+c)+a}\,(e\cos(dx+c))^p}{3\,a^3\cos(dx+c)^2-4\,a^3+\left(a^3\cos(dx+c)^2-4\,a^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(d*x + c) + a)*(e*cos(d*x + c))^p/(3*a^3*cos(d*x + c)^2
 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**(5/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(a\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(a*sin(d*x + c) + a)^(5/2), x)
```

3.342 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=114

$$-\frac{a2^{m+\frac{p}{2}+\frac{1}{2}}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{p+1}(\sin(c+dx)+1)^{\frac{1}{2}(-2m-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2m-p+1),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c$$
$$de(p+1)$$

```
[Out] -((2^(1/2 + m + p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(1 - 2*m
- p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((1
- 2*m - p)/2)*(a + a*Sin[c + d*x])^(-1 + m))/(d*e*(1 + p)))
```

Rubi [A] time = 0.115, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {2689, 70, 69}

$$-\frac{a2^{m+\frac{p}{2}+\frac{1}{2}}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{p+1}(\sin(c+dx)+1)^{\frac{1}{2}(-2m-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2m-p+1),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c$$
$$de(p+1)$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^m,x]
```


```
[Out] -((2^(1/2 + m + p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(1 - 2*m
- p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((1
- 2*m - p)/2)*(a + a*Sin[c + d*x])^(-1 + m))/(d*e*(1 + p)))
```


Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Sub}}{de}$$

$$= \frac{\left(2^{-\frac{1}{2}+m+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{-\frac{1}{2}+}\right.}{}$$

$$= -\frac{2^{\frac{1}{2}+m+\frac{p}{2}}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(1-2m-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(1+p)}$$

Mathematica [A] time = 0.18975, size = 112, normalized size = 0.98

$$-\frac{2^{\frac{1}{2}(2m+p+1)}\cos(c+dx)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^p(\sin(c+dx)+1)^{\frac{1}{2}(-2m-p-1)}\,{}_2F_1\left(\frac{1}{2}(-2m-p+1),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\right.}{d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((2^((1 + 2*m + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(1
 - 2*m - p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^((-1 - 2*m - p)/2)*(a*(1 + Sin[c + d*x]))^m)/(d*(1 + p)))
```

Maple [F] time = 0.882, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p (a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((e\cos(dx+c))^p (a\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^p*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sin(c+dx)+1))^m (e\cos(c+dx))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p (a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p*(a*sin(d*x + c) + a)^m, x)
```

3.343 $\int \cos^7(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=109

$$\frac{8(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}-\frac{12(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}+\frac{6(a\sin(c+dx)+a)^{m+6}}{a^6d(m+6)}-\frac{(a\sin(c+dx)+a)^{m+7}}{a^7d(m+7)}$$

```
[Out] (8*(a + a*Sin[c + d*x])^(4 + m))/(a^4*d*(4 + m)) - (12*(a + a*Sin[c + d*x])
^(5 + m))/(a^5*d*(5 + m)) + (6*(a + a*Sin[c + d*x])^(6 + m))/(a^6*d*(6 + m)
) - (a + a*Sin[c + d*x])^(7 + m)/(a^7*d*(7 + m))
```

Rubi [A] time = 0.0857299, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{8(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}-\frac{12(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}+\frac{6(a\sin(c+dx)+a)^{m+6}}{a^6d(m+6)}-\frac{(a\sin(c+dx)+a)^{m+7}}{a^7d(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(4 + m))/(a^4*d*(4 + m)) - (12*(a + a*Sin[c + d*x])
^(5 + m))/(a^5*d*(5 + m)) + (6*(a + a*Sin[c + d*x])^(6 + m))/(a^6*d*(6 + m)
) - (a + a*Sin[c + d*x])^(7 + m)/(a^7*d*(7 + m))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{3+m}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{3+m} - 12a^2(a+x)^{4+m} + 6a(a+x)^{5+m} - (a+x)^{6+m}\right)dx, x,$$
$$= \frac{8(a+a\sin(c+dx))^{4+m}}{a^4d(4+m)} - \frac{12(a+a\sin(c+dx))^{5+m}}{a^5d(5+m)} + \frac{6(a+a\sin(c+dx))^{6+m}}{a^6d(6+m)} -$$

Mathematica [A] time = 0.698101, size = 89, normalized size = 0.82

$$\frac{(a(\sin(c+dx)+1))^{m+4}\left(\frac{6a^3(\sin(c+dx)+1)^2}{m+6} - \frac{12a^3(\sin(c+dx)+1)}{m+5} + \frac{8a^3}{m+4} - \frac{(a\sin(c+dx)+a)^3}{m+7}\right)}{a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(4 + m)*((8*a^3)/(4 + m) - (12*a^3*(1 + Sin[c + d*x
]))/(5 + m) + (6*a^3*(1 + Sin[c + d*x])^2)/(6 + m) - (a + a*Sin[c + d*x])^3
/(7 + m)))/(a^7*d)
```

Maple [F] time = 3.554, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^7 (a+a\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.09511, size = 408, normalized size = 3.74

$$\frac{\left(\left(m^3+9\,m^2+20\,m\right)\cos\left(dx+c\right)^6+12\left(m^2+3\,m\right)\cos\left(dx+c\right)^4+96\,m\cos\left(dx+c\right)^2+\left(\left(m^3+15\,m^2+74\,m+120\right)\mathrm{co}\right.}{dm^4+22\,dm^3+179}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m^3 + 9*m^2 + 20*m)*cos(d*x + c)^6 + 12*(m^2 + 3*m)*cos(d*x + c)^4 + 96*m
*cos(d*x + c)^2 + ((m^3 + 15*m^2 + 74*m + 120)*cos(d*x + c)^6 + 12*(m^2 + 7
*m + 12)*cos(d*x + c)^4 + 96*(m + 2)*cos(d*x + c)^2 + 384)*sin(d*x + c) + 3
84)*(a*sin(d*x + c) + a)^m/(d*m^4 + 22*d*m^3 + 179*d*m^2 + 638*d*m + 840*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10298, size = 698, normalized size = 6.4

$$-\frac{\left(a\sin\left(dx+c\right)+a\right)^7\left(a\sin\left(dx+c\right)+a\right)^m m^3-6\left(a\sin\left(dx+c\right)+a\right)^6\left(a\sin\left(dx+c\right)+a\right)^m am^3+12\left(a\sin\left(dx+c\right)+a\right)^5(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] -((a*sin(d*x + c) + a)^7*(a*sin(d*x + c) + a)^m*m^3 - 6*(a*sin(d*x + c) + a
)^6*(a*sin(d*x + c) + a)^m*a*m^3 + 12*(a*sin(d*x + c) + a)^5*(a*sin(d*x + c
) + a)^m*a^2*m^3 - 8*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a^3*m^3
+ 15*(a*sin(d*x + c) + a)^7*(a*sin(d*x + c) + a)^m*m^2 - 96*(a*sin(d*x + c)
 + a)^6*(a*sin(d*x + c) + a)^m*a*m^2 + 204*(a*sin(d*x + c) + a)^5*(a*sin(d*
x + c) + a)^m*a^2*m^2 - 144*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a
^3*m^2 + 74*(a*sin(d*x + c) + a)^7*(a*sin(d*x + c) + a)^m*m - 498*(a*sin(d*
x + c) + a)^6*(a*sin(d*x + c) + a)^m*a*m + 1128*(a*sin(d*x + c) + a)^5*(a*s
in(d*x + c) + a)^m*a^2*m - 856*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^
m*a^3*m + 120*(a*sin(d*x + c) + a)^7*(a*sin(d*x + c) + a)^m - 840*(a*sin(d*
x + c) + a)^6*(a*sin(d*x + c) + a)^m*a + 2016*(a*sin(d*x + c) + a)^5*(a*sin
(d*x + c) + a)^m*a^2 - 1680*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a
^3)/((a^6*m^4 + 22*a^6*m^3 + 179*a^6*m^2 + 638*a^6*m + 840*a^6)*a*d)
```

3.344 $\int \cos^5(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=81

$$\frac{4(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}-\frac{4(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}+\frac{(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}$$

```
[Out] (4*(a + a*Sin[c + d*x])^(3 + m))/(a^3*d*(3 + m)) - (4*(a + a*Sin[c + d*x])^
(4 + m))/(a^4*d*(4 + m)) + (a + a*Sin[c + d*x])^(5 + m)/(a^5*d*(5 + m))
```

Rubi [A] time = 0.0690724, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{4(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}-\frac{4(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}+\frac{(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(3 + m))/(a^3*d*(3 + m)) - (4*(a + a*Sin[c + d*x])^
(4 + m))/(a^4*d*(4 + m)) + (a + a*Sin[c + d*x])^(5 + m)/(a^5*d*(5 + m))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{2+m}\,dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{2+m}-4a(a+x)^{3+m}+(a+x)^{4+m}\right)dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{4(a+a\sin(c+dx))^{3+m}}{a^3d(3+m)}-\frac{4(a+a\sin(c+dx))^{4+m}}{a^4d(4+m)}+\frac{(a+a\sin(c+dx))^{5+m}}{a^5d(5+m)}\end{aligned}$$

Mathematica [A] time = 0.321718, size = 68, normalized size = 0.84

$$\frac{(a(\sin(c+dx)+1))^{m+3}\left(-\frac{4a^2(\sin(c+dx)+1)}{m+4}+\frac{4a^2}{m+3}+\frac{(a\sin(c+dx)+a)^2}{m+5}\right)}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(3 + m)*((4*a^2)/(3 + m) - (4*a^2*(1 + Sin[c + d*x]
))/(4 + m) + (a + a*Sin[c + d*x])^2/(5 + m)))/(a^5*d)
```

Maple [F] time = 1.564, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^5\,(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 2.76582, size = 263, normalized size = 3.25

$$\frac{\left(\left(m^2+3\,m\right)\cos\left(dx+c\right)^4+8\,m\cos\left(dx+c\right)^2+\left(\left(m^2+7\,m+12\right)\cos\left(dx+c\right)^4+8\left(m+2\right)\cos\left(dx+c\right)^2+32\right)\sin\left(dx+\right.\right.}{dm^3+12\,dm^2+47\,dm+60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="fricas")

[Out] ((m^2 + 3*m)*cos(d*x + c)^4 + 8*m*cos(d*x + c)^2 + ((m^2 + 7*m + 12)*cos(d*
x + c)^4 + 8*(m + 2)*cos(d*x + c)^2 + 32)*sin(d*x + c) + 32)*(a*sin(d*x + c
) + a)^m/(d*m^3 + 12*d*m^2 + 47*d*m + 60*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**m,x)

[Out] Timed out

Giac [B] time = 1.09763, size = 397, normalized size = 4.9

$$\frac{\left(a\sin\left(dx+c\right)+a\right)^5\left(a\sin\left(dx+c\right)+a\right)^m m^2-4\left(a\sin\left(dx+c\right)+a\right)^4\left(a\sin\left(dx+c\right)+a\right)^m a m^2+4\left(a\sin\left(dx+c\right)+a\right)^3\left(a\,\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="giac")

```
[Out] ((a*sin(d*x + c) + a)^5*(a*sin(d*x + c) + a)^m*m^2 - 4*(a*sin(d*x + c) + a)
^4*(a*sin(d*x + c) + a)^m*a*m^2 + 4*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c)
+ a)^m*a^2*m^2 + 7*(a*sin(d*x + c) + a)^5*(a*sin(d*x + c) + a)^m*m - 32*(a*
sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a*m + 36*(a*sin(d*x + c) + a)^3*
(a*sin(d*x + c) + a)^m*a^2*m + 12*(a*sin(d*x + c) + a)^5*(a*sin(d*x + c) +
a)^m - 60*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a + 80*(a*sin(d*x +
 c) + a)^3*(a*sin(d*x + c) + a)^m*a^2)/((a^4*m^3 + 12*a^4*m^2 + 47*a^4*m +
60*a^4)*a*d)
```

3.345 $\int \cos^3(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=55

$$\frac{2(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}-\frac{(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}$$

```
[Out] (2*(a + a*Sin[c + d*x])^(2 + m))/(a^2*d*(2 + m)) - (a + a*Sin[c + d*x])^(3
+ m)/(a^3*d*(3 + m))
```

Rubi [A] time = 0.0576335, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 43}

$$\frac{2(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}-\frac{(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(2 + m))/(a^2*d*(2 + m)) - (a + a*Sin[c + d*x])^(3
+ m)/(a^3*d*(3 + m))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^{1+m}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(a+x)^{1+m}-(a+x)^{2+m}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{2(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)} - \frac{(a+a\sin(c+dx))^{3+m}}{a^3d(3+m)}$$

Mathematica [A] time = 0.111585, size = 52, normalized size = 0.95

$$-\frac{(\sin(c+dx)+1)^2((m+2)\sin(c+dx)-m-4)(a(\sin(c+dx)+1))^m}{d(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(((1 + Sin[c + d*x])^2*(a*(1 + Sin[c + d*x]))^m*(-4 - m + (2 + m)*Sin[c +
d*x]))/(d*(2 + m)*(3 + m)))
```

Maple [F] time = 0.837, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^3 (a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.67177, size = 153, normalized size = 2.78

$$\frac{\left(m\cos\left(dx+c\right)^{2}+\left(\left(m+2\right)\cos\left(dx+c\right)^{2}+4\right)\sin\left(dx+c\right)+4\right)\left(a\sin\left(dx+c\right)+a\right)^{m}}{dm^{2}+5\,dm+6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (m*cos(d*x + c)^2 + ((m + 2)*cos(d*x + c)^2 + 4)*sin(d*x + c) + 4)*(a*sin(d
*x + c) + a)^m/(d*m^2 + 5*d*m + 6*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.09057, size = 185, normalized size = 3.36

$$-\frac{\left(a\sin\left(dx+c\right)+a\right)^{3}\left(a\sin\left(dx+c\right)+a\right)^{m}m-2\left(a\sin\left(dx+c\right)+a\right)^{2}\left(a\sin\left(dx+c\right)+a\right)^{m}am+2\left(a\sin\left(dx+c\right)+a\right)^{3}\left(a\,\mathrm{s}\right.}{\left(a^{2}m^{2}+5\,a^{2}m+6\,a^{2}\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] -((a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m*m - 2*(a*sin(d*x + c) + a)^
2*(a*sin(d*x + c) + a)^m*a*m + 2*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a
```

```
)^m - 6*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a)/((a^2*m^2 + 5*a^2*
m + 6*a^2)*a*d)
```

3.346 $\int \cos(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=26

$$\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

```
[Out] (a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m))
```

Rubi [A] time = 0.0275411, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 32}

$$\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\mathrm{Subst}\left(\int (a+x)^m\,dx, x, a\sin(c+dx)\right)}{ad} = \frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)}$$

Mathematica [A] time = 0.0321921, size = 26, normalized size = 1.

$$\frac{(a(\sin(c+dx)+1))^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (a*(1 + Sin[c + d*x]))^(1 + m)/(a*d*(1 + m))
```

Maple [A] time = 0.001, size = 27, normalized size = 1.

$$\frac{(a+a\sin(dx+c))^{1+m}}{da(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] (a+a*sin(d*x+c))^(1+m)/a/d/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.29077, size = 72, normalized size = 2.77

$$\frac{(a\sin(dx+c)+a)^m(\sin(dx+c)+1)}{dm+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (a*sin(d*x + c) + a)^m*(sin(d*x + c) + 1)/(d*m + d)
```

Sympy [A] time = 2.64415, size = 80, normalized size = 3.08

$$\begin{cases} \frac{x\cos(c)}{a\sin(c)+a} & \text{for } d=0 \wedge m=-1 \\ x(a\sin(c)+a)^m\cos(c) & \text{for } d=0 \\ \frac{\log(\sin(c+dx)+1)}{ad} & \text{for } m=-1 \\ \frac{(a\sin(c+dx)+a)^m\sin(c+dx)}{dm+d}+\frac{(a\sin(c+dx)+a)^m}{dm+d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*cos(c)/(a*sin(c) + a), Eq(d, 0) & Eq(m, -1)), (x*(a*sin(c) + a
)**m*cos(c), Eq(d, 0)), (log(sin(c + d*x) + 1)/(a*d), Eq(m, -1)), ((a*sin(c
 + d*x) + a)**m*sin(c + d*x)/(d*m + d) + (a*sin(c + d*x) + a)**m/(d*m + d),
 True))
```

Giac [A] time = 1.07156, size = 35, normalized size = 1.35

$$\frac{(a\sin(dx+c)+a)^{m+1}}{ad(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] (a*sin(d*x + c) + a)^(m + 1)/(a*d*(m + 1))
```

3.347 $\int \sec(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=40

$$\frac{(a\sin(c+dx)+a)^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(\sin(c+dx)+1)\right)}{2dm}$$

```
[Out] (Hypergeometric2F1[1, m, 1 + m, (1 + Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^
m)/(2*d*m)
```

Rubi [A] time = 0.0487234, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 68}

$$\frac{(a\sin(c+dx)+a)^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(\sin(c+dx)+1)\right)}{2dm}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, m, 1 + m, (1 + Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^
m)/(2*d*m)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{a\,\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{{}_2F_1\left(1, m; 1+m; \frac{1}{2}(1+\sin(c+dx))\right)(a+a\sin(c+dx))^m}{2dm}$$

Mathematica [A] time = 0.0941581, size = 63, normalized size = 1.58

$$\frac{(a(\sin(c+dx)+1))^m\left(m(\sin(c+dx)+1)\,{}_2F_1\left(1, m+1; m+2; \frac{1}{2}(\sin(c+dx)+1)\right)+2(m+1)\right)}{4dm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^m*(2*(1 + m) + m*Hypergeometric2F1[1, 1 + m, 2 + m,
 (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x])))/(4*d*m*(1 + m))
```

Maple [F] time = 0.54, size = 0, normalized size = 0.

$$\int \sec(dx+c)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sin(dx+c)+a)^m\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a(\sin(c+dx)+1))^m\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(a\sin(dx+c)+a)^m\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

3.348 $\int \sec^3(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=47

$$-\frac{a(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(2,m-1;m;\frac{1}{2}(\sin(c+dx)+1)\right)}{4d(1-m)}$$

```
[Out] -(a*Hypergeometric2F1[2, -1 + m, m, (1 + Sin[c + d*x])/2]*(a + a*Sin[c + d*
x])^(-1 + m))/(4*d*(1 - m))
```

Rubi [A] time = 0.0545228, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 68}

$$-\frac{a(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(2,m-1;m;\frac{1}{2}(\sin(c+dx)+1)\right)}{4d(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(a*Hypergeometric2F1[2, -1 + m, m, (1 + Sin[c + d*x])/2]*(a + a*Sin[c + d*
x])^(-1 + m))/(4*d*(1 - m))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{a^3\,\mathrm{Subst}\left(\int\frac{(a+x)^{-2+m}}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\,{}_2F_1\left(2,-1+m;m;\frac{1}{2}(1+\sin(c+dx))\right)(a+a\sin(c+dx))^{-1+m}}{4d(1-m)}$$

Mathematica [B] time = 0.337913, size = 111, normalized size = 2.36

$$\frac{(a(\sin(c+dx)+1))^m\left(\frac{2(\sin(c+dx)+1)\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(c+dx)+1)\right)}{m+1}+\frac{(\sin(c+dx)+1)\,{}_2F_1\left(2,m+1;m+2;\frac{1}{2}(\sin(c+dx)+1)\right)}{m+1}+4\left(\frac{1}{(m-1)(\sin(c+dx)\right.\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^m*((2*Hypergeometric2F1[1, 1 + m, 2 + m, (1 + Sin[c
 + d*x])/2]*(1 + Sin[c + d*x]))/(1 + m) + (Hypergeometric2F1[2, 1 + m, 2 +
m, (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x]))/(1 + m) + 4*(m^(-1) + 1/((-1 +
 m)*(1 + Sin[c + d*x])))))/(16*d)
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^3 (a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m \sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sin(dx+c)+a)^m\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

3.349 $\int \sec^5(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=51

$$-\frac{a^2(a\sin(c+dx)+a)^{m-2}\,{}_2F_1\left(3,m-2;m-1;\frac{1}{2}(\sin(c+dx)+1)\right)}{8d(2-m)}$$

```
[Out] -(a^2*Hypergeometric2F1[3, -2 + m, -1 + m, (1 + Sin[c + d*x])/2]*(a + a*Sin
[c + d*x])^(-2 + m))/(8*d*(2 - m))
```

Rubi [A] time = 0.0561663, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 68}

$$-\frac{a^2(a\sin(c+dx)+a)^{m-2}\,{}_2F_1\left(3,m-2;m-1;\frac{1}{2}(\sin(c+dx)+1)\right)}{8d(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(a^2*Hypergeometric2F1[3, -2 + m, -1 + m, (1 + Sin[c + d*x])/2]*(a + a*Sin
[c + d*x])^(-2 + m))/(8*d*(2 - m))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{a^5\,\mathrm{Subst}\left(\int \frac{(a+x)^{-3+m}}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^2\,{}_2F_1\left(3,-2+m;-1+m;\frac{1}{2}(1+\sin(c+dx))\right)(a+a\sin(c+dx))^{-2+m}}{8d(2-m)}$$

Mathematica [B] time = 0.710449, size = 163, normalized size = 3.2

$$\frac{(a(\sin(c+dx)+1))^m\left(\frac{6(\sin(c+dx)+1)\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(c+dx)+1)\right)}{m+1}+\frac{3(\sin(c+dx)+1)\,{}_2F_1\left(2,m+1;m+2;\frac{1}{2}(\sin(c+dx)+1)\right)}{m+1}+\frac{(\sin(c+dx)+1)\,{}_2F_1\left(3,\right.}{}\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^m*(12/m + 8/((-2 + m)*(1 + Sin[c + d*x])^2) + 12/((
-1 + m)*(1 + Sin[c + d*x])) + (6*Hypergeometric2F1[1, 1 + m, 2 + m, (1 + Si
n[c + d*x])/2]*(1 + Sin[c + d*x]))/(1 + m) + (3*Hypergeometric2F1[2, 1 + m,
 2 + m, (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x]))/(1 + m) + (Hypergeometric
2F1[3, 1 + m, 2 + m, (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x]))/(1 + m)))/(6
4*d)
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^5\,(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m \sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sin(dx+c)+a)^m\sec(dx+c)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

3.350 $\int \cos^4(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=83

$$-\frac{a^2 2^{m+\frac{5}{2}} \cos^5(c+dx)(\sin(c+dx)+1)^{-m-\frac{1}{2}}(a\sin(c+dx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5d}$$

```
[Out] -(2^(5/2 + m)*a^2*Cos[c + d*x]^5*Hypergeometric2F1[5/2, -3/2 - m, 7/2, (1 -
 Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - m)*(a + a*Sin[c + d*x])^(-2 +
m))/(5*d)
```

Rubi [A] time = 0.0751118, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2689, 70, 69}

$$-\frac{a^2 2^{m+\frac{5}{2}} \cos^5(c+dx)(\sin(c+dx)+1)^{-m-\frac{1}{2}}(a\sin(c+dx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(2^(5/2 + m)*a^2*Cos[c + d*x]^5*Hypergeometric2F1[5/2, -3/2 - m, 7/2, (1 -
 Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - m)*(a + a*Sin[c + d*x])^(-2 +
m))/(5*d)
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2\cos^5(c+dx)\right)\text{Subst}\left(\int (a-ax)^{3/2}(a+ax)^{\frac{3}{2}+m}\,dx, x, \sin(c+dx)\right)}{d(a-a\sin(c+dx))^{5/2}(a+a\sin(c+dx))^{5/2}}$$

$$= \frac{\left(2^{\frac{3}{2}+m}a^3\cos^5(c+dx)(a+a\sin(c+dx))^{-2+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{-\frac{1}{2}-m}\right)\text{Subst}\left(\int \left(\frac{1}{2}\right.\right.}{d(a-a\sin(c+dx))^{5/2}}$$

$$= -\frac{2^{\frac{5}{2}+m}a^2\cos^5(c+dx)\,{}_2F_1\left(\frac{5}{2},-\frac{3}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-}}{5d}$$

Mathematica [A] time = 0.122965, size = 78, normalized size = 0.94

$$-\frac{2^{m+\frac{5}{2}}\cos^5(c+dx)(\sin(c+dx)+1)^{-m-\frac{5}{2}}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(2^(5/2 + m)*Cos[c + d*x]^5*Hypergeometric2F1[5/2, -3/2 - m, 7/2, (1 - Sin
[c + d*x])/2]*(1 + Sin[c + d*x])^(-5/2 - m)*(a*(1 + Sin[c + d*x]))^m)/(5*d)
```

Maple [F] time = 1.293, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^4(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m \cos(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sin(dx+c)+a)^m \cos(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m \cos(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

3.351 $\int \cos^2(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{a2^{m+\frac{3}{2}}\cos^3(c+dx)(\sin(c+dx)+1)^{-m-\frac{1}{2}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3d}$$

```
[Out] -(2^(3/2 + m)*a*Cos[c + d*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 - S
in[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - m)*(a + a*Sin[c + d*x])^(-1 + m)
)/(3*d)
```

Rubi [A] time = 0.0744746, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2689, 70, 69}

$$-\frac{a2^{m+\frac{3}{2}}\cos^3(c+dx)(\sin(c+dx)+1)^{-m-\frac{1}{2}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(2^(3/2 + m)*a*Cos[c + d*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 - S
in[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - m)*(a + a*Sin[c + d*x])^(-1 + m)
)/(3*d)
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2\cos^3(c+dx)\right)\text{Subst}\left(\int \sqrt{a-ax}(a+ax)^{\frac{1}{2}+m}\,dx, x, \sin(c+dx)\right)}{d(a-a\sin(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}$$

$$= \frac{\left(2^{\frac{1}{2}+m}a^2\cos^3(c+dx)(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{-\frac{1}{2}-m}\right)\text{Subst}\left(\int \left(\frac{1}{2}\right.\right.}{d(a-a\sin(c+dx))^{3/2}}$$

$$= -\frac{2^{\frac{3}{2}+m}a\cos^3(c+dx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{1}{2}}}{3d}$$

Mathematica [A] time = 0.0942731, size = 78, normalized size = 0.96

$$-\frac{2^{m+\frac{3}{2}}\cos^3(c+dx)(\sin(c+dx)+1)^{-m-\frac{3}{2}}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(2^(3/2 + m)*Cos[c + d*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 - Sin
[c + d*x])/2]*(1 + Sin[c + d*x])^(-3/2 - m)*(a*(1 + Sin[c + d*x]))^m)/(3*d)
```

Maple [F] time = 0.749, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin (dx+c)+a)^m \cos (dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sin (dx+c)+a)^m \cos (dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sin (c+dx)+1))^m \cos^2 (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*cos(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin (dx + c) + a)^m \cos (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

3.352 $\int \sec^2(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=73

$$\frac{2^{m-\frac{1}{2}}\sec(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

```
[Out] (2^(-1/2 + m)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[c + d*x])/2]*S
ec[c + d*x]*(1 + Sin[c + d*x])^(1/2 - m)*(a + a*Sin[c + d*x])^m)/d
```

Rubi [A] time = 0.083032, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2689, 70, 69}

$$\frac{2^{m-\frac{1}{2}}\sec(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(-1/2 + m)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[c + d*x])/2]*S
ec[c + d*x]*(1 + Sin[c + d*x])^(1/2 - m)*(a + a*Sin[c + d*x])^m)/d
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2\sec(c+dx)\sqrt{a-a\sin(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{3}{2}+m}}{(a-ax)^{3/2}}\,dx,x\right.}{d}$$

$$= \frac{\left(2^{-\frac{3}{2}+m}a\sec(c+dx)\sqrt{a-a\sin(c+dx)}(a+a\sin(c+dx))^m\left(\frac{a+a\sin(c+dx)}{a}\right)^{\frac{1}{2}-m}\right)}{d}$$

$$= \frac{2^{-\frac{1}{2}+m}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sec(c+dx)(1+\sin(c+dx))^{\frac{1}{2}-m}(a}{d}$$

Mathematica [C] time = 25.916, size = 5807, normalized size = 79.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2\,(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m \sec(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sin(dx+c)+a)^m \sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m \sec(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

3.353 $\int \sec^4(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=83

$$\frac{2^{m-\frac{3}{2}}\sec^3(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}-m}(a\sin(c+dx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3ad}$$

```
[Out] (2^(-3/2 + m)*Hypergeometric2F1[-3/2, 5/2 - m, -1/2, (1 - Sin[c + d*x])/2]*
Sec[c + d*x]^3*(1 + Sin[c + d*x])^(1/2 - m)*(a + a*Sin[c + d*x])^(1 + m))/(
3*a*d)
```

Rubi [A] time = 0.0841911, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2689, 70, 69}

$$\frac{2^{m-\frac{3}{2}}\sec^3(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}-m}(a\sin(c+dx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(-3/2 + m)*Hypergeometric2F1[-3/2, 5/2 - m, -1/2, (1 - Sin[c + d*x])/2]*
Sec[c + d*x]^3*(1 + Sin[c + d*x])^(1/2 - m)*(a + a*Sin[c + d*x])^(1 + m))/(
3*a*d)
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2\sec^3(c+dx)(a-a\sin(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{5}{2}+m}}{(a-ax)^{5/2}}\right.}{d}$$

$$= \frac{\left(2^{-\frac{5}{2}+m}\sec^3(c+dx)(a-a\sin(c+dx))^{3/2}(a+a\sin(c+dx))^{1+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{\frac{1}{2}}\right.}{d}$$

$$= \frac{2^{-\frac{3}{2}+m}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sec^3(c+dx)(1+\sin(c+dx))^{\frac{1}{2}-}}{3ad}$$

Mathematica [C] time = 13.1039, size = 307, normalized size = 3.7

$$-\frac{4\cos^2\left(\frac{1}{8}(2c+2dx-\pi)\right)\cot\left(\frac{1}{8}(2c+2dx-\pi)\right)\sec}{3d\left(2(2m-7)F_1\left(-\frac{1}{2};4-2m,2(m-3);\frac{1}{2};\tan^2\left(\frac{1}{8}(-2c-2dx+\pi)\right),-\tan^2\left(\frac{1}{8}(2c+2dx-\pi)\right)\right)+4(m-2)F_1\left(-\frac{1}{2};5-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (-4*AppellF1[-3/2, 4 - 2*m, -7 + 2*m, -1/2, Tan[(-2*c + Pi - 2*d*x)/8]^2, -
Tan[(2*c - Pi + 2*d*x)/8]^2]*Cos[(2*c - Pi + 2*d*x)/8]^2*Cot[(2*c - Pi + 2*
d*x)/8]*Sec[c + d*x]^4*(a*(1 + Sin[c + d*x]))^m)/(3*d*(2*(-7 + 2*m)*AppellF
1[-1/2, 4 - 2*m, 2*(-3 + m), 1/2, Tan[(-2*c + Pi - 2*d*x)/8]^2, -Tan[(2*c -
 Pi + 2*d*x)/8]^2] + 4*(-2 + m)*AppellF1[-1/2, 5 - 2*m, -7 + 2*m, 1/2, Tan[
(-2*c + Pi - 2*d*x)/8]^2, -Tan[(2*c - Pi + 2*d*x)/8]^2] + AppellF1[-3/2, 4
- 2*m, -7 + 2*m, -1/2, Tan[(-2*c + Pi - 2*d*x)/8]^2, -Tan[(2*c - Pi + 2*d*x
```

```
)/8]^2]*Cot[(2*c - Pi + 2*d*x)/8]^2))
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^4 (a+a\sin (dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin (dx+c)+a)^m \sec (dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sin (dx+c)+a)^m \sec (dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+a)^m \sec(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

3.354 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{a2^{m+\frac{11}{4}}(e\cos(c+dx))^{7/2}(\sin(c+dx)+1)^{-m-\frac{3}{4}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{7}{4},-m-\frac{3}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de}$$

```
[Out] -(2^(11/4 + m)*a*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, -3/4 - m, 11
/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-3/4 - m)*(a + a*Sin[c + d*x]
)^(-1 + m))/(7*d*e)
```

Rubi [A] time = 0.096131, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$-\frac{a2^{m+\frac{11}{4}}(e\cos(c+dx))^{7/2}(\sin(c+dx)+1)^{-m-\frac{3}{4}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{7}{4},-m-\frac{3}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^m,x]
```


```
[Out] -(2^(11/4 + m)*a*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, -3/4 - m, 11
/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-3/4 - m)*(a + a*Sin[c + d*x]
)^(-1 + m))/(7*d*e)
```


Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{7/2}\right)\operatorname{Subst}\left(\int (a-ax)^{3/4}(a+ax)^{\frac{3}{4}+m}\,dx,x,\sin(c+dx)\right)}{de(a-a\sin(c+dx))^{7/4}(a+a\sin(c+dx))^{7/4}}$$
$$= \frac{\left(2^{\frac{3}{4}+m}a^2(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{-\frac{3}{4}-m}\right)\operatorname{Subs}}{de(a-a\sin(c+dx))^{7/4}}$$
$$= -\frac{2^{\frac{11}{4}+m}a(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},-\frac{3}{4}-m;\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(}{7de}$$

Mathematica [A] time = 0.195887, size = 85, normalized size = 0.97

$$-\frac{2^{m+\frac{11}{4}}(e\cos(c+dx))^{7/2}(\sin(c+dx)+1)^{-m-\frac{7}{4}}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(\frac{7}{4},-m-\frac{3}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(2^(11/4 + m)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, -3/4 - m, 11/4
, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-7/4 - m)*(a*(1 + Sin[c + d*x])
)^m)/(7*d*e)
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos (dx+c))^{\frac{5}{2}} (a\sin (dx+c)+a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{e\cos (dx+c)}(a\sin (dx+c)+a)^m e^2 \cos (dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m*e^2*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(a*sin(d*x + c) + a)^m, x)
```

3.355 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{a2^{m+\frac{9}{4}}(e\cos(c+dx))^{5/2}(\sin(c+dx)+1)^{-m-\frac{1}{4}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de}$$

```
[Out] -(2^(9/4 + m)*a*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m, 9/4
, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/4 - m)*(a + a*Sin[c + d*x])^
(-1 + m))/(5*d*e)
```

Rubi [A] time = 0.0933461, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$-\frac{a2^{m+\frac{9}{4}}(e\cos(c+dx))^{5/2}(\sin(c+dx)+1)^{-m-\frac{1}{4}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^m,x]
```


```
[Out] -(2^(9/4 + m)*a*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m, 9/4
, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/4 - m)*(a + a*Sin[c + d*x])^
(-1 + m))/(5*d*e)
```


Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{5/2}\right)\text{Subst}\left(\int \sqrt[4]{a-ax}(a+ax)^{\frac{1}{4}+m}\,dx, x, \sin(c+dx)\right)}{de(a-a\sin(c+dx))^{5/4}(a+a\sin(c+dx))^{5/4}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{-\frac{1}{4}-m}\right)\text{Subs}}{de(a-a\sin(c+dx))^{5/4}}$$

$$= -\frac{2^{\frac{9}{4}+m}a(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},-\frac{1}{4}-m;\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c}{5de}$$

Mathematica [A] time = 0.129162, size = 85, normalized size = 0.97

$$-\frac{2^{m+\frac{9}{4}}(e\cos(c+dx))^{5/2}(\sin(c+dx)+1)^{-m-\frac{5}{4}}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(2^(9/4 + m)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m, 9/4,
(1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-5/4 - m)*(a*(1 + Sin[c + d*x]))^
m)/(5*d*e)
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^m e\cos(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m*e*cos(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos (dx+c))^{\frac{3}{2}} (a\sin (dx+c)+a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(a*sin(d*x + c) + a)^m, x)
```

3.356 $\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{a2^{m+\frac{7}{4}}(e\cos(c+dx))^{3/2}(\sin(c+dx)+1)^{\frac{1}{4}-m}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de}$$

```
[Out] -(2^(7/4 + m)*a*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 1/4 - m, 7/4,
 (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1/4 - m)*(a + a*Sin[c + d*x])^(-
1 + m))/(3*d*e)
```

Rubi [A] time = 0.0992657, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$-\frac{a2^{m+\frac{7}{4}}(e\cos(c+dx))^{3/2}(\sin(c+dx)+1)^{\frac{1}{4}-m}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(2^(7/4 + m)*a*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 1/4 - m, 7/4,
 (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1/4 - m)*(a + a*Sin[c + d*x])^(-
1 + m))/(3*d*e)
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{4}+m}}{\sqrt[4]{a-ax}}\,dx, x, \sin(c+dx)\right)}{de(a-a\sin(c+dx))^{3/4}(a+a\sin(c+dx))^{3/4}}$$

$$= \frac{\left(2^{-\frac{1}{4}+m}a^2(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{\frac{1}{4}-m}\right)\text{Subst}}{de(a-a\sin(c+dx))^{3/4}}$$

$$= -\frac{2^{\frac{7}{4}+m}a(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+d}{3de}$$

Mathematica [A] time = 0.0831012, size = 85, normalized size = 0.97

$$-\frac{2^{m+\frac{7}{4}}(e\cos(c+dx))^{3/2}(\sin(c+dx)+1)^{-m-\frac{3}{4}}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(2^(7/4 + m)*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 1/4 - m, 7/4, (
1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-3/4 - m)*(a*(1 + Sin[c + d*x]))^m
)/(3*d*e)
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}\,(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sin(c+dx)+1))^m\sqrt{e\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*sqrt(e*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e \cos (dx + c)}(a \sin (dx + c) + a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m, x)
```

3.357 $\int \frac{(a+a\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=86

$$-\frac{a2^{m+\frac{5}{4}}\sqrt{e\cos(c+dx)}(\sin(c+dx)+1)^{\frac{3}{4}-m}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de}$$

```
[Out] -((2^(5/4 + m)*a*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 3/4 - m, 5/4,
(1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(3/4 - m)*(a + a*Sin[c + d*x])^(-1
 + m))/(d*e))
```

Rubi [A] time = 0.0884103, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$-\frac{a2^{m+\frac{5}{4}}\sqrt{e\cos(c+dx)}(\sin(c+dx)+1)^{\frac{3}{4}-m}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] -((2^(5/4 + m)*a*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 3/4 - m, 5/4,
(1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(3/4 - m)*(a + a*Sin[c + d*x])^(-1
 + m))/(d*e))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx = \frac{\left(a^2\sqrt{e\cos(c+dx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{3}{4}+m}}{(a-ax)^{3/4}}\,dx, x, \sin(c+dx)\right)}{de\sqrt[4]{a-a\sin(c+dx)}\sqrt[4]{a+a\sin(c+dx)}}$$

$$= \frac{\left(2^{-\frac{3}{4}+m}a^2\sqrt{e\cos(c+dx)}(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{\frac{3}{4}-m}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{x}{2}\right)^{-\frac{3}{4}+m}}{(a-ax)^{3/4}}\,dx\right.}{de\sqrt[4]{a-a\sin(c+dx)}}$$

$$= -\frac{2^{\frac{5}{4}+m}a\sqrt{e\cos(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{3}{4}-m}(a+a\sin}{de}$$

Mathematica [A] time = 0.0770356, size = 83, normalized size = 0.97

$$-\frac{2^{m+\frac{5}{4}}\sqrt{e\cos(c+dx)}(\sin(c+dx)+1)^{-m-\frac{1}{4}}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(\frac{1}{4},\frac{3}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] -((2^(5/4 + m)*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 3/4 - m, 5/4, (1
 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/4 - m)*(a*(1 + Sin[c + d*x]))^m)
/(d*e))
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int (a+a\sin(dx+c))^m \frac{1}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^m}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/sqrt(e*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}(a\sin(dx+c)+a)^m}{e\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m/(e*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a(\sin(c+dx)+1))^m}{\sqrt{e\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m/sqrt(e*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^m}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/sqrt(e*cos(d*x + c)), x)
```

3.358 $\int \frac{(a+a\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{2^{m+\frac{3}{4}}(\sin(c+dx)+1)^{\frac{1}{4}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] (2^(3/4 + m)*Hypergeometric2F1[-1/4, 5/4 - m, 3/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(1/4 - m)*(a + a*Sin[c + d*x])^m)/(d*e*Sqrt[e*Cos[c + d*x]
])
```

Rubi [A] time = 0.0950441, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$\frac{2^{m+\frac{3}{4}}(\sin(c+dx)+1)^{\frac{1}{4}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2^(3/4 + m)*Hypergeometric2F1[-1/4, 5/4 - m, 3/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(1/4 - m)*(a + a*Sin[c + d*x])^m)/(d*e*Sqrt[e*Cos[c + d*x]
])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{\left(a^2\sqrt[4]{a-a\sin(c+dx)}\sqrt[4]{a+a\sin(c+dx)}\right)\mathrm{Subst}\left(\int \frac{(a+ax)^{-\frac{5}{4}+m}}{(a-ax)^{5/4}}\,dx, x, \sin(c+dx)\right)}{de\sqrt{e\cos(c+dx)}} \\
&= \frac{\left(2^{-\frac{5}{4}+m}a\sqrt[4]{a-a\sin(c+dx)}(a+a\sin(c+dx))^m\left(\frac{a+a\sin(c+dx)}{a}\right)^{\frac{1}{4}-m}\right)\mathrm{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{x}{2}\right)^{-\frac{5}{4}+m}}{(a-ax)^{5/4}}\,dx,\right.}{de\sqrt{e\cos(c+dx)}} \\
&= \frac{2^{\frac{3}{4}+m}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{4}-m}(a+a\sin(c+dx))^m}{de\sqrt{e\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.107823, size = 82, normalized size = 1.

$$
\frac{2^{m+\frac{3}{4}}(\sin(c+dx)+1)^{\frac{1}{4}-m}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt{e\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2^(3/4 + m)*Hypergeometric2F1[-1/4, 5/4 - m, 3/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(1/4 - m)*(a*(1 + Sin[c + d*x]))^m)/(d*e*Sqrt[e*Cos[c + d*
x]])
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int (a + a\sin(dx + c))^m (e\cos(dx + c))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx + c) + a)^m}{(e\cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx + c)}(a\sin(dx + c) + a)^m}{e^2\cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m/(e^2*cos(d*x + c)^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(3/2), x)
```

3.359 $\int \frac{(a+a\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2^{m+\frac{1}{4}}(\sin(c+dx)+1)^{\frac{3}{4}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] (2^(1/4 + m)*Hypergeometric2F1[-3/4, 7/4 - m, 1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(3/4 - m)*(a + a*Sin[c + d*x])^m)/(3*d*e*(e*Cos[c + d*x])^
(3/2))
```

Rubi [A] time = 0.0941946, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$\frac{2^{m+\frac{1}{4}}(\sin(c+dx)+1)^{\frac{3}{4}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2^(1/4 + m)*Hypergeometric2F1[-3/4, 7/4 - m, 1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(3/4 - m)*(a + a*Sin[c + d*x])^m)/(3*d*e*(e*Cos[c + d*x])^
(3/2))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx = \frac{\left(a^2(a-a\sin(c+dx))^{3/4}(a+a\sin(c+dx))^{3/4}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{7}{4}+m}}{(a-ax)^{7/4}}\,dx, x, \sin(c+dx)\right)}{de(e\cos(c+dx))^{3/2}}$$

$$= \frac{\left(2^{-\frac{7}{4}+m}a(a-a\sin(c+dx))^{3/4}(a+a\sin(c+dx))^m\left(\frac{a+a\sin(c+dx)}{a}\right)^{\frac{3}{4}-m}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{x}{2}\right)^{-\frac{7}{4}+m}}{(a-ax)^{7/4}}\,d\right.}{de(e\cos(c+dx))^{3/2}}$$

$$= \frac{2^{\frac{1}{4}+m}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{3}{4}-m}(a+a\sin(c+dx))^m}{3de(e\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.115325, size = 85, normalized size = 1.

$$\frac{2^{m+\frac{1}{4}}(\sin(c+dx)+1)^{\frac{3}{4}-m}(a(\sin(c+dx)+1))^m\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2^(1/4 + m)*Hypergeometric2F1[-3/4, 7/4 - m, 1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(3/4 - m)*(a*(1 + Sin[c + d*x]))^m)/(3*d*e*(e*Cos[c + d*x]
)^(3/2))
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int (a + a\sin(dx + c))^m (e\cos(dx + c))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx + c) + a)^m}{(e\cos(dx + c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\cos(dx + c)}(a\sin(dx + c) + a)^m}{e^3\cos(dx + c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(a*sin(d*x + c) + a)^m/(e^3*cos(d*x + c)^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(5/2), x)
```

3.360 $\int (e\cos(c+dx))^{-4-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=201

$$\frac{6(a\sin(c+dx)+a)^{m+2}(e\cos(c+dx))^{-m-3}}{a^2de(3-m)\left(1-m^2\right)}-\frac{6(a\sin(c+dx)+a)^{m+3}(e\cos(c+dx))^{-m-3}}{a^3de\left(m^4-10m^2+9\right)}-\frac{(a\sin(c+dx)+a)^m(e\cos(c+\ldots}{de(3-m)}$$

```
[Out] -((((e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^m)/(d*e*(3 - m))) - (3*(e
*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^(1 + m))/(a*d*e*(1 - m)*(3 - m
)) + (6*(e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^(2 + m))/(a^2*d*e*(3
 - m)*(1 - m^2)) - (6*(e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^(3 + m
))/(a^3*d*e*(9 - 10*m^2 + m^4))
```

Rubi [A] time = 0.321228, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{6(a\sin(c+dx)+a)^{m+2}(e\cos(c+dx))^{-m-3}}{a^2de(3-m)\left(1-m^2\right)}-\frac{6(a\sin(c+dx)+a)^{m+3}(e\cos(c+dx))^{-m-3}}{a^3de\left(m^4-10m^2+9\right)}-\frac{(a\sin(c+dx)+a)^m(e\cos(c+\ldots}{de(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-4 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((((e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^m)/(d*e*(3 - m))) - (3*(e
*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^(1 + m))/(a*d*e*(1 - m)*(3 - m
)) + (6*(e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^(2 + m))/(a^2*d*e*(3
 - m)*(1 - m^2)) - (6*(e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^(3 + m
))/(a^3*d*e*(9 - 10*m^2 + m^4))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{-4-m}(a+a\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m}{de(3-m)} + \frac{3\int (e\cos(c+dx))^{-4-m}(a+a}{a(3-m)}$$
$$= -\frac{(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m}{de(3-m)} - \frac{3(e\cos(c+dx))^{-3-m}(a+a\,\mathrm{s}}{ade(1-m)(3-}$$
$$= -\frac{(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m}{de(3-m)} - \frac{3(e\cos(c+dx))^{-3-m}(a+a\,\mathrm{s}}{ade(1-m)(3-}$$
$$= -\frac{(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m}{de(3-m)} - \frac{3(e\cos(c+dx))^{-3-m}(a+a\,\mathrm{s}}{ade(1-m)(3-}$$

Mathematica [A] time = 0.196414, size = 101, normalized size = 0.5

$$\frac{\sec^3(c+dx)\left(-3\left(m^2-3\right)\sin(c+dx)+6m\sin^2(c+dx)-6\sin^3(c+dx)+m\left(m^2-7\right)\right)(a(\sin(c+dx)+1))^m(e\cos(c+}{de^4(m-3)(m-1)(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-4 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Sec[c + d*x]^3*(a*(1 + Sin[c + d*x]))^m*(m*(-7 + m^2) - 3*(-3 + m^2)*Sin[c
 + d*x] + 6*m*Sin[c + d*x]^2 - 6*Sin[c + d*x]^3))/(d*e^4*(-3 + m)*(-1 + m)*
(1 + m)*(3 + m)*(e*Cos[c + d*x])^m)
```

Maple [F] time = 0.171, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-4-m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m-4}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 4)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [A] time = 2.37926, size = 247, normalized size = 1.23

$$-\frac{\left(6\,m\cos(dx+c)^3-\left(m^3-m\right)\cos(dx+c)-3\left(2\,\cos(dx+c)^3-\left(m^2-1\right)\cos(dx+c)\right)\sin(dx+c)\right)(e\cos(dx+c))^{-m-}}{dm^4-10\,dm^2+9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -(6*m*cos(d*x + c)^3 - (m^3 - m)*cos(d*x + c) - 3*(2*cos(d*x + c)^3 - (m^2
- 1)*cos(d*x + c))*sin(d*x + c))*(e*cos(d*x + c))^(-m - 4)*(a*sin(d*x + c)
+ a)^m/(d*m^4 - 10*d*m^2 + 9*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-4-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m-4}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 4)*(a*sin(d*x + c) + a)^m, x)
```

3.361 $\int (e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=142

$$-\frac{2(a\sin(c+dx)+a)^{m+2}(e\cos(c+dx))^{-m-2}}{a^2dem\left(4-m^2\right)}-\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m-2}}{de(2-m)}+\frac{2(a\sin(c+dx)+a)^{m+1}(e\cos(c}{ade(2-m)m}$$

```
[Out] -(((e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^m)/(d*e*(2 - m))) + (2*(e
*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^(1 + m))/(a*d*e*(2 - m)*m) - (
2*(e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^(2 + m))/(a^2*d*e*m*(4 - m
^2))
```

Rubi [A] time = 0.222483, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$-\frac{2(a\sin(c+dx)+a)^{m+2}(e\cos(c+dx))^{-m-2}}{a^2dem\left(4-m^2\right)}-\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m-2}}{de(2-m)}+\frac{2(a\sin(c+dx)+a)^{m+1}(e\cos(c}{ade(2-m)m}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^m)/(d*e*(2 - m))) + (2*(e
*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^(1 + m))/(a*d*e*(2 - m)*m) - (
2*(e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^(2 + m))/(a^2*d*e*m*(4 - m
^2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
```

```
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m}{de(2-m)} + \frac{2\int (e\cos(c+dx))^{-3-m}(a+\ldots}{a(2-m)\ldots}$$
$$= -\frac{(e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m}{de(2-m)} + \frac{2(e\cos(c+dx))^{-2-m}(a+a\,\mathrm{s}\ldots}{ade(2-m)m\ldots}$$
$$= -\frac{(e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m}{de(2-m)} + \frac{2(e\cos(c+dx))^{-2-m}(a+a\,\mathrm{s}\ldots}{ade(2-m)m\ldots}$$

Mathematica [A] time = 0.136861, size = 76, normalized size = 0.54

$$\frac{\sec^2(c+dx)\left(-2m\sin(c+dx)+2\sin^2(c+dx)+m^2-2\right)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-m}}{de^3(m-2)m(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Sec[c + d*x]^2*(a*(1 + Sin[c + d*x]))^m*(-2 + m^2 - 2*m*Sin[c + d*x] + 2*S
in[c + d*x]^2))/(d*e^3*(-2 + m)*m*(2 + m)*(e*Cos[c + d*x])^m)
```

Maple [F] time = 0.157, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-3-m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m-3}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 3)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [A] time = 2.33089, size = 184, normalized size = 1.3

$$\frac{\left(m^2\cos(dx+c)-2\cos(dx+c)^3-2m\cos(dx+c)\sin(dx+c)\right)(e\cos(dx+c))^{-m-3}(a\sin(dx+c)+a)^m}{dm^3-4dm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (m^2*cos(d*x + c) - 2*cos(d*x + c)^3 - 2*m*cos(d*x + c)*sin(d*x + c))*(e*co
s(d*x + c))^(-m - 3)*(a*sin(d*x + c) + a)^m/(d*m^3 - 4*d*m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-3-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m-3}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 3)*(a*sin(d*x + c) + a)^m, x)
```

3.362 $\int (e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=89

$$\frac{(a\sin(c+dx)+a)^{m+1}(e\cos(c+dx))^{-m-1}}{ade\left(1-m^2\right)}-\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m-1}}{de(1-m)}$$

```
[Out] -(((e*Cos[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^m)/(d*e*(1 - m))) + ((e*C
os[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^(1 + m))/(a*d*e*(1 - m^2))
```

Rubi [A] time = 0.124096, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2672, 2671}

$$\frac{(a\sin(c+dx)+a)^{m+1}(e\cos(c+dx))^{-m-1}}{ade\left(1-m^2\right)}-\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m-1}}{de(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^m)/(d*e*(1 - m))) + ((e*C
os[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^(1 + m))/(a*d*e*(1 - m^2))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^m}{de(1-m)} + \frac{\int (e\cos(c+dx))^{-2-m}(a+a\,\text{s}}{a(1-m)}$$
$$= -\frac{(e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^m}{de(1-m)} + \frac{(e\cos(c+dx))^{-1-m}(a+a\,\text{si}}{ade\left(1-m^2\right)}$$

Mathematica [A] time = 0.120738, size = 53, normalized size = 0.6

$$\frac{(m-\sin(c+dx))(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-m-1}}{de(m-1)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((e*Cos[c + d*x])^(-1 - m)*(m - Sin[c + d*x])*(a*(1 + Sin[c + d*x]))^m)/(d*
e*(-1 + m)*(1 + m))
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-2-m}(a+a\sin(dx+c))^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m-2}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 2)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [A] time = 2.37061, size = 144, normalized size = 1.62

$$\frac{(m\cos(dx+c)-\cos(dx+c)\sin(dx+c))\,(e\cos(dx+c))^{-m-2}\,(a\sin(dx+c)+a)^m}{dm^2-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (m*cos(d*x + c) - cos(d*x + c)*sin(d*x + c))*(e*cos(d*x + c))^(-m - 2)*(a*s
in(d*x + c) + a)^m/(d*m^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-2-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m-2}\,(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 2)*(a*sin(d*x + c) + a)^m, x)
```

3.363 $\int (e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=34

$$\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m}}{dem}$$

```
[Out] (a + a*Sin[c + d*x])^m/(d*e*m*(e*Cos[c + d*x])^m)
```

Rubi [A] time = 0.0506929, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2671}

$$\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m}}{dem}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (a + a*Sin[c + d*x])^m/(d*e*m*(e*Cos[c + d*x])^m)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^m\,dx=\frac{(e\cos(c+dx))^{-m}(a+a\sin(c+dx))^m}{dem}$$

Mathematica [A] time = 0.0531585, size = 34, normalized size = 1.

$$\frac{(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-m}}{dem}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (a*(1 + Sin[c + d*x]))^m/(d*e*m*(e*Cos[c + d*x])^m)
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int (e\cos (dx+c))^{-1-m} (a+a\sin (dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [A] time = 1.57164, size = 88, normalized size = 2.59

$$\frac{a^m e^{-m-1} e^{\left(m \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)-m \log\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)\right)}}{dm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] a^m*e^(-m - 1)*e^(m*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1) - m*log(-sin(d
*x + c)/(cos(d*x + c) + 1) + 1))/(d*m)
```

Fricas [A] time = 2.28788, size = 93, normalized size = 2.74

$$\frac{(e\cos (dx+c))^{-m-1} (a\sin (dx+c)+a)^m \cos (dx+c)}{dm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (e*cos(d*x + c))^(-m - 1)*(a*sin(d*x + c) + a)^m*cos(d*x + c)/(d*m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-1-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m-1} (a\sin(dx+c)+a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 1)*(a*sin(d*x + c) + a)^m, x)
```

3.364 $\int (e\cos(c+dx))^{-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=115

$$-\frac{a2^{\frac{m}{2}+\frac{1}{2}}(\sin(c+dx)+1)^{\frac{1-m}{2}}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{1-m}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-m}{2};\frac{3-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(1-m)}$$

```
[Out] -((2^(1/2 + m/2)*a*(e*Cos[c + d*x])^(1 - m)*Hypergeometric2F1[(1 - m)/2, (1
 - m)/2, (3 - m)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((1 - m)/2)*(a
 + a*Sin[c + d*x])^(-1 + m))/(d*e*(1 - m)))
```

Rubi [A] time = 0.108866, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2689, 70, 69}

$$-\frac{a2^{\frac{m}{2}+\frac{1}{2}}(\sin(c+dx)+1)^{\frac{1-m}{2}}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{1-m}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-m}{2};\frac{3-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^m,x]
```

```
[Out] -((2^(1/2 + m/2)*a*(e*Cos[c + d*x])^(1 - m)*Hypergeometric2F1[(1 - m)/2, (1
 - m)/2, (3 - m)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((1 - m)/2)*(a
 + a*Sin[c + d*x])^(-1 + m))/(d*e*(1 - m)))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^{-m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{1-m}(a-a\sin(c+dx))^{\frac{1}{2}(-1+m)}(a+a\sin(c+dx))^{\frac{1}{2}(-1+m)}\right)}{de}$$

$$= \frac{\left(2^{-\frac{1}{2}+\frac{m}{2}}a^2(e\cos(c+dx))^{1-m}(a-a\sin(c+dx))^{\frac{1}{2}(-1+m)}(a+a\sin(c+dx))^{-\frac{1}{2}}\right.}{}$$

$$= -\frac{2^{\frac{1}{2}+\frac{m}{2}}a(e\cos(c+dx))^{1-m}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-m}{2};\frac{3-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin}{de(1-m)}$$

Mathematica [A] time = 0.128852, size = 108, normalized size = 0.94

$$\frac{2^{\frac{m+1}{2}}\cos(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-m-1)}(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-m}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-m}{2};\frac{3-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^m,x]
```

```
[Out] (2^((1 + m)/2)*Cos[c + d*x]*Hypergeometric2F1[(1 - m)/2, (1 - m)/2, (3 - m)
/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - m)/2)*(a*(1 + Sin[c + d
*x]))^m)/(d*(-1 + m)*(e*Cos[c + d*x])^m)
```

Maple [F] time = 0.355, size = 0, normalized size = 0.

$$\int \frac{(a+a\sin(dx+c))^m}{(e\cos(dx+c))^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x)
```

```
[Out] int((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sin(c+dx)+1))^m (e\cos(c+dx))^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/((e*cos(d*x+c))**m),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**(-m), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx+c)+a)^m}{(e\cos(dx+c))^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^m, x)
```

3.365 $\int (e\cos(c+dx))^{1-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=97

$$\frac{2^{1-\frac{m}{2}}(1-\sin(c+dx))^{\frac{m}{2}-1}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{2-m}\,{}_2F_1\left(\frac{m}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de(m+2)}$$

```
[Out] (2^(1 - m/2)*(e*Cos[c + d*x])^(2 - m)*Hypergeometric2F1[m/2, (2 + m)/2, (4
+ m)/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-1 + m/2)*(a + a*Sin[c +
d*x])^m)/(d*e*(2 + m))
```

Rubi [A] time = 0.106128, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 70, 69}

$$\frac{2^{1-\frac{m}{2}}(1-\sin(c+dx))^{\frac{m}{2}-1}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{2-m}\,{}_2F_1\left(\frac{m}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(1 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(1 - m/2)*(e*Cos[c + d*x])^(2 - m)*Hypergeometric2F1[m/2, (2 + m)/2, (4
+ m)/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-1 + m/2)*(a + a*Sin[c +
d*x])^m)/(d*e*(2 + m))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^{1-m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{2-m}(a-a\sin(c+dx))^{\frac{1}{2}(-2+m)}(a+a\sin(c+dx))^{\frac{1}{2}(-2+m)}\right.}{de}$$

$$= \frac{\left(2^{-m/2}a^2(e\cos(c+dx))^{2-m}(a-a\sin(c+dx))^{\frac{1}{2}(-2+m)-\frac{m}{2}}\left(\frac{a-a\sin(c+dx)}{a}\right)^{m/2}\right.}{}$$

$$= \frac{2^{1-\frac{m}{2}}(e\cos(c+dx))^{2-m}\,{}_2F_1\left(\frac{m}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{1}{2}(1+\sin(c+dx))\right)(1-\sin(c+}{de(2+m)}$$

Mathematica [A] time = 0.254181, size = 97, normalized size = 1.

$$\frac{2^{\frac{m}{2}+1}(\sin(c+dx)+1)^{-\frac{m}{2}-1}(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{2-m}\,{}_2F_1\left(1-\frac{m}{2},-\frac{m}{2};2-\frac{m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(m-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(1 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(1 + m/2)*(e*Cos[c + d*x])^(2 - m)*Hypergeometric2F1[1 - m/2, -m/2, 2 -
m/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1 - m/2)*(a*(1 + Sin[c + d*
x]))^m)/(d*e*(-2 + m))
```

Maple [F] time = 0.15, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{1-m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m+1}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m + 1)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((e\cos(dx+c))^{-m+1}(a\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-m + 1)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m+1}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m + 1)*(a*sin(d*x + c) + a)^m, x)
```

3.366 $\int (e\cos(c+dx))^{2-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=115

$$-\frac{a2^{\frac{m}{2}+\frac{3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-m-1)}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{3-m}\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{3-m}{2};\frac{5-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(3-m)}$$

```
[Out] -((2^(3/2 + m/2)*a*(e*Cos[c + d*x])^(3 - m)*Hypergeometric2F1[(-1 - m)/2, (
3 - m)/2, (5 - m)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - m)/2)*
(a + a*Sin[c + d*x])^(-1 + m))/(d*e*(3 - m)))
```

Rubi [A] time = 0.116932, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 70, 69}

$$-\frac{a2^{\frac{m}{2}+\frac{3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-m-1)}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{3-m}\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{3-m}{2};\frac{5-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(2 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((2^(3/2 + m/2)*a*(e*Cos[c + d*x])^(3 - m)*Hypergeometric2F1[(-1 - m)/2, (
3 - m)/2, (5 - m)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - m)/2)*
(a + a*Sin[c + d*x])^(-1 + m))/(d*e*(3 - m)))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^{2-m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{3-m}(a-a\sin(c+dx))^{\frac{1}{2}(-3+m)}(a+a\sin(c+dx))^{\frac{1}{2}(-3+m)}\right.}{de}$$

$$= \frac{\left(2^{\frac{1}{2}+\frac{m}{2}}a^2(e\cos(c+dx))^{3-m}(a-a\sin(c+dx))^{\frac{1}{2}(-3+m)}(a+a\sin(c+dx))^{\frac{1}{2}+}\right.}{}$$

$$= -\frac{2^{\frac{3}{2}+\frac{m}{2}}a(e\cos(c+dx))^{3-m}\,{}_2F_1\left(\frac{1}{2}(-1-m),\frac{3-m}{2};\frac{5-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(3-m)}$$

Mathematica [A] time = 0.250281, size = 113, normalized size = 0.98

$$\frac{e^2 2^{\frac{m+3}{2}}\cos^3(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}(-m-3)}(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-m}\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{3-m}{2};\frac{5-m}{2};\frac{1}{2}(1-\sin(c\right.}{d(m-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(2 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^((3 + m)/2)*e^2*Cos[c + d*x]^3*Hypergeometric2F1[(-1 - m)/2, (3 - m)/2,
(5 - m)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-3 - m)/2)*(a*(1 + Si
n[c + d*x]))^m)/(d*(-3 + m)*(e*Cos[c + d*x])^m)
```

Maple [F] time = 0.152, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{2-m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m+2}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m + 2)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((e\cos(dx+c))^{-m+2}(a\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-m + 2)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(2-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m+2}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m + 2)*(a*sin(d*x + c) + a)^m, x)
```

3.367 $\int (e\cos(c+dx))^{5-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=150

$$-\frac{8a^3(a\sin(c+dx)+a)^{m-3}(e\cos(c+dx))^{6-2m}}{de(5-m)\left(m^2-7m+12\right)}-\frac{4a^2(a\sin(c+dx)+a)^{m-2}(e\cos(c+dx))^{6-2m}}{de\left(m^2-9m+20\right)}-\frac{a(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{6-2m}}{de(5-m)}$$

```
[Out] (-8*a^3*(e*Cos[c + d*x])^(6 - 2*m)*(a + a*Sin[c + d*x])^(-3 + m))/(d*e*(5 -
 m)*(12 - 7*m + m^2)) - (4*a^2*(e*Cos[c + d*x])^(6 - 2*m)*(a + a*Sin[c + d*
x])^(-2 + m))/(d*e*(20 - 9*m + m^2)) - (a*(e*Cos[c + d*x])^(6 - 2*m)*(a + a
*Sin[c + d*x])^(-1 + m))/(d*e*(5 - m))
```

Rubi [A] time = 0.240436, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2674, 2673}

$$-\frac{8a^3(a\sin(c+dx)+a)^{m-3}(e\cos(c+dx))^{6-2m}}{de(5-m)\left(m^2-7m+12\right)}-\frac{4a^2(a\sin(c+dx)+a)^{m-2}(e\cos(c+dx))^{6-2m}}{de\left(m^2-9m+20\right)}-\frac{a(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{6-2m}}{de(5-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```


```
[Out] (-8*a^3*(e*Cos[c + d*x])^(6 - 2*m)*(a + a*Sin[c + d*x])^(-3 + m))/(d*e*(5 -
 m)*(12 - 7*m + m^2)) - (4*a^2*(e*Cos[c + d*x])^(6 - 2*m)*(a + a*Sin[c + d*
x])^(-2 + m))/(d*e*(20 - 9*m + m^2)) - (a*(e*Cos[c + d*x])^(6 - 2*m)*(a + a
*Sin[c + d*x])^(-1 + m))/(d*e*(5 - m))
```


Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
```

```
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (e\cos(c+dx))^{5-2m}(a+a\sin(c+dx))^m\,dx = -\frac{a(e\cos(c+dx))^{6-2m}(a+a\sin(c+dx))^{-1+m}}{de(5-m)} + \frac{(4a)\int (e\cos(c+dx))^{5-2}}{5}$$

$$= -\frac{4a^2(e\cos(c+dx))^{6-2m}(a+a\sin(c+dx))^{-2+m}}{de\left(20-9m+m^2\right)} - \frac{a(e\cos(c+dx))^{6-2m}(a}{de(5}$$

$$= -\frac{8a^3(e\cos(c+dx))^{6-2m}(a+a\sin(c+dx))^{-3+m}}{de(3-m)\left(20-9m+m^2\right)} - \frac{4a^2(e\cos(c+dx))^{6-2n}}{de\left(20-\right.}$$

Mathematica [A] time = 0.434272, size = 105, normalized size = 0.7

$$\frac{e^5\cos^6(c+dx)\left(\left(m^2-7m+12\right)\sin^2(c+dx)+2\left(m^2-9m+18\right)\sin(c+dx)+m^2-11m+32\right)(a(\sin(c+dx)+1))^m(e}{d(m-5)(m-4)(m-3)(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (e^5*Cos[c + d*x]^6*(a*(1 + Sin[c + d*x]))^m*(32 - 11*m + m^2 + 2*(18 - 9*m
 + m^2)*Sin[c + d*x] + (12 - 7*m + m^2)*Sin[c + d*x]^2))/(d*(-5 + m)*(-4 +
m)*(-3 + m)*(e*Cos[c + d*x])^(2*m)*(1 + Sin[c + d*x])^3)
```

Maple [F] time = 0.914, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{5-2\,m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [B] time = 1.66588, size = 842, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((m^2 - 11*m + 32)*a^m*e^5 - 2*(m^2 - 15*m + 60)*a^m*e^5*sin(d*x + c)/(cos(
d*x + c) + 1) - (3*m^2 - m - 160)*a^m*e^5*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 8*(m^2 - 7*m - 20)*a^m*e^5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(m^
2 + 5*m + 160)*a^m*e^5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(3*m^2 - 13*
m + 116)*a^m*e^5*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*(m^2 + 5*m + 160)*
a^m*e^5*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 8*(m^2 - 7*m - 20)*a^m*e^5*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7 - (3*m^2 - m - 160)*a^m*e^5*sin(d*x + c)^
8/(cos(d*x + c) + 1)^8 - 2*(m^2 - 15*m + 60)*a^m*e^5*sin(d*x + c)^9/(cos(d*
x + c) + 1)^9 + (m^2 - 11*m + 32)*a^m*e^5*sin(d*x + c)^10/(cos(d*x + c) + 1
)^10)*e^(-2*m*log(-sin(d*x + c)/(cos(d*x + c) + 1) + 1) + m*log(sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 1))/(((m^3 - 12*m^2 + 47*m - 60)*e^(2*m) + 5*(m^
3 - 12*m^2 + 47*m - 60)*e^(2*m)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*(m
^3 - 12*m^2 + 47*m - 60)*e^(2*m)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*(
m^3 - 12*m^2 + 47*m - 60)*e^(2*m)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*(
m^3 - 12*m^2 + 47*m - 60)*e^(2*m)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + (m^
3 - 12*m^2 + 47*m - 60)*e^(2*m)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*d)
```

Fricas [B] time = 2.46202, size = 813, normalized size = 5.42

$$-\frac{\left(\left(m^2-7\,m+12\right)\cos\left(dx+c\right)^3-\left(m^2-11\,m+24\right)\cos\left(dx+c\right)^2-2\left(m\right.\right.}{4\,dm^3-\left(dm^3-12\,dm^2+47\,dm-60\,d\right)\cos\left(dx+c\right)^3-48\,dm^2-3\left(dm^3-12\,dm^2+47\,dm-60\,d\right)\cos\left(dx+c\right)^2+188\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -((m^2 - 7*m + 12)*cos(d*x + c)^3 - (m^2 - 11*m + 24)*cos(d*x + c)^2 - 2*(m
^2 - 9*m + 22)*cos(d*x + c) - ((m^2 - 7*m + 12)*cos(d*x + c)^2 + 2*(m^2 - 9
*m + 18)*cos(d*x + c) - 8)*sin(d*x + c) - 8)*(e*cos(d*x + c))^(-2*m + 5)*(a
*sin(d*x + c) + a)^m/(4*d*m^3 - (d*m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x
+ c)^3 - 48*d*m^2 - 3*(d*m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x + c)^2 + 1
88*d*m + 2*(d*m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x + c) + (4*d*m^3 - 48*
d*m^2 - (d*m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x + c)^2 + 188*d*m + 2*(d*
```

```
m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x + c) - 240*d)*sin(d*x + c) - 240*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-2m+5}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m + 5)*(a*sin(d*x + c) + a)^m, x)
```

3.368 $\int (e\cos(c+dx))^{3-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=94

$$-\frac{2a^2(a\sin(c+dx)+a)^{m-2}(e\cos(c+dx))^{4-2m}}{de\left(m^2-5m+6\right)}-\frac{a(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{4-2m}}{de(3-m)}$$

```
[Out] (-2*a^2*(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^(-2 + m))/(d*e*(6 -
 5*m + m^2)) - (a*(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^(-1 + m))
/(d*e*(3 - m))
```

Rubi [A] time = 0.140641, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2674, 2673}

$$-\frac{2a^2(a\sin(c+dx)+a)^{m-2}(e\cos(c+dx))^{4-2m}}{de\left(m^2-5m+6\right)}-\frac{a(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{4-2m}}{de(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (-2*a^2*(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^(-2 + m))/(d*e*(6 -
 5*m + m^2)) - (a*(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^(-1 + m))
/(d*e*(3 - m))
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (e\cos(c+dx))^{3-2m}(a+a\sin(c+dx))^m\,dx = -\frac{a(e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^{-1+m}}{de(3-m)} + \frac{(2a)\int (e\cos(c+dx))^{3-2}}{3}$$
$$= -\frac{2a^2(e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^{-2+m}}{de\left(6-5m+m^2\right)} - \frac{a(e\cos(c+dx))^{4-2m}(a}{de(3}$$

Mathematica [A] time = 0.219216, size = 72, normalized size = 0.77

$$\frac{e^3\cos^4(c+dx)((m-2)\sin(c+dx)+m-4)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m}}{d(m-3)(m-2)(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (e^3*Cos[c + d*x]^4*(a*(1 + Sin[c + d*x]))^m*(-4 + m + (-2 + m)*Sin[c + d*x
]))/(d*(-3 + m)*(-2 + m)*(e*Cos[c + d*x])^(2*m)*(1 + Sin[c + d*x])^2)
```

Maple [F] time = 0.935, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{3-2\,m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [B] time = 1.5684, size = 474, normalized size = 5.04

$$\frac{\left(a^m e^3(m-4) - \frac{2\,a^m e^3(m-6)\sin(dx+c)}{\cos(dx+c)+1} - \frac{a^m e^3(m+12)\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{4\,a^m e^3(m+2)\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{a^m e^3(m+12)\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{2\,a^m e^3(m-6)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right.}{\left(\left(m^2-5\,m+6\right)e^{2\,m} + \frac{3\left(m^2-5\,m+6\right)e^{2\,m}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3\left(m^2-5\,m+6\right)e^{2\,m}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + (m^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] (a^m*e^3*(m - 4) - 2*a^m*e^3*(m - 6)*sin(d*x + c)/(cos(d*x + c) + 1) - a^m*
e^3*(m + 12)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^m*e^3*(m + 2)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 - a^m*e^3*(m + 12)*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 - 2*a^m*e^3*(m - 6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^m*e^3
*(m - 4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*e^(-2*m*log(-sin(d*x + c)/(co
s(d*x + c) + 1) + 1) + m*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1))/(((m
^2 - 5*m + 6)*e^(2*m) + 3*(m^2 - 5*m + 6)*e^(2*m)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 3*(m^2 - 5*m + 6)*e^(2*m)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
+ (m^2 - 5*m + 6)*e^(2*m)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d)
```

Fricas [A] time = 2.37375, size = 440, normalized size = 4.68

$$\frac{\left((m-2)\cos\left(dx+c\right)^{2}+(m-4)\cos\left(dx+c\right)+((m-2)\cos\left(dx+c\right)+2)\sin\left(dx+c\right)-2\right)\left(e\cos\left(dx+c\right)\right)}{2\,dm^{2}-\left(dm^{2}-5\,dm+6\,d\right)\cos\left(dx+c\right)^{2}-10\,dm+\left(dm^{2}-5\,dm+6\,d\right)\cos\left(dx+c\right)+\left(2\,dm^{2}-10\,dm+\left(dm^{2}-5\,dm+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m - 2)*cos(d*x + c)^2 + (m - 4)*cos(d*x + c) + ((m - 2)*cos(d*x + c) + 2)
*sin(d*x + c) - 2)*(e*cos(d*x + c))^(-2*m + 3)*(a*sin(d*x + c) + a)^m/(2*d*
m^2 - (d*m^2 - 5*d*m + 6*d)*cos(d*x + c)^2 - 10*d*m + (d*m^2 - 5*d*m + 6*d)
*cos(d*x + c) + (2*d*m^2 - 10*d*m + (d*m^2 - 5*d*m + 6*d)*cos(d*x + c) + 12
*d)*sin(d*x + c) + 12*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-2m+3}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m + 3)*(a*sin(d*x + c) + a)^m, x)
```

3.369 $\int (e\cos(c+dx))^{1-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=44

$$-\frac{a(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{2-2m}}{de(1-m)}$$

```
[Out] -((a*(e*Cos[c + d*x])^(2 - 2*m)*(a + a*Sin[c + d*x])^(-1 + m))/(d*e*(1 - m)
))
```

Rubi [A] time = 0.0554556, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2673}

$$-\frac{a(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{2-2m}}{de(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(1 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((a*(e*Cos[c + d*x])^(2 - 2*m)*(a + a*Sin[c + d*x])^(-1 + m))/(d*e*(1 - m)
))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (e\cos(c+dx))^{1-2m}(a+a\sin(c+dx))^m\,dx=-\frac{a(e\cos(c+dx))^{2-2m}(a+a\sin(c+dx))^{-1+m}}{de(1-m)}$$

Mathematica [A] time = 0.153695, size = 43, normalized size = 0.98

$$-\frac{e(\sin(c+dx)-1)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m}}{d(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(1 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((e*(-1 + Sin[c + d*x])*(a*(1 + Sin[c + d*x]))^m)/(d*(-1 + m)*(e*Cos[c + d
*x])^(2*m)))
```

Maple [F] time = 1.25, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{1-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [B] time = 1.50738, size = 194, normalized size = 4.41

$$\frac{\left(a^m e - \frac{2\,a^m e\sin(dx+c)}{\cos(dx+c)+1} + \frac{a^m e\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)e^{\left(-2\,m\log\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)+m\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)\right)}}{\left(e^{2\,m}(m-1)+\frac{e^{2\,m}(m-1)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] (a^m*e - 2*a^m*e*sin(d*x + c)/(cos(d*x + c) + 1) + a^m*e*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2)*e^(-2*m*log(-sin(d*x + c)/(cos(d*x + c) + 1) + 1) + m*lo
g(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1))/((e^(2*m)*(m - 1) + e^(2*m)*(m
- 1)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d)
```

Fricas [A] time = 2.29197, size = 197, normalized size = 4.48

$$\frac{(e\cos(dx+c))^{-2\,m+1}(a\sin(dx+c)+a)^m(\cos(dx+c)-\sin(dx+c)+1)}{dm+(dm-d)\cos(dx+c)+(dm-d)\sin(dx+c)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (e*cos(d*x + c))^(-2*m + 1)*(a*sin(d*x + c) + a)^m*(cos(d*x + c) - sin(d*x
+ c) + 1)/(d*m + (d*m - d)*cos(d*x + c) + (d*m - d)*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-2m+1}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m + 1)*(a*sin(d*x + c) + a)^m, x)
```

3.370 $\int (e\cos(c+dx))^{-1-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=61

$$\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-2m}\,{}_2F_1\left(1,-m;1-m;\frac{1}{2}(1-\sin(c+dx))\right)}{2dem}$$

```
[Out] (Hypergeometric2F1[1, -m, 1 - m, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x])
^m)/(2*d*e*m*(e*Cos[c + d*x])^(2*m))
```

Rubi [A] time = 0.067174, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 7, 68}

$$\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-2m}\,{}_2F_1\left(1,-m;1-m;\frac{1}{2}(1-\sin(c+dx))\right)}{2dem}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-1 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, -m, 1 - m, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x])
^m)/(2*d*e*m*(e*Cos[c + d*x])^(2*m))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 && !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (e\cos(c+dx))^{-1-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{-2m}(a-a\sin(c+dx))^m(a+a\sin(c+dx))^m\right)\text{Subst}\left(\int\right.}{de}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{-2m}(a-a\sin(c+dx))^m(a+a\sin(c+dx))^m\right)\text{Subst}\left(\int\right.}{de}$$

$$= \frac{(e\cos(c+dx))^{-2m}\,{}_2F_1\left(1,-m;1-m;\frac{1}{2}(1-\sin(c+dx))\right)(a+a\sin(c+dx$$

$$2dem$$

Mathematica [A] time = 0.0674885, size = 61, normalized size = 1.

$$\frac{(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m}\,{}_2F_1\left(1,-m;1-m;\frac{1}{2}(1-\sin(c+dx))\right)}{2dem}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-1 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, -m, 1 - m, (1 - Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]
))^m)/(2*d*e*m*(e*Cos[c + d*x])^(2*m))
```

Maple [F] time = 0.773, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-1-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e \cos (dx + c))^{-2m-1} (a \sin (dx + c) + a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m - 1)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((e \cos (dx + c))^{-2m-1} (a \sin (dx + c) + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-2*m - 1)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-1-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e \cos (dx + c))^{-2m-1} (a \sin (dx + c) + a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m - 1)*(a*sin(d*x + c) + a)^m, x)
```

3.371 $\int (e\cos(c+dx))^{-3-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=70

$$\frac{(a\sin(c+dx)+a)^{m+1}(e\cos(c+dx))^{-2(m+1)}\,{}_2F_1\left(2,-m-1;-m;\frac{1}{2}(1-\sin(c+dx))\right)}{4ade(m+1)}$$

```
[Out] (Hypergeometric2F1[2, -1 - m, -m, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x]
)^(1 + m))/(4*a*d*e*(1 + m)*(e*Cos[c + d*x])^(2*(1 + m)))
```

Rubi [A] time = 0.0751064, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2689, 7, 68}

$$\frac{(a\sin(c+dx)+a)^{m+1}(e\cos(c+dx))^{-2(m+1)}\,{}_2F_1\left(2,-m-1;-m;\frac{1}{2}(1-\sin(c+dx))\right)}{4ade(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-3 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, -1 - m, -m, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x]
)^(1 + m))/(4*a*d*e*(1 + m)*(e*Cos[c + d*x])^(2*(1 + m)))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (e\cos(c+dx))^{-3-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{-2-2m}(a-a\sin(c+dx))^{\frac{1}{2}(2+2m)}(a+a\sin(c+dx))^{\frac{1}{2}(2+2}\right.}{de}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{-2-2m}(a-a\sin(c+dx))^{\frac{1}{2}(2+2m)}(a+a\sin(c+dx))^{\frac{1}{2}(2+2}\right.}{de}$$

$$= \frac{(e\cos(c+dx))^{-2(1+m)}\,{}_2F_1\left(2,-1-m;-m;\frac{1}{2}(1-\sin(c+dx))\right)(a+a\sin(c}{4ade(1+m)}$$

Mathematica [A] time = 0.136757, size = 76, normalized size = 1.09

$$\frac{\sec^2(c+dx)(a(\sin(c+dx)+1))^{m+1}(e\cos(c+dx))^{-2m}\,{}_2F_1\left(2,-m-1;-m;\frac{1}{2}(1-\sin(c+dx))\right)}{4ade^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-3 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, -1 - m, -m, (1 - Sin[c + d*x])/2]*Sec[c + d*x]^2*(a*(
1 + Sin[c + d*x]))^(1 + m))/(4*a*d*e^3*(1 + m)*(e*Cos[c + d*x])^(2*m))
```

Maple [F] time = 0.802, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-3-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-3-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-3-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-2m-3}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m - 3)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((e\cos(dx+c))^{-2m-3}(a\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-2*m - 3)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-3-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-2m-3}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m - 3)*(a*sin(d*x + c) + a)^m, x)
```

3.372 $\int (e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=89

$$\frac{2^{\frac{5}{2}-m}(1-\sin(c+dx))^{m-\frac{5}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{5-2m}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}(2m-3);\frac{7}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{5de}$$

```
[Out] (2^(5/2 - m)*(e*Cos[c + d*x])^(5 - 2*m)*Hypergeometric2F1[5/2, (-3 + 2*m)/2
, 7/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-5/2 + m)*(a + a*Sin[c + d
*x])^m)/(5*d*e)
```

Rubi [A] time = 0.0964986, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2689, 7, 70, 69}

$$\frac{2^{\frac{5}{2}-m}(1-\sin(c+dx))^{m-\frac{5}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{5-2m}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}(2m-3);\frac{7}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```


```
[Out] (2^(5/2 - m)*(e*Cos[c + d*x])^(5 - 2*m)*Hypergeometric2F1[5/2, (-3 + 2*m)/2
, 7/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-5/2 + m)*(a + a*Sin[c + d
*x])^m)/(5*d*e)
```


Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{5-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-5+2m)}(a+a\sin(c+dx))^{\frac{1}{2}(-5+2\ldots}\right.}{de}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{5-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-5+2m)}(a+a\sin(c+dx))^{\frac{1}{2}(-5+2\ldots}\right.}{de}$$

$$= \frac{\left(2^{\frac{3}{2}-m}a^3(e\cos(c+dx))^{5-2m}(a-a\sin(c+dx))^{\frac{1}{2}-m+\frac{1}{2}(-5+2m)}\left(\frac{a-a\sin(c+dx)}{a}\right)^{-\ldots}\right.}{}$$

$$= \frac{2^{\frac{5}{2}-m}(e\cos(c+dx))^{5-2m}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}(-3+2m);\frac{7}{2};\frac{1}{2}(1+\sin(c+dx))\right)(1-\text{si}\ldots}{5de}$$

Mathematica [A] time = 0.207738, size = 96, normalized size = 1.08

$$\frac{4\sqrt{2}e^4\cos^5(c+dx)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;\frac{7}{2}-m;\frac{1}{2}(1-\sin(c+dx))\right)}{d(2m-5)(\sin(c+dx)+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (4*Sqrt[2]*e^4*Cos[c + d*x]^5*Hypergeometric2F1[-3/2, 5/2 - m, 7/2 - m, (1
- Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^m)/(d*(-5 + 2*m)*(e*Cos[c + d*x])
```

```
^(2*m)*(1 + Sin[c + d*x])^(5/2))
```

Maple [F] time = 0.928, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{4-2m} (a+a\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-2m+4} (a\sin(dx+c)+a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m + 4)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((e\cos(dx+c))^{-2m+4} (a\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-2*m + 4)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(4-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-2m+4}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m + 4)*(a*sin(d*x + c) + a)^m, x)
```

3.373 $\int (e\cos(c+dx))^{2-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=89

$$\frac{2^{\frac{3}{2}-m}(1-\sin(c+dx))^{m-\frac{3}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{3-2m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(2m-1);\frac{5}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{3de}$$

```
[Out] (2^(3/2 - m)*(e*Cos[c + d*x])^(3 - 2*m)*Hypergeometric2F1[3/2, (-1 + 2*m)/2
, 5/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-3/2 + m)*(a + a*Sin[c + d
*x])^m)/(3*d*e)
```

Rubi [A] time = 0.0992501, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2689, 7, 70, 69}

$$\frac{2^{\frac{3}{2}-m}(1-\sin(c+dx))^{m-\frac{3}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{3-2m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(2m-1);\frac{5}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(2 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(3/2 - m)*(e*Cos[c + d*x])^(3 - 2*m)*Hypergeometric2F1[3/2, (-1 + 2*m)/2
, 5/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-3/2 + m)*(a + a*Sin[c + d
*x])^m)/(3*d*e)
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^{2-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{3-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-3+2m)}(a+a\sin(c+dx))^{\frac{1}{2}(-3+2}\right.}{de}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{3-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-3+2m)}(a+a\sin(c+dx))^{\frac{1}{2}(-3+2}\right.}{de}$$

$$= \frac{\left(2^{\frac{1}{2}-m}a^2(e\cos(c+dx))^{3-2m}(a-a\sin(c+dx))^{\frac{1}{2}-m+\frac{1}{2}(-3+2m)}\left(\frac{a-a\sin(c+dx)}{a}\right)^{-}\right.}{}$$

$$= \frac{2^{\frac{3}{2}-m}(e\cos(c+dx))^{3-2m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-1+2m);\frac{5}{2};\frac{1}{2}(1+\sin(c+dx))\right)(1-\sin}{3de}$$

Mathematica [A] time = 0.181125, size = 96, normalized size = 1.08

$$\frac{2\sqrt{2}e^2\cos^3(c+dx)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{5}{2}-m;\frac{1}{2}(1-\sin(c+dx))\right)}{d(2m-3)(\sin(c+dx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(2 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2*Sqrt[2]*e^2*Cos[c + d*x]^3*Hypergeometric2F1[-1/2, 3/2 - m, 5/2 - m, (1
- Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^m)/(d*(-3 + 2*m)*(e*Cos[c + d*x])
```

```
^(2*m)*(1 + Sin[c + d*x])^(3/2))
```

Maple [F] time = 0.927, size = 0, normalized size = 0.

$$\int (e\cos (dx+c))^{2-2m} (a+a\sin (dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos (dx+c))^{-2m+2} (a\sin (dx+c)+a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m + 2)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((e\cos (dx+c))^{-2m+2} (a\sin (dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-2*m + 2)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(2-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-2m+2}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m + 2)*(a*sin(d*x + c) + a)^m, x)
```

3.374 $\int (e\cos(c+dx))^{-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=86

$$\frac{2^{\frac{1}{2}-m}(1-\sin(c+dx))^{m-\frac{1}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{1-2m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(2m+1);\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de}$$

```
[Out] (2^(1/2 - m)*(e*Cos[c + d*x])^(1 - 2*m)*Hypergeometric2F1[1/2, (1 + 2*m)/2,
 3/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-1/2 + m)*(a + a*Sin[c + d*
x])^m)/(d*e)
```

Rubi [A] time = 0.0904185, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2689, 7, 70, 69}

$$\frac{2^{\frac{1}{2}-m}(1-\sin(c+dx))^{m-\frac{1}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{1-2m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(2m+1);\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(2*m),x]
```

```
[Out] (2^(1/2 - m)*(e*Cos[c + d*x])^(1 - 2*m)*Hypergeometric2F1[1/2, (1 + 2*m)/2,
 3/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-1/2 + m)*(a + a*Sin[c + d*
x])^m)/(d*e)
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 && !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^{-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{1-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-1+2m)}(a+a\sin(c+dx))^{\frac{1}{2}(-1+2m}\right.}{de}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{1-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-1+2m)}(a+a\sin(c+dx))^{\frac{1}{2}(-1+2m}\right.}{de}$$

$$= \frac{\left(2^{-\frac{1}{2}-m}a^2(e\cos(c+dx))^{1-2m}(a-a\sin(c+dx))^{-\frac{1}{2}-m+\frac{1}{2}(-1+2m)}\left(\frac{a-a\sin(c+dx)}{a}\right)\right.}{d}$$

$$= \frac{2^{\frac{1}{2}-m}(e\cos(c+dx))^{1-2m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+2m);\frac{3}{2};\frac{1}{2}(1+\sin(c+dx))\right)(1-\sin(c}{de}$$

Mathematica [A] time = 0.0854901, size = 90, normalized size = 1.05

$$\frac{\sqrt{2}\cos(c+dx)(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2}-m;\frac{1}{2}(1-\sin(c+dx))\right)}{d(2m-1)\sqrt{\sin(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(2*m),x]
```

```
[Out] (Sqrt[2]*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2 - m, (1 - Sin[c +
 d*x])/2]*(a*(1 + Sin[c + d*x]))^m)/(d*(-1 + 2*m)*(e*Cos[c + d*x])^(2*m)*Sq
```

```
rt[1 + Sin[c + d*x]])
```

Maple [F] time = 0.42, size = 0, normalized size = 0.

$$\int \frac{(a + a\sin(dx + c))^m}{(e\cos(dx + c))^{2m}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x)
```

```
[Out] int((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin(dx + c) + a)^m}{(e\cos(dx + c))^{2m}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(2*m), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(a\sin(dx + c) + a)^m}{(e\cos(dx + c))^{2m}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(2*m), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sin (c+dx)+1))^m (e\cos (c+dx))^{-2m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/((e*cos(d*x+c))**(2*m)),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**(-2*m), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sin (dx+c)+a)^m}{(e\cos (dx+c))^{2m}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(2*m), x)
```

3.375 $\int (e\cos(c+dx))^{-2-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=87

$$-\frac{2^{-m-\frac{1}{2}}(1-\sin(c+dx))^{m+\frac{1}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-2m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(2m+3);\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de}$$

```
[Out] -((2^(-1/2 - m)*(e*Cos[c + d*x])^(-1 - 2*m)*Hypergeometric2F1[-1/2, (3 + 2*
m)/2, 1/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(1/2 + m)*(a + a*Sin[c
+ d*x])^m)/(d*e))
```

Rubi [A] time = 0.0953067, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2689, 7, 70, 69}

$$-\frac{2^{-m-\frac{1}{2}}(1-\sin(c+dx))^{m+\frac{1}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-2m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(2m+3);\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-2 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((2^(-1/2 - m)*(e*Cos[c + d*x])^(-1 - 2*m)*Hypergeometric2F1[-1/2, (3 + 2*
m)/2, 1/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(1/2 + m)*(a + a*Sin[c
+ d*x])^m)/(d*e))
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^{-2-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{-1-2m}(a-a\sin(c+dx))^{\frac{1}{2}(1+2m)}(a+a\sin(c+dx))^{\frac{1}{2}(1+2}\right.}{de}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{-1-2m}(a-a\sin(c+dx))^{\frac{1}{2}(1+2m)}(a+a\sin(c+dx))^{\frac{1}{2}(1+2}\right.}{de}$$

$$= \frac{\left(2^{-\frac{3}{2}-m}a(e\cos(c+dx))^{-1-2m}(a-a\sin(c+dx))^{-\frac{1}{2}-m+\frac{1}{2}(1+2m)}\left(\frac{a-a\sin(c+dx)}{a}\right.\right.}{}$$

$$= -\frac{2^{-\frac{1}{2}-m}(e\cos(c+dx))^{-1-2m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(3+2m);\frac{1}{2};\frac{1}{2}(1+\sin(c+dx))\right)(1}{de}$$

Mathematica [A] time = 0.225398, size = 87, normalized size = 1.

$$\frac{\sqrt{\sin(c+dx)+1}(a(\sin(c+dx)+1))^m(e\cos(c+dx))^{-2m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{1}{2}-m;\frac{1}{2}(1-\sin(c+dx))\right)}{\sqrt{2}e(2dm+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-2 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((e*Cos[c + d*x])^(-1 - 2*m)*Hypergeometric2F1[3/2, -1/2 - m, 1/2 - m, (1 -
 Sin[c + d*x])/2]*Sqrt[1 + Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^m)/(Sqrt[2]
```

```
*e*(d + 2*d*m))
```

Maple [F] time = 0.766, size = 0, normalized size = 0.

$$\int (e\cos (dx+c))^{-2-2m} (a+a\sin (dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos (dx+c))^{-2m-2} (a\sin (dx+c)+a)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m - 2)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((e\cos (dx+c))^{-2m-2} (a\sin (dx+c)+a)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-2*m - 2)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-2-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-2m-2}(a\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-2*m - 2)*(a*sin(d*x + c) + a)^m, x)
```

3.376 $\int \cos^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^6(c+dx)}{6d}$$

```
[Out] -(b*Cos[c + d*x]^6)/(6*d) + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0422384, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2668, 641, 194}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^6)/(6*d) + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos^5(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}\\
&= -\frac{b\cos^6(c+dx)}{6d}+\frac{a\,\text{Subst}\left(\int \left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}\\
&= -\frac{b\cos^6(c+dx)}{6d}+\frac{a\,\text{Subst}\left(\int \left(b^4-2b^2x^2+x^4\right)\,dx, x, b\sin(c+dx)\right)}{b^5 d}\\
&= -\frac{b\cos^6(c+dx)}{6d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] time = 0.0232151, size = 60, normalized size = 1.

$$
\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^6(c+dx)}{6d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^6)/(6*d) + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.027, size = 46, normalized size = 0.8

$$
\frac{1}{d}\left(-\frac{b\left(\cos\left(dx+c\right)\right)^6}{6}+\frac{a\sin\left(dx+c\right)}{5}\left(\frac{8}{3}+\left(\cos\left(dx+c\right)\right)^4+\frac{4\left(\cos\left(dx+c\right)\right)^2}{3}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/6*b*cos(d*x+c)^6+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+
c))
```

Maxima [A] time = 0.948505, size = 95, normalized size = 1.58

$$\frac{5\,b\sin\left(dx+c\right)^{6}+6\,a\sin\left(dx+c\right)^{5}-15\,b\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}+15\,b\sin\left(dx+c\right)^{2}+30\,a\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(5*b*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 - 15*b*sin(d*x + c)^4 - 20*a*
sin(d*x + c)^3 + 15*b*sin(d*x + c)^2 + 30*a*sin(d*x + c))/d
```

Fricas [A] time = 2.3126, size = 128, normalized size = 2.13

$$-\frac{5\,b\cos\left(dx+c\right)^{6}-2\,\left(3\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(5*b*cos(d*x + c)^6 - 2*(3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*
a)*sin(d*x + c))/d
```

Sympy [A] time = 3.76841, size = 83, normalized size = 1.38

$$\begin{cases} \frac{8a\sin^{5}\left(c+dx\right)}{15d}+\frac{4a\sin^{3}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{3d}+\frac{a\sin\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d}-\frac{b\cos^{6}\left(c+dx\right)}{6d} & \text{for}\ d\neq 0 \\ x\left(a+b\sin\left(c\right)\right)\cos^{5}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + a*sin(c + d*x)*cos(c + d*x)**4/d - b*cos(c + d*x)**6/(6*d), Ne(d,
0)), (x*(a + b*sin(c))*cos(c)**5, True))
```

Giac [A] time = 1.0811, size = 119, normalized size = 1.98

$$-\frac{b\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{b\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,b\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{a\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{5\,a\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*b*cos(6*d*x + 6*c)/d - 1/32*b*cos(4*d*x + 4*c)/d - 5/64*b*cos(2*d*x
+ 2*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 5/8*a*si
n(d*x + c)/d
```

3.377 $\int \cos^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^4(c+dx)}{4d}$$

```
[Out] -(b*Cos[c + d*x]^4)/(4*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0318842, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 641}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^4)/(4*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)\left(b^2-x^2\right)\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= -\frac{b\cos^4(c+dx)}{4d} + \frac{a\,\text{Subst}\left(\int \left(b^2-x^2\right)\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= -\frac{b\cos^4(c+dx)}{4d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d}$$

Mathematica [A] time = 0.0126988, size = 44, normalized size = 1.

$$-\frac{a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} - \frac{b\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^4)/(4*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.022, size = 36, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{b\left(\cos\left(dx+c\right)\right)^4}{4} + \frac{a\left(2+\left(\cos\left(dx+c\right)\right)^2\right)\sin\left(dx+c\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/4*b*cos(d*x+c)^4+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.95303, size = 65, normalized size = 1.48

$$-\frac{3\,b\sin\left(dx+c\right)^4 + 4\,a\sin\left(dx+c\right)^3 - 6\,b\sin\left(dx+c\right)^2 - 12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*b*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*b*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

Fricas [A] time = 2.12835, size = 97, normalized size = 2.2

$$-\frac{3\,b\cos\left(dx+c\right)^4-4\left(a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b*cos(d*x + c)^4 - 4*(a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

Sympy [A] time = 1.08351, size = 82, normalized size = 1.86

$$\begin{cases} \frac{2a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)\cos^2(c+dx)}{d} + \frac{b\sin^4(c+dx)}{4d} + \frac{b\sin^2(c+dx)\cos^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a+b\sin(c)\right)\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d + b
*sin(c + d*x)**4/(4*d) + b*sin(c + d*x)**2*cos(c + d*x)**2/(2*d), Ne(d, 0))
, (x*(a + b*sin(c))*cos(c)**3, True))
```

Giac [A] time = 1.07937, size = 65, normalized size = 1.48

$$-\frac{3\,b\sin\left(dx+c\right)^4+4\,a\sin\left(dx+c\right)^3-6\,b\sin\left(dx+c\right)^2-12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*b*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*b*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

3.378 $\int \cos(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\sin(c+dx))^2}{2bd}$$

```
[Out] (a + b*Sin[c + d*x])^2/(2*b*d)
```

Rubi [A] time = 0.0164447, antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2668}

$$\frac{a\sin(c+dx)}{d}+\frac{b\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x])/d + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))\,dx&=\frac{\text{Subst}\left(\int(a+x)\,dx,x,b\sin(c+dx)\right)}{bd}\\&=\frac{a\sin(c+dx)}{d}+\frac{b\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0110288, size = 39, normalized size = 1.77

$$\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}-\frac{b\cos^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^2)/(2*d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.008, size = 25, normalized size = 1.1

$$\frac{1}{d}\left(\frac{(\sin(dx+c))^2 b}{2}+a\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/2*sin(d*x+c)^2*b+a*sin(d*x+c))
```

Maxima [A] time = 0.942197, size = 27, normalized size = 1.23

$$\frac{(b\sin(dx+c)+a)^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*sin(d*x + c) + a)^2/(b*d)
```

Fricas [A] time = 2.04339, size = 62, normalized size = 2.82

$$-\frac{b\cos(dx+c)^2-2\,a\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*cos(d*x + c)^2 - 2*a*sin(d*x + c))/d
```

Sympy [A] time = 0.210159, size = 34, normalized size = 1.55

$$\begin{cases} \frac{a\sin(c+dx)}{d} + \frac{b\sin^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a+b\sin(c))\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)/d + b*sin(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b
*sin(c))*cos(c), True))
```

Giac [A] time = 1.06324, size = 34, normalized size = 1.55

$$\frac{b\sin(dx+c)^2 + 2a\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b*sin(d*x + c)^2 + 2*a*sin(d*x + c))/d
```

3.379 $\int \sec(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{(a-b)\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)\log(1-\sin(c+dx))}{2d}$$

```
[Out] -((a + b)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)*Log[1 + Sin[c + d*x]])/(2
*d)
```

Rubi [A] time = 0.0400481, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2668, 633, 31}

$$\frac{(a-b)\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)\log(1-\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a + b)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)*Log[1 + Sin[c + d*x]])/(2
*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))\,dx = \frac{b\,\text{Subst}\left(\int \frac{a+x}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2d} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{b-x}\,dx, x, b\sin(c$$
$$= -\frac{(a+b)\log(1-\sin(c+dx))}{2d} + \frac{(a-b)\log(1+\sin(c+dx))}{2d}$$

Mathematica [A] time = 0.0108662, size = 26, normalized size = 0.6

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x]),x]

[Out] (a*ArcTanh[Sin[c + d*x]])/d - (b*Log[Cos[c + d*x]])/d
```

Maple [A] time = 0.026, size = 34, normalized size = 0.8

$$-\frac{b\ln(\cos(dx+c))}{d} + \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c)),x)

[Out] -1/d*b*ln(cos(d*x+c))+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.950124, size = 47, normalized size = 1.09

$$\frac{(a-b)\log(\sin(dx+c)+1) - (a+b)\log(\sin(dx+c)-1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((a - b)*log(sin(d*x + c) + 1) - (a + b)*log(sin(d*x + c) - 1))/d
```

Fricas [A] time = 2.29831, size = 97, normalized size = 2.26

$$\frac{(a-b)\log\left(\sin\left(dx+c\right)+1\right)-(a+b)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*log(sin(d*x + c) + 1) - (a + b)*log(-sin(d*x + c) + 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sin\left(c+dx\right))\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x), x)
```

Giac [A] time = 1.10903, size = 50, normalized size = 1.16

$$\frac{(a-b)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-(a+b)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((a - b)*log(abs(sin(d*x + c) + 1)) - (a + b)*log(abs(sin(d*x + c) - 1)
))/d
```

3.380 $\int \sec^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$\frac{\sec^2(c+dx)(a\sin(c+dx)+b)}{2d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(b + a*Sin[c + d*x]))/(2*
d)
```

Rubi [A] time = 0.037927, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2668, 639, 206}

$$\frac{\sec^2(c+dx)(a\sin(c+dx)+b)}{2d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(b + a*Sin[c + d*x]))/(2*
d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{a+x}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))}{2d} + \frac{(ab)\,\text{Subst}\left(\int \frac{1}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{2d}$$

$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))}{2d}$$

Mathematica [A] time = 0.0201198, size = 52, normalized size = 1.27

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d} + \frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^2)/(2*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.031, size = 54, normalized size = 1.3

$$\frac{a\sec(dx+c)\tan(dx+c)}{2d} + \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{b}{2d(\cos(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*b/cos
(d*x+c)^2
```

Maxima [A] time = 0.9563, size = 72, normalized size = 1.76

$$\frac{a\log\left(\sin\left(dx+c\right)+1\right)-a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,\left(a\sin\left(dx+c\right)+b\right)}{\sin\left(dx+c\right)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*(a*sin(d*x + c)
+ b)/(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 2.18825, size = 178, normalized size = 4.34

$$\frac{a\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,a\sin\left(dx+c\right)+2\,b}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*a*sin(d*x + c) + 2*b)/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(c+dx\right)\right)\sec^3\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x)**3, x)
```

Giac [A] time = 1.12968, size = 74, normalized size = 1.8

$$\frac{a\log\left(|\sin\left(dx+c\right)+1|\right)-a\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\,(a\sin(dx+c)+b)}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(a*log(abs(sin(d*x + c) + 1)) - a*log(abs(sin(d*x + c) - 1)) - 2*(a*sin
(d*x + c) + b)/(sin(d*x + c)^2 - 1))/d
```

3.381 $\int \sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{\sec^4(c+dx)(a\sin(c+dx)+b)}{4d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x]))/(
4*d) + (3*a*Sec[c + d*x]*Tan[c + d*x])/(8*d)
```

Rubi [A] time = 0.0438092, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2668, 639, 199, 206}

$$\frac{\sec^4(c+dx)(a\sin(c+dx)+b)}{4d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```


```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x]))/(
4*d) + (3*a*Sec[c + d*x]*Tan[c + d*x])/(8*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{a+x}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))}{4d} + \frac{\left(3ab^3\right)\text{Subst}\left(\int \frac{1}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))}{4d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{(3ab)\,\text{Subst}\left(\int \frac{}{b^2 \cdots}\right.}{}$$

$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c+dx)(b+a\sin(c+dx))}{4d} + \frac{3a\sec(c+dx)\tan(c \cdots}{8d}$$

Mathematica [A] time = 0.161547, size = 68, normalized size = 1.11

$$\frac{a\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{3a\left(\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\sec(c+dx)\right)}{8d} + \frac{b\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(Ar
cTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A] time = 0.033, size = 74, normalized size = 1.2

$$\frac{a\tan(dx+c)(\sec(dx+c))^3}{4d} + \frac{3a\sec(dx+c)\tan(dx+c)}{8d} + \frac{3a\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{b}{4d(\cos(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d*a*tan(d*x+c)*sec(d*x+c)^3+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(se
c(d*x+c)+tan(d*x+c))+1/4/d*b/cos(d*x+c)^4
```

Maxima [A] time = 0.964794, size = 105, normalized size = 1.72

$$\frac{3\,a\log\left(\sin\left(dx+c\right)+1\right)-3\,a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,a\sin(dx+c)^3-5\,a\sin(dx+c)-2\,b\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(3*a*log(sin(d*x + c) + 1) - 3*a*log(sin(d*x + c) - 1) - 2*(3*a*sin(d*
x + c)^3 - 5*a*sin(d*x + c) - 2*b)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))
/d
```

Fricas [A] time = 2.27274, size = 219, normalized size = 3.59

$$\frac{3\,a\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3\,a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx+c\right.}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(3*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(3*a*cos(d*x + c)^2 + 2*a)*sin(d*x + c) + 4*b)/(d*cos(d
*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14051, size = 95, normalized size = 1.56

$$\frac{3\,a\log\left(|\sin\left(dx+c\right)+1|\right)-3\,a\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(3\,a\sin\left(dx+c\right)^{3}-5\,a\sin\left(dx+c\right)-2\,b\right)}{\left(\sin\left(dx+c\right)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(3*a*log(abs(sin(d*x + c) + 1)) - 3*a*log(abs(sin(d*x + c) - 1)) - 2*(
3*a*sin(d*x + c)^3 - 5*a*sin(d*x + c) - 2*b)/(sin(d*x + c)^2 - 1)^2)/d
```

3.382 $\int \cos^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^5(c+dx)}{5d}$$

```
[Out] (3*a*x)/8 - (b*Cos[c + d*x]^5)/(5*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.0449726, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2669, 2635, 8}

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (b*Cos[c + d*x]^5)/(5*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(c+dx)(a+b\sin(c+dx))\,dx &= -\frac{b\cos^5(c+dx)}{5d} + a\int \cos^4(c+dx)\,dx \\
&= -\frac{b\cos^5(c+dx)}{5d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx \\
&= -\frac{b\cos^5(c+dx)}{5d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{8}(3a \\
&= \frac{3ax}{8} - \frac{b\cos^5(c+dx)}{5d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d}
\end{aligned}
$$

Mathematica [A] time = 0.110367, size = 62, normalized size = 0.95

$$\frac{3a(c+dx)}{8d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d} - \frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) - (b*Cos[c + d*x]^5)/(5*d) + (a*Sin[2*(c + d*x)])/(4*
d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.021, size = 52, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{b(\cos(dx+c))^5}{5} + a\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3 + \frac{3\cos(dx+c)}{2}\right) + \frac{3dx}{8} + \frac{3c}{8}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/5*b*cos(d*x+c)^5+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/
8*d*x+3/8*c))
```

Maxima [A] time = 0.963297, size = 65, normalized size = 1.

$$-\frac{32\,b\cos(dx+c)^5 - 5\,(12\,dx + 12\,c + \sin(4\,dx+4\,c) + 8\,\sin(2\,dx+2\,c))a}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/160*(32*b*cos(d*x + c)^5 - 5*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2
*d*x + 2*c))*a)/d
```

Fricas [A] time = 2.25246, size = 132, normalized size = 2.03

$$-\frac{8\,b\cos\left(dx+c\right)^{5}-15\,adx-5\left(2\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/40*(8*b*cos(d*x + c)^5 - 15*a*d*x - 5*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x
+ c))*sin(d*x + c))/d
```

Sympy [A] time = 2.22711, size = 124, normalized size = 1.91

$$\begin{cases}\frac{3ax\sin^{4}\left(c+dx\right)}{8}+\frac{3ax\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{4}+\frac{3ax\cos^{4}\left(c+dx\right)}{8}+\frac{3a\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{8d}+\frac{5a\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{8d}-\frac{b\cos^{5}\left(c+dx\right)}{5d} & \text{for } d \ne \\ x\left(a+b\sin\left(c\right)\right)\cos^{4}\left(c\right) & \text{otherw}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) - b*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x
*(a + b*sin(c))*cos(c)**4, True))
```

Giac [A] time = 1.07678, size = 104, normalized size = 1.6

$$\frac{3}{8}\,ax-\frac{b\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{b\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{b\cos\left(dx+c\right)}{8\,d}+\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x - 1/80*b*cos(5*d*x + 5*c)/d - 1/16*b*cos(3*d*x + 3*c)/d - 1/8*b*cos
(d*x + c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

3.383 $\int \cos^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\cos^3(c+dx)}{3d}$$

```
[Out] (a*x)/2 - (b*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0327304, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2669, 2635, 8}

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/2 - (b*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(c+dx)(a+b\sin(c+dx))\,dx &= -\frac{b\cos^3(c+dx)}{3d}+a\int \cos^2(c+dx)\,dx \\
&= -\frac{b\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx \\
&= \frac{ax}{2}-\frac{b\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.0594055, size = 46, normalized size = 1.07

$$
\frac{a(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}-\frac{b\cos^3(c+dx)}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) - (b*Cos[c + d*x]^3)/(3*d) + (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.02, size = 41, normalized size = 1.

$$
\frac{1}{d}\left(-\frac{b(\cos(dx+c))^3}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-1/3*b*cos(d*x+c)^3+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.964148, size = 50, normalized size = 1.16

$$
-\frac{4\,b\cos(dx+c)^3-3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a}{12\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*b*cos(d*x + c)^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a)/d
```

Fricas [A] time = 2.16566, size = 96, normalized size = 2.23

$$-\frac{2\,b\cos\left(dx+c\right)^{3}-3\,adx-3\,a\cos\left(dx+c\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*cos(d*x + c)^3 - 3*a*d*x - 3*a*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [A] time = 0.609546, size = 71, normalized size = 1.65

$$\begin{cases} \frac{ax\sin^{2}(c+dx)}{2}+\frac{ax\cos^{2}(c+dx)}{2}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}-\frac{b\cos^{3}(c+dx)}{3d} & \text{for}\ d\neq 0\\ x\left(a+b\sin\left(c\right)\right)\cos^{2}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*c
os(c + d*x)/(2*d) - b*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sin(c))*c
os(c)**2, True))
```

Giac [A] time = 1.06991, size = 63, normalized size = 1.47

$$\frac{1}{2}\,ax-\frac{b\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{b\cos\left(dx+c\right)}{4\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x - 1/12*b*cos(3*d*x + 3*c)/d - 1/4*b*cos(d*x + c)/d + 1/4*a*sin(2*d*
x + 2*c)/d
```

3.384 $\int \sec^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=23

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.0311392, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2669, 3767, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \sec^2(c+dx)(a+b\sin(c+dx))\,dx &= \frac{b\sec(c+dx)}{d} + a\int \sec^2(c+dx)\,dx \\
&= \frac{b\sec(c+dx)}{d} - \frac{a\,\mathrm{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d} \\
&= \frac{b\sec(c+dx)}{d} + \frac{a\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.0110951, size = 23, normalized size = 1.

$$
\frac{a\tan(c+dx)}{d} + \frac{b\sec(c+dx)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.028, size = 24, normalized size = 1.

$$
\frac{1}{d}\left(a\tan(dx+c) + \frac{b}{\cos(dx+c)}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*tan(d*x+c)+b/cos(d*x+c))
```

Maxima [A] time = 0.955777, size = 31, normalized size = 1.35

$$
\frac{a\tan(dx+c) + \frac{b}{\cos(dx+c)}}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*tan(d*x + c) + b/cos(d*x + c))/d
```

Fricas [A] time = 2.05694, size = 53, normalized size = 2.3

$$\frac{a\sin\left(dx+c\right)+b}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*sin(d*x + c) + b)/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(c+dx\right)\right)\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x)**2, x)
```

Giac [A] time = 1.09731, size = 45, normalized size = 1.96

$$-\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*(a*tan(1/2*d*x + 1/2*c) + b)/((tan(1/2*d*x + 1/2*c)^2 - 1)*d)
```

3.385 $\int \sec^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}$$

```
[Out] (b*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.035826, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2669, 3767}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))\,dx = \frac{b\sec^3(c+dx)}{3d} + a\int \sec^4(c+dx)\,dx$$
$$= \frac{b\sec^3(c+dx)}{3d} - \frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{d}$$
$$= \frac{b\sec^3(c+dx)}{3d} + \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d}$$

Mathematica [A] time = 0.0793873, size = 41, normalized size = 0.93

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d} + \frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^3)/(3*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.033, size = 38, normalized size = 0.9

$$\frac{1}{d}\left(-a\left(-\frac{2}{3}-\frac{(\sec(dx+c))^2}{3}\right)\tan(dx+c)+\frac{b}{3\,(\cos(dx+c))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/3*b/cos(d*x+c)^3)
```

Maxima [A] time = 0.967181, size = 47, normalized size = 1.07

$$\frac{\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a+\frac{b}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a + b/cos(d*x + c)^3)/d
```

Fricas [A] time = 1.98761, size = 92, normalized size = 2.09

$$\frac{\left(2\,a\cos\left(dx+c\right)^{2}+a\right)\sin\left(dx+c\right)+b}{3\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*((2*a*cos(d*x + c)^2 + a)*sin(d*x + c) + b)/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0921, size = 103, normalized size = 2.34

$$-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/3*(3*a*tan(1/2*d*x + 1/2*c)^5 + 3*b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2
*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c) + b)/((tan(1/2*d*x + 1/2*c)^2 -
1)^3*d)
```

3.386 $\int \sec^6(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^5(c+dx)}{5d}$$

```
[Out] (b*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d)
+ (a*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0400507, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2669, 3767}

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d)
+ (a*Tan[c + d*x]^5)/(5*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))\,dx = \frac{b\sec^5(c+dx)}{5d} + a\int \sec^6(c+dx)\,dx$$
$$= \frac{b\sec^5(c+dx)}{5d} - \frac{a\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(c+dx)\right)}{d}$$
$$= \frac{b\sec^5(c+dx)}{5d} + \frac{a\tan(c+dx)}{d} + \frac{2a\tan^3(c+dx)}{3d} + \frac{a\tan^5(c+dx)}{5d}$$

Mathematica [A] time = 0.179222, size = 53, normalized size = 0.88

$$\frac{a\left(\frac{1}{5}\tan^5(c+dx)+\frac{2}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{b\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^5)/(5*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c
+ d*x]^5/5))/d
```

Maple [A] time = 0.032, size = 48, normalized size = 0.8

$$\frac{1}{d}\left(-a\left(-\frac{8}{15}-\frac{(\sec(dx+c))^4}{5}-\frac{4\,(\sec(dx+c))^2}{15}\right)\tan(dx+c)+\frac{b}{5\,(\cos(dx+c))^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/5*b/cos(d*x
+c)^5)
```

Maxima [A] time = 0.951036, size = 65, normalized size = 1.08

$$\frac{\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a+\frac{3b}{\cos(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a + 3*b/cos(
d*x + c)^5)/d
```

Fricas [A] time = 2.09528, size = 127, normalized size = 2.12

$$\frac{\left(8\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+3\,a\right)\sin\left(dx+c\right)+3\,b}{15\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*((8*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 3*a)*sin(d*x + c) + 3*b)/(
d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10191, size = 162, normalized size = 2.7

$$-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-20\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+58\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+30\,b\tan\left(\frac{1}{2}\,dx+\right.\right.}{15\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/15*(15*a*tan(1/2*d*x + 1/2*c)^9 + 15*b*tan(1/2*d*x + 1/2*c)^8 - 20*a*tan
(1/2*d*x + 1/2*c)^7 + 58*a*tan(1/2*d*x + 1/2*c)^5 + 30*b*tan(1/2*d*x + 1/2*
c)^4 - 20*a*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*d*x + 1/2*c) + 3*b)/((tan
(1/2*d*x + 1/2*c)^2 - 1)^5*d)
```

3.387 $\int \cos^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=99

$$\frac{(a^2-2b^2)\sin^5(c+dx)}{5d}-\frac{(2a^2-b^2)\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{ab\cos^6(c+dx)}{3d}+\frac{b^2\sin^7(c+dx)}{7d}$$

```
[Out] -(a*b*Cos[c + d*x]^6)/(3*d) + (a^2*Sin[c + d*x])/d - ((2*a^2 - b^2)*Sin[c +
 d*x]^3)/(3*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^5)/(5*d) + (b^2*Sin[c + d*x]^7
)/(7*d)
```

Rubi [A] time = 0.089251, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2668, 696, 1810}

$$\frac{(a^2-2b^2)\sin^5(c+dx)}{5d}-\frac{(2a^2-b^2)\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{ab\cos^6(c+dx)}{3d}+\frac{b^2\sin^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a*b*Cos[c + d*x]^6)/(3*d) + (a^2*Sin[c + d*x])/d - ((2*a^2 - b^2)*Sin[c +
 d*x]^3)/(3*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^5)/(5*d) + (b^2*Sin[c + d*x]^7
)/(7*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 696

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*m*d^(m - 1)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= -\frac{ab\cos^6(c+dx)}{3d} + \frac{\text{Subst}\left(\int \left(b^2-x^2\right)^2\left(-2ax+(a+x)^2\right)\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= -\frac{ab\cos^6(c+dx)}{3d} + \frac{\text{Subst}\left(\int \left(a^2b^4+b^2\left(-2a^2+b^2\right)x^2+\left(a^2-2b^2\right)x^4+x^6\right)\,dx\right.}{b^5 d}$$
$$= -\frac{ab\cos^6(c+dx)}{3d} + \frac{a^2\sin(c+dx)}{d} - \frac{\left(2a^2-b^2\right)\sin^3(c+dx)}{3d} + \frac{\left(a^2-2b^2\right)\sin^5(c}{5d}$$

Mathematica [A] time = 0.204956, size = 104, normalized size = 1.05

$$\frac{\sin(c+dx)\left(21\left(a^2-2b^2\right)\sin^4(c+dx)+35\left(b^2-2a^2\right)\sin^2(c+dx)+105a^2+35ab\sin^5(c+dx)-105ab\sin^3(c+dx)+1\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]*(105*a^2 + 105*a*b*Sin[c + d*x] + 35*(-2*a^2 + b^2)*Sin[c + d
*x]^2 - 105*a*b*Sin[c + d*x]^3 + 21*(a^2 - 2*b^2)*Sin[c + d*x]^4 + 35*a*b*S
in[c + d*x]^5 + 15*b^2*Sin[c + d*x]^6))/(105*d)
```

Maple [A] time = 0.043, size = 98, normalized size = 1.

$$\frac{1}{d}\left(b^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^6}{7}+\frac{\sin(dx+c)}{35}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(dx+c))^2}{3}\right)\right)-\frac{ab\,(\cos(dx+c))^6}{3}+\frac{a^2\,\mathrm{si}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-1/3*a*b*cos(d*x+c)^6+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.960065, size = 143, normalized size = 1.44

$$\frac{15\,b^2\sin\left(dx+c\right)^7+35\,ab\sin\left(dx+c\right)^6-105\,ab\sin\left(dx+c\right)^4+21\left(a^2-2\,b^2\right)\sin\left(dx+c\right)^5+105\,ab\sin\left(dx+c\right)^2-35\,(\cdots}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*sin(d*x + c)^7 + 35*a*b*sin(d*x + c)^6 - 105*a*b*sin(d*x + c)
^4 + 21*(a^2 - 2*b^2)*sin(d*x + c)^5 + 105*a*b*sin(d*x + c)^2 - 35*(2*a^2 -
 b^2)*sin(d*x + c)^3 + 105*a^2*sin(d*x + c))/d
```

Fricas [A] time = 2.23771, size = 211, normalized size = 2.13

$$-\frac{35\,ab\cos\left(dx+c\right)^6+\left(15\,b^2\cos\left(dx+c\right)^6-3\left(7\,a^2+b^2\right)\cos\left(dx+c\right)^4-4\left(7\,a^2+b^2\right)\cos\left(dx+c\right)^2-56\,a^2-8\,b^2\right)\sin\left(\cdots\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(35*a*b*cos(d*x + c)^6 + (15*b^2*cos(d*x + c)^6 - 3*(7*a^2 + b^2)*co
s(d*x + c)^4 - 4*(7*a^2 + b^2)*cos(d*x + c)^2 - 56*a^2 - 8*b^2)*sin(d*x + c
))/d
```

Sympy [A] time = 8.25853, size = 202, normalized size = 2.04

$$\begin{cases}\frac{8a^2\sin^5(c+dx)}{15d}+\frac{4a^2\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{a^2\sin(c+dx)\cos^4(c+dx)}{d}+\frac{ab\sin^6(c+dx)}{3d}+\frac{ab\sin^4(c+dx)\cos^2(c+dx)}{d}+\frac{ab\sin^2(c+dx)\cos^4(c+dx)}{d} \cdots \\ x\left(a+b\sin\left(c\right)\right)^2\cos^5\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**5/(15*d) + 4*a**2*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**4/d + a*b*sin(c + d*x)**6/(3
*d) + a*b*sin(c + d*x)**4*cos(c + d*x)**2/d + a*b*sin(c + d*x)**2*cos(c + d
*x)**4/d + 8*b**2*sin(c + d*x)**7/(105*d) + 4*b**2*sin(c + d*x)**5*cos(c +
d*x)**2/(15*d) + b**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d), Ne(d, 0)), (x*
(a + b*sin(c))**2*cos(c)**5, True))
```

Giac [A] time = 1.08869, size = 184, normalized size = 1.86

$$-\frac{ab\cos(6\,dx+6\,c)}{96\,d}-\frac{ab\cos(4\,dx+4\,c)}{16\,d}-\frac{5\,ab\cos(2\,dx+2\,c)}{32\,d}-\frac{b^2\sin(7\,dx+7\,c)}{448\,d}+\frac{\left(4\,a^2-3\,b^2\right)\sin(5\,dx+5\,c)}{320\,d}+\frac{(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*a*b*cos(6*d*x + 6*c)/d - 1/16*a*b*cos(4*d*x + 4*c)/d - 5/32*a*b*cos(2
*d*x + 2*c)/d - 1/448*b^2*sin(7*d*x + 7*c)/d + 1/320*(4*a^2 - 3*b^2)*sin(5*
d*x + 5*c)/d + 1/192*(20*a^2 - b^2)*sin(3*d*x + 3*c)/d + 5/64*(8*a^2 + b^2)
*sin(d*x + c)/d
```

3.388 $\int \cos^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=77

$$-\frac{\left(a^2-b^2\right)(a+b\sin(c+dx))^3}{3b^3d}-\frac{(a+b\sin(c+dx))^5}{5b^3d}+\frac{a(a+b\sin(c+dx))^4}{2b^3d}$$

```
[Out] -((a^2 - b^2)*(a + b*Sin[c + d*x])^3)/(3*b^3*d) + (a*(a + b*Sin[c + d*x])^4
)/(2*b^3*d) - (a + b*Sin[c + d*x])^5/(5*b^3*d)
```

Rubi [A] time = 0.0707126, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{\left(a^2-b^2\right)(a+b\sin(c+dx))^3}{3b^3d}-\frac{(a+b\sin(c+dx))^5}{5b^3d}+\frac{a(a+b\sin(c+dx))^4}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2 - b^2)*(a + b*Sin[c + d*x])^3)/(3*b^3*d) + (a*(a + b*Sin[c + d*x])^4
)/(2*b^3*d) - (a + b*Sin[c + d*x])^5/(5*b^3*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(b^2-x^2\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^2+2a(a+x)^3-(a+x)^4\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= -\frac{\left(a^2-b^2\right)(a+b\sin(c+dx))^3}{3b^3d}+\frac{a(a+b\sin(c+dx))^4}{2b^3d}-\frac{(a+b\sin(c+dx))^5}{5b^3d}$$

Mathematica [A] time = 0.11586, size = 56, normalized size = 0.73

$$\frac{(a+b\sin(c+dx))^3\left(-a^2+3ab\sin(c+dx)+3b^2\cos(2(c+dx))+7b^2\right)}{30b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a + b*Sin[c + d*x])^3*(-a^2 + 7*b^2 + 3*b^2*Cos[2*(c + d*x)] + 3*a*b*Sin[
c + d*x]))/(30*b^3*d)
```

Maple [A] time = 0.042, size = 78, normalized size = 1.

$$\frac{1}{d}\left(b^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^4}{5}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{15}\right)-\frac{ab(\cos(dx+c))^4}{2}+\frac{a^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/
2*a*b*cos(d*x+c)^4+1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.961635, size = 99, normalized size = 1.29

$$-\frac{6\,b^2\sin(dx+c)^5+15\,ab\sin(dx+c)^4-30\,ab\sin(dx+c)^2+10\left(a^2-b^2\right)\sin(dx+c)^3-30\,a^2\sin(dx+c)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(6*b^2*sin(d*x + c)^5 + 15*a*b*sin(d*x + c)^4 - 30*a*b*sin(d*x + c)^2
 + 10*(a^2 - b^2)*sin(d*x + c)^3 - 30*a^2*sin(d*x + c))/d
```

Fricas [A] time = 2.2199, size = 163, normalized size = 2.12

$$-\frac{15\,ab\cos\left(dx+c\right)^{4}+2\left(3\,b^{2}\cos\left(dx+c\right)^{4}-\left(5\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{2}-10\,a^{2}-2\,b^{2}\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(15*a*b*cos(d*x + c)^4 + 2*(3*b^2*cos(d*x + c)^4 - (5*a^2 + b^2)*cos(
d*x + c)^2 - 10*a^2 - 2*b^2)*sin(d*x + c))/d
```

Sympy [A] time = 2.5956, size = 129, normalized size = 1.68

$$\begin{cases} \frac{2a^{2}\sin^{3}(c+dx)}{3d}+\frac{a^{2}\sin(c+dx)\cos^{2}(c+dx)}{d}+\frac{ab\sin^{4}(c+dx)}{2d}+\frac{ab\sin^{2}(c+dx)\cos^{2}(c+dx)}{d}+\frac{2b^{2}\sin^{5}(c+dx)}{15d}+\frac{b^{2}\sin^{3}(c+dx)\cos^{2}(c+dx)}{3d} & \text{for } d \neq \\ x\left(a+b\sin(c)\right)^{2}\cos^{3}(c) & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**2
/d + a*b*sin(c + d*x)**4/(2*d) + a*b*sin(c + d*x)**2*cos(c + d*x)**2/d + 2*
b**2*sin(c + d*x)**5/(15*d) + b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d), N
e(d, 0)), (x*(a + b*sin(c))**2*cos(c)**3, True))
```

Giac [A] time = 1.08543, size = 108, normalized size = 1.4

$$-\frac{6\,b^{2}\sin\left(dx+c\right)^{5}+15\,ab\sin\left(dx+c\right)^{4}+10\,a^{2}\sin\left(dx+c\right)^{3}-10\,b^{2}\sin\left(dx+c\right)^{3}-30\,ab\sin\left(dx+c\right)^{2}-30\,a^{2}\sin\left(dx+\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(6*b^2*sin(d*x + c)^5 + 15*a*b*sin(d*x + c)^4 + 10*a^2*sin(d*x + c)^3
 - 10*b^2*sin(d*x + c)^3 - 30*a*b*sin(d*x + c)^2 - 30*a^2*sin(d*x + c))/d
```

3.389 $\int \cos(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\sin(c+dx))^3}{3bd}$$

```
[Out] (a + b*Sin[c + d*x])^3/(3*b*d)
```

Rubi [A] time = 0.0269699, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 32}

$$\frac{(a+b\sin(c+dx))^3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a + b*Sin[c + d*x])^3/(3*b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{(a+b\sin(c+dx))^3}{3bd}\end{aligned}$$

Mathematica [B] time = 0.0135242, size = 46, normalized size = 2.09

$$\frac{a^2\sin(c+dx)}{d}+\frac{ab\sin^2(c+dx)}{d}+\frac{b^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d + (a*b*Sin[c + d*x]^2)/d + (b^2*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.015, size = 21, normalized size = 1.

$$\frac{(a+b\sin(dx+c))^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/3*(a+b*sin(d*x+c))^3/b/d
```

Maxima [A] time = 0.940937, size = 27, normalized size = 1.23

$$\frac{(b\sin(dx+c)+a)^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*sin(d*x + c) + a)^3/(b*d)
```

Fricas [B] time = 2.1854, size = 109, normalized size = 4.95

$$-\frac{3\,ab\cos(dx+c)^2+\left(b^2\cos(dx+c)^2-3\,a^2-b^2\right)\sin(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*b*cos(d*x + c)^2 + (b^2*cos(d*x + c)^2 - 3*a^2 - b^2)*sin(d*x + c
))/d
```

Sympy [A] time = 0.5765, size = 53, normalized size = 2.41

$$\begin{cases} \frac{a^2 \sin(c+dx)}{d} + \frac{ab \sin^2(c+dx)}{d} + \frac{b^2 \sin^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x (a + b \sin(c))^2 \cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)/d + a*b*sin(c + d*x)**2/d + b**2*sin(c + d*x)*
*3/(3*d), Ne(d, 0)), (x*(a + b*sin(c))**2*cos(c), True))
```

Giac [A] time = 1.08004, size = 27, normalized size = 1.23

$$\frac{(b \sin(dx + c) + a)^3}{3\, bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(b*sin(d*x + c) + a)^3/(b*d)
```

3.390 $\int \sec(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=61

$$\frac{(a-b)^2\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}-\frac{b^2\sin(c+dx)}{d}$$

```
[Out] -((a + b)^2*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)^2*Log[1 + Sin[c + d*x]]
)/(2*d) - (b^2*Sin[c + d*x])/d
```

Rubi [A] time = 0.0818299, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2668, 702, 633, 31}

$$\frac{(a-b)^2\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}-\frac{b^2\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a + b)^2*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)^2*Log[1 + Sin[c + d*x]]
)/(2*d) - (b^2*Sin[c + d*x])/d
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{b\,\mathrm{Subst}\left(\int \frac{(a+x)^2}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}\\
&= \frac{b\,\mathrm{Subst}\left(\int \left(-1+\frac{a^2+b^2+2ax}{b^2-x^2}\right)dx, x, b\sin(c+dx)\right)}{d}\\
&= -\frac{b^2\sin(c+dx)}{d}+\frac{b\,\mathrm{Subst}\left(\int \frac{a^2+b^2+2ax}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}\\
&= -\frac{b^2\sin(c+dx)}{d}-\frac{(a-b)^2\,\mathrm{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2d}+\frac{(a+b)^2\,\mathrm{Subst}}{}\\
&= -\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}+\frac{(a-b)^2\log(1+\sin(c+dx))}{2d}-\frac{b^2\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0625328, size = 54, normalized size = 0.89

$$\frac{(a-b)^2\log(\sin(c+dx)+1)-(a+b)^2\log(1-\sin(c+dx))-2b^2\sin(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-((a + b)^2*Log[1 - Sin[c + d*x]]) + (a - b)^2*Log[1 + Sin[c + d*x]] - 2*b
^2*Sin[c + d*x])/(2*d)
```

Maple [A] time = 0.043, size = 72, normalized size = 1.2

$$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{b^2\sin(dx+c)}{d}-2\,\frac{ab\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^2*ln(sec(d*x+c)+tan(d*x+c))-b^2*sin
(d*x+c)/d-2/d*a*b*ln(cos(d*x+c))
```

Maxima [A] time = 0.962998, size = 81, normalized size = 1.33

$$-\frac{2\,b^2\sin\left(dx+c\right)-\left(a^2-2\,ab+b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a^2+2\,ab+b^2\right)\log\left(\sin\left(dx+c\right)-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(sin(d*x + c) + 1) + (a^2
 + 2*a*b + b^2)*log(sin(d*x + c) - 1))/d
```

Fricas [A] time = 2.27886, size = 159, normalized size = 2.61

$$-\frac{2\,b^2\sin\left(dx+c\right)-\left(a^2-2\,ab+b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a^2+2\,ab+b^2\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(sin(d*x + c) + 1) + (a^2
 + 2*a*b + b^2)*log(-sin(d*x + c) + 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(c+dx\right)\right)^2\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sec(c + d*x), x)
```

Giac [A] time = 1.12927, size = 84, normalized size = 1.38

$$-\frac{2 b^2 \sin (dx + c) - \left(a^2 - 2 ab + b^2\right) \log (|\sin (dx + c) + 1|) + \left(a^2 + 2 ab + b^2\right) \log (|\sin (dx + c) - 1|)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(abs(sin(d*x + c) + 1)) +
 (a^2 + 2*a*b + b^2)*log(abs(sin(d*x + c) - 1)))/d
```

3.391 $\int \sec^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=59

$$\frac{(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{2d}$$

```
[Out] ((a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(b + a*Sin[c +
d*x])*(a + b*Sin[c + d*x]))/(2*d)
```

Rubi [A] time = 0.0612642, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2668, 723, 206}

$$\frac{(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(b + a*Sin[c +
d*x])*(a + b*Sin[c + d*x]))/(2*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{(a+x)^2}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{2d} + \frac{\left(b\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{b^2-\ldots}\right.}{2d}$$

$$= \frac{\left(a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+\ldots}{2d}$$

Mathematica [A] time = 0.942801, size = 113, normalized size = 1.92

$$\frac{\left(4ab^3-6a^3b\right)\tan^2(c+dx)+\left(a^2-b^2\right)^2(\log(1-\sin(c+dx))-\log(\sin(c+dx)+1))-2\left(a^4-b^4\right)\tan(c+dx)\sec(c+\ldots}{4d\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 - b^2)^2*(Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + 2*a^3*b*Se
c[c + d*x]^2 - 2*(a^4 - b^4)*Sec[c + d*x]*Tan[c + d*x] + (-6*a^3*b + 4*a*b^
3)*Tan[c + d*x]^2)/(4*(-a^2 + b^2)*d)
```

Maple [B] time = 0.055, size = 118, normalized size = 2.

$$\frac{a^2\sec(dx+c)\tan(dx+c)}{2d} + \frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{ab}{d(\cos(dx+c))^2} + \frac{b^2(\sin(dx+c))^3}{2d(\cos(dx+c))^2} + \frac{b^2\sin(dx+\ldots}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d*a^2*sec(d*x+c)*tan(d*x+c)+1/2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*b
/cos(d*x+c)^2+1/2/d*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*b^2*sin(d*x+c)/d-1/2/
d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.956641, size = 105, normalized size = 1.78

$$\frac{\left(a^2-b^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2-b^2\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(2\,ab+\left(a^2+b^2\right)\sin(dx+c)\right)}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((a^2 - b^2)*log(sin(d*x + c) + 1) - (a^2 - b^2)*log(sin(d*x + c) - 1)
- 2*(2*a*b + (a^2 + b^2)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 2.23095, size = 221, normalized size = 3.75

$$\frac{\left(a^2-b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2-b^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+4\,ab+2\left(a^2+b^2\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (a^2 - b^2)*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 4*a*b + 2*(a^2 + b^2)*sin(d*x + c))/(d*cos
(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15026, size = 116, normalized size = 1.97

$$\frac{\left(a^2-b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(a^2-b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(a^2\sin(dx+c)+b^2\sin(dx+c)+2\,ab\right)}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*((a^2 - b^2)*log(abs(sin(d*x + c) + 1)) - (a^2 - b^2)*log(abs(sin(d*x +
 c) - 1)) - 2*(a^2*sin(d*x + c) + b^2*sin(d*x + c) + 2*a*b)/(sin(d*x + c)^2
 - 1))/d
```

3.392 $\int \sec^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=99

$$\frac{(3a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^2(c+dx)\left((3a^2-b^2)\sin(c+dx)+2ab\right)}{8d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+b)(a+b\sin}{4d}$$

```
[Out] ((3*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(b + a*Sin[c
+ d*x])*(a + b*Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^2*(2*a*b + (3*a^2 - b^2
)*Sin[c + d*x]))/(8*d)
```

Rubi [A] time = 0.0878322, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2668, 739, 639, 206}

$$\frac{(3a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^2(c+dx)\left((3a^2-b^2)\sin(c+dx)+2ab\right)}{8d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+b)(a+b\sin}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((3*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(b + a*Sin[c
+ d*x])*(a + b*Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^2*(2*a*b + (3*a^2 - b^2
)*Sin[c + d*x]))/(8*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{(a+x)^2}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{4d} - \frac{b^3\,\text{Subst}\left(\int \frac{-3a^2+b^2-2ax}{\left(b^2-x^2\right)^2}\,dx \right.}{4d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{4d} + \frac{\sec^2(c+dx)\left(2ab+\left(3a^2-\right.\right.}{8d}$$

$$= \frac{\left(3a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+\right.}{4d}$$

Mathematica [A] time = 0.731291, size = 166, normalized size = 1.68

$$\frac{4\left(b^2-a^2\right)\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^3+\left(b^2-3a^2\right)\left(\left(4ab^3-6a^3b\right)\tan^2(c+dx)+\left(a^2-b^2\right)^2(\log\right.}{16d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4*(-a^2 + b^2)*Sec[c + d*x]^4*(b - a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3
+ (-3*a^2 + b^2)*((a^2 - b^2)^2*(Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*
x]]) + 2*a^3*b*Sec[c + d*x]^2 - 2*(a^4 - b^4)*Sec[c + d*x]*Tan[c + d*x] + (
-6*a^3*b + 4*a*b^3)*Tan[c + d*x]^2))/(16*(a^2 - b^2)^2*d)
```

Maple [A] time = 0.058, size = 165, normalized size = 1.7

$$\frac{a^2\tan(dx+c)(\sec(dx+c))^3}{4d}+\frac{3a^2\sec(dx+c)\tan(dx+c)}{8d}+\frac{3a^2\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{ab}{2d(\cos(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4/d*a^2*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^2*sec(d*x+c)*tan(d*x+c)+3/8/d*a^2
*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*b/cos(d*x+c)^4+1/4/d*b^2*sin(d*x+c)^3/co
s(d*x+c)^4+1/8/d*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/8*b^2*sin(d*x+c)/d-1/8/d*b
^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.968193, size = 155, normalized size = 1.57

$$\frac{(3a^2-b^2)\log(\sin(dx+c)+1)-(3a^2-b^2)\log(\sin(dx+c)-1)-\frac{2\left((3a^2-b^2)\sin(dx+c)^3-4ab-(5a^2+b^2)\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/16*((3*a^2 - b^2)*log(sin(d*x + c) + 1) - (3*a^2 - b^2)*log(sin(d*x + c)
- 1) - 2*((3*a^2 - b^2)*sin(d*x + c)^3 - 4*a*b - (5*a^2 + b^2)*sin(d*x + c)
)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.3231, size = 275, normalized size = 2.78

$$\frac{(3a^2-b^2)\cos(dx+c)^4\log(\sin(dx+c)+1)-(3a^2-b^2)\cos(dx+c)^4\log(-\sin(dx+c)+1)+8ab+2\left((3a^2-b^2)\cos\right.}{16d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*((3*a^2 - b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*a^2 - b^2)*co
s(d*x + c)^4*log(-sin(d*x + c) + 1) + 8*a*b + 2*((3*a^2 - b^2)*cos(d*x + c)
^2 + 2*a^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14714, size = 159, normalized size = 1.61

$$\frac{\left(3\,a^2-b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(3\,a^2-b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(3\,a^2\sin(dx+c)^3-b^2\sin(dx+c)^3-5\,a^2\sin(dx+c)-b^2\sin(dx+c)-\right.}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*((3*a^2 - b^2)*log(abs(sin(d*x + c) + 1)) - (3*a^2 - b^2)*log(abs(sin(
d*x + c) - 1)) - 2*(3*a^2*sin(d*x + c)^3 - b^2*sin(d*x + c)^3 - 5*a^2*sin(d
*x + c) - b^2*sin(d*x + c) - 4*a*b)/(sin(d*x + c)^2 - 1)^2)/d
```

3.393 $\int \cos^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=146

$$\frac{(8a^2+b^2)\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5(8a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5(8a^2+b^2)\sin(c+dx)\cos(c+dx)}{128d}+\frac{5}{128}$$

```
[Out] (5*(8*a^2 + b^2)*x)/128 - (9*a*b*Cos[c + d*x]^7)/(56*d) + (5*(8*a^2 + b^2)*
Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*(8*a^2 + b^2)*Cos[c + d*x]^3*Sin[c
+ d*x])/(192*d) + ((8*a^2 + b^2)*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (b*C
os[c + d*x]^7*(a + b*Sin[c + d*x]))/(8*d)
```

Rubi [A] time = 0.133561, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2692, 2669, 2635, 8}

$$\frac{(8a^2+b^2)\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5(8a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5(8a^2+b^2)\sin(c+dx)\cos(c+dx)}{128d}+\frac{5}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (5*(8*a^2 + b^2)*x)/128 - (9*a*b*Cos[c + d*x]^7)/(56*d) + (5*(8*a^2 + b^2)*
Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*(8*a^2 + b^2)*Cos[c + d*x]^3*Sin[c
+ d*x])/(192*d) + ((8*a^2 + b^2)*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (b*C
os[c + d*x]^7*(a + b*Sin[c + d*x]))/(8*d)
```


Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+b\sin(c+dx))^2\,dx &= -\frac{b\cos^7(c+dx)(a+b\sin(c+dx))}{8d}+\frac{1}{8}\int \cos^6(c+dx)\left(8a^2+b^2+9ab\sin(c+\right.\\ &= -\frac{9ab\cos^7(c+dx)}{56d}-\frac{b\cos^7(c+dx)(a+b\sin(c+dx))}{8d}+\frac{1}{8}\left(8a^2+b^2\right)\int \cos^6(c\\ &= -\frac{9ab\cos^7(c+dx)}{56d}+\frac{\left(8a^2+b^2\right)\cos^5(c+dx)\sin(c+dx)}{48d}-\frac{b\cos^7(c+dx)(a+}{8d}\\ &= -\frac{9ab\cos^7(c+dx)}{56d}+\frac{5\left(8a^2+b^2\right)\cos^3(c+dx)\sin(c+dx)}{192d}+\frac{\left(8a^2+b^2\right)\cos^5(c}{48}\\ &= -\frac{9ab\cos^7(c+dx)}{56d}+\frac{5\left(8a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{128d}+\frac{5\left(8a^2+b^2\right)\cos^3(c}{19}\\ &= \frac{5}{128}\left(8a^2+b^2\right)x-\frac{9ab\cos^7(c+dx)}{56d}+\frac{5\left(8a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{128d}+\end{aligned}$$

Mathematica [A] time = 0.374819, size = 141, normalized size = 0.97

$$\frac{840\left(8a^2+b^2\right)(c+dx)+336\left(15a^2+b^2\right)\sin(2(c+dx))+168\left(6a^2-b^2\right)\sin(4(c+dx))+112(a-b)(a+b)\sin(6(c+d}{2150}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (840*(8*a^2 + b^2)*(c + d*x) - 3360*a*b*Cos[c + d*x] - 2016*a*b*Cos[3*(c +
d*x)] - 672*a*b*Cos[5*(c + d*x)] - 96*a*b*Cos[7*(c + d*x)] + 336*(15*a^2 +
b^2)*Sin[2*(c + d*x)] + 168*(6*a^2 - b^2)*Sin[4*(c + d*x)] + 112*(a - b)*(a
```

```
+ b)*Sin[6*(c + d*x)] - 21*b^2*Sin[8*(c + d*x)])/(21504*d)
```

Maple [A] time = 0.043, size = 128, normalized size = 0.9

$$\frac{1}{d}\left(b^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^7}{8}+\frac{\sin(dx+c)}{48}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{128}+\frac{5c}{128}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-2/7*a*b*cos(d*x+c)^7+a^2*(1/
6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*
c))
```

Maxima [A] time = 0.975867, size = 154, normalized size = 1.05

$$-\frac{6144\,ab\cos(dx+c)^7+112\left(4\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\sin(4\,dx+4\,c)-48\sin(2\,dx+2\,c)\right)a^2-7\left(64\sin(2\right.}{21504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/21504*(6144*a*b*cos(d*x + c)^7 + 112*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60
*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 - 7*(64*sin(2*d*x + 2*c)
^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*b^2)/d
```

Fricas [A] time = 2.46034, size = 270, normalized size = 1.85

$$-\frac{768\,ab\cos(dx+c)^7-105\left(8\,a^2+b^2\right)dx+7\left(48\,b^2\cos(dx+c)^7-8\left(8\,a^2+b^2\right)\cos(dx+c)^5-10\left(8\,a^2+b^2\right)\cos(dx+c\right.}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2688*(768*a*b*cos(d*x + c)^7 - 105*(8*a^2 + b^2)*d*x + 7*(48*b^2*cos(d*x
 + c)^7 - 8*(8*a^2 + b^2)*cos(d*x + c)^5 - 10*(8*a^2 + b^2)*cos(d*x + c)^3
- 15*(8*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 13.7406, size = 398, normalized size = 2.73

$$\begin{cases} \frac{5a^2x\sin^6(c+dx)}{16} + \frac{15a^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15a^2x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5a^2x\cos^6(c+dx)}{16} + \frac{5a^2\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{5a^2\sin^3(c+dx)}{6d} \\ x(a+b\sin(c))^2\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**2*x*cos(c
+ d*x)**6/16 + 5*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**2*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - 2*a*b*cos(c + d*x)**7/(7*d) + 5*b**2*x*sin(c + d*x)**8/128 + 5*b**2*x*si
n(c + d*x)**6*cos(c + d*x)**2/32 + 15*b**2*x*sin(c + d*x)**4*cos(c + d*x)**
4/64 + 5*b**2*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 5*b**2*x*cos(c + d*x)*
*8/128 + 5*b**2*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*b**2*sin(c + d*x)
**5*cos(c + d*x)**3/(384*d) + 73*b**2*sin(c + d*x)**3*cos(c + d*x)**5/(384*
d) - 5*b**2*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sin(
c))**2*cos(c)**6, True))
```

Giac [A] time = 1.10233, size = 219, normalized size = 1.5

$$\frac{5}{128}\left(8a^2+b^2\right)x - \frac{ab\cos(7dx+7c)}{224d} - \frac{ab\cos(5dx+5c)}{32d} - \frac{3ab\cos(3dx+3c)}{32d} - \frac{5ab\cos(dx+c)}{32d} - \frac{b^2\sin(8dx+8}{1024d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/128*(8*a^2 + b^2)*x - 1/224*a*b*cos(7*d*x + 7*c)/d - 1/32*a*b*cos(5*d*x +
 5*c)/d - 3/32*a*b*cos(3*d*x + 3*c)/d - 5/32*a*b*cos(d*x + c)/d - 1/1024*b^
2*sin(8*d*x + 8*c)/d + 1/192*(a^2 - b^2)*sin(6*d*x + 6*c)/d + 1/128*(6*a^2
- b^2)*sin(4*d*x + 4*c)/d + 1/64*(15*a^2 + b^2)*sin(2*d*x + 2*c)/d
```

3.394 $\int \cos^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{(6a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(6a^2+b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(6a^2+b^2\right)-\frac{7ab\cos^5(c+dx)}{30d}-\frac{b\cos^5(c}{}$$

```
[Out] ((6*a^2 + b^2)*x)/16 - (7*a*b*Cos[c + d*x]^5)/(30*d) + ((6*a^2 + b^2)*Cos[c
 + d*x]*Sin[c + d*x])/(16*d) + ((6*a^2 + b^2)*Cos[c + d*x]^3*Sin[c + d*x])/
(24*d) - (b*Cos[c + d*x]^5*(a + b*Sin[c + d*x]))/(6*d)
```

Rubi [A] time = 0.115815, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2692, 2669, 2635, 8}

$$\frac{(6a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(6a^2+b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(6a^2+b^2\right)-\frac{7ab\cos^5(c+dx)}{30d}-\frac{b\cos^5(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((6*a^2 + b^2)*x)/16 - (7*a*b*Cos[c + d*x]^5)/(30*d) + ((6*a^2 + b^2)*Cos[c
 + d*x]*Sin[c + d*x])/(16*d) + ((6*a^2 + b^2)*Cos[c + d*x]^3*Sin[c + d*x])/
(24*d) - (b*Cos[c + d*x]^5*(a + b*Sin[c + d*x]))/(6*d)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+b\sin(c+dx))^2\,dx &= -\frac{b\cos^5(c+dx)(a+b\sin(c+dx))}{6d}+\frac{1}{6}\int \cos^4(c+dx)\left(6a^2+b^2+7ab\sin(c+\right.\\ &= -\frac{7ab\cos^5(c+dx)}{30d}-\frac{b\cos^5(c+dx)(a+b\sin(c+dx))}{6d}+\frac{1}{6}\left(6a^2+b^2\right)\int \cos^4(c\\ &= -\frac{7ab\cos^5(c+dx)}{30d}+\frac{\left(6a^2+b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{b\cos^5(c+dx)(a+}{6d}\\ &= -\frac{7ab\cos^5(c+dx)}{30d}+\frac{\left(6a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{16d}+\frac{\left(6a^2+b^2\right)\cos^3(c+}{24d}\\ &= \frac{1}{16}\left(6a^2+b^2\right)x-\frac{7ab\cos^5(c+dx)}{30d}+\frac{\left(6a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{16d}+\frac{(6a}{}\end{aligned}$$

Mathematica [A] time = 0.193004, size = 133, normalized size = 1.15

$$\frac{240a^2\sin(2(c+dx))+30a^2\sin(4(c+dx))+360a^2c+360a^2dx-240ab\cos(c+dx)-120ab\cos(3(c+dx))-24ab\cos(}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (360*a^2*c + 60*b^2*c + 360*a^2*d*x + 60*b^2*d*x - 240*a*b*Cos[c + d*x] - 1
20*a*b*Cos[3*(c + d*x)] - 24*a*b*Cos[5*(c + d*x)] + 240*a^2*Sin[2*(c + d*x)
] + 15*b^2*Sin[2*(c + d*x)] + 30*a^2*Sin[4*(c + d*x)] - 15*b^2*Sin[4*(c + d
*x)] - 5*b^2*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.043, size = 108, normalized size = 0.9

$$\frac{1}{d}\left(b^{2}\left(-\frac{\sin(dx+c)(\cos(dx+c))^{5}}{6}+\frac{\sin(dx+c)}{24}\left((\cos(dx+c))^{3}+\frac{3\cos(dx+c)}{2}\right)+\frac{dx}{16}+\frac{c}{16}\right)-\frac{2ab(\cos(dx+c))^{5}}{5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)-2/5*a*b*cos(d*x+c)^5+a^2*(1/4*(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 0.962841, size = 119, normalized size = 1.03

$$-\frac{384\,ab\cos(dx+c)^{5}-30\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^{2}-5\left(4\,\sin(2\,dx+2\,c)^{3}+12\,dx+12\,c-\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/960*(384*a*b*cos(d*x + c)^5 - 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*s
in(2*d*x + 2*c))*a^2 - 5*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*
x + 4*c))*b^2)/d
```

Fricas [A] time = 2.30337, size = 217, normalized size = 1.87

$$-\frac{96\,ab\cos(dx+c)^{5}-15\left(6\,a^{2}+b^{2}\right)dx+5\left(8\,b^{2}\cos(dx+c)^{5}-2\left(6\,a^{2}+b^{2}\right)\cos(dx+c)^{3}-3\left(6\,a^{2}+b^{2}\right)\cos(dx+c)\right)\sin}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/240*(96*a*b*cos(d*x + c)^5 - 15*(6*a^2 + b^2)*d*x + 5*(8*b^2*cos(d*x + c
)^5 - 2*(6*a^2 + b^2)*cos(d*x + c)^3 - 3*(6*a^2 + b^2)*cos(d*x + c))*sin(d*
x + c))/d
```

Sympy [A] time = 4.7273, size = 287, normalized size = 2.47

$$\begin{cases} \frac{3a^2x\sin^4(c+dx)}{8} + \frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^2x\cos^4(c+dx)}{8} + \frac{3a^2\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a^2\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{2ab\cos^5(c+dx)}{5d} + \\ x(a+b\sin(c))^2\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**2*x*cos(c + d*x)**4/8 + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*a*b*cos(c + d*x)**5/(
5*d) + b**2*x*sin(c + d*x)**6/16 + 3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2
/16 + 3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + b**2*x*cos(c + d*x)**6/
16 + b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + b**2*sin(c + d*x)**3*cos(c
+ d*x)**3/(6*d) - b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(
a + b*sin(c))**2*cos(c)**4, True))
```

Giac [A] time = 1.09804, size = 166, normalized size = 1.43

$$\frac{1}{16}\left(6a^2+b^2\right)x - \frac{ab\cos(5dx+5c)}{40d} - \frac{ab\cos(3dx+3c)}{8d} - \frac{ab\cos(dx+c)}{4d} - \frac{b^2\sin(6dx+6c)}{192d} + \frac{\left(2a^2-b^2\right)\sin(4dx}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(6*a^2 + b^2)*x - 1/40*a*b*cos(5*d*x + 5*c)/d - 1/8*a*b*cos(3*d*x + 3*
c)/d - 1/4*a*b*cos(d*x + c)/d - 1/192*b^2*sin(6*d*x + 6*c)/d + 1/64*(2*a^2
- b^2)*sin(4*d*x + 4*c)/d + 1/64*(16*a^2 + b^2)*sin(2*d*x + 2*c)/d
```

3.395 $\int \cos^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=86

$$\frac{(4a^2+b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2+b^2\right)-\frac{5ab\cos^3(c+dx)}{12d}-\frac{b\cos^3(c+dx)(a+b\sin(c+dx))}{4d}$$

```
[Out] ((4*a^2 + b^2)*x)/8 - (5*a*b*Cos[c + d*x]^3)/(12*d) + ((4*a^2 + b^2)*Cos[c
+ d*x]*Sin[c + d*x])/(8*d) - (b*Cos[c + d*x]^3*(a + b*Sin[c + d*x]))/(4*d)
```

Rubi [A] time = 0.0954186, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2692, 2669, 2635, 8}

$$\frac{(4a^2+b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2+b^2\right)-\frac{5ab\cos^3(c+dx)}{12d}-\frac{b\cos^3(c+dx)(a+b\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((4*a^2 + b^2)*x)/8 - (5*a*b*Cos[c + d*x]^3)/(12*d) + ((4*a^2 + b^2)*Cos[c
+ d*x]*Sin[c + d*x])/(8*d) - (b*Cos[c + d*x]^3*(a + b*Sin[c + d*x]))/(4*d)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{b\cos^3(c+dx)(a+b\sin(c+dx))}{4d}+\frac{1}{4}\int \cos^2(c+dx)\left(4a^2+b^2+5ab\sin(c+\right.$$

$$= -\frac{5ab\cos^3(c+dx)}{12d}-\frac{b\cos^3(c+dx)(a+b\sin(c+dx))}{4d}+\frac{1}{4}\left(4a^2+b^2\right)\int \cos^2(c$$

$$= -\frac{5ab\cos^3(c+dx)}{12d}+\frac{\left(4a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos^3(c+dx)(a+b}{4d}$$

$$= \frac{1}{8}\left(4a^2+b^2\right)x-\frac{5ab\cos^3(c+dx)}{12d}+\frac{\left(4a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos}{}$$

Mathematica [A] time = 0.236866, size = 85, normalized size = 0.99

$$\frac{3\left(8a^2\sin(2(c+dx))+16a^2c+16a^2dx-b^2\sin(4(c+dx))+4b^2c+4b^2dx\right)-48ab\cos(c+dx)-16ab\cos(3(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-48*a*b*Cos[c + d*x] - 16*a*b*Cos[3*(c + d*x)] + 3*(16*a^2*c + 4*b^2*c + 1
6*a^2*d*x + 4*b^2*d*x + 8*a^2*Sin[2*(c + d*x)] - b^2*Sin[4*(c + d*x)]))/(96
*d)
```

Maple [A] time = 0.042, size = 86, normalized size = 1.

$$\frac{1}{d}\left(b^2\left(-\frac{(\cos(dx+c))^3\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{2ab(\cos(dx+c))^3}{3}+a^2\left(\frac{\cos(dx+c)\sin(}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)-2/3*a*b*cos(d*x+c)^3+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.968966, size = 86, normalized size = 1.

$$-\frac{64\,ab\cos\left(dx+c\right)^{3}-24\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{2}-3\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)b^{2}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/96*(64*a*b*cos(d*x + c)^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 3*
(4*d*x + 4*c - sin(4*d*x + 4*c))*b^2)/d
```

Fricas [A] time = 2.26164, size = 167, normalized size = 1.94

$$-\frac{16\,ab\cos\left(dx+c\right)^{3}-3\left(4\,a^{2}+b^{2}\right)dx+3\left(2\,b^{2}\cos\left(dx+c\right)^{3}-\left(4\,a^{2}+b^{2}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/24*(16*a*b*cos(d*x + c)^3 - 3*(4*a^2 + b^2)*d*x + 3*(2*b^2*cos(d*x + c)^
3 - (4*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.31756, size = 180, normalized size = 2.09

$$\begin{cases} \frac{a^{2}x\sin^{2}\left(c+dx\right)}{2}+\frac{a^{2}x\cos^{2}\left(c+dx\right)}{2}+\frac{a^{2}\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}-\frac{2ab\cos^{3}\left(c+dx\right)}{3d}+\frac{b^{2}x\sin^{4}\left(c+dx\right)}{8}+\frac{b^{2}x\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{4}+\frac{b^{2}x\cos^{4}\left(c+dx\right)}{8}+ \\ x\left(a+b\sin\left(c\right)\right)^{2}\cos^{2}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c
 + d*x)*cos(c + d*x)/(2*d) - 2*a*b*cos(c + d*x)**3/(3*d) + b**2*x*sin(c + d
*x)**4/8 + b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + b**2*x*cos(c + d*x)**
4/8 + b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - b**2*sin(c + d*x)*cos(c + d
*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sin(c))**2*cos(c)**2, True))
```

Giac [A] time = 1.08505, size = 103, normalized size = 1.2

$$\frac{1}{8}\left(4\,a^2+b^2\right)x-\frac{ab\cos\left(3\,dx+3\,c\right)}{6\,d}-\frac{ab\cos\left(dx+c\right)}{2\,d}-\frac{b^2\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a^2\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*a^2 + b^2)*x - 1/6*a*b*cos(3*d*x + 3*c)/d - 1/2*a*b*cos(d*x + c)/d -
 1/32*b^2*sin(4*d*x + 4*c)/d + 1/4*a^2*sin(2*d*x + 2*c)/d
```

3.396 $\int \sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=49

$$\frac{ab\cos(c+dx)}{d}+\frac{\sec(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{d}+b^2(-x)$$

```
[Out] -(b^2*x) + (a*b*Cos[c + d*x])/d + (Sec[c + d*x]*(b + a*Sin[c + d*x])*(a + b
*Sin[c + d*x]))/d
```

Rubi [A] time = 0.0484278, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2691, 2638}

$$\frac{ab\cos(c+dx)}{d}+\frac{\sec(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{d}+b^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(b^2*x) + (a*b*Cos[c + d*x])/d + (Sec[c + d*x]*(b + a*Sin[c + d*x])*(a + b
*Sin[c + d*x]))/d
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{d} - \int \left(b^2+ab\sin(c+dx)\right)\,dx\\ &= -b^2x+\frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{d}-(ab)\int \sin(c+dx)\,d\\ &= -b^2x+\frac{ab\cos(c+dx)}{d}+\frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0586616, size = 55, normalized size = 1.12

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{b^2\tan^{-1}(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((b^2*ArcTan[Tan[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d + (a^2*Tan[c + d*x
])/d + (b^2*Tan[c + d*x])/d
```

Maple [A] time = 0.036, size = 46, normalized size = 0.9

$$\frac{1}{d}\left(a^2\tan(dx+c)+2\,\frac{ab}{\cos(dx+c)}+b^2(\tan(dx+c)-dx-c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*tan(d*x+c)+2*a*b/cos(d*x+c)+b^2*(tan(d*x+c)-d*x-c))
```

Maxima [A] time = 1.46466, size = 62, normalized size = 1.27

$$-\frac{(dx+c-\tan(dx+c))b^2-a^2\tan(dx+c)-\frac{2\,ab}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*b^2 - a^2*tan(d*x + c) - 2*a*b/cos(d*x + c))/d
```

Fricas [A] time = 2.09162, size = 104, normalized size = 2.12

$$-\frac{b^2 dx \cos(dx+c) - 2\,ab - \left(a^2+b^2\right)\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(b^2*d*x*cos(d*x + c) - 2*a*b - (a^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sin(c+dx))^2 \sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [A] time = 1.11494, size = 85, normalized size = 1.73

$$-\frac{(dx+c)b^2 + \frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)*b^2 + 2*(a^2*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2*d*x + 1/2*c) +
2*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.397 $\int \sec^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=75

$$\frac{(2a^2-b^2)\tan(c+dx)}{3d}+\frac{ab\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{3d}$$

```
[Out] (a*b*Sec[c + d*x])/(3*d) + (Sec[c + d*x]^3*(b + a*Sin[c + d*x])*(a + b*Sin[
c + d*x]))/(3*d) + ((2*a^2 - b^2)*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.0962427, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2691, 2669, 3767, 8}

$$\frac{(2a^2-b^2)\tan(c+dx)}{3d}+\frac{ab\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x])/(3*d) + (Sec[c + d*x]^3*(b + a*Sin[c + d*x])*(a + b*Sin[
c + d*x]))/(3*d) + ((2*a^2 - b^2)*Tan[c + d*x])/(3*d)
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{3d} - \frac{1}{3}\int \sec^2(c+dx)\left(-2a^2+b^2\right.$$
$$= \frac{ab\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{3d} - \frac{1}{3}\left(-2a^2+\right.$$
$$= \frac{ab\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{3d} - \frac{\left(2a^2-b^2\right)}{}$$
$$= \frac{ab\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{3d} + \frac{\left(2a^2-b^2\right)}{3}$$

Mathematica [A] time = 0.322364, size = 105, normalized size = 1.4

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(3\left(2a^2+b^2\right)\sin(c+dx)+\left(2a^2-b^2\right)\sin(3(c+dx))+8ab\right)}{12d(\sin(c+dx)-1)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(8*a*b + 3*(2*a^2 + b^2)*Sin[c + d*x
] + (2*a^2 - b^2)*Sin[3*(c + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^3*(-1 + Sin[c + d*x])^2)
```

Maple [A] time = 0.051, size = 62, normalized size = 0.8

$$\frac{1}{d}\left(-a^2\left(-\frac{2}{3}-\frac{(\sec(dx+c))^2}{3}\right)\tan(dx+c)+\frac{2ab}{3(\cos(dx+c))^3}+\frac{b^2(\sin(dx+c))^3}{3(\cos(dx+c))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2/3*a*b/cos(d*x+c)^3+1/3*b^2*s
in(d*x+c)^3/cos(d*x+c)^3)
```

Maxima [A] time = 0.972691, size = 69, normalized size = 0.92

$$\frac{b^2\tan(dx+c)^3+\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2+\frac{2ab}{\cos(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 2*a*b/cos
(d*x + c)^3)/d
```

Fricas [A] time = 2.12719, size = 122, normalized size = 1.63

$$\frac{2ab+\left(\left(2a^2-b^2\right)\cos(dx+c)^2+a^2+b^2\right)\sin(dx+c)}{3d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*b + ((2*a^2 - b^2)*cos(d*x + c)^2 + a^2 + b^2)*sin(d*x + c))/(d*co
s(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.11118, size = 138, normalized size = 1.84

$$-\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*a*b*tan(1/2*d*x + 1/2*c)^4 - 2*a^2*t
an(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x +
1/2*c) + 2*a*b)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*d)
```

3.398 $\int \sec^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=103

$$\frac{(4a^2-b^2)\tan^3(c+dx)}{15d}+\frac{(4a^2-b^2)\tan(c+dx)}{5d}+\frac{ab\sec^3(c+dx)}{5d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{5d}$$

```
[Out] (a*b*Sec[c + d*x]^3)/(5*d) + (Sec[c + d*x]^5*(b + a*Sin[c + d*x])*(a + b*Si
n[c + d*x]))/(5*d) + ((4*a^2 - b^2)*Tan[c + d*x])/(5*d) + ((4*a^2 - b^2)*Ta
n[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.101227, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2691, 2669, 3767}

$$\frac{(4a^2-b^2)\tan^3(c+dx)}{15d}+\frac{(4a^2-b^2)\tan(c+dx)}{5d}+\frac{ab\sec^3(c+dx)}{5d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x]^3)/(5*d) + (Sec[c + d*x]^5*(b + a*Sin[c + d*x])*(a + b*Si
n[c + d*x]))/(5*d) + ((4*a^2 - b^2)*Tan[c + d*x])/(5*d) + ((4*a^2 - b^2)*Ta
n[c + d*x]^3)/(15*d)
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{5d} - \frac{1}{5}\int \sec^4(c+dx)\left(-4a^2+b^2\right.$$
$$= \frac{ab\sec^3(c+dx)}{5d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{5d} - \frac{1}{5}\left(-4a^2+\right.$$
$$= \frac{ab\sec^3(c+dx)}{5d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{5d} - \frac{\left(4a^2-b^2\right.}{}$$
$$= \frac{ab\sec^3(c+dx)}{5d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{5d} + \frac{\left(4a^2-b^2\right.}{}$$

Mathematica [A] time = 0.433124, size = 84, normalized size = 0.82

$$\frac{\sec^5(c+dx)\left(20\left(2a^2+b^2\right)\sin(c+dx)+5\left(4a^2-b^2\right)\sin(3(c+dx))+4a^2\sin(5(c+dx))+48ab-b^2\sin(5(c+dx))\right)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^5*(48*a*b + 20*(2*a^2 + b^2)*Sin[c + d*x] + 5*(4*a^2 - b^2)*S
in[3*(c + d*x)] + 4*a^2*Sin[5*(c + d*x)] - b^2*Sin[5*(c + d*x)]))/(120*d)
```

Maple [A] time = 0.056, size = 92, normalized size = 0.9

$$\frac{1}{d}\left(-a^2\left(-\frac{8}{15}-\frac{(\sec(dx+c))^4}{5}-\frac{4(\sec(dx+c))^2}{15}\right)\tan(dx+c)+\frac{2ab}{5(\cos(dx+c))^5}+b^2\left(\frac{(\sin(dx+c))^3}{5(\cos(dx+c))^5}+\frac{2(\sin(dx}{15(\cos(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+2/5*a*b/cos
(d*x+c)^5+b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3
))
```

Maxima [A] time = 0.976974, size = 103, normalized size = 1.

$$\frac{\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a^{2}+\left(3\,\tan\left(dx+c\right)^{5}+5\,\tan\left(dx+c\right)^{3}\right)b^{2}+\frac{6\,ab}{\cos(dx+c)^{5}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + (3*tan
(d*x + c)^5 + 5*tan(d*x + c)^3)*b^2 + 6*a*b/cos(d*x + c)^5)/d
```

Fricas [A] time = 2.10586, size = 173, normalized size = 1.68

$$\frac{6\,ab+\left(2\left(4\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{4}+\left(4\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+3\,a^{2}+3\,b^{2}\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(6*a*b + (2*(4*a^2 - b^2)*cos(d*x + c)^4 + (4*a^2 - b^2)*cos(d*x + c)^
2 + 3*a^2 + 3*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11975, size = 244, normalized size = 2.37

$$-\frac{2\left(15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-20\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+20\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+58\,a^{2}\tan\left(\frac{1}{2}\,d\right.}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^2*tan(1/2*d*x + 1/2*c)^9 + 30*a*b*tan(1/2*d*x + 1/2*c)^8 - 20*a
^2*tan(1/2*d*x + 1/2*c)^7 + 20*b^2*tan(1/2*d*x + 1/2*c)^7 + 58*a^2*tan(1/2*
d*x + 1/2*c)^5 + 8*b^2*tan(1/2*d*x + 1/2*c)^5 + 60*a*b*tan(1/2*d*x + 1/2*c)
^4 - 20*a^2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 + 15*a^2
*tan(1/2*d*x + 1/2*c) + 6*a*b)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*d)
```

3.399 $\int \sec^8(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=129

$$\frac{(6a^2-b^2)\tan^5(c+dx)}{35d}+\frac{2(6a^2-b^2)\tan^3(c+dx)}{21d}+\frac{(6a^2-b^2)\tan(c+dx)}{7d}+\frac{ab\sec^5(c+dx)}{7d}+\frac{\sec^7(c+dx)(a\sin(c$$

```
[Out] (a*b*Sec[c + d*x]^5)/(7*d) + (Sec[c + d*x]^7*(b + a*Sin[c + d*x])*(a + b*Si
n[c + d*x]))/(7*d) + ((6*a^2 - b^2)*Tan[c + d*x])/(7*d) + (2*(6*a^2 - b^2)*
Tan[c + d*x]^3)/(21*d) + ((6*a^2 - b^2)*Tan[c + d*x]^5)/(35*d)
```

Rubi [A] time = 0.122862, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2691, 2669, 3767}

$$\frac{(6a^2-b^2)\tan^5(c+dx)}{35d}+\frac{2(6a^2-b^2)\tan^3(c+dx)}{21d}+\frac{(6a^2-b^2)\tan(c+dx)}{7d}+\frac{ab\sec^5(c+dx)}{7d}+\frac{\sec^7(c+dx)(a\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^2,x]
```



```
[Out] (a*b*Sec[c + d*x]^5)/(7*d) + (Sec[c + d*x]^7*(b + a*Sin[c + d*x])*(a + b*Si
n[c + d*x]))/(7*d) + ((6*a^2 - b^2)*Tan[c + d*x])/(7*d) + (2*(6*a^2 - b^2)*
Tan[c + d*x]^3)/(21*d) + ((6*a^2 - b^2)*Tan[c + d*x]^5)/(35*d)
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{7d}-\frac{1}{7}\int \sec^6(c+dx)\left(-6a^2+b^2\right.\\ &= \frac{ab\sec^5(c+dx)}{7d}+\frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{7d}-\frac{1}{7}\left(-6a^2+\right.\\ &= \frac{ab\sec^5(c+dx)}{7d}+\frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{7d}-\frac{\left(6a^2-b^2\right)}{}\\ &= \frac{ab\sec^5(c+dx)}{7d}+\frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{7d}+\frac{\left(6a^2-b^2\right.}{}\end{aligned}$$

Mathematica [A] time = 0.835627, size = 110, normalized size = 0.85

$$\frac{\sec^7(c+dx)\left(105\left(2a^2+b^2\right)\sin(c+dx)+21\left(6a^2-b^2\right)\sin(3(c+dx))+42a^2\sin(5(c+dx))+6a^2\sin(7(c+dx))+240ab\right.}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^7*(240*a*b + 105*(2*a^2 + b^2)*Sin[c + d*x] + 21*(6*a^2 - b^2
)*Sin[3*(c + d*x)] + 42*a^2*Sin[5*(c + d*x)] - 7*b^2*Sin[5*(c + d*x)] + 6*a
^2*Sin[7*(c + d*x)] - b^2*Sin[7*(c + d*x)]))/(840*d)
```

Maple [A] time = 0.062, size = 120, normalized size = 0.9

$$\frac{1}{d}\left(-a^2\left(-\frac{16}{35}-\frac{(\sec(dx+c))^6}{7}-\frac{6\,(\sec(dx+c))^4}{35}-\frac{8\,(\sec(dx+c))^2}{35}\right)\tan(dx+c)+\frac{2\,ab}{7\,(\cos(dx+c))^7}+b^2\left(\frac{(\sin(dx+}{7\,(\cos(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+2/7*a*b/cos(d*x+c)^7+b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*
x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3))
```

Maxima [A] time = 0.971905, size = 131, normalized size = 1.02

$$\frac{3\left(5\,\tan\left(dx+c\right)^{7}+21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+35\,\tan\left(dx+c\right)\right)a^{2}+\left(15\,\tan\left(dx+c\right)^{7}+42\,\tan\left(dx+c\right)^{5}+\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(3*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan
(d*x + c))*a^2 + (15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3
)*b^2 + 30*a*b/cos(d*x + c)^7)/d
```

Fricas [A] time = 2.20787, size = 225, normalized size = 1.74

$$\frac{30\,ab+\left(8\left(6\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{6}+4\left(6\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{4}+3\left(6\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+15\,a^{2}+15\,b^{2}\right)\sin\left(dx+c\right.}{105\,d\cos\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(30*a*b + (8*(6*a^2 - b^2)*cos(d*x + c)^6 + 4*(6*a^2 - b^2)*cos(d*x +
 c)^4 + 3*(6*a^2 - b^2)*cos(d*x + c)^2 + 15*a^2 + 15*b^2)*sin(d*x + c))/(d*
cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12001, size = 351, normalized size = 2.72

$$-\frac{2\left(105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+210\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-210\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+140\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+903\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/105*(105*a^2*tan(1/2*d*x + 1/2*c)^13 + 210*a*b*tan(1/2*d*x + 1/2*c)^12 -
 210*a^2*tan(1/2*d*x + 1/2*c)^11 + 140*b^2*tan(1/2*d*x + 1/2*c)^11 + 903*a^
2*tan(1/2*d*x + 1/2*c)^9 + 112*b^2*tan(1/2*d*x + 1/2*c)^9 + 1050*a*b*tan(1/
2*d*x + 1/2*c)^8 - 636*a^2*tan(1/2*d*x + 1/2*c)^7 + 456*b^2*tan(1/2*d*x + 1
/2*c)^7 + 903*a^2*tan(1/2*d*x + 1/2*c)^5 + 112*b^2*tan(1/2*d*x + 1/2*c)^5 +
 630*a*b*tan(1/2*d*x + 1/2*c)^4 - 210*a^2*tan(1/2*d*x + 1/2*c)^3 + 140*b^2*
tan(1/2*d*x + 1/2*c)^3 + 105*a^2*tan(1/2*d*x + 1/2*c) + 30*a*b)/((tan(1/2*d
*x + 1/2*c)^2 - 1)^7*d)
```

3.400 $\int \cos^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=144

$$\frac{(3a^2-b^2)(a+b\sin(c+dx))^6}{3b^5d}-\frac{4a(a^2-b^2)(a+b\sin(c+dx))^5}{5b^5d}+\frac{(a^2-b^2)^2(a+b\sin(c+dx))^4}{4b^5d}+\frac{(a+b\sin(c+dx))}{8b^5d}$$

```
[Out] ((a^2 - b^2)^2*(a + b*Sin[c + d*x])^4)/(4*b^5*d) - (4*a*(a^2 - b^2)*(a + b*
Sin[c + d*x])^5)/(5*b^5*d) + ((3*a^2 - b^2)*(a + b*Sin[c + d*x])^6)/(3*b^5*
d) - (4*a*(a + b*Sin[c + d*x])^7)/(7*b^5*d) + (a + b*Sin[c + d*x])^8/(8*b^5
*d)
```

Rubi [A] time = 0.132702, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{(3a^2-b^2)(a+b\sin(c+dx))^6}{3b^5d}-\frac{4a(a^2-b^2)(a+b\sin(c+dx))^5}{5b^5d}+\frac{(a^2-b^2)^2(a+b\sin(c+dx))^4}{4b^5d}+\frac{(a+b\sin(c+dx))}{8b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((a^2 - b^2)^2*(a + b*Sin[c + d*x])^4)/(4*b^5*d) - (4*a*(a^2 - b^2)*(a + b*
Sin[c + d*x])^5)/(5*b^5*d) + ((3*a^2 - b^2)*(a + b*Sin[c + d*x])^6)/(3*b^5*
d) - (4*a*(a + b*Sin[c + d*x])^7)/(7*b^5*d) + (a + b*Sin[c + d*x])^8/(8*b^5
*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int (a+x)^3\left(b^2-x^2\right)^2\,dx,x,b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2(a+x)^3-4\left(a^3-ab^2\right)(a+x)^4+2\left(3a^2-b^2\right)(a+x)^5-4a\right.\right.}{b^5d}$$
$$= \frac{\left(a^2-b^2\right)^2(a+b\sin(c+dx))^4}{4b^5d}-\frac{4a\left(a^2-b^2\right)(a+b\sin(c+dx))^5}{5b^5d}+\frac{\left(3a^2-b^2\right)(a}{}$$

Mathematica [A] time = 0.544047, size = 120, normalized size = 0.83

$$\frac{\frac{1}{3}\left(3a^2-b^2\right)(a+b\sin(c+dx))^6+\frac{1}{4}\left(a^2-b^2\right)^2(a+b\sin(c+dx))^4+\frac{1}{8}(a+b\sin(c+dx))^8-\frac{4}{7}a(a+b\sin(c+dx))^7-\frac{4}{5}a(}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (((a^2 - b^2)^2*(a + b*Sin[c + d*x])^4)/4 - (4*a*(a - b)*(a + b)*(a + b*Sin
[c + d*x])^5)/5 + ((3*a^2 - b^2)*(a + b*Sin[c + d*x])^6)/3 - (4*a*(a + b*Si
n[c + d*x])^7)/7 + (a + b*Sin[c + d*x])^8/8)/(b^5*d)
```

Maple [A] time = 0.057, size = 135, normalized size = 0.9

$$\frac{1}{d}\left(b^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^6}{8}-\frac{(\cos(dx+c))^6}{24}\right)+3\,ab^2\left(-1/7\,\sin(dx+c)(\cos(dx+c))^6+1/35\,\left(8/3+(\cos(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+3*a*b^2*(-1/7*s
in(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
-1/2*a^2*b*cos(d*x+c)^6+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x
+c))
```

Maxima [A] time = 0.94948, size = 194, normalized size = 1.35

$$\frac{105\,b^3\sin\left(dx+c\right)^8+360\,ab^2\sin\left(dx+c\right)^7+140\left(3\,a^2b-2\,b^3\right)\sin\left(dx+c\right)^6+168\left(a^3-6\,ab^2\right)\sin\left(dx+c\right)^5+1260\,a^2b}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(105*b^3*sin(d*x + c)^8 + 360*a*b^2*sin(d*x + c)^7 + 140*(3*a^2*b - 2
*b^3)*sin(d*x + c)^6 + 168*(a^3 - 6*a*b^2)*sin(d*x + c)^5 + 1260*a^2*b*sin(
d*x + c)^2 - 210*(6*a^2*b - b^3)*sin(d*x + c)^4 + 840*a^3*sin(d*x + c) - 28
0*(2*a^3 - 3*a*b^2)*sin(d*x + c)^3)/d
```

Fricas [A] time = 2.37953, size = 281, normalized size = 1.95

$$\frac{105\,b^3\cos\left(dx+c\right)^8-140\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)^6-8\left(45\,ab^2\cos\left(dx+c\right)^6-3\left(7\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^4-56\,a^3-2\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*b^3*cos(d*x + c)^8 - 140*(3*a^2*b + b^3)*cos(d*x + c)^6 - 8*(45*
a*b^2*cos(d*x + c)^6 - 3*(7*a^3 + 3*a*b^2)*cos(d*x + c)^4 - 56*a^3 - 24*a*b
^2 - 4*(7*a^3 + 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 13.3257, size = 280, normalized size = 1.94

$$\begin{cases} \frac{8a^3\sin^5(c+dx)}{15d}+\frac{4a^3\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{a^3\sin(c+dx)\cos^4(c+dx)}{d}+\frac{a^2b\sin^6(c+dx)}{2d}+\frac{3a^2b\sin^4(c+dx)\cos^2(c+dx)}{2d}+\frac{3a^2b\sin^2(c+dx)\cos^4(c}{2d} \\ x\left(a+b\sin(c)\right)^3\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**5/(15*d) + 4*a**3*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**4/d + a**2*b*sin(c + d*x)**6
```

```
/(2*d) + 3*a**2*b*sin(c + d*x)**4*cos(c + d*x)**2/(2*d) + 3*a**2*b*sin(c +
d*x)**2*cos(c + d*x)**4/(2*d) + 8*a*b**2*sin(c + d*x)**7/(35*d) + 4*a*b**2*
sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + a*b**2*sin(c + d*x)**3*cos(c + d*x)
**4/d + b**3*sin(c + d*x)**8/(24*d) + b**3*sin(c + d*x)**6*cos(c + d*x)**2/
(6*d) + b**3*sin(c + d*x)**4*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a + b*si
n(c))**3*cos(c)**5, True))
```

Giac [A] time = 1.11447, size = 250, normalized size = 1.74

$$\frac{b^3\cos(8\,dx+8\,c)}{1024\,d}-\frac{3\,ab^2\sin(7\,dx+7\,c)}{448\,d}-\frac{\left(6\,a^2b-b^3\right)\cos(6\,dx+6\,c)}{384\,d}-\frac{\left(24\,a^2b+b^3\right)\cos(4\,dx+4\,c)}{256\,d}-\frac{3\left(10\,a^2b+b^3\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1024*b^3*cos(8*d*x + 8*c)/d - 3/448*a*b^2*sin(7*d*x + 7*c)/d - 1/384*(6*a
^2*b - b^3)*cos(6*d*x + 6*c)/d - 1/256*(24*a^2*b + b^3)*cos(4*d*x + 4*c)/d
- 3/128*(10*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/320*(4*a^3 - 9*a*b^2)*sin(5
*d*x + 5*c)/d + 1/192*(20*a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 5/64*(8*a^3 +
 3*a*b^2)*sin(d*x + c)/d
```

3.401 $\int \cos^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=77

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^4}{4b^3d}-\frac{(a+b\sin(c+dx))^6}{6b^3d}+\frac{2a(a+b\sin(c+dx))^5}{5b^3d}$$

```
[Out] -((a^2 - b^2)*(a + b*Sin[c + d*x])^4)/(4*b^3*d) + (2*a*(a + b*Sin[c + d*x])
^5)/(5*b^3*d) - (a + b*Sin[c + d*x])^6/(6*b^3*d)
```

Rubi [A] time = 0.0802826, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^4}{4b^3d}-\frac{(a+b\sin(c+dx))^6}{6b^3d}+\frac{2a(a+b\sin(c+dx))^5}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((a^2 - b^2)*(a + b*Sin[c + d*x])^4)/(4*b^3*d) + (2*a*(a + b*Sin[c + d*x])
^5)/(5*b^3*d) - (a + b*Sin[c + d*x])^6/(6*b^3*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int (a+x)^3\left(b^2-x^2\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^3+2a(a+x)^4-(a+x)^5\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= -\frac{\left(a^2-b^2\right)(a+b\sin(c+dx))^4}{4b^3d}+\frac{2a(a+b\sin(c+dx))^5}{5b^3d}-\frac{(a+b\sin(c+dx))^6}{6b^3d}$$

Mathematica [A] time = 0.148069, size = 56, normalized size = 0.73

$$\frac{(a+b\sin(c+dx))^4\left(-a^2+4ab\sin(c+dx)+5b^2\cos(2(c+dx))+10b^2\right)}{60b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((a + b*Sin[c + d*x])^4*(-a^2 + 10*b^2 + 5*b^2*Cos[2*(c + d*x)] + 4*a*b*Sin
[c + d*x]))/(60*b^3*d)
```

Maple [A] time = 0.055, size = 115, normalized size = 1.5

$$\frac{1}{d}\left(b^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^4}{6}-\frac{(\cos(dx+c))^4}{12}\right)+3\,ab^2\left(-1/5\,\sin(dx+c)(\cos(dx+c))^4+1/15\,\left(2+(\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a*b^2*(-1/5*s
in(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/4*a^2*b*cos(d*x+
c)^4+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.965449, size = 135, normalized size = 1.75

$$-\frac{10\,b^3\sin(dx+c)^6+36\,ab^2\sin(dx+c)^5-90\,a^2b\sin(dx+c)^2+15\left(3\,a^2b-b^3\right)\sin(dx+c)^4-60\,a^3\sin(dx+c)+20\left(a\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(10*b^3*sin(d*x + c)^6 + 36*a*b^2*sin(d*x + c)^5 - 90*a^2*b*sin(d*x +
 c)^2 + 15*(3*a^2*b - b^3)*sin(d*x + c)^4 - 60*a^3*sin(d*x + c) + 20*(a^3 -
 3*a*b^2)*sin(d*x + c)^3)/d
```

Fricas [A] time = 2.25355, size = 221, normalized size = 2.87

$$\frac{10\,b^3\cos\left(dx+c\right)^6-15\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)^4-4\left(9\,ab^2\cos\left(dx+c\right)^4-10\,a^3-6\,ab^2-\left(5\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^3*cos(d*x + c)^6 - 15*(3*a^2*b + b^3)*cos(d*x + c)^4 - 4*(9*a*b^
2*cos(d*x + c)^4 - 10*a^3 - 6*a*b^2 - (5*a^3 + 3*a*b^2)*cos(d*x + c)^2)*sin
(d*x + c))/d
```

Sympy [A] time = 4.58979, size = 178, normalized size = 2.31

$$\begin{cases}\frac{2a^3\sin^3\left(c+dx\right)}{3d}+\frac{a^3\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{3a^2b\sin^4\left(c+dx\right)}{4d}+\frac{3a^2b\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{2d}+\frac{2ab^2\sin^5\left(c+dx\right)}{5d}+\frac{ab^2\sin^3\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{b^3\sin^6\left(c+dx\right)}{12d}+\frac{b^3\sin^4\left(c+dx\right)\cos^2\left(c+dx\right)}{4d} & \text{for}\: d \neq 0 \\ x\left(a+b\sin\left(c\right)\right)^3\cos^3\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2
/d + 3*a**2*b*sin(c + d*x)**4/(4*d) + 3*a**2*b*sin(c + d*x)**2*cos(c + d*x)
**2/(2*d) + 2*a*b**2*sin(c + d*x)**5/(5*d) + a*b**2*sin(c + d*x)**3*cos(c +
 d*x)**2/d + b**3*sin(c + d*x)**6/(12*d) + b**3*sin(c + d*x)**4*cos(c + d*x
)**2/(4*d), Ne(d, 0)), (x*(a + b*sin(c))**3*cos(c)**3, True))
```

Giac [A] time = 1.10707, size = 151, normalized size = 1.96

$$-\frac{10\,b^3\sin\left(dx+c\right)^6+36\,ab^2\sin\left(dx+c\right)^5+45\,a^2b\sin\left(dx+c\right)^4-15\,b^3\sin\left(dx+c\right)^4+20\,a^3\sin\left(dx+c\right)^3-60\,ab^2\sin\left(dx\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(10*b^3*sin(d*x + c)^6 + 36*a*b^2*sin(d*x + c)^5 + 45*a^2*b*sin(d*x +
 c)^4 - 15*b^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3 - 60*a*b^2*sin(d*x +
c)^3 - 90*a^2*b*sin(d*x + c)^2 - 60*a^3*sin(d*x + c))/d
```

3.402 $\int \cos(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\sin(c+dx))^4}{4bd}$$

```
[Out] (a + b*Sin[c + d*x])^4/(4*b*d)
```

Rubi [A] time = 0.0263097, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 32}

$$\frac{(a+b\sin(c+dx))^4}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a + b*Sin[c + d*x])^4/(4*b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{\operatorname{Subst}\left(\int (a+x)^3\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{(a+b\sin(c+dx))^4}{4bd}\end{aligned}$$

Mathematica [B] time = 0.0693703, size = 57, normalized size = 2.59

$$\frac{\sin(c+dx)\left(6a^2b\sin(c+dx)+4a^3+4ab^2\sin^2(c+dx)+b^3\sin^3(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sin[c + d*x]*(4*a^3 + 6*a^2*b*Sin[c + d*x] + 4*a*b^2*Sin[c + d*x]^2 + b^3*
Sin[c + d*x]^3))/(4*d)
```

Maple [A] time = 0.019, size = 21, normalized size = 1.

$$\frac{(a+b\sin(dx+c))^4}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4*(a+b*sin(d*x+c))^4/b/d
```

Maxima [A] time = 0.954898, size = 27, normalized size = 1.23

$$\frac{(b\sin(dx+c)+a)^4}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b*sin(d*x + c) + a)^4/(b*d)
```

Fricas [B] time = 2.24263, size = 158, normalized size = 7.18

$$\frac{b^3\cos(dx+c)^4-2\left(3\,a^2b+b^3\right)\cos(dx+c)^2-4\left(ab^2\cos(dx+c)^2-a^3-ab^2\right)\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*cos(d*x + c)^4 - 2*(3*a^2*b + b^3)*cos(d*x + c)^2 - 4*(a*b^2*cos(d
*x + c)^2 - a^3 - a*b^2)*sin(d*x + c))/d
```

Sympy [A] time = 1.22593, size = 97, normalized size = 4.41

$$\begin{cases} \frac{a^3\sin(c+dx)}{d} + \frac{3a^2b\sin^2(c+dx)}{2d} + \frac{ab^2\sin^3(c+dx)}{d} - \frac{b^3\sin^2(c+dx)\cos^2(c+dx)}{2d} - \frac{b^3\cos^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x\,(a + b\sin(c))^3\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)/d + 3*a**2*b*sin(c + d*x)**2/(2*d) + a*b**2*si
n(c + d*x)**3/d - b**3*sin(c + d*x)**2*cos(c + d*x)**2/(2*d) - b**3*cos(c +
 d*x)**4/(4*d), Ne(d, 0)), (x*(a + b*sin(c))**3*cos(c), True))
```

Giac [A] time = 1.09528, size = 27, normalized size = 1.23

$$\frac{(b\sin(dx+c)+a)^4}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(b*sin(d*x + c) + a)^4/(b*d)
```

3.403 $\int \sec(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=80

$$-\frac{3ab^2\sin(c+dx)}{d}+\frac{(a-b)^3\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}-\frac{b^3\sin^2(c+dx)}{2d}$$

```
[Out] -((a + b)^3*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)^3*Log[1 + Sin[c + d*x]]
)/(2*d) - (3*a*b^2*Sin[c + d*x])/d - (b^3*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.105492, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2668, 702, 633, 31}

$$-\frac{3ab^2\sin(c+dx)}{d}+\frac{(a-b)^3\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}-\frac{b^3\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```



```
[Out] -((a + b)^3*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)^3*Log[1 + Sin[c + d*x]]
)/(2*d) - (3*a*b^2*Sin[c + d*x])/d - (b^3*Sin[c + d*x]^2)/(2*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{b\,\mathrm{Subst}\left(\int \frac{(a+x)^3}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{b\,\mathrm{Subst}\left(\int \left(-3a-x+\frac{a^3+3ab^2+(3a^2+b^2)x}{b^2-x^2}\right)dx, x, b\sin(c+dx)\right)}{d}\\ &= -\frac{3ab^2\sin(c+dx)}{d}-\frac{b^3\sin^2(c+dx)}{2d}+\frac{b\,\mathrm{Subst}\left(\int \frac{a^3+3ab^2+(3a^2+b^2)x}{b^2-x^2}\,dx, x, b\sin(c\right.}{d}\\ &= -\frac{3ab^2\sin(c+dx)}{d}-\frac{b^3\sin^2(c+dx)}{2d}-\frac{(a-b)^3\,\mathrm{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx\right.}{2d}\\ &= -\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}+\frac{(a-b)^3\log(1+\sin(c+dx))}{2d}-\frac{3ab^2\sin(c+dx}{d}\end{aligned}$$

Mathematica [A] time = 0.118111, size = 67, normalized size = 0.84

$$-\frac{6ab^2\sin(c+dx)+(a-b)^3(-\log(\sin(c+dx)+1))+(a+b)^3\log(1-\sin(c+dx))+b^3\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((a + b)^3*Log[1 - Sin[c + d*x]] - (a - b)^3*Log[1 + Sin[c + d*x]] + 6*a*b
^2*Sin[c + d*x] + b^3*Sin[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.054, size = 108, normalized size = 1.4

$$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}-3\,\frac{a^2b\ln(\cos(dx+c))}{d}-3\,\frac{ab^2\sin(dx+c)}{d}+3\,\frac{ab^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))-3/d*a^2*b*ln(cos(d*x+c))-3*a*b^2*sin(d*x+
c)/d+3/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c))-1/2*b^3*sin(d*x+c)^2/d-1/d*b^3*ln(
cos(d*x+c))
```

Maxima [A] time = 0.945462, size = 123, normalized size = 1.54

$$-\frac{b^3\sin(dx+c)^2+6\,ab^2\sin(dx+c)-\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\log(\sin(dx+c)+1)+\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log(\sin}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*sin(d*x + c)^2 + 6*a*b^2*sin(d*x + c) - (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*log(sin(d*x + c) + 1) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(sin(d*x
+ c) - 1))/d
```

Fricas [A] time = 2.44291, size = 221, normalized size = 2.76

$$\frac{b^3\cos(dx+c)^2-6\,ab^2\sin(dx+c)+\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\log(\sin(dx+c)+1)-\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log(-\sin}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*cos(d*x + c)^2 - 6*a*b^2*sin(d*x + c) + (a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*log(sin(d*x + c) + 1) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(-sin(d*x
+ c) + 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sin(c+dx))^3\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*sec(c + d*x), x)
```

Giac [A] time = 1.14519, size = 126, normalized size = 1.58

$$-\frac{b^3 \sin(dx+c)^2 + 6\,ab^2 \sin(dx+c) - \left(a^3 - 3\,a^2b + 3\,ab^2 - b^3\right)\log\left(|\sin(dx+c)+1|\right) + \left(a^3 + 3\,a^2b + 3\,ab^2 + b^3\right)\log\left(|\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*sin(d*x + c)^2 + 6*a*b^2*sin(d*x + c) - (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*log(abs(sin(d*x + c) + 1)) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs
(sin(d*x + c) - 1)))/d
```

3.404 $\int \sec^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=111

$$\frac{ab^2\sin(c+dx)}{2d}+\frac{(a+2b)(a-b)^2\log(\sin(c+dx)+1)}{4d}-\frac{(a-2b)(a+b)^2\log(1-\sin(c+dx))}{4d}+\frac{\sec^2(c+dx)(a\sin(c+d$$

```
[Out] -((a - 2*b)*(a + b)^2*Log[1 - Sin[c + d*x]])/(4*d) + ((a - b)^2*(a + 2*b)*L
og[1 + Sin[c + d*x]])/(4*d) + (a*b^2*Sin[c + d*x])/(2*d) + (Sec[c + d*x]^2*
(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(2*d)
```

Rubi [A] time = 0.134295, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2668, 739, 774, 633, 31}

$$\frac{ab^2\sin(c+dx)}{2d}+\frac{(a+2b)(a-b)^2\log(\sin(c+dx)+1)}{4d}-\frac{(a-2b)(a+b)^2\log(1-\sin(c+dx))}{4d}+\frac{\sec^2(c+dx)(a\sin(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((a - 2*b)*(a + b)^2*Log[1 - Sin[c + d*x]])/(4*d) + ((a - b)^2*(a + 2*b)*L
og[1 + Sin[c + d*x]])/(4*d) + (a*b^2*Sin[c + d*x])/(2*d) + (Sec[c + d*x]^2*
(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(2*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^3\operatorname{Subst}\left(\int \frac{(a+x)^3}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{2d} - \frac{b\operatorname{Subst}\left(\int \frac{(a+x)\left(-a^2+2b^2+a\right.}{b^2-x^2}\right.}{2d}$$

$$= \frac{ab^2\sin(c+dx)}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{2d} + \frac{b\operatorname{Subs}}{}$$

$$= \frac{ab^2\sin(c+dx)}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{2d} + \frac{((a-2}{}$$

$$= -\frac{(a-2b)(a+b)^2\log(1-\sin(c+dx))}{4d} + \frac{(a-b)^2(a+2b)\log(1+\sin(c+dx))}{4d}$$

Mathematica [A] time = 1.30535, size = 176, normalized size = 1.59

$$\frac{\left(a^2-b^2\right)\left((a-2b)(a+b)^2\log(1-\sin(c+dx))-(a-b)^2(a+2b)\log(\sin(c+dx)+1)\right)+\tan^2(c+dx)\left(4a^2b^3-8a^4b-\right.}{4d\left(b^2-a\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((a^2 - b^2)*((a - 2*b)*(a + b)^2*Log[1 - Sin[c + d*x]] - (a - b)^2*(a + 2*
b)*Log[1 + Sin[c + d*x]]) + 2*a^4*b*Sec[c + d*x]^2 - a*(2*a^4 + 4*a^2*b^2 -
 7*b^4 + b^4*Cos[2*(c + d*x)])*Sec[c + d*x]*Tan[c + d*x] + (-8*a^4*b + 4*a^
2*b^3 + 2*b^5 - 2*a*b^4*Sin[c + d*x])*Tan[c + d*x]^2)/(4*(-a^2 + b^2)*d)
```

Maple [A] time = 0.067, size = 154, normalized size = 1.4

$$\frac{a^3 \sec(dx+c)\tan(dx+c)}{2d} + \frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{3a^2b}{2d(\cos(dx+c))^2} + \frac{3ab^2(\sin(dx+c))^3}{2d(\cos(dx+c))^2} + \frac{3ab^2 \sin(}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d*a^3*sec(d*x+c)*tan(d*x+c)+1/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*a
^2*b/cos(d*x+c)^2+3/2/d*a*b^2*sin(d*x+c)^3/cos(d*x+c)^2+3/2*a*b^2*sin(d*x+c
)/d-3/2/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*b^3*tan(d*x+c)^2+1/d*b^3*ln
(cos(d*x+c))
```

Maxima [A] time = 0.967185, size = 132, normalized size = 1.19

$$\frac{\left(a^3 - 3ab^2 + 2b^3\right)\log(\sin(dx+c)+1) - \left(a^3 - 3ab^2 - 2b^3\right)\log(\sin(dx+c)-1) - \frac{2\left(3a^2b+b^3+\left(a^3+3ab^2\right)\sin(dx+c)\right)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((a^3 - 3*a*b^2 + 2*b^3)*log(sin(d*x + c) + 1) - (a^3 - 3*a*b^2 - 2*b^3
)*log(sin(d*x + c) - 1) - 2*(3*a^2*b + b^3 + (a^3 + 3*a*b^2)*sin(d*x + c))/
(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 2.36893, size = 273, normalized size = 2.46

$$\frac{\left(a^3 - 3ab^2 + 2b^3\right)\cos(dx+c)^2\log(\sin(dx+c)+1) - \left(a^3 - 3ab^2 - 2b^3\right)\cos(dx+c)^2\log(-\sin(dx+c)+1) + 6a^2b +}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^3 - 3*a*b^2 + 2*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (a^3 -
3*a*b^2 - 2*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 6*a^2*b + 2*b^3 +
2*(a^3 + 3*a*b^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16731, size = 154, normalized size = 1.39

$$\frac{\left(a^3-3\,ab^2+2\,b^3\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(a^3-3\,ab^2-2\,b^3\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(b^3\sin(dx+c)^2+a^3\sin(dx+c)+3\,ab^2\sin(d\right.}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*((a^3 - 3*a*b^2 + 2*b^3)*log(abs(sin(d*x + c) + 1)) - (a^3 - 3*a*b^2 -
2*b^3)*log(abs(sin(d*x + c) - 1)) - 2*(b^3*sin(d*x + c)^2 + a^3*sin(d*x + c
) + 3*a*b^2*sin(d*x + c) + 3*a^2*b)/(sin(d*x + c)^2 - 1))/d
```

3.405 $\int \sec^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=94

$$\frac{3a\left(a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a\sec^2(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{8d}+\frac{\tan(c+dx)\sec^3(c+dx)(a+b}{4d}$$

```
[Out] (3*a*(a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (3*a*Sec[c + d*x]^2*(b + a*
Sin[c + d*x])*(a + b*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c +
d*x])^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.0800275, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2668, 729, 723, 206}

$$\frac{3a\left(a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a\sec^2(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{8d}+\frac{\tan(c+dx)\sec^3(c+dx)(a+b}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (3*a*(a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (3*a*Sec[c + d*x]^2*(b + a*
Sin[c + d*x])*(a + b*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c +
d*x])^3*Tan[c + d*x])/(4*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 729

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^m*(2*c*x)*(a + c*x^2)^(p + 1))/(4*a*c*(p + 1)), x] - Dist[(m*(2
*c*d))/(4*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /
; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3,
 0] && LtQ[p, -1]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5 \operatorname{Subst}\left(\int \frac{(a+x)^3}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)}{4d} + \frac{\left(3ab^3\right)\operatorname{Subst}\left(\int \frac{(a+x)^2}{\left(b^2-x^2\right)^2}\,dx, x, b\right.}{4d}$$

$$= \frac{3a\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{8d} + \frac{\sec^3(c+dx)(a+b\sin(c}{4d}$$

$$= \frac{3a\left(a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{3a\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin}{8d}$$

Mathematica [B] time = 4.11311, size = 318, normalized size = 3.38

$$\frac{16a^4b\left(3a^2-2b^2\right)\tan^2(c+dx)+8b^3\left(-5a^2b^2+4a^4+b^4\right)\tan^4(c+dx)-6a\left(a^2-b^2\right)^3(\log(1-\sin(c+dx))-\log(\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-6*a*(a^2 - b^2)^3*(Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + a*b*S
ec[c + d*x]^4*(-8*a^5 + 8*a^3*b^2 + (18*a^4*b - 11*a^2*b^3 + 5*b^5)*Sin[3*(
c + d*x)]) + a*(8*a^6 - 22*a^4*b^2 + 29*a^2*b^4 - 3*b^6)*Sec[c + d*x]^3*Tan
```

```
[c + d*x] + 16*a^4*b*(3*a^2 - 2*b^2)*Tan[c + d*x]^2 + 8*b^3*(4*a^4 - 5*a^2*
b^2 + b^4)*Tan[c + d*x]^4 + 4*a*Sec[c + d*x]*Tan[c + d*x]*(3*(a^6 - 5*a^4*b
^2) + 4*b^2*(3*a^4 - 5*a^2*b^2 + 2*b^4)*Tan[c + d*x]^2) + 16*a^2*b*Sec[c +
d*x]^2*(-a^4 + (2*a^4 - 5*a^2*b^2 + 3*b^4)*Tan[c + d*x]^2))/(32*(a^2 - b^2)
^2*d)
```

Maple [B] time = 0.072, size = 195, normalized size = 2.1

$$\frac{a^3\tan(dx+c)(\sec(dx+c))^3}{4d}+\frac{3a^3\sec(dx+c)\tan(dx+c)}{8d}+\frac{3a^3\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{3a^2b}{4d(\cos(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^3*sec(d*x+c)*tan(d*x+c)+3/8/d*a^3
*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*a^2*b/cos(d*x+c)^4+3/4/d*a*b^2*sin(d*x+c)^
3/cos(d*x+c)^4+3/8/d*a*b^2*sin(d*x+c)^3/cos(d*x+c)^2+3/8*a*b^2*sin(d*x+c)/d
-3/8/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*b^3*sin(d*x+c)^4/cos(d*x+c)^4
```

Maxima [A] time = 0.950487, size = 184, normalized size = 1.96

$$\frac{3\left(a^3-ab^2\right)\log(\sin(dx+c)+1)-3\left(a^3-ab^2\right)\log(\sin(dx+c)-1)+\frac{2\left(4b^3\sin(dx+c)^2-3\left(a^3-ab^2\right)\sin(dx+c)^3+6a^2b-2b^3+\left(5a^3+3ab\right.\right.}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(a^3 - a*b^2)*log(sin(d*x + c) + 1) - 3*(a^3 - a*b^2)*log(sin(d*x +
 c) - 1) + 2*(4*b^3*sin(d*x + c)^2 - 3*(a^3 - a*b^2)*sin(d*x + c)^3 + 6*a^2
*b - 2*b^3 + (5*a^3 + 3*a*b^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x +
c)^2 + 1))/d
```

Fricas [A] time = 2.40031, size = 332, normalized size = 3.53

$$\frac{3\left(a^3-ab^2\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(a^3-ab^2\right)\cos(dx+c)^4\log(-\sin(dx+c)+1)-8b^3\cos(dx+c)^2+1}{16d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(a^3 - a*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(a^3 - a*b^2
)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 8*b^3*cos(d*x + c)^2 + 12*a^2*b +
 4*b^3 + 2*(2*a^3 + 6*a*b^2 + 3*(a^3 - a*b^2)*cos(d*x + c)^2)*sin(d*x + c))
/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1669, size = 188, normalized size = 2.

$$\frac{3\left(a^3-ab^2\right)\log\left(|\sin\left(dx+c\right)+1|\right)-3\left(a^3-ab^2\right)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(3a^3\sin(dx+c)^3-3ab^2\sin(dx+c)^3-4b^3\sin(dx+c)^2-5a^3\mathrm{si}\right.}{\left(\sin(dx+c)^2-1\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*(a^3 - a*b^2)*log(abs(sin(d*x + c) + 1)) - 3*(a^3 - a*b^2)*log(abs(
sin(d*x + c) - 1)) - 2*(3*a^3*sin(d*x + c)^3 - 3*a*b^2*sin(d*x + c)^3 - 4*b
^3*sin(d*x + c)^2 - 5*a^3*sin(d*x + c) - 3*a*b^2*sin(d*x + c) - 6*a^2*b + 2
*b^3)/(sin(d*x + c)^2 - 1)^2)/d
```

3.406 $\int \cos^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=158

$$-\frac{b\left(17a^2+4b^2\right)\cos^5(c+dx)}{70d}+\frac{a\left(2a^2+b^2\right)\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3a\left(2a^2+b^2\right)\sin(c+dx)\cos(c+dx)}{16d}+\frac{3}{16}ax\left(2a\right.$$

```
[Out] (3*a*(2*a^2 + b^2)*x)/16 - (b*(17*a^2 + 4*b^2)*Cos[c + d*x]^5)/(70*d) + (3*
a*(2*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(2*a^2 + b^2)*Cos[c
+ d*x]^3*Sin[c + d*x])/(8*d) - (3*a*b*Cos[c + d*x]^5*(a + b*Sin[c + d*x]))/
(14*d) - (b*Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2)/(7*d)
```

Rubi [A] time = 0.216748, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2692, 2862, 2669, 2635, 8}

$$-\frac{b\left(17a^2+4b^2\right)\cos^5(c+dx)}{70d}+\frac{a\left(2a^2+b^2\right)\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3a\left(2a^2+b^2\right)\sin(c+dx)\cos(c+dx)}{16d}+\frac{3}{16}ax\left(2a\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*(2*a^2 + b^2)*x)/16 - (b*(17*a^2 + 4*b^2)*Cos[c + d*x]^5)/(70*d) + (3*
a*(2*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(2*a^2 + b^2)*Cos[c
+ d*x]^3*Sin[c + d*x])/(8*d) - (3*a*b*Cos[c + d*x]^5*(a + b*Sin[c + d*x]))/
(14*d) - (b*Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2)/(7*d)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
```

```
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(c+dx)(a+b\sin(c+dx))^3\,dx &= -\frac{b\cos^5(c+dx)(a+b\sin(c+dx))^2}{7d}+\frac{1}{7}\int \cos^4(c+dx)(a+b\sin(c+dx))\left(7a^2\right.\\
&= -\frac{3ab\cos^5(c+dx)(a+b\sin(c+dx))}{14d}-\frac{b\cos^5(c+dx)(a+b\sin(c+dx))^2}{7d}+\frac{1}{42}\\
&= -\frac{b\left(17a^2+4b^2\right)\cos^5(c+dx)}{70d}-\frac{3ab\cos^5(c+dx)(a+b\sin(c+dx))}{14d}-\frac{b\cos^5(c\\
&= -\frac{b\left(17a^2+4b^2\right)\cos^5(c+dx)}{70d}+\frac{a\left(2a^2+b^2\right)\cos^3(c+dx)\sin(c+dx)}{8d}-\frac{3ab\cos\\
&= -\frac{b\left(17a^2+4b^2\right)\cos^5(c+dx)}{70d}+\frac{3a\left(2a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a\left(2a^2\right.\\
&= \frac{3}{16}a\left(2a^2+b^2\right)x-\frac{b\left(17a^2+4b^2\right)\cos^5(c+dx)}{70d}+\frac{3a\left(2a^2+b^2\right)\cos(c+dx)\sin}{16d}
\end{aligned}
$$

Mathematica [A] time = 0.384893, size = 182, normalized size = 1.15

$$\frac{-105b\left(8a^2+b^2\right)\cos(c+dx)-35\left(12a^2b+b^3\right)\cos(3(c+dx))-84a^2b\cos(5(c+dx))+560a^3\sin(2(c+dx))+70a^3\sin(4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (840*a^3*c + 420*a*b^2*c + 840*a^3*d*x + 420*a*b^2*d*x - 105*b*(8*a^2 + b^2
)*Cos[c + d*x] - 35*(12*a^2*b + b^3)*Cos[3*(c + d*x)] - 84*a^2*b*Cos[5*(c +
 d*x)] + 7*b^3*Cos[5*(c + d*x)] + 5*b^3*Cos[7*(c + d*x)] + 560*a^3*Sin[2*(c
 + d*x)] + 105*a*b^2*Sin[2*(c + d*x)] + 70*a^3*Sin[4*(c + d*x)] - 105*a*b^2
*Sin[4*(c + d*x)] - 35*a*b^2*Sin[6*(c + d*x)])/(2240*d)
```

Maple [A] time = 0.053, size = 145, normalized size = 0.9

$$\frac{1}{d}\left(b^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^5}{7}-\frac{2(\cos(dx+c))^5}{35}\right)+3ab^2\left(-1/6\sin(dx+c)(\cos(dx+c))^5+1/24\left((\cos(dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*a*b^2*(-1/6*s
in(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d
*x+1/16*c)-3/5*a^2*b*cos(d*x+c)^5+a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*si
n(d*x+c)+3/8*d*x+3/8*c))
```

Maple [A] time = 0.959663, size = 158, normalized size = 1.

$$-\frac{1344a^2b\cos(dx+c)^5-70(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^3-35\left(4\sin(2dx+2c)^3+12dx+12c\right.}{2240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2240*(1344*a^2*b*cos(d*x + c)^5 - 70*(12*d*x + 12*c + sin(4*d*x + 4*c) +
 8*sin(2*d*x + 2*c))*a^3 - 35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin
(4*d*x + 4*c))*a*b^2 - 64*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b^3)/d
```

Fricas [A] time = 2.51896, size = 279, normalized size = 1.77

$$\frac{80\,b^3\cos\left(dx+c\right)^7-112\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)^5+105\left(2\,a^3+ab^2\right)dx-35\left(8\,ab^2\cos\left(dx+c\right)^5-2\left(2\,a^3+ab^2\right)\cos\left(d\right.\right.}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(80*b^3*cos(d*x + c)^7 - 112*(3*a^2*b + b^3)*cos(d*x + c)^5 + 105*(2*
a^3 + a*b^2)*d*x - 35*(8*a*b^2*cos(d*x + c)^5 - 2*(2*a^3 + a*b^2)*cos(d*x +
 c)^3 - 3*(2*a^3 + a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 8.27802, size = 348, normalized size = 2.2

$$\begin{cases} \frac{3a^3x\sin^4(c+dx)}{8}+\frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^3x\cos^4(c+dx)}{8}+\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{3a^2b\cos^5(c+dx)}{5d}+ \\ x\left(a+b\sin(c)\right)^3\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**3*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 3*a**2*b*cos(c + d*x)**
5/(5*d) + 3*a*b**2*x*sin(c + d*x)**6/16 + 9*a*b**2*x*sin(c + d*x)**4*cos(c
+ d*x)**2/16 + 9*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*a*b**2*x*c
os(c + d*x)**6/16 + 3*a*b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + a*b**2*s
in(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*a*b**2*sin(c + d*x)*cos(c + d*x)**
5/(16*d) - b**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 2*b**3*cos(c + d*x)
**7/(35*d), Ne(d, 0)), (x*(a + b*sin(c))**3*cos(c)**4, True))
```

Giac [A] time = 1.10315, size = 234, normalized size = 1.48

$$\frac{b^3\cos(7\,dx+7\,c)}{448\,d}-\frac{ab^2\sin(6\,dx+6\,c)}{64\,d}+\frac{3}{16}\left(2\,a^3+ab^2\right)x-\frac{\left(12\,a^2b-b^3\right)\cos(5\,dx+5\,c)}{320\,d}-\frac{\left(12\,a^2b+b^3\right)\cos(3\,dx+3}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/448*b^3*cos(7*d*x + 7*c)/d - 1/64*a*b^2*sin(6*d*x + 6*c)/d + 3/16*(2*a^3
+ a*b^2)*x - 1/320*(12*a^2*b - b^3)*cos(5*d*x + 5*c)/d - 1/64*(12*a^2*b + b
^3)*cos(3*d*x + 3*c)/d - 3/64*(8*a^2*b + b^3)*cos(d*x + c)/d + 1/64*(2*a^3
- 3*a*b^2)*sin(4*d*x + 4*c)/d + 1/64*(16*a^3 + 3*a*b^2)*sin(2*d*x + 2*c)/d
```

3.407 $\int \cos^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=131

$$-\frac{b\left(27a^2+8b^2\right)\cos^3(c+dx)}{60d}+\frac{a\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax\left(4a^2+3b^2\right)-\frac{b\cos^3(c+dx)(a+b\sin(c+}{5d}$$

```
[Out] (a*(4*a^2 + 3*b^2)*x)/8 - (b*(27*a^2 + 8*b^2)*Cos[c + d*x]^3)/(60*d) + (a*(
4*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (7*a*b*Cos[c + d*x]^3*(a
+ b*Sin[c + d*x]))/(20*d) - (b*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(5*d)
```

Rubi [A] time = 0.193578, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2692, 2862, 2669, 2635, 8}

$$-\frac{b\left(27a^2+8b^2\right)\cos^3(c+dx)}{60d}+\frac{a\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax\left(4a^2+3b^2\right)-\frac{b\cos^3(c+dx)(a+b\sin(c+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(4*a^2 + 3*b^2)*x)/8 - (b*(27*a^2 + 8*b^2)*Cos[c + d*x]^3)/(60*d) + (a*(
4*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (7*a*b*Cos[c + d*x]^3*(a
+ b*Sin[c + d*x]))/(20*d) - (b*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(5*d)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
 !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)(a+b\sin(c+dx))^3\,dx &= -\frac{b\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}+\frac{1}{5}\int \cos^2(c+dx)(a+b\sin(c+dx))\left(5a^2\right.\\
&= -\frac{7ab\cos^3(c+dx)(a+b\sin(c+dx))}{20d}-\frac{b\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}+\frac{1}{20}\\
&= -\frac{b\left(27a^2+8b^2\right)\cos^3(c+dx)}{60d}-\frac{7ab\cos^3(c+dx)(a+b\sin(c+dx))}{20d}-\frac{b\cos^3(c+}{}\\
&= -\frac{b\left(27a^2+8b^2\right)\cos^3(c+dx)}{60d}+\frac{a\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{8d}-\frac{7ab\cos^3}{}\\
&= \frac{1}{8}a\left(4a^2+3b^2\right)x-\frac{b\left(27a^2+8b^2\right)\cos^3(c+dx)}{60d}+\frac{a\left(4a^2+3b^2\right)\cos(c+dx)\sin(}{8d}
\end{aligned}$$

Mathematica [A] time = 0.539743, size = 107, normalized size = 0.82

$$\frac{15a\left(4\left(4a^2+3b^2\right)(c+dx)+8a^2\sin(2(c+dx))-3b^2\sin(4(c+dx))\right)-60b\left(6a^2+b^2\right)\cos(c+dx)-10\left(12a^2b+b^3\right)\cos(}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-60*b*(6*a^2 + b^2)*Cos[c + d*x] - 10*(12*a^2*b + b^3)*Cos[3*(c + d*x)] +
6*b^3*Cos[5*(c + d*x)] + 15*a*(4*(4*a^2 + 3*b^2)*(c + d*x) + 8*a^2*Sin[2*(c
 + d*x)] - 3*b^2*Sin[4*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.05, size = 123, normalized size = 0.9

$$\frac{1}{d}\left(b^{3}\left(-\frac{(\sin(dx+c))^{2}(\cos(dx+c))^{3}}{5}-\frac{2\,(\cos(dx+c))^{3}}{15}\right)+3\,ab^{2}\left(-1/4\,(\cos(dx+c))^{3}\sin(dx+c)+1/8\,\cos(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3*a*b^2*(-1/4*c
os(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-a^2*b*cos(d
*x+c)^3+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.961946, size = 126, normalized size = 0.96

$$-\frac{480\,a^{2}b\cos(dx+c)^{3}-120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^{3}-45\,(4\,dx+4\,c-\sin(4\,dx+4\,c))ab^{2}-32\left(3\,\cos(dx+c)^{5}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/480*(480*a^2*b*cos(d*x + c)^3 - 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3
 - 45*(4*d*x + 4*c - sin(4*d*x + 4*c))*a*b^2 - 32*(3*cos(d*x + c)^5 - 5*cos
(d*x + c)^3)*b^3)/d
```

Fricas [A] time = 2.23542, size = 232, normalized size = 1.77

$$\frac{24\,b^{3}\cos(dx+c)^{5}-40\left(3\,a^{2}b+b^{3}\right)\cos(dx+c)^{3}+15\left(4\,a^{3}+3\,ab^{2}\right)dx-15\left(6\,ab^{2}\cos(dx+c)^{3}-\left(4\,a^{3}+3\,ab^{2}\right)\cos(dx\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(24*b^3*cos(d*x + c)^5 - 40*(3*a^2*b + b^3)*cos(d*x + c)^3 + 15*(4*a^
3 + 3*a*b^2)*d*x - 15*(6*a*b^2*cos(d*x + c)^3 - (4*a^3 + 3*a*b^2)*cos(d*x +
 c))*sin(d*x + c))/d
```

Sympy [A] time = 2.59479, size = 236, normalized size = 1.8

$$\begin{cases} \frac{a^3x\sin^2(c+dx)}{2} + \frac{a^3x\cos^2(c+dx)}{2} + \frac{a^3\sin(c+dx)\cos(c+dx)}{2d} - \frac{a^2b\cos^3(c+dx)}{d} + \frac{3ab^2x\sin^4(c+dx)}{8} + \frac{3ab^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ab^2x\cos^4(c+ \\ x(a+b\sin(c))^3\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**2/2 + a**3*x*cos(c + d*x)**2/2 + a**3*sin(c
 + d*x)*cos(c + d*x)/(2*d) - a**2*b*cos(c + d*x)**3/d + 3*a*b**2*x*sin(c +
d*x)**4/8 + 3*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a*b**2*x*cos(c
 + d*x)**4/8 + 3*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a*b**2*sin(c
 + d*x)*cos(c + d*x)**3/(8*d) - b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d)
- 2*b**3*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sin(c))**3*cos(c)**2,
 True))
```

Giac [A] time = 1.08993, size = 153, normalized size = 1.17

$$\frac{b^3\cos(5\,dx+5\,c)}{80\,d} - \frac{3\,ab^2\sin(4\,dx+4\,c)}{32\,d} + \frac{a^3\sin(2\,dx+2\,c)}{4\,d} + \frac{1}{8}\left(4\,a^3+3\,ab^2\right)x - \frac{\left(12\,a^2b+b^3\right)\cos(3\,dx+3\,c)}{48\,d} - \frac{(6\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/80*b^3*cos(5*d*x + 5*c)/d - 3/32*a*b^2*sin(4*d*x + 4*c)/d + 1/4*a^3*sin(2
*d*x + 2*c)/d + 1/8*(4*a^3 + 3*a*b^2)*x - 1/48*(12*a^2*b + b^3)*cos(3*d*x +
 3*c)/d - 1/8*(6*a^2*b + b^3)*cos(d*x + c)/d
```

3.408 $\int \sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=79

$$\frac{2b\left(a^2+b^2\right)\cos(c+dx)}{d}+\frac{ab^2\sin(c+dx)\cos(c+dx)}{d}-3ab^2x+\frac{\sec(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{d}$$

```
[Out] -3*a*b^2*x + (2*b*(a^2 + b^2)*Cos[c + d*x])/d + (a*b^2*Cos[c + d*x]*Sin[c +
 d*x])/d + (Sec[c + d*x]*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/d
```

Rubi [A] time = 0.0704697, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2691, 2734}

$$\frac{2b\left(a^2+b^2\right)\cos(c+dx)}{d}+\frac{ab^2\sin(c+dx)\cos(c+dx)}{d}-3ab^2x+\frac{\sec(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] -3*a*b^2*x + (2*b*(a^2 + b^2)*Cos[c + d*x])/d + (a*b^2*Cos[c + d*x]*Sin[c +
 d*x])/d + (Sec[c + d*x]*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/d
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{d} - \int (a+b\sin(c+dx))\left(2b^2+\right.$$
$$= -3ab^2x + \frac{2b\left(a^2+b^2\right)\cos(c+dx)}{d} + \frac{ab^2\cos(c+dx)\sin(c+dx)}{d} + \frac{\sec(c+dx)(b}{}$$

Mathematica [A] time = 0.30729, size = 68, normalized size = 0.86

$$\frac{2a\left(a^2+3b^2\right)\tan(c+dx)+\sec(c+dx)\left(6a^2b+b^3\cos(2(c+dx))+3b^3\right)-6ab^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-6*a*b^2*(c + d*x) + (6*a^2*b + 3*b^3 + b^3*Cos[2*(c + d*x)])*Sec[c + d*x]
 + 2*a*(a^2 + 3*b^2)*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.046, size = 89, normalized size = 1.1

$$\frac{1}{d}\left(a^3\tan(dx+c)+3\,\frac{a^2b}{\cos(dx+c)}+3\,ab^2(\tan(dx+c)-dx-c)+b^3\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*tan(d*x+c)+3*a^2*b/cos(d*x+c)+3*a*b^2*(tan(d*x+c)-d*x-c)+b^3*(sin(
d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 1.44526, size = 95, normalized size = 1.2

$$-\frac{3\,(dx+c-\tan(dx+c))ab^2-b^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-a^3\tan(dx+c)-\frac{3\,a^2b}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(3*(d*x + c - tan(d*x + c))*a*b^2 - b^3*(1/cos(d*x + c) + cos(d*x + c)) -
a^3*tan(d*x + c) - 3*a^2*b/cos(d*x + c))/d
```

Fricas [A] time = 2.50838, size = 154, normalized size = 1.95

$$-\frac{3\,ab^2dx\cos\left(dx+c\right)-b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3-\left(a^3+3\,ab^2\right)\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*a*b^2*d*x*cos(d*x + c) - b^3*cos(d*x + c)^2 - 3*a^2*b - b^3 - (a^3 + 3*
a*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10934, size = 166, normalized size = 2.1

$$-\frac{3\left(dx+c\right)ab^2+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2b+2\,b^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(d*x + c)*a*b^2 + 2*(a^3*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^2*tan(1/2*d*x +
 1/2*c)^3 + 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) + 3*a
*b^2*tan(1/2*d*x + 1/2*c) + 3*a^2*b + 2*b^3)/(tan(1/2*d*x + 1/2*c)^4 - 1))/
d
```

3.409 $\int \sec^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=84

$$\frac{2a\left(a^2-b^2\right)\tan(c+dx)}{3d}+\frac{2b\left(a^2-b^2\right)\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{3d}$$

```
[Out] (2*b*(a^2 - b^2)*Sec[c + d*x])/(3*d) + (Sec[c + d*x]^3*(b + a*Sin[c + d*x])
*(a + b*Sin[c + d*x])^2)/(3*d) + (2*a*(a^2 - b^2)*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.088474, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2691, 12, 2669, 3767, 8}

$$\frac{2a\left(a^2-b^2\right)\tan(c+dx)}{3d}+\frac{2b\left(a^2-b^2\right)\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b*(a^2 - b^2)*Sec[c + d*x])/(3*d) + (Sec[c + d*x]^3*(b + a*Sin[c + d*x])
*(a + b*Sin[c + d*x])^2)/(3*d) + (2*a*(a^2 - b^2)*Tan[c + d*x])/(3*d)
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3d}-\frac{1}{3}\int\left(-2a^2+2b^2\right)\sec^2(c+\\ &= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3d}+\frac{1}{3}\left(2\left(a^2-b^2\right)\right)\int\sec^2(c+\\ &= \frac{2b\left(a^2-b^2\right)\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3d}+\\ &= \frac{2b\left(a^2-b^2\right)\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3d}-\\ &= \frac{2b\left(a^2-b^2\right)\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3d}+\end{aligned}$$

Mathematica [A] time = 0.412579, size = 136, normalized size = 1.62

$$\frac{\sec^3(c+dx)\left(\left(15b^3-9a^2b\right)\cos(c+dx)-3a^2b\cos(3(c+dx))+24a^2b+12a^3\sin(c+dx)+4a^3\sin(3(c+dx))+18ab^2\sin\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(24*a^2*b - 4*b^3 + (-9*a^2*b + 15*b^3)*Cos[c + d*x] - 12*b
^3*Cos[2*(c + d*x)] - 3*a^2*b*Cos[3*(c + d*x)] + 5*b^3*Cos[3*(c + d*x)] + 1
2*a^3*Sin[c + d*x] + 18*a*b^2*Sin[c + d*x] + 4*a^3*Sin[3*(c + d*x)] - 6*a*b
```

```
^2*Sin[3*(c + d*x)]))/(24*d)
```

Maple [A] time = 0.068, size = 122, normalized size = 1.5

$$\frac{1}{d}\left(-a^{3}\left(-\frac{2}{3}-\frac{(\sec(dx+c))^{2}}{3}\right)\tan(dx+c)+\frac{a^{2}b}{(\cos(dx+c))^{3}}+\frac{ab^{2}(\sin(dx+c))^{3}}{(\cos(dx+c))^{3}}+b^{3}\left(\frac{(\sin(dx+c))^{4}}{3(\cos(dx+c))^{3}}-\frac{(\sin(dx+c))^{4}}{3\cos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^2*b/cos(d*x+c)^3+a*b^2*sin(d
*x+c)^3/cos(d*x+c)^3+b^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/co
s(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 0.954564, size = 108, normalized size = 1.29

$$\frac{3\,ab^{2}\tan(dx+c)^{3}+\left(\tan(dx+c)^{3}+3\,\tan(dx+c)\right)a^{3}-\frac{\left(3\,\cos(dx+c)^{2}-1\right)b^{3}}{\cos(dx+c)^{3}}+\frac{3\,a^{2}b}{\cos(dx+c)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a*b^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - (3*co
s(d*x + c)^2 - 1)*b^3/cos(d*x + c)^3 + 3*a^2*b/cos(d*x + c)^3)/d
```

Fricas [A] time = 2.42204, size = 176, normalized size = 2.1

$$-\frac{3\,b^{3}\cos(dx+c)^{2}-3\,a^{2}b-b^{3}-\left(a^{3}+3\,ab^{2}+\left(2\,a^{3}-3\,ab^{2}\right)\cos(dx+c)^{2}\right)\sin(dx+c)}{3\,d\cos(dx+c)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^3*cos(d*x + c)^2 - 3*a^2*b - b^3 - (a^3 + 3*a*b^2 + (2*a^3 - 3*a*
b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11149, size = 173, normalized size = 2.06

$$-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,b^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^3*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 2*a^3
*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*b^3*tan(1/2*d
*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 3*a^2*b - 2*b^3)/((tan(1/2*d*x
 + 1/2*c)^2 - 1)^3*d)
```

3.410 $\int \sec^6(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=135

$$\frac{2a\left(4a^2-3b^2\right)\tan(c+dx)}{15d}+\frac{2b\left(2a^2-b^2\right)\sec(c+dx)}{15d}+\frac{2\sec^3(c+dx)(a+b\sin(c+dx))\left(\left(2a^2-b^2\right)\sin(c+dx)+ab\right)}{15d}$$

```
[Out] (2*b*(2*a^2 - b^2)*Sec[c + d*x])/(15*d) + (Sec[c + d*x]^5*(b + a*Sin[c + d*
x])*(a + b*Sin[c + d*x])^2)/(5*d) + (2*Sec[c + d*x]^3*(a + b*Sin[c + d*x])*
(a*b + (2*a^2 - b^2)*Sin[c + d*x]))/(15*d) + (2*a*(4*a^2 - 3*b^2)*Tan[c + d
*x])/(15*d)
```

Rubi [A] time = 0.19216, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2691, 2861, 2669, 3767, 8}

$$\frac{2a\left(4a^2-3b^2\right)\tan(c+dx)}{15d}+\frac{2b\left(2a^2-b^2\right)\sec(c+dx)}{15d}+\frac{2\sec^3(c+dx)(a+b\sin(c+dx))\left(\left(2a^2-b^2\right)\sin(c+dx)+ab\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b*(2*a^2 - b^2)*Sec[c + d*x])/(15*d) + (Sec[c + d*x]^5*(b + a*Sin[c + d*
x])*(a + b*Sin[c + d*x])^2)/(5*d) + (2*Sec[c + d*x]^3*(a + b*Sin[c + d*x])*
(a*b + (2*a^2 - b^2)*Sin[c + d*x]))/(15*d) + (2*a*(4*a^2 - 3*b^2)*Tan[c + d
*x])/(15*d)
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
```

```
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \sec^6(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5d} - \frac{1}{5}\int \sec^4(c+dx)(a+b\sin \\
&= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5d} + \frac{2\sec^3(c+dx)(a+b\sin(c - \\
&= \frac{2b\left(2a^2-b^2\right)\sec(c+dx)}{15d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5d} + \\
&= \frac{2b\left(2a^2-b^2\right)\sec(c+dx)}{15d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5d} + \\
&= \frac{2b\left(2a^2-b^2\right)\sec(c+dx)}{15d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5d} +
\end{aligned}$$

Mathematica [A] time = 0.623113, size = 190, normalized size = 1.41

$$\frac{\sec^5(c+dx)\left(\left(110b^3-270a^2b\right)\cos(c+dx)-135a^2b\cos(3(c+dx))-27a^2b\cos(5(c+dx))+1152a^2b+640a^3\sin(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^5*(1152*a^2*b + 64*b^3 + (-270*a^2*b + 110*b^3)*Cos[c + d*x]
- 320*b^3*Cos[2*(c + d*x)] - 135*a^2*b*Cos[3*(c + d*x)] + 55*b^3*Cos[3*(c +
 d*x)] - 27*a^2*b*Cos[5*(c + d*x)] + 11*b^3*Cos[5*(c + d*x)] + 640*a^3*Sin[
c + d*x] + 960*a*b^2*Sin[c + d*x] + 320*a^3*Sin[3*(c + d*x)] - 240*a*b^2*Si
n[3*(c + d*x)] + 64*a^3*Sin[5*(c + d*x)] - 48*a*b^2*Sin[5*(c + d*x)]))/(192
0*d)
```

Maple [A] time = 0.085, size = 173, normalized size = 1.3

$$\frac{1}{d}\left(-a^{3}\left(-\frac{8}{15}-\frac{(\sec(dx+c))^{4}}{5}-\frac{4\,(\sec(dx+c))^{2}}{15}\right)\tan(dx+c)+\frac{3\,a^{2}b}{5\,(\cos(dx+c))^{5}}+3\,ab^{2}\left(1/5\,\frac{(\sin(dx+c))^{3}}{(\cos(dx+c))^{5}}+2/15\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+3/5*a^2*b/c
os(d*x+c)^5+3*a*b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*
x+c)^3)+b^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1
/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 0.972075, size = 142, normalized size = 1.05

$$\frac{\left(3\,\tan(dx+c)^{5}+10\,\tan(dx+c)^{3}+15\,\tan(dx+c)\right)a^{3}+3\left(3\,\tan(dx+c)^{5}+5\,\tan(dx+c)^{3}\right)ab^{2}-\frac{\left(5\,\cos(dx+c)^{2}-3\right)b^{3}}{\cos(dx+c)^{5}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 3*(3*t
an(d*x + c)^5 + 5*tan(d*x + c)^3)*a*b^2 - (5*cos(d*x + c)^2 - 3)*b^3/cos(d*
x + c)^5 + 9*a^2*b/cos(d*x + c)^5)/d
```

Fricas [A] time = 2.39549, size = 232, normalized size = 1.72

$$-\frac{5\,b^{3}\cos\left(dx+c\right)^{2}-9\,a^{2}b-3\,b^{3}-\left(2\left(4\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{4}+3\,a^{3}+9\,ab^{2}+\left(4\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right.}{15\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^3*cos(d*x + c)^2 - 9*a^2*b - 3*b^3 - (2*(4*a^3 - 3*a*b^2)*cos(d*
x + c)^4 + 3*a^3 + 9*a*b^2 + (4*a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c)
)/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10546, size = 328, normalized size = 2.43

$$-\frac{2\left(15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+45\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-20\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+60\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+30\,b^{3}\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^3*tan(1/2*d*x + 1/2*c)^9 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 20
*a^3*tan(1/2*d*x + 1/2*c)^7 + 60*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 30*b^3*tan(
1/2*d*x + 1/2*c)^6 + 58*a^3*tan(1/2*d*x + 1/2*c)^5 + 24*a*b^2*tan(1/2*d*x +
```

```
1/2*c)^5 + 90*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 10*b^3*tan(1/2*d*x + 1/2*c)^4
 - 20*a^3*tan(1/2*d*x + 1/2*c)^3 + 60*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 10*b^3
*tan(1/2*d*x + 1/2*c)^2 + 15*a^3*tan(1/2*d*x + 1/2*c) + 9*a^2*b - 2*b^3)/((
tan(1/2*d*x + 1/2*c)^2 - 1)^5*d)
```

3.411 $\int \sec^8(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=165

$$\frac{4a(2a^2-b^2)\tan^3(c+dx)}{35d}+\frac{12a(2a^2-b^2)\tan(c+dx)}{35d}+\frac{2b(3a^2-b^2)\sec^3(c+dx)}{35d}+\frac{2\sec^5(c+dx)(a+b\sin(c+dx))}{35d}$$

```
[Out] (2*b*(3*a^2 - b^2)*Sec[c + d*x]^3)/(35*d) + (Sec[c + d*x]^7*(b + a*Sin[c +
d*x])*(a + b*Sin[c + d*x])^2)/(7*d) + (2*Sec[c + d*x]^5*(a + b*Sin[c + d*x]
)*(2*a*b + (3*a^2 - b^2)*Sin[c + d*x]))/(35*d) + (12*a*(2*a^2 - b^2)*Tan[c
+ d*x])/(35*d) + (4*a*(2*a^2 - b^2)*Tan[c + d*x]^3)/(35*d)
```

Rubi [A] time = 0.207002, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2691, 2861, 2669, 3767}

$$\frac{4a(2a^2-b^2)\tan^3(c+dx)}{35d}+\frac{12a(2a^2-b^2)\tan(c+dx)}{35d}+\frac{2b(3a^2-b^2)\sec^3(c+dx)}{35d}+\frac{2\sec^5(c+dx)(a+b\sin(c+dx))}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b*(3*a^2 - b^2)*Sec[c + d*x]^3)/(35*d) + (Sec[c + d*x]^7*(b + a*Sin[c +
d*x])*(a + b*Sin[c + d*x])^2)/(7*d) + (2*Sec[c + d*x]^5*(a + b*Sin[c + d*x]
)*(2*a*b + (3*a^2 - b^2)*Sin[c + d*x]))/(35*d) + (12*a*(2*a^2 - b^2)*Tan[c
+ d*x])/(35*d) + (4*a*(2*a^2 - b^2)*Tan[c + d*x]^3)/(35*d)
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
```

```
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7d} - \frac{1}{7}\int \sec^6(c+dx)(a+b\,\mathrm{si}\\ &= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7d} + \frac{2\sec^5(c+dx)(a+b\sin(c}{}\\ &= \frac{2b\left(3a^2-b^2\right)\sec^3(c+dx)}{35d} + \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7d}\\ &= \frac{2b\left(3a^2-b^2\right)\sec^3(c+dx)}{35d} + \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7d}\\ &= \frac{2b\left(3a^2-b^2\right)\sec^3(c+dx)}{35d} + \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7d}\end{aligned}$$

Mathematica [A] time = 0.90584, size = 245, normalized size = 1.48

$$\frac{\sec^7(c+dx)\left(35b\left(17b^2-75a^2\right)\cos(c+dx)-1575a^2b\cos(3(c+dx))-525a^2b\cos(5(c+dx))-75a^2b\cos(7(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^7*(15360*a^2*b + 1536*b^3 + 35*b*(-75*a^2 + 17*b^2)*Cos[c + d
*x] - 3584*b^3*Cos[2*(c + d*x)] - 1575*a^2*b*Cos[3*(c + d*x)] + 357*b^3*Cos
[3*(c + d*x)] - 525*a^2*b*Cos[5*(c + d*x)] + 119*b^3*Cos[5*(c + d*x)] - 75*
a^2*b*Cos[7*(c + d*x)] + 17*b^3*Cos[7*(c + d*x)] + 8960*a^3*Sin[c + d*x] +
13440*a*b^2*Sin[c + d*x] + 5376*a^3*Sin[3*(c + d*x)] - 2688*a*b^2*Sin[3*(c
+ d*x)] + 1792*a^3*Sin[5*(c + d*x)] - 896*a*b^2*Sin[5*(c + d*x)] + 256*a^3*
Sin[7*(c + d*x)] - 128*a*b^2*Sin[7*(c + d*x)]))/(35840*d)
```

Maple [A] time = 0.082, size = 219, normalized size = 1.3

$$\frac{1}{d}\left(-a^3\left(-\frac{16}{35}-\frac{(\sec(dx+c))^6}{7}-\frac{6\ (\sec(dx+c))^4}{35}-\frac{8\ (\sec(dx+c))^2}{35}\right)\tan(dx+c)+\frac{3\,a^2b}{7\ (\cos(dx+c))^7}+3\,ab^2\left(1/7\,\frac{(\sin}{(\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+3/7*a^2*b/cos(d*x+c)^7+3*a*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*
sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/7*sin(d*x
+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x
+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 0.968928, size = 167, normalized size = 1.01

$$\frac{\left(5\ \tan(dx+c)^7+21\ \tan(dx+c)^5+35\ \tan(dx+c)^3+35\ \tan(dx+c)\right)a^3+\left(15\ \tan(dx+c)^7+42\ \tan(dx+c)^5+35\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/35*((5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*
x + c))*a^3 + (15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a
*b^2 - (7*cos(d*x + c)^2 - 5)*b^3/cos(d*x + c)^7 + 15*a^2*b/cos(d*x + c)^7)
/d
```

Fricas [A] time = 2.36043, size = 279, normalized size = 1.69

$$-\frac{7\,b^3\cos\left(dx+c\right)^2-15\,a^2b-5\,b^3-\left(8\left(2\,a^3-ab^2\right)\cos\left(dx+c\right)^6+4\left(2\,a^3-ab^2\right)\cos\left(dx+c\right)^4+5\,a^3+15\,ab^2+3\left(2\,a^3\right.\right.}{35\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(7*b^3*cos(d*x + c)^2 - 15*a^2*b - 5*b^3 - (8*(2*a^3 - a*b^2)*cos(d*x
 + c)^6 + 4*(2*a^3 - a*b^2)*cos(d*x + c)^4 + 5*a^3 + 15*a*b^2 + 3*(2*a^3 -
a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13117, size = 483, normalized size = 2.93

$$-\frac{2\left(35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+105\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-70\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+140\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+70\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*a^3*tan(1/2*d*x + 1/2*c)^13 + 105*a^2*b*tan(1/2*d*x + 1/2*c)^12 -
 70*a^3*tan(1/2*d*x + 1/2*c)^11 + 140*a*b^2*tan(1/2*d*x + 1/2*c)^11 + 70*b^
3*tan(1/2*d*x + 1/2*c)^10 + 301*a^3*tan(1/2*d*x + 1/2*c)^9 + 112*a*b^2*tan(
1/2*d*x + 1/2*c)^9 + 525*a^2*b*tan(1/2*d*x + 1/2*c)^8 + 70*b^3*tan(1/2*d*x
```

```
+ 1/2*c)^8 - 212*a^3*tan(1/2*d*x + 1/2*c)^7 + 456*a*b^2*tan(1/2*d*x + 1/2*c
)^7 + 140*b^3*tan(1/2*d*x + 1/2*c)^6 + 301*a^3*tan(1/2*d*x + 1/2*c)^5 + 112
*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 315*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 28*b^3*t
an(1/2*d*x + 1/2*c)^4 - 70*a^3*tan(1/2*d*x + 1/2*c)^3 + 140*a*b^2*tan(1/2*d
*x + 1/2*c)^3 + 14*b^3*tan(1/2*d*x + 1/2*c)^2 + 35*a^3*tan(1/2*d*x + 1/2*c)
 + 15*a^2*b - 2*b^3)/((tan(1/2*d*x + 1/2*c)^2 - 1)^7*d)
```

3.412 $\int \sec^{10}(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=192

$$\frac{2a(8a^2-3b^2)\tan^5(c+dx)}{105d}+\frac{4a(8a^2-3b^2)\tan^3(c+dx)}{63d}+\frac{2a(8a^2-3b^2)\tan(c+dx)}{21d}+\frac{2b(4a^2-b^2)\sec^5(c+dx)}{63d}+$$

```
[Out] (2*b*(4*a^2 - b^2)*Sec[c + d*x]^5)/(63*d) + (Sec[c + d*x]^9*(b + a*Sin[c +
d*x])*(a + b*Sin[c + d*x])^2)/(9*d) + (2*Sec[c + d*x]^7*(a + b*Sin[c + d*x]
)*(3*a*b + (4*a^2 - b^2)*Sin[c + d*x]))/(63*d) + (2*a*(8*a^2 - 3*b^2)*Tan[c
 + d*x])/(21*d) + (4*a*(8*a^2 - 3*b^2)*Tan[c + d*x]^3)/(63*d) + (2*a*(8*a^2
 - 3*b^2)*Tan[c + d*x]^5)/(105*d)
```

Rubi [A] time = 0.221397, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2691, 2861, 2669, 3767}

$$\frac{2a(8a^2-3b^2)\tan^5(c+dx)}{105d}+\frac{4a(8a^2-3b^2)\tan^3(c+dx)}{63d}+\frac{2a(8a^2-3b^2)\tan(c+dx)}{21d}+\frac{2b(4a^2-b^2)\sec^5(c+dx)}{63d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (2*b*(4*a^2 - b^2)*Sec[c + d*x]^5)/(63*d) + (Sec[c + d*x]^9*(b + a*Sin[c +
d*x])*(a + b*Sin[c + d*x])^2)/(9*d) + (2*Sec[c + d*x]^7*(a + b*Sin[c + d*x]
)*(3*a*b + (4*a^2 - b^2)*Sin[c + d*x]))/(63*d) + (2*a*(8*a^2 - 3*b^2)*Tan[c
 + d*x])/(21*d) + (4*a*(8*a^2 - 3*b^2)*Tan[c + d*x]^3)/(63*d) + (2*a*(8*a^2
 - 3*b^2)*Tan[c + d*x]^5)/(105*d)
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{10}(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{9d}-\frac{1}{9}\int \sec^8(c+dx)(a+b\sin\\ &= \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{9d}+\frac{2\sec^7(c+dx)(a+b\sin(c\\ &= \frac{2b\left(4a^2-b^2\right)\sec^5(c+dx)}{63d}+\frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{9d}\\ &= \frac{2b\left(4a^2-b^2\right)\sec^5(c+dx)}{63d}+\frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{9d}\\ &= \frac{2b\left(4a^2-b^2\right)\sec^5(c+dx)}{63d}+\frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{9d}\end{aligned}$$

Mathematica [A] time = 1.59264, size = 299, normalized size = 1.56

$$\frac{\sec^9(c+dx)\left(3150b\left(23b^2-147a^2\right)\cos(c+dx)-308700a^2b\cos(3(c+dx))-132300a^2b\cos(5(c+dx))-33075a^2b\cos(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^9*(3440640*a^2*b + 409600*b^3 + 3150*b*(-147*a^2 + 23*b^2)*Co
s[c + d*x] - 737280*b^3*Cos[2*(c + d*x)] - 308700*a^2*b*Cos[3*(c + d*x)] +
48300*b^3*Cos[3*(c + d*x)] - 132300*a^2*b*Cos[5*(c + d*x)] + 20700*b^3*Cos[
5*(c + d*x)] - 33075*a^2*b*Cos[7*(c + d*x)] + 5175*b^3*Cos[7*(c + d*x)] - 3
675*a^2*b*Cos[9*(c + d*x)] + 575*b^3*Cos[9*(c + d*x)] + 2064384*a^3*Sin[c +
 d*x] + 3096576*a*b^2*Sin[c + d*x] + 1376256*a^3*Sin[3*(c + d*x)] - 516096*
a*b^2*Sin[3*(c + d*x)] + 589824*a^3*Sin[5*(c + d*x)] - 221184*a*b^2*Sin[5*(
c + d*x)] + 147456*a^3*Sin[7*(c + d*x)] - 55296*a*b^2*Sin[7*(c + d*x)] + 16
384*a^3*Sin[9*(c + d*x)] - 6144*a*b^2*Sin[9*(c + d*x)]))/(10321920*d)
```

Maple [A] time = 0.118, size = 265, normalized size = 1.4

$$\frac{1}{d}\left(-a^3\left(-\frac{128}{315}-\frac{(\sec(dx+c))^8}{9}-\frac{8\,(\sec(dx+c))^6}{63}-\frac{16\,(\sec(dx+c))^4}{105}-\frac{64\,(\sec(dx+c))^2}{315}\right)\tan(dx+c)+\frac{a^2b}{3\,(\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-128/315-1/9*sec(d*x+c)^8-8/63*sec(d*x+c)^6-16/105*sec(d*x+c)^4-
64/315*sec(d*x+c)^2)*tan(d*x+c)+1/3*a^2*b/cos(d*x+c)^9+3*a*b^2*(1/9*sin(d*x
+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*
x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/9*sin(d*x+c)^4/cos(d*x+c)^9
+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/cos(d*x+c)^5+1/63*sin(d*x
+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/63*(2+sin(d*x+c)^2)*cos(d
*x+c)))
```

Maxima [A] time = 0.981288, size = 196, normalized size = 1.02

$$\frac{\left(35\tan(dx+c)^9+180\tan(dx+c)^7+378\tan(dx+c)^5+420\tan(dx+c)^3+315\tan(dx+c)\right)a^3+3\left(35\tan(dx+\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/315*((35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 378*tan(d*x + c)^5 + 420*t
an(d*x + c)^3 + 315*tan(d*x + c))*a^3 + 3*(35*tan(d*x + c)^9 + 135*tan(d*x
+ c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*a*b^2 - 5*(9*cos(d*x + c)
^2 - 7)*b^3/cos(d*x + c)^9 + 105*a^2*b/cos(d*x + c)^9)/d
```

Fricas [A] time = 2.38494, size = 347, normalized size = 1.81

$$-\frac{45\,b^3\cos\left(dx+c\right)^2-105\,a^2b-35\,b^3-\left(16\left(8\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^8+8\left(8\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^6+6\left(8\,a^3-3\,ab^2\right)\right.}{315\,d\cos\left(dx+c\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/315*(45*b^3*cos(d*x + c)^2 - 105*a^2*b - 35*b^3 - (16*(8*a^3 - 3*a*b^2)*
cos(d*x + c)^8 + 8*(8*a^3 - 3*a*b^2)*cos(d*x + c)^6 + 6*(8*a^3 - 3*a*b^2)*c
os(d*x + c)^4 + 35*a^3 + 105*a*b^2 + 5*(8*a^3 - 3*a*b^2)*cos(d*x + c)^2)*si
n(d*x + c))/(d*cos(d*x + c)^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13864, size = 639, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/315*(315*a^3*tan(1/2*d*x + 1/2*c)^17 + 945*a^2*b*tan(1/2*d*x + 1/2*c)^16
 - 840*a^3*tan(1/2*d*x + 1/2*c)^15 + 1260*a*b^2*tan(1/2*d*x + 1/2*c)^15 + 6
30*b^3*tan(1/2*d*x + 1/2*c)^14 + 4788*a^3*tan(1/2*d*x + 1/2*c)^13 + 1512*a*
b^2*tan(1/2*d*x + 1/2*c)^13 + 8820*a^2*b*tan(1/2*d*x + 1/2*c)^12 + 1050*b^3
*tan(1/2*d*x + 1/2*c)^12 - 5112*a^3*tan(1/2*d*x + 1/2*c)^11 + 8532*a*b^2*ta
n(1/2*d*x + 1/2*c)^11 + 3150*b^3*tan(1/2*d*x + 1/2*c)^10 + 10658*a^3*tan(1/
2*d*x + 1/2*c)^9 + 4272*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 13230*a^2*b*tan(1/2*
d*x + 1/2*c)^8 + 1890*b^3*tan(1/2*d*x + 1/2*c)^8 - 5112*a^3*tan(1/2*d*x + 1
/2*c)^7 + 8532*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 1890*b^3*tan(1/2*d*x + 1/2*c)
^6 + 4788*a^3*tan(1/2*d*x + 1/2*c)^5 + 1512*a*b^2*tan(1/2*d*x + 1/2*c)^5 +
3780*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 270*b^3*tan(1/2*d*x + 1/2*c)^4 - 840*a^
3*tan(1/2*d*x + 1/2*c)^3 + 1260*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 90*b^3*tan(1
/2*d*x + 1/2*c)^2 + 315*a^3*tan(1/2*d*x + 1/2*c) + 105*a^2*b - 10*b^3)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)^9*d)
```

3.413 $\int \cos^5(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=144

$$\frac{2\left(3a^2-b^2\right)(a+b\sin(c+dx))^{11}}{11b^5d}-\frac{2a\left(a^2-b^2\right)(a+b\sin(c+dx))^{10}}{5b^5d}+\frac{\left(a^2-b^2\right)^2(a+b\sin(c+dx))^9}{9b^5d}+\frac{(a+b\sin(c+dx}{13b^5d}$$

```
[Out] ((a^2 - b^2)^2*(a + b*Sin[c + d*x])^9)/(9*b^5*d) - (2*a*(a^2 - b^2)*(a + b*
Sin[c + d*x])^10)/(5*b^5*d) + (2*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^11)/(11
*b^5*d) - (a*(a + b*Sin[c + d*x])^12)/(3*b^5*d) + (a + b*Sin[c + d*x])^13/(
13*b^5*d)
```

Rubi [A] time = 0.221048, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{2\left(3a^2-b^2\right)(a+b\sin(c+dx))^{11}}{11b^5d}-\frac{2a\left(a^2-b^2\right)(a+b\sin(c+dx))^{10}}{5b^5d}+\frac{\left(a^2-b^2\right)^2(a+b\sin(c+dx))^9}{9b^5d}+\frac{(a+b\sin(c+dx}{13b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^8,x]
```


```
[Out] ((a^2 - b^2)^2*(a + b*Sin[c + d*x])^9)/(9*b^5*d) - (2*a*(a^2 - b^2)*(a + b*
Sin[c + d*x])^10)/(5*b^5*d) + (2*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^11)/(11
*b^5*d) - (a*(a + b*Sin[c + d*x])^12)/(3*b^5*d) + (a + b*Sin[c + d*x])^13/(
13*b^5*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\text{Subst}\left(\int (a+x)^8\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2(a+x)^8-4\left(a^3-ab^2\right)(a+x)^9+2\left(3a^2-b^2\right)(a+x)^{10}-\right.}{b^5d}$$
$$= \frac{\left(a^2-b^2\right)^2(a+b\sin(c+dx))^9}{9b^5d}-\frac{2a\left(a^2-b^2\right)(a+b\sin(c+dx))^{10}}{5b^5d}+\frac{2\left(3a^2-b\right.}{}$$

Mathematica [A] time = 2.06117, size = 120, normalized size = 0.83

$$\frac{\frac{2}{11}\left(3a^2-b^2\right)(a+b\sin(c+dx))^{11}+\frac{1}{9}\left(a^2-b^2\right)^2(a+b\sin(c+dx))^9+\frac{1}{13}(a+b\sin(c+dx))^{13}-\frac{1}{3}a(a+b\sin(c+dx))^{12}}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (((a^2 - b^2)^2*(a + b*Sin[c + d*x])^9)/9 - (2*a*(a - b)*(a + b)*(a + b*Sin
[c + d*x])^10)/5 + (2*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^11)/11 - (a*(a + b
*Sin[c + d*x])^12)/3 + (a + b*Sin[c + d*x])^13/13)/(b^5*d)
```

Maple [B] time = 0.086, size = 530, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(b^8*(-1/13*sin(d*x+c)^7*cos(d*x+c)^6-7/143*sin(d*x+c)^5*cos(d*x+c)^6-3
5/1287*sin(d*x+c)^3*cos(d*x+c)^6-5/429*sin(d*x+c)*cos(d*x+c)^6+1/429*(8/3+c
os(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+8*a*b^7*(-1/12*sin(d*x+c)^6*cos(d
*x+c)^6-1/20*sin(d*x+c)^4*cos(d*x+c)^6-1/40*sin(d*x+c)^2*cos(d*x+c)^6-1/120
*cos(d*x+c)^6)+28*a^2*b^6*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^
3*cos(d*x+c)^6-5/231*sin(d*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*co
s(d*x+c)^2)*sin(d*x+c))+56*a^3*b^5*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*si
```

```
n(d*x+c)^2*cos(d*x+c)^6-1/60*cos(d*x+c)^6)+70*a^4*b^4*(-1/9*sin(d*x+c)^3*co
s(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c))+56*a^5*b^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+
c)^6)+28*a^6*b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c))-4/3*a^7*b*cos(d*x+c)^6+1/5*a^8*(8/3+cos(d*x+c)^4+4
/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 0.957436, size = 420, normalized size = 2.92

$$\frac{495\,b^8\sin(dx+c)^{13}+4290\,ab^7\sin(dx+c)^{12}+1170\left(14\,a^2b^6-b^8\right)\sin(dx+c)^{11}+5148\left(7\,a^3b^5-2\,ab^7\right)\sin(dx+c)^{10}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/6435*(495*b^8*sin(d*x + c)^13 + 4290*a*b^7*sin(d*x + c)^12 + 1170*(14*a^2
*b^6 - b^8)*sin(d*x + c)^11 + 5148*(7*a^3*b^5 - 2*a*b^7)*sin(d*x + c)^10 +
25740*a^7*b*sin(d*x + c)^2 + 715*(70*a^4*b^4 - 56*a^2*b^6 + b^8)*sin(d*x +
c)^9 + 6435*a^8*sin(d*x + c) + 6435*(7*a^5*b^3 - 14*a^3*b^5 + a*b^7)*sin(d*
x + c)^8 + 25740*(a^6*b^2 - 5*a^4*b^4 + a^2*b^6)*sin(d*x + c)^7 + 8580*(a^7
*b - 14*a^5*b^3 + 7*a^3*b^5)*sin(d*x + c)^6 + 1287*(a^8 - 56*a^6*b^2 + 70*a
^4*b^4)*sin(d*x + c)^5 - 12870*(2*a^7*b - 7*a^5*b^3)*sin(d*x + c)^4 - 4290*
(a^8 - 14*a^6*b^2)*sin(d*x + c)^3)/d
```

Fricas [B] time = 3.13386, size = 879, normalized size = 6.1

$$\frac{4290\,ab^7\cos(dx+c)^{12}-5148\left(7\,a^3b^5+3\,ab^7\right)\cos(dx+c)^{10}+6435\left(7\,a^5b^3+14\,a^3b^5+3\,ab^7\right)\cos(dx+c)^8-8580\left(a^7b\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/6435*(4290*a*b^7*cos(d*x + c)^12 - 5148*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c
)^10 + 6435*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^8 - 8580*(a^7*b
 + 7*a^5*b^3 + 7*a^3*b^5 + a*b^7)*cos(d*x + c)^6 + (495*b^8*cos(d*x + c)^12
 - 180*(91*a^2*b^6 + 10*b^8)*cos(d*x + c)^10 + 10*(5005*a^4*b^4 + 4186*a^2*
b^6 + 229*b^8)*cos(d*x + c)^8 + 3432*a^8 + 13728*a^6*b^2 + 11440*a^4*b^4 +
2080*a^2*b^6 + 40*b^8 - 20*(1287*a^6*b^2 + 3575*a^4*b^4 + 1469*a^2*b^6 + 53
```

```
*b^8)*cos(d*x + c)^6 + 3*(429*a^8 + 1716*a^6*b^2 + 1430*a^4*b^4 + 260*a^2*b
^6 + 5*b^8)*cos(d*x + c)^4 + 4*(429*a^8 + 1716*a^6*b^2 + 1430*a^4*b^4 + 260
*a^2*b^6 + 5*b^8)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 122.701, size = 614, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((8*a**8*sin(c + d*x)**5/(15*d) + 4*a**8*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**8*sin(c + d*x)*cos(c + d*x)**4/d - 4*a**7*b*cos(c + d*x)*
*6/(3*d) + 32*a**6*b**2*sin(c + d*x)**7/(15*d) + 112*a**6*b**2*sin(c + d*x)
**5*cos(c + d*x)**2/(15*d) + 28*a**6*b**2*sin(c + d*x)**3*cos(c + d*x)**4/(
3*d) - 28*a**5*b**3*sin(c + d*x)**2*cos(c + d*x)**6/(3*d) - 7*a**5*b**3*cos
(c + d*x)**8/(3*d) + 16*a**4*b**4*sin(c + d*x)**9/(9*d) + 8*a**4*b**4*sin(c
 + d*x)**7*cos(c + d*x)**2/d + 14*a**4*b**4*sin(c + d*x)**5*cos(c + d*x)**4
/d - 28*a**3*b**5*sin(c + d*x)**4*cos(c + d*x)**6/(3*d) - 14*a**3*b**5*sin(
c + d*x)**2*cos(c + d*x)**8/(3*d) - 14*a**3*b**5*cos(c + d*x)**10/(15*d) +
32*a**2*b**6*sin(c + d*x)**11/(99*d) + 16*a**2*b**6*sin(c + d*x)**9*cos(c +
 d*x)**2/(9*d) + 4*a**2*b**6*sin(c + d*x)**7*cos(c + d*x)**4/d - 4*a*b**7*s
in(c + d*x)**6*cos(c + d*x)**6/(3*d) - a*b**7*sin(c + d*x)**4*cos(c + d*x)*
*8/d - 2*a*b**7*sin(c + d*x)**2*cos(c + d*x)**10/(5*d) - a*b**7*cos(c + d*x
)**12/(15*d) + 8*b**8*sin(c + d*x)**13/(1287*d) + 4*b**8*sin(c + d*x)**11*c
os(c + d*x)**2/(99*d) + b**8*sin(c + d*x)**9*cos(c + d*x)**4/(9*d), Ne(d, 0
)), (x*(a + b*sin(c))**8*cos(c)**5, True))
```

Giac [B] time = 1.2091, size = 626, normalized size = 4.35

$$\frac{ab^7\cos(12\,dx+12\,c)}{3072\,d}+\frac{b^8\sin(13\,dx+13\,c)}{53248\,d}-\frac{\left(14\,a^3b^5+ab^7\right)\cos(10\,dx+10\,c)}{1280\,d}+\frac{\left(28\,a^5b^3-ab^7\right)\cos(8\,dx+8\,c)}{512\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/3072*a*b^7*cos(12*d*x + 12*c)/d + 1/53248*b^8*sin(13*d*x + 13*c)/d - 1/12
80*(14*a^3*b^5 + a*b^7)*cos(10*d*x + 10*c)/d + 1/512*(28*a^5*b^3 - a*b^7)*c
```

```
os(8*d*x + 8*c)/d - 1/768*(32*a^7*b - 112*a^5*b^3 - 70*a^3*b^5 - 5*a*b^7)*c
os(6*d*x + 6*c)/d - 1/1024*(256*a^7*b + 224*a^5*b^3 - 5*a*b^7)*cos(4*d*x +
4*c)/d - 1/128*(80*a^7*b + 168*a^5*b^3 + 70*a^3*b^5 + 5*a*b^7)*cos(2*d*x +
2*c)/d - 1/45056*(112*a^2*b^6 + 3*b^8)*sin(11*d*x + 11*c)/d + 1/18432*(560*
a^4*b^4 + 56*a^2*b^6 - b^8)*sin(9*d*x + 9*c)/d - 1/2048*(128*a^6*b^2 - 80*a
^4*b^4 - 40*a^2*b^6 - b^8)*sin(7*d*x + 7*c)/d + 1/20480*(256*a^8 - 5376*a^6
*b^2 - 4480*a^4*b^4 - 560*a^2*b^6 - 5*b^8)*sin(5*d*x + 5*c)/d + 1/12288*(12
80*a^8 - 1792*a^6*b^2 - 4480*a^4*b^4 - 1120*a^2*b^6 - 25*b^8)*sin(3*d*x + 3
*c)/d + 5/1024*(128*a^8 + 448*a^6*b^2 + 336*a^4*b^4 + 56*a^2*b^6 + b^8)*sin
(d*x + c)/d
```

3.414 $\int \cos^3(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=77

$$-\frac{\left(a^2-b^2\right)(a+b\sin(c+dx))^9}{9b^3d}-\frac{(a+b\sin(c+dx))^{11}}{11b^3d}+\frac{a(a+b\sin(c+dx))^{10}}{5b^3d}$$

```
[Out] -((a^2 - b^2)*(a + b*Sin[c + d*x])^9)/(9*b^3*d) + (a*(a + b*Sin[c + d*x])^1
0)/(5*b^3*d) - (a + b*Sin[c + d*x])^11/(11*b^3*d)
```

Rubi [A] time = 0.150984, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{\left(a^2-b^2\right)(a+b\sin(c+dx))^9}{9b^3d}-\frac{(a+b\sin(c+dx))^{11}}{11b^3d}+\frac{a(a+b\sin(c+dx))^{10}}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -((a^2 - b^2)*(a + b*Sin[c + d*x])^9)/(9*b^3*d) + (a*(a + b*Sin[c + d*x])^1
0)/(5*b^3*d) - (a + b*Sin[c + d*x])^11/(11*b^3*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\text{Subst}\left(\int (a+x)^8\left(b^2-x^2\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^8+2a(a+x)^9-(a+x)^{10}\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= -\frac{\left(a^2-b^2\right)(a+b\sin(c+dx))^9}{9b^3d}+\frac{a(a+b\sin(c+dx))^{10}}{5b^3d}-\frac{(a+b\sin(c+dx))^{11}}{11b^3d}$$

Mathematica [A] time = 0.884818, size = 56, normalized size = 0.73

$$\frac{(a+b\sin(c+dx))^9\left(-2a^2+18ab\sin(c+dx)+45b^2\cos(2(c+dx))+65b^2\right)}{990b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((a + b*Sin[c + d*x])^9*(-2*a^2 + 65*b^2 + 45*b^2*Cos[2*(c + d*x)] + 18*a*b
*Sin[c + d*x]))/(990*b^3*d)
```

Maple [B] time = 0.085, size = 480, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(b^8*(-1/11*sin(d*x+c)^7*cos(d*x+c)^4-7/99*sin(d*x+c)^5*cos(d*x+c)^4-5/
99*sin(d*x+c)^3*cos(d*x+c)^4-1/33*sin(d*x+c)*cos(d*x+c)^4+1/99*(2+cos(d*x+c
)^2)*sin(d*x+c))+8*a*b^7*(-1/10*sin(d*x+c)^6*cos(d*x+c)^4-3/40*sin(d*x+c)^4
*cos(d*x+c)^4-1/20*sin(d*x+c)^2*cos(d*x+c)^4-1/40*cos(d*x+c)^4)+28*a^2*b^6*
(-1/9*sin(d*x+c)^5*cos(d*x+c)^4-5/63*sin(d*x+c)^3*cos(d*x+c)^4-1/21*sin(d*x
+c)*cos(d*x+c)^4+1/63*(2+cos(d*x+c)^2)*sin(d*x+c))+56*a^3*b^5*(-1/8*sin(d*x
+c)^4*cos(d*x+c)^4-1/12*sin(d*x+c)^2*cos(d*x+c)^4-1/24*cos(d*x+c)^4)+70*a^4
*b^4*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(2+c
os(d*x+c)^2)*sin(d*x+c))+56*a^5*b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*co
s(d*x+c)^4)+28*a^6*b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*
```

```
sin(d*x+c))-2*a^7*b*cos(d*x+c)^4+1/3*a^8*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 0.971706, size = 315, normalized size = 4.09

$$-\frac{45\,b^8\sin\left(dx+c\right)^{11}+396\,ab^7\sin\left(dx+c\right)^{10}-1980\,a^7b\sin\left(dx+c\right)^2+55\left(28\,a^2b^6-b^8\right)\sin\left(dx+c\right)^9-495\,a^8\sin\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/495*(45*b^8*sin(d*x + c)^11 + 396*a*b^7*sin(d*x + c)^10 - 1980*a^7*b*sin
(d*x + c)^2 + 55*(28*a^2*b^6 - b^8)*sin(d*x + c)^9 - 495*a^8*sin(d*x + c) +
 495*(7*a^3*b^5 - a*b^7)*sin(d*x + c)^8 + 990*(5*a^4*b^4 - 2*a^2*b^6)*sin(d
*x + c)^7 + 4620*(a^5*b^3 - a^3*b^5)*sin(d*x + c)^6 + 1386*(2*a^6*b^2 - 5*a
^4*b^4)*sin(d*x + c)^5 + 990*(a^7*b - 7*a^5*b^3)*sin(d*x + c)^4 + 165*(a^8
- 28*a^6*b^2)*sin(d*x + c)^3)/d
```

Fricas [B] time = 3.02639, size = 740, normalized size = 9.61

$$\frac{396\,ab^7\cos\left(dx+c\right)^{10}-495\left(7\,a^3b^5+3\,ab^7\right)\cos\left(dx+c\right)^8+660\left(7\,a^5b^3+14\,a^3b^5+3\,ab^7\right)\cos\left(dx+c\right)^6-990\left(a^7b+7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/495*(396*a*b^7*cos(d*x + c)^10 - 495*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^8
 + 660*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^6 - 990*(a^7*b + 7*a
^5*b^3 + 7*a^3*b^5 + a*b^7)*cos(d*x + c)^4 + (45*b^8*cos(d*x + c)^10 - 10*(
154*a^2*b^6 + 17*b^8)*cos(d*x + c)^8 + 330*a^8 + 1848*a^6*b^2 + 1980*a^4*b^
4 + 440*a^2*b^6 + 10*b^8 + 10*(495*a^4*b^4 + 418*a^2*b^6 + 23*b^8)*cos(d*x
+ c)^6 - 12*(231*a^6*b^2 + 660*a^4*b^4 + 275*a^2*b^6 + 10*b^8)*cos(d*x + c)
^4 + (165*a^8 + 924*a^6*b^2 + 990*a^4*b^4 + 220*a^2*b^6 + 5*b^8)*cos(d*x +
c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 60.1598, size = 493, normalized size = 6.4

$$\begin{cases} \frac{2a^8\sin^3(c+dx)}{3d} + \frac{a^8\sin(c+dx)\cos^2(c+dx)}{d} + \frac{2a^7b\sin^4(c+dx)}{d} + \frac{4a^7b\sin^2(c+dx)\cos^2(c+dx)}{d} + \frac{56a^6b^2\sin^5(c+dx)}{15d} + \frac{28a^6b^2\sin^3(c+dx)\cos^2(c+dx)}{3d} \\ x(a+b\sin(c))^8\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((2*a**8*sin(c + d*x)**3/(3*d) + a**8*sin(c + d*x)*cos(c + d*x)**2
/d + 2*a**7*b*sin(c + d*x)**4/d + 4*a**7*b*sin(c + d*x)**2*cos(c + d*x)**2/
d + 56*a**6*b**2*sin(c + d*x)**5/(15*d) + 28*a**6*b**2*sin(c + d*x)**3*cos(
c + d*x)**2/(3*d) - 14*a**5*b**3*sin(c + d*x)**2*cos(c + d*x)**4/d - 14*a**
5*b**3*cos(c + d*x)**6/(3*d) + 4*a**4*b**4*sin(c + d*x)**7/d + 14*a**4*b**4
*sin(c + d*x)**5*cos(c + d*x)**2/d - 14*a**3*b**5*sin(c + d*x)**4*cos(c + d
*x)**4/d - 28*a**3*b**5*sin(c + d*x)**2*cos(c + d*x)**6/(3*d) - 7*a**3*b**5
*cos(c + d*x)**8/(3*d) + 8*a**2*b**6*sin(c + d*x)**9/(9*d) + 4*a**2*b**6*si
n(c + d*x)**7*cos(c + d*x)**2/d - 2*a*b**7*sin(c + d*x)**6*cos(c + d*x)**4/
d - 2*a*b**7*sin(c + d*x)**4*cos(c + d*x)**6/d - a*b**7*sin(c + d*x)**2*cos
(c + d*x)**8/d - a*b**7*cos(c + d*x)**10/(5*d) + 2*b**8*sin(c + d*x)**11/(9
9*d) + b**8*sin(c + d*x)**9*cos(c + d*x)**2/(9*d), Ne(d, 0)), (x*(a + b*sin
(c))**8*cos(c)**3, True))
```

Giac [B] time = 1.16422, size = 367, normalized size = 4.77

$$-\frac{45\,b^8\sin(dx+c)^{11} + 396\,ab^7\sin(dx+c)^{10} + 1540\,a^2b^6\sin(dx+c)^9 - 55\,b^8\sin(dx+c)^9 + 3465\,a^3b^5\sin(dx+c)^8 - 49}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/495*(45*b^8*sin(d*x + c)^11 + 396*a*b^7*sin(d*x + c)^10 + 1540*a^2*b^6*s
in(d*x + c)^9 - 55*b^8*sin(d*x + c)^9 + 3465*a^3*b^5*sin(d*x + c)^8 - 495*a
*b^7*sin(d*x + c)^8 + 4950*a^4*b^4*sin(d*x + c)^7 - 1980*a^2*b^6*sin(d*x +
c)^7 + 4620*a^5*b^3*sin(d*x + c)^6 - 4620*a^3*b^5*sin(d*x + c)^6 + 2772*a^6
*b^2*sin(d*x + c)^5 - 6930*a^4*b^4*sin(d*x + c)^5 + 990*a^7*b*sin(d*x + c)^
4 - 6930*a^5*b^3*sin(d*x + c)^4 + 165*a^8*sin(d*x + c)^3 - 4620*a^6*b^2*sin
(d*x + c)^3 - 1980*a^7*b*sin(d*x + c)^2 - 495*a^8*sin(d*x + c))/d
```

3.415 $\int \cos(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\sin(c+dx))^9}{9bd}$$

```
[Out] (a + b*Sin[c + d*x])^9/(9*b*d)
```

Rubi [A] time = 0.0263247, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 32}

$$\frac{(a+b\sin(c+dx))^9}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a + b*Sin[c + d*x])^9/(9*b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^8\,dx &= \frac{\operatorname{Subst}\left(\int (a+x)^8\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{(a+b\sin(c+dx))^9}{9bd}\end{aligned}$$

Mathematica [B] time = 0.359453, size = 137, normalized size = 6.23

$$\frac{\sin(c+dx)\left(84a^6b^2\sin^2(c+dx)+126a^5b^3\sin^3(c+dx)+126a^4b^4\sin^4(c+dx)+84a^3b^5\sin^5(c+dx)+36a^2b^6\sin^6(c+d\right.}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sin[c + d*x]*(9*a^8 + 36*a^7*b*Sin[c + d*x] + 84*a^6*b^2*Sin[c + d*x]^2 +
126*a^5*b^3*Sin[c + d*x]^3 + 126*a^4*b^4*Sin[c + d*x]^4 + 84*a^3*b^5*Sin[c
+ d*x]^5 + 36*a^2*b^6*Sin[c + d*x]^6 + 9*a*b^7*Sin[c + d*x]^7 + b^8*Sin[c +
 d*x]^8))/(9*d)
```

Maple [A] time = 0.029, size = 21, normalized size = 1.

$$\frac{(a+b\sin(dx+c))^9}{9\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/9*(a+b*sin(d*x+c))^9/b/d
```

Maxima [A] time = 0.943892, size = 27, normalized size = 1.23

$$\frac{(b\sin(dx+c)+a)^9}{9\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/9*(b*sin(d*x + c) + a)^9/(b*d)
```

Fricas [B] time = 3.10267, size = 582, normalized size = 26.45

$$\frac{9\,ab^7\cos(dx+c)^8 - 12\left(7\,a^3b^5 + 3\,ab^7\right)\cos(dx+c)^6 + 18\left(7\,a^5b^3 + 14\,a^3b^5 + 3\,ab^7\right)\cos(dx+c)^4 - 36\left(a^7b + 7\,a^5b^3 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a*b^7*cos(d*x + c)^8 - 12*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^6 + 18*
(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 - 36*(a^7*b + 7*a^5*b^3 +
 7*a^3*b^5 + a*b^7)*cos(d*x + c)^2 + (b^8*cos(d*x + c)^8 + 9*a^8 + 84*a^6*b
^2 + 126*a^4*b^4 + 36*a^2*b^6 + b^8 - 4*(9*a^2*b^6 + b^8)*cos(d*x + c)^6 +
6*(21*a^4*b^4 + 18*a^2*b^6 + b^8)*cos(d*x + c)^4 - 4*(21*a^6*b^2 + 63*a^4*b
^4 + 27*a^2*b^6 + b^8)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 20.9601, size = 168, normalized size = 7.64

$$\begin{cases} \frac{a^8\sin(c+dx)}{d} + \frac{4a^7b\sin^2(c+dx)}{d} + \frac{28a^6b^2\sin^3(c+dx)}{3d} + \frac{14a^5b^3\sin^4(c+dx)}{d} + \frac{14a^4b^4\sin^5(c+dx)}{d} + \frac{28a^3b^5\sin^6(c+dx)}{3d} + \frac{4a^2b^6\sin^7(c+dx)}{d} + \frac{ab^7}{} \\ x\left(a + b\sin(c)\right)^8\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((a**8*sin(c + d*x)/d + 4*a**7*b*sin(c + d*x)**2/d + 28*a**6*b**2*
sin(c + d*x)**3/(3*d) + 14*a**5*b**3*sin(c + d*x)**4/d + 14*a**4*b**4*sin(c
 + d*x)**5/d + 28*a**3*b**5*sin(c + d*x)**6/(3*d) + 4*a**2*b**6*sin(c + d*x
)**7/d + a*b**7*sin(c + d*x)**8/d + b**8*sin(c + d*x)**9/(9*d), Ne(d, 0)),
(x*(a + b*sin(c))**8*cos(c), True))
```

Giac [A] time = 1.12865, size = 27, normalized size = 1.23

$$\frac{\left(b\sin(dx+c) + a\right)^9}{9\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/9*(b*sin(d*x + c) + a)^9/(b*d)
```

3.416 $\int \sec(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=245

$$-\frac{b^6\left(28a^2+b^2\right)\sin^5(c+dx)}{5d}-\frac{2ab^5\left(7a^2+b^2\right)\sin^4(c+dx)}{d}-\frac{b^4\left(28a^2b^2+70a^4+b^4\right)\sin^3(c+dx)}{3d}-\frac{4ab^3\left(7a^2b^2+7a\right.}{}$$

```
[Out] -((a + b)^8*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)^8*Log[1 + Sin[c + d*x]]
)/(2*d) - (b^2*(28*a^6 + 70*a^4*b^2 + 28*a^2*b^4 + b^6)*Sin[c + d*x])/d - (
4*a*b^3*(7*a^4 + 7*a^2*b^2 + b^4)*Sin[c + d*x]^2)/d - (b^4*(70*a^4 + 28*a^2
*b^2 + b^4)*Sin[c + d*x]^3)/(3*d) - (2*a*b^5*(7*a^2 + b^2)*Sin[c + d*x]^4)/
d - (b^6*(28*a^2 + b^2)*Sin[c + d*x]^5)/(5*d) - (4*a*b^7*Sin[c + d*x]^6)/(3
*d) - (b^8*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.182292, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2668, 702, 633, 31}

$$-\frac{b^6\left(28a^2+b^2\right)\sin^5(c+dx)}{5d}-\frac{2ab^5\left(7a^2+b^2\right)\sin^4(c+dx)}{d}-\frac{b^4\left(28a^2b^2+70a^4+b^4\right)\sin^3(c+dx)}{3d}-\frac{4ab^3\left(7a^2b^2+7a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^8,x]
```


```
[Out] -((a + b)^8*Log[1 - Sin[c + d*x]])/(2*d) + ((a - b)^8*Log[1 + Sin[c + d*x]]
)/(2*d) - (b^2*(28*a^6 + 70*a^4*b^2 + 28*a^2*b^4 + b^6)*Sin[c + d*x])/d - (
4*a*b^3*(7*a^4 + 7*a^2*b^2 + b^4)*Sin[c + d*x]^2)/d - (b^4*(70*a^4 + 28*a^2
*b^2 + b^4)*Sin[c + d*x]^3)/(3*d) - (2*a*b^5*(7*a^2 + b^2)*Sin[c + d*x]^4)/
d - (b^6*(28*a^2 + b^2)*Sin[c + d*x]^5)/(5*d) - (4*a*b^7*Sin[c + d*x]^6)/(3
*d) - (b^8*Sin[c + d*x]^7)/(7*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^8}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \left(-28a^6-70a^4b^2-28a^2b^4-b^6-8a\left(7a^4+7a^2b^2+b^4\right)x-\left(70a^4+2\right.\right.\right.}{}$$

$$= -\frac{b^2\left(28a^6+70a^4b^2+28a^2b^4+b^6\right)\sin(c+dx)}{d}-\frac{4ab^3\left(7a^4+7a^2b^2+b^4\right)\sin^2(c}{d}$$

$$= -\frac{b^2\left(28a^6+70a^4b^2+28a^2b^4+b^6\right)\sin(c+dx)}{d}-\frac{4ab^3\left(7a^4+7a^2b^2+b^4\right)\sin^2(c}{d}$$

$$= -\frac{(a+b)^8\log(1-\sin(c+dx))}{2d}+\frac{(a-b)^8\log(1+\sin(c+dx))}{2d}-\frac{b^2\left(28a^6+70a^4b\right.}{}$$

Mathematica [A] time = 0.216993, size = 227, normalized size = 0.93

$$\frac{b\left(-\frac{1}{5}b^5\left(28a^2+b^2\right)\sin^5(c+dx)-2ab^4\left(7a^2+b^2\right)\sin^4(c+dx)-\frac{1}{3}b^3\left(28a^2b^2+70a^4+b^4\right)\sin^3(c+dx)-4ab^2\left(7a^2b^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (b*(-((a + b)^8*Log[1 - Sin[c + d*x]])/(2*b) + ((a - b)^8*Log[1 + Sin[c + d
*x]])/(2*b) - b*(28*a^6 + 70*a^4*b^2 + 28*a^2*b^4 + b^6)*Sin[c + d*x] - 4*a
*b^2*(7*a^4 + 7*a^2*b^2 + b^4)*Sin[c + d*x]^2 - (b^3*(70*a^4 + 28*a^2*b^2 +
 b^4)*Sin[c + d*x]^3)/3 - 2*a*b^4*(7*a^2 + b^2)*Sin[c + d*x]^4 - (b^5*(28*a
^2 + b^2)*Sin[c + d*x]^5)/5 - (4*a*b^6*Sin[c + d*x]^6)/3 - (b^7*Sin[c + d*x
]^7)/7))/d
```

Maple [A] time = 0.103, size = 465, normalized size = 1.9

$$\frac{a^8 \ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{b^8 \sin(dx+c)}{d}-\frac{b^8 (\sin(dx+c))^3}{3d}+\frac{b^8 \ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{b^8 (\sin(\ldots}{5\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*a^8*ln(sec(d*x+c)+tan(d*x+c))-1/d*b^8*sin(d*x+c)-1/3/d*b^8*sin(d*x+c)^3
+1/d*b^8*ln(sec(d*x+c)+tan(d*x+c))-1/5/d*b^8*sin(d*x+c)^5-1/7*b^8*sin(d*x+c
)^7/d-70/d*a^4*b^4*sin(d*x+c)+70/d*a^4*b^4*ln(sec(d*x+c)+tan(d*x+c))-14/d*a
^3*b^5*sin(d*x+c)^4-28/d*a^3*b^5*sin(d*x+c)^2-56/d*a^3*b^5*ln(cos(d*x+c))-2
8/5/d*a^2*b^6*sin(d*x+c)^5-28/3/d*a^2*b^6*sin(d*x+c)^3-28/d*a^2*b^6*sin(d*x
+c)+28/d*a^2*b^6*ln(sec(d*x+c)+tan(d*x+c))-2/d*a*b^7*sin(d*x+c)^4-4/d*a*b^7
*sin(d*x+c)^2-8/d*a*b^7*ln(cos(d*x+c))-8/d*a^7*b*ln(cos(d*x+c))-28/d*a^6*b^
2*sin(d*x+c)+28/d*a^6*b^2*ln(sec(d*x+c)+tan(d*x+c))-28/d*a^5*b^3*sin(d*x+c)
^2-56/d*a^5*b^3*ln(cos(d*x+c))-70/3/d*a^4*b^4*sin(d*x+c)^3-4/3*a*b^7*sin(d*
x+c)^6/d
```

Maxima [A] time = 0.955783, size = 428, normalized size = 1.75

$$-\frac{30\,b^8 \sin(dx+c)^7+280\,ab^7 \sin(dx+c)^6+42\left(28\,a^2b^6+b^8\right)\sin(dx+c)^5+420\left(7\,a^3b^5+ab^7\right)\sin(dx+c)^4+70\left(70\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/210*(30*b^8*sin(d*x + c)^7 + 280*a*b^7*sin(d*x + c)^6 + 42*(28*a^2*b^6 +
 b^8)*sin(d*x + c)^5 + 420*(7*a^3*b^5 + a*b^7)*sin(d*x + c)^4 + 70*(70*a^4*
b^4 + 28*a^2*b^6 + b^8)*sin(d*x + c)^3 + 840*(7*a^5*b^3 + 7*a^3*b^5 + a*b^7
)*sin(d*x + c)^2 - 105*(a^8 - 8*a^7*b + 28*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^
```

```
4 - 56*a^3*b^5 + 28*a^2*b^6 - 8*a*b^7 + b^8)*log(sin(d*x + c) + 1) + 105*(a
^8 + 8*a^7*b + 28*a^6*b^2 + 56*a^5*b^3 + 70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b
^6 + 8*a*b^7 + b^8)*log(sin(d*x + c) - 1) + 210*(28*a^6*b^2 + 70*a^4*b^4 +
28*a^2*b^6 + b^8)*sin(d*x + c))/d
```

Fricas [A] time = 3.42162, size = 784, normalized size = 3.2

$$\frac{280\,ab^7\cos(dx+c)^6 - 420\left(7\,a^3b^5 + 3\,ab^7\right)\cos(dx+c)^4 + 840\left(7\,a^5b^3 + 14\,a^3b^5 + 3\,ab^7\right)\cos(dx+c)^2 + 105\left(a^8 - 8\,a^7b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/210*(280*a*b^7*cos(d*x + c)^6 - 420*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4
+ 840*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + 105*(a^8 - 8*a^7*
b + 28*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^4 - 56*a^3*b^5 + 28*a^2*b^6 - 8*a*b^
7 + b^8)*log(sin(d*x + c) + 1) - 105*(a^8 + 8*a^7*b + 28*a^6*b^2 + 56*a^5*b
^3 + 70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b^6 + 8*a*b^7 + b^8)*log(-sin(d*x + c
) + 1) + 2*(15*b^8*cos(d*x + c)^6 - 2940*a^6*b^2 - 9800*a^4*b^4 - 4508*a^2*
b^6 - 176*b^8 - 6*(98*a^2*b^6 + 11*b^8)*cos(d*x + c)^4 + 2*(1225*a^4*b^4 +
1078*a^2*b^6 + 61*b^8)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1722, size = 510, normalized size = 2.08

$$-\frac{30\,b^8\sin(dx+c)^7 + 280\,ab^7\sin(dx+c)^6 + 1176\,a^2b^6\sin(dx+c)^5 + 42\,b^8\sin(dx+c)^5 + 2940\,a^3b^5\sin(dx+c)^4 + 420}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/210*(30*b^8*sin(d*x + c)^7 + 280*a*b^7*sin(d*x + c)^6 + 1176*a^2*b^6*sin
(d*x + c)^5 + 42*b^8*sin(d*x + c)^5 + 2940*a^3*b^5*sin(d*x + c)^4 + 420*a*b
^7*sin(d*x + c)^4 + 4900*a^4*b^4*sin(d*x + c)^3 + 1960*a^2*b^6*sin(d*x + c)
^3 + 70*b^8*sin(d*x + c)^3 + 5880*a^5*b^3*sin(d*x + c)^2 + 5880*a^3*b^5*sin
(d*x + c)^2 + 840*a*b^7*sin(d*x + c)^2 + 5880*a^6*b^2*sin(d*x + c) + 14700*
a^4*b^4*sin(d*x + c) + 5880*a^2*b^6*sin(d*x + c) + 210*b^8*sin(d*x + c) - 1
05*(a^8 - 8*a^7*b + 28*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^4 - 56*a^3*b^5 + 28*
a^2*b^6 - 8*a*b^7 + b^8)*log(abs(sin(d*x + c) + 1)) + 105*(a^8 + 8*a^7*b +
28*a^6*b^2 + 56*a^5*b^3 + 70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b^6 + 8*a*b^7 +
b^8)*log(abs(sin(d*x + c) - 1)))/d
```

3.417 $\int \sec^3(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=284

$$\frac{7b^6\left(5a^2+b^2\right)\sin^5(c+dx)}{10d}+\frac{3ab^5\left(7a^2+4b^2\right)\sin^4(c+dx)}{2d}+\frac{7b^4\left(20a^2b^2+15a^4+b^4\right)\sin^3(c+dx)}{6d}+\frac{ab^3\left(112a^2b^2+35\right.}{}$$

```
[Out] -((a - 7*b)*(a + b)^7*Log[1 - Sin[c + d*x]])/(4*d) + ((a - b)^7*(a + 7*b)*L
og[1 + Sin[c + d*x]])/(4*d) + (7*b^2*(3*a^6 + 30*a^4*b^2 + 20*a^2*b^4 + b^6
)*Sin[c + d*x])/(2*d) + (a*b^3*(35*a^4 + 112*a^2*b^2 + 24*b^4)*Sin[c + d*x]
^2)/(2*d) + (7*b^4*(15*a^4 + 20*a^2*b^2 + b^4)*Sin[c + d*x]^3)/(6*d) + (3*a
*b^5*(7*a^2 + 4*b^2)*Sin[c + d*x]^4)/(2*d) + (7*b^6*(5*a^2 + b^2)*Sin[c + d
*x]^5)/(10*d) + (a*b^7*Sin[c + d*x]^6)/(2*d) + (Sec[c + d*x]^2*(b + a*Sin[c
 + d*x])*(a + b*Sin[c + d*x])^7)/(2*d)
```

Rubi [A] time = 0.242127, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2668, 739, 801, 633, 31}

$$\frac{7b^6\left(5a^2+b^2\right)\sin^5(c+dx)}{10d}+\frac{3ab^5\left(7a^2+4b^2\right)\sin^4(c+dx)}{2d}+\frac{7b^4\left(20a^2b^2+15a^4+b^4\right)\sin^3(c+dx)}{6d}+\frac{ab^3\left(112a^2b^2+35\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^8,x]
```


```
[Out] -((a - 7*b)*(a + b)^7*Log[1 - Sin[c + d*x]])/(4*d) + ((a - b)^7*(a + 7*b)*L
og[1 + Sin[c + d*x]])/(4*d) + (7*b^2*(3*a^6 + 30*a^4*b^2 + 20*a^2*b^4 + b^6
)*Sin[c + d*x])/(2*d) + (a*b^3*(35*a^4 + 112*a^2*b^2 + 24*b^4)*Sin[c + d*x]
^2)/(2*d) + (7*b^4*(15*a^4 + 20*a^2*b^2 + b^4)*Sin[c + d*x]^3)/(6*d) + (3*a
*b^5*(7*a^2 + 4*b^2)*Sin[c + d*x]^4)/(2*d) + (7*b^6*(5*a^2 + b^2)*Sin[c + d
*x]^5)/(10*d) + (a*b^7*Sin[c + d*x]^6)/(2*d) + (Sec[c + d*x]^2*(b + a*Sin[c
 + d*x])*(a + b*Sin[c + d*x])^7)/(2*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{b^3\,\mathrm{Subst}\left(\int \frac{(a+x)^8}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{2d} - \frac{b\,\mathrm{Subst}\left(\int \frac{(a+x)^6\left(-a^2+7b^2+6a\right.}{b^2-x^2}\right.}{2d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{2d} - \frac{b\,\mathrm{Subst}\left(\int\left(-7\left(3a^6+30a\right.\right.\right.}{}$$

$$= \frac{7b^2\left(3a^6+30a^4b^2+20a^2b^4+b^6\right)\sin(c+dx)}{2d} + \frac{ab^3\left(35a^4+112a^2b^2+24b^4\right)\sin}{2d}$$

$$= \frac{7b^2\left(3a^6+30a^4b^2+20a^2b^4+b^6\right)\sin(c+dx)}{2d} + \frac{ab^3\left(35a^4+112a^2b^2+24b^4\right)\sin}{2d}$$

$$= -\frac{(a-7b)(a+b)^7\log(1-\sin(c+dx))}{4d} + \frac{(a-b)^7(a+7b)\log(1+\sin(c+dx))}{4d} +$$

Mathematica [A] time = 2.35673, size = 366, normalized size = 1.29

$$\frac{b^9\left(b^2-9a^2\right)\sin^7(c+dx) - 4ab^8\left(9a^2-2b^2\right)\sin^6(c+dx) + \frac{7}{5}b^7\left(19a^2b^2-60a^4+b^4\right)\sin^5(c+dx) - 2ab^6\left(-22a^2b^2+63a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((b*(a^2 - b^2)*((a - 7*b)*(a + b)^7*Log[1 - Sin[c + d*x]] - (a - b)^7*(a +
 7*b)*Log[1 + Sin[c + d*x]]))/2 + b^3*(-36*a^8 - 182*a^6*b^2 + 70*a^4*b^4 +
 133*a^2*b^6 + 7*b^8)*Sin[c + d*x] - 4*a*b^4*(21*a^6 + 14*a^4*b^2 - 22*a^2*
b^4 - 6*b^6)*Sin[c + d*x]^2 + (7*b^5*(-54*a^6 + 10*a^4*b^2 + 19*a^2*b^4 + b
^6)*Sin[c + d*x]^3)/3 - 2*a*b^6*(63*a^4 - 22*a^2*b^2 - 6*b^4)*Sin[c + d*x]^
4 + (7*b^7*(-60*a^4 + 19*a^2*b^2 + b^4)*Sin[c + d*x]^5)/5 - 4*a*b^8*(9*a^2
- 2*b^2)*Sin[c + d*x]^6 + b^9*(-9*a^2 + b^2)*Sin[c + d*x]^7 - a*b^10*Sin[c
+ d*x]^8 + b*Sec[c + d*x]^2*(b - a*Sin[c + d*x])*(a + b*Sin[c + d*x])^9)/(2
*b*(-a^2 + b^2)*d)
```

Maple [B] time = 0.127, size = 645, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 4/d*a^7*b/cos(d*x+c)^2+28/d*a^5*b^3*tan(d*x+c)^2+1/2/d*b^8*sin(d*x+c)^9/cos
(d*x+c)^2+1/2/d*a^8*ln(sec(d*x+c)+tan(d*x+c))+7/2/d*b^8*sin(d*x+c)+7/6/d*b^
8*sin(d*x+c)^3-7/2/d*b^8*ln(sec(d*x+c)+tan(d*x+c))+7/10/d*b^8*sin(d*x+c)^5+
1/2*b^8*sin(d*x+c)^7/d+105/d*a^4*b^4*sin(d*x+c)-105/d*a^4*b^4*ln(sec(d*x+c)
+tan(d*x+c))+28/d*a^3*b^5*sin(d*x+c)^4+56/d*a^3*b^5*sin(d*x+c)^2+112/d*a^3*
b^5*ln(cos(d*x+c))+14/d*a^2*b^6*sin(d*x+c)^5+70/3/d*a^2*b^6*sin(d*x+c)^3+70
/d*a^2*b^6*sin(d*x+c)-70/d*a^2*b^6*ln(sec(d*x+c)+tan(d*x+c))+6/d*a*b^7*sin(
d*x+c)^4+12/d*a*b^7*sin(d*x+c)^2+24/d*a*b^7*ln(cos(d*x+c))+14/d*a^6*b^2*sin
(d*x+c)-14/d*a^6*b^2*ln(sec(d*x+c)+tan(d*x+c))+56/d*a^5*b^3*ln(cos(d*x+c))+
35/d*a^4*b^4*sin(d*x+c)^3+14/d*a^2*b^6*sin(d*x+c)^7/cos(d*x+c)^2+4/d*a*b^7*
sin(d*x+c)^8/cos(d*x+c)^2+14/d*a^6*b^2*sin(d*x+c)^3/cos(d*x+c)^2+35/d*a^4*b
^4*sin(d*x+c)^5/cos(d*x+c)^2+28/d*a^3*b^5*sin(d*x+c)^6/cos(d*x+c)^2+1/2/d*a
^8*sec(d*x+c)*tan(d*x+c)+4*a*b^7*sin(d*x+c)^6/d
```

Maxima [A] time = 0.97204, size = 436, normalized size = 1.54

$$12\,b^8\sin(dx+c)^5 + 120\,ab^7\sin(dx+c)^4 + 40\left(14\,a^2b^6 + b^8\right)\sin(dx+c)^3 + 240\left(7\,a^3b^5 + 2\,ab^7\right)\sin(dx+c)^2 + 15\left(a^8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/60*(12*b^8*sin(d*x + c)^5 + 120*a*b^7*sin(d*x + c)^4 + 40*(14*a^2*b^6 + b
^8)*sin(d*x + c)^3 + 240*(7*a^3*b^5 + 2*a*b^7)*sin(d*x + c)^2 + 15*(a^8 - 2
8*a^6*b^2 + 112*a^5*b^3 - 210*a^4*b^4 + 224*a^3*b^5 - 140*a^2*b^6 + 48*a*b^
7 - 7*b^8)*log(sin(d*x + c) + 1) - 15*(a^8 - 28*a^6*b^2 - 112*a^5*b^3 - 210
*a^4*b^4 - 224*a^3*b^5 - 140*a^2*b^6 - 48*a*b^7 - 7*b^8)*log(sin(d*x + c) -
 1) + 60*(70*a^4*b^4 + 56*a^2*b^6 + 3*b^8)*sin(d*x + c) - 30*(8*a^7*b + 56*
a^5*b^3 + 56*a^3*b^5 + 8*a*b^7 + (a^8 + 28*a^6*b^2 + 70*a^4*b^4 + 28*a^2*b^
6 + b^8)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 3.36943, size = 894, normalized size = 3.15

$$120\,ab^7\cos(dx+c)^6 + 240\,a^7b + 1680\,a^5b^3 + 1680\,a^3b^5 + 240\,ab^7 - 240\left(7\,a^3b^5 + 3\,ab^7\right)\cos(dx+c)^4 + 15\left(a^8 - 28\,a^6b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/60*(120*a*b^7*cos(d*x + c)^6 + 240*a^7*b + 1680*a^5*b^3 + 1680*a^3*b^5 +
240*a*b^7 - 240*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 + 15*(a^8 - 28*a^6*b^2
 + 112*a^5*b^3 - 210*a^4*b^4 + 224*a^3*b^5 - 140*a^2*b^6 + 48*a*b^7 - 7*b^8
)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 15*(a^8 - 28*a^6*b^2 - 112*a^5*b^3
 - 210*a^4*b^4 - 224*a^3*b^5 - 140*a^2*b^6 - 48*a*b^7 - 7*b^8)*cos(d*x + c)
^2*log(-sin(d*x + c) + 1) + 105*(8*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + 2*(6
*b^8*cos(d*x + c)^6 + 15*a^8 + 420*a^6*b^2 + 1050*a^4*b^4 + 420*a^2*b^6 + 1
5*b^8 - 8*(35*a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + 4*(525*a^4*b^4 + 490*a^2*b^
6 + 29*b^8)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19351, size = 551, normalized size = 1.94

$$\frac{12\,b^8\sin(dx+c)^5+120\,ab^7\sin(dx+c)^4+560\,a^2b^6\sin(dx+c)^3+40\,b^8\sin(dx+c)^3+1680\,a^3b^5\sin(dx+c)^2+480\,ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/60*(12*b^8*sin(d*x + c)^5 + 120*a*b^7*sin(d*x + c)^4 + 560*a^2*b^6*sin(d*
x + c)^3 + 40*b^8*sin(d*x + c)^3 + 1680*a^3*b^5*sin(d*x + c)^2 + 480*a*b^7*
sin(d*x + c)^2 + 4200*a^4*b^4*sin(d*x + c) + 3360*a^2*b^6*sin(d*x + c) + 18
0*b^8*sin(d*x + c) + 15*(a^8 - 28*a^6*b^2 + 112*a^5*b^3 - 210*a^4*b^4 + 224
*a^3*b^5 - 140*a^2*b^6 + 48*a*b^7 - 7*b^8)*log(abs(sin(d*x + c) + 1)) - 15*
```

```
(a^8 - 28*a^6*b^2 - 112*a^5*b^3 - 210*a^4*b^4 - 224*a^3*b^5 - 140*a^2*b^6 -
 48*a*b^7 - 7*b^8)*log(abs(sin(d*x + c) - 1)) - 30*(56*a^5*b^3*sin(d*x + c)
^2 + 112*a^3*b^5*sin(d*x + c)^2 + 24*a*b^7*sin(d*x + c)^2 + a^8*sin(d*x + c
) + 28*a^6*b^2*sin(d*x + c) + 70*a^4*b^4*sin(d*x + c) + 28*a^2*b^6*sin(d*x
+ c) + b^8*sin(d*x + c) + 8*a^7*b - 56*a^3*b^5 - 16*a*b^7)/(sin(d*x + c)^2
- 1))/d
```

3.418 $\int \sec^5(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=320

$$-\frac{ab^7\left(13-\frac{3a^2}{b^2}\right)\sin^4(c+dx)}{8d}+\frac{5b^4\left(-42a^2b^2+9a^4-7b^4\right)\sin^3(c+dx)}{24d}+\frac{ab^3\left(-77a^2b^2+15a^4-48b^4\right)\sin^2(c+dx)}{4d}+\frac{5b}{}$$

```
[Out] -((a + b)^6*(3*a^2 - 18*a*b + 35*b^2)*Log[1 - Sin[c + d*x]])/(16*d) + ((a -
 b)^6*(3*a^2 + 18*a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*d) + (5*b^2*(6*a
^6 - 35*a^4*b^2 - 84*a^2*b^4 - 7*b^6)*Sin[c + d*x])/(8*d) + (a*b^3*(15*a^4
- 77*a^2*b^2 - 48*b^4)*Sin[c + d*x]^2)/(4*d) + (5*b^4*(9*a^4 - 42*a^2*b^2 -
 7*b^4)*Sin[c + d*x]^3)/(24*d) - (a*(13 - (3*a^2)/b^2)*b^7*Sin[c + d*x]^4)/
(8*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^7)/(4*d)
- (Sec[c + d*x]^2*(a + b*Sin[c + d*x])^5*(b*(a^2 + 7*b^2) - a*(3*a^2 - 11*b
^2)*Sin[c + d*x]))/(8*d)
```

Rubi [A] time = 0.303829, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2668, 739, 819, 801, 633, 31}

$$-\frac{ab^7\left(13-\frac{3a^2}{b^2}\right)\sin^4(c+dx)}{8d}+\frac{5b^4\left(-42a^2b^2+9a^4-7b^4\right)\sin^3(c+dx)}{24d}+\frac{ab^3\left(-77a^2b^2+15a^4-48b^4\right)\sin^2(c+dx)}{4d}+\frac{5b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^8,x]
```


```
[Out] -((a + b)^6*(3*a^2 - 18*a*b + 35*b^2)*Log[1 - Sin[c + d*x]])/(16*d) + ((a -
 b)^6*(3*a^2 + 18*a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*d) + (5*b^2*(6*a
^6 - 35*a^4*b^2 - 84*a^2*b^4 - 7*b^6)*Sin[c + d*x])/(8*d) + (a*b^3*(15*a^4
- 77*a^2*b^2 - 48*b^4)*Sin[c + d*x]^2)/(4*d) + (5*b^4*(9*a^4 - 42*a^2*b^2 -
 7*b^4)*Sin[c + d*x]^3)/(24*d) - (a*(13 - (3*a^2)/b^2)*b^7*Sin[c + d*x]^4)/
(8*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^7)/(4*d)
- (Sec[c + d*x]^2*(a + b*Sin[c + d*x])^5*(b*(a^2 + 7*b^2) - a*(3*a^2 - 11*b
^2)*Sin[c + d*x]))/(8*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
```

```
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{(a+x)^8}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{4d} - \frac{b^3\,\text{Subst}\left(\int \frac{(a+x)^6\left(-3a^2+7b^2+\right.}{\left(b^2-x^2\right)^2}\right.}{4d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+}{}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+}{}$$

$$= \frac{5b^2\left(6a^6-35a^4b^2-84a^2b^4-7b^6\right)\sin(c+dx)}{8d} + \frac{ab^3\left(15a^4-77a^2b^2-48b^4\right)\sin}{4d}$$

$$= \frac{5b^2\left(6a^6-35a^4b^2-84a^2b^4-7b^6\right)\sin(c+dx)}{8d} + \frac{ab^3\left(15a^4-77a^2b^2-48b^4\right)\sin}{4d}$$

$$= -\frac{(a+b)^6\left(3a^2-18ab+35b^2\right)\log(1-\sin(c+dx))}{16d} + \frac{(a-b)^6\left(3a^2+18ab+35b\right.}{16d}$$

Mathematica [A] time = 4.07519, size = 514, normalized size = 1.61

$$-\frac{-6ab^9\left(3a^2+11b^2\right)\sin^8(c+dx)+6b^8\left(-90a^2b^2-27a^4+5b^4\right)\sin^7(c+dx)-24ab^7\left(79a^2b^2+27a^4-8b^4\right)\sin^6(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -(3*(a^2 - b^2)^2*((a + b)^6*(3*a^2 - 18*a*b + 35*b^2)*Log[1 - Sin[c + d*x]
] - (a - b)^6*(3*a^2 + 18*a*b + 35*b^2)*Log[1 + Sin[c + d*x]]) + 6*b^2*(-10
8*a^10 + 234*a^8*b^2 - 28*a^6*b^4 - 595*a^4*b^6 + 350*a^2*b^8 + 35*b^10)*Si
n[c + d*x] - 24*a*b^3*(63*a^8 - 21*a^6*b^2 + 88*a^4*b^4 - 8*a^2*b^6 - 24*b^
8)*Sin[c + d*x]^2 + 14*b^4*(-162*a^8 - 144*a^6*b^2 - 85*a^4*b^4 + 50*a^2*b^
6 + 5*b^8)*Sin[c + d*x]^3 - 12*a*b^5*(189*a^6 + 333*a^4*b^2 - 8*a^2*b^4 - 2
4*b^6)*Sin[c + d*x]^4 + 42*b^6*(-36*a^6 - 87*a^4*b^2 + 10*a^2*b^4 + b^6)*Si
n[c + d*x]^5 - 24*a*b^7*(27*a^4 + 79*a^2*b^2 - 8*b^4)*Sin[c + d*x]^6 + 6*b^
8*(-27*a^4 - 90*a^2*b^2 + 5*b^4)*Sin[c + d*x]^7 - 6*a*b^9*(3*a^2 + 11*b^2)*
Sin[c + d*x]^8 + 12*(a^2 - b^2)*Sec[c + d*x]^4*(b - a*Sin[c + d*x])*(a + b*
```

```
Sin[c + d*x])^9 + 6*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^9*(9*a^2*b + 5*b^3
- a*(3*a^2 + 11*b^2)*Sin[c + d*x]))/(48*(a^2 - b^2)^2*d)
```

Maple [B] time = 0.139, size = 760, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^8,x)
```

```
[Out] -5/8/d*b^8*sin(d*x+c)^9/cos(d*x+c)^2+1/4/d*a^8*tan(d*x+c)*sec(d*x+c)^3+3/8/
d*a^8*ln(sec(d*x+c)+tan(d*x+c))-35/8/d*b^8*sin(d*x+c)-35/24/d*b^8*sin(d*x+c
)^3+35/8/d*b^8*ln(sec(d*x+c)+tan(d*x+c))-7/8/d*b^8*sin(d*x+c)^5-5/8*b^8*sin
(d*x+c)^7/d+7/d*a^6*b^2*sin(d*x+c)^3/cos(d*x+c)^4-105/4/d*a^4*b^4*sin(d*x+c
)+105/4/d*a^4*b^4*ln(sec(d*x+c)+tan(d*x+c))-56/d*a^3*b^5*ln(cos(d*x+c))-21/
2/d*a^2*b^6*sin(d*x+c)^5-35/2/d*a^2*b^6*sin(d*x+c)^3-105/2/d*a^2*b^6*sin(d*
x+c)+105/2/d*a^2*b^6*ln(sec(d*x+c)+tan(d*x+c))-6/d*a*b^7*sin(d*x+c)^4-12/d*
a*b^7*sin(d*x+c)^2-24/d*a*b^7*ln(cos(d*x+c))+7/2/d*a^6*b^2*sin(d*x+c)-7/2/d
*a^6*b^2*ln(sec(d*x+c)+tan(d*x+c))-35/4/d*a^4*b^4*sin(d*x+c)^3-21/2/d*a^2*b
^6*sin(d*x+c)^7/cos(d*x+c)^2-4/d*a*b^7*sin(d*x+c)^8/cos(d*x+c)^2+7/2/d*a^6*
b^2*sin(d*x+c)^3/cos(d*x+c)^2-35/4/d*a^4*b^4*sin(d*x+c)^5/cos(d*x+c)^2+3/8/
d*a^8*sec(d*x+c)*tan(d*x+c)+1/4/d*b^8*sin(d*x+c)^9/cos(d*x+c)^4+2/d*a^7*b/c
os(d*x+c)^4+14/d*a^3*b^5*tan(d*x+c)^4-28/d*a^3*b^5*tan(d*x+c)^2-4*a*b^7*sin
(d*x+c)^6/d+35/2/d*a^4*b^4*sin(d*x+c)^5/cos(d*x+c)^4+7/d*a^2*b^6*sin(d*x+c)
^7/cos(d*x+c)^4+2/d*a*b^7*sin(d*x+c)^8/cos(d*x+c)^4+14/d*a^5*b^3*sin(d*x+c)
^4/cos(d*x+c)^4
```

Maxima [A] time = 0.980675, size = 470, normalized size = 1.47

$$-\frac{16\,b^{8}\sin\left(dx+c\right)^{3}+192\,ab^{7}\sin\left(dx+c\right)^{2}-3\left(3\,a^{8}-28\,a^{6}b^{2}+210\,a^{4}b^{4}-448\,a^{3}b^{5}+420\,a^{2}b^{6}-192\,ab^{7}+35\,b^{8}\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/48*(16*b^8*sin(d*x + c)^3 + 192*a*b^7*sin(d*x + c)^2 - 3*(3*a^8 - 28*a^6
*b^2 + 210*a^4*b^4 - 448*a^3*b^5 + 420*a^2*b^6 - 192*a*b^7 + 35*b^8)*log(si
```

```
n(d*x + c) + 1) + 3*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 + 448*a^3*b^5 + 420*a
^2*b^6 + 192*a*b^7 + 35*b^8)*log(sin(d*x + c) - 1) + 48*(28*a^2*b^6 + 3*b^8
)*sin(d*x + c) - 6*(16*a^7*b - 112*a^5*b^3 - 336*a^3*b^5 - 80*a*b^7 - (3*a^
8 - 28*a^6*b^2 - 350*a^4*b^4 - 252*a^2*b^6 - 13*b^8)*sin(d*x + c)^3 + 32*(7
*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*sin(d*x + c)^2 + (5*a^8 + 28*a^6*b^2 - 210
*a^4*b^4 - 196*a^2*b^6 - 11*b^8)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
+ c)^2 + 1))/d
```

Fricas [A] time = 3.37023, size = 879, normalized size = 2.75

$$\frac{192\,ab^{7}\cos\left(dx+c\right)^{6}-96\,ab^{7}\cos\left(dx+c\right)^{4}+96\,a^{7}b+672\,a^{5}b^{3}+672\,a^{3}b^{5}+96\,ab^{7}+3\left(3\,a^{8}-28\,a^{6}b^{2}+210\,a^{4}b^{4}-448\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/48*(192*a*b^7*cos(d*x + c)^6 - 96*a*b^7*cos(d*x + c)^4 + 96*a^7*b + 672*a
^5*b^3 + 672*a^3*b^5 + 96*a*b^7 + 3*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 - 448
*a^3*b^5 + 420*a^2*b^6 - 192*a*b^7 + 35*b^8)*cos(d*x + c)^4*log(sin(d*x + c
) + 1) - 3*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 + 448*a^3*b^5 + 420*a^2*b^6 +
192*a*b^7 + 35*b^8)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 192*(7*a^5*b^3
+ 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + 2*(8*b^8*cos(d*x + c)^6 + 6*a^8 +
168*a^6*b^2 + 420*a^4*b^4 + 168*a^2*b^6 + 6*b^8 - 16*(42*a^2*b^6 + 5*b^8)*c
os(d*x + c)^4 + 3*(3*a^8 - 28*a^6*b^2 - 350*a^4*b^4 - 252*a^2*b^6 - 13*b^8)
*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21373, size = 579, normalized size = 1.81

$$-\frac{16\,b^8\sin(dx+c)^3+192\,ab^7\sin(dx+c)^2+1344\,a^2b^6\sin(dx+c)+144\,b^8\sin(dx+c)-3\left(3\,a^8-28\,a^6b^2+210\,a^4b^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/48*(16*b^8*sin(d*x + c)^3 + 192*a*b^7*sin(d*x + c)^2 + 1344*a^2*b^6*sin(
d*x + c) + 144*b^8*sin(d*x + c) - 3*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 - 448
*a^3*b^5 + 420*a^2*b^6 - 192*a*b^7 + 35*b^8)*log(abs(sin(d*x + c) + 1)) + 3
*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 + 448*a^3*b^5 + 420*a^2*b^6 + 192*a*b^7
+ 35*b^8)*log(abs(sin(d*x + c) - 1)) - 6*(336*a^3*b^5*sin(d*x + c)^4 + 144*
a*b^7*sin(d*x + c)^4 - 3*a^8*sin(d*x + c)^3 + 28*a^6*b^2*sin(d*x + c)^3 + 3
50*a^4*b^4*sin(d*x + c)^3 + 252*a^2*b^6*sin(d*x + c)^3 + 13*b^8*sin(d*x + c
)^3 + 224*a^5*b^3*sin(d*x + c)^2 - 224*a^3*b^5*sin(d*x + c)^2 - 192*a*b^7*s
in(d*x + c)^2 + 5*a^8*sin(d*x + c) + 28*a^6*b^2*sin(d*x + c) - 210*a^4*b^4*
sin(d*x + c) - 196*a^2*b^6*sin(d*x + c) - 11*b^8*sin(d*x + c) + 16*a^7*b -
112*a^5*b^3 + 64*a*b^7)/(sin(d*x + c)^2 - 1)^2/d
```

3.419 $\int \cos^2(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=423

$$-\frac{11ab\left(10536a^4b^2+9588a^2b^4+1792a^6+1289b^6\right)\cos^3(c+dx)}{40320d}-\frac{b\left(64a^2+21b^2\right)\cos^3(c+dx)(a+b\sin(c+dx))^5}{240d}-\frac{ab}{}$$

```
[Out] ((128*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*x)/256 - (11*
a*b*(1792*a^6 + 10536*a^4*b^2 + 9588*a^2*b^4 + 1289*b^6)*Cos[c + d*x]^3)/(4
0320*d) + ((128*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*Cos
[c + d*x]*Sin[c + d*x])/(256*d) - (b*(6272*a^6 + 28088*a^4*b^2 + 15956*a^2*
b^4 + 735*b^6)*Cos[c + d*x]^3*(a + b*Sin[c + d*x]))/(13440*d) - (13*a*b*(11
2*a^4 + 348*a^2*b^2 + 101*b^4)*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(3360
*d) - (b*(784*a^4 + 1500*a^2*b^2 + 147*b^4)*Cos[c + d*x]^3*(a + b*Sin[c + d
*x])^3)/(2016*d) - (a*b*(112*a^2 + 109*b^2)*Cos[c + d*x]^3*(a + b*Sin[c + d
*x])^4)/(336*d) - (b*(64*a^2 + 21*b^2)*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^
5)/(240*d) - (17*a*b*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^6)/(90*d) - (b*Cos
[c + d*x]^3*(a + b*Sin[c + d*x])^7)/(10*d)
```

Rubi [A] time = 1.21706, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2692, 2862, 2669, 2635, 8}

$$-\frac{11ab\left(10536a^4b^2+9588a^2b^4+1792a^6+1289b^6\right)\cos^3(c+dx)}{40320d}-\frac{b\left(64a^2+21b^2\right)\cos^3(c+dx)(a+b\sin(c+dx))^5}{240d}-\frac{ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^8,x]
```



```
[Out] ((128*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*x)/256 - (11*
a*b*(1792*a^6 + 10536*a^4*b^2 + 9588*a^2*b^4 + 1289*b^6)*Cos[c + d*x]^3)/(4
0320*d) + ((128*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*Cos
[c + d*x]*Sin[c + d*x])/(256*d) - (b*(6272*a^6 + 28088*a^4*b^2 + 15956*a^2*
b^4 + 735*b^6)*Cos[c + d*x]^3*(a + b*Sin[c + d*x]))/(13440*d) - (13*a*b*(11
2*a^4 + 348*a^2*b^2 + 101*b^4)*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(3360
*d) - (b*(784*a^4 + 1500*a^2*b^2 + 147*b^4)*Cos[c + d*x]^3*(a + b*Sin[c + d
*x])^3)/(2016*d) - (a*b*(112*a^2 + 109*b^2)*Cos[c + d*x]^3*(a + b*Sin[c + d
*x])^4)/(336*d) - (b*(64*a^2 + 21*b^2)*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^
5)/(240*d) - (17*a*b*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^6)/(90*d) - (b*Cos
[c + d*x]^3*(a + b*Sin[c + d*x])^7)/(10*d)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^8\,dx = -\frac{b\cos^3(c+dx)(a+b\sin(c+dx))^7}{10d} + \frac{1}{10}\int \cos^2(c+dx)(a+b\sin(c+dx))^6\,(10$$

$$= -\frac{17ab\cos^3(c+dx)(a+b\sin(c+dx))^6}{90d} - \frac{b\cos^3(c+dx)(a+b\sin(c+dx))^7}{10d} + \frac{1}{90}$$

$$= -\frac{b\left(64a^2+21b^2\right)\cos^3(c+dx)(a+b\sin(c+dx))^5}{240d} - \frac{17ab\cos^3(c+dx)(a+b\sin}{90d}$$

$$= -\frac{ab\left(112a^2+109b^2\right)\cos^3(c+dx)(a+b\sin(c+dx))^4}{336d} - \frac{b\left(64a^2+21b^2\right)\cos^3(c+}{240}$$

$$= -\frac{b\left(784a^4+1500a^2b^2+147b^4\right)\cos^3(c+dx)(a+b\sin(c+dx))^3}{2016d} - \frac{ab\left(112a^2+1\right.}{}$$

$$= -\frac{13ab\left(112a^4+348a^2b^2+101b^4\right)\cos^3(c+dx)(a+b\sin(c+dx))^2}{3360d} - \frac{b\left(784a^4+\right.}{}$$

$$= -\frac{b\left(6272a^6+28088a^4b^2+15956a^2b^4+735b^6\right)\cos^3(c+dx)(a+b\sin(c+dx))}{13440d}$$

$$= -\frac{11ab\left(1792a^6+10536a^4b^2+9588a^2b^4+1289b^6\right)\cos^3(c+dx)}{40320d} - \frac{b\left(6272a^6+2\right.}{}$$

$$= -\frac{11ab\left(1792a^6+10536a^4b^2+9588a^2b^4+1289b^6\right)\cos^3(c+dx)}{40320d} + \frac{\left(128a^8+896\right.}{}$$

$$= \frac{1}{256}\left(128a^8+896a^6b^2+1120a^4b^4+280a^2b^6+7b^8\right)x - \frac{11ab\left(1792a^6+10536a\right.}{}$$

Mathematica [A] time = 1.05263, size = 457, normalized size = 1.08

$$\frac{-564480a^6b^2\sin(4(c+dx)) - 705600a^4b^4\sin(2(c+dx)) - 705600a^4b^4\sin(4(c+dx)) + 235200a^4b^4\sin(6(c+dx)) - 282}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (322560*a^8*c + 2257920*a^6*b^2*c + 2822400*a^4*b^4*c + 705600*a^2*b^6*c +
17640*b^8*c + 322560*a^8*d*x + 2257920*a^6*b^2*d*x + 2822400*a^4*b^4*d*x +
705600*a^2*b^6*d*x + 17640*b^8*d*x - 40320*a*b*(32*a^6 + 112*a^4*b^2 + 70*a
^2*b^4 + 7*b^6)*Cos[c + d*x] - 26880*(16*a^7*b + 28*a^5*b^3 + 7*a^3*b^5)*Co
s[3*(c + d*x)] + 451584*a^5*b^3*Cos[5*(c + d*x)] + 338688*a^3*b^5*Cos[5*(c
+ d*x)] + 32256*a*b^7*Cos[5*(c + d*x)] - 80640*a^3*b^5*Cos[7*(c + d*x)] - 1
4400*a*b^7*Cos[7*(c + d*x)] + 2240*a*b^7*Cos[9*(c + d*x)] + 161280*a^8*Sin[
2*(c + d*x)] - 705600*a^4*b^4*Sin[2*(c + d*x)] - 282240*a^2*b^6*Sin[2*(c +
d*x)] - 8820*b^8*Sin[2*(c + d*x)] - 564480*a^6*b^2*Sin[4*(c + d*x)] - 70560
```

```
0*a^4*b^4*Sin[4*(c + d*x)] - 141120*a^2*b^6*Sin[4*(c + d*x)] - 2520*b^8*Sin
[4*(c + d*x)] + 235200*a^4*b^4*Sin[6*(c + d*x)] + 94080*a^2*b^6*Sin[6*(c +
d*x)] + 2730*b^8*Sin[6*(c + d*x)] - 17640*a^2*b^6*Sin[8*(c + d*x)] - 945*b^
8*Sin[8*(c + d*x)] + 126*b^8*Sin[10*(c + d*x)])/(645120*d)
```

Maple [A] time = 0.082, size = 497, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(b^8*(-1/10*sin(d*x+c)^7*cos(d*x+c)^3-7/80*sin(d*x+c)^5*cos(d*x+c)^3-7/
96*sin(d*x+c)^3*cos(d*x+c)^3-7/128*cos(d*x+c)^3*sin(d*x+c)+7/256*cos(d*x+c)
*sin(d*x+c)+7/256*d*x+7/256*c)+8*a*b^7*(-1/9*sin(d*x+c)^6*cos(d*x+c)^3-2/21
*sin(d*x+c)^4*cos(d*x+c)^3-8/105*sin(d*x+c)^2*cos(d*x+c)^3-16/315*cos(d*x+c
)^3)+28*a^2*b^6*(-1/8*sin(d*x+c)^5*cos(d*x+c)^3-5/48*sin(d*x+c)^3*cos(d*x+c
)^3-5/64*cos(d*x+c)^3*sin(d*x+c)+5/128*cos(d*x+c)*sin(d*x+c)+5/128*d*x+5/12
8*c)+56*a^3*b^5*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c
)^3-8/105*cos(d*x+c)^3)+70*a^4*b^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(
d*x+c)^3*sin(d*x+c)+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+56*a^5*b^3*
(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+28*a^6*b^2*(-1/4*cos(d*x
+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-8/3*a^7*b*cos(d*x
+c)^3+a^8*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.01069, size = 454, normalized size = 1.07

$$-\frac{1720320\, a^7 b \cos(dx+c)^3 - 161280\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))a^8 - 564480\,(4\,dx + 4\,c - \sin(4\,dx + 4\,c))a^6 b^2 - 2408}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/645120*(1720320*a^7*b*cos(d*x + c)^3 - 161280*(2*d*x + 2*c + sin(2*d*x +
 2*c))*a^8 - 564480*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^6*b^2 - 2408448*(3*c
os(d*x + c)^5 - 5*cos(d*x + c)^3)*a^5*b^3 + 235200*(4*sin(2*d*x + 2*c)^3 -
12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^4*b^4 + 344064*(15*cos(d*x + c)^7 - 4
```

```
2*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^3*b^5 + 5880*(64*sin(2*d*x + 2*c)^3
 - 120*d*x - 120*c + 3*sin(8*d*x + 8*c) + 24*sin(4*d*x + 4*c))*a^2*b^6 - 16
384*(35*cos(d*x + c)^9 - 135*cos(d*x + c)^7 + 189*cos(d*x + c)^5 - 105*cos(
d*x + c)^3)*a*b^7 - 21*(96*sin(2*d*x + 2*c)^5 - 640*sin(2*d*x + 2*c)^3 + 84
0*d*x + 840*c - 45*sin(8*d*x + 8*c) - 120*sin(4*d*x + 4*c))*b^8)/d
```

Fricas [A] time = 3.26176, size = 780, normalized size = 1.84

$$\frac{71680\,ab^7\cos(dx+c)^9 - 92160\left(7\,a^3b^5+3\,ab^7\right)\cos(dx+c)^7 + 129024\left(7\,a^5b^3+14\,a^3b^5+3\,ab^7\right)\cos(dx+c)^5 - 215040}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/80640*(71680*a*b^7*cos(d*x + c)^9 - 92160*(7*a^3*b^5 + 3*a*b^7)*cos(d*x +
 c)^7 + 129024*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^5 - 215040*(
a^7*b + 7*a^5*b^3 + 7*a^3*b^5 + a*b^7)*cos(d*x + c)^3 + 315*(128*a^8 + 896*
a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*d*x + 21*(384*b^8*cos(d*x + c
)^9 - 48*(280*a^2*b^6 + 31*b^8)*cos(d*x + c)^7 + 8*(5600*a^4*b^4 + 4760*a^2
*b^6 + 263*b^8)*cos(d*x + c)^5 - 10*(2688*a^6*b^2 + 7840*a^4*b^4 + 3304*a^2
*b^6 + 121*b^8)*cos(d*x + c)^3 + 15*(128*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 +
 280*a^2*b^6 + 7*b^8)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 44.4758, size = 1115, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((a**8*x*sin(c + d*x)**2/2 + a**8*x*cos(c + d*x)**2/2 + a**8*sin(c
 + d*x)*cos(c + d*x)/(2*d) - 8*a**7*b*cos(c + d*x)**3/(3*d) + 7*a**6*b**2*x
*sin(c + d*x)**4/2 + 7*a**6*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2 + 7*a**6
*b**2*x*cos(c + d*x)**4/2 + 7*a**6*b**2*sin(c + d*x)**3*cos(c + d*x)/(2*d)
- 7*a**6*b**2*sin(c + d*x)*cos(c + d*x)**3/(2*d) - 56*a**5*b**3*sin(c + d*x
)**2*cos(c + d*x)**3/(3*d) - 112*a**5*b**3*cos(c + d*x)**5/(15*d) + 35*a**4
*b**4*x*sin(c + d*x)**6/8 + 105*a**4*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2
```

```
/8 + 105*a**4*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + 35*a**4*b**4*x*cos
(c + d*x)**6/8 + 35*a**4*b**4*sin(c + d*x)**5*cos(c + d*x)/(8*d) - 35*a**4*
b**4*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 35*a**4*b**4*sin(c + d*x)*cos(
c + d*x)**5/(8*d) - 56*a**3*b**5*sin(c + d*x)**4*cos(c + d*x)**3/(3*d) - 22
4*a**3*b**5*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) - 64*a**3*b**5*cos(c + d
*x)**7/(15*d) + 35*a**2*b**6*x*sin(c + d*x)**8/32 + 35*a**2*b**6*x*sin(c +
d*x)**6*cos(c + d*x)**2/8 + 105*a**2*b**6*x*sin(c + d*x)**4*cos(c + d*x)**4
/16 + 35*a**2*b**6*x*sin(c + d*x)**2*cos(c + d*x)**6/8 + 35*a**2*b**6*x*cos
(c + d*x)**8/32 + 35*a**2*b**6*sin(c + d*x)**7*cos(c + d*x)/(32*d) - 511*a*
*2*b**6*sin(c + d*x)**5*cos(c + d*x)**3/(96*d) - 385*a**2*b**6*sin(c + d*x)
**3*cos(c + d*x)**5/(96*d) - 35*a**2*b**6*sin(c + d*x)*cos(c + d*x)**7/(32*
d) - 8*a*b**7*sin(c + d*x)**6*cos(c + d*x)**3/(3*d) - 16*a*b**7*sin(c + d*x
)**4*cos(c + d*x)**5/(5*d) - 64*a*b**7*sin(c + d*x)**2*cos(c + d*x)**7/(35*
d) - 128*a*b**7*cos(c + d*x)**9/(315*d) + 7*b**8*x*sin(c + d*x)**10/256 + 3
5*b**8*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 35*b**8*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 35*b**8*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 35*b**8
*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 7*b**8*x*cos(c + d*x)**10/256 + 7*
b**8*sin(c + d*x)**9*cos(c + d*x)/(256*d) - 79*b**8*sin(c + d*x)**7*cos(c +
 d*x)**3/(384*d) - 7*b**8*sin(c + d*x)**5*cos(c + d*x)**5/(30*d) - 49*b**8*
sin(c + d*x)**3*cos(c + d*x)**7/(384*d) - 7*b**8*sin(c + d*x)*cos(c + d*x)*
*9/(256*d), Ne(d, 0)), (x*(a + b*sin(c))**8*cos(c)**2, True))
```

Giac [A] time = 1.17001, size = 491, normalized size = 1.16

$$\frac{ab^7\cos(9\,dx+9\,c)}{288\,d}+\frac{b^8\sin(10\,dx+10\,c)}{5120\,d}+\frac{1}{256}\left(128\,a^8+896\,a^6b^2+1120\,a^4b^4+280\,a^2b^6+7\,b^8\right)x-\frac{\left(28\,a^3b^5+5\,ab^7\right.}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/288*a*b^7*cos(9*d*x + 9*c)/d + 1/5120*b^8*sin(10*d*x + 10*c)/d + 1/256*(1
28*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*x - 1/224*(28*a^
3*b^5 + 5*a*b^7)*cos(7*d*x + 7*c)/d + 1/40*(28*a^5*b^3 + 21*a^3*b^5 + 2*a*b
^7)*cos(5*d*x + 5*c)/d - 1/24*(16*a^7*b + 28*a^5*b^3 + 7*a^3*b^5)*cos(3*d*x
 + 3*c)/d - 1/16*(32*a^7*b + 112*a^5*b^3 + 70*a^3*b^5 + 7*a*b^7)*cos(d*x +
c)/d - 1/2048*(56*a^2*b^6 + 3*b^8)*sin(8*d*x + 8*c)/d + 1/3072*(1120*a^4*b^
4 + 448*a^2*b^6 + 13*b^8)*sin(6*d*x + 6*c)/d - 1/256*(224*a^6*b^2 + 280*a^4
*b^4 + 56*a^2*b^6 + b^8)*sin(4*d*x + 4*c)/d + 1/512*(128*a^8 - 560*a^4*b^4
- 224*a^2*b^6 - 7*b^8)*sin(2*d*x + 2*c)/d
```

3.420 $\int \sec^2(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=349

$$\frac{ab\left(1664a^4b^2+2789a^2b^4+40a^6+512b^6\right)\cos(c+dx)}{20d}+\frac{b\left(6a^2+7b^2\right)\cos(c+dx)(a+b\sin(c+dx))^5}{6d}+ab\left(30a^2+113b\right.$$

```
[Out] (-7*b^2*(64*a^6 + 240*a^4*b^2 + 120*a^2*b^4 + 5*b^6)*x)/16 + (a*b*(40*a^6 +
 1664*a^4*b^2 + 2789*a^2*b^4 + 512*b^6)*Cos[c + d*x])/(20*d) + (b^2*(80*a^6
 + 2248*a^4*b^2 + 2502*a^2*b^4 + 175*b^6)*Cos[c + d*x]*Sin[c + d*x])/(80*d)
 + (a*b*(40*a^4 + 624*a^2*b^2 + 337*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^
2)/(40*d) + (b*(120*a^4 + 992*a^2*b^2 + 175*b^4)*Cos[c + d*x]*(a + b*Sin[c
+ d*x])^3)/(120*d) + (a*b*(30*a^2 + 113*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*
x])^4)/(30*d) + (b*(6*a^2 + 7*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^5)/(6*
d) + (a*b*Cos[c + d*x]*(a + b*Sin[c + d*x])^6)/d + (Sec[c + d*x]*(b + a*Sin
[c + d*x])*(a + b*Sin[c + d*x])^7)/d
```

Rubi [A] time = 0.563405, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2691, 2753, 2734}

$$\frac{ab\left(1664a^4b^2+2789a^2b^4+40a^6+512b^6\right)\cos(c+dx)}{20d}+\frac{b\left(6a^2+7b^2\right)\cos(c+dx)(a+b\sin(c+dx))^5}{6d}+ab\left(30a^2+113b\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (-7*b^2*(64*a^6 + 240*a^4*b^2 + 120*a^2*b^4 + 5*b^6)*x)/16 + (a*b*(40*a^6 +
 1664*a^4*b^2 + 2789*a^2*b^4 + 512*b^6)*Cos[c + d*x])/(20*d) + (b^2*(80*a^6
 + 2248*a^4*b^2 + 2502*a^2*b^4 + 175*b^6)*Cos[c + d*x]*Sin[c + d*x])/(80*d)
 + (a*b*(40*a^4 + 624*a^2*b^2 + 337*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^
2)/(40*d) + (b*(120*a^4 + 992*a^2*b^2 + 175*b^4)*Cos[c + d*x]*(a + b*Sin[c
+ d*x])^3)/(120*d) + (a*b*(30*a^2 + 113*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*
x])^4)/(30*d) + (b*(6*a^2 + 7*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^5)/(6*
d) + (a*b*Cos[c + d*x]*(a + b*Sin[c + d*x])^6)/d + (Sec[c + d*x]*(b + a*Sin
[c + d*x])*(a + b*Sin[c + d*x])^7)/d
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
```

```
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c+dx)(a+b\sin(c+dx))^8\,dx &= \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{d} - \int (a+b\sin(c+dx))^6\left(7b^2\right.\\
&= \frac{ab\cos(c+dx)(a+b\sin(c+dx))^6}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c}{d}\\
&= \frac{b\left(6a^2+7b^2\right)\cos(c+dx)(a+b\sin(c+dx))^5}{6d} + \frac{ab\cos(c+dx)(a+b\sin(c+dx}{d}\\
&= \frac{ab\left(30a^2+113b^2\right)\cos(c+dx)(a+b\sin(c+dx))^4}{30d} + \frac{b\left(6a^2+7b^2\right)\cos(c+dx)(}{6d}\\
&= \frac{b\left(120a^4+992a^2b^2+175b^4\right)\cos(c+dx)(a+b\sin(c+dx))^3}{120d} + \frac{ab\left(30a^2+113b\right.}{}\\
&= \frac{ab\left(40a^4+624a^2b^2+337b^4\right)\cos(c+dx)(a+b\sin(c+dx))^2}{40d} + \frac{b\left(120a^4+992\right.}{}\\
&= -\frac{7}{16}b^2\left(64a^6+240a^4b^2+120a^2b^4+5b^6\right)x + \frac{ab\left(40a^6+1664a^4b^2+2789a^2b^4\right.}{20d}
\end{aligned}$$

Mathematica [A] time = 1.12468, size = 313, normalized size = 0.9

$$\frac{\sec(c+dx)\left(53760a^6b^2\sin(c+dx)+151200a^4b^4\sin(c+dx)+16800a^4b^4\sin(3(c+dx))+67200a^2b^6\sin(c+dx)+1260\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]*(15360*a^7*b + 161280*a^5*b^3 + 201600*a^3*b^5 + 33600*a*b^7
- 840*b^2*(64*a^6 + 240*a^4*b^2 + 120*a^2*b^4 + 5*b^6)*(c + d*x)*Cos[c + d*
x] + 1120*(48*a^5*b^3 + 80*a^3*b^5 + 15*a*b^7)*Cos[2*(c + d*x)] - 4480*a^3*
b^5*Cos[4*(c + d*x)] - 1344*a*b^7*Cos[4*(c + d*x)] + 96*a*b^7*Cos[6*(c + d*
x)] + 1920*a^8*Sin[c + d*x] + 53760*a^6*b^2*Sin[c + d*x] + 151200*a^4*b^4*S
in[c + d*x] + 67200*a^2*b^6*Sin[c + d*x] + 2625*b^8*Sin[c + d*x] + 16800*a^
4*b^4*Sin[3*(c + d*x)] + 12600*a^2*b^6*Sin[3*(c + d*x)] + 630*b^8*Sin[3*(c
+ d*x)] - 840*a^2*b^6*Sin[5*(c + d*x)] - 70*b^8*Sin[5*(c + d*x)] + 5*b^8*Si
n[7*(c + d*x)]))/(1920*d)
```

Maple [A] time = 0.066, size = 406, normalized size = 1.2

$$\frac{1}{d}\left(a^8\tan(dx+c)+8\,\frac{a^7b}{\cos(dx+c)}+28\,a^6b^2(\tan(dx+c)-dx-c)+56\,a^5b^3\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*tan(d*x+c)+8*a^7*b/cos(d*x+c)+28*a^6*b^2*(tan(d*x+c)-d*x-c)+56*a^5
*b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+70*a^4*b^4*(sin(
d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)
+56*a^3*b^5*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*co
s(d*x+c))+28*a^2*b^6*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^
3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+8*a*b^7*(sin(d*x+c)^8/cos(d*
x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+b^8*
(sin(d*x+c)^9/cos(d*x+c)+(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+
35/16*sin(d*x+c))*cos(d*x+c)-35/16*d*x-35/16*c))
```

Maxima [A] time = 1.48209, size = 470, normalized size = 1.35

$$-\frac{6720\,(dx+c-\tan(dx+c))a^6b^2+8400\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)a^4b^4+4480\left(\cos(dx+c)^3-\frac{3}{\cos(dx\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/240*(6720*(d*x + c - tan(d*x + c))*a^6*b^2 + 8400*(3*d*x + 3*c - tan(d*x
 + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^4*b^4 + 4480*(cos(d*x + c)^3
 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^3*b^5 + 840*(15*d*x + 15*c - (9*tan(d
*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 8*tan
(d*x + c))*a^2*b^6 - 384*(cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 5/cos(d*x + c
) + 15*cos(d*x + c))*a*b^7 + 5*(105*d*x + 105*c - (87*tan(d*x + c)^5 + 136*
tan(d*x + c)^3 + 57*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*ta
n(d*x + c)^2 + 1) - 48*tan(d*x + c))*b^8 - 13440*a^5*b^3*(1/cos(d*x + c) +
cos(d*x + c)) - 240*a^8*tan(d*x + c) - 1920*a^7*b/cos(d*x + c))/d
```

Fricas [A] time = 2.81196, size = 651, normalized size = 1.87

$$\frac{384\,ab^7\cos(dx+c)^6+1920\,a^7b+13440\,a^5b^3+13440\,a^3b^5+1920\,ab^7-640\left(7\,a^3b^5+3\,ab^7\right)\cos(dx+c)^4-105\left(64\,a^{6}\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(384*a*b^7*cos(d*x + c)^6 + 1920*a^7*b + 13440*a^5*b^3 + 13440*a^3*b^
5 + 1920*a*b^7 - 640*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 - 105*(64*a^6*b^2
 + 240*a^4*b^4 + 120*a^2*b^6 + 5*b^8)*d*x*cos(d*x + c) + 1920*(7*a^5*b^3 +
14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + 5*(8*b^8*cos(d*x + c)^6 + 48*a^8 + 1
344*a^6*b^2 + 3360*a^4*b^4 + 1344*a^2*b^6 + 48*b^8 - 2*(168*a^2*b^6 + 19*b^
8)*cos(d*x + c)^4 + 3*(560*a^4*b^4 + 504*a^2*b^6 + 29*b^8)*cos(d*x + c)^2)*
sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17102, size = 1079, normalized size = 3.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/240*(105*(64*a^6*b^2 + 240*a^4*b^4 + 120*a^2*b^6 + 5*b^8)*(d*x + c) + 48
0*(a^8*tan(1/2*d*x + 1/2*c) + 28*a^6*b^2*tan(1/2*d*x + 1/2*c) + 70*a^4*b^4*
tan(1/2*d*x + 1/2*c) + 28*a^2*b^6*tan(1/2*d*x + 1/2*c) + b^8*tan(1/2*d*x +
1/2*c) + 8*a^7*b + 56*a^5*b^3 + 56*a^3*b^5 + 8*a*b^7)/(tan(1/2*d*x + 1/2*c)
^2 - 1) + 2*(8400*a^4*b^4*tan(1/2*d*x + 1/2*c)^11 + 5880*a^2*b^6*tan(1/2*d*
x + 1/2*c)^11 + 285*b^8*tan(1/2*d*x + 1/2*c)^11 - 13440*a^5*b^3*tan(1/2*d*x
 + 1/2*c)^10 - 13440*a^3*b^5*tan(1/2*d*x + 1/2*c)^10 - 1920*a*b^7*tan(1/2*d
*x + 1/2*c)^10 + 25200*a^4*b^4*tan(1/2*d*x + 1/2*c)^9 + 24360*a^2*b^6*tan(1
/2*d*x + 1/2*c)^9 + 1295*b^8*tan(1/2*d*x + 1/2*c)^9 - 67200*a^5*b^3*tan(1/2
*d*x + 1/2*c)^8 - 94080*a^3*b^5*tan(1/2*d*x + 1/2*c)^8 - 13440*a*b^7*tan(1/
2*d*x + 1/2*c)^8 + 16800*a^4*b^4*tan(1/2*d*x + 1/2*c)^7 + 18480*a^2*b^6*tan
(1/2*d*x + 1/2*c)^7 + 1650*b^8*tan(1/2*d*x + 1/2*c)^7 - 134400*a^5*b^3*tan(
1/2*d*x + 1/2*c)^6 - 224000*a^3*b^5*tan(1/2*d*x + 1/2*c)^6 - 42240*a*b^7*ta
n(1/2*d*x + 1/2*c)^6 - 16800*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 18480*a^2*b^6
*tan(1/2*d*x + 1/2*c)^5 - 1650*b^8*tan(1/2*d*x + 1/2*c)^5 - 134400*a^5*b^3*
tan(1/2*d*x + 1/2*c)^4 - 241920*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 49920*a*b^
7*tan(1/2*d*x + 1/2*c)^4 - 25200*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 24360*a^2
*b^6*tan(1/2*d*x + 1/2*c)^3 - 1295*b^8*tan(1/2*d*x + 1/2*c)^3 - 67200*a^5*b
^3*tan(1/2*d*x + 1/2*c)^2 - 120960*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 - 23424*a
*b^7*tan(1/2*d*x + 1/2*c)^2 - 8400*a^4*b^4*tan(1/2*d*x + 1/2*c) - 5880*a^2*
b^6*tan(1/2*d*x + 1/2*c) - 285*b^8*tan(1/2*d*x + 1/2*c) - 13440*a^5*b^3 - 2
2400*a^3*b^5 - 4224*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.421 $\int \sec^4(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=369

$$\frac{ab\left(-104a^4b^2-803a^2b^4+8a^6-256b^6\right)\cos(c+dx)}{6d}+\frac{b\left(2a^2-7b^2\right)\cos(c+dx)(a+b\sin(c+dx))^5}{3d}+\frac{ab\left(2a^2-13b^2\right)c}{}$$

```
[Out] (35*b^4*(16*a^4 + 16*a^2*b^2 + b^4)*x)/8 + (a*b*(8*a^6 - 104*a^4*b^2 - 803*
a^2*b^4 - 256*b^6)*Cos[c + d*x])/(6*d) + (b^2*(16*a^6 - 200*a^4*b^2 - 866*a
^2*b^4 - 105*b^6)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (a*b*(8*a^4 - 88*a^2*
b^2 - 151*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(12*d) + (b*(8*a^4 - 72
*a^2*b^2 - 35*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(12*d) + (a*b*(2*a^
2 - 13*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^4)/(3*d) + (b*(2*a^2 - 7*b^2)
*Cos[c + d*x]*(a + b*Sin[c + d*x])^5)/(3*d) + (Sec[c + d*x]^3*(b + a*Sin[c
+ d*x])*(a + b*Sin[c + d*x])^7)/(3*d) - (Sec[c + d*x]*(a + b*Sin[c + d*x])^
6*(5*a*b - (2*a^2 - 7*b^2)*Sin[c + d*x]))/(3*d)
```

Rubi [A] time = 0.645708, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2691, 2861, 2753, 2734}

$$\frac{ab\left(-104a^4b^2-803a^2b^4+8a^6-256b^6\right)\cos(c+dx)}{6d}+\frac{b\left(2a^2-7b^2\right)\cos(c+dx)(a+b\sin(c+dx))^5}{3d}+\frac{ab\left(2a^2-13b^2\right)c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^8,x]
```


```
[Out] (35*b^4*(16*a^4 + 16*a^2*b^2 + b^4)*x)/8 + (a*b*(8*a^6 - 104*a^4*b^2 - 803*
a^2*b^4 - 256*b^6)*Cos[c + d*x])/(6*d) + (b^2*(16*a^6 - 200*a^4*b^2 - 866*a
^2*b^4 - 105*b^6)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (a*b*(8*a^4 - 88*a^2*
b^2 - 151*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(12*d) + (b*(8*a^4 - 72
*a^2*b^2 - 35*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(12*d) + (a*b*(2*a^
2 - 13*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^4)/(3*d) + (b*(2*a^2 - 7*b^2)
*Cos[c + d*x]*(a + b*Sin[c + d*x])^5)/(3*d) + (Sec[c + d*x]^3*(b + a*Sin[c
+ d*x])*(a + b*Sin[c + d*x])^7)/(3*d) - (Sec[c + d*x]*(a + b*Sin[c + d*x])^
6*(5*a*b - (2*a^2 - 7*b^2)*Sin[c + d*x]))/(3*d)
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
```

```
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{3d} - \frac{1}{3}\int \sec^2(c+dx)(a+b\,\mathrm{si}$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{3d} - \frac{\sec(c+dx)(a+b\sin(c+}{}$$

$$= \frac{b\left(2a^2-7b^2\right)\cos(c+dx)(a+b\sin(c+dx))^5}{3d} + \frac{\sec^3(c+dx)(b+a\sin(c+dx))}{3d}$$

$$= \frac{ab\left(2a^2-13b^2\right)\cos(c+dx)(a+b\sin(c+dx))^4}{3d} + \frac{b\left(2a^2-7b^2\right)\cos(c+dx)(a+}{3d}$$

$$= \frac{b\left(8a^4-72a^2b^2-35b^4\right)\cos(c+dx)(a+b\sin(c+dx))^3}{12d} + \frac{ab\left(2a^2-13b^2\right)\cos(}{}$$

$$= \frac{ab\left(8a^4-88a^2b^2-151b^4\right)\cos(c+dx)(a+b\sin(c+dx))^2}{12d} + \frac{b\left(8a^4-72a^2b^2-\right.}{}$$

$$= \frac{35}{8}b^4\left(16a^4+16a^2b^2+b^4\right)x + \frac{ab\left(8a^6-104a^4b^2-803a^2b^4-256b^6\right)\cos(c+d}{6d}$$

Mathematica [A] time = 1.16431, size = 414, normalized size = 1.12

$$\frac{\sec^3(c+dx)\left(5376a^6b^2\sin(c+dx)-1792a^6b^2\sin(3(c+dx))-17920a^4b^4\sin(3(c+dx))-6720a^2b^6\sin(c+dx)-1456\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^3*(2048*a^7*b - 7168*a^5*b^3 - 44800*a^3*b^5 - 13440*a*b^7 +
40320*a^4*b^4*(c + d*x)*Cos[c + d*x] + 40320*a^2*b^6*(c + d*x)*Cos[c + d*x]
 + 2520*b^8*(c + d*x)*Cos[c + d*x] - 21504*a^5*b^3*Cos[2*(c + d*x)] - 64512
*a^3*b^5*Cos[2*(c + d*x)] - 17472*a*b^7*Cos[2*(c + d*x)] + 13440*a^4*b^4*(c
 + d*x)*Cos[3*(c + d*x)] + 13440*a^2*b^6*(c + d*x)*Cos[3*(c + d*x)] + 840*b
^8*(c + d*x)*Cos[3*(c + d*x)] - 5376*a^3*b^5*Cos[4*(c + d*x)] - 1920*a*b^7*
Cos[4*(c + d*x)] + 64*a*b^7*Cos[6*(c + d*x)] + 384*a^8*Sin[c + d*x] + 5376*
a^6*b^2*Sin[c + d*x] - 6720*a^2*b^6*Sin[c + d*x] - 525*b^8*Sin[c + d*x] + 1
28*a^8*Sin[3*(c + d*x)] - 1792*a^6*b^2*Sin[3*(c + d*x)] - 17920*a^4*b^4*Sin
[3*(c + d*x)] - 14560*a^2*b^6*Sin[3*(c + d*x)] - 847*b^8*Sin[3*(c + d*x)] -
 672*a^2*b^6*Sin[5*(c + d*x)] - 63*b^8*Sin[5*(c + d*x)] + 3*b^8*Sin[7*(c +
d*x)]))/(768*d)
```

Maple [A] time = 0.134, size = 495, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-a^8*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+8/3*a^7*b/cos(d*x+c)^3+28/3*a^
6*b^2*sin(d*x+c)^3/cos(d*x+c)^3+56*a^5*b^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1
/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+70*a^4*b^4*(1/3
*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+56*a^3*b^5*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-s
in(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+28*a
^2*b^6*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(
d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+8*a*b^
7*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16/5+sin(
d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+b^8*(1/3*sin(d*x+c)
^9/cos(d*x+c)^3-2*sin(d*x+c)^9/cos(d*x+c)-2*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+
35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/8*d*x+35/8*c))
```

Maxima [A] time = 1.47095, size = 443, normalized size = 1.2

$$\frac{224\,a^6b^2\tan(dx+c)^3+8\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^8+560\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^4b^4+1\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/24*(224*a^6*b^2*tan(d*x + c)^3 + 8*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^8
+ 560*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^4*b^4 + 112*(2*tan(
d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d
*x + c))*a^2*b^6 + 64*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)
^3 - 9*cos(d*x + c))*a*b^7 + (8*tan(d*x + c)^3 + 105*d*x + 105*c - 3*(13*ta
n(d*x + c)^3 + 11*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 7
2*tan(d*x + c))*b^8 - 448*a^3*b^5*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 +
3*cos(d*x + c)) - 448*(3*cos(d*x + c)^2 - 1)*a^5*b^3/cos(d*x + c)^3 + 64*a^
7*b/cos(d*x + c)^3)/d
```

Fricas [A] time = 2.91977, size = 633, normalized size = 1.72

$$\frac{64\,ab^7\cos\left(dx+c\right)^6+64\,a^7b+448\,a^5b^3+448\,a^3b^5+64\,ab^7+105\left(16\,a^4b^4+16\,a^2b^6+b^8\right)dx\cos\left(dx+c\right)^3-192\left(7\,a^3b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/24*(64*a*b^7*cos(d*x + c)^6 + 64*a^7*b + 448*a^5*b^3 + 448*a^3*b^5 + 64*a
*b^7 + 105*(16*a^4*b^4 + 16*a^2*b^6 + b^8)*d*x*cos(d*x + c)^3 - 192*(7*a^3*
b^5 + 3*a*b^7)*cos(d*x + c)^4 - 192*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(
d*x + c)^2 + (6*b^8*cos(d*x + c)^6 + 8*a^8 + 224*a^6*b^2 + 560*a^4*b^4 + 22
4*a^2*b^6 + 8*b^8 - 3*(112*a^2*b^6 + 13*b^8)*cos(d*x + c)^4 + 16*(a^8 - 14*
a^6*b^2 - 140*a^4*b^4 - 98*a^2*b^6 - 5*b^8)*cos(d*x + c)^2)*sin(d*x + c))/(
d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18347, size = 923, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/24*(105*(16*a^4*b^4 + 16*a^2*b^6 + b^8)*(d*x + c) - 16*(3*a^8*tan(1/2*d*x
 + 1/2*c)^5 - 210*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 168*a^2*b^6*tan(1/2*d*x
+ 1/2*c)^5 - 9*b^8*tan(1/2*d*x + 1/2*c)^5 + 24*a^7*b*tan(1/2*d*x + 1/2*c)^4
 - 168*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 48*a*b^7*tan(1/2*d*x + 1/2*c)^4 - 2
```

```
*a^8*tan(1/2*d*x + 1/2*c)^3 + 112*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 700*a^4*
b^4*tan(1/2*d*x + 1/2*c)^3 + 448*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 22*b^8*ta
n(1/2*d*x + 1/2*c)^3 + 336*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 672*a^3*b^5*tan
(1/2*d*x + 1/2*c)^2 + 144*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 3*a^8*tan(1/2*d*x
+ 1/2*c) - 210*a^4*b^4*tan(1/2*d*x + 1/2*c) - 168*a^2*b^6*tan(1/2*d*x + 1/2
*c) - 9*b^8*tan(1/2*d*x + 1/2*c) + 8*a^7*b - 112*a^5*b^3 - 280*a^3*b^5 - 64
*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3 + 2*(336*a^2*b^6*tan(1/2*d*x + 1/2*c
)^7 + 33*b^8*tan(1/2*d*x + 1/2*c)^7 - 1344*a^3*b^5*tan(1/2*d*x + 1/2*c)^6 -
 384*a*b^7*tan(1/2*d*x + 1/2*c)^6 + 336*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 57
*b^8*tan(1/2*d*x + 1/2*c)^5 - 4032*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 1536*a*
b^7*tan(1/2*d*x + 1/2*c)^4 - 336*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 57*b^8*ta
n(1/2*d*x + 1/2*c)^3 - 4032*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 - 1664*a*b^7*tan
(1/2*d*x + 1/2*c)^2 - 336*a^2*b^6*tan(1/2*d*x + 1/2*c) - 33*b^8*tan(1/2*d*x
 + 1/2*c) - 1344*a^3*b^5 - 512*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.422 $\int \sec^6(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=381

$$\frac{2ab\left(-48a^4b^2+163a^2b^4+8a^6+192b^6\right)\cos(c+dx)}{15d}+\frac{4ab\left(2a^2+b^2\right)\cos(c+dx)(a+b\sin(c+dx))^4}{15d}+\frac{b\left(-16a^2b^2+8\right.}{}$$

```
[Out] (-7*b^6*(8*a^2 + b^2)*x)/2 + (2*a*b*(8*a^6 - 48*a^4*b^2 + 163*a^2*b^4 + 192
*b^6)*Cos[c + d*x])/(15*d) + (b^2*(16*a^6 - 88*a^4*b^2 + 282*a^2*b^4 + 105*
b^6)*Cos[c + d*x]*Sin[c + d*x])/(30*d) + (a*b*(8*a^4 - 32*a^2*b^2 + 87*b^4)
*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(15*d) + (b*(8*a^4 - 16*a^2*b^2 + 35*
b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(15*d) + (4*a*b*(2*a^2 + b^2)*Cos
[c + d*x]*(a + b*Sin[c + d*x])^4)/(15*d) + (Sec[c + d*x]^5*(b + a*Sin[c + d
*x])*(a + b*Sin[c + d*x])^7)/(5*d) - (Sec[c + d*x]^3*(a + b*Sin[c + d*x])^6
*(3*a*b - (4*a^2 - 7*b^2)*Sin[c + d*x]))/(15*d) - (4*Sec[c + d*x]*(a + b*Si
n[c + d*x])^5*(b*(4*a^2 - 7*b^2) - a*(2*a^2 + b^2)*Sin[c + d*x]))/(15*d)
```

Rubi [A] time = 0.723576, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2691, 2861, 2753, 2734}

$$\frac{2ab\left(-48a^4b^2+163a^2b^4+8a^6+192b^6\right)\cos(c+dx)}{15d}+\frac{4ab\left(2a^2+b^2\right)\cos(c+dx)(a+b\sin(c+dx))^4}{15d}+\frac{b\left(-16a^2b^2+8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^8,x]
```


```
[Out] (-7*b^6*(8*a^2 + b^2)*x)/2 + (2*a*b*(8*a^6 - 48*a^4*b^2 + 163*a^2*b^4 + 192
*b^6)*Cos[c + d*x])/(15*d) + (b^2*(16*a^6 - 88*a^4*b^2 + 282*a^2*b^4 + 105*
b^6)*Cos[c + d*x]*Sin[c + d*x])/(30*d) + (a*b*(8*a^4 - 32*a^2*b^2 + 87*b^4)
*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(15*d) + (b*(8*a^4 - 16*a^2*b^2 + 35*
b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(15*d) + (4*a*b*(2*a^2 + b^2)*Cos
[c + d*x]*(a + b*Sin[c + d*x])^4)/(15*d) + (Sec[c + d*x]^5*(b + a*Sin[c + d
*x])*(a + b*Sin[c + d*x])^7)/(5*d) - (Sec[c + d*x]^3*(a + b*Sin[c + d*x])^6
*(3*a*b - (4*a^2 - 7*b^2)*Sin[c + d*x]))/(15*d) - (4*Sec[c + d*x]*(a + b*Si
n[c + d*x])^5*(b*(4*a^2 - 7*b^2) - a*(2*a^2 + b^2)*Sin[c + d*x]))/(15*d)
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
```

```
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{5d} - \frac{1}{5}\int \sec^4(c+dx)(a+b\,\mathrm{si}$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{5d} - \frac{\sec^3(c+dx)(a+b\sin(c+}{}$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{5d} - \frac{\sec^3(c+dx)(a+b\sin(c+}{}$$

$$= \frac{4ab\left(2a^2+b^2\right)\cos(c+dx)(a+b\sin(c+dx))^4}{15d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx)}{5d}$$

$$= \frac{b\left(8a^4-16a^2b^2+35b^4\right)\cos(c+dx)(a+b\sin(c+dx))^3}{15d} + \frac{4ab\left(2a^2+b^2\right)\cos(c}{}$$

$$= \frac{ab\left(8a^4-32a^2b^2+87b^4\right)\cos(c+dx)(a+b\sin(c+dx))^2}{15d} + \frac{b\left(8a^4-16a^2b^2+3\right.}{}$$

$$= -\frac{7}{2}b^6\left(8a^2+b^2\right)x + \frac{2ab\left(8a^6-48a^4b^2+163a^2b^4+192b^6\right)\cos(c+dx)}{15d} + \frac{b^2\left(1\right.}{}$$

Mathematica [A] time = 1.29166, size = 472, normalized size = 1.24

$$\frac{\sec^5(c+dx)\left(8960a^6b^2\sin(c+dx) - 2240a^6b^2\sin(3(c+dx)) - 448a^6b^2\sin(5(c+dx)) + 16800a^4b^4\sin(c+dx) - 8400a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^5*(3072*a^7*b + 3584*a^5*b^3 + 25984*a^3*b^5 + 17472*a*b^7 -
33600*a^2*b^6*(c + d*x)*Cos[c + d*x] - 4200*b^8*(c + d*x)*Cos[c + d*x] - 17
920*a^5*b^3*Cos[2*(c + d*x)] + 17920*a^3*b^5*Cos[2*(c + d*x)] + 22560*a*b^7
*Cos[2*(c + d*x)] - 16800*a^2*b^6*(c + d*x)*Cos[3*(c + d*x)] - 2100*b^8*(c
+ d*x)*Cos[3*(c + d*x)] + 13440*a^3*b^5*Cos[4*(c + d*x)] + 8640*a*b^7*Cos[4
*(c + d*x)] - 3360*a^2*b^6*(c + d*x)*Cos[5*(c + d*x)] - 420*b^8*(c + d*x)*C
os[5*(c + d*x)] + 480*a*b^7*Cos[6*(c + d*x)] + 640*a^8*Sin[c + d*x] + 8960*
a^6*b^2*Sin[c + d*x] + 16800*a^4*b^4*Sin[c + d*x] + 11200*a^2*b^6*Sin[c + d
*x] + 875*b^8*Sin[c + d*x] + 320*a^8*Sin[3*(c + d*x)] - 2240*a^6*b^2*Sin[3*
(c + d*x)] - 8400*a^4*b^4*Sin[3*(c + d*x)] + 5600*a^2*b^6*Sin[3*(c + d*x)]
+ 1015*b^8*Sin[3*(c + d*x)] + 64*a^8*Sin[5*(c + d*x)] - 448*a^6*b^2*Sin[5*(
c + d*x)] + 1680*a^4*b^4*Sin[5*(c + d*x)] + 5152*a^2*b^6*Sin[5*(c + d*x)] +
 539*b^8*Sin[5*(c + d*x)] + 15*b^8*Sin[7*(c + d*x)]))/(1920*d)
```

Maple [A] time = 0.126, size = 544, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-a^8*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+8/5*a^7*b/c
os(d*x+c)^5+28*a^6*b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos
(d*x+c)^3)+56*a^5*b^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(
d*x+c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+14*
a^4*b^4*sin(d*x+c)^5/cos(d*x+c)^5+56*a^3*b^5*(1/5*sin(d*x+c)^6/cos(d*x+c)^5
-1/15*sin(d*x+c)^6/cos(d*x+c)^3+1/5*sin(d*x+c)^6/cos(d*x+c)+1/5*(8/3+sin(d*
x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+28*a^2*b^6*(1/5*tan(d*x+c)^5-1/3*tan(d
*x+c)^3+tan(d*x+c)-d*x-c)+8*a*b^7*(1/5*sin(d*x+c)^8/cos(d*x+c)^5-1/5*sin(d*
x+c)^8/cos(d*x+c)^3+sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+
c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+b^8*(1/5*sin(d*x+c)^9/cos(d*x+c)^5-4/15*
sin(d*x+c)^9/cos(d*x+c)^3+8/5*sin(d*x+c)^9/cos(d*x+c)+8/5*(sin(d*x+c)^7+7/6
*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)-7/2*d*x-7/2*c
))
```

Maxima [A] time = 1.46795, size = 425, normalized size = 1.12

$$\frac{420\,a^4b^4\tan\left(dx+c\right)^5 + 2\left(3\,\tan\left(dx+c\right)^5 + 10\,\tan\left(dx+c\right)^3 + 15\,\tan\left(dx+c\right)\right)a^8 + 56\left(3\,\tan\left(dx+c\right)^5 + 5\,\tan\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/30*(420*a^4*b^4*tan(d*x + c)^5 + 2*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3
+ 15*tan(d*x + c))*a^8 + 56*(3*tan(d*x + c)^5 + 5*tan(d*x + c)^3)*a^6*b^2 +
 56*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))
*a^2*b^6 + (6*tan(d*x + c)^5 - 20*tan(d*x + c)^3 - 105*d*x - 105*c + 15*tan
(d*x + c)/(tan(d*x + c)^2 + 1) + 90*tan(d*x + c))*b^8 + 48*a*b^7*((15*cos(d
*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^5 + 5*cos(d*x + c)) - 112*(5
*cos(d*x + c)^2 - 3)*a^5*b^3/cos(d*x + c)^5 + 112*(15*cos(d*x + c)^4 - 10*c
os(d*x + c)^2 + 3)*a^3*b^5/cos(d*x + c)^5 + 48*a^7*b/cos(d*x + c)^5)/d
```

Fricas [A] time = 3.01358, size = 660, normalized size = 1.73

$$\frac{240\,ab^7\cos(dx+c)^6+48\,a^7b+336\,a^5b^3+336\,a^3b^5+48\,ab^7-105\left(8\,a^2b^6+b^8\right)dx\cos(dx+c)^5+240\left(7\,a^3b^5+3\,ab^7\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/30*(240*a*b^7*cos(d*x + c)^6 + 48*a^7*b + 336*a^5*b^3 + 336*a^3*b^5 + 48*
a*b^7 - 105*(8*a^2*b^6 + b^8)*d*x*cos(d*x + c)^5 + 240*(7*a^3*b^5 + 3*a*b^7
)*cos(d*x + c)^4 - 80*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + (
15*b^8*cos(d*x + c)^6 + 6*a^8 + 168*a^6*b^2 + 420*a^4*b^4 + 168*a^2*b^6 + 6
*b^8 + 4*(4*a^8 - 28*a^6*b^2 + 105*a^4*b^4 + 322*a^2*b^6 + 29*b^8)*cos(d*x
+ c)^4 + 8*(a^8 - 7*a^6*b^2 - 105*a^4*b^4 - 77*a^2*b^6 - 4*b^8)*cos(d*x + c
)^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19863, size = 895, normalized size = 2.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/30*(105*(8*a^2*b^6 + b^8)*(d*x + c) + 30*(b^8*tan(1/2*d*x + 1/2*c)^3 - 1
6*a*b^7*tan(1/2*d*x + 1/2*c)^2 - b^8*tan(1/2*d*x + 1/2*c) - 16*a*b^7)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2 + 4*(15*a^8*tan(1/2*d*x + 1/2*c)^9 + 420*a^2*b^6*
tan(1/2*d*x + 1/2*c)^9 + 45*b^8*tan(1/2*d*x + 1/2*c)^9 + 120*a^7*b*tan(1/2*
```

```
d*x + 1/2*c)^8 + 120*a*b^7*tan(1/2*d*x + 1/2*c)^8 - 20*a^8*tan(1/2*d*x + 1/
2*c)^7 + 560*a^6*b^2*tan(1/2*d*x + 1/2*c)^7 - 2240*a^2*b^6*tan(1/2*d*x + 1/
2*c)^7 - 220*b^8*tan(1/2*d*x + 1/2*c)^7 + 1680*a^5*b^3*tan(1/2*d*x + 1/2*c)
^6 - 720*a*b^7*tan(1/2*d*x + 1/2*c)^6 + 58*a^8*tan(1/2*d*x + 1/2*c)^5 + 224
*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 3360*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 498
4*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 398*b^8*tan(1/2*d*x + 1/2*c)^5 + 240*a^7
*b*tan(1/2*d*x + 1/2*c)^4 + 560*a^5*b^3*tan(1/2*d*x + 1/2*c)^4 + 4480*a^3*b
^5*tan(1/2*d*x + 1/2*c)^4 + 1920*a*b^7*tan(1/2*d*x + 1/2*c)^4 - 20*a^8*tan(
1/2*d*x + 1/2*c)^3 + 560*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 2240*a^2*b^6*tan(
1/2*d*x + 1/2*c)^3 - 220*b^8*tan(1/2*d*x + 1/2*c)^3 + 560*a^5*b^3*tan(1/2*d
*x + 1/2*c)^2 - 2240*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 - 1200*a*b^7*tan(1/2*d*
x + 1/2*c)^2 + 15*a^8*tan(1/2*d*x + 1/2*c) + 420*a^2*b^6*tan(1/2*d*x + 1/2*
c) + 45*b^8*tan(1/2*d*x + 1/2*c) + 24*a^7*b - 112*a^5*b^3 + 448*a^3*b^5 + 2
64*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.423 $\int \sec^8(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=404

$$\frac{4ab\left(-88a^4b^2+125a^2b^4+24a^6-96b^6\right)\cos(c+dx)}{105d}+\frac{b^2\left(-152a^4b^2+174a^2b^4+48a^6-105b^6\right)\sin(c+dx)\cos(c+dx)}{105d}$$

```
[Out] b^8*x + (4*a*b*(24*a^6 - 88*a^4*b^2 + 125*a^2*b^4 - 96*b^6)*Cos[c + d*x])/(
105*d) + (b^2*(48*a^6 - 152*a^4*b^2 + 174*a^2*b^4 - 105*b^6)*Cos[c + d*x]*S
in[c + d*x])/(105*d) + (2*a*b*(24*a^4 - 40*a^2*b^2 + 9*b^4)*Cos[c + d*x]*(a
 + b*Sin[c + d*x])^2)/(105*d) + (2*b*(24*a^4 + 8*a^2*b^2 - 35*b^4)*Cos[c +
d*x]*(a + b*Sin[c + d*x])^3)/(105*d) + (Sec[c + d*x]^7*(b + a*Sin[c + d*x])
*(a + b*Sin[c + d*x])^7)/(7*d) - (2*Sec[c + d*x]^3*(a + b*Sin[c + d*x])^5*(
b*(6*a^2 - 7*b^2) - a*(12*a^2 - 11*b^2)*Sin[c + d*x]))/(105*d) - (Sec[c + d
*x]^5*(a + b*Sin[c + d*x])^6*(a*b - (6*a^2 - 7*b^2)*Sin[c + d*x]))/(35*d) -
 (2*Sec[c + d*x]*(a + b*Sin[c + d*x])^4*(3*a*b*(12*a^2 - 11*b^2) - (24*a^4
+ 8*a^2*b^2 - 35*b^4)*Sin[c + d*x]))/(105*d)
```

Rubi [A] time = 0.820547, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2691, 2861, 2753, 2734}

$$\frac{4ab\left(-88a^4b^2+125a^2b^4+24a^6-96b^6\right)\cos(c+dx)}{105d}+\frac{b^2\left(-152a^4b^2+174a^2b^4+48a^6-105b^6\right)\sin(c+dx)\cos(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^8,x]
```


```
[Out] b^8*x + (4*a*b*(24*a^6 - 88*a^4*b^2 + 125*a^2*b^4 - 96*b^6)*Cos[c + d*x])/(
105*d) + (b^2*(48*a^6 - 152*a^4*b^2 + 174*a^2*b^4 - 105*b^6)*Cos[c + d*x]*S
in[c + d*x])/(105*d) + (2*a*b*(24*a^4 - 40*a^2*b^2 + 9*b^4)*Cos[c + d*x]*(a
 + b*Sin[c + d*x])^2)/(105*d) + (2*b*(24*a^4 + 8*a^2*b^2 - 35*b^4)*Cos[c +
d*x]*(a + b*Sin[c + d*x])^3)/(105*d) + (Sec[c + d*x]^7*(b + a*Sin[c + d*x])
*(a + b*Sin[c + d*x])^7)/(7*d) - (2*Sec[c + d*x]^3*(a + b*Sin[c + d*x])^5*(
b*(6*a^2 - 7*b^2) - a*(12*a^2 - 11*b^2)*Sin[c + d*x]))/(105*d) - (Sec[c + d
*x]^5*(a + b*Sin[c + d*x])^6*(a*b - (6*a^2 - 7*b^2)*Sin[c + d*x]))/(35*d) -
 (2*Sec[c + d*x]*(a + b*Sin[c + d*x])^4*(3*a*b*(12*a^2 - 11*b^2) - (24*a^4
+ 8*a^2*b^2 - 35*b^4)*Sin[c + d*x]))/(105*d)
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{7d} - \frac{1}{7}\int \sec^6(c+dx)(a+b\sin$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{7d} - \frac{\sec^5(c+dx)(a+b\sin(c+$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{7d} - \frac{2\sec^3(c+dx)(a+b\sin(c$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{7d} - \frac{2\sec^3(c+dx)(a+b\sin(c$$

$$= \frac{2b\left(24a^4+8a^2b^2-35b^4\right)\cos(c+dx)(a+b\sin(c+dx))^3}{105d} + \frac{\sec^7(c+dx)(b+a$$

$$= \frac{2ab\left(24a^4-40a^2b^2+9b^4\right)\cos(c+dx)(a+b\sin(c+dx))^2}{105d} + \frac{2b\left(24a^4+8a^2b^2\right.}{}$$

$$= b^8x + \frac{4ab\left(24a^6-88a^4b^2+125a^2b^4-96b^6\right)\cos(c+dx)}{105d} + \frac{b^2\left(48a^6-152a^4b^2\right.}{}$$

Mathematica [A] time = 1.48119, size = 479, normalized size = 1.19

$$\frac{\sec^7(c+dx)\left(23520a^6b^2\sin(c+dx)-4704a^6b^2\sin(3(c+dx))-1568a^6b^2\sin(5(c+dx))-224a^6b^2\sin(7(c+dx))+44\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^7*(7680*a^7*b + 16128*a^5*b^3 + 25536*a^3*b^5 - 5088*a*b^7 +
3675*b^8*(c + d*x)*Cos[c + d*x] - 37632*a^5*b^3*Cos[2*(c + d*x)] - 12544*a^
3*b^5*Cos[2*(c + d*x)] - 14448*a*b^7*Cos[2*(c + d*x)] + 2205*b^8*(c + d*x)*
Cos[3*(c + d*x)] + 15680*a^3*b^5*Cos[4*(c + d*x)] - 3360*a*b^7*Cos[4*(c + d
*x)] + 735*b^8*(c + d*x)*Cos[5*(c + d*x)] - 1680*a*b^7*Cos[6*(c + d*x)] + 1
05*b^8*(c + d*x)*Cos[7*(c + d*x)] + 1680*a^8*Sin[c + d*x] + 23520*a^6*b^2*S
in[c + d*x] + 44100*a^4*b^4*Sin[c + d*x] + 14700*a^2*b^6*Sin[c + d*x] + 100
8*a^8*Sin[3*(c + d*x)] - 4704*a^6*b^2*Sin[3*(c + d*x)] - 20580*a^4*b^4*Sin[
3*(c + d*x)] - 8820*a^2*b^6*Sin[3*(c + d*x)] - 1176*b^8*Sin[3*(c + d*x)] +
336*a^8*Sin[5*(c + d*x)] - 1568*a^6*b^2*Sin[5*(c + d*x)] + 2940*a^4*b^4*Sin
[5*(c + d*x)] + 2940*a^2*b^6*Sin[5*(c + d*x)] - 392*b^8*Sin[5*(c + d*x)] +
48*a^8*Sin[7*(c + d*x)] - 224*a^6*b^2*Sin[7*(c + d*x)] + 420*a^4*b^4*Sin[7*
(c + d*x)] - 420*a^2*b^6*Sin[7*(c + d*x)] - 176*b^8*Sin[7*(c + d*x)]))/(672
0*d)
```

Maple [A] time = 0.14, size = 567, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-a^8*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+8/7*a^7*b/cos(d*x+c)^7+28*a^6*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/
35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+56*a^5*b^3*(1
/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)
^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+
c))+70*a^4*b^4*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^
5)+56*a^3*b^5*(1/7*sin(d*x+c)^6/cos(d*x+c)^7+1/35*sin(d*x+c)^6/cos(d*x+c)^5
-1/105*sin(d*x+c)^6/cos(d*x+c)^3+1/35*sin(d*x+c)^6/cos(d*x+c)+1/35*(8/3+sin
(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+4*a^2*b^6*sin(d*x+c)^7/cos(d*x+c)^7
+8*a*b^7*(1/7*sin(d*x+c)^8/cos(d*x+c)^7-1/35*sin(d*x+c)^8/cos(d*x+c)^5+1/35
*sin(d*x+c)^8/cos(d*x+c)^3-1/7*sin(d*x+c)^8/cos(d*x+c)-1/7*(16/5+sin(d*x+c)
^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+b^8*(1/7*tan(d*x+c)^7-1/5
*tan(d*x+c)^5+1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A] time = 1.48954, size = 419, normalized size = 1.04

$$\frac{420\,a^2b^6\tan\left(dx+c\right)^7+3\left(5\,\tan\left(dx+c\right)^7+21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+35\,\tan\left(dx+c\right)\right)a^8+28\left(15\,\tan\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/105*(420*a^2*b^6*tan(d*x + c)^7 + 3*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5
 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*a^8 + 28*(15*tan(d*x + c)^7 + 42*ta
n(d*x + c)^5 + 35*tan(d*x + c)^3)*a^6*b^2 + 210*(5*tan(d*x + c)^7 + 7*tan(d
*x + c)^5)*a^4*b^4 + (15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x +
c)^3 + 105*d*x + 105*c - 105*tan(d*x + c))*b^8 - 168*(7*cos(d*x + c)^2 - 5)
*a^5*b^3/cos(d*x + c)^7 + 56*(35*cos(d*x + c)^4 - 42*cos(d*x + c)^2 + 15)*a
^3*b^5/cos(d*x + c)^7 - 24*(35*cos(d*x + c)^6 - 35*cos(d*x + c)^4 + 21*cos(
d*x + c)^2 - 5)*a*b^7/cos(d*x + c)^7 + 120*a^7*b/cos(d*x + c)^7)/d
```

Fricas [A] time = 2.90773, size = 732, normalized size = 1.81

$$\frac{105\, b^8 dx \cos(dx+c)^7 - 840\, ab^7 \cos(dx+c)^6 + 120\, a^7 b + 840\, a^5 b^3 + 840\, a^3 b^5 + 120\, ab^7 + 280\left(7\, a^3 b^5 + 3\, ab^7\right)\cos(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*b^8*d*x*cos(d*x + c)^7 - 840*a*b^7*cos(d*x + c)^6 + 120*a^7*b +
840*a^5*b^3 + 840*a^3*b^5 + 120*a*b^7 + 280*(7*a^3*b^5 + 3*a*b^7)*cos(d*x +
 c)^4 - 168*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + (15*a^8 + 4
20*a^6*b^2 + 1050*a^4*b^4 + 420*a^2*b^6 + 15*b^8 + 4*(12*a^8 - 56*a^6*b^2 +
 105*a^4*b^4 - 105*a^2*b^6 - 44*b^8)*cos(d*x + c)^6 + 2*(12*a^8 - 56*a^6*b^
2 + 105*a^4*b^4 + 630*a^2*b^6 + 61*b^8)*cos(d*x + c)^4 + 6*(3*a^8 - 14*a^6*
b^2 - 280*a^4*b^4 - 210*a^2*b^6 - 11*b^8)*cos(d*x + c)^2)*sin(d*x + c))/(d*
cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19368, size = 980, normalized size = 2.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/105*(105*(d*x + c)*b^8 - 2*(105*a^8*tan(1/2*d*x + 1/2*c)^13 - 105*b^8*tan
(1/2*d*x + 1/2*c)^13 + 840*a^7*b*tan(1/2*d*x + 1/2*c)^12 - 210*a^8*tan(1/2*
```

```
d*x + 1/2*c)^11 + 3920*a^6*b^2*tan(1/2*d*x + 1/2*c)^11 + 770*b^8*tan(1/2*d*
x + 1/2*c)^11 + 11760*a^5*b^3*tan(1/2*d*x + 1/2*c)^10 + 903*a^8*tan(1/2*d*x
 + 1/2*c)^9 + 3136*a^6*b^2*tan(1/2*d*x + 1/2*c)^9 + 23520*a^4*b^4*tan(1/2*d
*x + 1/2*c)^9 - 2471*b^8*tan(1/2*d*x + 1/2*c)^9 + 4200*a^7*b*tan(1/2*d*x +
1/2*c)^8 + 11760*a^5*b^3*tan(1/2*d*x + 1/2*c)^8 + 31360*a^3*b^5*tan(1/2*d*x
 + 1/2*c)^8 - 636*a^8*tan(1/2*d*x + 1/2*c)^7 + 12768*a^6*b^2*tan(1/2*d*x +
1/2*c)^7 + 20160*a^4*b^4*tan(1/2*d*x + 1/2*c)^7 + 26880*a^2*b^6*tan(1/2*d*x
 + 1/2*c)^7 + 4572*b^8*tan(1/2*d*x + 1/2*c)^7 + 23520*a^5*b^3*tan(1/2*d*x +
 1/2*c)^6 + 15680*a^3*b^5*tan(1/2*d*x + 1/2*c)^6 + 13440*a*b^7*tan(1/2*d*x
+ 1/2*c)^6 + 903*a^8*tan(1/2*d*x + 1/2*c)^5 + 3136*a^6*b^2*tan(1/2*d*x + 1/
2*c)^5 + 23520*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 2471*b^8*tan(1/2*d*x + 1/2*
c)^5 + 2520*a^7*b*tan(1/2*d*x + 1/2*c)^4 + 4704*a^5*b^3*tan(1/2*d*x + 1/2*c
)^4 + 9408*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 8064*a*b^7*tan(1/2*d*x + 1/2*c)
^4 - 210*a^8*tan(1/2*d*x + 1/2*c)^3 + 3920*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 +
 770*b^8*tan(1/2*d*x + 1/2*c)^3 + 2352*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 - 313
6*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 + 2688*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 105*
a^8*tan(1/2*d*x + 1/2*c) - 105*b^8*tan(1/2*d*x + 1/2*c) + 120*a^7*b - 336*a
^5*b^3 + 448*a^3*b^5 - 384*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

3.424 $\int \sec^{10}(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=236

$$\frac{128a^2(a^2-b^2)^3\tan(c+dx)}{315d}+\frac{128ab(a^2-b^2)^3\sec(c+dx)}{315d}+\frac{\sec^7(c+dx)(a+b\sin(c+dx))^6((8a^2-7b^2)\sin(c+dx)}{63d}$$

```
[Out] (128*a*b*(a^2 - b^2)^3*Sec[c + d*x])/(315*d) + (64*a*(a^2 - b^2)^2*Sec[c +
d*x]^3*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(315*d) + (16*a*(a^2 -
b^2)*Sec[c + d*x]^5*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^4)/(105*d) +
(Sec[c + d*x]^9*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^7)/(9*d) + (Sec[c
 + d*x]^7*(a + b*Sin[c + d*x])^6*(a*b + (8*a^2 - 7*b^2)*Sin[c + d*x]))/(63*
d) + (128*a^2*(a^2 - b^2)^3*Tan[c + d*x])/(315*d)
```

Rubi [A] time = 0.384936, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2691, 2861, 12, 2669, 3767, 8}

$$\frac{128a^2(a^2-b^2)^3\tan(c+dx)}{315d}+\frac{128ab(a^2-b^2)^3\sec(c+dx)}{315d}+\frac{\sec^7(c+dx)(a+b\sin(c+dx))^6((8a^2-7b^2)\sin(c+dx)}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + b*Sin[c + d*x])^8,x]
```


```
[Out] (128*a*b*(a^2 - b^2)^3*Sec[c + d*x])/(315*d) + (64*a*(a^2 - b^2)^2*Sec[c +
d*x]^3*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(315*d) + (16*a*(a^2 -
b^2)*Sec[c + d*x]^5*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^4)/(105*d) +
(Sec[c + d*x]^9*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^7)/(9*d) + (Sec[c
 + d*x]^7*(a + b*Sin[c + d*x])^6*(a*b + (8*a^2 - 7*b^2)*Sin[c + d*x]))/(63*
d) + (128*a^2*(a^2 - b^2)^3*Tan[c + d*x])/(315*d)
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{9d} - \frac{1}{9}\int \sec^8(c+dx)(a+b\,\mathrm{s}$$

$$= \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{9d} + \frac{\sec^7(c+dx)(a+b\sin(c}{}$$

$$= \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{9d} + \frac{\sec^7(c+dx)(a+b\sin(c}{}$$

$$= \frac{16a\left(a^2-b^2\right)\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^4}{105d} + \frac{\sec^9(c+dx}{}$$

$$= \frac{16a\left(a^2-b^2\right)\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^4}{105d} + \frac{\sec^9(c+dx}{}$$

$$= \frac{64a\left(a^2-b^2\right)^2\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{315d} + \frac{16a\left(a^2-\right.}{}$$

$$= \frac{64a\left(a^2-b^2\right)^2\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{315d} + \frac{16a\left(a^2-\right.}{}$$

$$= \frac{128ab\left(a^2-b^2\right)^3\sec(c+dx)}{315d} + \frac{64a\left(a^2-b^2\right)^2\sec^3(c+dx)(b+a\sin(c+dx))(}{315d}$$

$$= \frac{128ab\left(a^2-b^2\right)^3\sec(c+dx)}{315d} + \frac{64a\left(a^2-b^2\right)^2\sec^3(c+dx)(b+a\sin(c+dx))(}{315d}$$

$$= \frac{128ab\left(a^2-b^2\right)^3\sec(c+dx)}{315d} + \frac{64a\left(a^2-b^2\right)^2\sec^3(c+dx)(b+a\sin(c+dx))(}{315d}$$

Mathematica [A] time = 4.49955, size = 313, normalized size = 1.33

$$\frac{\cos(c+dx)\left(\frac{a(8(a-b)(1-\sin(c+dx))((a-b)(1-\sin(c+dx))(2(a-b)(1-\sin(c+dx))((a-b)(1-\sin(c+dx))(35(a+b\sin(c+dx))^4-4(a-b)(1-\sin(c+dx))((a+b)(\sin(c}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Cos[c + d*x]*(-(Sec[c + d*x]^10*(a + b*Sin[c + d*x])^9) + (a*(35*(a + b*Si
n[c + d*x])^8 + 8*(a - b)*(1 - Sin[c + d*x])*(5*(a + b*Sin[c + d*x])^7 + (a
 - b)*(1 - Sin[c + d*x])*(7*(a + b*Sin[c + d*x])^6 + 2*(a - b)*(1 - Sin[c +
 d*x])*(7*(a + b*Sin[c + d*x])^5 + (a - b)*(1 - Sin[c + d*x])*(35*(a + b*Si
n[c + d*x])^4 - 4*(a - b)*(1 - Sin[c + d*x])*(5*(a + b*Sin[c + d*x])^3 + (a
 + b)*(1 + Sin[c + d*x])*(7*a^2 + 6*a*b + 2*b^2 + 6*(a^2 + 3*a*b + b^2)*Sin
```

```
[c + d*x] + (2*a^2 + 6*a*b + 7*b^2)*Sin[c + d*x]^2))))))))/(35*(1 - Sin[c +
 d*x])^5*(1 + Sin[c + d*x])^4)))/(9*(a - b)*d)
```

Maple [B] time = 0.135, size = 662, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-a^8*(-128/315-1/9*sec(d*x+c)^8-8/63*sec(d*x+c)^6-16/105*sec(d*x+c)^4-
64/315*sec(d*x+c)^2)*tan(d*x+c)+8/9*a^7*b/cos(d*x+c)^9+28*a^6*b^2*(1/9*sin(
d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos
(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+56*a^5*b^3*(1/9*sin(d*x+c)^4/co
s(d*x+c)^9+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/cos(d*x+c)^5+1/
63*sin(d*x+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/63*(2+sin(d*x+c
)^2)*cos(d*x+c))+70*a^4*b^4*(1/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^
5/cos(d*x+c)^7+8/315*sin(d*x+c)^5/cos(d*x+c)^5)+56*a^3*b^5*(1/9*sin(d*x+c)^
6/cos(d*x+c)^9+1/21*sin(d*x+c)^6/cos(d*x+c)^7+1/105*sin(d*x+c)^6/cos(d*x+c)
^5-1/315*sin(d*x+c)^6/cos(d*x+c)^3+1/105*sin(d*x+c)^6/cos(d*x+c)+1/105*(8/3
+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+28*a^2*b^6*(1/9*sin(d*x+c)^7/co
s(d*x+c)^9+2/63*sin(d*x+c)^7/cos(d*x+c)^7)+8*a*b^7*(1/9*sin(d*x+c)^8/cos(d*
x+c)^9+1/63*sin(d*x+c)^8/cos(d*x+c)^7-1/315*sin(d*x+c)^8/cos(d*x+c)^5+1/315
*sin(d*x+c)^8/cos(d*x+c)^3-1/63*sin(d*x+c)^8/cos(d*x+c)-1/63*(16/5+sin(d*x+
c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+1/9*b^8*sin(d*x+c)^9/co
s(d*x+c)^9)
```

Maxima [A] time = 1.00529, size = 425, normalized size = 1.8

$$\frac{35\,b^8\tan\left(dx+c\right)^9+\left(35\,\tan\left(dx+c\right)^9+180\,\tan\left(dx+c\right)^7+378\,\tan\left(dx+c\right)^5+420\,\tan\left(dx+c\right)^3+315\,\tan\left(dx+c\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^8*tan(d*x + c)^9 + (35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 37
8*tan(d*x + c)^5 + 420*tan(d*x + c)^3 + 315*tan(d*x + c))*a^8 + 28*(35*tan(
```

```
d*x + c)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*
a^6*b^2 + 70*(35*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*a^
4*b^4 + 140*(7*tan(d*x + c)^9 + 9*tan(d*x + c)^7)*a^2*b^6 - 280*(9*cos(d*x
+ c)^2 - 7)*a^5*b^3/cos(d*x + c)^9 + 56*(63*cos(d*x + c)^4 - 90*cos(d*x + c
)^2 + 35)*a^3*b^5/cos(d*x + c)^9 - 8*(105*cos(d*x + c)^6 - 189*cos(d*x + c)
^4 + 135*cos(d*x + c)^2 - 35)*a*b^7/cos(d*x + c)^9 + 280*a^7*b/cos(d*x + c)
^9)/d
```

Fricas [A] time = 3.14994, size = 795, normalized size = 3.37

$$-\frac{840\,ab^{7}\cos\left(dx+c\right)^{6}-280\,a^{7}b-1960\,a^{5}b^{3}-1960\,a^{3}b^{5}-280\,ab^{7}-504\left(7\,a^{3}b^{5}+3\,ab^{7}\right)\cos\left(dx+c\right)^{4}+360\left(7\,a^{5}b^{3}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/315*(840*a*b^7*cos(d*x + c)^6 - 280*a^7*b - 1960*a^5*b^3 - 1960*a^3*b^5
- 280*a*b^7 - 504*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 + 360*(7*a^5*b^3 + 1
4*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 - ((128*a^8 - 448*a^6*b^2 + 560*a^4*b^4
 - 280*a^2*b^6 + 35*b^8)*cos(d*x + c)^8 + 35*a^8 + 980*a^6*b^2 + 2450*a^4*b
^4 + 980*a^2*b^6 + 35*b^8 + 4*(16*a^8 - 56*a^6*b^2 + 70*a^4*b^4 - 35*a^2*b^
6 - 35*b^8)*cos(d*x + c)^6 + 6*(8*a^8 - 28*a^6*b^2 + 35*a^4*b^4 + 350*a^2*b
^6 + 35*b^8)*cos(d*x + c)^4 + 20*(2*a^8 - 7*a^6*b^2 - 175*a^4*b^4 - 133*a^2
*b^6 - 7*b^8)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19109, size = 1204, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/315*(315*a^8*tan(1/2*d*x + 1/2*c)^17 + 2520*a^7*b*tan(1/2*d*x + 1/2*c)^1
6 - 840*a^8*tan(1/2*d*x + 1/2*c)^15 + 11760*a^6*b^2*tan(1/2*d*x + 1/2*c)^15
 + 35280*a^5*b^3*tan(1/2*d*x + 1/2*c)^14 + 4788*a^8*tan(1/2*d*x + 1/2*c)^13
 + 14112*a^6*b^2*tan(1/2*d*x + 1/2*c)^13 + 70560*a^4*b^4*tan(1/2*d*x + 1/2*
c)^13 + 23520*a^7*b*tan(1/2*d*x + 1/2*c)^12 + 58800*a^5*b^3*tan(1/2*d*x + 1
/2*c)^12 + 94080*a^3*b^5*tan(1/2*d*x + 1/2*c)^12 - 5112*a^8*tan(1/2*d*x + 1
/2*c)^11 + 79632*a^6*b^2*tan(1/2*d*x + 1/2*c)^11 + 120960*a^4*b^4*tan(1/2*d
*x + 1/2*c)^11 + 80640*a^2*b^6*tan(1/2*d*x + 1/2*c)^11 + 176400*a^5*b^3*tan
(1/2*d*x + 1/2*c)^10 + 141120*a^3*b^5*tan(1/2*d*x + 1/2*c)^10 + 40320*a*b^7
*tan(1/2*d*x + 1/2*c)^10 + 10658*a^8*tan(1/2*d*x + 1/2*c)^9 + 39872*a^6*b^2
*tan(1/2*d*x + 1/2*c)^9 + 244160*a^4*b^4*tan(1/2*d*x + 1/2*c)^9 + 89600*a^2
*b^6*tan(1/2*d*x + 1/2*c)^9 + 8960*b^8*tan(1/2*d*x + 1/2*c)^9 + 35280*a^7*b
*tan(1/2*d*x + 1/2*c)^8 + 105840*a^5*b^3*tan(1/2*d*x + 1/2*c)^8 + 197568*a^
3*b^5*tan(1/2*d*x + 1/2*c)^8 + 24192*a*b^7*tan(1/2*d*x + 1/2*c)^8 - 5112*a^
8*tan(1/2*d*x + 1/2*c)^7 + 79632*a^6*b^2*tan(1/2*d*x + 1/2*c)^7 + 120960*a^
4*b^4*tan(1/2*d*x + 1/2*c)^7 + 80640*a^2*b^6*tan(1/2*d*x + 1/2*c)^7 + 10584
0*a^5*b^3*tan(1/2*d*x + 1/2*c)^6 + 56448*a^3*b^5*tan(1/2*d*x + 1/2*c)^6 + 1
0752*a*b^7*tan(1/2*d*x + 1/2*c)^6 + 4788*a^8*tan(1/2*d*x + 1/2*c)^5 + 14112
*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 70560*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 10
080*a^7*b*tan(1/2*d*x + 1/2*c)^4 + 15120*a^5*b^3*tan(1/2*d*x + 1/2*c)^4 + 1
6128*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 4608*a*b^7*tan(1/2*d*x + 1/2*c)^4 - 8
40*a^8*tan(1/2*d*x + 1/2*c)^3 + 11760*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 5040
*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 - 4032*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 + 115
2*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 315*a^8*tan(1/2*d*x + 1/2*c) + 280*a^7*b -
 560*a^5*b^3 + 448*a^3*b^5 - 128*a*b^7)/((tan(1/2*d*x + 1/2*c)^2 - 1)^9*d)
```

3.425 $\int \frac{\cos^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{(a^2-2b^2)\sin^2(c+dx)}{2b^3d}-\frac{a(a^2-2b^2)\sin(c+dx)}{b^4d}+\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^5d}-\frac{a\sin^3(c+dx)}{3b^2d}+\frac{\sin^4(c+dx)}{4bd}$$

```
[Out] ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^5*d) - (a*(a^2 - 2*b^2)*Sin[c +
d*x])/(b^4*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^2)/(2*b^3*d) - (a*Sin[c + d*x]^
3)/(3*b^2*d) + Sin[c + d*x]^4/(4*b*d)
```

Rubi [A] time = 0.105313, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{(a^2-2b^2)\sin^2(c+dx)}{2b^3d}-\frac{a(a^2-2b^2)\sin(c+dx)}{b^4d}+\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^5d}-\frac{a\sin^3(c+dx)}{3b^2d}+\frac{\sin^4(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```



```
[Out] ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^5*d) - (a*(a^2 - 2*b^2)*Sin[c +
d*x])/(b^4*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^2)/(2*b^3*d) - (a*Sin[c + d*x]^
3)/(3*b^2*d) + Sin[c + d*x]^4/(4*b*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^3\left(1-\frac{2b^2}{a^2}\right)+\left(a^2-2b^2\right)x - ax^2 + x^3 + \frac{\left(a^2-b^2\right)^2}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\left(a^2-b^2\right)^2 \log(a+b\sin(c+dx))}{b^5 d} - \frac{a\left(a^2-2b^2\right)\sin(c+dx)}{b^4 d} + \frac{\left(a^2-2b^2\right)\sin^2(c+dx)}{2b^3 d} - \frac{a\sin^3(}{3b}$$

Mathematica [A] time = 0.195237, size = 103, normalized size = 0.87

$$\frac{6b^2\left(a^2-b^2\right)\sin^2(c+dx) - 12ab\left(a^2-2b^2\right)\sin(c+dx) + 12\left(a^2-b^2\right)^2\log(a+b\sin(c+dx)) - 4ab^3\sin^3(c+dx) + 3b^4 \text{co}}{12b^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b^4*Cos[c + d*x]^4 + 12*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]] - 12*a*b*(
a^2 - 2*b^2)*Sin[c + d*x] + 6*b^2*(a^2 - b^2)*Sin[c + d*x]^2 - 4*a*b^3*Sin[
c + d*x]^3)/(12*b^5*d)
```

Maple [A] time = 0.036, size = 163, normalized size = 1.4

$$\frac{(\sin(dx+c))^4}{4bd} - \frac{a(\sin(dx+c))^3}{3b^2 d} + \frac{(\sin(dx+c))^2 a^2}{2db^3} - \frac{(\sin(dx+c))^2}{bd} - \frac{a^3\sin(dx+c)}{db^4} + 2\frac{a\sin(dx+c)}{b^2 d} + \frac{\ln(a+b\sin}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/4*sin(d*x+c)^4/b/d-1/3*a*sin(d*x+c)^3/b^2/d+1/2/d/b^3*sin(d*x+c)^2*a^2-si
n(d*x+c)^2/b/d-1/d/b^4*a^3*sin(d*x+c)+2*a*sin(d*x+c)/b^2/d+1/d/b^5*ln(a+b*s
in(d*x+c))*a^4-2/d/b^3*ln(a+b*sin(d*x+c))*a^2+ln(a+b*sin(d*x+c))/b/d
```

Maxima [A] time = 0.963665, size = 146, normalized size = 1.24

$$\frac{\frac{3\,b^3\sin(dx+c)^4-4\,ab^2\sin(dx+c)^3+6\left(a^2b-2\,b^3\right)\sin(dx+c)^2-12\left(a^3-2\,ab^2\right)\sin(dx+c)}{b^4}+\frac{12\left(a^4-2\,a^2b^2+b^4\right)\log(b\sin(dx+c)+a)}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*(a^2*b - 2*b^3)*si
n(d*x + c)^2 - 12*(a^3 - 2*a*b^2)*sin(d*x + c))/b^4 + 12*(a^4 - 2*a^2*b^2 +
 b^4)*log(b*sin(d*x + c) + a)/b^5)/d
```

Fricas [A] time = 2.71323, size = 250, normalized size = 2.12

$$\frac{3\,b^4\cos\left(dx+c\right)^4-6\left(a^2b^2-b^4\right)\cos\left(dx+c\right)^2+12\left(a^4-2\,a^2b^2+b^4\right)\log\left(b\sin\left(dx+c\right)+a\right)+4\left(ab^3\cos\left(dx+c\right)^2-3\,a\right.}{12\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*cos(d*x + c)^4 - 6*(a^2*b^2 - b^4)*cos(d*x + c)^2 + 12*(a^4 - 2
*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a) + 4*(a*b^3*cos(d*x + c)^2 - 3*a^3*b
 + 5*a*b^3)*sin(d*x + c))/(b^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.125, size = 162, normalized size = 1.37

$$\frac{\frac{3\,b^3\sin(dx+c)^4-4\,ab^2\sin(dx+c)^3+6\,a^2b\sin(dx+c)^2-12\,b^3\sin(dx+c)^2-12\,a^3\sin(dx+c)+24\,ab^2\sin(dx+c)}{b^4}+\frac{12\left(a^4-2\,a^2b^2+b^4\right)\log(|b\sin(dx+c)+a|)}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*((3*b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*a^2*b*sin(d*x + c)
^2 - 12*b^3*sin(d*x + c)^2 - 12*a^3*sin(d*x + c) + 24*a*b^2*sin(d*x + c))/b
^4 + 12*(a^4 - 2*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/b^5)/d
```

3.426 $\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=61

$$-\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}$$

```
[Out] -(((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^3*d)) + (a*Sin[c + d*x])/(b^2*d)
 - Sin[c + d*x]^2/(2*b*d)
```

Rubi [A] time = 0.066362, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^3*d)) + (a*Sin[c + d*x])/(b^2*d)
 - Sin[c + d*x]^2/(2*b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2+b^2}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= -\frac{\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{b^3d} + \frac{a\sin(c+dx)}{b^2d} - \frac{\sin^2(c+dx)}{2bd}$$

Mathematica [A] time = 0.0671992, size = 54, normalized size = 0.89

$$\frac{-\left(a^2-b^2\right)\log(a+b\sin(c+dx))+ab\sin(c+dx)-\frac{1}{2}b^2\sin^2(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-((a^2 - b^2)*Log[a + b*Sin[c + d*x]]) + a*b*Sin[c + d*x] - (b^2*Sin[c + d
*x]^2)/2)/(b^3*d)
```

Maple [A] time = 0.033, size = 72, normalized size = 1.2

$$-\frac{(\sin(dx+c))^2}{2\,bd}+\frac{a\sin(dx+c)}{b^2d}-\frac{\ln(a+b\sin(dx+c))\,a^2}{db^3}+\frac{\ln(a+b\sin(dx+c))}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2*sin(d*x+c)^2/b/d+a*sin(d*x+c)/b^2/d-1/d/b^3*ln(a+b*sin(d*x+c))*a^2+ln(
a+b*sin(d*x+c))/b/d
```

Maxima [A] time = 0.944457, size = 74, normalized size = 1.21

$$-\frac{\frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2}+\frac{2\left(a^2-b^2\right)\log(b\sin(dx+c)+a)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2 + 2*(a^2 - b^2)*log(b*sin(d
*x + c) + a)/b^3)/d
```

Fricas [A] time = 2.54508, size = 128, normalized size = 2.1

$$\frac{b^{2} \cos \left(dx + c\right)^{2} + 2 \, ab \sin \left(dx + c\right) - 2 \, \left(a^{2} - b^{2}\right) \log \left(b \sin \left(dx + c\right) + a\right)}{2 \, b^{3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cos(d*x + c)^2 + 2*a*b*sin(d*x + c) - 2*(a^2 - b^2)*log(b*sin(d*x
+ c) + a))/(b^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10446, size = 76, normalized size = 1.25

$$-\frac{\frac{b \sin\left(dx+c\right)^{2} - 2 \, a \sin\left(dx+c\right)}{b^{2}} + \frac{2 \, \left(a^{2} - b^{2}\right) \log\left({\left| b \sin\left(dx+c\right) + a \right|}\right)}{b^{3}}}{2 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2 + 2*(a^2 - b^2)*log(abs(b*s
in(d*x + c) + a))/b^3)/d
```

3.427 $\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

```
[Out] Log[a + b*Sin[c + d*x]]/(b*d)
```

Rubi [A] time = 0.0267588, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 31}

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[a + b*Sin[c + d*x]]/(b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\sin(c+dx))}{bd}\end{aligned}$$

Mathematica [A] time = 0.0067112, size = 18, normalized size = 1.

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[a + b*Sin[c + d*x]]/(b*d)
```

Maple [A] time = 0.013, size = 19, normalized size = 1.1

$$\frac{\ln(a+b\sin(dx+c))}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] ln(a+b*sin(d*x+c))/b/d
```

Maxima [A] time = 0.932619, size = 24, normalized size = 1.33

$$\frac{\log(b\sin(dx+c)+a)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(b*sin(d*x + c) + a)/(b*d)
```

Fricas [A] time = 2.37353, size = 42, normalized size = 2.33

$$\frac{\log(b\sin(dx+c)+a)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] log(b*sin(d*x + c) + a)/(b*d)
```

Sympy [A] time = 0.618084, size = 41, normalized size = 2.28

$$\begin{cases} \frac{x\cos(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{x\cos(c)}{a+b\sin(c)} & \text{for } d = 0 \\ \frac{\sin(c+dx)}{ad} & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b}+\sin(c+dx)\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (x*cos(c)/(a + b*sin(c)), Eq(d
, 0)), (sin(c + d*x)/(a*d), Eq(b, 0)), (log(a/b + sin(c + d*x))/(b*d), True
))
```

Giac [A] time = 1.10082, size = 26, normalized size = 1.44

$$\frac{\log(|b\sin(dx+c)+a|)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*sin(d*x + c) + a))/(b*d)
```

3.428 $\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$-\frac{b\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
- (b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.0829564, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2668, 706, 31, 633}

$$-\frac{b\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
- (b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{b\,\mathrm{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d} \\
&= -\frac{b\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{\left(a^2-b^2\right)d} - \frac{b\,\mathrm{Subst}\left(\int \frac{-a+x}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{\left(a^2-b^2\right)d} \\
&= -\frac{b\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)d} - \frac{\mathrm{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2(a-b)d} + \frac{\mathrm{Subst}\left(\int \frac{1}{b-x}\,dx, x, b\sin(c\right.}{2(a+b)d} \\
&= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)d}
\end{aligned}
$$

Mathematica [A] time = 0.0546337, size = 64, normalized size = 0.85

$$
\frac{(b-a)\log(1-\sin(c+dx)) + (a+b)\log(\sin(c+dx)+1) - 2b\log(a+b\sin(c+dx))}{2d(a-b)(a+b)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a + b)*Log[1 - Sin[c + d*x]] + (a + b)*Log[1 + Sin[c + d*x]] - 2*b*Log[a
 + b*Sin[c + d*x]])/(2*(a - b)*(a + b)*d)
```

Maple [A] time = 0.043, size = 76, normalized size = 1.

$$
-\frac{b\ln(a+b\sin(dx+c))}{d(a+b)(a-b)} - \frac{\ln(\sin(dx+c)-1)}{d(2a+2b)} + \frac{\ln(1+\sin(dx+c))}{d(2a-2b)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*b/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-1/d/(2*a+2*b)*ln(sin(d*x+c)-1)+1/d/(2
*a-2*b)*ln(1+sin(d*x+c))
```

Maxima [A] time = 0.94883, size = 86, normalized size = 1.15

$$-\frac{\frac{2\,b\log(b\sin(dx+c)+a)}{a^2-b^2}-\frac{\log(\sin(dx+c)+1)}{a-b}+\frac{\log(\sin(dx+c)-1)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*log(b*sin(d*x + c) + a)/(a^2 - b^2) - log(sin(d*x + c) + 1)/(a -
b) + log(sin(d*x + c) - 1)/(a + b))/d
```

Fricas [A] time = 2.66229, size = 158, normalized size = 2.11

$$-\frac{2\,b\log\left(b\sin\left(dx+c\right)+a\right)-(a+b)\log\left(\sin\left(dx+c\right)+1\right)+(a-b)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*log(b*sin(d*x + c) + a) - (a + b)*log(sin(d*x + c) + 1) + (a - b)
*log(-sin(d*x + c) + 1))/((a^2 - b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.14144, size = 96, normalized size = 1.28

$$-\frac{\frac{2\,b^2\log(|b\sin(dx+c)+a|)}{a^2b-b^3}-\frac{\log(|\sin(dx+c)+1|)}{a-b}+\frac{\log(|\sin(dx+c)-1|)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x +
c) + 1))/(a - b) + log(abs(sin(d*x + c) - 1))/(a + b))/d
```

3.429 $\int \frac{\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=123

$$\frac{b^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)}-\frac{(a+2b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(a-2b)\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

```
[Out] -((a + 2*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((a - 2*b)*Log[1 + Sin
[c + d*x]])/(4*(a - b)^2*d) + (b^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*
d) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.163535, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2668, 741, 801}

$$\frac{b^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)}-\frac{(a+2b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(a-2b)\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```


```
[Out] -((a + 2*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((a - 2*b)*Log[1 + Sin
[c + d*x]])/(4*(a - b)^2*d) + (b^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*
d) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
```

```
LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d} + \frac{b\,\text{Subst}\left(\int \frac{a^2-2b^2+ax}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d} + \frac{b\,\text{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right) dx,\right.}{2\left(a^2-b^2\right)d}$$

$$= -\frac{(a+2b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-2b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{b^3\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2d} - $$

Mathematica [A] time = 0.589515, size = 170, normalized size = 1.38

$$\frac{\frac{4b^3\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2} + \frac{1}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{1}{(a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{2(a+2b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^2} + }{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*(a + 2*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^2 + (2*(a
- 2*b)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b)^2 + (4*b^3*Log[a +
 b*Sin[c + d*x]])/(a^2 - b^2)^2 + 1/((a + b)*(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^2) - 1/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))/(4*d)
```

Maple [A] time = 0.067, size = 164, normalized size = 1.3

$$\frac{b^3 \ln(a+b\sin(dx+c))}{d(a+b)^2(a-b)^2} - \frac{1}{d(4a+4b)(\sin(dx+c)-1)} - \frac{\ln(\sin(dx+c)-1)a}{4d(a+b)^2} - \frac{\ln(\sin(dx+c)-1)b}{2d(a+b)^2} - \frac{}{d(4a-4b)(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*b^3/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))-1/d/(4*a+4*b)/(sin(d*x+c)-1)-1/4
/d/(a+b)^2*ln(sin(d*x+c)-1)*a-1/2/d/(a+b)^2*ln(sin(d*x+c)-1)*b-1/d/(4*a-4*b
)/(1+sin(d*x+c))+1/4/d/(a-b)^2*ln(1+sin(d*x+c))*a-1/2/d/(a-b)^2*ln(1+sin(d*
x+c))*b
```

Maxima [A] time = 0.96123, size = 188, normalized size = 1.53

$$\frac{\frac{4b^3\log(b\sin(dx+c)+a)}{a^4-2a^2b^2+b^4} + \frac{(a-2b)\log(\sin(dx+c)+1)}{a^2-2ab+b^2} - \frac{(a+2b)\log(\sin(dx+c)-1)}{a^2+2ab+b^2} - \frac{2(a\sin(dx+c)-b)}{(a^2-b^2)\sin(dx+c)^2-a^2+b^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*b^3*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) + (a - 2*b)*log(
sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (a + 2*b)*log(sin(d*x + c) - 1)/(a^
2 + 2*a*b + b^2) - 2*(a*sin(d*x + c) - b)/((a^2 - b^2)*sin(d*x + c)^2 - a^2
 + b^2))/d
```

Fricas [A] time = 3.26639, size = 366, normalized size = 2.98

$$\frac{4b^3\cos(dx+c)^2\log(b\sin(dx+c)+a) + (a^3-3ab^2-2b^3)\cos(dx+c)^2\log(\sin(dx+c)+1) - (a^3-3ab^2+2b^3)\cos}{4(a^4-2a^2b^2+b^4)d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^3*cos(d*x + c)^2*log(b*sin(d*x + c) + a) + (a^3 - 3*a*b^2 - 2*b^3)
*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (a^3 - 3*a*b^2 + 2*b^3)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c)
)/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.17158, size = 239, normalized size = 1.94

$$\frac{\frac{4\,b^4\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5}+\frac{(a-2\,b)\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2}-\frac{(a+2\,b)\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2}+\frac{2\left(b^3\sin(dx+c)^2-a^3\sin(dx+c)+ab^2\sin(dx+c)+a^2b-2\,b^3\right)}{\left(a^4-2\,a^2b^2+b^4\right)\left(\sin(dx+c)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(4*b^4*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) + (a - 2*
b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - (a + 2*b)*log(abs(sin(d
*x + c) - 1))/(a^2 + 2*a*b + b^2) + 2*(b^3*sin(d*x + c)^2 - a^3*sin(d*x + c
) + a*b^2*sin(d*x + c) + a^2*b - 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x +
 c)^2 - 1)))/d
```

3.430 $\int \frac{\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=195

$$-\frac{b^5\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\left(3a^2+9ab+8b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(3a^2-9ab+8b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}-\frac{\sec^4(c+$$

```
[Out] -((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((3*a^2
 - 9*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (b^5*Log[a + b*
Sin[c + d*x]])/((a^2 - b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4
*(a^2 - b^2)*d) + (Sec[c + d*x]^2*(4*b^3 + a*(3*a^2 - 7*b^2)*Sin[c + d*x]))
/(8*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.254862, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2668, 741, 823, 801}

$$-\frac{b^5\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\left(3a^2+9ab+8b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{\left(3a^2-9ab+8b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}-\frac{\sec^4(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```


```
[Out] -((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((3*a^2
 - 9*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (b^5*Log[a + b*
Sin[c + d*x]])/((a^2 - b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4
*(a^2 - b^2)*d) + (Sec[c + d*x]^2*(4*b^3 + a*(3*a^2 - 7*b^2)*Sin[c + d*x]))
/(8*(a^2 - b^2)^2*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{b^3\,\text{Subst}\left(\int \frac{3a^2-4b^2+3ax}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx)\left(4b^3+a\left(3a^2-7b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} - \frac{b\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx)\left(4b^3+a\left(3a^2-7b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d} - \frac{b\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\left(3a^2+9ab+8b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3 d} + \frac{\left(3a^2-9ab+8b^2\right)\log(1+\sin(c+dx))}{16(a-b)^3 d} - \frac{b^5\log(a+}{\left(a^2\right.}$$

Mathematica [A] time = 0.953295, size = 266, normalized size = 1.36

$$\frac{\frac{16b^5\log(a+b\sin(c+dx))}{\left(b^2-a^2\right)^3} - \frac{2\left(3a^2+9ab+8b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^3} + \frac{2\left(3a^2-9ab+8b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^3} + \frac{3a}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx\right.\right.}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*(3*a^2 + 9*a*b + 8*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a +
 b)^3 + (2*(3*a^2 - 9*a*b + 8*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
)/(a - b)^3 + (16*b^5*Log[a + b*Sin[c + d*x]])/(-a^2 + b^2)^3 + 1/((a + b)*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4) + (3*a + 5*b)/((a + b)^2*(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])^2) - 1/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^4) + (-3*a + 5*b)/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
2))/(16*d)
```

Maple [A] time = 0.061, size = 305, normalized size = 1.6

$$-\frac{b^5\ln(a+b\sin(dx+c))}{d(a+b)^3(a-b)^3} + \frac{1}{2d(8a+8b)(\sin(dx+c)-1)^2} - \frac{3a}{16d(a+b)^2(\sin(dx+c)-1)} - \frac{5b}{16d(a+b)^2(\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*b^5/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^
2-3/16/d/(a+b)^2/(sin(d*x+c)-1)*a-5/16/d/(a+b)^2/(sin(d*x+c)-1)*b-3/16/d/(a
+b)^3*ln(sin(d*x+c)-1)*a^2-9/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a*b-1/2/d/(a+b)^
3*ln(sin(d*x+c)-1)*b^2-1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2-3/16/d/(a-b)^2/(1+s
in(d*x+c))*a+5/16/d/(a-b)^2/(1+sin(d*x+c))*b+3/16/d/(a-b)^3*ln(1+sin(d*x+c)
)*a^2-9/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b+1/2/d/(a-b)^3*ln(1+sin(d*x+c))*b^
2
```

Maxima [A] time = 0.985268, size = 375, normalized size = 1.92

$$-\frac{\frac{16\,b^5 \log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{\left(3\,a^2-9\,ab+8\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3} + \frac{\left(3\,a^2+9\,ab+8\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{2\left(4\,b^3\sin(dx+c)^2+\left(3\,a^3-7\,ab^2\right)\sin(dx+c)^3+2\,a^2\cdots\right.}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+c)^4+a^4-2\,a^2b^2+b^4-2\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*b^5*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
 (3*a^2 - 9*a*b + 8*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b
^3) + (3*a^2 + 9*a*b + 8*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^
2 + b^3) + 2*(4*b^3*sin(d*x + c)^2 + (3*a^3 - 7*a*b^2)*sin(d*x + c)^3 + 2*a
^2*b - 6*b^3 - (5*a^3 - 9*a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin
(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c
)^2))/d
```

Fricas [A] time = 4.30634, size = 579, normalized size = 2.97

$$-\frac{16\,b^5\cos(dx+c)^4\log(b\sin(dx+c)+a) - \left(3\,a^5-10\,a^3b^2+15\,ab^4+8\,b^5\right)\cos(dx+c)^4\log(\sin(dx+c)+1) + \left(3\,a^5\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*b^5*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (3*a^5 - 10*a^3*b^2
+ 15*a*b^4 + 8*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (3*a^5 - 10*a^3*
b^2 + 15*a*b^4 - 8*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^4*b - 8
*a^2*b^3 + 4*b^5 - 8*(a^2*b^3 - b^5)*cos(d*x + c)^2 - 2*(2*a^5 - 4*a^3*b^2
+ 2*a*b^4 + (3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.16194, size = 448, normalized size = 2.3

$$-\frac{\frac{16\,b^6\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{\left(3\,a^2-9\,ab+8\,b^2\right)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(3\,a^2+9\,ab+8\,b^2\right)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(6\,b^5\sin(dx+c)^4+3\,a^5\sin(dx+c)^3-10\,a^3b^2\sin(\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*b^6*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7) - (3*a^2 - 9*a*b + 8*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b +
 3*a*b^2 - b^3) + (3*a^2 + 9*a*b + 8*b^2)*log(abs(sin(d*x + c) - 1))/(a^3 +
 3*a^2*b + 3*a*b^2 + b^3) + 2*(6*b^5*sin(d*x + c)^4 + 3*a^5*sin(d*x + c)^3
- 10*a^3*b^2*sin(d*x + c)^3 + 7*a*b^4*sin(d*x + c)^3 + 4*a^2*b^3*sin(d*x +
c)^2 - 16*b^5*sin(d*x + c)^2 - 5*a^5*sin(d*x + c) + 14*a^3*b^2*sin(d*x + c)
 - 9*a*b^4*sin(d*x + c) + 2*a^4*b - 8*a^2*b^3 + 12*b^5)/((a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

3.431 $\int \frac{\cos^6(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=188

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}-\frac{\cos^3(c+dx)\left(4\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{12b^3d}+\frac{\cos(c+dx)\left(8\left(a^2-b^2\right)^2-ab\left(4\right.\right.}{8b^5d}$$

```
[Out] (a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x)/(8*b^6) - (2*(a^2 - b^2)^(5/2)*ArcTan[(
b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) + Cos[c + d*x]^5/(5*b*d)
- (Cos[c + d*x]^3*(4*(a^2 - b^2) - 3*a*b*Sin[c + d*x]))/(12*b^3*d) + (Cos[c
 + d*x]*(8*(a^2 - b^2)^2 - a*b*(4*a^2 - 7*b^2)*Sin[c + d*x]))/(8*b^5*d)
```

Rubi [A] time = 0.461873, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2695, 2865, 2735, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}-\frac{\cos^3(c+dx)\left(4\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{12b^3d}+\frac{\cos(c+dx)\left(8\left(a^2-b^2\right)^2-ab\left(4\right.\right.}{8b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```


```
[Out] (a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x)/(8*b^6) - (2*(a^2 - b^2)^(5/2)*ArcTan[(
b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) + Cos[c + d*x]^5/(5*b*d)
- (Cos[c + d*x]^3*(4*(a^2 - b^2) - 3*a*b*Sin[c + d*x]))/(12*b^3*d) + (Cos[c
 + d*x]*(8*(a^2 - b^2)^2 - a*b*(4*a^2 - 7*b^2)*Sin[c + d*x]))/(8*b^5*d)
```


Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\cos^5(c+dx)}{5bd} + \frac{\int \frac{\cos^4(c+dx)(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{12b^3d} + \frac{\int \frac{\cos^2(c+dx)\left(-b\left(a^2-4b^2\right)-a\left(4a^2-7b^2\right)\right.}{a+b\sin(c+dx)}}{4b^3}$$

$$= \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{12b^3d} + \frac{\cos(c+dx)\left(8\left(a^2-b^2\right)^2-ab\right.}{8b^5d}$$

$$= \frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} + \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{12b^3d} +$$

$$= \frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} + \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{12b^3d} +$$

$$= \frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} + \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{12b^3d} +$$

$$= \frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} - \frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d} + \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+}{}$$

Mathematica [B] time = 6.29307, size = 2843, normalized size = 15.12

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^5*((8*Sqrt[2]*b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(5/
2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) -
 (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((5/(16*(1 + ((a - b)*(-(b/(a - b)
) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(8*(1 + ((a - b)*(-(b/(a - b))
 - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*
Sin[c + d*x])/(a - b)))/(2*b))^(-1))/2 - (15*b^3*(((a - b)*(-(b/(a - b)) -
(b*Sin[c + d*x])/(a - b)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/
(a - b))^2)/(3*b^2) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a
 - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) -
(b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Si
n[c + d*x])/(a - b)))/(2*b)])))/(64*(a - b)^3*(-(b/(a - b)) - (b*Sin[c + d*
x])/(a - b))^3*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*
```

```
b))^3)))/(5*(a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))
/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*((8*Sqrt[2]*b*(-(b/(a - b)) - (b*S
in[c + d*x])/(a - b))^(3/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 +
 ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((3*(5/(8
*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(6*
(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + (
(a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1)))/8 + (15*b^
2*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - (Sqrt[2]*Sqrt[a
- b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(S
qrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sq
rt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(64*(a
 - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2*(1 + ((a - b)*(-(b/(a -
 b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(3*(a + b)^2*Sqrt[((a + b)*(b
/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b
))*((8*Sqrt[2]*b*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a +
b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x
])/(a - b)))/(2*b))^(7/2)*((5*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)
) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(8*Sqrt[2]*Sqrt[a - b]*S
qrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) -
(b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)) + (15/(8*(1 + ((a - b)*(-(b/(a - b
)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(4*(1 + ((a - b)*(-(b/(a - b)
) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b
*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/6))/((a + b)^2*Sqrt[((a + b)*(b/(a +
b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*(-(
((-((a*b)/(a + b)) - b^2/(a + b))*(-(((-((a*b)/(a + b)) - b^2/(a + b))*((-2
*(-((a*b)/(a + b)) - b^2/(a + b))*ArcTan[(Sqrt[(a*b)/(a + b) + b^2/(a + b)]
*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[-((a*b)/(a - b)) + b^
2/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[-((a*b)/(a
 - b)) + b^2/(a - b)]*Sqrt[(a*b)/(a + b) + b^2/(a + b)]) + (2*Sqrt[a - b]*A
rcTanh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[a
+ b]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[a + b])))/b) + (
2*Sqrt[2]*(a - b)*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a +
 b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*
x])/(a - b)))/(2*b))^(3/2)*((Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b))
 - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - b]*Sqrt
[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*
Sin[c + d*x])/(a - b)))/(2*b))^(3/2)) + 1/(2*(1 + ((a - b)*(-(b/(a - b)) -
(b*Sin[c + d*x])/(a - b)))/(2*b)))))/(b*(a + b)*Sqrt[((a + b)*(b/(a + b) -
(b*Sin[c + d*x])/(a + b)))/b])))/b) + (4*Sqrt[2]*(a - b)*Sqrt[-(b/(a - b))
- (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 +
 ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)*((3*Sqrt[
b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqr
t[2]*Sqrt[b])])/(4*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])
/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(
5/2)) + (3/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b
```

```
))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1
))/4))/((a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b])
))/b))/b))/b))/(d*(1 - (a + b*Sin[c + d*x])/(a - b))^(5/2)*(1 - (a + b*Sin[
c + d*x])/(a + b))^(5/2))
```

Maple [B] time = 0.058, size = 1055, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] 28/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^2+12/d/b/(1+tan(1/2*
d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6+56/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^5*ta
n(1/2*d*x+1/2*c)^4+2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*a^4-14/3/d/b^3/(1+tan
(1/2*d*x+1/2*c)^2)^5*a^2-5/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^3+2/d/b^6*arc
tan(tan(1/2*d*x+1/2*c))*a^5+6/d/b/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/
2*c)^8+15/4/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))-6/d/b^2/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2+12/d/b^5/(1+tan(1/
2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4*a^4+46/15/d/b/(1+tan(1/2*d*x+1/2*c)^
2)^5+2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^
(1/2))+6/d/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2
-b^2)^(1/2))*a^4-80/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^4
*a^2-2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a^3+5/2/d/b^2/
(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^3*a+8/d/b^5/(1+tan(1/2*d*x+1/
2*c)^2)^5*tan(1/2*d*x+1/2*c)^2*a^4-52/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*ta
n(1/2*d*x+1/2*c)^2*a^2-6/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c
)^8*a^2+2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a^3-5/2/d/b
^2/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^7*a+8/d/b^5/(1+tan(1/2*d*x
+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)^6*a^4-9/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^5*
tan(1/2*d*x+1/2*c)^9*a+2/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c
)^8*a^4-1/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a^3+9/4/d/b^2
/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1/2*d*x+1/2*c)*a-20/d/b^3/(1+tan(1/2*d*x+1/
2*c)^2)^5*tan(1/2*d*x+1/2*c)^6*a^2+1/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^5*tan(1
/2*d*x+1/2*c)^9*a^3-2/d/b^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2
*c)+2*b)/(a^2-b^2)^(1/2))*a^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.07267, size = 1121, normalized size = 5.96

$$\left[\frac{24\,b^5\cos\left(dx+c\right)^5-40\left(a^2b^3-b^5\right)\cos\left(dx+c\right)^3+15\left(8\,a^5-20\,a^3b^2+15\,ab^4\right)dx+60\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}\log\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/120*(24*b^5*cos(d*x + c)^5 - 40*(a^2*b^3 - b^5)*cos(d*x + c)^3 + 15*(8*a
^5 - 20*a^3*b^2 + 15*a*b^4)*d*x + 60*(a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^
2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(
a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 120*(a^4*b - 2*a^2*b^3 + b^5)
*cos(d*x + c) + 15*(2*a*b^4*cos(d*x + c)^3 - (4*a^3*b^2 - 7*a*b^4)*cos(d*x
+ c))*sin(d*x + c))/(b^6*d), 1/120*(24*b^5*cos(d*x + c)^5 - 40*(a^2*b^3 - b
^5)*cos(d*x + c)^3 + 15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*d*x + 120*(a^4 - 2*
a^2*b^2 + b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x + c))) + 120*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c) + 15*(2*a*b^4
*cos(d*x + c)^3 - (4*a^3*b^2 - 7*a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^6*d)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12566, size = 670, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*(d*x + c)/b^6 - 240*(a^6 - 3*a^4*
b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 2*(60*
a^3*b*tan(1/2*d*x + 1/2*c)^9 - 135*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*a^4*t
an(1/2*d*x + 1/2*c)^8 - 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 360*b^4*tan(1/
2*d*x + 1/2*c)^8 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 150*a*b^3*tan(1/2*d*x
 + 1/2*c)^7 + 480*a^4*tan(1/2*d*x + 1/2*c)^6 - 1200*a^2*b^2*tan(1/2*d*x + 1
/2*c)^6 + 720*b^4*tan(1/2*d*x + 1/2*c)^6 + 720*a^4*tan(1/2*d*x + 1/2*c)^4 -
 1600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 1120*b^4*tan(1/2*d*x + 1/2*c)^4 - 12
0*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 150*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 480*a^4
*tan(1/2*d*x + 1/2*c)^2 - 1040*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 560*b^4*tan
(1/2*d*x + 1/2*c)^2 - 60*a^3*b*tan(1/2*d*x + 1/2*c) + 135*a*b^3*tan(1/2*d*x
 + 1/2*c) + 120*a^4 - 280*a^2*b^2 + 184*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^
5*b^5))/d
```

3.432 $\int \frac{\cos^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}-\frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{2b^3d}-\frac{ax\left(2a^2-3b^2\right)}{2b^4}+\frac{\cos^3(c+dx)}{3bd}$$

```
[Out] -(a*(2*a^2 - 3*b^2)*x)/(2*b^4) + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c
+ d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*d) + Cos[c + d*x]^3/(3*b*d) - (Cos[c + d*
x]*(2*(a^2 - b^2) - a*b*Sin[c + d*x]))/(2*b^3*d)
```

Rubi [A] time = 0.252221, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2695, 2865, 2735, 2660, 618, 204}

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}-\frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{2b^3d}-\frac{ax\left(2a^2-3b^2\right)}{2b^4}+\frac{\cos^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*(2*a^2 - 3*b^2)*x)/(2*b^4) + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c
+ d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*d) + Cos[c + d*x]^3/(3*b*d) - (Cos[c + d*
x]*(2*(a^2 - b^2) - a*b*Sin[c + d*x]))/(2*b^3*d)
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
```

```
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\cos^3(c+dx)}{3bd} + \frac{\int \frac{\cos^2(c+dx)(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{2b^3d} + \frac{\int \frac{-b\left(a^2-2b^2\right)-a\left(2a^2-3b^2\right)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b^3}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{2b^3d} + \frac{\left(a^2-b^2\right)^2 \int \frac{}{a+}}{b^4}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{2b^3d} + \frac{\left(2\left(a^2-b^2\right)^2\right)\mathrm{S}}{}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{2b^3d} - \frac{\left(4\left(a^2-b^2\right)^2\right)\mathrm{S}}{}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d} + \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+dx)\left(2\left(a^2-\right.\right.}{2}$$

Mathematica [B] time = 6.13175, size = 1685, normalized size = 13.27

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^3*((4*Sqrt[2]*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(3/2)
*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (
b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)*((3/(4*(1 + ((a - b)*(-(b/(a - b)) -
 (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Si
n[c + d*x])/(a - b)))/(2*b))^(-1))/2 + (3*b^2*(((a - b)*(-(b/(a - b)) - (b*
Sin[c + d*x])/(a - b)))/b - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[
-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a -
b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) -
 (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(8*(a - b)^2*(-(b/(a - b)) - (b*Sin[c
 + d*x])/(a - b))^2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))
)/(2*b))^2)))/(3*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b
)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a - b)) + b^2/(a
- b))*(-(((-((a*b)/(a + b)) - b^2/(a + b))*((-2*(-((a*b)/(a + b)) - b^2/(a
+ b))*ArcTan[(Sqrt[(a*b)/(a + b) + b^2/(a + b)]*Sqrt[-(b/(a - b)) - (b*Sin[
c + d*x])/(a - b)])/(Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[b/(a + b) -
```

```
(b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[(
a*b)/(a + b) + b^2/(a + b)]) + (2*Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sqrt[-(b
/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[a + b]*Sqrt[b/(a + b) - (b*Sin
[c + d*x])/(a + b)])])/(b*Sqrt[a + b])))/b) + (2*Sqrt[2]*(a - b)*Sqrt[-(b/(
a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a +
b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)*(
(Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]
)/(Sqrt[2]*Sqrt[b])]])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b
))^(3/2)) + 1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(
2*b)))))/(b*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b
])))/b) + (4*Sqrt[2]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(
a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c +
 d*x])/(a - b)))/(2*b))^(5/2)*((3*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]])/(4*Sqrt[2]*Sqrt[a -
b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)
) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)) + (3/(2*(1 + ((a - b)*(-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b))
- (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/4))/((a + b)*Sqrt[((a + b)*(b/(a
+ b) - (b*Sin[c + d*x])/(a + b)))/b])))/b))/(d*(1 - (a + b*Sin[c + d*x])/(a
 - b))^(3/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(3/2))
```

Maple [B] time = 0.045, size = 450, normalized size = 3.5

$$-\frac{a}{db^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-3}-2\,\frac{(\tan(1/2\,dx+c/2))^4\,a^2}{db^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+4\,\frac{(\tan(1/2\,dx+c/2))^4}{bd\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)^5-2/d/b^3/(1+tan(1
/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2+4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^
3*tan(1/2*d*x+1/2*c)^4-4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c
)^2*a^2+4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2+1/d/b^2/(1+ta
n(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)-2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)
^3*a^2+8/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3-2/d/b^4*arctan(tan(1/2*d*x+1/2*c)
)*a^3+3/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))+2/d/b^4/(a^2-b^2)^(1/2)*arctan(1
/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^4-4/d/b^2/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2+2/d/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.72894, size = 768, normalized size = 6.05

$$\left[\frac{2\,b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)-3\left(2\,a^3-3\,ab^2\right)dx-3\left(a^2-b^2\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\,\mathrm{s}}{}\right.}{6\,b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)*sin(d*x + c) - 3*(2*a^3 -
 3*a*b^2)*d*x - 3*(a^2 - b^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x +
 c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b
*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2)) - 6*(a^2*b - b^3)*cos(d*x + c))/(b^4*d), 1/6*(2*b^3*cos(d*x +
c)^3 + 3*a*b^2*cos(d*x + c)*sin(d*x + c) - 3*(2*a^3 - 3*a*b^2)*d*x - 6*(a^2
 - b^2)^(3/2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))
- 6*(a^2*b - b^3)*cos(d*x + c))/(b^4*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11415, size = 305, normalized size = 2.4

$$-\frac{\frac{3\left(2a^3-3ab^2\right)(dx+c)}{b^4}-\frac{12\left(a^4-2a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}+\frac{2\left(3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-12b^2\tan\left(\frac{1}{2}\right.\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 - 3*a*b^2)*(d*x + c)/b^4 - 12*(a^4 - 2*a^2*b^2 + b^4)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sq
rt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^4) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6
*a^2*tan(1/2*d*x + 1/2*c)^4 - 12*b^2*tan(1/2*d*x + 1/2*c)^4 + 12*a^2*tan(1/
2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*
c) + 6*a^2 - 8*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

3.433 $\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=70

$$-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}$$

```
[Out] (a*x)/b^2 - (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(b^2*d) + Cos[c + d*x]/(b*d)
```

Rubi [A] time = 0.114138, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2695, 2735, 2660, 618, 204}

$$-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```



```
[Out] (a*x)/b^2 - (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(b^2*d) + Cos[c + d*x]/(b*d)
```


Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\cos(c+dx)}{bd}+\frac{\int \frac{b+a\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}\\
&= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}-\frac{\left(a^2-b^2\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^2}\\
&= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}-\frac{\left(2\left(a^2-b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\
&= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}+\frac{\left(4\left(a^2-b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\
&= \frac{ax}{b^2}-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{\cos(c+dx)}{bd}
\end{aligned}
$$

Mathematica [B] time = 2.21671, size = 398, normalized size = 5.69

$$
\frac{b\cos(c+dx)\left(\sqrt{a+b}\left(2\sqrt{-b^2}\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sinh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(\sin(c+dx)+1)}{a-b}}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{a-b}\sqrt{1-\sin(c+dx)}\left(\sqrt{-b}\sqrt{-\frac{b(\sin(c+dx)-}{a+b}}\right.\right.\right.}{(-b)^{5/2}d\sqrt{a-b}\sqrt{a+b}\sqrt{1-\sin(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Cos[c + d*x]*(-2*Sqrt[-b]*(-a + b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + S
in[c + d*x]))/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b
))])]*Sqrt[1 - Sin[c + d*x]] + Sqrt[a + b]*(2*Sqrt[-b^2]*ArcSinh[(Sqrt[a -
b]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(Sqrt[2]*Sqrt[b])]*Sqrt[-((b*(-
1 + Sin[c + d*x]))/(a + b))] + Sqrt[a - b]*Sqrt[1 - Sin[c + d*x]]*(2*Sqrt[b
]*ArcTan[(Sqrt[b]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(Sqrt[-b]*Sqrt[-((
b*(-1 + Sin[c + d*x]))/(a + b))])] + Sqrt[-b]*Sqrt[-((b*(-1 + Sin[c + d*x])
)/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]))))/(Sqrt[a - b]*(-b)^(5/
2)*Sqrt[a + b]*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a +
 b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])
```

Maple [B] time = 0.037, size = 142, normalized size = 2.

$$2\,\frac{1}{bd\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}\right)}+2\,\frac{a\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{db^{2}}-2\,\frac{a^{2}}{db^{2}\sqrt{a^{2}-b^{2}}}\arctan\left(1/2\,\frac{2\,a\tan\left(1/2\,dx+c/2\right)+2\,b}{\sqrt{a^{2}-b^{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 2/d/b/(1+tan(1/2*d*x+1/2*c)^2)+2/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))-2/d/b^2
/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a
^2+2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.85851, size = 498, normalized size = 7.11

$$\left[\frac{2\,adx + 2\,b\cos\left(dx+c\right) + \sqrt{-a^2+b^2}\log\left(\frac{\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2 - 2\,ab\sin\left(dx+c\right) - a^2 - b^2 + 2\,\left(a\cos\left(dx+c\right)\sin\left(dx+c\right) + b\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx+c\right)^2 - 2\,ab\sin\left(dx+c\right) - a^2 - b^2}\right)}{2\,b^2d},\ ad\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*d*x + 2*b*cos(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2)))/(b^2*d), (a*d*x + b*cos(d*x + c) + sqrt(a^2 - b^2)*arcta
n(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))))/(b^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12663, size = 128, normalized size = 1.83

$$\frac{\frac{(dx+c)a}{b^2} - \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2} + \frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a/b^2 - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2 + 2/((tan(1/
2*d*x + 1/2*c)^2 + 1)*b))/d
```

3.434 $\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\left(a^2-b^2\right)}$$

```
[Out] (-2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (Sec[c + d*x]*(b - a*Sin[c + d*x]))/((a^2 - b^2)*d)
```

Rubi [A] time = 0.0939213, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2696, 12, 2660, 618, 204}

$$-\frac{2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (Sec[c + d*x]*(b - a*Sin[c + d*x]))/((a^2 - b^2)*d)
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx &= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d}+\frac{\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}\\
&= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d}-\frac{b^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2}\\
&= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d}-\frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}\\
&= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d}+\frac{\left(4b^2\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}\\
&= -\frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{\left(a^2-b^2\right)d}
\end{aligned}
$$

Mathematica [A] time = 0.293336, size = 152, normalized size = 1.81

$$\frac{\sqrt{a^2-b^2}(-a\sin(c+dx)+b(-\cos(c+dx))+b)+2b^2\cos(c+dx)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(b-a)(a+b)\sqrt{a^2-b^2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqrt
[a^2 - b^2]*(b - b*Cos[c + d*x] - a*Sin[c + d*x]))/((-a + b)*(a + b)*Sqrt[a
^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]))
```

Maple [A] time = 0.046, size = 117, normalized size = 1.4

$$-2\frac{1}{d(2a-2b)(\tan(1/2dx+c/2)+1)}-2\frac{b^2}{d(a-b)(a+b)\sqrt{a^2-b^2}}\arctan\left(1/2\frac{2a\tan(1/2dx+c/2)+2b}{\sqrt{a^2-b^2}}\right)-2\frac{}{d(2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)-2/d*b^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*a
rctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/(2*a+2*b)/(tan(
1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.84104, size = 684, normalized size = 8.14

$$\left[\frac{\sqrt{-a^2+b^2}b^2\cos(dx+c)\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^2b+2b^3+}{2\left(a^4-2a^2b^2+b^4\right)d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*b^2*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 +
 b^4)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*b^2*arctan(-(a*sin(d*x + c) + b)/(s
qrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*si
n(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.11943, size = 144, normalized size = 1.71

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b}{\left(a^2-b^2\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*b^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c) -
 b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.435 $\int \frac{\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3d\left(a^2-b^2\right)}+\frac{\sec(c+dx)\left(a\left(2a^2-5b^2\right)\sin(c+dx)+3b^3\right)}{3d\left(a^2-b^2\right)^2}$$

```
[Out] (2*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) - (Sec[c + d*x]^3*(b - a*Sin[c + d*x]))/(3*(a^2 - b^2)*d) + (Sec[c + d*
x]*(3*b^3 + a*(2*a^2 - 5*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.250985, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2696, 2866, 12, 2660, 618, 204}

$$\frac{2b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3d\left(a^2-b^2\right)}+\frac{\sec(c+dx)\left(a\left(2a^2-5b^2\right)\sin(c+dx)+3b^3\right)}{3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) - (Sec[c + d*x]^3*(b - a*Sin[c + d*x]))/(3*(a^2 - b^2)*d) + (Sec[c + d*
x]*(3*b^3 + a*(2*a^2 - 5*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^2*d)
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\int \frac{\sec^2(c+dx)\left(-2a^2+3b^2-2ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3\left(a^2-b^2\right)d} + \frac{\sec(c+dx)\left(3b^3+a\left(2a^2-5b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{\int \frac{3b^4}{a+b\sin(c+d}}{3\left(a^2-b^2\right.}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3\left(a^2-b^2\right)d} + \frac{\sec(c+dx)\left(3b^3+a\left(2a^2-5b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{b^4\int \frac{1}{a+b\sin(}}{\left(a^2-b\right.}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3\left(a^2-b^2\right)d} + \frac{\sec(c+dx)\left(3b^3+a\left(2a^2-5b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{\left(2b^4\right)\mathrm{Subs}}{}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3\left(a^2-b^2\right)d} + \frac{\sec(c+dx)\left(3b^3+a\left(2a^2-5b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} - \frac{\left(4b^4\right)\mathrm{Subs}}{}$$

$$= \frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} - \frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3\left(a^2-b^2\right)d} + \frac{\sec(c+dx)\left(3b^3+a\left(2a^2-5b^2\right.\right.}{3\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 1.26235, size = 202, normalized size = 1.47

$$\frac{\frac{24b^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{\sec^3(c+dx)\left(\frac{3}{2}b\left(a^2-7b^2\right)\cos(c+dx)+\frac{1}{2}a^2b\cos(3(c+dx))-4a^2b+6a^3\sin(c+dx)+2a^3\sin(3(c+dx))-9ab^2\sin(c+dx)-5ab^2\sin(3(c+\right.}{(a-b)^2(a+b)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((24*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
) + (Sec[c + d*x]^3*(-4*a^2*b + 10*b^3 + (3*b*(a^2 - 7*b^2)*Cos[c + d*x])/2
 + 6*b^3*Cos[2*(c + d*x)] + (a^2*b*Cos[3*(c + d*x)])/2 - (7*b^3*Cos[3*(c +
d*x)])/2 + 6*a^3*Sin[c + d*x] - 9*a*b^2*Sin[c + d*x] + 2*a^3*Sin[3*(c + d*x
)] - 5*a*b^2*Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(12*d)
```

Maple [B] time = 0.059, size = 270, normalized size = 2.

$$-\frac{2}{3d\left(2a+2b\right)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{1}{d\left(2a+2b\right)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}-\frac{a}{d\left(a+b\right)^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{3}{2d(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/3/d/(tan(1/2*d*x+1/2*c)-1)^3/(2*a+2*b)-1/d/(2*a+2*b)/(tan(1/2*d*x+1/2*c)
-1)^2-1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a-3/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c
)-1)*b-2/3/d/(tan(1/2*d*x+1/2*c)+1)^3/(2*a-2*b)+1/d/(2*a-2*b)/(tan(1/2*d*x+
1/2*c)+1)^2-1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*a+3/2/d/(a-b)^2/(tan(1/2*d*x
+1/2*c)+1)*b+2/d*b^4/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.80367, size = 1027, normalized size = 7.5

$$\left[-\frac{3\sqrt{-a^{2}+b^{2}}b^{4}\cos\left(dx+c\right)^{3}\log\left(\frac{\left(2a^{2}-b^{2}\right)\cos(dx+c)^{2}-2ab\sin(dx+c)-a^{2}-b^{2}+2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^{2}+b^{2}}}{b^{2}\cos(dx+c)^{2}-2ab\sin(dx+c)-a^{2}-b^{2}}\right)+2a^{4}b-4}{6\left(a^{6}-3a^{4}b^{2}+3a^{2}b^{4}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*b^4*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
```

```
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(a^2*b^3 - b^5)*cos(d*x + c)^2
 - 2*(a^5 - 2*a^3*b^2 + a*b^4 + (2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^
2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^3), -1
/3*(3*sqrt(a^2 - b^2)*b^4*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos
(d*x + c)))*cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*(a^2*b^3 - b^5)*co
s(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 + (2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

Giac [B] time = 1.13423, size = 369, normalized size = 2.69

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^4}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}-\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+6b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a^3\tan\left(\cdots\right.}{\left(a^4-2a^2b^2+b\cdots\right.}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*b^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
- (3*a^3*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*b*
tan(1/2*d*x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^4 - 2*a^3*tan(1/2*d*x +
 1/2*c)^3 + 8*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*b^3*tan(1/2*d*x + 1/2*c)^2 +
 3*a^3*tan(1/2*d*x + 1/2*c) - 6*a*b^2*tan(1/2*d*x + 1/2*c) - a^2*b + 4*b^3)
/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

3.436 $\int \frac{\sec^6(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=197

$$-\frac{2b^6\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5d\left(a^2-b^2\right)}+\frac{\sec^3(c+dx)\left(a\left(4a^2-9b^2\right)\sin(c+dx)+5b^3\right)}{15d\left(a^2-b^2\right)^2}-\frac{\sec(c}{}$$

```
[Out] (-2*b^6*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2
)*d) - (Sec[c + d*x]^5*(b - a*Sin[c + d*x]))/(5*(a^2 - b^2)*d) + (Sec[c + d
*x]^3*(5*b^3 + a*(4*a^2 - 9*b^2)*Sin[c + d*x]))/(15*(a^2 - b^2)^2*d) - (Sec
[c + d*x]*(15*b^5 - a*(8*a^4 - 26*a^2*b^2 + 33*b^4)*Sin[c + d*x]))/(15*(a^2
 - b^2)^3*d)
```

Rubi [A] time = 0.495377, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2696, 2866, 12, 2660, 618, 204}

$$-\frac{2b^6\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5d\left(a^2-b^2\right)}+\frac{\sec^3(c+dx)\left(a\left(4a^2-9b^2\right)\sin(c+dx)+5b^3\right)}{15d\left(a^2-b^2\right)^2}-\frac{\sec(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```



```
[Out] (-2*b^6*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2
)*d) - (Sec[c + d*x]^5*(b - a*Sin[c + d*x]))/(5*(a^2 - b^2)*d) + (Sec[c + d
*x]^3*(5*b^3 + a*(4*a^2 - 9*b^2)*Sin[c + d*x]))/(15*(a^2 - b^2)^2*d) - (Sec
[c + d*x]*(15*b^5 - a*(8*a^4 - 26*a^2*b^2 + 33*b^4)*Sin[c + d*x]))/(15*(a^2
 - b^2)^3*d)
```


Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
```

```
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} - \frac{\int \frac{\sec^4(c+dx)\left(-4a^2+5b^2-4ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{5\left(a^2-b^2\right)}$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a\left(4a^2-9b^2\right)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d} + \frac{\int \frac{\sec^2(c+}{}}{}$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a\left(4a^2-9b^2\right)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d} - \frac{\sec(c+d}{}$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a\left(4a^2-9b^2\right)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d} - \frac{\sec(c+d}{}$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a\left(4a^2-9b^2\right)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d} - \frac{\sec(c+d}{}$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a\left(4a^2-9b^2\right)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d} - \frac{\sec(c+d}{}$$

$$= -\frac{2b^6\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} - \frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a\left(4a^2-\right.\right.}{15\left(a^2-b^2\right)^2}$$

Mathematica [A] time = 2.54924, size = 370, normalized size = 1.88

$$\frac{\sec^5(c+dx)\left(-1600a^3b^2\sin(c+dx)-1040a^3b^2\sin(3(c+dx))-208a^3b^2\sin(5(c+dx))-190a^2b^3\cos(3(c+dx))-38a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^6*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2
)*d) + (Sec[c + d*x]^5*(-384*a^4*b + 1088*a^2*b^3 - 1424*b^5 + 10*b*(9*a^4
- 38*a^2*b^2 + 149*b^4)*Cos[c + d*x] + 320*b^3*(a^2 - 4*b^2)*Cos[2*(c + d*x
)] + 45*a^4*b*Cos[3*(c + d*x)] - 190*a^2*b^3*Cos[3*(c + d*x)] + 745*b^5*Cos
[3*(c + d*x)] - 240*b^5*Cos[4*(c + d*x)] + 9*a^4*b*Cos[5*(c + d*x)] - 38*a^
```

```
2*b^3*Cos[5*(c + d*x)] + 149*b^5*Cos[5*(c + d*x)] + 640*a^5*Sin[c + d*x] -
1600*a^3*b^2*Sin[c + d*x] + 1200*a*b^4*Sin[c + d*x] + 320*a^5*Sin[3*(c + d*
x)] - 1040*a^3*b^2*Sin[3*(c + d*x)] + 1080*a*b^4*Sin[3*(c + d*x)] + 64*a^5*
Sin[5*(c + d*x)] - 208*a^3*b^2*Sin[5*(c + d*x)] + 264*a*b^4*Sin[5*(c + d*x)
]))/(1920*(a - b)^3*(a + b)^3*d)
```

Maple [B] time = 0.063, size = 525, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/5/d/(2*a+2*b)/(tan(1/2*d*x+1/2*c)-1)^5-1/2/d/(a+b)/(tan(1/2*d*x+1/2*c)-1
)^4-7/8/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^2*a-9/8/d/(a+b)^2/(tan(1/2*d*x+1/2
*c)-1)^2*b-11/12/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3*a-13/12/d/(a+b)^2/(tan(
1/2*d*x+1/2*c)-1)^3*b-1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)*a^2-21/8/d/(a+b)^3
/(tan(1/2*d*x+1/2*c)-1)*a*b-15/8/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)*b^2-2/5/d
/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)^5+1/2/d/(a-b)/(tan(1/2*d*x+1/2*c)+1)^4+7/
8/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^2*a-9/8/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)
^2*b-11/12/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3*a+13/12/d/(a-b)^2/(tan(1/2*d*
x+1/2*c)+1)^3*b-1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)*a^2+21/8/d/(a-b)^3/(tan(
1/2*d*x+1/2*c)+1)*a*b-15/8/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)*b^2-2/d*b^6/(a-
b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.28354, size = 1482, normalized size = 7.52

$$\left[\frac{15\,\sqrt{-a^2+b^2}b^6\cos\left(dx+c\right)^5\log\left(\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)-6\,a^6b+1\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(-a^2 + b^2)*b^6*cos(d*x + c)^5*log(((2*a^2 - b^2)*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*
cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
a^2 - b^2)) - 6*a^6*b + 18*a^4*b^3 - 18*a^2*b^5 + 6*b^7 - 30*(a^2*b^5 - b^7
)*cos(d*x + c)^4 + 10*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2 + 2*(3*a^7
 - 9*a^5*b^2 + 9*a^3*b^4 - 3*a*b^6 + (8*a^7 - 34*a^5*b^2 + 59*a^3*b^4 - 33*
a*b^6)*cos(d*x + c)^4 + (4*a^7 - 17*a^5*b^2 + 22*a^3*b^4 - 9*a*b^6)*cos(d*x
 + c)^2)*sin(d*x + c))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d*c
os(d*x + c)^5), 1/15*(15*sqrt(a^2 - b^2)*b^6*arctan(-(a*sin(d*x + c) + b)/(
sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c)^5 - 3*a^6*b + 9*a^4*b^3 - 9*a^2
*b^5 + 3*b^7 - 15*(a^2*b^5 - b^7)*cos(d*x + c)^4 + 5*(a^4*b^3 - 2*a^2*b^5 +
 b^7)*cos(d*x + c)^2 + (3*a^7 - 9*a^5*b^2 + 9*a^3*b^4 - 3*a*b^6 + (8*a^7 -
34*a^5*b^2 + 59*a^3*b^4 - 33*a*b^6)*cos(d*x + c)^4 + (4*a^7 - 17*a^5*b^2 +
22*a^3*b^4 - 9*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 - 4*a^6*b^2 + 6*a
^4*b^4 - 4*a^2*b^6 + b^8)*d*cos(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16254, size = 788, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/15*(15*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
+ 1/2*c) + b)/sqrt(a^2 - b^2)))*b^6/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sq
rt(a^2 - b^2)) + (15*a^5*tan(1/2*d*x + 1/2*c)^9 - 45*a^3*b^2*tan(1/2*d*x +
1/2*c)^9 + 45*a*b^4*tan(1/2*d*x + 1/2*c)^9 - 15*a^4*b*tan(1/2*d*x + 1/2*c)^
8 + 45*a^2*b^3*tan(1/2*d*x + 1/2*c)^8 - 45*b^5*tan(1/2*d*x + 1/2*c)^8 - 20*
a^5*tan(1/2*d*x + 1/2*c)^7 + 80*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*a*b^4*
tan(1/2*d*x + 1/2*c)^7 - 30*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 + 90*b^5*tan(1/2
*d*x + 1/2*c)^6 + 58*a^5*tan(1/2*d*x + 1/2*c)^5 - 166*a^3*b^2*tan(1/2*d*x +
 1/2*c)^5 + 198*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 30*a^4*b*tan(1/2*d*x + 1/2*c
)^4 + 80*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 - 140*b^5*tan(1/2*d*x + 1/2*c)^4 -
20*a^5*tan(1/2*d*x + 1/2*c)^3 + 80*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a*b
^4*tan(1/2*d*x + 1/2*c)^3 - 10*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 70*b^5*tan(
1/2*d*x + 1/2*c)^2 + 15*a^5*tan(1/2*d*x + 1/2*c) - 45*a^3*b^2*tan(1/2*d*x +
 1/2*c) + 45*a*b^4*tan(1/2*d*x + 1/2*c) - 3*a^4*b + 11*a^2*b^3 - 23*b^5)/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)^5))/d
```

3.437 $\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=184

$$-\frac{\left(a^2-b^2\right)\sin^3(c+dx)}{b^4d}+\frac{a\left(2a^2-3b^2\right)\sin^2(c+dx)}{b^5d}-\frac{\left(-9a^2b^2+5a^4+3b^4\right)\sin(c+dx)}{b^6d}+\frac{\left(a^2-b^2\right)^3}{b^7d(a+b\sin(c+dx))}+\frac{6a$$

```
[Out] (6*a*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^7*d) - ((5*a^4 - 9*a^2*b^2 +
 3*b^4)*Sin[c + d*x])/(b^6*d) + (a*(2*a^2 - 3*b^2)*Sin[c + d*x]^2)/(b^5*d)
- ((a^2 - b^2)*Sin[c + d*x]^3)/(b^4*d) + (a*Sin[c + d*x]^4)/(2*b^3*d) - Sin
[c + d*x]^5/(5*b^2*d) + (a^2 - b^2)^3/(b^7*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.172894, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{\left(a^2-b^2\right)\sin^3(c+dx)}{b^4d}+\frac{a\left(2a^2-3b^2\right)\sin^2(c+dx)}{b^5d}-\frac{\left(-9a^2b^2+5a^4+3b^4\right)\sin(c+dx)}{b^6d}+\frac{\left(a^2-b^2\right)^3}{b^7d(a+b\sin(c+dx))}+\frac{6a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (6*a*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^7*d) - ((5*a^4 - 9*a^2*b^2 +
 3*b^4)*Sin[c + d*x])/(b^6*d) + (a*(2*a^2 - 3*b^2)*Sin[c + d*x]^2)/(b^5*d)
- ((a^2 - b^2)*Sin[c + d*x]^3)/(b^4*d) + (a*Sin[c + d*x]^4)/(2*b^3*d) - Sin
[c + d*x]^5/(5*b^2*d) + (a^2 - b^2)^3/(b^7*d*(a + b*Sin[c + d*x]))
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(-5a^4\left(1+\frac{3b^2\left(-3a^2+b^2\right)}{5a^4}\right)+2a\left(2a^2-3b^2\right)x-3\left(a^2-b^2\right)x^2+2ax^3-x^4-\frac{\left(a^2-b^2\right)^3}{(a+x)^2}\right.\right.}{b^7 d}$$
$$= \frac{6a\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{b^7 d}-\frac{\left(5a^4-9a^2b^2+3b^4\right)\sin(c+dx)}{b^6 d}+\frac{a\left(2a^2-3b^2\right)\sin^2(c}{b^5 d}$$

Mathematica [A] time = 0.517787, size = 235, normalized size = 1.28

$$\frac{-4a^2b^4\sin^4(c+dx)+2ab^3\left(5a^2-7b^2\right)\sin^3(c+dx)-2b^2\left(-29a^2b^2+15a^4+8b^4\right)\sin^2(c+dx)+4\left(a^2-b^2\right)^2\left(15a^2\log(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b^6*Cos[c + d*x]^6 + 4*(a^2 - b^2)^2*(4*a^2 - 4*b^2 + 15*a^2*Log[a + b*S
in[c + d*x]]) + 4*a*b*(-11*a^4 + 18*a^2*b^2 - 4*b^4 + 15*(a^2 - b^2)^2*Log[
a + b*Sin[c + d*x]])*Sin[c + d*x] - 2*b^2*(15*a^4 - 29*a^2*b^2 + 8*b^4)*Sin
[c + d*x]^2 + 2*a*b^3*(5*a^2 - 7*b^2)*Sin[c + d*x]^3 - 4*a^2*b^4*Sin[c + d*
x]^4 + b^4*Cos[c + d*x]^4*(-a^2 + 4*b^2 + 3*a*b*Sin[c + d*x]))/(10*b^7*d*(a
 + b*Sin[c + d*x]))
```

Maple [A] time = 0.076, size = 305, normalized size = 1.7

$$-\frac{(\sin(dx+c))^5}{5b^2d}+\frac{a(\sin(dx+c))^4}{2b^3d}-\frac{(\sin(dx+c))^3a^2}{db^4}+\frac{(\sin(dx+c))^3}{b^2d}+2\frac{(\sin(dx+c))^2a^3}{db^5}-3\frac{(\sin(dx+c))^2a}{b^3d}-5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/5*sin(d*x+c)^5/b^2/d+1/2*a*sin(d*x+c)^4/b^3/d-1/d/b^4*sin(d*x+c)^3*a^2+s
in(d*x+c)^3/b^2/d+2/d/b^5*sin(d*x+c)^2*a^3-3*a*sin(d*x+c)^2/b^3/d-5/d/b^6*a
```

```
^4*sin(d*x+c)+9/d/b^4*a^2*sin(d*x+c)-3*sin(d*x+c)/b^2/d+6/d*a^5/b^7*ln(a+b*
sin(d*x+c))-12/d*a^3/b^5*ln(a+b*sin(d*x+c))+6*a*ln(a+b*sin(d*x+c))/b^3/d+1/
d/b^7/(a+b*sin(d*x+c))*a^6-3/d/b^5/(a+b*sin(d*x+c))*a^4+3/d/b^3/(a+b*sin(d*
x+c))*a^2-1/b/d/(a+b*sin(d*x+c))
```

Maxima [A] time = 0.955161, size = 257, normalized size = 1.4

$$\frac{\frac{10\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}{b^8\sin(dx+c)+ab^7}-\frac{2b^4\sin(dx+c)^5-5ab^3\sin(dx+c)^4+10\left(a^2b^2-b^4\right)\sin(dx+c)^3-10\left(2a^3b-3ab^3\right)\sin(dx+c)^2+10\left(5a^4-9a^2b^2+3b^4\right)\sin(dx+c)}{b^6}+\frac{60}{}}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/10*(10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)/(b^8*sin(d*x + c) + a*b^7) - (
2*b^4*sin(d*x + c)^5 - 5*a*b^3*sin(d*x + c)^4 + 10*(a^2*b^2 - b^4)*sin(d*x
+ c)^3 - 10*(2*a^3*b - 3*a*b^3)*sin(d*x + c)^2 + 10*(5*a^4 - 9*a^2*b^2 + 3*
b^4)*sin(d*x + c))/b^6 + 60*(a^5 - 2*a^3*b^2 + a*b^4)*log(b*sin(d*x + c) +
a)/b^7)/d
```

Fricas [A] time = 3.24837, size = 571, normalized size = 3.1

$$\frac{16\,b^6\cos(dx+c)^6+80\,a^6-560\,a^4b^2+785\,a^2b^4-256\,b^6-8\left(5\,a^2b^4-4\,b^6\right)\cos(dx+c)^4+16\left(15\,a^4b^2-25\,a^2b^4+8\,b^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/80*(16*b^6*cos(d*x + c)^6 + 80*a^6 - 560*a^4*b^2 + 785*a^2*b^4 - 256*b^6
- 8*(5*a^2*b^4 - 4*b^6)*cos(d*x + c)^4 + 16*(15*a^4*b^2 - 25*a^2*b^4 + 8*b^
6)*cos(d*x + c)^2 + 480*(a^6 - 2*a^4*b^2 + a^2*b^4 + (a^5*b - 2*a^3*b^3 + a
*b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) + (24*a*b^5*cos(d*x + c)^4 - 40
0*a^5*b + 720*a^3*b^3 - 271*a*b^5 - 16*(5*a^3*b^3 - 7*a*b^5)*cos(d*x + c)^2
)*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11342, size = 339, normalized size = 1.84

$$\frac{\frac{60\left(a^5-2a^3b^2+ab^4\right)\log(|b\sin(dx+c)+a|)}{b^7}-\frac{10\left(6a^5b\sin(dx+c)-12a^3b^3\sin(dx+c)+6ab^5\sin(dx+c)+5a^6-9a^4b^2+3a^2b^4+b^6\right)}{(b\sin(dx+c)+a)b^7}-\frac{2b^8\sin(dx+c)^5-5ab^7\sin(dx+c}{}}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/10*(60*(a^5 - 2*a^3*b^2 + a*b^4)*log(abs(b*sin(d*x + c) + a))/b^7 - 10*(6
*a^5*b*sin(d*x + c) - 12*a^3*b^3*sin(d*x + c) + 6*a*b^5*sin(d*x + c) + 5*a^
6 - 9*a^4*b^2 + 3*a^2*b^4 + b^6)/((b*sin(d*x + c) + a)*b^7) - (2*b^8*sin(d*
x + c)^5 - 5*a*b^7*sin(d*x + c)^4 + 10*a^2*b^6*sin(d*x + c)^3 - 10*b^8*sin(
d*x + c)^3 - 20*a^3*b^5*sin(d*x + c)^2 + 30*a*b^7*sin(d*x + c)^2 + 50*a^4*b
^4*sin(d*x + c) - 90*a^2*b^6*sin(d*x + c) + 30*b^8*sin(d*x + c))/b^10)/d
```

3.438 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{(3a^2-2b^2)\sin(c+dx)}{b^4d}-\frac{(a^2-b^2)^2}{b^5d(a+b\sin(c+dx))}-\frac{4a(a^2-b^2)\log(a+b\sin(c+dx))}{b^5d}-\frac{a\sin^2(c+dx)}{b^3d}+\frac{\sin^3(c+dx)}{3b^2d}$$

```
[Out] (-4*a*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^5*d) + ((3*a^2 - 2*b^2)*Sin[c
 + d*x])/(b^4*d) - (a*Sin[c + d*x]^2)/(b^3*d) + Sin[c + d*x]^3/(3*b^2*d) -
(a^2 - b^2)^2/(b^5*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.101275, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{(3a^2-2b^2)\sin(c+dx)}{b^4d}-\frac{(a^2-b^2)^2}{b^5d(a+b\sin(c+dx))}-\frac{4a(a^2-b^2)\log(a+b\sin(c+dx))}{b^5d}-\frac{a\sin^2(c+dx)}{b^3d}+\frac{\sin^3(c+dx)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*a*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^5*d) + ((3*a^2 - 2*b^2)*Sin[c
 + d*x])/(b^4*d) - (a*Sin[c + d*x]^2)/(b^3*d) + Sin[c + d*x]^3/(3*b^2*d) -
(a^2 - b^2)^2/(b^5*d*(a + b*Sin[c + d*x]))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)-2ax+x^2+\frac{(a^2-b^2)^2}{(a+x)^2}-\frac{4(a^3-ab^2)}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= -\frac{4a(a^2-b^2)\log(a+b\sin(c+dx))}{b^5 d}+\frac{(3a^2-2b^2)\sin(c+dx)}{b^4 d}-\frac{a\sin^2(c+dx)}{b^3 d}+\frac{\sin^3(c+a}{3b^2 d}$$

Mathematica [A] time = 0.634727, size = 127, normalized size = 1.06

$$\frac{(8a^2b-4b^3)\sin(c+dx)+\frac{b^4\cos^4(c+dx)-4(a^2-b^2)(3a^2\log(a+b\sin(c+dx))+a^2+3ab\sin(c+dx)\log(a+b\sin(c+dx))-b^2)}{a+b\sin(c+dx)}-2ab^2\sin^2(c+dx)}{3b^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((8*a^2*b - 4*b^3)*Sin[c + d*x] - 2*a*b^2*Sin[c + d*x]^2 + (b^4*Cos[c + d*x
]^4 - 4*(a^2 - b^2)*(a^2 - b^2 + 3*a^2*Log[a + b*Sin[c + d*x]] + 3*a*b*Log[
a + b*Sin[c + d*x]]*Sin[c + d*x]))/(a + b*Sin[c + d*x]))/(3*b^5*d)
```

Maple [A] time = 0.079, size = 174, normalized size = 1.5

$$\frac{(\sin(dx+c))^3}{3b^2d}-\frac{(\sin(dx+c))^2 a}{b^3d}+3\frac{a^2\sin(dx+c)}{db^4}-2\frac{\sin(dx+c)}{b^2d}-4\frac{a^3\ln(a+b\sin(dx+c))}{db^5}+4\frac{a\ln(a+b\sin(dx}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/3*sin(d*x+c)^3/b^2/d-a*sin(d*x+c)^2/b^3/d+3/d/b^4*a^2*sin(d*x+c)-2*sin(d*
x+c)/b^2/d-4/d*a^3/b^5*ln(a+b*sin(d*x+c))+4*a*ln(a+b*sin(d*x+c))/b^3/d-1/d/
b^5/(a+b*sin(d*x+c))*a^4+2/d/b^3/(a+b*sin(d*x+c))*a^2-1/b/d/(a+b*sin(d*x+c)
)
```

Maxima [A] time = 0.953096, size = 157, normalized size = 1.31

$$-\frac{\frac{3\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)}{b^{6}\sin(dx+c)+ab^{5}}-\frac{b^{2}\sin(dx+c)^{3}-3\,ab\sin(dx+c)^{2}+3\left(3\,a^{2}-2\,b^{2}\right)\sin(dx+c)}{b^{4}}+\frac{12\left(a^{3}-ab^{2}\right)\log(b\sin(dx+c)+a)}{b^{5}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a^4 - 2*a^2*b^2 + b^4)/(b^6*sin(d*x + c) + a*b^5) - (b^2*sin(d*x +
 c)^3 - 3*a*b*sin(d*x + c)^2 + 3*(3*a^2 - 2*b^2)*sin(d*x + c))/b^4 + 12*(a^
3 - a*b^2)*log(b*sin(d*x + c) + a)/b^5)/d
```

Fricas [A] time = 2.81643, size = 359, normalized size = 2.99

$$\frac{2\,b^{4}\cos\left(dx+c\right)^{4}-6\,a^{4}+27\,a^{2}b^{2}-16\,b^{4}-4\left(3\,a^{2}b^{2}-2\,b^{4}\right)\cos\left(dx+c\right)^{2}-24\left(a^{4}-a^{2}b^{2}+\left(a^{3}b-ab^{3}\right)\sin\left(dx+c\right)\right)\log}{6\left(b^{6}d\sin\left(dx+c\right)+ab^{5}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^4*cos(d*x + c)^4 - 6*a^4 + 27*a^2*b^2 - 16*b^4 - 4*(3*a^2*b^2 - 2*
b^4)*cos(d*x + c)^2 - 24*(a^4 - a^2*b^2 + (a^3*b - a*b^3)*sin(d*x + c))*log
(b*sin(d*x + c) + a) + (4*a*b^3*cos(d*x + c)^2 + 18*a^3*b - 13*a*b^3)*sin(d
*x + c))/(b^6*d*sin(d*x + c) + a*b^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12152, size = 203, normalized size = 1.69

$$-\frac{\frac{12\left(a^{3}-ab^{2}\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{b^{5}}-\frac{b^{4}\sin\left(dx+c\right)^{3}-3\,ab^{3}\sin\left(dx+c\right)^{2}+9\,a^{2}b^{2}\sin\left(dx+c\right)-6\,b^{4}\sin\left(dx+c\right)}{b^{6}}-\frac{3\left(4\,a^{3}b\sin\left(dx+c\right)-4\,ab^{3}\sin\left(dx+c\right)+3\,a^{4}-2\,a^{2}b^{2}-b^{4}\right)}{\left(b\sin\left(dx+c\right)+a\right)b^{5}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^3 - a*b^2)*log(abs(b*sin(d*x + c) + a))/b^5 - (b^4*sin(d*x + c)
^3 - 3*a*b^3*sin(d*x + c)^2 + 9*a^2*b^2*sin(d*x + c) - 6*b^4*sin(d*x + c))/
b^6 - 3*(4*a^3*b*sin(d*x + c) - 4*a*b^3*sin(d*x + c) + 3*a^4 - 2*a^2*b^2 -
b^4)/((b*sin(d*x + c) + a)*b^5))/d
```

3.439 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=63

$$\frac{a^2-b^2}{b^3d(a+b\sin(c+dx))}+\frac{2a\log(a+b\sin(c+dx))}{b^3d}-\frac{\sin(c+dx)}{b^2d}$$

```
[Out] (2*a*Log[a + b*Sin[c + d*x]])/(b^3*d) - Sin[c + d*x]/(b^2*d) + (a^2 - b^2)/
(b^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.0651441, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{a^2-b^2}{b^3d(a+b\sin(c+dx))}+\frac{2a\log(a+b\sin(c+dx))}{b^3d}-\frac{\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*Log[a + b*Sin[c + d*x]])/(b^3*d) - Sin[c + d*x]/(b^2*d) + (a^2 - b^2)/
(b^3*d*(a + b*Sin[c + d*x]))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{b^2-x^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(-1+\frac{-a^2+b^2}{(a+x)^2}+\frac{2a}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{2a\log(a+b\sin(c+dx))}{b^3d} - \frac{\sin(c+dx)}{b^2d} + \frac{a^2-b^2}{b^3d(a+b\sin(c+dx))}$$

Mathematica [A] time = 0.0376845, size = 52, normalized size = 0.83

$$\frac{\frac{(a-b)(a+b)}{a+b\sin(c+dx)} + 2a\log(a+b\sin(c+dx)) - b\sin(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*Log[a + b*Sin[c + d*x]] - b*Sin[c + d*x] + ((a - b)*(a + b))/(a + b*Si
n[c + d*x]))/(b^3*d)
```

Maple [A] time = 0.075, size = 78, normalized size = 1.2

$$-\frac{\sin(dx+c)}{b^2d} + 2\,\frac{a\ln(a+b\sin(dx+c))}{b^3d} + \frac{a^2}{b^3d\,(a+b\sin(dx+c))} - \frac{1}{bd\,(a+b\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -sin(d*x+c)/b^2/d+2*a*ln(a+b*sin(d*x+c))/b^3/d+1/d/b^3/(a+b*sin(d*x+c))*a^2
-1/b/d/(a+b*sin(d*x+c))
```

Maxima [A] time = 0.943103, size = 82, normalized size = 1.3

$$\frac{\frac{a^2-b^2}{b^4\sin(dx+c)+ab^3} + \frac{2\,a\log(b\sin(dx+c)+a)}{b^3} - \frac{\sin(dx+c)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] ((a^2 - b^2)/(b^4*sin(d*x + c) + a*b^3) + 2*a*log(b*sin(d*x + c) + a)/b^3 -
 sin(d*x + c)/b^2)/d
```

Fricas [A] time = 2.805, size = 188, normalized size = 2.98

$$\frac{b^{2} \cos \left(dx+c\right)^{2}-ab \sin \left(dx+c\right)+a^{2}-2 \, b^{2}+2 \, \left(ab \sin \left(dx+c\right)+a^{2}\right) \log \left(b \sin \left(dx+c\right)+a\right)}{b^{4} d \sin \left(dx+c\right)+a b^{3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (b^2*cos(d*x + c)^2 - a*b*sin(d*x + c) + a^2 - 2*b^2 + 2*(a*b*sin(d*x + c)
+ a^2)*log(b*sin(d*x + c) + a))/(b^4*d*sin(d*x + c) + a*b^3*d)
```

Sympy [A] time = 2.13483, size = 316, normalized size = 5.02

$$\begin{cases} \frac{\tilde{\infty} x \cos^{3}{\left(c \right)}}{\sin^{2}{\left(c \right)}} \\ \frac{- \frac{2 \sin{\left(c+dx \right)}}{d} - \frac{\cos^{2}{\left(c+dx \right)}}{d \sin{\left(c+dx \right)}}}{b^{2}} \\ \frac{\frac{2 \sin^{3}{\left(c+dx \right)}}{3d} + \frac{\sin{\left(c+dx \right)} \cos^{2}{\left(c+dx \right)}}{d}}{a^{2}} \\ \frac{x \cos^{3}{\left(c \right)}}{\left(a+b \sin{\left(c \right)}\right)^{2}} \\ \frac{2 a^{3} \log{\left(\frac{a}{b}+\sin{\left(c+dx \right)} \right)}}{a^{2} b^{3} d+a b^{4} d \sin{\left(c+dx \right)}} + \frac{2 a^{3}}{a^{2} b^{3} d+a b^{4} d \sin{\left(c+dx \right)}} + \frac{2 a^{2} b \log{\left(\frac{a}{b}+\sin{\left(c+dx \right)} \right)} \sin{\left(c+dx \right)}}{a^{2} b^{3} d+a b^{4} d \sin{\left(c+dx \right)}} - \frac{a b^{2} \sin^{2}{\left(c+dx \right)}}{a^{2} b^{3} d+a b^{4} d \sin{\left(c+dx \right)}} + \frac{b^{3} \sin^{3}{\left(c+dx \right)}}{a^{2} b^{3} d+a b^{4} d \sin{\left(c+dx \right)}} + \frac{b^{3} \sin{\left(c \cdots}{a^{2} b^{3} d+ \cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x*cos(c)**3/sin(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-2
*sin(c + d*x)/d - cos(c + d*x)**2/(d*sin(c + d*x)))/b**2, Eq(a, 0)), ((2*si
n(c + d*x)**3/(3*d) + sin(c + d*x)*cos(c + d*x)**2/d)/a**2, Eq(b, 0)), (x*c
os(c)**3/(a + b*sin(c))**2, Eq(d, 0)), (2*a**3*log(a/b + sin(c + d*x))/(a**
```

```
2*b**3*d + a*b**4*d*sin(c + d*x)) + 2*a**3/(a**2*b**3*d + a*b**4*d*sin(c +
d*x)) + 2*a**2*b*log(a/b + sin(c + d*x))*sin(c + d*x)/(a**2*b**3*d + a*b**4
*d*sin(c + d*x)) - a*b**2*sin(c + d*x)**2/(a**2*b**3*d + a*b**4*d*sin(c + d
*x)) + b**3*sin(c + d*x)**3/(a**2*b**3*d + a*b**4*d*sin(c + d*x)) + b**3*si
n(c + d*x)*cos(c + d*x)**2/(a**2*b**3*d + a*b**4*d*sin(c + d*x)), True))
```

Giac [A] time = 1.10733, size = 123, normalized size = 1.95

$$-\frac{\frac{2\,a\log\left(\frac{|b\sin(dx+c)+a|}{(b\sin(dx+c)+a)^2|b|}\right)}{b^3}+\frac{b\sin(dx+c)+a}{b^3}-\frac{a^2}{(b\sin(dx+c)+a)b^3}+\frac{1}{(b\sin(dx+c)+a)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a*log(abs(b*sin(d*x + c) + a)/((b*sin(d*x + c) + a)^2*abs(b)))/b^3 + (b
*sin(d*x + c) + a)/b^3 - a^2/((b*sin(d*x + c) + a)*b^3) + 1/((b*sin(d*x + c
) + a)*b))/d
```

3.440 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{bd(a+b\sin(c+dx))}$$

```
[Out] -(1/(b*d*(a + b*Sin[c + d*x])))
```

Rubi [A] time = 0.0266825, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 32}

$$-\frac{1}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(1/(b*d*(a + b*Sin[c + d*x])))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{1}{bd(a+b\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.023615, size = 20, normalized size = 1.

$$-\frac{1}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(1/(b*d*(a + b*Sin[c + d*x])))
```

Maple [A] time = 0.023, size = 21, normalized size = 1.1

$$-\frac{1}{bd\left(a+b\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/b/d/(a+b*sin(d*x+c))
```

Maxima [A] time = 0.932654, size = 27, normalized size = 1.35

$$-\frac{1}{\left(b\sin\left(dx+c\right)+a\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/((b*sin(d*x + c) + a)*b*d)
```

Fricas [A] time = 2.37971, size = 45, normalized size = 2.25

$$-\frac{1}{b^2d\sin\left(dx+c\right)+abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*d*sin(d*x + c) + a*b*d)
```

Sympy [A] time = 1.14095, size = 51, normalized size = 2.55

$$\begin{cases} \frac{x\cos(c)}{a^2} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sin(c+dx)}{a^2 d} & \text{for } b = 0 \\ \frac{x\cos(c)}{(a+b\sin(c))^2} & \text{for } d = 0 \\ -\frac{1}{abd+b^2 d\sin(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*cos(c)/a**2, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**2*d), Eq(
b, 0)), (x*cos(c)/(a + b*sin(c))**2, Eq(d, 0)), (-1/(a*b*d + b**2*d*sin(c +
 d*x)), True))
```

Giac [A] time = 1.10155, size = 27, normalized size = 1.35

$$-\frac{1}{(b\sin(dx+c)+a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/((b*sin(d*x + c) + a)*b*d)
```

3.441 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{b}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{2ab\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^2}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^2*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2
*d) - (2*a*b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) + b/((a^2 - b^2)*d*
(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.114654, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2668, 710, 801}

$$\frac{b}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{2ab\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^2}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^2*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2
*d) - (2*a*b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) + b/((a^2 - b^2)*d*
(a + b*Sin[c + d*x]))
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{(a+x)^2(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{b\,\text{Subst}\left(\int \frac{a-x}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{(a^2-b^2)\,d}$$

$$= \frac{b}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{b\,\text{Subst}\left(\int \left(\frac{a-b}{2b(a+b)(b-x)} - \frac{2a}{(a-b)(a+b)(a+x)} + \frac{a+b}{2(a-b)b(b+x)}\right) dx, x\right.}{(a^2-b^2)\,d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} - \frac{2ab\log(a+b\sin(c+dx))}{(a^2-b^2)^2\,d} + \frac{}{(a^2-b^2)\,d(a}$$

Mathematica [A] time = 0.205833, size = 102, normalized size = 0.98

$$\frac{b\left(\frac{1}{(a^2-b^2)(a+b\sin(c+dx))} - \frac{\log(1-\sin(c+dx))}{2b(a+b)^2} + \frac{\log(\sin(c+dx)+1)}{2b(a-b)^2} - \frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (b*(-Log[1 - Sin[c + d*x]]/(2*b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*(a -
b)^2*b) - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^2 - b
^2)*(a + b*Sin[c + d*x]))))/d
```

Maple [A] time = 0.091, size = 101, normalized size = 1.

$$\frac{b}{d(a+b)(a-b)(a+b\sin(dx+c))} - 2\,\frac{ab\ln(a+b\sin(dx+c))}{d(a+b)^2(a-b)^2} - \frac{\ln(\sin(dx+c)-1)}{2d(a+b)^2} + \frac{\ln(1+\sin(dx+c))}{2(a-b)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*b/(a+b)/(a-b)/(a+b*sin(d*x+c))-2/d*a*b/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c
))-1/2/d/(a+b)^2*ln(sin(d*x+c)-1)+1/2*ln(1+sin(d*x+c))/(a-b)^2/d
```

Maxima [A] time = 1.05678, size = 159, normalized size = 1.53

$$-\frac{\frac{4\,ab\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}-\frac{2\,b}{a^3-ab^2+\left(a^2b-b^3\right)\sin(dx+c)}-\frac{\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*a*b*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - 2*b/(a^3 - a*
b^2 + (a^2*b - b^3)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^
2) + log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

Fricas [A] time = 3.20017, size = 443, normalized size = 4.26

$$\frac{2\,a^2b-2\,b^3-4\left(ab^2\sin\left(dx+c\right)+a^2b\right)\log\left(b\sin\left(dx+c\right)+a\right)+\left(a^3+2\,a^2b+ab^2+\left(a^2b+2\,ab^2+b^3\right)\sin\left(dx+c\right)\right)\log(}{2\left(\left(a^4b-2\,a^2b^3+b^5\right)d\sin\left(dx+c\right)+\left(a^5-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*b - 2*b^3 - 4*(a*b^2*sin(d*x + c) + a^2*b)*log(b*sin(d*x + c) +
a) + (a^3 + 2*a^2*b + a*b^2 + (a^2*b + 2*a*b^2 + b^3)*sin(d*x + c))*log(sin
(d*x + c) + 1) - (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*sin(d*x +
 c))*log(-sin(d*x + c) + 1))/((a^4*b - 2*a^2*b^3 + b^5)*d*sin(d*x + c) + (a
^5 - 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.14376, size = 198, normalized size = 1.9

$$-\frac{\frac{4\,ab^2\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5}-\frac{\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2}+\frac{\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2}-\frac{2\left(2\,ab^2\sin(dx+c)+3\,a^2b-b^3\right)}{\left(a^4-2\,a^2b^2+b^4\right)(b\sin(dx+c)+a)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b^2*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - log(
abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + log(abs(sin(d*x + c) - 1))/(a^
2 + 2*a*b + b^2) - 2*(2*a*b^2*sin(d*x + c) + 3*a^2*b - b^3)/((a^4 - 2*a^2*b
^2 + b^4)*(b*sin(d*x + c) + a)))/d
```

3.442 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=177

$$-\frac{b\left(a^2+3b^2\right)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{4ab^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{(a+3b)\log(1-\sin(}{4d(a+b)^3}$$

```
[Out] -((a + 3*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^3*d) + ((a - 3*b)*Log[1 + Sin
[c + d*x]])/(4*(a - b)^3*d) + (4*a*b^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2
)^3*d) - (b*(a^2 + 3*b^2))/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (Sec[
c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.20884, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2668, 741, 801}

$$-\frac{b\left(a^2+3b^2\right)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{4ab^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{(a+3b)\log(1-\sin(}{4d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -((a + 3*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^3*d) + ((a - 3*b)*Log[1 + Sin
[c + d*x]])/(4*(a - b)^3*d) + (4*a*b^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2
)^3*d) - (b*(a^2 + 3*b^2))/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (Sec[
c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
```

```
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(b^2-x^2\right)^2}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{b\,\text{Subst}\left(\int \frac{a^2-3b^2+2ax}{(a+x)^2\left(b^2-x^2\right)}\,dx,x,b\sin(c+dx)\right)}{2\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{b\,\text{Subst}\left(\int \left(\frac{(a-b)(a+3b)}{2b(a+b)^2(b-x)} + \frac{a^2+3b^2}{(a-b)(a+b)(a+x)^2} + \frac{8ab^2}{(a-b)^2(a+b)^2(a}\right.\right.}{2\left(a^2-b^2\right)d}$$

$$= -\frac{(a+3b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-3b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{4ab^3\log(a+b\sin(c+dx}{\left(a^2-b^2\right)^3d}$$

Mathematica [A] time = 1.73611, size = 222, normalized size = 1.25

$$\frac{-b\left(-a^2-3b^2\right)\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))} - \frac{\log(1-\sin(c+dx))}{2b(a+b)^2} + \frac{\log(\sin(c+dx)+1)}{2b(a-b)^2} - \frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}\right) + \frac{a((a-b)\log(1-\sin(c+dx))-(a+b)\log(}{(a-b}}{2d\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a*((a - b)*Log[1 - Sin[c + d*x]] - (a + b)*Log[1 + Sin[c + d*x]] + 2*b*Lo
g[a + b*Sin[c + d*x]]))/((a - b)*(a + b)) + (Sec[c + d*x]^2*(b - a*Sin[c +
d*x]))/(a + b*Sin[c + d*x]) - b*(-a^2 - 3*b^2)*(-Log[1 - Sin[c + d*x]]/(2*b
*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*b) - (2*a*Log[a + b*Sin[c
+ d*x]])/((a - b)^2*(a + b)^2) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x]))))/(2*
```

```
(-a^2 + b^2)*d)
```

Maple [A] time = 0.108, size = 192, normalized size = 1.1

$$-\frac{b^3}{d(a+b)^2(a-b)^2(a+b\sin(dx+c))}+4\frac{ab^3\ln(a+b\sin(dx+c))}{d(a+b)^3(a-b)^3}-\frac{1}{4d(a+b)^2(\sin(dx+c)-1)}-\frac{\ln(\sin(dx+c)-}{4d(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*b^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))+4/d*b^3*a/(a+b)^3/(a-b)^3*ln(a+b*
sin(d*x+c))-1/4/d/(a+b)^2/(sin(d*x+c)-1)-1/4/d/(a+b)^3*ln(sin(d*x+c)-1)*a-3
/4/d/(a+b)^3*ln(sin(d*x+c)-1)*b-1/4/d/(a-b)^2/(1+sin(d*x+c))+1/4/d/(a-b)^3*
ln(1+sin(d*x+c))*a-3/4/d/(a-b)^3*ln(1+sin(d*x+c))*b
```

Maxima [A] time = 0.97815, size = 371, normalized size = 2.1

$$\frac{\frac{16ab^3\log(b\sin(dx+c)+a)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{(a-3b)\log(\sin(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3}-\frac{(a+3b)\log(\sin(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3}-\frac{2\left(2a^2b+2b^3-\left(a^2b+3b^3\right)\sin(dx+c)^2-\left(a^3-ab^2\right)\mathrm{si}}{a^5-2a^3b^2+ab^4-\left(a^4b-2a^2b^3+b^5\right)\sin(dx+c)^3-\left(a^5-2a^3b^2+ab^4\right)\sin(dx+}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(16*a*b^3*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) +
 (a - 3*b)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (a + 3*b
)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(2*a^2*b + 2*b^
3 - (a^2*b + 3*b^3)*sin(d*x + c)^2 - (a^3 - a*b^2)*sin(d*x + c))/(a^5 - 2*a
^3*b^2 + a*b^4 - (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)^3 - (a^5 - 2*a^3*b^
2 + a*b^4)*sin(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)))/d
```

Fricas [B] time = 3.82479, size = 867, normalized size = 4.9

$$-\frac{2a^4b-4a^2b^3+2b^5+2\left(a^4b+2a^2b^3-3b^5\right)\cos(dx+c)^2-16\left(ab^4\cos(dx+c)^2\sin(dx+c)+a^2b^3\cos(dx+c)^2\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(a^4*b + 2*a^2*b^3 - 3*b^5)*cos(d*x +
 c)^2 - 16*(a*b^4*cos(d*x + c)^2*sin(d*x + c) + a^2*b^3*cos(d*x + c)^2)*log
(b*sin(d*x + c) + a) - ((a^4*b - 6*a^2*b^3 - 8*a*b^4 - 3*b^5)*cos(d*x + c)^
2*sin(d*x + c) + (a^5 - 6*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^2)*lo
g(sin(d*x + c) + 1) + ((a^4*b - 6*a^2*b^3 + 8*a*b^4 - 3*b^5)*cos(d*x + c)^2
*sin(d*x + c) + (a^5 - 6*a^3*b^2 + 8*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^2)*log
(-sin(d*x + c) + 1) - 2*(a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3
*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)^2*sin(d*x + c) + (a^7 - 3*a^5*b^
2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.1665, size = 329, normalized size = 1.86

$$\frac{\frac{16\,ab^4\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}+\frac{(a-3\,b)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{(a+3\,b)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(a^2b\sin(dx+c)^2+3\,b^3\sin(dx+c)^2+a^3\sin(dx+c)-ab^2\sin(dx+c)-2\cdots\right)}{\left(a^4-2\,a^2b^2+b^4\right)\left(b\sin(dx+c)^3+a\sin(dx+c)^2-b\sin(dx+c)-\cdots\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*(16*a*b^4*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7) + (a - 3*b)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3
) - (a + 3*b)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) -
2*(a^2*b*sin(d*x + c)^2 + 3*b^3*sin(d*x + c)^2 + a^3*sin(d*x + c) - a*b^2*s
```

```
in(d*x + c) - 2*a^2*b - 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c)^3 +
 a*sin(d*x + c)^2 - b*sin(d*x + c) - a)))/d
```

3.443 $\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=269

$$-\frac{3b\left(-4a^2b^2+a^4-5b^4\right)}{8d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{6ab^5\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{3\left(a^2+4ab+5b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^4}+\frac{3\left(a^2-4ab\right.}{}$$

```
[Out] (-3*(a^2 + 4*a*b + 5*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^4*d) + (3*(a^2
 - 4*a*b + 5*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^4*d) - (6*a*b^5*Log[a
+ b*Sin[c + d*x]])/((a^2 - b^2)^4*d) - (3*b*(a^4 - 4*a^2*b^2 - 5*b^4))/(8*(
a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x])
)/(4*(a^2 - b^2)*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(b*(a^2 + 5*b^2)
 + 3*a*(a^2 - 3*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])
)
```

Rubi [A] time = 0.321401, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2668, 741, 823, 801}

$$-\frac{3b\left(-4a^2b^2+a^4-5b^4\right)}{8d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{6ab^5\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{3\left(a^2+4ab+5b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^4}+\frac{3\left(a^2-4ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*(a^2 + 4*a*b + 5*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^4*d) + (3*(a^2
 - 4*a*b + 5*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^4*d) - (6*a*b^5*Log[a
+ b*Sin[c + d*x]])/((a^2 - b^2)^4*d) - (3*b*(a^4 - 4*a^2*b^2 - 5*b^4))/(8*(
a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x])
)/(4*(a^2 - b^2)*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(b*(a^2 + 5*b^2)
 + 3*a*(a^2 - 3*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])
)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
```

```
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b^5\,\mathrm{Subst}\left(\int \frac{1}{(a+x)^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{b^3\,\mathrm{Subst}\left(\int \frac{3a^2-5b^2+4ax}{(a+x)^2\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\left(b\left(a^2+5b^2\right)+3a\left(a^2-3b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\left(b\left(a^2+5b^2\right)+3a\left(a^2-3b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))}$$

$$= -\frac{3\left(a^2+4ab+5b^2\right)\log(1-\sin(c+dx))}{16(a+b)^4 d} + \frac{3\left(a^2-4ab+5b^2\right)\log(1+\sin(c+dx))}{16(a-b)^4 d} - \frac{6ab^5}{}$$

Mathematica [A] time = 6.11299, size = 406, normalized size = 1.51

$$\frac{b^5\left(\frac{\sec^4(c+dx)\left(b^2-ab\sin(c+dx)\right)}{4b^6\left(b^2-a^2\right)(a+b\sin(c+dx))} - \frac{\frac{\left(6a^2\left(a^2-3b^2\right)-3\left(-2a^2b^2+a^4+5b^4\right)\right)\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2}+\frac{\log(\sin(c+dx)+1)}{2b(a-b)^2}-\frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}\right)-6a\left(a^2-3b^2\right)\left(\right.}{2b^2\left(b^2-a^2\right)}}{4b^2\left(b^2-\right.}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (b^5*((Sec[c + d*x]^4*(b^2 - a*b*Sin[c + d*x]))/(4*b^6*(-a^2 + b^2)*(a + b*
Sin[c + d*x])) - (-(Sec[c + d*x]^2*(4*a^2*b^2 - b^2*(3*a^2 - 5*b^2) - b*(4*
a*b^2 - a*(3*a^2 - 5*b^2))*Sin[c + d*x]))/(2*b^4*(-a^2 + b^2)*(a + b*Sin[c
+ d*x])) + (-6*a*(a^2 - 3*b^2)*(-Log[1 - Sin[c + d*x]]/(2*b*(a + b)) + Log[
1 + Sin[c + d*x]]/(2*(a - b)*b) - Log[a + b*Sin[c + d*x]]/(a^2 - b^2)) + (6
*a^2*(a^2 - 3*b^2) - 3*(a^4 - 2*a^2*b^2 + 5*b^4))*(-Log[1 - Sin[c + d*x]]/(
2*b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*b) - (2*a*Log[a + b*Sin
[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x]))))/
(2*b^2*(-a^2 + b^2)))/(4*b^2*(-a^2 + b^2))))/d
```

Maple [A] time = 0.118, size = 331, normalized size = 1.2

$$\frac{b^5}{d(a+b)^3(a-b)^3(a+b\sin(dx+c))}-6\frac{b^5a\ln(a+b\sin(dx+c))}{d(a+b)^4(a-b)^4}+\frac{1}{16d(a+b)^2(\sin(dx+c)-1)^2}-\frac{7b}{16d(a+b)^3(\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*b^5/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))-6/d*b^5*a/(a+b)^4/(a-b)^4*ln(a+b*s
in(d*x+c))+1/16/d/(a+b)^2/(sin(d*x+c)-1)^2-7/16/d/(a+b)^3/(sin(d*x+c)-1)*b-
3/16/d/(a+b)^3/(sin(d*x+c)-1)*a-3/16/d/(a+b)^4*ln(sin(d*x+c)-1)*a^2-3/4/d/(
a+b)^4*ln(sin(d*x+c)-1)*a*b-15/16/d/(a+b)^4*ln(sin(d*x+c)-1)*b^2-1/16/d/(a-
b)^2/(1+sin(d*x+c))^2+7/16/d/(a-b)^3/(1+sin(d*x+c))*b-3/16/d/(a-b)^3/(1+sin
(d*x+c))*a+3/16/d/(a-b)^4*ln(1+sin(d*x+c))*a^2-3/4/d/(a-b)^4*ln(1+sin(d*x+c
))*a*b+15/16/d/(a-b)^4*ln(1+sin(d*x+c))*b^2
```

Maxima [A] time = 1.03183, size = 682, normalized size = 2.54

$$-\frac{\frac{96\,ab^5\log(b\sin(dx+c)+a)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{3\left(a^2-4\,ab+5\,b^2\right)\log(\sin(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}+\frac{3\left(a^2+4\,ab+5\,b^2\right)\log(\sin(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\left(4\,a^4b-20\,a^2b^3-8\,b\right.}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6+\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/16*(96*a*b^5*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^
2*b^6 + b^8) - 3*(a^2 - 4*a*b + 5*b^2)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b
 + 6*a^2*b^2 - 4*a*b^3 + b^4) + 3*(a^2 + 4*a*b + 5*b^2)*log(sin(d*x + c) -
1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(4*a^4*b - 20*a^2*b^3 -
8*b^5 + 3*(a^4*b - 4*a^2*b^3 - 5*b^5)*sin(d*x + c)^4 + 3*(a^5 - 4*a^3*b^2 +
 3*a*b^4)*sin(d*x + c)^3 - (5*a^4*b - 28*a^2*b^3 - 25*b^5)*sin(d*x + c)^2 -
 (5*a^5 - 16*a^3*b^2 + 11*a*b^4)*sin(d*x + c))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4
 - a*b^6 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)^5 + (a^7 - 3*
a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)^4 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*
b^5 - b^7)*sin(d*x + c)^3 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x
 + c)^2 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)))/d
```

Fricas [B] time = 5.57136, size = 1195, normalized size = 4.44

$$-\frac{4\,a^6b-12\,a^4b^3+12\,a^2b^5-4\,b^7+6\left(a^6b-5\,a^4b^3-a^2b^5+5\,b^7\right)\cos\left(dx+c\right)^4-2\left(a^6b+3\,a^4b^3-9\,a^2b^5+5\,b^7\right)\cos\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 6*(a^6*b - 5*a^4*b^3 - a
^2*b^5 + 5*b^7)*cos(d*x + c)^4 - 2*(a^6*b + 3*a^4*b^3 - 9*a^2*b^5 + 5*b^7)*
cos(d*x + c)^2 + 96*(a*b^6*cos(d*x + c)^4*sin(d*x + c) + a^2*b^5*cos(d*x +
c)^4)*log(b*sin(d*x + c) + a) - 3*((a^6*b - 5*a^4*b^3 + 15*a^2*b^5 + 16*a*b
^6 + 5*b^7)*cos(d*x + c)^4*sin(d*x + c) + (a^7 - 5*a^5*b^2 + 15*a^3*b^4 + 1
6*a^2*b^5 + 5*a*b^6)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) + 3*((a^6*b - 5*
a^4*b^3 + 15*a^2*b^5 - 16*a*b^6 + 5*b^7)*cos(d*x + c)^4*sin(d*x + c) + (a^7
 - 5*a^5*b^2 + 15*a^3*b^4 - 16*a^2*b^5 + 5*a*b^6)*cos(d*x + c)^4)*log(-sin(
d*x + c) + 1) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + 3*(a^7 - 5*a^5
*b^2 + 7*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b
^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^4*sin(d*x + c) + (a^9 - 4*
a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18301, size = 621, normalized size = 2.31

$$-\frac{\frac{96\,ab^6\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^8b-4\,a^6b^3+6\,a^4b^5-4\,a^2b^7+b^9}-\frac{3\left(a^2-4\,ab+5\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}+\frac{3\left(a^2+4\,ab+5\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{16\left(6\,ab^6\sin\left(dx+c\right)+7\,a^2b^5-b^7\right)}{\left(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8\right)\left(b\sin\left(dx+c\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(96*a*b^6*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3 + 6*a^4*b^5
 - 4*a^2*b^7 + b^9) - 3*(a^2 - 4*a*b + 5*b^2)*log(abs(sin(d*x + c) + 1))/(a
^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 3*(a^2 + 4*a*b + 5*b^2)*log(abs
(sin(d*x + c) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 16*(6*a*b
^6*sin(d*x + c) + 7*a^2*b^5 - b^7)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^
6 + b^8)*(b*sin(d*x + c) + a)) + 2*(36*a*b^5*sin(d*x + c)^4 + 3*a^6*sin(d*x
 + c)^3 - 15*a^4*b^2*sin(d*x + c)^3 + 5*a^2*b^4*sin(d*x + c)^3 + 7*b^6*sin(
d*x + c)^3 + 16*a^3*b^3*sin(d*x + c)^2 - 88*a*b^5*sin(d*x + c)^2 - 5*a^6*si
n(d*x + c) + 17*a^4*b^2*sin(d*x + c) - 3*a^2*b^4*sin(d*x + c) - 9*b^6*sin(d
*x + c) + 4*a^5*b - 24*a^3*b^3 + 56*a*b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 -
4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)^2))/d
```

3.444 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=187

$$\frac{10a\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}-\frac{5\cos(c+dx)\left(8a\left(a^2-b^2\right)-b\left(4a^2-3b^2\right)\sin(c+dx)\right)}{8b^5d}-\frac{5x\left(-12a^2b^2+8a^4\right.}{8b^6}$$

```
[Out] (-5*(8*a^4 - 12*a^2*b^2 + 3*b^4)*x)/(8*b^6) + (10*a*(a^2 - b^2)^(3/2)*ArcTa
n[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) + (5*Cos[c + d*x]^3*(4
*a - 3*b*Sin[c + d*x]))/(12*b^3*d) - Cos[c + d*x]^5/(b*d*(a + b*Sin[c + d*x
])) - (5*Cos[c + d*x]*(8*a*(a^2 - b^2) - b*(4*a^2 - 3*b^2)*Sin[c + d*x]))/(
8*b^5*d)
```

Rubi [A] time = 0.371367, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2693, 2865, 2735, 2660, 618, 204}

$$\frac{10a\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}-\frac{5\cos(c+dx)\left(8a\left(a^2-b^2\right)-b\left(4a^2-3b^2\right)\sin(c+dx)\right)}{8b^5d}-\frac{5x\left(-12a^2b^2+8a^4\right.}{8b^6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-5*(8*a^4 - 12*a^2*b^2 + 3*b^4)*x)/(8*b^6) + (10*a*(a^2 - b^2)^(3/2)*ArcTa
n[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) + (5*Cos[c + d*x]^3*(4
*a - 3*b*Sin[c + d*x]))/(12*b^3*d) - Cos[c + d*x]^5/(b*d*(a + b*Sin[c + d*x
])) - (5*Cos[c + d*x]*(8*a*(a^2 - b^2) - b*(4*a^2 - 3*b^2)*Sin[c + d*x]))/(
8*b^5*d)
```


Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos^5(c+dx)}{bd(a+b\sin(c+dx))} - \frac{5\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c+dx)}{bd(a+b\sin(c+dx))} - \frac{5\int \frac{\cos^2(c+dx)\left(-ab-\left(4a^2-3b^2\right)\sin(}{a+b\sin(c+dx)}}{4b^3}$$

$$= \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c+dx)}{bd(a+b\sin(c+dx))} - \frac{5\cos(c+dx)\left(8a\left(a^2-b^2\right)-\right.}{8b^5}$$

$$= -\frac{5\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} + \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c+dx)}{bd(a+b\sin(c+dx))} -$$

$$= -\frac{5\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} + \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c+dx)}{bd(a+b\sin(c+dx))} -$$

$$= -\frac{5\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} + \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c+dx)}{bd(a+b\sin(c+dx))} -$$

$$= -\frac{5\left(8a^4-12a^2b^2+3b^4\right)x}{8b^6} + \frac{10a\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d} + \frac{5\cos^3(c+dx)(4a-}{12b^3}$$

<u>Mathematica</u> [B] time = 6.54191, size = 3695, normalized size = 19.76

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]^5*(-((b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(7/2)*(b/(a
 + b) - (b*Sin[c + d*x])/(a + b))^(7/2))/(((a*b)/(a - b) - b^2/(a - b))*((a
*b)/(a + b) + b^2/(a + b))*(a + b*Sin[c + d*x]))) - ((48*Sqrt[2]*(a - b)*b^
3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(7/2)*Sqrt[b/(a + b) - (b*Sin[c
 + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/
(2*b))^(7/2)*((7*(3/(16*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a -
 b)))/(2*b))^3) + 1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a -
b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(
2*b))^(-1)))/12 + (35*b^4*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b
)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b^2) + (
2*(a - b)^3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3)/(15*b^3) - (Sqrt[2
```

]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a
- b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(S
qrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])
))/(128*(a - b)^4*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^4*(1 + ((a - b)
*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(7*(a + b)^2*(a^2 -
 b^2)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) + (5*a*b^2*
((8*Sqrt[2]*b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(5/2)*Sqrt[b/(a + b
) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x]
)/(a - b)))/(2*b))^(7/2)*((5/(16*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)))/(2*b))^3) + 5/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*
x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a
 - b)))/(2*b))^(-1))/2 - (15*b^3*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])
/(a - b)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b
^2) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[
c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])
/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a -
 b)))/(2*b)])))/(64*(a - b)^3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3*(
1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(5*(a +
 b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)
/(a - b)) + b^2/(a - b))*((8*Sqrt[2]*b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a
- b))^(3/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(
a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((3*(5/(8*(1 + ((a - b)*(
-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(6*(1 + ((a - b)*(-
(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1)))/8 + (15*b^2*(((a - b)*(-(b
/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sq
rt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]
*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*
(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(64*(a - b)^2*(-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b))^2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c
+ d*x])/(a - b)))/(2*b))^3)))/(3*(a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Si
n[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*((8*Sqrt[2]*b
*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c +
d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*
b))^(7/2)*((5*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(8*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b))
 - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])
/(a - b)))/(2*b))^(7/2)) + (15/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c +
d*x])/(a - b)))/(2*b))^3) + 5/(4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(
a - b)))/(2*b))^(-1))/6))/((a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c +
d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a + b
)) - b^2/(a + b))*(-(((-((a*b)/(a + b)) - b^2/(a + b))*((-2*(-((a*b)/(a + b
)) - b^2/(a + b))*ArcTan[(Sqrt[(a*b)/(a + b) + b^2/(a + b)]*Sqrt[-(b/(a - b
)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[

```
b/(a + b) - (b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[-((a*b)/(a - b)) + b^2/(a
- b)]*Sqrt[(a*b)/(a + b) + b^2/(a + b)]) + (2*Sqrt[a - b]*ArcTanh[(Sqrt[a -
 b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[a + b]*Sqrt[b/(a +
 b) - (b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[a + b])))/b) + (2*Sqrt[2]*(a - b
)*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c +
 d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2
*b))^(3/2)*((Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*
x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) -
(b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a
 - b)))/(2*b))^(3/2)) + 1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])
/(a - b)))/(2*b)))))/(b*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])
/(a + b)))/b])))/b) + (4*Sqrt[2]*(a - b)*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x
])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(
a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)*((3*Sqrt[b]*ArcSinh[(Sqrt
[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/
(4*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + (
(a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)) + (3/(2*(1
 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a
 - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/4))/((a + b)^
2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b])))/b))/b))/b))/(
a^2 - b^2))/(((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b)))))
/(d*(1 - (a + b*Sin[c + d*x])/(a - b))^(5/2)*(1 - (a + b*Sin[c + d*x])/(a +
 b))^(5/2))
```

Maple [B] time = 0.093, size = 1021, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 28/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*a-10/d/b^6*arctan(tan(1/2*d*x+1/2*c))
*a^4+15/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^2-2/d/b^5/(tan(1/2*d*x+1/2*c)^2*
a+2*tan(1/2*d*x+1/2*c)*b+a)*a^4+4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)*a^2+9/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^
7+1/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-1/4/d/b^2/(1+ta
n(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-9/4/d/b^2/(1+tan(1/2*d*x+1/2*c)^
2)^4*tan(1/2*d*x+1/2*c)-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)/a*tan(1/2*d*x+1/2*c)-8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*a^3+28/d/b^3/(1+
tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a+3/d/b^4/(1+tan(1/2*d*x+1/2*c
)^2)^4*tan(1/2*d*x+1/2*c)^3*a^2-24/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2
```

```
*d*x+1/2*c)^2*a^3+76/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^
2*a+3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*a^2-2/d/b^4/(tan(
1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^3*tan(1/2*d*x+1/2*c)-24/d/b^
5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a^3-15/4/d/b^2*arctan(tan
(1/2*d*x+1/2*c))-2/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)+4/
d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a*tan(1/2*d*x+1/2*c
)+10/d/b^6*a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2
-b^2)^(1/2))-20/d/b^4*a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))+10/d/b^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*
d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2
*d*x+1/2*c)^7*a^2-8/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6*a
^3+12/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6*a-3/d/b^4/(1+ta
n(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.24932, size = 1364, normalized size = 7.29

$$\left[\frac{6\,b^5\cos\left(dx+c\right)^5-5\left(4\,a^2b^3-3\,b^5\right)\cos\left(dx+c\right)^3-15\left(8\,a^5-12\,a^3b^2+3\,ab^4\right)dx-60\left(a^4-a^2b^2+\left(a^3b-ab^3\right)\sin\left(dx+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/24*(6*b^5*cos(d*x + c)^5 - 5*(4*a^2*b^3 - 3*b^5)*cos(d*x + c)^3 - 15*(8*
a^5 - 12*a^3*b^2 + 3*a*b^4)*d*x - 60*(a^4 - a^2*b^2 + (a^3*b - a*b^3)*sin(d
*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-
a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 15*(8*
```

```
a^4*b - 12*a^2*b^3 + 3*b^5)*cos(d*x + c) + 5*(2*a*b^4*cos(d*x + c)^3 - 3*(8
*a^4*b - 12*a^2*b^3 + 3*b^5)*d*x - 3*(4*a^3*b^2 - 5*a*b^4)*cos(d*x + c))*si
n(d*x + c))/(b^7*d*sin(d*x + c) + a*b^6*d), 1/24*(6*b^5*cos(d*x + c)^5 - 5*
(4*a^2*b^3 - 3*b^5)*cos(d*x + c)^3 - 15*(8*a^5 - 12*a^3*b^2 + 3*a*b^4)*d*x
- 120*(a^4 - a^2*b^2 + (a^3*b - a*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan
(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 15*(8*a^4*b - 12*a
^2*b^3 + 3*b^5)*cos(d*x + c) + 5*(2*a*b^4*cos(d*x + c)^3 - 3*(8*a^4*b - 12*
a^2*b^3 + 3*b^5)*d*x - 3*(4*a^3*b^2 - 5*a*b^4)*cos(d*x + c))*sin(d*x + c))/
(b^7*d*sin(d*x + c) + a*b^6*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12455, size = 633, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(15*(8*a^4 - 12*a^2*b^2 + 3*b^4)*(d*x + c)/b^6 - 240*(a^5 - 2*a^3*b^2
 + a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
+ 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 48*(a^4*b*tan(1/2*d
*x + 1/2*c) - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + a
^5 - 2*a^3*b^2 + a*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*
c) + a)*a*b^5) + 2*(36*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 27*b^3*tan(1/2*d*x +
1/2*c)^7 + 96*a^3*tan(1/2*d*x + 1/2*c)^6 - 144*a*b^2*tan(1/2*d*x + 1/2*c)^6
 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^5 + 288*a^3
*tan(1/2*d*x + 1/2*c)^4 - 336*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1
/2*d*x + 1/2*c)^3 + 3*b^3*tan(1/2*d*x + 1/2*c)^3 + 288*a^3*tan(1/2*d*x + 1/
2*c)^2 - 304*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 36*a^2*b*tan(1/2*d*x + 1/2*c) +
```

```
27*b^3*tan(1/2*d*x + 1/2*c) + 96*a^3 - 112*a*b^2)/((tan(1/2*d*x + 1/2*c)^2
+ 1)^4*b^5))/d
```

3.445 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=128

$$-\frac{6a\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}+\frac{3x\left(2a^2-b^2\right)}{2b^4}+\frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d}-\frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))}$$

```
[Out] (3*(2*a^2 - b^2)*x)/(2*b^4) - (6*a*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(b^4*d) + (3*Cos[c + d*x]*(2*a - b*Sin[c + d*x]))
/(2*b^3*d) - Cos[c + d*x]^3/(b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.21329, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2693, 2865, 2735, 2660, 618, 204}

$$-\frac{6a\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}+\frac{3x\left(2a^2-b^2\right)}{2b^4}+\frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d}-\frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*(2*a^2 - b^2)*x)/(2*b^4) - (6*a*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(b^4*d) + (3*Cos[c + d*x]*(2*a - b*Sin[c + d*x]))
/(2*b^3*d) - Cos[c + d*x]^3/(b*d*(a + b*Sin[c + d*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
```

```
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{3\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d} - \frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{3\int \frac{-ab-\left(2a^2-b^2\right)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b^3}$$

$$= \frac{3\left(2a^2-b^2\right)x}{2b^4} + \frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d} - \frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\left(3a\left(a^2-b^2\right)\right)}{}$$

$$= \frac{3\left(2a^2-b^2\right)x}{2b^4} + \frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d} - \frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\left(6a\left(a^2-b^2\right)\right)}{}$$

$$= \frac{3\left(2a^2-b^2\right)x}{2b^4} + \frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d} - \frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} + \frac{\left(12a\left(a^2-b^2\right)\right.}{}$$

$$= \frac{3\left(2a^2-b^2\right)x}{2b^4} - \frac{6a\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d} + \frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d} -$$

<u>Mathematica</u> [B] time = 6.2631, size = 2447, normalized size = 19.12

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]^3*(-((b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(5/2)*(b/(a
 + b) - (b*Sin[c + d*x])/(a + b))^(5/2))/(((a*b)/(a - b) - b^2/(a - b))*((a
*b)/(a + b) + b^2/(a + b))*(a + b*Sin[c + d*x]))) - ((16*Sqrt[2]*(a - b)*b^
2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(5/2)*Sqrt[b/(a + b) - (b*Sin[c
 + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/
(2*b))^(5/2)*((5*(1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a -
b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(
2*b))^(-1)))/8 - (15*b^3*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)
))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b^2) - (S
qrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x]
)/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)
])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2
*b)])))/(32*(a - b)^3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3*(1 + ((a
- b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2)))/(5*(a + b)*(a^2
```

```
 - b^2)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) + (3*a*b^
2*((4*Sqrt[2]*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(3/2)*Sqrt[b/(a + b
) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x]
)/(a - b)))/(2*b))^(5/2)*((3/(4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*
x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a
 - b)))/(2*b))^(-1))/2 + (3*b^2*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/
(a - b)))/b - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) -
 (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c
 + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*
x])/(a - b)))/(2*b)])))/(8*(a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a -
b))^2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2)))/
(3*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-(
(a*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a - b)) + b^2/(a - b))*(-(((-((
a*b)/(a + b)) - b^2/(a + b))*((-2*(-((a*b)/(a + b)) - b^2/(a + b))*ArcTan[(
Sqrt[(a*b)/(a + b) + b^2/(a + b)]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a -
 b)])/(Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x
])/(a + b)])])/(b*Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[(a*b)/(a + b) +
 b^2/(a + b)]) + (2*Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b
*Sin[c + d*x])/(a - b)])/(Sqrt[a + b]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a
+ b)])])/(b*Sqrt[a + b])))/b) + (2*Sqrt[2]*(a - b)*Sqrt[-(b/(a - b)) - (b*S
in[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a -
 b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)*((Sqrt[b]*ArcSi
nh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqr
t[b])])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*
(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)) + 1/
(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)))))/(b*(a
 + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b])))/b) + (4*S
qrt[2]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Si
n[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)
))/(2*b))^(5/2)*((3*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin
[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(4*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a
 - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c +
 d*x])/(a - b)))/(2*b))^(5/2)) + (3/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin
[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)))/(2*b))^(-1))/4))/((a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[
c + d*x])/(a + b)))/b])))/b))/(a^2 - b^2))/(((a*b)/(a - b) - b^2/(a - b))*(
(a*b)/(a + b) + b^2/(a + b))))))/(d*(1 - (a + b*Sin[c + d*x])/(a - b))^(3/2)
*(1 - (a + b*Sin[c + d*x])/(a + b))^(3/2))
```

Maple [B] time = 0.081, size = 385, normalized size = 3.

$$\frac{1}{db^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}+4\frac{(\tan(1/2\,dx+c/2))^2\,a}{db^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}-\frac{1}{db^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+4/d/b^3/(1+tan(1/2*
d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2*a-1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*t
an(1/2*d*x+1/2*c)+4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*a+6/d/b^4*arctan(tan(1
/2*d*x+1/2*c))*a^2-3/d/b^2*arctan(tan(1/2*d*x+1/2*c))+2/d/b^2/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a*tan(1/2*d*x+1/2*c)-2/d/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)/a*tan(1/2*d*x+1/2*c)+2/d/b^3/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*a^2-2/d/b/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)-6/d/b^4*a*(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.9217, size = 938, normalized size = 7.33

$$\left[\frac{b^3\cos(dx+c)^3+3\left(2a^3-ab^2\right)dx+3\left(ab\sin(dx+c)+a^2\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2\left(a\cos(d\right.}{b^2\cos(dx+c)^2-2ab\sin(dx+\right.}}{2\left(b^5d\sin(dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(b^3*cos(d*x + c)^3 + 3*(2*a^3 - a*b^2)*d*x + 3*(a*b*sin(d*x + c) + a^
2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
- a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 +
b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 3*(2*a^2*b -
 b^3)*cos(d*x + c) + 3*(a*b^2*cos(d*x + c) + (2*a^2*b - b^3)*d*x)*sin(d*x +
 c))/(b^5*d*sin(d*x + c) + a*b^4*d), 1/2*(b^3*cos(d*x + c)^3 + 3*(2*a^3 - a
*b^2)*d*x + 6*(a*b*sin(d*x + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
 c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 3*(2*a^2*b - b^3)*cos(d*x + c) +
 3*(a*b^2*cos(d*x + c) + (2*a^2*b - b^3)*d*x)*sin(d*x + c))/(b^5*d*sin(d*x
+ c) + a*b^4*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11879, size = 317, normalized size = 2.48

$$\frac{\frac{3\left(2a^2-b^2\right)(dx+c)}{b^4}-\frac{12\left(a^3-ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4a\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2b^3}+\frac{4\left(\cdots\right.}{\left(\cdots\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(2*a^2 - b^2)*(d*x + c)/b^4 - 12*(a^3 - a*b^2)*(pi*floor(1/2*(d*x +
c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))
/(sqrt(a^2 - b^2)*b^4) + 2*(b*tan(1/2*d*x + 1/2*c)^3 + 4*a*tan(1/2*d*x + 1/
2*c)^2 - b*tan(1/2*d*x + 1/2*c) + 4*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^3)
```

```
 + 4*(a^2*b*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c) + a^3 - a*b^2)/
((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a*b^3))/d
```

3.446 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=84

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\sqrt{a^2-b^2}}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}-\frac{x}{b^2}$$

```
[Out] -(x/b^2) + (2*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*Sqrt
[a^2 - b^2]*d) - Cos[c + d*x]/(b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.110158, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2693, 2735, 2660, 618, 204}

$$\frac{2a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\sqrt{a^2-b^2}}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}-\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```



```
[Out] -(x/b^2) + (2*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*Sqrt
[a^2 - b^2]*d) - Cos[c + d*x]/(b*d*(a + b*Sin[c + d*x]))
```


Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= -\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}-\frac{\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}\\
&= -\frac{x}{b^2}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}+\frac{a\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^2}\\
&= -\frac{x}{b^2}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\
&= -\frac{x}{b^2}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}-\frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\
&= -\frac{x}{b^2}+\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}
\end{aligned}
$$

Mathematica [B] time = 5.30223, size = 494, normalized size = 5.88

$$\cos(c+dx)\left(\sqrt{a+b}\left(\sqrt{a-b}\sqrt{1-\sin(c+dx)}\left(b\left(b^2-a^2\right)\sqrt{-\frac{b(\sin(c+dx)-1)}{a+b}}\sqrt{\frac{b(\sin(c+dx)+1)}{b-a}}+2a\left(a\sqrt{-b^2}+\sqrt{-b}b^{3/2}\sin(c+\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]*(-2*a*(a - b)*b*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[c + d
*x]))/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))])]*Sq
rt[1 - Sin[c + d*x]]*(a + b*Sin[c + d*x]) + Sqrt[a + b]*(2*ArcSinh[(Sqrt[a
- b]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(Sqrt[2]*Sqrt[b])]*Sqrt[-((b*
(-1 + Sin[c + d*x]))/(a + b))]*(a*(b^(5/2) + a*Sqrt[-b]*Sqrt[-b^2]) - b^2*(
a*Sqrt[b] + Sqrt[-b]*Sqrt[-b^2])*Sin[c + d*x]) + Sqrt[a - b]*Sqrt[1 - Sin[c
 + d*x]]*(b*(-a^2 + b^2)*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[(b*(
1 + Sin[c + d*x]))/(-a + b)] + 2*a*ArcTan[(Sqrt[b]*Sqrt[(b*(1 + Sin[c + d*x
]))/(-a + b)])/(Sqrt[-b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))])]*(a*Sqrt
[-b^2] + Sqrt[-b]*b^(3/2)*Sin[c + d*x])))))/((a - b)^(3/2)*b^2*(a + b)^(3/2
)*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[-(
(b*(1 + Sin[c + d*x]))/(a - b))]*(a + b*Sin[c + d*x]))
```

Maple [A] time = 0.072, size = 153, normalized size = 1.8

$$-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{db^2}-2\,\frac{\tan\left(1/2\,dx+c/2\right)}{d\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2a+2\,\tan\left(1/2\,dx+c/2\right)b+a\right)a}-2\,\frac{}{bd\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -2/d/b^2*arctan(tan(1/2*d*x+1/2*c))-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)/a*tan(1/2*d*x+1/2*c)-2/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)+2/d/b^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.76553, size = 852, normalized size = 10.14

$$\left[-\frac{2\left(a^2b-b^3\right)dx\sin\left(dx+c\right)+2\left(a^3-ab^2\right)dx+\left(ab\sin\left(dx+c\right)+a^2\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}{b^2\cos(dx+c)^2-}\right.}{2\left(\left(a^2b^3-b^5\right)d\sin\left(dx+c\right)+\left(a^3b^2-ab^4\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b - b^3)*d*x*sin(d*x + c) + 2*(a^3 - a*b^2)*d*x + (a*b*sin(d*
x + c) + a^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*si
n(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*s
qrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2
*(a^2*b - b^3)*cos(d*x + c))/((a^2*b^3 - b^5)*d*sin(d*x + c) + (a^3*b^2 - a
*b^4)*d), -((a^2*b - b^3)*d*x*sin(d*x + c) + (a^3 - a*b^2)*d*x + (a*b*sin(d
*x + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x + c))) + (a^2*b - b^3)*cos(d*x + c))/((a^2*b^3 - b^5)*d*sin(d*x +
 c) + (a^3*b^2 - a*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

[Out] Timed out

Giac [A] time = 1.10802, size = 170, normalized size = 2.02

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}b^2}-\frac{dx+c}{b^2}-\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*a/(sqrt(a^2 - b^2)*b^2) - (d*x + c)/b^2 - 2*(b*tan
(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c
) + a)*a*b))/d
```

3.447 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=130

$$-\frac{6ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\sec(c+dx)\left(3ab-\left(a^2+2b^2\right)\sin(c+dx)\right)}{d\left(a^2-b^2\right)^2}$$

```
[Out] (-6*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5
/2)*d) + (b*Sec[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (Sec[c + d
*x]*(3*a*b - (a^2 + 2*b^2)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Rubi [A] time = 0.20817, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2694, 2866, 12, 2660, 618, 204}

$$-\frac{6ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\sec(c+dx)\left(3ab-\left(a^2+2b^2\right)\sin(c+dx)\right)}{d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-6*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5
/2)*d) + (b*Sec[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (Sec[c + d
*x]*(3*a*b - (a^2 + 2*b^2)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^2(c+dx)(-a+2b\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$

$$= \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab-\left(a^2+2b^2\right)\sin(c+dx)\right)}{\left(a^2-b^2\right)^2 d} + \frac{\int -\frac{3ab^2}{a+b\sin(c+d}}{\left(a^2-b^2\right)}$$

$$= \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab-\left(a^2+2b^2\right)\sin(c+dx)\right)}{\left(a^2-b^2\right)^2 d} - \frac{\left(3ab^2\right)\int \frac{}{a+b\,\mathrm{s}}}{\left(a^2-b\right.}$$

$$= \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab-\left(a^2+2b^2\right)\sin(c+dx)\right)}{\left(a^2-b^2\right)^2 d} - \frac{\left(6ab^2\right)\mathrm{Subst}}{}$$

$$= \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab-\left(a^2+2b^2\right)\sin(c+dx)\right)}{\left(a^2-b^2\right)^2 d} + \frac{\left(12ab^2\right)\mathrm{Subs}}{}$$

$$= -\frac{6ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2} d} + \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab-\left(a^2+2\right.\right.}{\left(a^2-b^2\right)^2}$$

Mathematica [A] time = 1.10177, size = 162, normalized size = 1.25

$$\frac{-\frac{6ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{b^3\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\mathrm{s}\right.}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-6*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5
/2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
 + 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - (b^3*Cos[c + d*x]
)/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

Maple [A] time = 0.082, size = 222, normalized size = 1.7

$$-\frac{1}{d(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{1}{d(a-b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-2\,\frac{b^4\tan(1/2\,dx+c/2)}{d(a-b)^2(a+b)^2\left((\tan(1/2\,dx+c/2))^2a+2\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)-2/d*
b^4/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)/a*tan
(1/2*d*x+1/2*c)-2/d*b^3/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d
*x+1/2*c)*b+a)-6/d*b^2/(a-b)^2/(a+b)^2*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.8186, size = 1207, normalized size = 9.28

$$\left[-\frac{2\,a^4b-4\,a^2b^3+2\,b^5+2\left(a^4b+a^2b^3-2\,b^5\right)\cos(dx+c)^2+3\left(ab^3\cos(dx+c)\sin(dx+c)+a^2b^2\cos(dx+c)\right)\sqrt{-a^2+}}{2\left(\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right)d\cos(dx+c)\sin(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x +
c)^2 + 3*(a*b^3*cos(d*x + c)*sin(d*x + c) + a^2*b^2*cos(d*x + c))*sqrt(-a^2
```

```
 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^5 - 2*a^3*b^2 + a*
b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*si
n(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)), -(a^4*b
 - 2*a^2*b^3 + b^5 + (a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^2 - 3*(a*b^3*co
s(d*x + c)*sin(d*x + c) + a^2*b^2*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*
sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^5 - 2*a^3*b^2 + a*b^
4)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(
d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [B] time = 1.13913, size = 366, normalized size = 2.82

$$-\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab^2}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a^4\tan}{(a^5-2a^3b^2+ab^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/
2*c) + b)/sqrt(a^2 - b^2)))*a*b^2/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
 + (a^4*tan(1/2*d*x + 1/2*c)^3 + a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + b^4*tan(1
/2*d*x + 1/2*c)^3 + 3*a*b^3*tan(1/2*d*x + 1/2*c)^2 + a^4*tan(1/2*d*x + 1/2*
c) - 3*a^2*b^2*tan(1/2*d*x + 1/2*c) - b^4*tan(1/2*d*x + 1/2*c) - 2*a^3*b -
a*b^3)/((a^5 - 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d
```

```
*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)))/d
```

3.448 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=193

$$\frac{10ab^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{b\sec^3(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^2}+\frac{\sec(c+$$

```
[Out] (10*a*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7
/2)*d) + (b*Sec[c + d*x]^3)/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (Sec[c +
 d*x]^3*(5*a*b - (a^2 + 4*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^2*d) + (Sec[c
+ d*x]*(15*a*b^3 + (2*a^4 - 9*a^2*b^2 - 8*b^4)*Sin[c + d*x]))/(3*(a^2 - b^2
)^3*d)
```

Rubi [A] time = 0.36678, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2694, 2866, 12, 2660, 618, 204}

$$\frac{10ab^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{b\sec^3(c+dx)}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^2}+\frac{\sec(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (10*a*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7
/2)*d) + (b*Sec[c + d*x]^3)/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (Sec[c +
 d*x]^3*(5*a*b - (a^2 + 4*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^2*d) + (Sec[c
+ d*x]*(15*a*b^3 + (2*a^4 - 9*a^2*b^2 - 8*b^4)*Sin[c + d*x]))/(3*(a^2 - b^2
)^3*d)
```


Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
```

```
0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^4(c+dx)(-a+4b\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$

$$= \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \int \frac{\sec^2(c+dx)(\ldots}{}$$

$$= \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{\sec(c+dx)}{}$$

$$= \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{\sec(c+dx)}{}$$

$$= \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{\sec(c+dx)}{}$$

$$= \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5ab-\left(a^2+4b^2\right)\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{\sec(c+dx)}{}$$

$$= \frac{10ab^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{b\sec^3(c+dx)}{\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5ab-\left(a^2+\ldots\right.\right.}{3\left(a^2-b^2\right)\ldots}$$

Mathematica [A] time = 1.9094, size = 336, normalized size = 1.74

$$\frac{\frac{120ab^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{12b^5\cos(c+dx)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))} + \frac{4(2a+5b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((120*a*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(
7/2) + 1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (2*Sin[(c +
d*x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (4*(2*a + 5*
b)*Sin[(c + d*x)/2])/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2
```

```
*Sin[(c + d*x)/2])/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - 1/
((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(2*a - 5*b)*Sin[(c
 + d*x)/2])/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (12*b^5*Cos
[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])))/(12*d)
```

Maple [B] time = 0.118, size = 370, normalized size = 1.9

$$-\frac{1}{3\,d\,(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{1}{2\,d\,(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}-\frac{a}{d\,(a+b)^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-2\,\frac{}{d\,(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1
)^2-1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)*a-2/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)
*b-1/3/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)
+1)^2-1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)*a+2/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+
1)*b+2/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b
+a)/a*tan(1/2*d*x+1/2*c)+2/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*
tan(1/2*d*x+1/2*c)*b+a)+10/d*b^4/(a-b)^3/(a+b)^3*a/(a^2-b^2)^(1/2)*arctan(1
/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.38134, size = 1728, normalized size = 8.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + 2*(2*a^6*b - 11*a^4*b^3 +
a^2*b^5 + 8*b^7)*cos(d*x + c)^4 - 2*(a^6*b + 2*a^4*b^3 - 7*a^2*b^5 + 4*b^7)
*cos(d*x + c)^2 - 15*(a*b^5*cos(d*x + c)^3*sin(d*x + c) + a^2*b^4*cos(d*x +
 c)^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + (2*a^7 - 11*a^5*b^2 + 16*a^3*b^4 - 7*a*b^
6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^
7 + b^9)*d*cos(d*x + c)^3*sin(d*x + c) + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a
^3*b^6 + a*b^8)*d*cos(d*x + c)^3), -1/3*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7 + (2*a^6*b - 11*a^4*b^3 + a^2*b^5 + 8*b^7)*cos(d*x + c)^4 - (a^6*b + 2*a^
4*b^3 - 7*a^2*b^5 + 4*b^7)*cos(d*x + c)^2 + 15*(a*b^5*cos(d*x + c)^3*sin(d*
x + c) + a^2*b^4*cos(d*x + c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 +
 (2*a^7 - 11*a^5*b^2 + 16*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/
((a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^3*sin(d*x
 + c) + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x))**2, x)
```

Giac [B] time = 1.14284, size = 576, normalized size = 2.98

$$2\left(\frac{15\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab^4}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{3\left(b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+ab^5\right)}{\left(a^7-3a^5b^2+3a^3b^4-ab^6\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}-\frac{3a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(15*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*a*b^4/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sq
rt(a^2 - b^2)) + 3*(b^6*tan(1/2*d*x + 1/2*c) + a*b^5)/((a^7 - 3*a^5*b^2 + 3
*a^3*b^4 - a*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)
) - (3*a^4*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*b^
4*tan(1/2*d*x + 1/2*c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^4 + 18*a*b^3*tan(1/
2*d*x + 1/2*c)^4 - 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 18*a^2*b^2*tan(1/2*d*x +
1/2*c)^3 + 8*b^4*tan(1/2*d*x + 1/2*c)^3 - 24*a*b^3*tan(1/2*d*x + 1/2*c)^2 +
 3*a^4*tan(1/2*d*x + 1/2*c) - 9*a^2*b^2*tan(1/2*d*x + 1/2*c) - 6*b^4*tan(1/
2*d*x + 1/2*c) - 2*a^3*b + 14*a*b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(
tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

3.449 $\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=190

$$-\frac{3\left(2a^2-b^2\right)\sin^2(c+dx)}{2b^5d}+\frac{a\left(10a^2-9b^2\right)\sin(c+dx)}{b^6d}-\frac{6a\left(a^2-b^2\right)^2}{b^7d(a+b\sin(c+dx))}+\frac{\left(a^2-b^2\right)^3}{2b^7d(a+b\sin(c+dx))^2}-\frac{3\left(-6a^2b\right.}{}$$

```
[Out] (-3*(5*a^4 - 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(b^7*d) + (a*(10*a^2
 - 9*b^2)*Sin[c + d*x])/(b^6*d) - (3*(2*a^2 - b^2)*Sin[c + d*x]^2)/(2*b^5*d
) + (a*Sin[c + d*x]^3)/(b^4*d) - Sin[c + d*x]^4/(4*b^3*d) + (a^2 - b^2)^3/(
2*b^7*d*(a + b*Sin[c + d*x])^2) - (6*a*(a^2 - b^2)^2)/(b^7*d*(a + b*Sin[c +
 d*x]))
```

Rubi [A] time = 0.158582, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{3\left(2a^2-b^2\right)\sin^2(c+dx)}{2b^5d}+\frac{a\left(10a^2-9b^2\right)\sin(c+dx)}{b^6d}-\frac{6a\left(a^2-b^2\right)^2}{b^7d(a+b\sin(c+dx))}+\frac{\left(a^2-b^2\right)^3}{2b^7d(a+b\sin(c+dx))^2}-\frac{3\left(-6a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (-3*(5*a^4 - 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(b^7*d) + (a*(10*a^2
 - 9*b^2)*Sin[c + d*x])/(b^6*d) - (3*(2*a^2 - b^2)*Sin[c + d*x]^2)/(2*b^5*d
) + (a*Sin[c + d*x]^3)/(b^4*d) - Sin[c + d*x]^4/(4*b^3*d) + (a^2 - b^2)^3/(
2*b^7*d*(a + b*Sin[c + d*x])^2) - (6*a*(a^2 - b^2)^2)/(b^7*d*(a + b*Sin[c +
 d*x]))
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(10a^3\left(1-\frac{9b^2}{10a^2}\right)-3\left(2a^2-b^2\right)x+3ax^2-x^3-\frac{\left(a^2-b^2\right)^3}{(a+x)^3}+\frac{6a\left(a^2-b^2\right)^2}{(a+x)^2}-\frac{3\left(5a^4-6a^2b^2+b^4\right)}{a+x}\right.\right.}{b^7 d}$$

$$= -\frac{3\left(5a^4-6a^2b^2+b^4\right)\log(a+b\sin(c+dx))}{b^7 d}+\frac{a\left(10a^2-9b^2\right)\sin(c+dx)}{b^6 d}-\frac{3\left(2a^2-b^2\right)\sin}{2b^5 d}$$

Mathematica [A] time = 0.660965, size = 282, normalized size = 1.48

$$\frac{-2\left(2a^2b^4\sin^4(c+dx)-10ab^3\left(a^2-b^2\right)\sin^3(c+dx)+2b^2\sin^2(c+dx)\left(3\left(-6a^2b^2+5a^4+b^4\right)\log(a+b\sin(c+dx))+1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b^6*Cos[c + d*x]^6 + b^4*Cos[c + d*x]^4*(-a^2 + 3*b^2 + 2*a*b*Sin[c + d*x]
) - 2*((a^2 - b^2)*(19*a^4 - 16*a^2*b^2 - 3*b^4 + 6*a^2*(5*a^2 - b^2)*Log[a
 + b*Sin[c + d*x]]) + 2*a*b*(4*a^4 - 17*a^2*b^2 + 11*b^4 + 6*(5*a^4 - 6*a^2
*b^2 + b^4)*Log[a + b*Sin[c + d*x]])*Sin[c + d*x] + 2*b^2*(-13*a^4 + 10*a^2
*b^2 + 3*(5*a^4 - 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])*Sin[c + d*x]^2
- 10*a*b^3*(a^2 - b^2)*Sin[c + d*x]^3 + 2*a^2*b^4*Sin[c + d*x]^4))/(4*b^7*d
*(a + b*Sin[c + d*x])^2)
```

Maple [A] time = 0.098, size = 320, normalized size = 1.7

$$-\frac{(\sin(dx+c))^4}{4b^3d}+\frac{a(\sin(dx+c))^3}{b^4d}-3\frac{(\sin(dx+c))^2a^2}{db^5}+\frac{3(\sin(dx+c))^2}{2b^3d}+10\frac{a^3\sin(dx+c)}{db^6}-9\frac{a\sin(dx+c)}{b^4d}-15$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/4*sin(d*x+c)^4/b^3/d+a*sin(d*x+c)^3/b^4/d-3/d/b^5*sin(d*x+c)^2*a^2+3/2*s
in(d*x+c)^2/b^3/d+10/d/b^6*sin(d*x+c)*a^3-9*a*sin(d*x+c)/b^4/d-15/d/b^7*ln(
a+b*sin(d*x+c))*a^4+18/d/b^5*ln(a+b*sin(d*x+c))*a^2-3*ln(a+b*sin(d*x+c))/b^
3/d+1/2/d/b^7/(a+b*sin(d*x+c))^2*a^6-3/2/d/b^5/(a+b*sin(d*x+c))^2*a^4+3/2/d
/b^3/(a+b*sin(d*x+c))^2*a^2-1/2/b/d/(a+b*sin(d*x+c))^2-6/d*a^5/b^7/(a+b*sin
(d*x+c))+12/d*a^3/b^5/(a+b*sin(d*x+c))-6*a/b^3/d/(a+b*sin(d*x+c))
```

Maxima [A] time = 0.952573, size = 270, normalized size = 1.42

$$-\frac{\frac{2\left(11\,a^6-21\,a^4b^2+9\,a^2b^4+b^6+12\left(a^5b-2\,a^3b^3+ab^5\right)\sin(dx+c)\right)}{b^9\sin(dx+c)^2+2\,ab^8\sin(dx+c)+a^2b^7}+\frac{b^3\sin(dx+c)^4-4\,ab^2\sin(dx+c)^3+6\left(2\,a^2b-b^3\right)\sin(dx+c)^2-4\left(10\,a^3-9\,ab^2\right)\sin(dx+c)}{b^6}+\frac{12}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(11*a^6 - 21*a^4*b^2 + 9*a^2*b^4 + b^6 + 12*(a^5*b - 2*a^3*b^3 + a*
b^5)*sin(d*x + c))/(b^9*sin(d*x + c)^2 + 2*a*b^8*sin(d*x + c) + a^2*b^7) +
(b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*(2*a^2*b - b^3)*sin(d*x +
c)^2 - 4*(10*a^3 - 9*a*b^2)*sin(d*x + c))/b^6 + 12*(5*a^4 - 6*a^2*b^2 + b^4
)*log(b*sin(d*x + c) + a)/b^7)/d
```

Fricas [A] time = 3.42958, size = 693, normalized size = 3.65

$$-\frac{8\,b^6\cos\left(dx+c\right)^6-176\,a^6+928\,a^4b^2-685\,a^2b^4+3\,b^6-8\left(5\,a^2b^4-3\,b^6\right)\cos\left(dx+c\right)^4-\left(544\,a^4b^2-560\,a^2b^4+51\,b^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*b^6*cos(d*x + c)^6 - 176*a^6 + 928*a^4*b^2 - 685*a^2*b^4 + 3*b^6 -
 8*(5*a^2*b^4 - 3*b^6)*cos(d*x + c)^4 - (544*a^4*b^2 - 560*a^2*b^4 + 51*b^6
)*cos(d*x + c)^2 - 96*(5*a^6 - a^4*b^2 - 5*a^2*b^4 + b^6 - (5*a^4*b^2 - 6*a
^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(5*a^5*b - 6*a^3*b^3 + a*b^5)*sin(d*x + c)
)*log(b*sin(d*x + c) + a) + 2*(8*a*b^5*cos(d*x + c)^4 + 64*a^5*b + 176*a^3*
```

```
b^3 - 205*a*b^5 - 80*(a^3*b^3 - a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(b^9*d
*cos(d*x + c)^2 - 2*a*b^8*d*sin(d*x + c) - (a^2*b^7 + b^9)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1362, size = 331, normalized size = 1.74

$$-\frac{\frac{12\left(5a^4-6a^2b^2+b^4\right)\log(|b\sin(dx+c)+a|)}{b^7}-\frac{2\left(45a^4b^2\sin(dx+c)^2-54a^2b^4\sin(dx+c)^2+9b^6\sin(dx+c)^2+78a^5b\sin(dx+c)-84a^3b^3\sin(dx+c)+6ab^5\sin(dx+c)+\cdots\right.}{(b\sin(dx+c)+a)^2b^7}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(12*(5*a^4 - 6*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/b^7 - 2*(45
*a^4*b^2*sin(d*x + c)^2 - 54*a^2*b^4*sin(d*x + c)^2 + 9*b^6*sin(d*x + c)^2
+ 78*a^5*b*sin(d*x + c) - 84*a^3*b^3*sin(d*x + c) + 6*a*b^5*sin(d*x + c) +
34*a^6 - 33*a^4*b^2 - b^6)/((b*sin(d*x + c) + a)^2*b^7) + (b^9*sin(d*x + c)
^4 - 4*a*b^8*sin(d*x + c)^3 + 12*a^2*b^7*sin(d*x + c)^2 - 6*b^9*sin(d*x + c
)^2 - 40*a^3*b^6*sin(d*x + c) + 36*a*b^8*sin(d*x + c))/b^12)/d
```

3.450 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$-\frac{\left(a^2-b^2\right)^2}{2b^5d(a+b\sin(c+dx))^2}+\frac{4a\left(a^2-b^2\right)}{b^5d(a+b\sin(c+dx))}+\frac{2\left(3a^2-b^2\right)\log(a+b\sin(c+dx))}{b^5d}-\frac{3a\sin(c+dx)}{b^4d}+\frac{\sin^2(c+dx)}{2b^3d}$$

```
[Out] (2*(3*a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^5*d) - (3*a*Sin[c + d*x])/(b^4
*d) + Sin[c + d*x]^2/(2*b^3*d) - (a^2 - b^2)^2/(2*b^5*d*(a + b*Sin[c + d*x]
)^2) + (4*a*(a^2 - b^2))/(b^5*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.104938, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{\left(a^2-b^2\right)^2}{2b^5d(a+b\sin(c+dx))^2}+\frac{4a\left(a^2-b^2\right)}{b^5d(a+b\sin(c+dx))}+\frac{2\left(3a^2-b^2\right)\log(a+b\sin(c+dx))}{b^5d}-\frac{3a\sin(c+dx)}{b^4d}+\frac{\sin^2(c+dx)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*(3*a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^5*d) - (3*a*Sin[c + d*x])/(b^4
*d) + Sin[c + d*x]^2/(2*b^3*d) - (a^2 - b^2)^2/(2*b^5*d*(a + b*Sin[c + d*x]
)^2) + (4*a*(a^2 - b^2))/(b^5*d*(a + b*Sin[c + d*x]))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a+x+\frac{\left(a^2-b^2\right)^2}{(a+x)^3}-\frac{4\left(a^3-ab^2\right)}{(a+x)^2}+\frac{2\left(3a^2-b^2\right)}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{2\left(3a^2-b^2\right)\log(a+b\sin(c+dx))}{b^5 d}-\frac{3a\sin(c+dx)}{b^4 d}+\frac{\sin^2(c+dx)}{2b^3 d}-\frac{\left(a^2-b^2\right)^2}{2b^5 d(a+b\sin(c+dx)}$$

Mathematica [A] time = 0.959813, size = 143, normalized size = 1.13

$$\frac{2\left(b^2-a^2\right)\left(-\frac{3a^2+4ab\sin(c+dx)+b^2}{2(a+b\sin(c+dx))^2}-\log(a+b\sin(c+dx))\right)+\frac{b^4\cos^4(c+dx)}{2(a+b\sin(c+dx))^2}+2a\left(\frac{(a-b)(a+b)}{a+b\sin(c+dx)}+2a\log(a+b\sin(c+dx))-\right.}{b^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((b^4*Cos[c + d*x]^4)/(2*(a + b*Sin[c + d*x])^2) + 2*a*(2*a*Log[a + b*Sin[c
 + d*x]] - b*Sin[c + d*x] + ((a - b)*(a + b))/(a + b*Sin[c + d*x])) + 2*(-a
^2 + b^2)*(-Log[a + b*Sin[c + d*x]] - (3*a^2 + b^2 + 4*a*b*Sin[c + d*x])/(2
*(a + b*Sin[c + d*x])^2)))/(b^5*d)
```

Maple [A] time = 0.093, size = 183, normalized size = 1.4

$$\frac{(\sin(dx+c))^2}{2b^3d}-3\frac{a\sin(dx+c)}{b^4d}+6\frac{\ln(a+b\sin(dx+c))a^2}{db^5}-2\frac{\ln(a+b\sin(dx+c))}{b^3d}-\frac{a^4}{2db^5(a+b\sin(dx+c))^2}+\frac{}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2*sin(d*x+c)^2/b^3/d-3*a*sin(d*x+c)/b^4/d+6/d/b^5*ln(a+b*sin(d*x+c))*a^2-
2*ln(a+b*sin(d*x+c))/b^3/d-1/2/d/b^5/(a+b*sin(d*x+c))^2*a^4+1/d/b^3/(a+b*si
n(d*x+c))^2*a^2-1/2/b/d/(a+b*sin(d*x+c))^2+4/d*a^3/b^5/(a+b*sin(d*x+c))-4*a
/b^3/d/(a+b*sin(d*x+c))
```

Maxima [A] time = 0.956052, size = 177, normalized size = 1.39

$$\frac{\frac{7\,a^4-6\,a^2b^2-b^4+8\,\left(a^3b-ab^3\right)\sin(dx+c)}{b^7\sin(dx+c)^2+2\,ab^6\sin(dx+c)+a^2b^5}+\frac{b\sin(dx+c)^2-6\,a\sin(dx+c)}{b^4}+\frac{4\,\left(3\,a^2-b^2\right)\log(b\sin(dx+c)+a)}{b^5}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((7*a^4 - 6*a^2*b^2 - b^4 + 8*(a^3*b - a*b^3)*sin(d*x + c))/(b^7*sin(d*
x + c)^2 + 2*a*b^6*sin(d*x + c) + a^2*b^5) + (b*sin(d*x + c)^2 - 6*a*sin(d*
x + c))/b^4 + 4*(3*a^2 - b^2)*log(b*sin(d*x + c) + a)/b^5)/d
```

Fricas [A] time = 2.92335, size = 473, normalized size = 3.72

$$-\frac{2\,b^4\cos(dx+c)^4+14\,a^4-35\,a^2b^2-b^4+\left(22\,a^2b^2-3\,b^4\right)\cos(dx+c)^2+8\,(3\,a^4+2\,a^2b^2-b^4-\left(3\,a^2b^2-b^4\right)\cos(dx}{4\,(b^7d\cos(dx+c)^2-2\,ab^6d\,\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^4*cos(d*x + c)^4 + 14*a^4 - 35*a^2*b^2 - b^4 + (22*a^2*b^2 - 3*b^
4)*cos(d*x + c)^2 + 8*(3*a^4 + 2*a^2*b^2 - b^4 - (3*a^2*b^2 - b^4)*cos(d*x
+ c)^2 + 2*(3*a^3*b - a*b^3)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 2*(4*a
*b^3*cos(d*x + c)^2 + 2*a^3*b - 13*a*b^3)*sin(d*x + c))/(b^7*d*cos(d*x + c)
^2 - 2*a*b^6*d*sin(d*x + c) - (a^2*b^5 + b^7)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14714, size = 192, normalized size = 1.51

$$\frac{\frac{4\left(3\,a^2-b^2\right)\log\left(|b\sin\left(dx+c\right)+a|\right)}{b^5}+\frac{b^3\sin\left(dx+c\right)^2-6\,ab^2\sin\left(dx+c\right)}{b^6}-\frac{18\,a^2b^2\sin\left(dx+c\right)^2-6\,b^4\sin\left(dx+c\right)^2+28\,a^3b\sin\left(dx+c\right)-4\,ab^3\sin\left(dx+c\right)+11\,a^4+b^4}{\left(b\sin\left(dx+c\right)+a\right)^2b^5}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*(3*a^2 - b^2)*log(abs(b*sin(d*x + c) + a))/b^5 + (b^3*sin(d*x + c)^2
 - 6*a*b^2*sin(d*x + c))/b^6 - (18*a^2*b^2*sin(d*x + c)^2 - 6*b^4*sin(d*x +
 c)^2 + 28*a^3*b*sin(d*x + c) - 4*a*b^3*sin(d*x + c) + 11*a^4 + b^4)/((b*si
n(d*x + c) + a)^2*b^5))/d
```

3.451 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=72

$$\frac{a^2-b^2}{2b^3d(a+b\sin(c+dx))^2}-\frac{2a}{b^3d(a+b\sin(c+dx))}-\frac{\log(a+b\sin(c+dx))}{b^3d}$$

```
[Out] -(Log[a + b*Sin[c + d*x]]/(b^3*d)) + (a^2 - b^2)/(2*b^3*d*(a + b*Sin[c + d*
x])^2) - (2*a)/(b^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.071842, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{a^2-b^2}{2b^3d(a+b\sin(c+dx))^2}-\frac{2a}{b^3d(a+b\sin(c+dx))}-\frac{\log(a+b\sin(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(Log[a + b*Sin[c + d*x]]/(b^3*d)) + (a^2 - b^2)/(2*b^3*d*(a + b*Sin[c + d*
x])^2) - (2*a)/(b^3*d*(a + b*Sin[c + d*x]))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{-a^2+b^2}{(a+x)^3}+\frac{2a}{(a+x)^2}\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= -\frac{\log(a+b\sin(c+dx))}{b^3d}+\frac{a^2-b^2}{2b^3d(a+b\sin(c+dx))^2}-\frac{2a}{b^3d(a+b\sin(c+dx))}$$

Mathematica [A] time = 0.127896, size = 55, normalized size = 0.76

$$-\frac{\frac{3a^2+4ab\sin(c+dx)+b^2}{2(a+b\sin(c+dx))^2}+\log(a+b\sin(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((Log[a + b*Sin[c + d*x]] + (3*a^2 + b^2 + 4*a*b*Sin[c + d*x])/(2*(a + b*S
in[c + d*x])^2))/(b^3*d))
```

Maple [A] time = 0.092, size = 85, normalized size = 1.2

$$-\frac{\ln(a+b\sin(dx+c))}{b^3d}+\frac{a^2}{2\,b^3d\,(a+b\sin(dx+c))^2}-\frac{1}{2\,bd\,(a+b\sin(dx+c))^2}-2\,\frac{a}{b^3d\,(a+b\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -ln(a+b*sin(d*x+c))/b^3/d+1/2/d/b^3/(a+b*sin(d*x+c))^2*a^2-1/2/b/d/(a+b*sin
(d*x+c))^2-2*a/b^3/d/(a+b*sin(d*x+c))
```

Maxima [A] time = 0.954816, size = 103, normalized size = 1.43

$$-\frac{\frac{4\,ab\sin(dx+c)+3\,a^2+b^2}{b^5\sin(dx+c)^2+2\,ab^4\sin(dx+c)+a^2b^3}+\frac{2\,\log(b\sin(dx+c)+a)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*b*sin(d*x + c) + 3*a^2 + b^2)/(b^5*sin(d*x + c)^2 + 2*a*b^4*sin(
d*x + c) + a^2*b^3) + 2*log(b*sin(d*x + c) + a)/b^3)/d
```

Fricas [A] time = 2.77337, size = 257, normalized size = 3.57

$$\frac{4\,ab\sin\left(dx+c\right)+3\,a^{2}+b^{2}-2\left(b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}\right)\log\left(b\sin\left(dx+c\right)+a\right)}{2\left(b^{5}d\cos\left(dx+c\right)^{2}-2\,ab^{4}d\sin\left(dx+c\right)-\left(a^{2}b^{3}+b^{5}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*sin(d*x + c) + 3*a^2 + b^2 - 2*(b^2*cos(d*x + c)^2 - 2*a*b*sin(d
*x + c) - a^2 - b^2)*log(b*sin(d*x + c) + a))/(b^5*d*cos(d*x + c)^2 - 2*a*b
^4*d*sin(d*x + c) - (a^2*b^3 + b^5)*d)
```

Sympy [A] time = 2.74073, size = 670, normalized size = 9.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((zoo*x*cos(c)**3/sin(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-l
og(sin(c + d*x))/d - cos(c + d*x)**2/(2*d*sin(c + d*x)**2))/b**3, Eq(a, 0))
, ((2*sin(c + d*x)**3/(3*d) + sin(c + d*x)*cos(c + d*x)**2/d)/a**3, Eq(b, 0
)), (x*cos(c)**3/(a + b*sin(c))**3, Eq(d, 0)), (-2*a**4*log(a/b + sin(c + d
*x))/(2*a**4*b**3*d + 4*a**3*b**4*d*sin(c + d*x) + 2*a**2*b**5*d*sin(c + d*
x)**2) - a**4/(2*a**4*b**3*d + 4*a**3*b**4*d*sin(c + d*x) + 2*a**2*b**5*d*s
in(c + d*x)**2) - 4*a**3*b*log(a/b + sin(c + d*x))*sin(c + d*x)/(2*a**4*b**
3*d + 4*a**3*b**4*d*sin(c + d*x) + 2*a**2*b**5*d*sin(c + d*x)**2) - 2*a**2*
b**2*log(a/b + sin(c + d*x))*sin(c + d*x)**2/(2*a**4*b**3*d + 4*a**3*b**4*d
*sin(c + d*x) + 2*a**2*b**5*d*sin(c + d*x)**2) + 2*a**2*b**2*sin(c + d*x)**
2/(2*a**4*b**3*d + 4*a**3*b**4*d*sin(c + d*x) + 2*a**2*b**5*d*sin(c + d*x)*
```

```
*2) + 2*a*b**3*sin(c + d*x)**3/(2*a**4*b**3*d + 4*a**3*b**4*d*sin(c + d*x)
+ 2*a**2*b**5*d*sin(c + d*x)**2) + 2*a*b**3*sin(c + d*x)*cos(c + d*x)**2/(2
*a**4*b**3*d + 4*a**3*b**4*d*sin(c + d*x) + 2*a**2*b**5*d*sin(c + d*x)**2)
+ b**4*sin(c + d*x)**4/(2*a**4*b**3*d + 4*a**3*b**4*d*sin(c + d*x) + 2*a**2
*b**5*d*sin(c + d*x)**2) + b**4*sin(c + d*x)**2*cos(c + d*x)**2/(2*a**4*b**
3*d + 4*a**3*b**4*d*sin(c + d*x) + 2*a**2*b**5*d*sin(c + d*x)**2), True))
```

Giac [A] time = 1.12366, size = 84, normalized size = 1.17

$$-\frac{\frac{2\,\log\left(|b\sin\left(dx+c\right)+a|\right)}{b^3}+\frac{4\,a\sin\left(dx+c\right)+\frac{3\,a^2+b^2}{b}}{\left(b\sin\left(dx+c\right)+a\right)^2b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(b*sin(d*x + c) + a))/b^3 + (4*a*sin(d*x + c) + (3*a^2 + b^2
)/b)/((b*sin(d*x + c) + a)^2*b^2))/d
```

3.452 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2bd(a+b\sin(c+dx))^2}$$

```
[Out] -1/(2*b*d*(a + b*Sin[c + d*x])^2)
```

Rubi [A] time = 0.0263888, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 32}

$$-\frac{1}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/(2*b*d*(a + b*Sin[c + d*x])^2)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{1}{2bd(a+b\sin(c+dx))^2}\end{aligned}$$

Mathematica [A] time = 0.0244853, size = 22, normalized size = 1.

$$-\frac{1}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/(2*b*d*(a + b*Sin[c + d*x])^2)
```

Maple [A] time = 0.026, size = 21, normalized size = 1.

$$-\frac{1}{2\,bd\left(a+b\sin\left(dx+c\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/b/d/(a+b*sin(d*x+c))^2
```

Maxima [A] time = 0.941628, size = 27, normalized size = 1.23

$$-\frac{1}{2\left(b\sin\left(dx+c\right)+a\right)^{2}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*sin(d*x + c) + a)^2*b*d)
```

Fricas [B] time = 2.4696, size = 96, normalized size = 4.36

$$\frac{1}{2\left(b^{3}d\cos\left(dx+c\right)^{2}-2\,ab^{2}d\sin\left(dx+c\right)-\left(a^{2}b+b^{3}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2/(b^3*d*cos(d*x + c)^2 - 2*a*b^2*d*sin(d*x + c) - (a^2*b + b^3)*d)
```

Sympy [A] time = 2.03525, size = 73, normalized size = 3.32

$$\begin{cases} \frac{x\cos(c)}{a^3} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sin(c+dx)}{a^3 d} & \text{for } b = 0 \\ \frac{x\cos(c)}{(a+b\sin(c))^3} & \text{for } d = 0 \\ -\frac{1}{2a^2bd+4ab^2d\sin(c+dx)+2b^3d\sin^2(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((x*cos(c)/a**3, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**3*d), Eq(
b, 0)), (x*cos(c)/(a + b*sin(c))**3, Eq(d, 0)), (-1/(2*a**2*b*d + 4*a*b**2*
d*sin(c + d*x) + 2*b**3*d*sin(c + d*x)**2), True))
```

Giac [A] time = 1.10792, size = 27, normalized size = 1.23

$$-\frac{1}{2\,(b\sin(dx+c)+a)^2 bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*sin(d*x + c) + a)^2*b*d)
```

3.453 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$\frac{2ab}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{b\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\log(1-\sin(c+}{2d(a+b)^3}$$

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^3*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)^3
*d) - (b*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) + b/(2*(a
^2 - b^2)*d*(a + b*Sin[c + d*x])^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Sin[c
 + d*x]))
```

Rubi [A] time = 0.153961, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2668, 710, 801}

$$\frac{2ab}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{b\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\log(1-\sin(c+}{2d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^3*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)^3
*d) - (b*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) + b/(2*(a
^2 - b^2)*d*(a + b*Sin[c + d*x])^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Sin[c
 + d*x]))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
```

```
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{(a+x)^3\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{b\,\text{Subst}\left(\int \frac{a-x}{(a+x)^2\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{\left(a^2-b^2\right)d}$$

$$= \frac{b}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{b\,\text{Subst}\left(\int \left(\frac{a-b}{2b(a+b)^2(b-x)} - \frac{2a}{(a-b)(a+b)(a+x)^2} + \frac{-3a^2-b^2}{(a-b)^2(a+b)^2(a+}\right.\right.}{\left(a^2-b^2\right)d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^3d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^3d} - \frac{b\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^3 d} + \frac{}{2\left(a^2\right.}$$

Mathematica [A] time = 0.549629, size = 135, normalized size = 0.93

$$\frac{b\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))^2} - \frac{2\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} + \frac{4a}{(a-b)^2(a+b)^2(a+b\sin(c+dx))} - \frac{\log(1-\sin(c+dx))}{b(a+b)^3} + \frac{\log(\sin(c+dx)+1)}{b(a-b)^3}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(-(Log[1 - Sin[c + d*x]]/(b*(a + b)^3)) + Log[1 + Sin[c + d*x]]/((a - b)
^3*b) - (2*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a + b)^3) + 1
/((a^2 - b^2)*(a + b*Sin[c + d*x])^2) + (4*a)/((a - b)^2*(a + b)^2*(a + b*S
in[c + d*x]))))/(2*d)
```

Maple [A] time = 0.119, size = 166, normalized size = 1.1

$$\frac{b}{2\,d\,(a+b)\,(a-b)\,(a+b\sin(dx+c))^{2}}+2\,\frac{ab}{d\,(a+b)^{2}\,(a-b)^{2}\,(a+b\sin(dx+c))}-3\,\frac{b\ln(a+b\sin(dx+c))\,a^{2}}{d\,(a+b)^{3}\,(a-b)^{3}}-\frac{b^{3}\ln(a\cdots}{d\,(a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d*b/(a+b)/(a-b)/(a+b*sin(d*x+c))^2+2/d*a*b/(a+b)^2/(a-b)^2/(a+b*sin(d*x
+c))-3/d*b/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))*a^2-1/d*b^3/(a+b)^3/(a-b)^3*l
n(a+b*sin(d*x+c))-1/2/d/(a+b)^3*ln(sin(d*x+c)-1)+1/2*ln(1+sin(d*x+c))/(a-b)
^3/d
```

Maxima [A] time = 0.971178, size = 301, normalized size = 2.08

$$-\frac{\frac{2\left(3\,a^{2}b+b^{3}\right)\log(b\sin(dx+c)+a)}{a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}}-\frac{4\,ab^{2}\sin(dx+c)+5\,a^{2}b-b^{3}}{a^{6}-2\,a^{4}b^{2}+a^{2}b^{4}+\left(a^{4}b^{2}-2\,a^{2}b^{4}+b^{6}\right)\sin(dx+c)^{2}+2\left(a^{5}b-2\,a^{3}b^{3}+ab^{5}\right)\sin(dx+c)}-\frac{\log(\sin(dx+c)+1)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{\log(\sin(dx+c)-\cdots}{a^{3}+3\,a^{2}b+3\,ab^{2}\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(3*a^2*b + b^3)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) - (4*a*b^2*sin(d*x + c) + 5*a^2*b - b^3)/(a^6 - 2*a^4*b^2 + a^2*b^
4 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b
^5)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) +
 log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/d
```

Fricas [B] time = 3.65521, size = 999, normalized size = 6.89

$$-\frac{5\,a^{4}b-6\,a^{2}b^{3}+b^{5}-2\left(3\,a^{4}b+4\,a^{2}b^{3}+b^{5}-\left(3\,a^{2}b^{3}+b^{5}\right)\cos(dx+c)^{2}+2\left(3\,a^{3}b^{2}+ab^{4}\right)\sin(dx+c)\right)\log(b\sin(dx+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(5*a^4*b - 6*a^2*b^3 + b^5 - 2*(3*a^4*b + 4*a^2*b^3 + b^5 - (3*a^2*b^3
 + b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 + a*b^4)*sin(d*x + c))*log(b*sin(d*x
+ c) + a) + (a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b
^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a
^2*b^3 + a*b^4)*sin(d*x + c))*log(sin(d*x + c) + 1) - (a^5 - 3*a^4*b + 4*a^
3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*c
os(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*sin(d*x + c))*log
(-sin(d*x + c) + 1) + 4*(a^3*b^2 - a*b^4)*sin(d*x + c))/((a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d*sin(d*x + c) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 1.16443, size = 327, normalized size = 2.26

$$-\frac{\frac{2\left(3a^2b^2+b^4\right)\log(|b\sin(dx+c)+a|)}{a^6b-3a^4b^3+3a^2b^5-b^7}-\frac{\log(|\sin(dx+c)+1|)}{a^3-3a^2b+3ab^2-b^3}+\frac{\log(|\sin(dx+c)-1|)}{a^3+3a^2b+3ab^2+b^3}-\frac{9a^2b^3\sin(dx+c)^2+3b^5\sin(dx+c)^2+22a^3b^2\sin(dx+c)+2ab^4\sin(dx+c)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)(b\sin(dx+c)+a)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(3*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 +
 3*a^2*b^5 - b^7) - log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b
^3) + log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (9*a^2*b
^3*sin(d*x + c)^2 + 3*b^5*sin(d*x + c)^2 + 22*a^3*b^2*sin(d*x + c) + 2*a*b^
4*sin(d*x + c) + 14*a^4*b - 3*a^2*b^3 + b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*(b*sin(d*x + c) + a)^2))/d
```

3.454 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=226

$$-\frac{ab\left(a^2+11b^2\right)}{2d\left(a^2-b^2\right)^3\left(a+b\sin(c+dx)\right)}-\frac{b\left(a^2+2b^2\right)}{2d\left(a^2-b^2\right)^2\left(a+b\sin(c+dx)\right)^2}+\frac{2b^3\left(5a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{\sec^2(c+}{2d\left(a^2-\right.}$$

```
[Out] -((a + 4*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^4*d) + ((a - 4*b)*Log[1 + Sin
[c + d*x]])/(4*(a - b)^4*d) + (2*b^3*(5*a^2 + b^2)*Log[a + b*Sin[c + d*x]])
/((a^2 - b^2)^4*d) - (b*(a^2 + 2*b^2))/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*
x])^2) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[
c + d*x])^2) - (a*b*(a^2 + 11*b^2))/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])
)
```

Rubi [A] time = 0.276535, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2668, 741, 801}

$$-\frac{ab\left(a^2+11b^2\right)}{2d\left(a^2-b^2\right)^3\left(a+b\sin(c+dx)\right)}-\frac{b\left(a^2+2b^2\right)}{2d\left(a^2-b^2\right)^2\left(a+b\sin(c+dx)\right)^2}+\frac{2b^3\left(5a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{\sec^2(c+}{2d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((a + 4*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^4*d) + ((a - 4*b)*Log[1 + Sin
[c + d*x]])/(4*(a - b)^4*d) + (2*b^3*(5*a^2 + b^2)*Log[a + b*Sin[c + d*x]])
/((a^2 - b^2)^4*d) - (b*(a^2 + 2*b^2))/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*
x])^2) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[
c + d*x])^2) - (a*b*(a^2 + 11*b^2))/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])
)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{1}{(a+x)^3\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{b\,\text{Subst}\left(\int \frac{a^2-4b^2+3ax}{(a+x)^3\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{b\,\text{Subst}\left(\int\left(\frac{(a-b)(a+4b)}{2b(a+b)^3(b-x)} + \frac{2\left(a^2+2b^2\right)}{(a-b)(a+b)(a+x)^3} + \frac{a\left(a^2+11b\right.}{(a-b)^2(a+b)^2(}\right.\right.}{2\left(a\right.}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^4d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^4d} + \frac{2b^3\left(5a^2+b^2\right)\log(a+b\,\text{s}}{\left(a^2-b^2\right)^4d}$$

Mathematica [A] time = 4.10495, size = 283, normalized size = 1.25

$$\frac{b\left(a^2+2b^2\right)\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))^2} - \frac{2\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} + \frac{4a}{(a-b)^2(a+b)^2(a+b\sin(c+dx))} - \frac{\log(1-\sin(c+dx))}{b(a+b)^3} + \frac{\log(\sin(c+dx)+1)}{b(a-b)^3}\right)+}{2d\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2 + b*(a^2 + 2*
b^2)*(-(Log[1 - Sin[c + d*x]]/(b*(a + b)^3)) + Log[1 + Sin[c + d*x]]/((a -
b)^3*b) - (2*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a + b)^3) +
 1/((a^2 - b^2)*(a + b*Sin[c + d*x])^2) + (4*a)/((a - b)^2*(a + b)^2*(a + b
*Sin[c + d*x]))) + (3*a*(Log[1 - Sin[c + d*x]]/(a + b)^2 - Log[1 + Sin[c +
d*x]]/(a - b)^2 + (4*a*b*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + (
2*b)/((-a^2 + b^2)*(a + b*Sin[c + d*x]))))/2)/(2*(-a^2 + b^2)*d)
```

Maple [A] time = 0.141, size = 258, normalized size = 1.1

$$-\frac{b^3}{2\,d\,(a+b)^2\,(a-b)^2\,(a+b\sin(dx+c))^2}-4\,\frac{ab^3}{d\,(a+b)^3\,(a-b)^3\,(a+b\sin(dx+c))}+10\,\frac{b^3\ln(a+b\sin(dx+c))\,a^2}{d\,(a+b)^4\,(a-b)^4}+2\,\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*b^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))^2-4/d*a*b^3/(a+b)^3/(a-b)^3/(a+
b*sin(d*x+c))+10/d*b^3/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*a^2+2/d*b^5/(a+b)
^4/(a-b)^4*ln(a+b*sin(d*x+c))-1/4/d/(a+b)^3/(sin(d*x+c)-1)-1/4/d/(a+b)^4*ln
(sin(d*x+c)-1)*a-1/d/(a+b)^4*ln(sin(d*x+c)-1)*b-1/4/d/(a-b)^3/(1+sin(d*x+c)
)+1/4/d/(a-b)^4*ln(1+sin(d*x+c))*a-1/d/(a-b)^4*ln(1+sin(d*x+c))*b
```

Maxima [B] time = 1.02886, size = 591, normalized size = 2.62

$$\frac{\frac{8\left(5\,a^2b^3+b^5\right)\log(b\sin(dx+c)+a)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}+\frac{(a-4\,b)\log(\sin(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{(a+4\,b)\log(\sin(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{2\left(3\,a^4b+10\,a^2b^3-b^5-(a\right.}{a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6-\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)\sin(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*(5*a^2*b^3 + b^5)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^4*b
^4 - 4*a^2*b^6 + b^8) + (a - 4*b)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b + 6*
a^2*b^2 - 4*a*b^3 + b^4) - (a + 4*b)*log(sin(d*x + c) - 1)/(a^4 + 4*a^3*b +
 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(3*a^4*b + 10*a^2*b^3 - b^5 - (a^3*b^2 + 11
*a*b^4)*sin(d*x + c)^3 - 2*(a^4*b + 6*a^2*b^3 - b^5)*sin(d*x + c)^2 - (a^5
- 3*a^3*b^2 - 10*a*b^4)*sin(d*x + c))/(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^
6 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*sin(d*x + c)^4 - 2*(a^7*b - 3*a
```

```
^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(d*x + c)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 -
 4*a^2*b^6 + b^8)*sin(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7
)*sin(d*x + c)))/d
```

Fricas [B] time = 5.52251, size = 1577, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + 4*(a^6*b + 5*a^4*b^3 - 7*a^2
*b^5 + b^7)*cos(d*x + c)^2 + 8*((5*a^2*b^5 + b^7)*cos(d*x + c)^4 - 2*(5*a^3
*b^4 + a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (5*a^4*b^3 + 6*a^2*b^5 + b^7)*c
os(d*x + c)^2)*log(b*sin(d*x + c) + a) + ((a^5*b^2 - 10*a^3*b^4 - 20*a^2*b^
5 - 15*a*b^6 - 4*b^7)*cos(d*x + c)^4 - 2*(a^6*b - 10*a^4*b^3 - 20*a^3*b^4 -
 15*a^2*b^5 - 4*a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (a^7 - 9*a^5*b^2 - 20*
a^4*b^3 - 25*a^3*b^4 - 24*a^2*b^5 - 15*a*b^6 - 4*b^7)*cos(d*x + c)^2)*log(s
in(d*x + c) + 1) - ((a^5*b^2 - 10*a^3*b^4 + 20*a^2*b^5 - 15*a*b^6 + 4*b^7)*
cos(d*x + c)^4 - 2*(a^6*b - 10*a^4*b^3 + 20*a^3*b^4 - 15*a^2*b^5 + 4*a*b^6)
*cos(d*x + c)^2*sin(d*x + c) - (a^7 - 9*a^5*b^2 + 20*a^4*b^3 - 25*a^3*b^4 +
 24*a^2*b^5 - 15*a*b^6 + 4*b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*
(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (a^5*b^2 + 10*a^3*b^4 - 11*a*b^6)*co
s(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 +
 b^10)*d*cos(d*x + c)^4 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*
b^9)*d*cos(d*x + c)^2*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*
b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c + dx)}{(a + b\sin(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 1.69925, size = 558, normalized size = 2.47

$$\frac{\frac{8\left(5\,a^2b^4+b^6\right)\log(|b\sin(dx+c)+a|)}{a^8b-4\,a^6b^3+6\,a^4b^5-4\,a^2b^7+b^9}+\frac{(a-4\,b)\log(|\sin(dx+c)+1|)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{(a+4\,b)\log(|\sin(dx+c)-1|)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\left(10\,a^2b^3\sin(dx+c)^2+2\,b^5\sin(dx+c)^2-a^5\sin(dx+c)-2\,a^3b^2}{\left(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(8*(5*a^2*b^4 + b^6)*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3 +
6*a^4*b^5 - 4*a^2*b^7 + b^9) + (a - 4*b)*log(abs(sin(d*x + c) + 1))/(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (a + 4*b)*log(abs(sin(d*x + c) - 1))
/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(10*a^2*b^3*sin(d*x + c)^2
 + 2*b^5*sin(d*x + c)^2 - a^5*sin(d*x + c) - 2*a^3*b^2*sin(d*x + c) + 3*a*b
^4*sin(d*x + c) + 3*a^4*b - 12*a^2*b^3 - 3*b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b
^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)) - 2*(30*a^2*b^5*sin(d*x + c)^2
+ 6*b^7*sin(d*x + c)^2 + 68*a^3*b^4*sin(d*x + c) + 4*a*b^6*sin(d*x + c) + 3
9*a^4*b^3 - 4*a^2*b^5 + b^7)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^
8)*(b*sin(d*x + c) + a)^2))/d
```

3.455 $\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=328

$$-\frac{3ab\left(-6a^2b^2+a^4-27b^4\right)}{8d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}-\frac{3b\left(-5a^2b^2+a^4-4b^4\right)}{8d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2}-\frac{3b^5\left(7a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^5}-\frac{3\left(a^2+\right.}{}$$

```
[Out] (-3*(a^2 + 5*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^5*d) + (3*(a^2
 - 5*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^5*d) - (3*b^5*(7*a^2 +
 b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^5*d) - (3*b*(a^4 - 5*a^2*b^2 -
4*b^4))/(8*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) - (Sec[c + d*x]^4*(b - a
*Sin[c + d*x]))/(4*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) - (3*a*b*(a^4 - 6*
a^2*b^2 - 27*b^4))/(8*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]
^2*(2*b*(a^2 + 3*b^2) + a*(3*a^2 - 11*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*
d*(a + b*Sin[c + d*x])^2)
```

Rubi [A] time = 0.420186, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2668, 741, 823, 801}

$$-\frac{3ab\left(-6a^2b^2+a^4-27b^4\right)}{8d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}-\frac{3b\left(-5a^2b^2+a^4-4b^4\right)}{8d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2}-\frac{3b^5\left(7a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^5}-\frac{3\left(a^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (-3*(a^2 + 5*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^5*d) + (3*(a^2
 - 5*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^5*d) - (3*b^5*(7*a^2 +
 b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^5*d) - (3*b*(a^4 - 5*a^2*b^2 -
4*b^4))/(8*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) - (Sec[c + d*x]^4*(b - a
*Sin[c + d*x]))/(4*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) - (3*a*b*(a^4 - 6*
a^2*b^2 - 27*b^4))/(8*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]
^2*(2*b*(a^2 + 3*b^2) + a*(3*a^2 - 11*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*
d*(a + b*Sin[c + d*x])^2)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b^5\,\mathrm{Subst}\left(\int \frac{1}{(a+x)^3\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{b^3\,\mathrm{Subst}\left(\int \frac{3\left(a^2-2b^2\right)+5ax}{(a+x)^3\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\sec^2(c+dx)\left(2b\left(a^2+3b^2\right)+a\left(3a^2-11b^2\right)\sin(c+dx\right.}{8\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^2}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\sec^2(c+dx)\left(2b\left(a^2+3b^2\right)+a\left(3a^2-11b^2\right)\sin(c+dx\right.}{8\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^2}$$

$$= -\frac{3\left(a^2+5ab+8b^2\right)\log(1-\sin(c+dx))}{16(a+b)^5 d} + \frac{3\left(a^2-5ab+8b^2\right)\log(1+\sin(c+dx))}{16(a-b)^5 d} - \frac{3b^5\,(}{}$$

Mathematica [A] time = 2.6111, size = 388, normalized size = 1.18

$$-\frac{\frac{b\left(3\left(-5a^2b^2+a^4-4b^4\right)\left(\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{2\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3}+\frac{4a}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}-\frac{\log(1-\sin(c+dx))}{b(a+b)^3}+\frac{\log(\sin(c+dx)+1)}{b(a-b)^3}\right)-3a\left(3a^2-11b^2\right)\left(\frac{}{\left(a^2-\right.}\right.\right.}{b^2-a^2}}{8d\left(b\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2 + (Sec[c +
d*x]^2*(2*b*(a^2 + 3*b^2) + a*(3*a^2 - 11*b^2)*Sin[c + d*x]))/((-a^2 + b^2)
*(a + b*Sin[c + d*x])^2) - (b*(3*(a^4 - 5*a^2*b^2 - 4*b^4)*(-(Log[1 - Sin[c
 + d*x]]/(b*(a + b)^3)) + Log[1 + Sin[c + d*x]]/((a - b)^3*b) - (2*(3*a^2 +
 b^2)*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a + b)^3) + 1/((a^2 - b^2)*(a +
b*Sin[c + d*x])^2) + (4*a)/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x]))) - 3*
a*(3*a^2 - 11*b^2)*(-Log[1 - Sin[c + d*x]]/(2*b*(a + b)^2) + Log[1 + Sin[c
+ d*x]]/(2*(a - b)^2*b) - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^
2) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x])))))/(-a^2 + b^2))/(8*(-a^2 + b^2)*
d)
```

Maple [A] time = 0.142, size = 398, normalized size = 1.2

$$\frac{b^5}{2d(a+b)^3(a-b)^3(a+b\sin(dx+c))^2}+6\frac{b^5a}{d(a+b)^4(a-b)^4(a+b\sin(dx+c))}-21\frac{b^5\ln(a+b\sin(dx+c))a^2}{d(a+b)^5(a-b)^5}-3\frac{b^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d*b^5/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))^2+6/d*b^5*a/(a+b)^4/(a-b)^4/(a+b
*sin(d*x+c))-21/d*b^5/(a+b)^5/(a-b)^5*ln(a+b*sin(d*x+c))*a^2-3/d*b^7/(a+b)^
5/(a-b)^5*ln(a+b*sin(d*x+c))+1/16/d/(a+b)^3/(sin(d*x+c)-1)^2-3/16/d/(a+b)^4
/(sin(d*x+c)-1)*a-9/16/d/(a+b)^4/(sin(d*x+c)-1)*b-3/16/d/(a+b)^5*ln(sin(d*x
+c)-1)*a^2-15/16/d/(a+b)^5*ln(sin(d*x+c)-1)*a*b-3/2/d/(a+b)^5*ln(sin(d*x+c)
-1)*b^2-1/16/d/(a-b)^3/(1+sin(d*x+c))^2-3/16/d/(a-b)^4/(1+sin(d*x+c))*a+9/1
6/d/(a-b)^4/(1+sin(d*x+c))*b+3/16/d/(a-b)^5*ln(1+sin(d*x+c))*a^2-15/16/d/(a
-b)^5*ln(1+sin(d*x+c))*a*b+3/2/d/(a-b)^5*ln(1+sin(d*x+c))*b^2
```

Maxima [B] time = 1.04404, size = 979, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/16*(48*(7*a^2*b^5 + b^7)*log(b*sin(d*x + c) + a)/(a^10 - 5*a^8*b^2 + 10*
a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - 3*(a^2 - 5*a*b + 8*b^2)*log(sin(
d*x + c) + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5) + 3
*(a^2 + 5*a*b + 8*b^2)*log(sin(d*x + c) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 +
10*a^2*b^3 + 5*a*b^4 + b^5) + 2*(6*a^6*b - 44*a^4*b^3 - 62*a^2*b^5 + 4*b^7
+ 3*(a^5*b^2 - 6*a^3*b^4 - 27*a*b^6)*sin(d*x + c)^5 + 6*(a^6*b - 6*a^4*b^3
- 13*a^2*b^5 + 2*b^7)*sin(d*x + c)^4 + (3*a^7 - 23*a^5*b^2 + 61*a^3*b^4 + 1
51*a*b^6)*sin(d*x + c)^3 - 2*(5*a^6*b - 37*a^4*b^3 - 73*a^2*b^5 + 9*b^7)*si
n(d*x + c)^2 - (5*a^7 - 26*a^5*b^2 + 49*a^3*b^4 + 68*a*b^6)*sin(d*x + c))/(
a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8 + (a^8*b^2 - 4*a^6*b^4 +
 6*a^4*b^6 - 4*a^2*b^8 + b^10)*sin(d*x + c)^6 + 2*(a^9*b - 4*a^7*b^3 + 6*a^
5*b^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)^5 + (a^10 - 6*a^8*b^2 + 14*a^6*b^4
- 16*a^4*b^6 + 9*a^2*b^8 - 2*b^10)*sin(d*x + c)^4 - 4*(a^9*b - 4*a^7*b^3 +
6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)^3 - (2*a^10 - 9*a^8*b^2 + 16*a^
```

```
6*b^4 - 14*a^4*b^6 + 6*a^2*b^8 - b^10)*sin(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^
3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)))/d
```

Fricas [B] time = 9.12643, size = 1995, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^8*b - 16*a^6*b^3 + 24*a^4*b^5 - 16*a^2*b^7 + 4*b^9 + 12*(a^8*b -
7*a^6*b^3 - 7*a^4*b^5 + 15*a^2*b^7 - 2*b^9)*cos(d*x + c)^4 - 4*(a^8*b - 6*a
^4*b^5 + 8*a^2*b^7 - 3*b^9)*cos(d*x + c)^2 - 48*((7*a^2*b^7 + b^9)*cos(d*x
+ c)^6 - 2*(7*a^3*b^6 + a*b^8)*cos(d*x + c)^4*sin(d*x + c) - (7*a^4*b^5 + 8
*a^2*b^7 + b^9)*cos(d*x + c)^4)*log(b*sin(d*x + c) + a) + 3*((a^7*b^2 - 7*a
^5*b^4 + 35*a^3*b^6 + 56*a^2*b^7 + 35*a*b^8 + 8*b^9)*cos(d*x + c)^6 - 2*(a^
8*b - 7*a^6*b^3 + 35*a^4*b^5 + 56*a^3*b^6 + 35*a^2*b^7 + 8*a*b^8)*cos(d*x +
 c)^4*sin(d*x + c) - (a^9 - 6*a^7*b^2 + 28*a^5*b^4 + 56*a^4*b^5 + 70*a^3*b^
6 + 64*a^2*b^7 + 35*a*b^8 + 8*b^9)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) -
3*((a^7*b^2 - 7*a^5*b^4 + 35*a^3*b^6 - 56*a^2*b^7 + 35*a*b^8 - 8*b^9)*cos(d
*x + c)^6 - 2*(a^8*b - 7*a^6*b^3 + 35*a^4*b^5 - 56*a^3*b^6 + 35*a^2*b^7 - 8
*a*b^8)*cos(d*x + c)^4*sin(d*x + c) - (a^9 - 6*a^7*b^2 + 28*a^5*b^4 - 56*a^
4*b^5 + 70*a^3*b^6 - 64*a^2*b^7 + 35*a*b^8 - 8*b^9)*cos(d*x + c)^4)*log(-si
n(d*x + c) + 1) - 2*(2*a^9 - 8*a^7*b^2 + 12*a^5*b^4 - 8*a^3*b^6 + 2*a*b^8 -
 3*(a^7*b^2 - 7*a^5*b^4 - 21*a^3*b^6 + 27*a*b^8)*cos(d*x + c)^4 + (3*a^9 -
20*a^7*b^2 + 42*a^5*b^4 - 36*a^3*b^6 + 11*a*b^8)*cos(d*x + c)^2)*sin(d*x +
c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d
*cos(d*x + c)^6 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b
^9 - a*b^11)*d*cos(d*x + c)^4*sin(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a^8*b^4
 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20583, size = 776, normalized size = 2.37

$$-\frac{\frac{48\left(7a^2b^6+b^8\right)\log(|b\sin(dx+c)+a|)}{a^{10}b-5a^8b^3+10a^6b^5-10a^4b^7+5a^2b^9-b^{11}}-\frac{3\left(a^2-5ab+8b^2\right)\log(|\sin(dx+c)+1|)}{a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5}+\frac{3\left(a^2+5ab+8b^2\right)\log(|\sin(dx+c)-1|)}{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5}+\frac{2\left(3a^5b^2\sin(dx+c)^5-18a^3b^4\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(48*(7*a^2*b^6 + b^8)*log(abs(b*sin(d*x + c) + a))/(a^10*b - 5*a^8*b^
3 + 10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11) - 3*(a^2 - 5*a*b + 8*b^2)*l
og(abs(sin(d*x + c) + 1))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^
4 - b^5) + 3*(a^2 + 5*a*b + 8*b^2)*log(abs(sin(d*x + c) - 1))/(a^5 + 5*a^4*
b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) + 2*(3*a^5*b^2*sin(d*x + c)^5
- 18*a^3*b^4*sin(d*x + c)^5 - 81*a*b^6*sin(d*x + c)^5 + 6*a^6*b*sin(d*x + c
)^4 - 36*a^4*b^3*sin(d*x + c)^4 - 78*a^2*b^5*sin(d*x + c)^4 + 12*b^7*sin(d*
x + c)^4 + 3*a^7*sin(d*x + c)^3 - 23*a^5*b^2*sin(d*x + c)^3 + 61*a^3*b^4*si
n(d*x + c)^3 + 151*a*b^6*sin(d*x + c)^3 - 10*a^6*b*sin(d*x + c)^2 + 74*a^4*
b^3*sin(d*x + c)^2 + 146*a^2*b^5*sin(d*x + c)^2 - 18*b^7*sin(d*x + c)^2 - 5
*a^7*sin(d*x + c) + 26*a^5*b^2*sin(d*x + c) - 49*a^3*b^4*sin(d*x + c) - 68*
a*b^6*sin(d*x + c) + 6*a^6*b - 44*a^4*b^3 - 62*a^2*b^5 + 4*b^7)/((a^8 - 4*a
^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(b*sin(d*x + c)^3 + a*sin(d*x + c)^2
- b*sin(d*x + c) - a)^2))/d
```

3.456 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=197

$$-\frac{5\left(-5a^2b^2+4a^4+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d\sqrt{a^2-b^2}}+\frac{5\cos(c+dx)\left(4a^2-2ab\sin(c+dx)-b^2\right)}{2b^5d}+\frac{5ax\left(4a^2-3b^2\right)}{2b^6}-\frac{5\cos}{}$$

```
[Out] (5*a*(4*a^2 - 3*b^2)*x)/(2*b^6) - (5*(4*a^4 - 5*a^2*b^2 + b^4)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*Sqrt[a^2 - b^2]*d) - Cos[c + d*x
]^5/(2*b*d*(a + b*Sin[c + d*x])^2) - (5*Cos[c + d*x]^3*(4*a + b*Sin[c + d*x
]))/(6*b^3*d*(a + b*Sin[c + d*x])) + (5*Cos[c + d*x]*(4*a^2 - b^2 - 2*a*b*S
in[c + d*x]))/(2*b^5*d)
```

Rubi [A] time = 0.366881, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2693, 2863, 2865, 2735, 2660, 618, 204}

$$-\frac{5\left(-5a^2b^2+4a^4+b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d\sqrt{a^2-b^2}}+\frac{5\cos(c+dx)\left(4a^2-2ab\sin(c+dx)-b^2\right)}{2b^5d}+\frac{5ax\left(4a^2-3b^2\right)}{2b^6}-\frac{5\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (5*a*(4*a^2 - 3*b^2)*x)/(2*b^6) - (5*(4*a^4 - 5*a^2*b^2 + b^4)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*Sqrt[a^2 - b^2]*d) - Cos[c + d*x
]^5/(2*b*d*(a + b*Sin[c + d*x])^2) - (5*Cos[c + d*x]^3*(4*a + b*Sin[c + d*x
]))/(6*b^3*d*(a + b*Sin[c + d*x])) + (5*Cos[c + d*x]*(4*a^2 - b^2 - 2*a*b*S
in[c + d*x]))/(2*b^5*d)
```


Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
```

```
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{2b}$$

$$= -\frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))} + \frac{5\int \frac{\cos^2(c+dx)(-b-4a\sin(c+dx)}{a+b\sin(c+dx)}}{2b^3}$$

$$= -\frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))} + \frac{5\cos(c+dx)\left(4a^2-b^2-\right.}{2b^5d}$$

$$= \frac{5a\left(4a^2-3b^2\right)x}{2b^6} - \frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))} + \frac{5\cos(c}{}$$

$$= \frac{5a\left(4a^2-3b^2\right)x}{2b^6} - \frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))} + \frac{5\cos(c}{}$$

$$= \frac{5a\left(4a^2-3b^2\right)x}{2b^6} - \frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))} + \frac{5\cos(c}{}$$

$$= \frac{5a\left(4a^2-3b^2\right)x}{2b^6} - \frac{5\left(4a^4-5a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}\,d} - \frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2}$$

Mathematica [B] time = 6.58301, size = 3905, normalized size = 19.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]^5*(-(b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(7/2)*(b/(a
+ b) - (b*Sin[c + d*x])/(a + b))^(7/2))/(2*((a*b)/(a - b) - b^2/(a - b))*((
a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[c + d*x])^2) - ((-3*a*b^3*(-(b/(a -
b)) - (b*Sin[c + d*x])/(a - b))^(7/2)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)
```

)^(7/2))/((a^2 - b^2)*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[c + d*x])) - ((144*Sqrt[2]*a*b^5*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(7/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((7*(3/(16*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1)))/12 + (35*b^4*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b^2) + (2*(a - b)^3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3)/(15*b^3) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(128*(a - b)^4*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(7*(a - b)*(a + b)^4*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) + (((18*a^2*b^5)/((a - b)^2*(a + b)^2) + (b^5*(2*a^2 - 5*b^2))/((a - b)^2*(a + b)^2))*((8*Sqrt[2]*b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(5/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((5/(16*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/2 - (15*b^3*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b^2) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(64*(a - b)^3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(5*(a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*((8*Sqrt[2]*b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(3/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((3*(5/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(6*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1)))/8 + (15*b^2*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(64*(a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(3*(a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*((8*Sqrt[2]*b*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((5*Sqrt[b]*ArcSinh[

```
(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b
])])/(8*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(
1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)) + (15
/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/
(4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1
+ ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/6))/((a
+ b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b
)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a + b)) - b^2/(a + b))*(-(((-((a*b)
/(a + b)) - b^2/(a + b))*((-2*(-((a*b)/(a + b)) - b^2/(a + b))*ArcTan[(Sqrt
[(a*b)/(a + b) + b^2/(a + b)]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]
)/(Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(
a + b)])])/(b*Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[(a*b)/(a + b) + b^2
/(a + b)]) + (2*Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin
[c + d*x])/(a - b)])/(Sqrt[a + b]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)
])])/(b*Sqrt[a + b])))/b) + (2*Sqrt[2]*(a - b)*Sqrt[-(b/(a - b)) - (b*Sin[c
 + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*
(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)*((Sqrt[b]*ArcSinh[(
Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b]
)])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 +
 ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)) + 1/(2*(
1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)))))/(b*(a + b
)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b])))/b) + (4*Sqrt[
2]*(a - b)*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (
b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a
- b)))/(2*b))^(5/2)*((3*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b
*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(4*Sqrt[2]*Sqrt[a - b]*Sqrt[-(
b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin
[c + d*x])/(a - b)))/(2*b))^(5/2)) + (3/(2*(1 + ((a - b)*(-(b/(a - b)) - (b
*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c
 + d*x])/(a - b)))/(2*b))^(-1))/4))/((a + b)^2*Sqrt[((a + b)*(b/(a + b) - (
b*Sin[c + d*x])/(a + b)))/b])))/b))/b))/b))/b)/(((a*b)/(a - b) - b^2/(a - b
))*((a*b)/(a + b) + b^2/(a + b))))/(2*((a*b)/(a - b) - b^2/(a - b))*((a*b)/
(a + b) + b^2/(a + b)))))/(d*(1 - (a + b*Sin[c + d*x])/(a - b))^(5/2)*(1 -
(a + b*Sin[c + d*x])/(a + b))^(5/2))
```

Maple [B] time = 0.121, size = 1060, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -14/3/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3-1/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(
1/2*d*x+1/2*c)*b+a)^2+12/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^3*a^2+20/d/b^6*arct
an(tan(1/2*d*x+1/2*c))*a^3-15/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a-15/d/b/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2+8/d/b
^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^4-7/d/b^3/(tan(1/2
*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2-5/d/b^2/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/d/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^3-2/d/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)-6/d/b^3/(1+tan(
1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4-8/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3
*tan(1/2*d*x+1/2*c)^2+3/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*a*tan(1/2*d*x+1/2*
c)^5+12/d/b^5/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^4*a^2+24/d/b^5/
(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2*a^2-3/d/b^4/(1+tan(1/2*d*x+
1/2*c)^2)^3*a*tan(1/2*d*x+1/2*c)+7/d/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2*a^3*tan(1/2*d*x+1/2*c)^3-5/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)^2*a*tan(1/2*d*x+1/2*c)^3+8/d/b^5/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^4*tan(1/2*d*x+1/2*c)^2+9/d/b^3/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2*tan(1/2*d*x+1/2*c)^2-2/d*
b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a^2*tan(1/2*d*x+1/2*c
)^2+25/d/b^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^3*tan(1/
2*d*x+1/2*c)-23/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a
*tan(1/2*d*x+1/2*c)-20/d/b^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/
2*c)+2*b)/(a^2-b^2)^(1/2))*a^4+25/d/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan
(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.82276, size = 1669, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*b^5*cos(d*x + c)^5 - 30*(4*a^3*b^2 - 3*a*b^4)*d*x*cos(d*x + c)^2
- 20*(2*a^2*b^3 - b^5)*cos(d*x + c)^3 + 30*(4*a^5 + a^3*b^2 - 3*a*b^4)*d*x
- 15*(4*a^4 + 3*a^2*b^2 - b^4 - (4*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2*(4*a^3
*b - a*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*co
s(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^
2 - b^2)) + 30*(4*a^4*b - a^2*b^3 - b^5)*cos(d*x + c) + 10*(a*b^4*cos(d*x +
 c)^3 + 6*(4*a^4*b - 3*a^2*b^3)*d*x + 6*(3*a^3*b^2 - 2*a*b^4)*cos(d*x + c))
*sin(d*x + c))/(b^8*d*cos(d*x + c)^2 - 2*a*b^7*d*sin(d*x + c) - (a^2*b^6 +
b^8)*d), -1/6*(2*b^5*cos(d*x + c)^5 - 15*(4*a^3*b^2 - 3*a*b^4)*d*x*cos(d*x
+ c)^2 - 10*(2*a^2*b^3 - b^5)*cos(d*x + c)^3 + 15*(4*a^5 + a^3*b^2 - 3*a*b^
4)*d*x + 15*(4*a^4 + 3*a^2*b^2 - b^4 - (4*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2
*(4*a^3*b - a*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 15*(4*a^4*b - a^2*b^3 - b^5)*cos(d*x +
 c) + 5*(a*b^4*cos(d*x + c)^3 + 6*(4*a^4*b - 3*a^2*b^3)*d*x + 6*(3*a^3*b^2
- 2*a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^8*d*cos(d*x + c)^2 - 2*a*b^7*d*si
n(d*x + c) - (a^2*b^6 + b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16336, size = 617, normalized size = 3.13

$$\frac{15\left(4a^3-3ab^2\right)(dx+c)}{b^6}-\frac{30\left(4a^4-5a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6}+\frac{2\left(9ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+36a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-18b^2\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*(4*a^3 - 3*a*b^2)*(d*x + c)/b^6 - 30*(4*a^4 - 5*a^2*b^2 + b^4)*(pi*
floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/
sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 2*(9*a*b*tan(1/2*d*x + 1/2*c)^5 +
 36*a^2*tan(1/2*d*x + 1/2*c)^4 - 18*b^2*tan(1/2*d*x + 1/2*c)^4 + 72*a^2*tan
(1/2*d*x + 1/2*c)^2 - 24*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*a*b*tan(1/2*d*x + 1
/2*c) + 36*a^2 - 14*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^5) + 6*(7*a^5*b*
tan(1/2*d*x + 1/2*c)^3 - 5*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^5*tan(1/2
*d*x + 1/2*c)^3 + 8*a^6*tan(1/2*d*x + 1/2*c)^2 + 9*a^4*b^2*tan(1/2*d*x + 1/
2*c)^2 - 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 2*b^6*tan(1/2*d*x + 1/2*c)^2 +
 25*a^5*b*tan(1/2*d*x + 1/2*c) - 23*a^3*b^3*tan(1/2*d*x + 1/2*c) - 2*a*b^5*
tan(1/2*d*x + 1/2*c) + 8*a^6 - 7*a^4*b^2 - a^2*b^4)/((a*tan(1/2*d*x + 1/2*c
)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^2*b^5))/d
```

3.457 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=139

$$\frac{3\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d\sqrt{a^2-b^2}}-\frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))}-\frac{3ax}{b^4}-\frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2}$$

```
[Out] (-3*a*x)/b^4 + (3*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(b^4*Sqrt[a^2 - b^2]*d) - Cos[c + d*x]^3/(2*b*d*(a + b*Sin[c + d*x])
^2) - (3*Cos[c + d*x]*(2*a + b*Sin[c + d*x]))/(2*b^3*d*(a + b*Sin[c + d*x])
)
```

Rubi [A] time = 0.208345, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2693, 2863, 2735, 2660, 618, 204}

$$\frac{3\left(2a^2-b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d\sqrt{a^2-b^2}}-\frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))}-\frac{3ax}{b^4}-\frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (-3*a*x)/b^4 + (3*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(b^4*Sqrt[a^2 - b^2]*d) - Cos[c + d*x]^3/(2*b*d*(a + b*Sin[c + d*x])
^2) - (3*Cos[c + d*x]*(2*a + b*Sin[c + d*x]))/(2*b^3*d*(a + b*Sin[c + d*x])
)
```


Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}dx}{2b}$$
$$= -\frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))} + \frac{3\int \frac{-b-2a\sin(c+dx)}{a+b\sin(c+dx)}dx}{2b^3}$$
$$= -\frac{3ax}{b^4} - \frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))} + \frac{\left(3\left(2a^2-b^2\right)\right)\int \frac{}{a+b}}{2b^4}$$
$$= -\frac{3ax}{b^4} - \frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))} + \frac{\left(3\left(2a^2-b^2\right)\right)\text{Subs}}{}$$
$$= -\frac{3ax}{b^4} - \frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))} - \frac{\left(6\left(2a^2-b^2\right)\right)\text{Subs}}{}$$
$$= -\frac{3ax}{b^4} + \frac{3\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}d} - \frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+dx)(2a}{2b^3d(a+b\sin}$$

Mathematica [B] time = 6.2807, size = 2657, normalized size = 19.12

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]^3*(-(b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(5/2)*(b/(a
+ b) - (b*Sin[c + d*x])/(a + b))^(5/2))/(2*((a*b)/(a - b) - b^2/(a - b))*((
a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[c + d*x])^2) - (-((a*b^3*(-(b/(a - b
)) - (b*Sin[c + d*x])/(a - b))^(5/2)*(b/(a + b) - (b*Sin[c + d*x])/(a + b))
^(5/2))/((a^2 - b^2)*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a
+ b))*(a + b*Sin[c + d*x]))) - ((16*Sqrt[2]*a*b^4*(-(b/(a - b)) - (b*Sin[c
+ d*x])/(a - b))^(5/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a
- b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)*((5*(1/(2*(1 +
 ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a -
 b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1)))/8 - (15*b^3*((
(a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - ((a - b)^2*(-(b/(a -
 b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b^2) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[
(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b
])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a -
```

b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)]])))/(32*(a - b)^3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2)))/(5*(a - b)*(a + b)^3*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) + (((4*a^2*b^5)/((a - b)^2*(a + b)^2) + (b^5*(2*a^2 - 3*b^2))/((a - b)^2*(a + b)^2))*((4*Sqrt[2]*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(3/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)*((3/(4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/2 + (3*b^2*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(8*(a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2)))/(3*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a + b)) - b^2/(a + b))*((-2*(-((a*b)/(a + b)) - b^2/(a + b))*ArcTan[(Sqrt[(a*b)/(a + b) + b^2/(a + b)]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[(a*b)/(a + b) + b^2/(a + b)]) + (2*Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[a + b]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])]])/(b*Sqrt[a + b])))/b) + (2*Sqrt[2]*(a - b)*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)*((Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)) + 1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)))))/(b*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b])))/b) + (4*Sqrt[2]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)*((3*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(4*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)) + (3/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/4))/((a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b])))/b))/b)/(((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b))))/(2*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b)))))/(d*(1 - (a + b*Sin[c + d*x])/(a - b))^(3/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(3/2))

Maple [B] time = 0.103, size = 560, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)-6/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a-3/d/
b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a*tan(1/2*d*x+1/2*c
)^3-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1
/2*c)^3-4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2*tan
(1/2*d*x+1/2*c)^2-9/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2
*tan(1/2*d*x+1/2*c)^2-2/d*b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2/a^2*tan(1/2*d*x+1/2*c)^2-13/d/b^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)^2*a*tan(1/2*d*x+1/2*c)-2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*
d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)-4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a+2*ta
n(1/2*d*x+1/2*c)*b+a)^2*a^2-1/d/b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2+6/d/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2))*a^2-3/d/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/
2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.50634, size = 1515, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(12*(a^3*b^2 - a*b^4)*d*x*cos(d*x + c)^2 + 4*(a^2*b^3 - b^5)*cos(d*x
+ c)^3 - 12*(a^5 - a*b^4)*d*x + 3*(2*a^4 + a^2*b^2 - b^4 - (2*a^2*b^2 - b^4
)*cos(d*x + c)^2 + 2*(2*a^3*b - a*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(2*a^4*b - a^2*b^3 - b^5)*cos(d*x
+ c) - 6*(4*(a^4*b - a^2*b^3)*d*x + 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d
*x + c))/((a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(a^3*b^5 - a*b^7)*d*sin(d*x
+ c) - (a^4*b^4 - b^8)*d), -1/2*(6*(a^3*b^2 - a*b^4)*d*x*cos(d*x + c)^2 + 2
*(a^2*b^3 - b^5)*cos(d*x + c)^3 - 6*(a^5 - a*b^4)*d*x - 3*(2*a^4 + a^2*b^2
- b^4 - (2*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2*(2*a^3*b - a*b^3)*sin(d*x + c)
)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c
))) - 3*(2*a^4*b - a^2*b^3 - b^5)*cos(d*x + c) - 3*(4*(a^4*b - a^2*b^3)*d*x
 + 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^6 - b^8)*d*cos(d
*x + c)^2 - 2*(a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^4*b^4 - b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15764, size = 367, normalized size = 2.64

$$-\frac{\frac{3\,(dx+c)a}{b^4} - \frac{3\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)(2\,a^2-b^2)}{\sqrt{a^2-b^2}b^4} + \frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b^3} + \frac{3\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^4\,\mathrm{t}}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(d*x + c)*a/b^4 - 3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((
a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*(2*a^2 - b^2)/(sqrt(a^2 - b^2
```

```
)*b^4) + 2/((tan(1/2*d*x + 1/2*c)^2 + 1)*b^3) + (3*a^3*b*tan(1/2*d*x + 1/2*
c)^3 + 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*tan(1/2*d*x + 1/2*c)^2 + 9*a^
2*b^2*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x + 1/2*c)^2 + 13*a^3*b*tan(
1/2*d*x + 1/2*c) + 2*a*b^3*tan(1/2*d*x + 1/2*c) + 4*a^4 + a^2*b^2)/((a*tan(
1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^2*b^3))/d
```

3.458 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=115

$$\frac{\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{a\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}$$

```
[Out] ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]/((a^2 - b^2)^(3/2)*d) - Co
s[c + d*x]/(2*b*d*(a + b*Sin[c + d*x])^2) + (a*Cos[c + d*x])/(2*b*(a^2 - b^
2)*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.127862, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2693, 2754, 12, 2660, 618, 204}

$$\frac{\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{a\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]/((a^2 - b^2)^(3/2)*d) - Co
s[c + d*x]/(2*b*d*(a + b*Sin[c + d*x])^2) + (a*Cos[c + d*x])/(2*b*(a^2 - b^
2)*d*(a + b*Sin[c + d*x]))
```


Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\int \frac{\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{2b}$$

$$= -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{b}{a+b\sin(c+dx)}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{1}{a+b\sin(c+dx)}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\right.}{\left(a^2-b^2\right)d}$$

$$= -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{2\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x,\right.}{\left(a^2-b^2\right.}$$

$$= \frac{\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d} - \frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))}$$

Mathematica [A] time = 0.305711, size = 93, normalized size = 0.81

$$\frac{\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{\cos(c+dx)(a\sin(c+dx)+b)}{(a+b\sin(c+dx))^2}}{2d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (Co
s[c + d*x]*(b + a*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2)/(2*(a - b)*(a + b)
*d)
```

Maple [B] time = 0.095, size = 443, normalized size = 3.9

$$-\frac{a}{d\left(a^2-b^2\right)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a + 2\tan(1/2\,dx+c/2)\,b + a\right)^{-2} + 2\,\frac{(\tan(1/2\,dx\,-}{d\left(\left(\tan(1/2\,dx+c/2)\right)^2 a + 2\tan(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a/(a^2-b^2)*tan(1/
2*d*x+1/2*c)^3+2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a/(a
^2-b^2)*tan(1/2*d*x+1/2*c)^3*b^2+1/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)^2*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2+2/d/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)/a^2*tan(1/2*d*x+1/2*c)^2+1/d/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a/(a^2-b^2)*tan(1/2*d*x+1/2
*c)+2/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a/(a^2-b^2)*tan
(1/2*d*x+1/2*c)*b^2+1/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2
/(a^2-b^2)*b+1/d/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a
^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.63783, size = 1107, normalized size = 9.63

$$\left[-\frac{2\left(a^{3}-ab^{2}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}\right)\sqrt{-a^{2}+b^{2}}\log\left(-\frac{\left(2\,a^{2}-b^{2}\right)\cos(dx+c}{}\right.}{4\left(\left(a^{4}b^{2}-2\,a^{2}b^{4}+b^{6}\right)d\cos\left(dx+c\right)^{2}-2\left(a^{5}b-2\,a^{3}b^{3}+ab^{5}\right)d\sin\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^3 - a*b^2)*cos(d*x + c)*sin(d*x + c) - (b^2*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) +
```

```
b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
- a^2 - b^2)) + 2*(a^2*b - b^3)*cos(d*x + c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*
d*cos(d*x + c)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*sin(d*x + c) - (a^6 - a^
4*b^2 - a^2*b^4 + b^6)*d), -1/2*((a^3 - a*b^2)*cos(d*x + c)*sin(d*x + c) +
(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(a^2 - b^2)*arcta
n(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (a^2*b - b^3)*cos
(d*x + c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*d*cos(d*x + c)^2 - 2*(a^5*b - 2*a^3
*b^3 + a*b^5)*d*sin(d*x + c) - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14356, size = 279, normalized size = 2.43

$$\frac{\frac{\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2a}{\left(a^4-a^2b^2\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c)
+ b)/sqrt(a^2 - b^2)))/(a^2 - b^2)^(3/2) - (a^3*tan(1/2*d*x + 1/2*c)^3 - 2*
a*b^2*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*d*x + 1/2*c)^2 - 2*b^3*tan(1/2
*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 2*a*b^2*tan(1/2*d*x + 1/2*c) -
 a^2*b)/((a^4 - a^2*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*
c) + a)^2))/d
```

3.459 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=192

$$-\frac{3b^2\left(4a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{5ab\sec(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\sec(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{\sec(c+}{}$$

```
[Out] (-3*b^2*(4*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a
^2 - b^2)^(7/2)*d) + (b*Sec[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
^2) + (5*a*b*Sec[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (Sec[
c + d*x]*(3*b*(4*a^2 + b^2) - a*(2*a^2 + 13*b^2)*Sin[c + d*x]))/(2*(a^2 - b
^2)^3*d)
```

Rubi [A] time = 0.391201, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2694, 2864, 2866, 12, 2660, 618, 204}

$$-\frac{3b^2\left(4a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{5ab\sec(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\sec(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{\sec(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b^2*(4*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a
^2 - b^2)^(7/2)*d) + (b*Sec[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
^2) + (5*a*b*Sec[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (Sec[
c + d*x]*(3*b*(4*a^2 + b^2) - a*(2*a^2 + 13*b^2)*Sin[c + d*x]))/(2*(a^2 - b
^2)^3*d)
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
```

```
0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)(-2a+3b\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^2(c+dx)\left(2a^2+3b^2-10\right.}{a+b\sin(c+}}{2\left(a^2-b^2\right.}$$

$$= \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3b\left(4a^2\right.\right.}{}$$

$$= \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3b\left(4a^2\right.\right.}{}$$

$$= \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3b\left(4a^2\right.\right.}{}$$

$$= \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3b\left(4a^2\right.\right.}{}$$

$$= -\frac{3b^2\left(4a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{5ab\,\mathrm{s}}{2\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 3.05529, size = 193, normalized size = 1.01

$$\frac{-\frac{6b^2\left(4a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{b^3\cos(c+dx)\left(-8a^2-7ab\sin(c+dx)+b^2\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a-b)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{}{(a+b)^3}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-6*b^2*(4*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (b^3*
Cos[c + d*x]*(-8*a^2 + b^2 - 7*a*b*Sin[c + d*x]))/((a - b)^3*(a + b)^3*(a +
 b*Sin[c + d*x])^2))/(2*d)
```

Maple [B] time = 0.092, size = 705, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)-1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)-9/d*
b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a*t
an(1/2*d*x+1/2*c)^3+2/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1
/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^3-8/d*b^3/(a-b)^3/(a+b)^3/(tan(1/
2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2*tan(1/2*d*x+1/2*c)^2-15/d*
b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan
(1/2*d*x+1/2*c)^2+2/d*b^7/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2
*d*x+1/2*c)*b+a)^2/a^2*tan(1/2*d*x+1/2*c)^2-23/d*b^4/(a-b)^3/(a+b)^3/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a*tan(1/2*d*x+1/2*c)+2/d*b^6/
(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1
/2*d*x+1/2*c)-8/d*b^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x
+1/2*c)*b+a)^2*a^2+1/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)^2-12/d*b^2/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2-3/d*b^4/(a-b)^3/(a+b)^3/(a^2
-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.31216, size = 1983, normalized size = 10.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(4*a^6*b + 10*a^4*b^3 -
 17*a^2*b^5 + 3*b^7)*cos(d*x + c)^2 + 3*((4*a^2*b^4 + b^6)*cos(d*x + c)^3 -
 2*(4*a^3*b^3 + a*b^5)*cos(d*x + c)*sin(d*x + c) - (4*a^4*b^2 + 5*a^2*b^4 +
 b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x +
 c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 - (2*a^5*b^2 + 11*a^3*b^4 -
 13*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6
- 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4
*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b
^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)), 1/2*(2*a^6*b - 6*a^4*b^
3 + 6*a^2*b^5 - 2*b^7 + (4*a^6*b + 10*a^4*b^3 - 17*a^2*b^5 + 3*b^7)*cos(d*x
 + c)^2 + 3*((4*a^2*b^4 + b^6)*cos(d*x + c)^3 - 2*(4*a^3*b^3 + a*b^5)*cos(d
*x + c)*sin(d*x + c) - (4*a^4*b^2 + 5*a^2*b^4 + b^6)*cos(d*x + c))*sqrt(a^2
 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a
^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 - (2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6)*
cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8
 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 +
a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*
b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [B] time = 1.18032, size = 520, normalized size = 2.71

$$-\frac{\frac{3\left(4a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{2\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^2b-b^3\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{9a^3b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab^6\tan}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(4*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*
tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*sqrt(a^2 - b^2)) + 2*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan(1/2*d*x
+ 1/2*c) - 3*a^2*b - b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x
 + 1/2*c)^2 - 1)) + (9*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^6*tan(1/2*d*x
 + 1/2*c)^3 + 8*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 + 15*a^2*b^5*tan(1/2*d*x + 1
/2*c)^2 - 2*b^7*tan(1/2*d*x + 1/2*c)^2 + 23*a^3*b^4*tan(1/2*d*x + 1/2*c) -
2*a*b^6*tan(1/2*d*x + 1/2*c) + 8*a^4*b^3 - a^2*b^5)/((a^8 - 3*a^6*b^2 + 3*a
^4*b^4 - a^2*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)
^2))/d
```

3.460 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=264

$$\frac{5b^4\left(6a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}+\frac{7ab\sec^3(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\sec^3(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\sec^3(c+$$

```
[Out] (5*b^4*(6*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^
2 - b^2)^(9/2)*d) + (b*Sec[c + d*x]^3)/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]
)^2) + (7*a*b*Sec[c + d*x]^3)/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (S
ec[c + d*x]^3*(5*b*(6*a^2 + b^2) - a*(2*a^2 + 33*b^2)*Sin[c + d*x]))/(6*(a^
2 - b^2)^3*d) + (Sec[c + d*x]*(15*b^3*(6*a^2 + b^2) + a*(4*a^4 - 28*a^2*b^2
 - 81*b^4)*Sin[c + d*x]))/(6*(a^2 - b^2)^4*d)
```

Rubi [A] time = 0.640153, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2694, 2864, 2866, 12, 2660, 618, 204}

$$\frac{5b^4\left(6a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}+\frac{7ab\sec^3(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\sec^3(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\sec^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*b^4*(6*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^
2 - b^2)^(9/2)*d) + (b*Sec[c + d*x]^3)/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]
)^2) + (7*a*b*Sec[c + d*x]^3)/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (S
ec[c + d*x]^3*(5*b*(6*a^2 + b^2) - a*(2*a^2 + 33*b^2)*Sin[c + d*x]))/(6*(a^
2 - b^2)^3*d) + (Sec[c + d*x]*(15*b^3*(6*a^2 + b^2) + a*(4*a^4 - 28*a^2*b^2
 - 81*b^4)*Sin[c + d*x]))/(6*(a^2 - b^2)^4*d)
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
```

```
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} - \frac{\int \frac{\sec^4(c+dx)(-2a+5b\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^4(c+dx)\left(2a^2+5b^2-28\right.}{a+b\sin(c+a}}{2\left(a^2-b^2\right.}$$

$$= \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5b\left(6a^2\right.\right.}{}$$

$$= \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5b\left(6a^2\right.\right.}{}$$

$$= \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5b\left(6a^2\right.\right.}{}$$

$$= \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5b\left(6a^2\right.\right.}{}$$

$$= \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\left(5b\left(6a^2\right.\right.}{}$$

$$= \frac{5b^4\left(6a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d} + \frac{b\sec^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{7ab\sec}{2\left(a^2-b^2\right)^2 d(}$$

Mathematica [A] time = 2.81134, size = 380, normalized size = 1.44

$$\frac{60b^4\left(6a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}} + \frac{66ab^5\cos(c+dx)}{(a-b)^4(a+b)^4(a+b\sin(c+dx))} + \frac{6b^5\cos(c+dx)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2} + \frac{2(4a+13b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^4\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{}{(a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((60*b^4*(6*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(9/2) + 1/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
2*Sin[(c + d*x)/2])/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (
2*(4*a + 13*b)*Sin[(c + d*x)/2])/((a + b)^4*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])) + (2*Sin[(c + d*x)/2])/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^3) - 1/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (2*(4*a -
 13*b)*Sin[(c + d*x)/2])/((a - b)^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
+ (6*b^5*Cos[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^2) + (66*a
*b^5*Cos[c + d*x])/((a - b)^4*(a + b)^4*(a + b*Sin[c + d*x])))/(12*d)
```

Maple [B] time = 0.151, size = 854, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/3/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1
)^2-1/d/(a+b)^4/(tan(1/2*d*x+1/2*c)-1)*a-5/2/d/(a+b)^4/(tan(1/2*d*x+1/2*c)-
1)*b-1/3/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/(a-b)^3/(tan(1/2*d*x+1/2*
c)+1)^2-1/d/(a-b)^4/(tan(1/2*d*x+1/2*c)+1)*a+5/2/d/(a-b)^4/(tan(1/2*d*x+1/2
*c)+1)*b+13/d*b^6/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*a*tan(1/2*d*x+1/2*c)^3-2/d*b^8/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a*tan(1/2*d*x+1/2*c)^3+12/d*b^5/(a-b)^4/
(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2*tan(1/2*d*x
+1/2*c)^2+23/d*b^7/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2-2/d*b^9/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c
)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a^2*tan(1/2*d*x+1/2*c)^2+35/d*b^6/(a-b)^4
/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a*tan(1/2*d*x+
1/2*c)-2/d*b^8/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)
*b+a)^2/a*tan(1/2*d*x+1/2*c)+12/d*b^5/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2-1/d*b^7/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2+30/d*b^4/(a-b)^4/(a+b)^4/(a^2-b^2)^(1/2
)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2+5/d*b^6/(a-b
)^4/(a+b)^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^
2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.68124, size = 2643, normalized size = 10.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^8*b - 16*a^6*b^3 + 24*a^4*b^5 - 16*a^2*b^7 + 4*b^9 + 2*(8*a^8*b
- 64*a^6*b^3 - 16*a^4*b^5 + 87*a^2*b^7 - 15*b^9)*cos(d*x + c)^4 - 4*(2*a^8*
b - a^6*b^3 - 9*a^4*b^5 + 13*a^2*b^7 - 5*b^9)*cos(d*x + c)^2 - 15*((6*a^2*b
^6 + b^8)*cos(d*x + c)^5 - 2*(6*a^3*b^5 + a*b^7)*cos(d*x + c)^3*sin(d*x + c
) - (6*a^4*b^4 + 7*a^2*b^6 + b^8)*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(((2*
a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x +
 c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^9 - 8*a^7*b^2 + 12*a^5*b^4 - 8*a^
3*b^6 + 2*a*b^8 - (4*a^7*b^2 - 32*a^5*b^4 - 53*a^3*b^6 + 81*a*b^8)*cos(d*x
+ c)^4 + 2*(2*a^9 - 15*a^7*b^2 + 33*a^5*b^4 - 29*a^3*b^6 + 9*a*b^8)*cos(d*x
 + c)^2)*sin(d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5
*a^2*b^10 - b^12)*d*cos(d*x + c)^5 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 1
0*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3*sin(d*x + c) - (a^12 - 4*a
^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^3), 1/6
*(2*a^8*b - 8*a^6*b^3 + 12*a^4*b^5 - 8*a^2*b^7 + 2*b^9 + (8*a^8*b - 64*a^6*
b^3 - 16*a^4*b^5 + 87*a^2*b^7 - 15*b^9)*cos(d*x + c)^4 - 2*(2*a^8*b - a^6*b
^3 - 9*a^4*b^5 + 13*a^2*b^7 - 5*b^9)*cos(d*x + c)^2 - 15*((6*a^2*b^6 + b^8)
*cos(d*x + c)^5 - 2*(6*a^3*b^5 + a*b^7)*cos(d*x + c)^3*sin(d*x + c) - (6*a^
4*b^4 + 7*a^2*b^6 + b^8)*cos(d*x + c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^9 - 8*a^7*b^2 + 12*a^5*b^
4 - 8*a^3*b^6 + 2*a*b^8 - (4*a^7*b^2 - 32*a^5*b^4 - 53*a^3*b^6 + 81*a*b^8)*
cos(d*x + c)^4 + 2*(2*a^9 - 15*a^7*b^2 + 33*a^5*b^4 - 29*a^3*b^6 + 9*a*b^8)
```

```
*cos(d*x + c)^2)*sin(d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4
*b^8 + 5*a^2*b^10 - b^12)*d*cos(d*x + c)^5 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7
*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3*sin(d*x + c) - (a^
12 - 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)
^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x))**3, x)
```

Giac [B] time = 1.22838, size = 840, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(15*(6*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^8 - 4*a^6*b^2 + 6*a^4*b
^4 - 4*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) + 3*(13*a^3*b^6*tan(1/2*d*x + 1/2*c)
^3 - 2*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*b^5*tan(1/2*d*x + 1/2*c)^2 + 2
3*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 - 2*b^9*tan(1/2*d*x + 1/2*c)^2 + 35*a^3*b^
6*tan(1/2*d*x + 1/2*c) - 2*a*b^8*tan(1/2*d*x + 1/2*c) + 12*a^4*b^5 - a^2*b^
7)/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*(a*tan(1/2*d*x + 1
/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) - 2*(3*a^5*tan(1/2*d*x + 1/2*c)^
5 - 12*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 9
*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 + 9*b^5*t
an(1/2*d*x + 1/2*c)^4 - 2*a^5*tan(1/2*d*x + 1/2*c)^3 + 32*a^3*b^2*tan(1/2*d
*x + 1/2*c)^3 + 42*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 60*a^2*b^3*tan(1/2*d*x +
1/2*c)^2 - 12*b^5*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*tan(1/2*d*x + 1/2*c) - 12*
a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*a*b^4*tan(1/2*d*x + 1/2*c) - 3*a^4*b + 32
```

```
*a^2*b^3 + 7*b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(tan(1/2
*d*x + 1/2*c)^2 - 1)^3))/d
```

3.461 $\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=207

$$\frac{\left(a^2-b^2\right)^3}{7b^7d(a+b\sin(c+dx))^7}-\frac{a\left(a^2-b^2\right)^2}{b^7d(a+b\sin(c+dx))^6}+\frac{5a^2-b^2}{b^7d(a+b\sin(c+dx))^3}-\frac{a\left(5a^2-3b^2\right)}{b^7d(a+b\sin(c+dx))^4}+\frac{3\left(-6a^2b^2+5\cdots\right.}{5b^7d(a+b\sin(\cdots}$$

```
[Out] (a^2 - b^2)^3/(7*b^7*d*(a + b*Sin[c + d*x])^7) - (a*(a^2 - b^2)^2)/(b^7*d*(
a + b*Sin[c + d*x])^6) + (3*(5*a^4 - 6*a^2*b^2 + b^4))/(5*b^7*d*(a + b*Sin[
c + d*x])^5) - (a*(5*a^2 - 3*b^2))/(b^7*d*(a + b*Sin[c + d*x])^4) + (5*a^2
- b^2)/(b^7*d*(a + b*Sin[c + d*x])^3) - (3*a)/(b^7*d*(a + b*Sin[c + d*x])^2
) + 1/(b^7*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.171168, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{\left(a^2-b^2\right)^3}{7b^7d(a+b\sin(c+dx))^7}-\frac{a\left(a^2-b^2\right)^2}{b^7d(a+b\sin(c+dx))^6}+\frac{5a^2-b^2}{b^7d(a+b\sin(c+dx))^3}-\frac{a\left(5a^2-3b^2\right)}{b^7d(a+b\sin(c+dx))^4}+\frac{3\left(-6a^2b^2+5\cdots\right.}{5b^7d(a+b\sin(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a^2 - b^2)^3/(7*b^7*d*(a + b*Sin[c + d*x])^7) - (a*(a^2 - b^2)^2)/(b^7*d*(
a + b*Sin[c + d*x])^6) + (3*(5*a^4 - 6*a^2*b^2 + b^4))/(5*b^7*d*(a + b*Sin[
c + d*x])^5) - (a*(5*a^2 - 3*b^2))/(b^7*d*(a + b*Sin[c + d*x])^4) + (5*a^2
- b^2)/(b^7*d*(a + b*Sin[c + d*x])^3) - (3*a)/(b^7*d*(a + b*Sin[c + d*x])^2
) + 1/(b^7*d*(a + b*Sin[c + d*x]))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{(a+x)^8}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\left(a^2-b^2\right)^3}{(a+x)^8} + \frac{6a\left(a^2-b^2\right)^2}{(a+x)^7} - \frac{3\left(5a^4-6a^2b^2+b^4\right)}{(a+x)^6} + \frac{4\left(5a^3-3ab^2\right)}{(a+x)^5} - \frac{3\left(5a^2-b^2\right)}{(a+x)^4} + \frac{6a}{(a+x)^3} - \frac{1}{(a+x)^2}\right)\right.}{b^7 d}$$
$$= \frac{\left(a^2-b^2\right)^3}{7b^7 d(a+b\sin(c+dx))^7} - \frac{a\left(a^2-b^2\right)^2}{b^7 d(a+b\sin(c+dx))^6} + \frac{3\left(5a^4-6a^2b^2+b^4\right)}{5b^7 d(a+b\sin(c+dx))^5} - \frac{a\left(5a\right.}{b^7 d(a+b\right.}$$

Mathematica [A] time = 1.02218, size = 171, normalized size = 0.83

$$\frac{\frac{\left(a^2-b^2\right)^3}{7(a+b\sin(c+dx))^7} - \frac{a\left(a^2-b^2\right)^2}{(a+b\sin(c+dx))^6} + \frac{5a^2-b^2}{(a+b\sin(c+dx))^3} - \frac{a\left(5a^2-3b^2\right)}{(a+b\sin(c+dx))^4} + \frac{3\left(-6a^2b^2+5a^4+b^4\right)}{5(a+b\sin(c+dx))^5} + \frac{1}{a+b\sin(c+dx)} - \frac{3a}{(a+b\sin(c+dx))^2}}{b^7 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((a^2 - b^2)^3/(7*(a + b*Sin[c + d*x])^7) - (a*(a^2 - b^2)^2)/(a + b*Sin[c
+ d*x])^6 + (3*(5*a^4 - 6*a^2*b^2 + b^4))/(5*(a + b*Sin[c + d*x])^5) - (a*(
5*a^2 - 3*b^2))/(a + b*Sin[c + d*x])^4 + (5*a^2 - b^2)/(a + b*Sin[c + d*x])
^3 - (3*a)/(a + b*Sin[c + d*x])^2 + (a + b*Sin[c + d*x])^(-1))/(b^7*d)
```

Maple [A] time = 0.154, size = 208, normalized size = 1.

$$\frac{1}{d}\left(-\frac{-a^6+3a^4b^2-3a^2b^4+b^6}{7b^7(a+b\sin(dx+c))^7} - \frac{a\left(a^4-2a^2b^2+b^4\right)}{b^7(a+b\sin(dx+c))^6} + \frac{1}{b^7(a+b\sin(dx+c))} - 3\frac{a}{b^7(a+b\sin(dx+c))^2} - \frac{-1}{3b^7(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-1/7*(-a^6+3*a^4*b^2-3*a^2*b^4+b^6)/b^7/(a+b*sin(d*x+c))^7-a*(a^4-2*a^
2*b^2+b^4)/b^7/(a+b*sin(d*x+c))^6+1/b^7/(a+b*sin(d*x+c))-3*a/b^7/(a+b*sin(d
*x+c))^2-1/3*(-15*a^2+3*b^2)/b^7/(a+b*sin(d*x+c))^3-a*(5*a^2-3*b^2)/b^7/(a+
b*sin(d*x+c))^4-1/5*(-15*a^4+18*a^2*b^2-3*b^4)/b^7/(a+b*sin(d*x+c))^5)
```

Maxima [A] time = 1.01252, size = 377, normalized size = 1.82

$$\frac{35\,b^6\sin(dx+c)^6+105\,ab^5\sin(dx+c)^5+5\,a^6-a^4b^2+a^2b^4-5\,b^6+35\left(5\,a^2b^4-b^6\right)\sin(dx+c)^4+35\left(5\,a^3b^3-ab^5\right)\mathrm{s}}{35\left(b^{14}\sin(dx+c)^7+7\,ab^{13}\sin(dx+c)^6+21\,a^2b^{12}\sin(dx+c)^5+35\,a^3b^{11}\sin(dx+c)^4+35\,a^4b^{10}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/35*(35*b^6*sin(d*x + c)^6 + 105*a*b^5*sin(d*x + c)^5 + 5*a^6 - a^4*b^2 +
a^2*b^4 - 5*b^6 + 35*(5*a^2*b^4 - b^6)*sin(d*x + c)^4 + 35*(5*a^3*b^3 - a*b
^5)*sin(d*x + c)^3 + 21*(5*a^4*b^2 - a^2*b^4 + b^6)*sin(d*x + c)^2 + 7*(5*a
^5*b - a^3*b^3 + a*b^5)*sin(d*x + c))/((b^14*sin(d*x + c)^7 + 7*a*b^13*sin(
d*x + c)^6 + 21*a^2*b^12*sin(d*x + c)^5 + 35*a^3*b^11*sin(d*x + c)^4 + 35*a
^4*b^10*sin(d*x + c)^3 + 21*a^5*b^9*sin(d*x + c)^2 + 7*a^6*b^8*sin(d*x + c)
 + a^7*b^7)*d)
```

Fricas [A] time = 4.18101, size = 882, normalized size = 4.26

$$\frac{35\,b^6\cos(dx+c)^6-5\,a^6-104\,a^4b^2-155\,a^2b^4-16\,b^6-35\left(5\,a^2b^4+2\,b^6\right)\cos(dx}{35\left(7\,ab^{13}d\cos(dx+c)^6-7\left(5\,a^3b^{11}+3\,ab^{13}\right)d\cos(dx+c)^4+7\left(3\,a^5b^9+10\,a^3b^{11}+3\,ab^{13}\right)d\cos(dx+c)^2-\left(a^7b^7+21\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^6*cos(d*x + c)^6 - 5*a^6 - 104*a^4*b^2 - 155*a^2*b^4 - 16*b^6 -
35*(5*a^2*b^4 + 2*b^6)*cos(d*x + c)^4 + 7*(15*a^4*b^2 + 47*a^2*b^4 + 8*b^6)
*cos(d*x + c)^2 - 7*(15*a*b^5*cos(d*x + c)^4 + 5*a^5*b + 24*a^3*b^3 + 11*a*
b^5 - 25*(a^3*b^3 + a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(7*a*b^13*d*cos(d*
x + c)^6 - 7*(5*a^3*b^11 + 3*a*b^13)*d*cos(d*x + c)^4 + 7*(3*a^5*b^9 + 10*a
^3*b^11 + 3*a*b^13)*d*cos(d*x + c)^2 - (a^7*b^7 + 21*a^5*b^9 + 35*a^3*b^11
+ 7*a*b^13)*d + (b^14*d*cos(d*x + c)^6 - 3*(7*a^2*b^12 + b^14)*d*cos(d*x +
c)^4 + (35*a^4*b^10 + 42*a^2*b^12 + 3*b^14)*d*cos(d*x + c)^2 - (7*a^6*b^8 +
```

```
35*a^4*b^10 + 21*a^2*b^12 + b^14)*d)*sin(d*x + c))
```

Sympy [A] time = 69.0541, size = 2718, normalized size = 13.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((zoo*x*cos(c)**7/sin(c)**8, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((16
/(35*d*sin(c + d*x)) - 8*cos(c + d*x)**2/(35*d*sin(c + d*x)**3) + 6*cos(c +
 d*x)**4/(35*d*sin(c + d*x)**5) - cos(c + d*x)**6/(7*d*sin(c + d*x)**7))/b*
*8, Eq(a, 0)), ((16*sin(c + d*x)**7/(35*d) + 8*sin(c + d*x)**5*cos(c + d*x)
**2/(5*d) + 2*sin(c + d*x)**3*cos(c + d*x)**4/d + sin(c + d*x)*cos(c + d*x)
**6/d)/a**8, Eq(b, 0)), (x*cos(c)**7/(a + b*sin(c))**8, Eq(d, 0)), (a**5*si
n(c + d*x)**4/(35*a**10*b**3*d + 245*a**9*b**4*d*sin(c + d*x) + 735*a**8*b*
*5*d*sin(c + d*x)**2 + 1225*a**7*b**6*d*sin(c + d*x)**3 + 1225*a**6*b**7*d*
sin(c + d*x)**4 + 735*a**5*b**8*d*sin(c + d*x)**5 + 245*a**4*b**9*d*sin(c +
 d*x)**6 + 35*a**3*b**10*d*sin(c + d*x)**7) + 2*a**5*sin(c + d*x)**2*cos(c
+ d*x)**2/(35*a**10*b**3*d + 245*a**9*b**4*d*sin(c + d*x) + 735*a**8*b**5*d
*sin(c + d*x)**2 + 1225*a**7*b**6*d*sin(c + d*x)**3 + 1225*a**6*b**7*d*sin(
c + d*x)**4 + 735*a**5*b**8*d*sin(c + d*x)**5 + 245*a**4*b**9*d*sin(c + d*x
)**6 + 35*a**3*b**10*d*sin(c + d*x)**7) + a**5*cos(c + d*x)**4/(35*a**10*b*
*3*d + 245*a**9*b**4*d*sin(c + d*x) + 735*a**8*b**5*d*sin(c + d*x)**2 + 122
5*a**7*b**6*d*sin(c + d*x)**3 + 1225*a**6*b**7*d*sin(c + d*x)**4 + 735*a**5
*b**8*d*sin(c + d*x)**5 + 245*a**4*b**9*d*sin(c + d*x)**6 + 35*a**3*b**10*d
*sin(c + d*x)**7) + 7*a**4*b*sin(c + d*x)**5/(35*a**10*b**3*d + 245*a**9*b*
*4*d*sin(c + d*x) + 735*a**8*b**5*d*sin(c + d*x)**2 + 1225*a**7*b**6*d*sin(
c + d*x)**3 + 1225*a**6*b**7*d*sin(c + d*x)**4 + 735*a**5*b**8*d*sin(c + d*
x)**5 + 245*a**4*b**9*d*sin(c + d*x)**6 + 35*a**3*b**10*d*sin(c + d*x)**7)
+ 14*a**4*b*sin(c + d*x)**3*cos(c + d*x)**2/(35*a**10*b**3*d + 245*a**9*b**
4*d*sin(c + d*x) + 735*a**8*b**5*d*sin(c + d*x)**2 + 1225*a**7*b**6*d*sin(c
 + d*x)**3 + 1225*a**6*b**7*d*sin(c + d*x)**4 + 735*a**5*b**8*d*sin(c + d*x
)**5 + 245*a**4*b**9*d*sin(c + d*x)**6 + 35*a**3*b**10*d*sin(c + d*x)**7) +
 7*a**4*b*sin(c + d*x)*cos(c + d*x)**4/(35*a**10*b**3*d + 245*a**9*b**4*d*s
in(c + d*x) + 735*a**8*b**5*d*sin(c + d*x)**2 + 1225*a**7*b**6*d*sin(c + d*
x)**3 + 1225*a**6*b**7*d*sin(c + d*x)**4 + 735*a**5*b**8*d*sin(c + d*x)**5
+ 245*a**4*b**9*d*sin(c + d*x)**6 + 35*a**3*b**10*d*sin(c + d*x)**7) + 16*a
**3*b**2*sin(c + d*x)**6/(35*a**10*b**3*d + 245*a**9*b**4*d*sin(c + d*x) +
735*a**8*b**5*d*sin(c + d*x)**2 + 1225*a**7*b**6*d*sin(c + d*x)**3 + 1225*a
**6*b**7*d*sin(c + d*x)**4 + 735*a**5*b**8*d*sin(c + d*x)**5 + 245*a**4*b**
```

```
9*d*sin(c + d*x)**6 + 35*a**3*b**10*d*sin(c + d*x)**7) + 27*a**3*b**2*sin(c
 + d*x)**4*cos(c + d*x)**2/(35*a**10*b**3*d + 245*a**9*b**4*d*sin(c + d*x)
+ 735*a**8*b**5*d*sin(c + d*x)**2 + 1225*a**7*b**6*d*sin(c + d*x)**3 + 1225
*a**6*b**7*d*sin(c + d*x)**4 + 735*a**5*b**8*d*sin(c + d*x)**5 + 245*a**4*b
**9*d*sin(c + d*x)**6 + 35*a**3*b**10*d*sin(c + d*x)**7) + 6*a**3*b**2*sin(
c + d*x)**2*cos(c + d*x)**4/(35*a**10*b**3*d + 245*a**9*b**4*d*sin(c + d*x)
 + 735*a**8*b**5*d*sin(c + d*x)**2 + 1225*a**7*b**6*d*sin(c + d*x)**3 + 122
5*a**6*b**7*d*sin(c + d*x)**4 + 735*a**5*b**8*d*sin(c + d*x)**5 + 245*a**4*
b**9*d*sin(c + d*x)**6 + 35*a**3*b**10*d*sin(c + d*x)**7) - 5*a**3*b**2*cos
(c + d*x)**6/(35*a**10*b**3*d + 245*a**9*b**4*d*sin(c + d*x) + 735*a**8*b**
5*d*sin(c + d*x)**2 + 1225*a**7*b**6*d*sin(c + d*x)**3 + 1225*a**6*b**7*d*s
in(c + d*x)**4 + 735*a**5*b**8*d*sin(c + d*x)**5 + 245*a**4*b**9*d*sin(c +
d*x)**6 + 35*a**3*b**10*d*sin(c + d*x)**7) + 16*a**2*b**3*sin(c + d*x)**7/(
35*a**10*b**3*d + 245*a**9*b**4*d*sin(c + d*x) + 735*a**8*b**5*d*sin(c + d*
x)**2 + 1225*a**7*b**6*d*sin(c + d*x)**3 + 1225*a**6*b**7*d*sin(c + d*x)**4
 + 735*a**5*b**8*d*sin(c + d*x)**5 + 245*a**4*b**9*d*sin(c + d*x)**6 + 35*a
**3*b**10*d*sin(c + d*x)**7) + 21*a**2*b**3*sin(c + d*x)**5*cos(c + d*x)**2
/(35*a**10*b**3*d + 245*a**9*b**4*d*sin(c + d*x) + 735*a**8*b**5*d*sin(c +
d*x)**2 + 1225*a**7*b**6*d*sin(c + d*x)**3 + 1225*a**6*b**7*d*sin(c + d*x)*
*4 + 735*a**5*b**8*d*sin(c + d*x)**5 + 245*a**4*b**9*d*sin(c + d*x)**6 + 35
*a**3*b**10*d*sin(c + d*x)**7) + 7*a*b**4*sin(c + d*x)**8/(35*a**10*b**3*d
+ 245*a**9*b**4*d*sin(c + d*x) + 735*a**8*b**5*d*sin(c + d*x)**2 + 1225*a**
7*b**6*d*sin(c + d*x)**3 + 1225*a**6*b**7*d*sin(c + d*x)**4 + 735*a**5*b**8
*d*sin(c + d*x)**5 + 245*a**4*b**9*d*sin(c + d*x)**6 + 35*a**3*b**10*d*sin(
c + d*x)**7) + 7*a*b**4*sin(c + d*x)**6*cos(c + d*x)**2/(35*a**10*b**3*d +
245*a**9*b**4*d*sin(c + d*x) + 735*a**8*b**5*d*sin(c + d*x)**2 + 1225*a**7*
b**6*d*sin(c + d*x)**3 + 1225*a**6*b**7*d*sin(c + d*x)**4 + 735*a**5*b**8*d
*sin(c + d*x)**5 + 245*a**4*b**9*d*sin(c + d*x)**6 + 35*a**3*b**10*d*sin(c
+ d*x)**7) + b**5*sin(c + d*x)**9/(35*a**10*b**3*d + 245*a**9*b**4*d*sin(c
+ d*x) + 735*a**8*b**5*d*sin(c + d*x)**2 + 1225*a**7*b**6*d*sin(c + d*x)**3
 + 1225*a**6*b**7*d*sin(c + d*x)**4 + 735*a**5*b**8*d*sin(c + d*x)**5 + 245
*a**4*b**9*d*sin(c + d*x)**6 + 35*a**3*b**10*d*sin(c + d*x)**7) + b**5*sin(
c + d*x)**7*cos(c + d*x)**2/(35*a**10*b**3*d + 245*a**9*b**4*d*sin(c + d*x)
 + 735*a**8*b**5*d*sin(c + d*x)**2 + 1225*a**7*b**6*d*sin(c + d*x)**3 + 122
5*a**6*b**7*d*sin(c + d*x)**4 + 735*a**5*b**8*d*sin(c + d*x)**5 + 245*a**4*
b**9*d*sin(c + d*x)**6 + 35*a**3*b**10*d*sin(c + d*x)**7), True))
```

Giac [A] time = 1.44447, size = 290, normalized size = 1.4

$$\frac{35\,b^6\sin(dx+c)^6 + 105\,ab^5\sin(dx+c)^5 + 175\,a^2b^4\sin(dx+c)^4 - 35\,b^6\sin(dx+c)^4 + 175\,a^3b^3\sin(dx+c)^3 - 35\,ab^5\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/35*(35*b^6*sin(d*x + c)^6 + 105*a*b^5*sin(d*x + c)^5 + 175*a^2*b^4*sin(d*
x + c)^4 - 35*b^6*sin(d*x + c)^4 + 175*a^3*b^3*sin(d*x + c)^3 - 35*a*b^5*si
n(d*x + c)^3 + 105*a^4*b^2*sin(d*x + c)^2 - 21*a^2*b^4*sin(d*x + c)^2 + 21*
b^6*sin(d*x + c)^2 + 35*a^5*b*sin(d*x + c) - 7*a^3*b^3*sin(d*x + c) + 7*a*b
^5*sin(d*x + c) + 5*a^6 - a^4*b^2 + a^2*b^4 - 5*b^6)/((b*sin(d*x + c) + a)^
7*b^7*d)
```

3.462 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=141

$$-\frac{\left(a^2-b^2\right)^2}{7b^5d(a+b\sin(c+dx))^7}+\frac{2a\left(a^2-b^2\right)}{3b^5d(a+b\sin(c+dx))^6}-\frac{2\left(3a^2-b^2\right)}{5b^5d(a+b\sin(c+dx))^5}-\frac{1}{3b^5d(a+b\sin(c+dx))^3}+\frac{}{b^5d(a+b\,\mathrm{s}}$$

```
[Out] -(a^2 - b^2)^2/(7*b^5*d*(a + b*Sin[c + d*x])^7) + (2*a*(a^2 - b^2))/(3*b^5*
d*(a + b*Sin[c + d*x])^6) - (2*(3*a^2 - b^2))/(5*b^5*d*(a + b*Sin[c + d*x])
^5) + a/(b^5*d*(a + b*Sin[c + d*x])^4) - 1/(3*b^5*d*(a + b*Sin[c + d*x])^3)
```

Rubi [A] time = 0.108766, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{\left(a^2-b^2\right)^2}{7b^5d(a+b\sin(c+dx))^7}+\frac{2a\left(a^2-b^2\right)}{3b^5d(a+b\sin(c+dx))^6}-\frac{2\left(3a^2-b^2\right)}{5b^5d(a+b\sin(c+dx))^5}-\frac{1}{3b^5d(a+b\sin(c+dx))^3}+\frac{}{b^5d(a+b\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -(a^2 - b^2)^2/(7*b^5*d*(a + b*Sin[c + d*x])^7) + (2*a*(a^2 - b^2))/(3*b^5*
d*(a + b*Sin[c + d*x])^6) - (2*(3*a^2 - b^2))/(5*b^5*d*(a + b*Sin[c + d*x])
^5) + a/(b^5*d*(a + b*Sin[c + d*x])^4) - 1/(3*b^5*d*(a + b*Sin[c + d*x])^3)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{(a+x)^8}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a^2-b^2)^2}{(a+x)^8} - \frac{4(a^3-ab^2)}{(a+x)^7} + \frac{2(3a^2-b^2)}{(a+x)^6} - \frac{4a}{(a+x)^5} + \frac{1}{(a+x)^4}\right)dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= -\frac{(a^2-b^2)^2}{7b^5d(a+b\sin(c+dx))^7} + \frac{2a(a^2-b^2)}{3b^5d(a+b\sin(c+dx))^6} - \frac{2(3a^2-b^2)}{5b^5d(a+b\sin(c+dx))^5} + \frac{}{b^5d(a+}$$

Mathematica [A] time = 0.285082, size = 107, normalized size = 0.76

$$-\frac{21b^2(a^2-2b^2)\sin^2(c+dx) + 7ab(a^2-2b^2)\sin(c+dx) - 2a^2b^2 + a^4 + 35ab^3\sin^3(c+dx) + 35b^4\sin^4(c+dx) + 15b^4}{105b^5d(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -(a^4 - 2*a^2*b^2 + 15*b^4 + 7*a*b*(a^2 - 2*b^2)*Sin[c + d*x] + 21*b^2*(a^2
 - 2*b^2)*Sin[c + d*x]^2 + 35*a*b^3*Sin[c + d*x]^3 + 35*b^4*Sin[c + d*x]^4)
/(105*b^5*d*(a + b*Sin[c + d*x])^7)
```

Maple [A] time = 0.159, size = 127, normalized size = 0.9

$$\frac{1}{d}\left(-\frac{a^4-2a^2b^2+b^4}{7b^5(a+b\sin(dx+c))^7} - \frac{6a^2-2b^2}{5b^5(a+b\sin(dx+c))^5} - \frac{1}{3b^5(a+b\sin(dx+c))^3} + \frac{2a(a^2-b^2)}{3b^5(a+b\sin(dx+c))^6} + \frac{}{b^5(a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-1/7*(a^4-2*a^2*b^2+b^4)/b^5/(a+b*sin(d*x+c))^7-1/5*(6*a^2-2*b^2)/b^5/
(a+b*sin(d*x+c))^5-1/3/b^5/(a+b*sin(d*x+c))^3+2/3*a*(a^2-b^2)/b^5/(a+b*sin(
d*x+c))^6+a/b^5/(a+b*sin(d*x+c))^4)
```

Maxima [A] time = 1.00109, size = 278, normalized size = 1.97

$$-\frac{35\,b^4\sin\left(dx+c\right)^4+35\,ab^3\sin\left(dx+c\right)^3+a^4-2\,a^2b^2+15\,b^4+21\,\left(a^2b^2-2\,b^4\right)\sin\left(dx+c\right)^2+7}{105\,\left(b^{12}\sin\left(dx+c\right)^7+7\,ab^{11}\sin\left(dx+c\right)^6+21\,a^2b^{10}\sin\left(dx+c\right)^5+35\,a^3b^9\sin\left(dx+c\right)^4+35\,a^4b^8\sin\left(dx+c\right)^3+21\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^4*sin(d*x + c)^4 + 35*a*b^3*sin(d*x + c)^3 + a^4 - 2*a^2*b^2 +
 15*b^4 + 21*(a^2*b^2 - 2*b^4)*sin(d*x + c)^2 + 7*(a^3*b - 2*a*b^3)*sin(d*x
 + c))/((b^12*sin(d*x + c)^7 + 7*a*b^11*sin(d*x + c)^6 + 21*a^2*b^10*sin(d*
x + c)^5 + 35*a^3*b^9*sin(d*x + c)^4 + 35*a^4*b^8*sin(d*x + c)^3 + 21*a^5*b
^7*sin(d*x + c)^2 + 7*a^6*b^6*sin(d*x + c) + a^7*b^5)*d)
```

Fricas [B] time = 4.10491, size = 705, normalized size = 5.

$$\frac{35\,b^4\cos\left(dx+c\right)^4+a^4+19\,a^2b^2}{105\,\left(7\,ab^{11}d\cos\left(dx+c\right)^6-7\,\left(5\,a^3b^9+3\,ab^{11}\right)d\cos\left(dx+c\right)^4+7\,\left(3\,a^5b^7+10\,a^3b^9+3\,ab^{11}\right)d\cos\left(dx+c\right)^2-\left(a^7b^5+21\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(35*b^4*cos(d*x + c)^4 + a^4 + 19*a^2*b^2 + 8*b^4 - 7*(3*a^2*b^2 + 4*
b^4)*cos(d*x + c)^2 - 7*(5*a*b^3*cos(d*x + c)^2 - a^3*b - 3*a*b^3)*sin(d*x
+ c))/(7*a*b^11*d*cos(d*x + c)^6 - 7*(5*a^3*b^9 + 3*a*b^11)*d*cos(d*x + c)^
4 + 7*(3*a^5*b^7 + 10*a^3*b^9 + 3*a*b^11)*d*cos(d*x + c)^2 - (a^7*b^5 + 21*
a^5*b^7 + 35*a^3*b^9 + 7*a*b^11)*d + (b^12*d*cos(d*x + c)^6 - 3*(7*a^2*b^10
 + b^12)*d*cos(d*x + c)^4 + (35*a^4*b^8 + 42*a^2*b^10 + 3*b^12)*d*cos(d*x +
 c)^2 - (7*a^6*b^6 + 35*a^4*b^8 + 21*a^2*b^10 + b^12)*d)*sin(d*x + c))
```

Sympy [A] time = 65.6567, size = 2181, normalized size = 15.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((zoo*x*cos(c)**5/sin(c)**8, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-8
/(105*d*sin(c + d*x)**3) + 4*cos(c + d*x)**2/(35*d*sin(c + d*x)**5) - cos(c
 + d*x)**4/(7*d*sin(c + d*x)**7))/b**8, Eq(a, 0)), ((8*sin(c + d*x)**5/(15*
d) + 4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + sin(c + d*x)*cos(c + d*x)**4
/d)/a**8, Eq(b, 0)), (x*cos(c)**5/(a + b*sin(c))**8, Eq(d, 0)), (-15*a**5*s
in(c + d*x)**4/(105*a**12*b*d + 735*a**11*b**2*d*sin(c + d*x) + 2205*a**10*
b**3*d*sin(c + d*x)**2 + 3675*a**9*b**4*d*sin(c + d*x)**3 + 3675*a**8*b**5*
d*sin(c + d*x)**4 + 2205*a**7*b**6*d*sin(c + d*x)**5 + 735*a**6*b**7*d*sin(
c + d*x)**6 + 105*a**5*b**8*d*sin(c + d*x)**7) - 30*a**5*sin(c + d*x)**2*co
s(c + d*x)**2/(105*a**12*b*d + 735*a**11*b**2*d*sin(c + d*x) + 2205*a**10*b
**3*d*sin(c + d*x)**2 + 3675*a**9*b**4*d*sin(c + d*x)**3 + 3675*a**8*b**5*d
*sin(c + d*x)**4 + 2205*a**7*b**6*d*sin(c + d*x)**5 + 735*a**6*b**7*d*sin(c
 + d*x)**6 + 105*a**5*b**8*d*sin(c + d*x)**7) - 15*a**5*cos(c + d*x)**4/(10
5*a**12*b*d + 735*a**11*b**2*d*sin(c + d*x) + 2205*a**10*b**3*d*sin(c + d*x
)**2 + 3675*a**9*b**4*d*sin(c + d*x)**3 + 3675*a**8*b**5*d*sin(c + d*x)**4
+ 2205*a**7*b**6*d*sin(c + d*x)**5 + 735*a**6*b**7*d*sin(c + d*x)**6 + 105*
a**5*b**8*d*sin(c + d*x)**7) - 49*a**4*b*sin(c + d*x)**5/(105*a**12*b*d + 7
35*a**11*b**2*d*sin(c + d*x) + 2205*a**10*b**3*d*sin(c + d*x)**2 + 3675*a**
9*b**4*d*sin(c + d*x)**3 + 3675*a**8*b**5*d*sin(c + d*x)**4 + 2205*a**7*b**
6*d*sin(c + d*x)**5 + 735*a**6*b**7*d*sin(c + d*x)**6 + 105*a**5*b**8*d*sin
(c + d*x)**7) - 70*a**4*b*sin(c + d*x)**3*cos(c + d*x)**2/(105*a**12*b*d +
735*a**11*b**2*d*sin(c + d*x) + 2205*a**10*b**3*d*sin(c + d*x)**2 + 3675*a*
*9*b**4*d*sin(c + d*x)**3 + 3675*a**8*b**5*d*sin(c + d*x)**4 + 2205*a**7*b*
*6*d*sin(c + d*x)**5 + 735*a**6*b**7*d*sin(c + d*x)**6 + 105*a**5*b**8*d*si
n(c + d*x)**7) - 63*a**3*b**2*sin(c + d*x)**6/(105*a**12*b*d + 735*a**11*b*
*2*d*sin(c + d*x) + 2205*a**10*b**3*d*sin(c + d*x)**2 + 3675*a**9*b**4*d*si
n(c + d*x)**3 + 3675*a**8*b**5*d*sin(c + d*x)**4 + 2205*a**7*b**6*d*sin(c +
 d*x)**5 + 735*a**6*b**7*d*sin(c + d*x)**6 + 105*a**5*b**8*d*sin(c + d*x)**
7) - 70*a**3*b**2*sin(c + d*x)**4*cos(c + d*x)**2/(105*a**12*b*d + 735*a**1
1*b**2*d*sin(c + d*x) + 2205*a**10*b**3*d*sin(c + d*x)**2 + 3675*a**9*b**4*
d*sin(c + d*x)**3 + 3675*a**8*b**5*d*sin(c + d*x)**4 + 2205*a**7*b**6*d*sin
(c + d*x)**5 + 735*a**6*b**7*d*sin(c + d*x)**6 + 105*a**5*b**8*d*sin(c + d*
x)**7) - 41*a**2*b**3*sin(c + d*x)**7/(105*a**12*b*d + 735*a**11*b**2*d*sin
(c + d*x) + 2205*a**10*b**3*d*sin(c + d*x)**2 + 3675*a**9*b**4*d*sin(c + d*
x)**3 + 3675*a**8*b**5*d*sin(c + d*x)**4 + 2205*a**7*b**6*d*sin(c + d*x)**5
 + 735*a**6*b**7*d*sin(c + d*x)**6 + 105*a**5*b**8*d*sin(c + d*x)**7) - 42*
a**2*b**3*sin(c + d*x)**5*cos(c + d*x)**2/(105*a**12*b*d + 735*a**11*b**2*d
*sin(c + d*x) + 2205*a**10*b**3*d*sin(c + d*x)**2 + 3675*a**9*b**4*d*sin(c
+ d*x)**3 + 3675*a**8*b**5*d*sin(c + d*x)**4 + 2205*a**7*b**6*d*sin(c + d*x
)**5 + 735*a**6*b**7*d*sin(c + d*x)**6 + 105*a**5*b**8*d*sin(c + d*x)**7) -
 14*a*b**4*sin(c + d*x)**8/(105*a**12*b*d + 735*a**11*b**2*d*sin(c + d*x) +
 2205*a**10*b**3*d*sin(c + d*x)**2 + 3675*a**9*b**4*d*sin(c + d*x)**3 + 367
```

```
5*a**8*b**5*d*sin(c + d*x)**4 + 2205*a**7*b**6*d*sin(c + d*x)**5 + 735*a**6
*b**7*d*sin(c + d*x)**6 + 105*a**5*b**8*d*sin(c + d*x)**7) - 14*a*b**4*sin(
c + d*x)**6*cos(c + d*x)**2/(105*a**12*b*d + 735*a**11*b**2*d*sin(c + d*x)
+ 2205*a**10*b**3*d*sin(c + d*x)**2 + 3675*a**9*b**4*d*sin(c + d*x)**3 + 36
75*a**8*b**5*d*sin(c + d*x)**4 + 2205*a**7*b**6*d*sin(c + d*x)**5 + 735*a**
6*b**7*d*sin(c + d*x)**6 + 105*a**5*b**8*d*sin(c + d*x)**7) - 2*b**5*sin(c
+ d*x)**9/(105*a**12*b*d + 735*a**11*b**2*d*sin(c + d*x) + 2205*a**10*b**3*
d*sin(c + d*x)**2 + 3675*a**9*b**4*d*sin(c + d*x)**3 + 3675*a**8*b**5*d*sin
(c + d*x)**4 + 2205*a**7*b**6*d*sin(c + d*x)**5 + 735*a**6*b**7*d*sin(c + d
*x)**6 + 105*a**5*b**8*d*sin(c + d*x)**7) - 2*b**5*sin(c + d*x)**7*cos(c +
d*x)**2/(105*a**12*b*d + 735*a**11*b**2*d*sin(c + d*x) + 2205*a**10*b**3*d*
sin(c + d*x)**2 + 3675*a**9*b**4*d*sin(c + d*x)**3 + 3675*a**8*b**5*d*sin(c
 + d*x)**4 + 2205*a**7*b**6*d*sin(c + d*x)**5 + 735*a**6*b**7*d*sin(c + d*x
)**6 + 105*a**5*b**8*d*sin(c + d*x)**7), True))
```

Giac [A] time = 1.44224, size = 158, normalized size = 1.12

$$-\frac{35\, b^4 \sin\left(dx+c\right)^4 + 35\, ab^3 \sin\left(dx+c\right)^3 + 21\, a^2b^2 \sin\left(dx+c\right)^2 - 42\, b^4 \sin\left(dx+c\right)^2 + 7\, a^3b \sin\left(dx+c\right) - 14\, ab^3 \sin\left(dx\right.}{105\left(b\sin\left(dx+c\right)+a\right)^7 b^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^4*sin(d*x + c)^4 + 35*a*b^3*sin(d*x + c)^3 + 21*a^2*b^2*sin(d*
x + c)^2 - 42*b^4*sin(d*x + c)^2 + 7*a^3*b*sin(d*x + c) - 14*a*b^3*sin(d*x
+ c) + a^4 - 2*a^2*b^2 + 15*b^4)/((b*sin(d*x + c) + a)^7*b^5*d)
```

3.463 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=77

$$\frac{a^2-b^2}{7b^3d(a+b\sin(c+dx))^7}-\frac{a}{3b^3d(a+b\sin(c+dx))^6}+\frac{1}{5b^3d(a+b\sin(c+dx))^5}$$

```
[Out] (a^2 - b^2)/(7*b^3*d*(a + b*Sin[c + d*x])^7) - a/(3*b^3*d*(a + b*Sin[c + d*
x])^6) + 1/(5*b^3*d*(a + b*Sin[c + d*x])^5)
```

Rubi [A] time = 0.0715896, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{a^2-b^2}{7b^3d(a+b\sin(c+dx))^7}-\frac{a}{3b^3d(a+b\sin(c+dx))^6}+\frac{1}{5b^3d(a+b\sin(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a^2 - b^2)/(7*b^3*d*(a + b*Sin[c + d*x])^7) - a/(3*b^3*d*(a + b*Sin[c + d*
x])^6) + 1/(5*b^3*d*(a + b*Sin[c + d*x])^5)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^8}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{(a+x)^8} + \frac{2a}{(a+x)^7} - \frac{1}{(a+x)^6}\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{a^2-b^2}{7b^3d(a+b\sin(c+dx))^7} - \frac{a}{3b^3d(a+b\sin(c+dx))^6} + \frac{1}{5b^3d(a+b\sin(c+dx))^5}$$

Mathematica [A] time = 0.195438, size = 54, normalized size = 0.7

$$\frac{a^2+7ab\sin(c+dx)+21b^2\sin^2(c+dx)-15b^2}{105b^3d(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a^2 - 15*b^2 + 7*a*b*Sin[c + d*x] + 21*b^2*Sin[c + d*x]^2)/(105*b^3*d*(a +
 b*Sin[c + d*x])^7)
```

Maple [A] time = 0.155, size = 67, normalized size = 0.9

$$\frac{1}{d}\left(-\frac{-a^2+b^2}{7\,b^3\,(a+b\sin(dx+c))^7} + \frac{1}{5\,b^3\,(a+b\sin(dx+c))^5} - \frac{a}{3\,b^3\,(a+b\sin(dx+c))^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/d*(-1/7*(-a^2+b^2)/b^3/(a+b*sin(d*x+c))^7+1/5/b^3/(a+b*sin(d*x+c))^5-1/3*
a/b^3/(a+b*sin(d*x+c))^6)
```

Maxima [B] time = 0.989478, size = 204, normalized size = 2.65

$$\frac{21\,b^2\sin(dx+c)^2+7\,ab\sin(dx+c)+a^2-15\,b^2}{105\left(b^{10}\sin(dx+c)^7+7\,ab^9\sin(dx+c)^6+21\,a^2b^8\sin(dx+c)^5+35\,a^3b^7\sin(dx+c)^4+35\,a^4b^6\sin(dx+c)^3+21\,a^5b^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/105*(21*b^2*sin(d*x + c)^2 + 7*a*b*sin(d*x + c) + a^2 - 15*b^2)/((b^10*si
n(d*x + c)^7 + 7*a*b^9*sin(d*x + c)^6 + 21*a^2*b^8*sin(d*x + c)^5 + 35*a^3*
b^7*sin(d*x + c)^4 + 35*a^4*b^6*sin(d*x + c)^3 + 21*a^5*b^5*sin(d*x + c)^2
+ 7*a^6*b^4*sin(d*x + c) + a^7*b^3)*d)
```

Fricas [B] time = 4.19691, size = 572, normalized size = 7.43

$$\overline{105\left(7\,ab^9d\cos\left(dx+c\right)^6-7\left(5\,a^3b^7+3\,ab^9\right)d\cos\left(dx+c\right)^4+7\left(3\,a^5b^5+10\,a^3b^7+3\,ab^9\right)d\cos\left(dx+c\right)^2-\left(a^7b^3+21\,a^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(21*b^2*cos(d*x + c)^2 - 7*a*b*sin(d*x + c) - a^2 - 6*b^2)/(7*a*b^9*d
*cos(d*x + c)^6 - 7*(5*a^3*b^7 + 3*a*b^9)*d*cos(d*x + c)^4 + 7*(3*a^5*b^5 +
 10*a^3*b^7 + 3*a*b^9)*d*cos(d*x + c)^2 - (a^7*b^3 + 21*a^5*b^5 + 35*a^3*b^
7 + 7*a*b^9)*d + (b^10*d*cos(d*x + c)^6 - 3*(7*a^2*b^8 + b^10)*d*cos(d*x +
c)^4 + (35*a^4*b^6 + 42*a^2*b^8 + 3*b^10)*d*cos(d*x + c)^2 - (7*a^6*b^4 + 3
5*a^4*b^6 + 21*a^2*b^8 + b^10)*d)*sin(d*x + c))
```

Sympy [A] time = 63.344, size = 2632, normalized size = 34.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((zoo*x*cos(c)**3/sin(c)**8, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((2/
(35*d*sin(c + d*x)**5) - cos(c + d*x)**2/(7*d*sin(c + d*x)**7))/b**8, Eq(a,
 0)), ((2*sin(c + d*x)**3/(3*d) + sin(c + d*x)*cos(c + d*x)**2/d)/a**8, Eq(
b, 0)), (x*cos(c)**3/(a + b*sin(c))**8, Eq(d, 0)), (-2*a**9/(105*a**14*b**3
*d + 735*a**13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)**2 + 36
75*a**11*b**6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4 + 2205*
```

```
a**9*b**8*d*sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105*a**7*b*
*10*d*sin(c + d*x)**7) - 14*a**8*b*sin(c + d*x)/(105*a**14*b**3*d + 735*a**
13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)**2 + 3675*a**11*b**
6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4 + 2205*a**9*b**8*d*
sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105*a**7*b**10*d*sin(c
+ d*x)**7) - 42*a**7*b**2*sin(c + d*x)**2/(105*a**14*b**3*d + 735*a**13*b**
4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)**2 + 3675*a**11*b**6*d*si
n(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4 + 2205*a**9*b**8*d*sin(c
+ d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105*a**7*b**10*d*sin(c + d*x)
**7) + 105*a**6*b**3*sin(c + d*x)*cos(c + d*x)**2/(105*a**14*b**3*d + 735*a
**13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)**2 + 3675*a**11*b
**6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4 + 2205*a**9*b**8*
d*sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105*a**7*b**10*d*sin(
c + d*x)**7) + 210*a**5*b**4*sin(c + d*x)**4/(105*a**14*b**3*d + 735*a**13*
b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)**2 + 3675*a**11*b**6*d
*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4 + 2205*a**9*b**8*d*sin
(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105*a**7*b**10*d*sin(c + d
*x)**7) + 315*a**5*b**4*sin(c + d*x)**2*cos(c + d*x)**2/(105*a**14*b**3*d +
 735*a**13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)**2 + 3675*a
**11*b**6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4 + 2205*a**9
*b**8*d*sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105*a**7*b**10*
d*sin(c + d*x)**7) + 462*a**4*b**5*sin(c + d*x)**5/(105*a**14*b**3*d + 735*
a**13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)**2 + 3675*a**11*
b**6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4 + 2205*a**9*b**8
*d*sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105*a**7*b**10*d*sin
(c + d*x)**7) + 525*a**4*b**5*sin(c + d*x)**3*cos(c + d*x)**2/(105*a**14*b*
*3*d + 735*a**13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)**2 +
3675*a**11*b**6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4 + 220
5*a**9*b**8*d*sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105*a**7*
b**10*d*sin(c + d*x)**7) + 504*a**3*b**6*sin(c + d*x)**6/(105*a**14*b**3*d
+ 735*a**13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)**2 + 3675*
a**11*b**6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4 + 2205*a**
9*b**8*d*sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105*a**7*b**10
*d*sin(c + d*x)**7) + 525*a**3*b**6*sin(c + d*x)**4*cos(c + d*x)**2/(105*a*
*14*b**3*d + 735*a**13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)
**2 + 3675*a**11*b**6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4
 + 2205*a**9*b**8*d*sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105
*a**7*b**10*d*sin(c + d*x)**7) + 312*a**2*b**7*sin(c + d*x)**7/(105*a**14*b
**3*d + 735*a**13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)**2 +
 3675*a**11*b**6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4 + 22
05*a**9*b**8*d*sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105*a**7
*b**10*d*sin(c + d*x)**7) + 315*a**2*b**7*sin(c + d*x)**5*cos(c + d*x)**2/(
105*a**14*b**3*d + 735*a**13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c
+ d*x)**2 + 3675*a**11*b**6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d
*x)**4 + 2205*a**9*b**8*d*sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6
```

```
 + 105*a**7*b**10*d*sin(c + d*x)**7) + 105*a*b**8*sin(c + d*x)**8/(105*a**1
4*b**3*d + 735*a**13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)**
2 + 3675*a**11*b**6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4 +
 2205*a**9*b**8*d*sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105*a
**7*b**10*d*sin(c + d*x)**7) + 105*a*b**8*sin(c + d*x)**6*cos(c + d*x)**2/(
105*a**14*b**3*d + 735*a**13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c
+ d*x)**2 + 3675*a**11*b**6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d
*x)**4 + 2205*a**9*b**8*d*sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6
 + 105*a**7*b**10*d*sin(c + d*x)**7) + 15*b**9*sin(c + d*x)**9/(105*a**14*b
**3*d + 735*a**13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)**2 +
 3675*a**11*b**6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4 + 22
05*a**9*b**8*d*sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105*a**7
*b**10*d*sin(c + d*x)**7) + 15*b**9*sin(c + d*x)**7*cos(c + d*x)**2/(105*a*
*14*b**3*d + 735*a**13*b**4*d*sin(c + d*x) + 2205*a**12*b**5*d*sin(c + d*x)
**2 + 3675*a**11*b**6*d*sin(c + d*x)**3 + 3675*a**10*b**7*d*sin(c + d*x)**4
 + 2205*a**9*b**8*d*sin(c + d*x)**5 + 735*a**8*b**9*d*sin(c + d*x)**6 + 105
*a**7*b**10*d*sin(c + d*x)**7), True))
```

Giac [A] time = 1.42951, size = 70, normalized size = 0.91

$$\frac{21\,b^2\sin(dx+c)^2+7\,ab\sin(dx+c)+a^2-15\,b^2}{105\,(b\sin(dx+c)+a)^7b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/105*(21*b^2*sin(d*x + c)^2 + 7*a*b*sin(d*x + c) + a^2 - 15*b^2)/((b*sin(d
*x + c) + a)^7*b^3*d)
```

3.464 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{7bd(a+b\sin(c+dx))^7}$$

```
[Out] -1/(7*b*d*(a + b*Sin[c + d*x])^7)
```

Rubi [A] time = 0.0269824, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 32}

$$-\frac{1}{7bd(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/(7*b*d*(a + b*Sin[c + d*x])^7)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^8}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^8}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{1}{7bd(a+b\sin(c+dx))^7}\end{aligned}$$

Mathematica [A] time = 0.0780185, size = 22, normalized size = 1.

$$-\frac{1}{7bd(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/(7*b*d*(a + b*Sin[c + d*x])^7)
```

Maple [A] time = 0.047, size = 21, normalized size = 1.

$$-\frac{1}{7\,bd\,(a+b\sin(dx+c))^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^8,x)
```

```
[Out] -1/7/b/d/(a+b*sin(d*x+c))^7
```

Maxima [A] time = 0.964449, size = 27, normalized size = 1.23

$$-\frac{1}{7\,(b\sin(dx+c)+a)^7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/7/((b*sin(d*x + c) + a)^7*b*d)
```

Fricas [B] time = 4.18221, size = 482, normalized size = 21.91

$$\overline{7\left(7\,ab^7d\cos(dx+c)^6-7\left(5\,a^3b^5+3\,ab^7\right)d\cos(dx+c)^4+7\left(3\,a^5b^3+10\,a^3b^5+3\,ab^7\right)d\cos(dx+c)^2-\left(a^7b+21\,a^5b^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/7/(7*a*b^7*d*cos(d*x + c)^6 - 7*(5*a^3*b^5 + 3*a*b^7)*d*cos(d*x + c)^4 +
7*(3*a^5*b^3 + 10*a^3*b^5 + 3*a*b^7)*d*cos(d*x + c)^2 - (a^7*b + 21*a^5*b^3
 + 35*a^3*b^5 + 7*a*b^7)*d + (b^8*d*cos(d*x + c)^6 - 3*(7*a^2*b^6 + b^8)*d*
cos(d*x + c)^4 + (35*a^4*b^4 + 42*a^2*b^6 + 3*b^8)*d*cos(d*x + c)^2 - (7*a^
6*b^2 + 35*a^4*b^4 + 21*a^2*b^6 + b^8)*d)*sin(d*x + c))
```

Sympy [A] time = 55.0178, size = 167, normalized size = 7.59

$$\begin{cases} \frac{x\cos(c)}{a^8} \\ \frac{\sin(c+dx)}{a^8 d} \\ \frac{x\cos(c)}{(a+b\sin(c))^8} \\ -\frac{1}{7a^7bd+49a^6b^2d\sin(c+dx)+147a^5b^3d\sin^2(c+dx)+245a^4b^4d\sin^3(c+dx)+245a^3b^5d\sin^4(c+dx)+147a^2b^6d\sin^5(c+dx)+49ab^7d\sin^6(c+dx)+7b^8d\sin^7(c+dx)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((x*cos(c)/a**8, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**8*d), Eq(
b, 0)), (x*cos(c)/(a + b*sin(c))**8, Eq(d, 0)), (-1/(7*a**7*b*d + 49*a**6*b
**2*d*sin(c + d*x) + 147*a**5*b**3*d*sin(c + d*x)**2 + 245*a**4*b**4*d*sin(
c + d*x)**3 + 245*a**3*b**5*d*sin(c + d*x)**4 + 147*a**2*b**6*d*sin(c + d*x
)**5 + 49*a*b**7*d*sin(c + d*x)**6 + 7*b**8*d*sin(c + d*x)**7), True))
```

Giac [A] time = 1.39745, size = 27, normalized size = 1.23

$$-\frac{1}{7\,(b\sin(dx+c)+a)^7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/7/((b*sin(d*x + c) + a)^7*b*d)
```

3.465 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=385

$$\frac{b\left(35a^4b^2+21a^2b^4+7a^6+b^6\right)}{d\left(a^2-b^2\right)^7(a+b\sin(c+dx))}+\frac{ab\left(3a^2+b^2\right)\left(a^2+3b^2\right)}{d\left(a^2-b^2\right)^6(a+b\sin(c+dx))^2}+\frac{b\left(10a^2b^2+5a^4+b^4\right)}{3d\left(a^2-b^2\right)^5(a+b\sin(c+dx))^3}+\frac{ab\,(\ldots}{d\left(a^2-b^2\right)^4(a\ldots}$$

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^8*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)^8
*d) - (8*a*b*(a^2 + b^2)*(a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(
(a^2 - b^2)^8*d) + b/(7*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) + (a*b)/(3*(a
^2 - b^2)^2*d*(a + b*Sin[c + d*x])^6) + (b*(3*a^2 + b^2))/(5*(a^2 - b^2)^3*
d*(a + b*Sin[c + d*x])^5) + (a*b*(a^2 + b^2))/((a^2 - b^2)^4*d*(a + b*Sin[c
 + d*x])^4) + (b*(5*a^4 + 10*a^2*b^2 + b^4))/(3*(a^2 - b^2)^5*d*(a + b*Sin[
c + d*x])^3) + (a*b*(3*a^2 + b^2)*(a^2 + 3*b^2))/((a^2 - b^2)^6*d*(a + b*Si
n[c + d*x])^2) + (b*(7*a^6 + 35*a^4*b^2 + 21*a^2*b^4 + b^6))/((a^2 - b^2)^7
*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.528099, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2668, 710, 801}

$$\frac{b\left(35a^4b^2+21a^2b^4+7a^6+b^6\right)}{d\left(a^2-b^2\right)^7(a+b\sin(c+dx))}+\frac{ab\left(3a^2+b^2\right)\left(a^2+3b^2\right)}{d\left(a^2-b^2\right)^6(a+b\sin(c+dx))^2}+\frac{b\left(10a^2b^2+5a^4+b^4\right)}{3d\left(a^2-b^2\right)^5(a+b\sin(c+dx))^3}+\frac{ab\,(\ldots}{d\left(a^2-b^2\right)^4(a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^8*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)^8
*d) - (8*a*b*(a^2 + b^2)*(a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(
(a^2 - b^2)^8*d) + b/(7*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) + (a*b)/(3*(a
^2 - b^2)^2*d*(a + b*Sin[c + d*x])^6) + (b*(3*a^2 + b^2))/(5*(a^2 - b^2)^3*
d*(a + b*Sin[c + d*x])^5) + (a*b*(a^2 + b^2))/((a^2 - b^2)^4*d*(a + b*Sin[c
 + d*x])^4) + (b*(5*a^4 + 10*a^2*b^2 + b^4))/(3*(a^2 - b^2)^5*d*(a + b*Sin[
c + d*x])^3) + (a*b*(3*a^2 + b^2)*(a^2 + 3*b^2))/((a^2 - b^2)^6*d*(a + b*Si
n[c + d*x])^2) + (b*(7*a^6 + 35*a^4*b^2 + 21*a^2*b^4 + b^6))/((a^2 - b^2)^7
*d*(a + b*Sin[c + d*x]))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{1}{(a+x)^8\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{b\,\mathrm{Subst}\left(\int \frac{a-x}{(a+x)^7\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{\left(a^2-b^2\right)d}$$

$$= \frac{b}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{b\,\mathrm{Subst}\left(\int \left(\frac{a-b}{2b(a+b)^7(b-x)} - \frac{2a}{(a-b)(a+b)(a+x)^7} + \frac{-3a^2-b^2}{(a-b)^2(a+b)^2(a+x}\right.\right.}{}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^8d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^8d} - \frac{8ab\left(a^2+b^2\right)\left(a^4+6a^2b^2+b^4\right)\log(a+b\sin}{\left(a^2-b^2\right)^8d}$$

Mathematica [A] time = 2.54058, size = 365, normalized size = 0.95

$$\frac{b\left(\frac{a\left(3a^2+b^2\right)\left(a^2+3b^2\right)}{(a-b)^6(a+b)^6(a+b\sin(c+dx))^2} + \frac{a\left(a^2+b^2\right)}{(a-b)^4(a+b)^4(a+b\sin(c+dx))^4} + \frac{35a^4b^2+21a^2b^4+7a^6+b^6}{(a-b)^7(a+b)^7(a+b\sin(c+dx))} + \frac{10a^2b^2+5a^4+b^4}{3(a-b)^5(a+b)^5(a+b\sin(c+dx))^3} + \frac{3a^2+b^2}{5(a-b)^3(a+b)^3(a+b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (b*(-Log[1 - Sin[c + d*x]]/(2*b*(a + b)^8) + Log[1 + Sin[c + d*x]]/(2*(a -
b)^8*b) - (8*a*(a^2 + b^2)*(a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])
/((a - b)^8*(a + b)^8) + 1/(7*(a^2 - b^2)*(a + b*Sin[c + d*x])^7) + a/(3*(a
 - b)^2*(a + b)^2*(a + b*Sin[c + d*x])^6) + (3*a^2 + b^2)/(5*(a - b)^3*(a +
 b)^3*(a + b*Sin[c + d*x])^5) + (a*(a^2 + b^2))/((a - b)^4*(a + b)^4*(a + b
*Sin[c + d*x])^4) + (5*a^4 + 10*a^2*b^2 + b^4)/(3*(a - b)^5*(a + b)^5*(a +
b*Sin[c + d*x])^3) + (a*(3*a^2 + b^2)*(a^2 + 3*b^2))/((a - b)^6*(a + b)^6*(
a + b*Sin[c + d*x])^2) + (7*a^6 + 35*a^4*b^2 + 21*a^2*b^4 + b^6)/((a - b)^7
*(a + b)^7*(a + b*Sin[c + d*x]))))/d
```

Maple [A] time = 0.253, size = 699, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^8,x)
```

```
[Out] 1/7/d*b/(a+b)/(a-b)/(a+b*sin(d*x+c))^7+1/3/d*a*b/(a+b)^2/(a-b)^2/(a+b*sin(d
*x+c))^6+3/5/d*b/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))^5*a^2+1/5/d*b^3/(a+b)^3/(
a-b)^3/(a+b*sin(d*x+c))^5+5/3/d*b/(a+b)^5/(a-b)^5/(a+b*sin(d*x+c))^3*a^4+10
/3/d*b^3/(a+b)^5/(a-b)^5/(a+b*sin(d*x+c))^3*a^2+1/3/d*b^5/(a+b)^5/(a-b)^5/(
a+b*sin(d*x+c))^3+7/d*b/(a+b)^7/(a-b)^7/(a+b*sin(d*x+c))*a^6+35/d*b^3/(a+b)
^7/(a-b)^7/(a+b*sin(d*x+c))*a^4+21/d*b^5/(a+b)^7/(a-b)^7/(a+b*sin(d*x+c))*a
^2+1/d*b^7/(a+b)^7/(a-b)^7/(a+b*sin(d*x+c))+1/d*b*a^3/(a+b)^4/(a-b)^4/(a+b*
sin(d*x+c))^4+1/d*b^3*a/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))^4+3/d*b*a^5/(a+b)^
6/(a-b)^6/(a+b*sin(d*x+c))^2+10/d*b^3*a^3/(a+b)^6/(a-b)^6/(a+b*sin(d*x+c))^
2+3/d*b^5*a/(a+b)^6/(a-b)^6/(a+b*sin(d*x+c))^2-8/d*b*a^7/(a+b)^8/(a-b)^8*ln
(a+b*sin(d*x+c))-56/d*b^3*a^5/(a+b)^8/(a-b)^8*ln(a+b*sin(d*x+c))-56/d*b^5*a
^3/(a+b)^8/(a-b)^8*ln(a+b*sin(d*x+c))-8/d*b^7*a/(a+b)^8/(a-b)^8*ln(a+b*sin(
d*x+c))-1/2/d/(a+b)^8*ln(sin(d*x+c)-1)+1/2*ln(1+sin(d*x+c))/(a-b)^8/d
```

Maxima [B] time = 1.24815, size = 1566, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/210*(1680*(a^7*b + 7*a^5*b^3 + 7*a^3*b^5 + a*b^7)*log(b*sin(d*x + c) + a
)/(a^16 - 8*a^14*b^2 + 28*a^12*b^4 - 56*a^10*b^6 + 70*a^8*b^8 - 56*a^6*b^10
 + 28*a^4*b^12 - 8*a^2*b^14 + b^16) - 2*(1443*a^12*b + 3704*a^10*b^3 + 1849
*a^8*b^5 - 496*a^6*b^7 + 309*a^4*b^9 - 104*a^2*b^11 + 15*b^13 + 105*(7*a^6*
b^7 + 35*a^4*b^9 + 21*a^2*b^11 + b^13)*sin(d*x + c)^6 + 105*(45*a^7*b^6 + 2
17*a^5*b^8 + 119*a^3*b^10 + 3*a*b^12)*sin(d*x + c)^5 + 35*(365*a^8*b^5 + 16
80*a^6*b^7 + 826*a^4*b^9 + 8*a^2*b^11 + b^13)*sin(d*x + c)^4 + 35*(533*a^9*
b^4 + 2304*a^7*b^6 + 994*a^5*b^8 + 8*a^3*b^10 + a*b^12)*sin(d*x + c)^3 + 21
*(743*a^10*b^3 + 2934*a^8*b^5 + 1099*a^6*b^7 + 29*a^4*b^9 - 6*a^2*b^11 + b^
13)*sin(d*x + c)^2 + 7*(1023*a^11*b^2 + 3494*a^9*b^4 + 1219*a^7*b^6 + 29*a^
5*b^8 - 6*a^3*b^10 + a*b^12)*sin(d*x + c))/(a^21 - 7*a^19*b^2 + 21*a^17*b^4
 - 35*a^15*b^6 + 35*a^13*b^8 - 21*a^11*b^10 + 7*a^9*b^12 - a^7*b^14 + (a^14
*b^7 - 7*a^12*b^9 + 21*a^10*b^11 - 35*a^8*b^13 + 35*a^6*b^15 - 21*a^4*b^17
+ 7*a^2*b^19 - b^21)*sin(d*x + c)^7 + 7*(a^15*b^6 - 7*a^13*b^8 + 21*a^11*b^
10 - 35*a^9*b^12 + 35*a^7*b^14 - 21*a^5*b^16 + 7*a^3*b^18 - a*b^20)*sin(d*x
 + c)^6 + 21*(a^16*b^5 - 7*a^14*b^7 + 21*a^12*b^9 - 35*a^10*b^11 + 35*a^8*b
^13 - 21*a^6*b^15 + 7*a^4*b^17 - a^2*b^19)*sin(d*x + c)^5 + 35*(a^17*b^4 -
7*a^15*b^6 + 21*a^13*b^8 - 35*a^11*b^10 + 35*a^9*b^12 - 21*a^7*b^14 + 7*a^5
*b^16 - a^3*b^18)*sin(d*x + c)^4 + 35*(a^18*b^3 - 7*a^16*b^5 + 21*a^14*b^7
- 35*a^12*b^9 + 35*a^10*b^11 - 21*a^8*b^13 + 7*a^6*b^15 - a^4*b^17)*sin(d*x
 + c)^3 + 21*(a^19*b^2 - 7*a^17*b^4 + 21*a^15*b^6 - 35*a^13*b^8 + 35*a^11*b
^10 - 21*a^9*b^12 + 7*a^7*b^14 - a^5*b^16)*sin(d*x + c)^2 + 7*(a^20*b - 7*a
^18*b^3 + 21*a^16*b^5 - 35*a^14*b^7 + 35*a^12*b^9 - 21*a^10*b^11 + 7*a^8*b^
13 - a^6*b^15)*sin(d*x + c)) - 105*log(sin(d*x + c) + 1)/(a^8 - 8*a^7*b + 2
8*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^4 - 56*a^3*b^5 + 28*a^2*b^6 - 8*a*b^7 + b
^8) + 105*log(sin(d*x + c) - 1)/(a^8 + 8*a^7*b + 28*a^6*b^2 + 56*a^5*b^3 +
70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b^6 + 8*a*b^7 + b^8))/d
```

Fricas [B] time = 26.269, size = 7699, normalized size = 20.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/210*(2886*a^14*b + 35728*a^12*b^3 + 113862*a^10*b^5 + 11760*a^8*b^7 - 97
230*a^6*b^9 - 62496*a^4*b^11 - 4158*a^2*b^13 - 352*b^15 - 210*(7*a^8*b^7 +
28*a^6*b^9 - 14*a^4*b^11 - 20*a^2*b^13 - b^15)*cos(d*x + c)^6 + 70*(365*a^1
0*b^5 + 1378*a^8*b^7 - 602*a^6*b^9 - 944*a^4*b^11 - 187*a^2*b^13 - 10*b^15)
```

*cos(d*x + c)^4 - 14*(2229*a^12*b^3 + 10223*a^10*b^5 + 7960*a^8*b^7 - 10490
*a^6*b^9 - 8915*a^4*b^11 - 949*a^2*b^13 - 58*b^15)*cos(d*x + c)^2 - 1680*(a
^14*b + 28*a^12*b^3 + 189*a^10*b^5 + 400*a^8*b^7 + 315*a^6*b^9 + 84*a^4*b^1
1 + 7*a^2*b^13 - 7*(a^8*b^7 + 7*a^6*b^9 + 7*a^4*b^11 + a^2*b^13)*cos(d*x +
c)^6 + 7*(5*a^10*b^5 + 38*a^8*b^7 + 56*a^6*b^9 + 26*a^4*b^11 + 3*a^2*b^13)*
cos(d*x + c)^4 - 7*(3*a^12*b^3 + 31*a^10*b^5 + 94*a^8*b^7 + 94*a^6*b^9 + 31
*a^4*b^11 + 3*a^2*b^13)*cos(d*x + c)^2 + (7*a^13*b^2 + 84*a^11*b^4 + 315*a^
9*b^6 + 400*a^7*b^8 + 189*a^5*b^10 + 28*a^3*b^12 + a*b^14 - (a^7*b^8 + 7*a^
5*b^10 + 7*a^3*b^12 + a*b^14)*cos(d*x + c)^6 + 3*(7*a^9*b^6 + 50*a^7*b^8 +
56*a^5*b^10 + 14*a^3*b^12 + a*b^14)*cos(d*x + c)^4 - (35*a^11*b^4 + 287*a^9
*b^6 + 542*a^7*b^8 + 350*a^5*b^10 + 63*a^3*b^12 + 3*a*b^14)*cos(d*x + c)^2)
*sin(d*x + c))*log(b*sin(d*x + c) + a) + 105*(a^15 + 8*a^14*b + 49*a^13*b^2
 + 224*a^12*b^3 + 693*a^11*b^4 + 1512*a^10*b^5 + 2485*a^9*b^6 + 3200*a^8*b^
7 + 3235*a^7*b^8 + 2520*a^6*b^9 + 1491*a^5*b^10 + 672*a^4*b^11 + 231*a^3*b^
12 + 56*a^2*b^13 + 7*a*b^14 - 7*(a^9*b^6 + 8*a^8*b^7 + 28*a^7*b^8 + 56*a^6*
b^9 + 70*a^5*b^10 + 56*a^4*b^11 + 28*a^3*b^12 + 8*a^2*b^13 + a*b^14)*cos(d*
x + c)^6 + 7*(5*a^11*b^4 + 40*a^10*b^5 + 143*a^9*b^6 + 304*a^8*b^7 + 434*a^
7*b^8 + 448*a^6*b^9 + 350*a^5*b^10 + 208*a^4*b^11 + 89*a^3*b^12 + 24*a^2*b^
13 + 3*a*b^14)*cos(d*x + c)^4 - 7*(3*a^13*b^2 + 24*a^12*b^3 + 94*a^11*b^4 +
 248*a^10*b^5 + 493*a^9*b^6 + 752*a^8*b^7 + 868*a^7*b^8 + 752*a^6*b^9 + 493
*a^5*b^10 + 248*a^4*b^11 + 94*a^3*b^12 + 24*a^2*b^13 + 3*a*b^14)*cos(d*x +
c)^2 + (7*a^14*b + 56*a^13*b^2 + 231*a^12*b^3 + 672*a^11*b^4 + 1491*a^10*b^
5 + 2520*a^9*b^6 + 3235*a^8*b^7 + 3200*a^7*b^8 + 2485*a^6*b^9 + 1512*a^5*b^
10 + 693*a^4*b^11 + 224*a^3*b^12 + 49*a^2*b^13 + 8*a*b^14 + b^15 - (a^8*b^7
 + 8*a^7*b^8 + 28*a^6*b^9 + 56*a^5*b^10 + 70*a^4*b^11 + 56*a^3*b^12 + 28*a^
2*b^13 + 8*a*b^14 + b^15)*cos(d*x + c)^6 + 3*(7*a^10*b^5 + 56*a^9*b^6 + 197
*a^8*b^7 + 400*a^7*b^8 + 518*a^6*b^9 + 448*a^5*b^10 + 266*a^4*b^11 + 112*a^
3*b^12 + 35*a^2*b^13 + 8*a*b^14 + b^15)*cos(d*x + c)^4 - (35*a^12*b^3 + 280
*a^11*b^4 + 1022*a^10*b^5 + 2296*a^9*b^6 + 3629*a^8*b^7 + 4336*a^7*b^8 + 40
04*a^6*b^9 + 2800*a^5*b^10 + 1421*a^4*b^11 + 504*a^3*b^12 + 126*a^2*b^13 +
24*a*b^14 + 3*b^15)*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 1
05*(a^15 - 8*a^14*b + 49*a^13*b^2 - 224*a^12*b^3 + 693*a^11*b^4 - 1512*a^10
*b^5 + 2485*a^9*b^6 - 3200*a^8*b^7 + 3235*a^7*b^8 - 2520*a^6*b^9 + 1491*a^5
*b^10 - 672*a^4*b^11 + 231*a^3*b^12 - 56*a^2*b^13 + 7*a*b^14 - 7*(a^9*b^6 -
 8*a^8*b^7 + 28*a^7*b^8 - 56*a^6*b^9 + 70*a^5*b^10 - 56*a^4*b^11 + 28*a^3*b
^12 - 8*a^2*b^13 + a*b^14)*cos(d*x + c)^6 + 7*(5*a^11*b^4 - 40*a^10*b^5 + 1
43*a^9*b^6 - 304*a^8*b^7 + 434*a^7*b^8 - 448*a^6*b^9 + 350*a^5*b^10 - 208*a
^4*b^11 + 89*a^3*b^12 - 24*a^2*b^13 + 3*a*b^14)*cos(d*x + c)^4 - 7*(3*a^13*
b^2 - 24*a^12*b^3 + 94*a^11*b^4 - 248*a^10*b^5 + 493*a^9*b^6 - 752*a^8*b^7
 + 868*a^7*b^8 - 752*a^6*b^9 + 493*a^5*b^10 - 248*a^4*b^11 + 94*a^3*b^12 - 2
4*a^2*b^13 + 3*a*b^14)*cos(d*x + c)^2 + (7*a^14*b - 56*a^13*b^2 + 231*a^12*
b^3 - 672*a^11*b^4 + 1491*a^10*b^5 - 2520*a^9*b^6 + 3235*a^8*b^7 - 3200*a^7
*b^8 + 2485*a^6*b^9 - 1512*a^5*b^10 + 693*a^4*b^11 - 224*a^3*b^12 + 49*a^2*
b^13 - 8*a*b^14 + b^15 - (a^8*b^7 - 8*a^7*b^8 + 28*a^6*b^9 - 56*a^5*b^10 +
70*a^4*b^11 - 56*a^3*b^12 + 28*a^2*b^13 - 8*a*b^14 + b^15)*cos(d*x + c)^6 +

```
 3*(7*a^10*b^5 - 56*a^9*b^6 + 197*a^8*b^7 - 400*a^7*b^8 + 518*a^6*b^9 - 448
*a^5*b^10 + 266*a^4*b^11 - 112*a^3*b^12 + 35*a^2*b^13 - 8*a*b^14 + b^15)*co
s(d*x + c)^4 - (35*a^12*b^3 - 280*a^11*b^4 + 1022*a^10*b^5 - 2296*a^9*b^6 +
 3629*a^8*b^7 - 4336*a^7*b^8 + 4004*a^6*b^9 - 2800*a^5*b^10 + 1421*a^4*b^11
 - 504*a^3*b^12 + 126*a^2*b^13 - 24*a*b^14 + 3*b^15)*cos(d*x + c)^2)*sin(d*
x + c))*log(-sin(d*x + c) + 1) + 14*(1023*a^13*b^2 + 5136*a^11*b^4 + 7255*a
^9*b^6 - 5160*a^7*b^8 - 6435*a^5*b^10 - 1768*a^3*b^12 - 51*a*b^14 + 15*(45*
a^9*b^6 + 172*a^7*b^8 - 98*a^5*b^10 - 116*a^3*b^12 - 3*a*b^14)*cos(d*x + c)
^4 - 5*(533*a^11*b^4 + 2041*a^9*b^6 - 278*a^7*b^8 - 1574*a^5*b^10 - 703*a^3
*b^12 - 19*a*b^14)*cos(d*x + c)^2)*sin(d*x + c))/(7*(a^17*b^6 - 8*a^15*b^8
+ 28*a^13*b^10 - 56*a^11*b^12 + 70*a^9*b^14 - 56*a^7*b^16 + 28*a^5*b^18 - 8
*a^3*b^20 + a*b^22)*d*cos(d*x + c)^6 - 7*(5*a^19*b^4 - 37*a^17*b^6 + 116*a^
15*b^8 - 196*a^13*b^10 + 182*a^11*b^12 - 70*a^9*b^14 - 28*a^7*b^16 + 44*a^5
*b^18 - 19*a^3*b^20 + 3*a*b^22)*d*cos(d*x + c)^4 + 7*(3*a^21*b^2 - 14*a^19*
b^4 + 7*a^17*b^6 + 88*a^15*b^8 - 266*a^13*b^10 + 364*a^11*b^12 - 266*a^9*b^
14 + 88*a^7*b^16 + 7*a^5*b^18 - 14*a^3*b^20 + 3*a*b^22)*d*cos(d*x + c)^2 -
(a^23 + 13*a^21*b^2 - 105*a^19*b^4 + 259*a^17*b^6 - 182*a^15*b^8 - 350*a^13
*b^10 + 910*a^11*b^12 - 890*a^9*b^14 + 421*a^7*b^16 - 63*a^5*b^18 - 21*a^3*
b^20 + 7*a*b^22)*d + ((a^16*b^7 - 8*a^14*b^9 + 28*a^12*b^11 - 56*a^10*b^13
+ 70*a^8*b^15 - 56*a^6*b^17 + 28*a^4*b^19 - 8*a^2*b^21 + b^23)*d*cos(d*x +
c)^6 - 3*(7*a^18*b^5 - 55*a^16*b^7 + 188*a^14*b^9 - 364*a^12*b^11 + 434*a^1
0*b^13 - 322*a^8*b^15 + 140*a^6*b^17 - 28*a^4*b^19 - a^2*b^21 + b^23)*d*cos
(d*x + c)^4 + (35*a^20*b^3 - 238*a^18*b^5 + 647*a^16*b^7 - 808*a^14*b^9 + 1
82*a^12*b^11 + 812*a^10*b^13 - 1162*a^8*b^15 + 728*a^6*b^17 - 217*a^4*b^19
+ 18*a^2*b^21 + 3*b^23)*d*cos(d*x + c)^2 - (7*a^22*b - 21*a^20*b^3 - 63*a^1
8*b^5 + 421*a^16*b^7 - 890*a^14*b^9 + 910*a^12*b^11 - 350*a^10*b^13 - 182*a
^8*b^15 + 259*a^6*b^17 - 105*a^4*b^19 + 13*a^2*b^21 + b^23)*d)*sin(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.5458, size = 1364, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/210*(1680*(a^7*b^2 + 7*a^5*b^4 + 7*a^3*b^6 + a*b^8)*log(abs(b*sin(d*x +
c) + a))/(a^16*b - 8*a^14*b^3 + 28*a^12*b^5 - 56*a^10*b^7 + 70*a^8*b^9 - 56
*a^6*b^11 + 28*a^4*b^13 - 8*a^2*b^15 + b^17) - 105*log(abs(sin(d*x + c) + 1
))/(a^8 - 8*a^7*b + 28*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^4 - 56*a^3*b^5 + 28*
a^2*b^6 - 8*a*b^7 + b^8) + 105*log(abs(sin(d*x + c) - 1))/(a^8 + 8*a^7*b +
28*a^6*b^2 + 56*a^5*b^3 + 70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b^6 + 8*a*b^7 +
b^8) - 2*(2178*a^7*b^8*sin(d*x + c)^7 + 15246*a^5*b^10*sin(d*x + c)^7 + 152
46*a^3*b^12*sin(d*x + c)^7 + 2178*a*b^14*sin(d*x + c)^7 + 15981*a^8*b^7*sin
(d*x + c)^6 + 109662*a^6*b^9*sin(d*x + c)^6 + 105252*a^4*b^11*sin(d*x + c)^
6 + 13146*a^2*b^13*sin(d*x + c)^6 - 105*b^15*sin(d*x + c)^6 + 50463*a^9*b^6
*sin(d*x + c)^5 + 338226*a^7*b^8*sin(d*x + c)^5 + 309876*a^5*b^10*sin(d*x +
 c)^5 + 33558*a^3*b^12*sin(d*x + c)^5 - 315*a*b^14*sin(d*x + c)^5 + 89005*a
^10*b^5*sin(d*x + c)^4 + 579635*a^8*b^7*sin(d*x + c)^4 + 503720*a^6*b^9*sin
(d*x + c)^4 + 47600*a^4*b^11*sin(d*x + c)^4 - 245*a^2*b^13*sin(d*x + c)^4 -
 35*b^15*sin(d*x + c)^4 + 94885*a^11*b^4*sin(d*x + c)^3 + 595595*a^9*b^6*si
n(d*x + c)^3 + 487760*a^7*b^8*sin(d*x + c)^3 + 41720*a^5*b^10*sin(d*x + c)^
3 - 245*a^3*b^12*sin(d*x + c)^3 - 35*a*b^14*sin(d*x + c)^3 + 61341*a^12*b^3
*sin(d*x + c)^2 + 366177*a^10*b^5*sin(d*x + c)^2 + 281631*a^8*b^7*sin(d*x +
 c)^2 + 23268*a^6*b^9*sin(d*x + c)^2 - 735*a^4*b^11*sin(d*x + c)^2 + 147*a^
2*b^13*sin(d*x + c)^2 - 21*b^15*sin(d*x + c)^2 + 22407*a^13*b^2*sin(d*x + c
) + 124019*a^11*b^4*sin(d*x + c) + 90797*a^9*b^6*sin(d*x + c) + 6916*a^7*b^
8*sin(d*x + c) - 245*a^5*b^10*sin(d*x + c) + 49*a^3*b^12*sin(d*x + c) - 7*a
*b^14*sin(d*x + c) + 3621*a^14*b + 17507*a^12*b^3 + 13391*a^10*b^5 - 167*a^
8*b^7 + 805*a^6*b^9 - 413*a^4*b^11 + 119*a^2*b^13 - 15*b^15)/((a^16 - 8*a^1
4*b^2 + 28*a^12*b^4 - 56*a^10*b^6 + 70*a^8*b^8 - 56*a^6*b^10 + 28*a^4*b^12
- 8*a^2*b^14 + b^16)*(b*sin(d*x + c) + a)^7))/d
```

3.466 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=527

$$-\frac{b\left(196a^6b^2+574a^4b^4+244a^2b^6+a^8+9b^8\right)}{2d\left(a^2-b^2\right)^8(a+b\sin(c+dx))}-\frac{ab\left(77a^4b^2+147a^2b^4+a^6+31b^6\right)}{2d\left(a^2-b^2\right)^7(a+b\sin(c+dx))^2}-\frac{b\left(115a^4b^2+129a^2b^4+3a^6+9b^6\right)}{6d\left(a^2-b^2\right)^6(a+b\sin(c+dx))^3}$$

```
[Out] -((a + 9*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^9*d) + ((a - 9*b)*Log[1 + Sin
[c + d*x]])/(4*(a - b)^9*d) + (8*a*b^3*(15*a^6 + 63*a^4*b^2 + 45*a^2*b^4 +
5*b^6)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^9*d) - (b*(7*a^2 + 9*b^2))/(14
*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^7) - (Sec[c + d*x]^2*(b - a*Sin[c + d
*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) - (a*b*(3*a^2 + 13*b^2))/(6*
(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^6) - (b*(5*a^4 + 50*a^2*b^2 + 9*b^4))/
(10*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^5) - (a*b*(a^4 + 20*a^2*b^2 + 11*b
^4))/(2*(a^2 - b^2)^5*d*(a + b*Sin[c + d*x])^4) - (b*(3*a^6 + 115*a^4*b^2 +
 129*a^2*b^4 + 9*b^6))/(6*(a^2 - b^2)^6*d*(a + b*Sin[c + d*x])^3) - (a*b*(a
^6 + 77*a^4*b^2 + 147*a^2*b^4 + 31*b^6))/(2*(a^2 - b^2)^7*d*(a + b*Sin[c +
d*x])^2) - (b*(a^8 + 196*a^6*b^2 + 574*a^4*b^4 + 244*a^2*b^6 + 9*b^8))/(2*(
a^2 - b^2)^8*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.73562, antiderivative size = 527, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2668, 741, 801}

$$-\frac{b\left(196a^6b^2+574a^4b^4+244a^2b^6+a^8+9b^8\right)}{2d\left(a^2-b^2\right)^8(a+b\sin(c+dx))}-\frac{ab\left(77a^4b^2+147a^2b^4+a^6+31b^6\right)}{2d\left(a^2-b^2\right)^7(a+b\sin(c+dx))^2}-\frac{b\left(115a^4b^2+129a^2b^4+3a^6+9b^6\right)}{6d\left(a^2-b^2\right)^6(a+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^8,x]
```


```
[Out] -((a + 9*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^9*d) + ((a - 9*b)*Log[1 + Sin
[c + d*x]])/(4*(a - b)^9*d) + (8*a*b^3*(15*a^6 + 63*a^4*b^2 + 45*a^2*b^4 +
5*b^6)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^9*d) - (b*(7*a^2 + 9*b^2))/(14
*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^7) - (Sec[c + d*x]^2*(b - a*Sin[c + d
*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) - (a*b*(3*a^2 + 13*b^2))/(6*
(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^6) - (b*(5*a^4 + 50*a^2*b^2 + 9*b^4))/
(10*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^5) - (a*b*(a^4 + 20*a^2*b^2 + 11*b
^4))/(2*(a^2 - b^2)^5*d*(a + b*Sin[c + d*x])^4) - (b*(3*a^6 + 115*a^4*b^2 +
 129*a^2*b^4 + 9*b^6))/(6*(a^2 - b^2)^6*d*(a + b*Sin[c + d*x])^3) - (a*b*(a
```

```
^6 + 77*a^4*b^2 + 147*a^2*b^4 + 31*b^6))/(2*(a^2 - b^2)^7*d*(a + b*Sin[c +
d*x])^2) - (b*(a^8 + 196*a^6*b^2 + 574*a^4*b^4 + 244*a^2*b^6 + 9*b^8))/(2*(
a^2 - b^2)^8*d*(a + b*Sin[c + d*x]))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{1}{(a+x)^8\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{b\,\text{Subst}\left(\int \frac{a^2-9b^2+8ax}{(a+x)^8\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{b\,\text{Subst}\left(\int \left(\frac{(a-b)(a+9b)}{2b(a+b)^8(b-x)} + \frac{7a^2+9b^2}{(a-b)(a+b)(a+x)^8} + \frac{2\left(3a^3+13a}{(a-b)^2(a+b)^2(}\right.\right.}{}$$

$$= -\frac{(a+9b)\log(1-\sin(c+dx))}{4(a+b)^9d} + \frac{(a-9b)\log(1+\sin(c+dx))}{4(a-b)^9d} + \frac{8ab^3\left(15a^6+63a^4b^2+45\right.}{(a}$$

Mathematica [A] time = 6.72323, size = 770, normalized size = 1.46

$$b^3 \left(\frac{\sec^2(c+dx)\left(b^2-ab\sin(c+dx)\right)}{2b^4\left(b^2-a^2\right)(a+b\sin(c+dx))^7} - \frac{8a\left(\frac{2a\left(3a^2+b^2\right)\left(a^2+3b^2\right)}{(a-b)^6(a+b)^6(a+b\sin(c+dx))}+\frac{4a\left(a^2+b^2\right)}{3(a-b)^4(a+b)^4(a+b\sin(c+dx))^3}+\frac{10a^2b^2+5a^4+b^4}{2(a-b)^5(a+b)^5(a+b\sin(c+dx))^2}+\frac{3a^2+b^2}{4(a-b)^3(a+b)^3(a+b\sin(c+dx))^4}+\frac{\cdots}{6\left(a^2-\cdots\right.}\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (b^3*((Sec[c + d*x]^2*(b^2 - a*b*Sin[c + d*x]))/(2*b^4*(-a^2 + b^2)*(a + b*
Sin[c + d*x])^7) - (8*a*(-Log[1 - Sin[c + d*x]]/(2*b*(a + b)^7) + Log[1 + S
in[c + d*x]]/(2*(a - b)^7*b) - ((7*a^6 + 35*a^4*b^2 + 21*a^2*b^4 + b^6)*Log
[a + b*Sin[c + d*x]])/((a - b)^7*(a + b)^7) + 1/(6*(a^2 - b^2)*(a + b*Sin[c
 + d*x])^6) + (2*a)/(5*(a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])^5) + (3*a^2
 + b^2)/(4*(a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^4) + (4*a*(a^2 + b^2))/
(3*(a - b)^4*(a + b)^4*(a + b*Sin[c + d*x])^3) + (5*a^4 + 10*a^2*b^2 + b^4)
/(2*(a - b)^5*(a + b)^5*(a + b*Sin[c + d*x])^2) + (2*a*(3*a^2 + b^2)*(a^2 +
 3*b^2))/((a - b)^6*(a + b)^6*(a + b*Sin[c + d*x]))) + (-7*a^2 - 9*b^2)*(-L
og[1 - Sin[c + d*x]]/(2*b*(a + b)^8) + Log[1 + Sin[c + d*x]]/(2*(a - b)^8*b
) - (8*a*(a^2 + b^2)*(a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/((a -
 b)^8*(a + b)^8) + 1/(7*(a^2 - b^2)*(a + b*Sin[c + d*x])^7) + a/(3*(a - b)^
2*(a + b)^2*(a + b*Sin[c + d*x])^6) + (3*a^2 + b^2)/(5*(a - b)^3*(a + b)^3*
(a + b*Sin[c + d*x])^5) + (a*(a^2 + b^2))/((a - b)^4*(a + b)^4*(a + b*Sin[c
 + d*x])^4) + (5*a^4 + 10*a^2*b^2 + b^4)/(3*(a - b)^5*(a + b)^5*(a + b*Sin[
c + d*x])^3) + (a*(3*a^2 + b^2)*(a^2 + 3*b^2))/((a - b)^6*(a + b)^6*(a + b*
Sin[c + d*x])^2) + (7*a^6 + 35*a^4*b^2 + 21*a^2*b^4 + b^6)/((a - b)^7*(a +
b)^7*(a + b*Sin[c + d*x]))))/(2*b^2*(-a^2 + b^2))))/d
```

Maple [A] time = 0.276, size = 804, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^8,x)
```

```
[Out] -1/4/d/(a+b)^9*ln(sin(d*x+c)-1)*a-9/4/d/(a+b)^9*ln(sin(d*x+c)-1)*b+1/4/d/(a
-b)^9*ln(1+sin(d*x+c))*a-9/4/d/(a-b)^9*ln(1+sin(d*x+c))*b-1/4/d/(a+b)^8/(si
n(d*x+c)-1)-1/4/d/(a-b)^8/(1+sin(d*x+c))-2/d*b^3/(a+b)^4/(a-b)^4/(a+b*sin(d
```

```
*x+c))^5*a^2-84/d*b^3/(a+b)^8/(a-b)^8/(a+b*sin(d*x+c))*a^6-252/d*b^5/(a+b)^
8/(a-b)^8/(a+b*sin(d*x+c))*a^4-108/d*b^7/(a+b)^8/(a-b)^8/(a+b*sin(d*x+c))*a
^2-4/d*b^9/(a+b)^8/(a-b)^8/(a+b*sin(d*x+c))-1/7/d*b^3/(a+b)^2/(a-b)^2/(a+b*
sin(d*x+c))^7-1/d*b^7/(a+b)^6/(a-b)^6/(a+b*sin(d*x+c))^3-2/5/d*b^5/(a+b)^4/
(a-b)^4/(a+b*sin(d*x+c))^5-5/d*b^3*a^3/(a+b)^5/(a-b)^5/(a+b*sin(d*x+c))^4-3
/d*b^5*a/(a+b)^5/(a-b)^5/(a+b*sin(d*x+c))^4-28/d*b^3*a^5/(a+b)^7/(a-b)^7/(a
+b*sin(d*x+c))^2-56/d*b^5*a^3/(a+b)^7/(a-b)^7/(a+b*sin(d*x+c))^2-12/d*b^7*a
/(a+b)^7/(a-b)^7/(a+b*sin(d*x+c))^2+120/d*b^3*a^7/(a+b)^9/(a-b)^9*ln(a+b*si
n(d*x+c))+504/d*b^5*a^5/(a+b)^9/(a-b)^9*ln(a+b*sin(d*x+c))+360/d*b^7*a^3/(a
+b)^9/(a-b)^9*ln(a+b*sin(d*x+c))+40/d*b^9*a/(a+b)^9/(a-b)^9*ln(a+b*sin(d*x+
c))-35/3/d*b^3/(a+b)^6/(a-b)^6/(a+b*sin(d*x+c))^3*a^4-14/d*b^5/(a+b)^6/(a-b
)^6/(a+b*sin(d*x+c))^3*a^2-2/3/d*a*b^3/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))^6
```

Maxima [B] time = 1.43342, size = 2255, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/420*(3360*(15*a^7*b^3 + 63*a^5*b^5 + 45*a^3*b^7 + 5*a*b^9)*log(b*sin(d*x
+ c) + a)/(a^18 - 9*a^16*b^2 + 36*a^14*b^4 - 84*a^12*b^6 + 126*a^10*b^8 - 1
26*a^8*b^10 + 84*a^6*b^12 - 36*a^4*b^14 + 9*a^2*b^16 - b^18) + 105*(a - 9*b
)*log(sin(d*x + c) + 1)/(a^9 - 9*a^8*b + 36*a^7*b^2 - 84*a^6*b^3 + 126*a^5*
b^4 - 126*a^4*b^5 + 84*a^3*b^6 - 36*a^2*b^7 + 9*a*b^8 - b^9) - 105*(a + 9*b
)*log(sin(d*x + c) - 1)/(a^9 + 9*a^8*b + 36*a^7*b^2 + 84*a^6*b^3 + 126*a^5*
b^4 + 126*a^4*b^5 + 84*a^3*b^6 + 36*a^2*b^7 + 9*a*b^8 + b^9) - 2*(840*a^14*
b + 33490*a^12*b^3 + 57724*a^10*b^5 + 16354*a^8*b^7 - 1496*a^6*b^9 + 814*a^
4*b^11 - 236*a^2*b^13 + 30*b^15 - 105*(a^8*b^7 + 196*a^6*b^9 + 574*a^4*b^11
 + 244*a^2*b^13 + 9*b^15)*sin(d*x + c)^8 - 105*(7*a^9*b^6 + 1252*a^7*b^8 +
3514*a^5*b^10 + 1348*a^3*b^12 + 23*a*b^14)*sin(d*x + c)^7 - 35*(63*a^10*b^5
 + 10066*a^8*b^7 + 26194*a^6*b^9 + 7384*a^4*b^11 - 681*a^2*b^13 - 18*b^15)*
sin(d*x + c)^6 - 35*(105*a^11*b^4 + 14506*a^9*b^6 + 32254*a^7*b^8 + 160*a^5
*b^10 - 3951*a^3*b^12 - 66*a*b^14)*sin(d*x + c)^5 - 7*(525*a^12*b^3 + 59310
*a^10*b^5 + 83812*a^8*b^7 - 98528*a^6*b^9 - 44663*a^4*b^11 - 438*a^2*b^13 -
 18*b^15)*sin(d*x + c)^4 - 7*(315*a^13*b^2 + 25930*a^11*b^4 - 20896*a^9*b^6
 - 166336*a^7*b^8 - 53641*a^5*b^10 - 386*a^3*b^12 - 26*a*b^14)*sin(d*x + c)
^3 - (735*a^14*b + 30550*a^12*b^3 - 361856*a^10*b^5 - 919070*a^8*b^7 - 2528
45*a^6*b^9 - 3050*a^4*b^11 + 310*a^2*b^13 - 54*b^15)*sin(d*x + c)^2 - 7*(15
*a^15 - 420*a^13*b^2 - 26140*a^11*b^4 - 52264*a^9*b^6 - 13189*a^7*b^8 - 184
*a^5*b^10 + 26*a^3*b^12 - 4*a*b^14)*sin(d*x + c))/(a^23 - 8*a^21*b^2 + 28*a
```

```
^19*b^4 - 56*a^17*b^6 + 70*a^15*b^8 - 56*a^13*b^10 + 28*a^11*b^12 - 8*a^9*b
^14 + a^7*b^16 - (a^16*b^7 - 8*a^14*b^9 + 28*a^12*b^11 - 56*a^10*b^13 + 70*
a^8*b^15 - 56*a^6*b^17 + 28*a^4*b^19 - 8*a^2*b^21 + b^23)*sin(d*x + c)^9 -
7*(a^17*b^6 - 8*a^15*b^8 + 28*a^13*b^10 - 56*a^11*b^12 + 70*a^9*b^14 - 56*a
^7*b^16 + 28*a^5*b^18 - 8*a^3*b^20 + a*b^22)*sin(d*x + c)^8 - (21*a^18*b^5
- 169*a^16*b^7 + 596*a^14*b^9 - 1204*a^12*b^11 + 1526*a^10*b^13 - 1246*a^8*
b^15 + 644*a^6*b^17 - 196*a^4*b^19 + 29*a^2*b^21 - b^23)*sin(d*x + c)^7 - 7
*(5*a^19*b^4 - 41*a^17*b^6 + 148*a^15*b^8 - 308*a^13*b^10 + 406*a^11*b^12 -
 350*a^9*b^14 + 196*a^7*b^16 - 68*a^5*b^18 + 13*a^3*b^20 - a*b^22)*sin(d*x
+ c)^6 - 7*(5*a^20*b^3 - 43*a^18*b^5 + 164*a^16*b^7 - 364*a^14*b^9 + 518*a^
12*b^11 - 490*a^10*b^13 + 308*a^8*b^15 - 124*a^6*b^17 + 29*a^4*b^19 - 3*a^2
*b^21)*sin(d*x + c)^5 - 7*(3*a^21*b^2 - 29*a^19*b^4 + 124*a^17*b^6 - 308*a^
15*b^8 + 490*a^13*b^10 - 518*a^11*b^12 + 364*a^9*b^14 - 164*a^7*b^16 + 43*a
^5*b^18 - 5*a^3*b^20)*sin(d*x + c)^4 - 7*(a^22*b - 13*a^20*b^3 + 68*a^18*b^
5 - 196*a^16*b^7 + 350*a^14*b^9 - 406*a^12*b^11 + 308*a^10*b^13 - 148*a^8*b
^15 + 41*a^6*b^17 - 5*a^4*b^19)*sin(d*x + c)^3 - (a^23 - 29*a^21*b^2 + 196*
a^19*b^4 - 644*a^17*b^6 + 1246*a^15*b^8 - 1526*a^13*b^10 + 1204*a^11*b^12 -
 596*a^9*b^14 + 169*a^7*b^16 - 21*a^5*b^18)*sin(d*x + c)^2 + 7*(a^22*b - 8*
a^20*b^3 + 28*a^18*b^5 - 56*a^16*b^7 + 70*a^14*b^9 - 56*a^12*b^11 + 28*a^10
*b^13 - 8*a^8*b^15 + a^6*b^17)*sin(d*x + c)))/d
```

Fricas [B] time = 44.7096, size = 9441, normalized size = 17.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/420*(210*a^16*b - 1680*a^14*b^3 + 5880*a^12*b^5 - 11760*a^10*b^7 + 14700*
a^8*b^9 - 11760*a^6*b^11 + 5880*a^4*b^13 - 1680*a^2*b^15 + 210*b^17 - 210*(
a^10*b^7 + 195*a^8*b^9 + 378*a^6*b^11 - 330*a^4*b^13 - 235*a^2*b^15 - 9*b^1
7)*cos(d*x + c)^8 + 70*(63*a^12*b^5 + 10015*a^10*b^7 + 18468*a^8*b^9 - 1427
4*a^6*b^11 - 12025*a^4*b^13 - 2157*a^2*b^15 - 90*b^17)*cos(d*x + c)^6 - 14*
(525*a^14*b^3 + 59730*a^12*b^5 + 174637*a^10*b^7 + 77130*a^8*b^9 - 194265*a
^6*b^11 - 106450*a^4*b^13 - 10785*a^2*b^15 - 522*b^17)*cos(d*x + c)^4 + 2*(
735*a^16*b + 37165*a^14*b^3 + 437199*a^12*b^5 + 836549*a^10*b^7 - 111195*a^
8*b^9 - 812385*a^6*b^11 - 362915*a^4*b^13 - 23569*a^2*b^15 - 1584*b^17)*cos
(d*x + c)^2 + 3360*(7*(15*a^8*b^9 + 63*a^6*b^11 + 45*a^4*b^13 + 5*a^2*b^15)
*cos(d*x + c)^8 - 7*(75*a^10*b^7 + 360*a^8*b^9 + 414*a^6*b^11 + 160*a^4*b^1
3 + 15*a^2*b^15)*cos(d*x + c)^6 + 7*(45*a^12*b^5 + 339*a^10*b^7 + 810*a^8*b
^9 + 654*a^6*b^11 + 185*a^4*b^13 + 15*a^2*b^15)*cos(d*x + c)^4 - (15*a^14*b
```

^3 + 378*a^12*b^5 + 1893*a^10*b^7 + 3260*a^8*b^9 + 2121*a^6*b^11 + 490*a^4*
b^13 + 35*a^2*b^15)*cos(d*x + c)^2 + ((15*a^7*b^10 + 63*a^5*b^12 + 45*a^3*b
^14 + 5*a*b^16)*cos(d*x + c)^8 - 3*(105*a^9*b^8 + 456*a^7*b^10 + 378*a^5*b^
12 + 80*a^3*b^14 + 5*a*b^16)*cos(d*x + c)^6 + (525*a^11*b^6 + 2835*a^9*b^8
+ 4266*a^7*b^10 + 2254*a^5*b^12 + 345*a^3*b^14 + 15*a*b^16)*cos(d*x + c)^4
- (105*a^13*b^4 + 966*a^11*b^6 + 2835*a^9*b^8 + 2948*a^7*b^10 + 1183*a^5*b^
12 + 150*a^3*b^14 + 5*a*b^16)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x +
 c) + a) + 105*(7*(a^11*b^6 - 45*a^9*b^8 - 240*a^8*b^9 - 630*a^7*b^10 - 100
8*a^6*b^11 - 1050*a^5*b^12 - 720*a^4*b^13 - 315*a^3*b^14 - 80*a^2*b^15 - 9*
a*b^16)*cos(d*x + c)^8 - 7*(5*a^13*b^4 - 222*a^11*b^6 - 1200*a^10*b^7 - 328
5*a^9*b^8 - 5760*a^8*b^9 - 7140*a^7*b^10 - 6624*a^6*b^11 - 4725*a^5*b^12 -
2560*a^4*b^13 - 990*a^3*b^14 - 240*a^2*b^15 - 27*a*b^16)*cos(d*x + c)^6 + 7
*(3*a^15*b^2 - 125*a^13*b^4 - 720*a^12*b^5 - 2337*a^11*b^6 - 5424*a^10*b^7
- 9585*a^9*b^8 - 12960*a^8*b^9 - 13335*a^7*b^10 - 10464*a^6*b^11 - 6327*a^5
*b^12 - 2960*a^4*b^13 - 1035*a^3*b^14 - 240*a^2*b^15 - 27*a*b^16)*cos(d*x +
 c)^4 - (a^17 - 24*a^15*b^2 - 240*a^14*b^3 - 1540*a^13*b^4 - 6048*a^12*b^5
- 15848*a^11*b^6 - 30288*a^10*b^7 - 44730*a^9*b^8 - 52160*a^8*b^9 - 47784*a
^7*b^10 - 33936*a^6*b^11 - 18564*a^5*b^12 - 7840*a^4*b^13 - 2520*a^3*b^14 -
 560*a^2*b^15 - 63*a*b^16)*cos(d*x + c)^2 + ((a^10*b^7 - 45*a^8*b^9 - 240*a
^7*b^10 - 630*a^6*b^11 - 1008*a^5*b^12 - 1050*a^4*b^13 - 720*a^3*b^14 - 315
*a^2*b^15 - 80*a*b^16 - 9*b^17)*cos(d*x + c)^8 - 3*(7*a^12*b^5 - 314*a^10*b
^7 - 1680*a^9*b^8 - 4455*a^8*b^9 - 7296*a^7*b^10 - 7980*a^6*b^11 - 6048*a^5
*b^12 - 3255*a^4*b^13 - 1280*a^3*b^14 - 378*a^2*b^15 - 80*a*b^16 - 9*b^17)*
cos(d*x + c)^6 + (35*a^14*b^3 - 1533*a^12*b^5 - 8400*a^11*b^6 - 23937*a^10*
b^7 - 45360*a^9*b^8 - 63345*a^8*b^9 - 68256*a^7*b^10 - 57015*a^6*b^11 - 360
64*a^5*b^12 - 16695*a^4*b^13 - 5520*a^3*b^14 - 1323*a^2*b^15 - 240*a*b^16 -
 27*b^17)*cos(d*x + c)^4 - (7*a^16*b - 280*a^14*b^3 - 1680*a^13*b^4 - 5964*
a^12*b^5 - 15456*a^11*b^6 - 30344*a^10*b^7 - 45360*a^9*b^8 - 52230*a^8*b^9
- 47168*a^7*b^10 - 33768*a^6*b^11 - 18928*a^5*b^12 - 7980*a^4*b^13 - 2400*a
^3*b^14 - 504*a^2*b^15 - 80*a*b^16 - 9*b^17)*cos(d*x + c)^2)*sin(d*x + c))*
log(sin(d*x + c) + 1) - 105*(7*(a^11*b^6 - 45*a^9*b^8 + 240*a^8*b^9 - 630*a
^7*b^10 + 1008*a^6*b^11 - 1050*a^5*b^12 + 720*a^4*b^13 - 315*a^3*b^14 + 80*
a^2*b^15 - 9*a*b^16)*cos(d*x + c)^8 - 7*(5*a^13*b^4 - 222*a^11*b^6 + 1200*a
^10*b^7 - 3285*a^9*b^8 + 5760*a^8*b^9 - 7140*a^7*b^10 + 6624*a^6*b^11 - 472
5*a^5*b^12 + 2560*a^4*b^13 - 990*a^3*b^14 + 240*a^2*b^15 - 27*a*b^16)*cos(d
x + c)^6 + 7(3*a^15*b^2 - 125*a^13*b^4 + 720*a^12*b^5 - 2337*a^11*b^6 + 5
424*a^10*b^7 - 9585*a^9*b^8 + 12960*a^8*b^9 - 13335*a^7*b^10 + 10464*a^6*b^
11 - 6327*a^5*b^12 + 2960*a^4*b^13 - 1035*a^3*b^14 + 240*a^2*b^15 - 27*a*b^
16)*cos(d*x + c)^4 - (a^17 - 24*a^15*b^2 + 240*a^14*b^3 - 1540*a^13*b^4 + 6
048*a^12*b^5 - 15848*a^11*b^6 + 30288*a^10*b^7 - 44730*a^9*b^8 + 52160*a^8*
b^9 - 47784*a^7*b^10 + 33936*a^6*b^11 - 18564*a^5*b^12 + 7840*a^4*b^13 - 25
20*a^3*b^14 + 560*a^2*b^15 - 63*a*b^16)*cos(d*x + c)^2 + ((a^10*b^7 - 45*a^
8*b^9 + 240*a^7*b^10 - 630*a^6*b^11 + 1008*a^5*b^12 - 1050*a^4*b^13 + 720*a
^3*b^14 - 315*a^2*b^15 + 80*a*b^16 - 9*b^17)*cos(d*x + c)^8 - 3*(7*a^12*b^5
 - 314*a^10*b^7 + 1680*a^9*b^8 - 4455*a^8*b^9 + 7296*a^7*b^10 - 7980*a^6*b^

```
11 + 6048*a^5*b^12 - 3255*a^4*b^13 + 1280*a^3*b^14 - 378*a^2*b^15 + 80*a*b^
16 - 9*b^17)*cos(d*x + c)^6 + (35*a^14*b^3 - 1533*a^12*b^5 + 8400*a^11*b^6
- 23937*a^10*b^7 + 45360*a^9*b^8 - 63345*a^8*b^9 + 68256*a^7*b^10 - 57015*a
^6*b^11 + 36064*a^5*b^12 - 16695*a^4*b^13 + 5520*a^3*b^14 - 1323*a^2*b^15 +
 240*a*b^16 - 27*b^17)*cos(d*x + c)^4 - (7*a^16*b - 280*a^14*b^3 + 1680*a^1
3*b^4 - 5964*a^12*b^5 + 15456*a^11*b^6 - 30344*a^10*b^7 + 45360*a^9*b^8 - 5
2230*a^8*b^9 + 47168*a^7*b^10 - 33768*a^6*b^11 + 18928*a^5*b^12 - 7980*a^4*
b^13 + 2400*a^3*b^14 - 504*a^2*b^15 + 80*a*b^16 - 9*b^17)*cos(d*x + c)^2)*s
in(d*x + c))*log(-sin(d*x + c) + 1) - 14*(15*a^17 - 120*a^15*b^2 + 420*a^13
*b^4 - 840*a^11*b^6 + 1050*a^9*b^8 - 840*a^7*b^10 + 420*a^5*b^12 - 120*a^3*
b^14 + 15*a*b^16 - 15*(7*a^11*b^6 + 1245*a^9*b^8 + 2262*a^7*b^10 - 2166*a^5
*b^12 - 1325*a^3*b^14 - 23*a*b^16)*cos(d*x + c)^6 + 5*(105*a^13*b^4 + 14464
*a^11*b^6 + 28953*a^9*b^8 - 11736*a^7*b^10 - 23605*a^5*b^12 - 8040*a^3*b^14
 - 141*a*b^16)*cos(d*x + c)^4 - (315*a^15*b^2 + 26665*a^13*b^4 + 97499*a^11
*b^6 + 88065*a^9*b^8 - 106455*a^7*b^10 - 85325*a^5*b^12 - 20415*a^3*b^14 -
349*a*b^16)*cos(d*x + c)^2)*sin(d*x + c))/(7*(a^19*b^6 - 9*a^17*b^8 + 36*a^
15*b^10 - 84*a^13*b^12 + 126*a^11*b^14 - 126*a^9*b^16 + 84*a^7*b^18 - 36*a^
5*b^20 + 9*a^3*b^22 - a*b^24)*d*cos(d*x + c)^8 - 7*(5*a^21*b^4 - 42*a^19*b^
6 + 153*a^17*b^8 - 312*a^15*b^10 + 378*a^13*b^12 - 252*a^11*b^14 + 42*a^9*b
^16 + 72*a^7*b^18 - 63*a^5*b^20 + 22*a^3*b^22 - 3*a*b^24)*d*cos(d*x + c)^6
+ 7*(3*a^23*b^2 - 17*a^21*b^4 + 21*a^19*b^6 + 81*a^17*b^8 - 354*a^15*b^10 +
 630*a^13*b^12 - 630*a^11*b^14 + 354*a^9*b^16 - 81*a^7*b^18 - 21*a^5*b^20 +
 17*a^3*b^22 - 3*a*b^24)*d*cos(d*x + c)^4 - (a^25 + 12*a^23*b^2 - 118*a^21*
b^4 + 364*a^19*b^6 - 441*a^17*b^8 - 168*a^15*b^10 + 1260*a^13*b^12 - 1800*a
^11*b^14 + 1311*a^9*b^16 - 484*a^7*b^18 + 42*a^5*b^20 + 28*a^3*b^22 - 7*a*b
^24)*d*cos(d*x + c)^2 + ((a^18*b^7 - 9*a^16*b^9 + 36*a^14*b^11 - 84*a^12*b^
13 + 126*a^10*b^15 - 126*a^8*b^17 + 84*a^6*b^19 - 36*a^4*b^21 + 9*a^2*b^23
- b^25)*d*cos(d*x + c)^8 - 3*(7*a^20*b^5 - 62*a^18*b^7 + 243*a^16*b^9 - 552
*a^14*b^11 + 798*a^12*b^13 - 756*a^10*b^15 + 462*a^8*b^17 - 168*a^6*b^19 +
27*a^4*b^21 + 2*a^2*b^23 - b^25)*d*cos(d*x + c)^6 + (35*a^22*b^3 - 273*a^20
*b^5 + 885*a^18*b^7 - 1455*a^16*b^9 + 990*a^14*b^11 + 630*a^12*b^13 - 1974*
a^10*b^15 + 1890*a^8*b^17 - 945*a^6*b^19 + 235*a^4*b^21 - 15*a^2*b^23 - 3*b
^25)*d*cos(d*x + c)^4 - (7*a^24*b - 28*a^22*b^3 - 42*a^20*b^5 + 484*a^18*b^
7 - 1311*a^16*b^9 + 1800*a^14*b^11 - 1260*a^12*b^13 + 168*a^10*b^15 + 441*a
^8*b^17 - 364*a^6*b^19 + 118*a^4*b^21 - 12*a^2*b^23 - b^25)*d*cos(d*x + c)^
2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.53095, size = 1791, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/420*(3360*(15*a^7*b^4 + 63*a^5*b^6 + 45*a^3*b^8 + 5*a*b^10)*log(abs(b*sin
(d*x + c) + a))/(a^18*b - 9*a^16*b^3 + 36*a^14*b^5 - 84*a^12*b^7 + 126*a^10
*b^9 - 126*a^8*b^11 + 84*a^6*b^13 - 36*a^4*b^15 + 9*a^2*b^17 - b^19) + 105*
(a - 9*b)*log(abs(sin(d*x + c) + 1))/(a^9 - 9*a^8*b + 36*a^7*b^2 - 84*a^6*b
^3 + 126*a^5*b^4 - 126*a^4*b^5 + 84*a^3*b^6 - 36*a^2*b^7 + 9*a*b^8 - b^9) -
 105*(a + 9*b)*log(abs(sin(d*x + c) - 1))/(a^9 + 9*a^8*b + 36*a^7*b^2 + 84*
a^6*b^3 + 126*a^5*b^4 + 126*a^4*b^5 + 84*a^3*b^6 + 36*a^2*b^7 + 9*a*b^8 + b
^9) + 210*(120*a^7*b^3*sin(d*x + c)^2 + 504*a^5*b^5*sin(d*x + c)^2 + 360*a^
3*b^7*sin(d*x + c)^2 + 40*a*b^9*sin(d*x + c)^2 - a^10*sin(d*x + c) - 27*a^8
*b^2*sin(d*x + c) - 42*a^6*b^4*sin(d*x + c) + 42*a^4*b^6*sin(d*x + c) + 27*
a^2*b^8*sin(d*x + c) + b^10*sin(d*x + c) + 8*a^9*b - 72*a^7*b^3 - 504*a^5*b
^5 - 408*a^3*b^7 - 48*a*b^9)/((a^18 - 9*a^16*b^2 + 36*a^14*b^4 - 84*a^12*b^
6 + 126*a^10*b^8 - 126*a^8*b^10 + 84*a^6*b^12 - 36*a^4*b^14 + 9*a^2*b^16 -
b^18)*(sin(d*x + c)^2 - 1)) - 4*(32670*a^7*b^10*sin(d*x + c)^7 + 137214*a^5
*b^12*sin(d*x + c)^7 + 98010*a^3*b^14*sin(d*x + c)^7 + 10890*a*b^16*sin(d*x
 + c)^7 + 237510*a^8*b^9*sin(d*x + c)^6 + 978138*a^6*b^11*sin(d*x + c)^6 +
670950*a^4*b^13*sin(d*x + c)^6 + 65310*a^2*b^15*sin(d*x + c)^6 - 420*b^17*s
in(d*x + c)^6 + 741930*a^9*b^8*sin(d*x + c)^5 + 2987334*a^7*b^10*sin(d*x +
c)^5 + 1959930*a^5*b^12*sin(d*x + c)^5 + 166530*a^3*b^14*sin(d*x + c)^5 - 1
260*a*b^16*sin(d*x + c)^5 + 1291675*a^10*b^7*sin(d*x + c)^4 + 5064885*a^8*b
^9*sin(d*x + c)^4 + 3165120*a^6*b^11*sin(d*x + c)^4 + 237020*a^4*b^13*sin(d
*x + c)^4 - 1155*a^2*b^15*sin(d*x + c)^4 - 105*b^17*sin(d*x + c)^4 + 135467
5*a^11*b^6*sin(d*x + c)^3 + 5144685*a^9*b^8*sin(d*x + c)^3 + 3051720*a^7*b^
10*sin(d*x + c)^3 + 207620*a^5*b^12*sin(d*x + c)^3 - 1155*a^3*b^14*sin(d*x
+ c)^3 - 105*a*b^16*sin(d*x + c)^3 + 856905*a^12*b^5*sin(d*x + c)^2 + 31265
01*a^10*b^7*sin(d*x + c)^2 + 1759590*a^8*b^9*sin(d*x + c)^2 + 113400*a^6*b^
11*sin(d*x + c)^2 - 2205*a^4*b^13*sin(d*x + c)^2 + 315*a^2*b^15*sin(d*x + c
)^2 - 42*b^17*sin(d*x + c)^2 + 303275*a^13*b^4*sin(d*x + c) + 1049727*a^11*
b^6*sin(d*x + c) + 565530*a^9*b^8*sin(d*x + c) + 33600*a^7*b^10*sin(d*x + c
) - 735*a^5*b^12*sin(d*x + c) + 105*a^3*b^14*sin(d*x + c) - 14*a*b^16*sin(d
```

```
*x + c) + 46475*a^14*b^3 + 149331*a^12*b^5 + 79845*a^10*b^7 + 2385*a^8*b^9
+ 1155*a^6*b^11 - 525*a^4*b^13 + 133*a^2*b^15 - 15*b^17)/((a^18 - 9*a^16*b^
2 + 36*a^14*b^4 - 84*a^12*b^6 + 126*a^10*b^8 - 126*a^8*b^10 + 84*a^6*b^12 -
 36*a^4*b^14 + 9*a^2*b^16 - b^18)*(b*sin(d*x + c) + a)^7))/d
```

3.467 $\int \frac{\cos^8(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=491

$$-\frac{a\left(-56a^4b^2+70a^2b^4+16a^6-35b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8b^8d\left(a^2-b^2\right)^{7/2}}+\frac{a\cos^7(c+dx)}{6bd\left(a^2-b^2\right)(a+b\sin(c+dx))^6}+\frac{\cos^5(c+dx)\left(6\left(a^2\cdots\right.\right.}{30b^3d\left(a^2-b^2\cdots\right.}$$

```
[Out] x/b^8 - (a*(16*a^6 - 56*a^4*b^2 + 70*a^2*b^4 - 35*b^6)*ArcTan[(b + a*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]])/(8*b^8*(a^2 - b^2)^(7/2)*d) - Cos[c + d*x]^7/
(7*b*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]^7)/(6*b*(a^2 - b^2)*d*(a +
 b*Sin[c + d*x])^6) - (a*(6*a^2 - 11*b^2)*Cos[c + d*x]^5)/(24*b^3*(a^2 - b^
2)^2*d*(a + b*Sin[c + d*x])^4) + (a*(8*a^4 - 22*a^2*b^2 + 19*b^4)*Cos[c + d
*x]^3)/(16*b^5*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) + (Cos[c + d*x]^5*(6
*(a^2 - b^2) + 5*a*b*Sin[c + d*x]))/(30*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*
x])^5) - (Cos[c + d*x]^3*(8*(a^2 - b^2)^2 + a*b*(6*a^2 - 11*b^2)*Sin[c + d*
x]))/(24*b^5*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^3) + (Cos[c + d*x]*(16*(a
^2 - b^2)^3 + a*b*(8*a^4 - 22*a^2*b^2 + 19*b^4)*Sin[c + d*x]))/(16*b^7*(a^2
 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.27382, antiderivative size = 491, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2693, 2864, 2863, 2735, 2660, 618, 204}

$$-\frac{a\left(-56a^4b^2+70a^2b^4+16a^6-35b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8b^8d\left(a^2-b^2\right)^{7/2}}+\frac{a\cos^7(c+dx)}{6bd\left(a^2-b^2\right)(a+b\sin(c+dx))^6}+\frac{\cos^5(c+dx)\left(6\left(a^2\cdots\right.\right.}{30b^3d\left(a^2-b^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] x/b^8 - (a*(16*a^6 - 56*a^4*b^2 + 70*a^2*b^4 - 35*b^6)*ArcTan[(b + a*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]])/(8*b^8*(a^2 - b^2)^(7/2)*d) - Cos[c + d*x]^7/
(7*b*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]^7)/(6*b*(a^2 - b^2)*d*(a +
 b*Sin[c + d*x])^6) - (a*(6*a^2 - 11*b^2)*Cos[c + d*x]^5)/(24*b^3*(a^2 - b^
2)^2*d*(a + b*Sin[c + d*x])^4) + (a*(8*a^4 - 22*a^2*b^2 + 19*b^4)*Cos[c + d
*x]^3)/(16*b^5*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) + (Cos[c + d*x]^5*(6
*(a^2 - b^2) + 5*a*b*Sin[c + d*x]))/(30*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*
```

```
x])^5) - (Cos[c + d*x]^3*(8*(a^2 - b^2)^2 + a*b*(6*a^2 - 11*b^2)*Sin[c + d*
x]))/(24*b^5*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^3) + (Cos[c + d*x]*(16*(a
^2 - b^2)^3 + a*b*(8*a^4 - 22*a^2*b^2 + 19*b^4)*Sin[c + d*x]))/(16*b^7*(a^2
 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(a+b\sin(c+dx))^8}\,dx = -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^7}\,dx}{b}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\int \frac{\cos^6(c+dx)(6b+a\sin(c+dx))}{(a+b\sin(c+dx))^6}\,dx}{6b\left(a^2-b^2\right)}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\cos^5(c+dx)\left(6\left(a^2-b^2\right)+5\right.}{30b^3\left(a^2-b^2\right)d(a+b\,\mathrm{si}}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-11b^2\right)\cos^5(c+}{24b^3\left(a^2-b^2\right)^2 d(a+b\sin(c}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-11b^2\right)\cos^5(c+}{24b^3\left(a^2-b^2\right)^2 d(a+b\sin(c}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-11b^2\right)\cos^5(c+}{24b^3\left(a^2-b^2\right)^2 d(a+b\sin(c}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-11b^2\right)\cos^5(c+}{24b^3\left(a^2-b^2\right)^2 d(a+b\sin(c}$$

$$= \frac{x}{b^8} - \frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-11b^2\right)\cos^5(}{24b^3\left(a^2-b^2\right)^2 d(a+b\,\mathrm{s}}$$

$$= \frac{x}{b^8} - \frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-11b^2\right)\cos^5(}{24b^3\left(a^2-b^2\right)^2 d(a+b\,\mathrm{s}}$$

$$= \frac{x}{b^8} - \frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} - \frac{a\left(6a^2-11b^2\right)\cos^5(}{24b^3\left(a^2-b^2\right)^2 d(a+b\,\mathrm{s}}$$

$$= \frac{x}{b^8} - \frac{a\left(16a^6-56a^4b^2+70a^2b^4-35b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8b^8\left(a^2-b^2\right)^{7/2}d} - \frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} +$$

Mathematica [B] time = 8.54895, size = 6586, normalized size = 13.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^8/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.227, size = 9454, normalized size = 19.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+b*sin(d*x+c))^8,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 10.6953, size = 8992, normalized size = 18.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [1/3360*(23520*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*x
*cos(d*x + c)^6 + 2*(4356*a^8*b^7 - 16864*a^6*b^9 + 24001*a^4*b^11 - 14309*
```

a^2*b^13 + 2816*b^15)*cos(d*x + c)^7 - 23520*(5*a^11*b^4 - 17*a^9*b^6 + 18*
a^7*b^8 - 2*a^5*b^10 - 7*a^3*b^12 + 3*a*b^14)*d*x*cos(d*x + c)^4 - 28*(2754
*a^10*b^5 - 9717*a^8*b^7 + 11528*a^6*b^9 - 3782*a^4*b^11 - 1247*a^2*b^13 +
464*b^15)*cos(d*x + c)^5 + 23520*(3*a^13*b^2 - 2*a^11*b^4 - 19*a^9*b^6 + 36
*a^7*b^8 - 19*a^5*b^10 - 2*a^3*b^12 + 3*a*b^14)*d*x*cos(d*x + c)^2 + 70*(85
6*a^12*b^3 - 1090*a^10*b^5 - 3477*a^8*b^7 + 7907*a^6*b^9 - 4423*a^4*b^11 +
67*a^2*b^13 + 160*b^15)*cos(d*x + c)^3 - 3360*(a^15 + 17*a^13*b^2 - 43*a^11
*b^4 - 11*a^9*b^6 + 99*a^7*b^8 - 77*a^5*b^10 + 7*a^3*b^12 + 7*a*b^14)*d*x +
105*(16*a^14 + 280*a^12*b^2 - 546*a^10*b^4 - 413*a^8*b^6 + 1323*a^6*b^8 -
735*a^4*b^10 - 245*a^2*b^12 - 7*(16*a^8*b^6 - 56*a^6*b^8 + 70*a^4*b^10 - 35
*a^2*b^12)*cos(d*x + c)^6 + 7*(80*a^10*b^4 - 232*a^8*b^6 + 182*a^6*b^8 + 35
*a^4*b^10 - 105*a^2*b^12)*cos(d*x + c)^4 - 7*(48*a^12*b^2 - 8*a^10*b^4 - 30
2*a^8*b^6 + 427*a^6*b^8 - 140*a^4*b^10 - 105*a^2*b^12)*cos(d*x + c)^2 + (11
2*a^13*b + 168*a^11*b^3 - 1134*a^9*b^5 + 1045*a^7*b^7 + 189*a^5*b^9 - 665*a
^3*b^11 - 35*a*b^13 - (16*a^7*b^7 - 56*a^5*b^9 + 70*a^3*b^11 - 35*a*b^13)*c
os(d*x + c)^6 + 3*(112*a^9*b^5 - 376*a^7*b^7 + 434*a^5*b^9 - 175*a^3*b^11 -
35*a*b^13)*cos(d*x + c)^4 - (560*a^11*b^3 - 1288*a^9*b^5 + 146*a^7*b^7 + 1
547*a^5*b^9 - 1260*a^3*b^11 - 105*a*b^13)*cos(d*x + c)^2)*sin(d*x + c))*sqr
t(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
- b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))
/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 420*(8*a^14*b + 1
12*a^12*b^3 - 322*a^10*b^5 + 63*a^8*b^7 + 479*a^6*b^9 - 379*a^4*b^11 + 31*a
^2*b^13 + 8*b^15)*cos(d*x + c) + 14*(240*(a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11
- 4*a^2*b^13 + b^15)*d*x*cos(d*x + c)^6 - 720*(7*a^10*b^5 - 27*a^8*b^7 + 38
*a^6*b^9 - 22*a^4*b^11 + 3*a^2*b^13 + b^15)*d*x*cos(d*x + c)^4 - (2676*a^9*
b^6 - 10264*a^7*b^8 + 14371*a^5*b^10 - 8204*a^3*b^12 + 1421*a*b^14)*cos(d*x
+ c)^5 + 240*(35*a^12*b^3 - 98*a^10*b^5 + 45*a^8*b^7 + 100*a^6*b^9 - 115*a
^4*b^11 + 30*a^2*b^13 + 3*b^15)*d*x*cos(d*x + c)^2 + 10*(638*a^11*b^4 - 192
5*a^9*b^6 + 1427*a^7*b^8 + 861*a^5*b^10 - 1253*a^3*b^12 + 252*a*b^14)*cos(d
x + c)^3 - 240(7*a^14*b + 7*a^12*b^3 - 77*a^10*b^5 + 99*a^8*b^7 - 11*a^6*
b^9 - 43*a^4*b^11 + 17*a^2*b^13 + b^15)*d*x - 15*(104*a^13*b^2 + 26*a^11*b^
4 - 897*a^9*b^6 + 1306*a^7*b^8 - 308*a^5*b^10 - 308*a^3*b^12 + 77*a*b^14)*c
os(d*x + c))*sin(d*x + c))/(7*(a^9*b^14 - 4*a^7*b^16 + 6*a^5*b^18 - 4*a^3*b
^20 + a*b^22)*d*cos(d*x + c)^6 - 7*(5*a^11*b^12 - 17*a^9*b^14 + 18*a^7*b^16
- 2*a^5*b^18 - 7*a^3*b^20 + 3*a*b^22)*d*cos(d*x + c)^4 + 7*(3*a^13*b^10 -
2*a^11*b^12 - 19*a^9*b^14 + 36*a^7*b^16 - 19*a^5*b^18 - 2*a^3*b^20 + 3*a*b^
22)*d*cos(d*x + c)^2 - (a^15*b^8 + 17*a^13*b^10 - 43*a^11*b^12 - 11*a^9*b^1
4 + 99*a^7*b^16 - 77*a^5*b^18 + 7*a^3*b^20 + 7*a*b^22)*d + ((a^8*b^15 - 4*a
^6*b^17 + 6*a^4*b^19 - 4*a^2*b^21 + b^23)*d*cos(d*x + c)^6 - 3*(7*a^10*b^13
- 27*a^8*b^15 + 38*a^6*b^17 - 22*a^4*b^19 + 3*a^2*b^21 + b^23)*d*cos(d*x +
c)^4 + (35*a^12*b^11 - 98*a^10*b^13 + 45*a^8*b^15 + 100*a^6*b^17 - 115*a^4
*b^19 + 30*a^2*b^21 + 3*b^23)*d*cos(d*x + c)^2 - (7*a^14*b^9 + 7*a^12*b^11
- 77*a^10*b^13 + 99*a^8*b^15 - 11*a^6*b^17 - 43*a^4*b^19 + 17*a^2*b^21 + b^
23)*d)*sin(d*x + c)), 1/1680*(11760*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a
^3*b^12 + a*b^14)*d*x*cos(d*x + c)^6 + (4356*a^8*b^7 - 16864*a^6*b^9 + 2400

1*a^4*b^11 - 14309*a^2*b^13 + 2816*b^15)*cos(d*x + c)^7 - 11760*(5*a^11*b^4
 - 17*a^9*b^6 + 18*a^7*b^8 - 2*a^5*b^10 - 7*a^3*b^12 + 3*a*b^14)*d*x*cos(d*
x + c)^4 - 14*(2754*a^10*b^5 - 9717*a^8*b^7 + 11528*a^6*b^9 - 3782*a^4*b^11
 - 1247*a^2*b^13 + 464*b^15)*cos(d*x + c)^5 + 11760*(3*a^13*b^2 - 2*a^11*b^
4 - 19*a^9*b^6 + 36*a^7*b^8 - 19*a^5*b^10 - 2*a^3*b^12 + 3*a*b^14)*d*x*cos(
d*x + c)^2 + 35*(856*a^12*b^3 - 1090*a^10*b^5 - 3477*a^8*b^7 + 7907*a^6*b^9
 - 4423*a^4*b^11 + 67*a^2*b^13 + 160*b^15)*cos(d*x + c)^3 - 1680*(a^15 + 17
*a^13*b^2 - 43*a^11*b^4 - 11*a^9*b^6 + 99*a^7*b^8 - 77*a^5*b^10 + 7*a^3*b^1
2 + 7*a*b^14)*d*x - 105*(16*a^14 + 280*a^12*b^2 - 546*a^10*b^4 - 413*a^8*b^
6 + 1323*a^6*b^8 - 735*a^4*b^10 - 245*a^2*b^12 - 7*(16*a^8*b^6 - 56*a^6*b^8
 + 70*a^4*b^10 - 35*a^2*b^12)*cos(d*x + c)^6 + 7*(80*a^10*b^4 - 232*a^8*b^6
 + 182*a^6*b^8 + 35*a^4*b^10 - 105*a^2*b^12)*cos(d*x + c)^4 - 7*(48*a^12*b^
2 - 8*a^10*b^4 - 302*a^8*b^6 + 427*a^6*b^8 - 140*a^4*b^10 - 105*a^2*b^12)*c
os(d*x + c)^2 + (112*a^13*b + 168*a^11*b^3 - 1134*a^9*b^5 + 1045*a^7*b^7 +
189*a^5*b^9 - 665*a^3*b^11 - 35*a*b^13 - (16*a^7*b^7 - 56*a^5*b^9 + 70*a^3*
b^11 - 35*a*b^13)*cos(d*x + c)^6 + 3*(112*a^9*b^5 - 376*a^7*b^7 + 434*a^5*b
^9 - 175*a^3*b^11 - 35*a*b^13)*cos(d*x + c)^4 - (560*a^11*b^3 - 1288*a^9*b^
5 + 146*a^7*b^7 + 1547*a^5*b^9 - 1260*a^3*b^11 - 105*a*b^13)*cos(d*x + c)^2
)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^
2)*cos(d*x + c))) - 210*(8*a^14*b + 112*a^12*b^3 - 322*a^10*b^5 + 63*a^8*b^
7 + 479*a^6*b^9 - 379*a^4*b^11 + 31*a^2*b^13 + 8*b^15)*cos(d*x + c) + 7*(24
0*(a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d*x*cos(d*x + c)^6
 - 720*(7*a^10*b^5 - 27*a^8*b^7 + 38*a^6*b^9 - 22*a^4*b^11 + 3*a^2*b^13 + b
^15)*d*x*cos(d*x + c)^4 - (2676*a^9*b^6 - 10264*a^7*b^8 + 14371*a^5*b^10 -
8204*a^3*b^12 + 1421*a*b^14)*cos(d*x + c)^5 + 240*(35*a^12*b^3 - 98*a^10*b^
5 + 45*a^8*b^7 + 100*a^6*b^9 - 115*a^4*b^11 + 30*a^2*b^13 + 3*b^15)*d*x*cos
(d*x + c)^2 + 10*(638*a^11*b^4 - 1925*a^9*b^6 + 1427*a^7*b^8 + 861*a^5*b^10
 - 1253*a^3*b^12 + 252*a*b^14)*cos(d*x + c)^3 - 240*(7*a^14*b + 7*a^12*b^3
- 77*a^10*b^5 + 99*a^8*b^7 - 11*a^6*b^9 - 43*a^4*b^11 + 17*a^2*b^13 + b^15)
*d*x - 15*(104*a^13*b^2 + 26*a^11*b^4 - 897*a^9*b^6 + 1306*a^7*b^8 - 308*a^
5*b^10 - 308*a^3*b^12 + 77*a*b^14)*cos(d*x + c))*sin(d*x + c))/(7*(a^9*b^14
 - 4*a^7*b^16 + 6*a^5*b^18 - 4*a^3*b^20 + a*b^22)*d*cos(d*x + c)^6 - 7*(5*a
^11*b^12 - 17*a^9*b^14 + 18*a^7*b^16 - 2*a^5*b^18 - 7*a^3*b^20 + 3*a*b^22)*
d*cos(d*x + c)^4 + 7*(3*a^13*b^10 - 2*a^11*b^12 - 19*a^9*b^14 + 36*a^7*b^16
 - 19*a^5*b^18 - 2*a^3*b^20 + 3*a*b^22)*d*cos(d*x + c)^2 - (a^15*b^8 + 17*a
^13*b^10 - 43*a^11*b^12 - 11*a^9*b^14 + 99*a^7*b^16 - 77*a^5*b^18 + 7*a^3*b
^20 + 7*a*b^22)*d + ((a^8*b^15 - 4*a^6*b^17 + 6*a^4*b^19 - 4*a^2*b^21 + b^2
3)*d*cos(d*x + c)^6 - 3*(7*a^10*b^13 - 27*a^8*b^15 + 38*a^6*b^17 - 22*a^4*b
^19 + 3*a^2*b^21 + b^23)*d*cos(d*x + c)^4 + (35*a^12*b^11 - 98*a^10*b^13 +
45*a^8*b^15 + 100*a^6*b^17 - 115*a^4*b^19 + 30*a^2*b^21 + 3*b^23)*d*cos(d*x
 + c)^2 - (7*a^14*b^9 + 7*a^12*b^11 - 77*a^10*b^13 + 99*a^8*b^15 - 11*a^6*b
^17 - 43*a^4*b^19 + 17*a^2*b^21 + b^23)*d)*sin(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.5888, size = 3140, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/840*(105*(16*a^7 - 56*a^5*b^2 + 70*a^3*b^4 - 35*a*b^6)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^
2)))/((a^6*b^8 - 3*a^4*b^10 + 3*a^2*b^12 - b^14)*sqrt(a^2 - b^2)) - (840*a^
18*b*tan(1/2*d*x + 1/2*c)^13 - 2310*a^16*b^3*tan(1/2*d*x + 1/2*c)^13 + 1995
*a^14*b^5*tan(1/2*d*x + 1/2*c)^13 - 1680*a^12*b^7*tan(1/2*d*x + 1/2*c)^13 +
 5040*a^10*b^9*tan(1/2*d*x + 1/2*c)^13 - 5040*a^8*b^11*tan(1/2*d*x + 1/2*c)
^13 + 1680*a^6*b^13*tan(1/2*d*x + 1/2*c)^13 + 1680*a^19*tan(1/2*d*x + 1/2*c
)^12 + 5880*a^17*b^2*tan(1/2*d*x + 1/2*c)^12 - 24990*a^15*b^4*tan(1/2*d*x +
 1/2*c)^12 + 24255*a^13*b^6*tan(1/2*d*x + 1/2*c)^12 - 10080*a^11*b^8*tan(1/
2*d*x + 1/2*c)^12 + 30240*a^9*b^10*tan(1/2*d*x + 1/2*c)^12 - 30240*a^7*b^12
*tan(1/2*d*x + 1/2*c)^12 + 10080*a^5*b^14*tan(1/2*d*x + 1/2*c)^12 + 26880*a
^18*b*tan(1/2*d*x + 1/2*c)^11 - 19320*a^16*b^3*tan(1/2*d*x + 1/2*c)^11 - 87
640*a^14*b^5*tan(1/2*d*x + 1/2*c)^11 + 118790*a^12*b^7*tan(1/2*d*x + 1/2*c)
^11 - 26880*a^10*b^9*tan(1/2*d*x + 1/2*c)^11 + 94080*a^8*b^11*tan(1/2*d*x +
 1/2*c)^11 - 98560*a^6*b^13*tan(1/2*d*x + 1/2*c)^11 + 33600*a^4*b^15*tan(1/
2*d*x + 1/2*c)^11 + 10080*a^19*tan(1/2*d*x + 1/2*c)^10 + 144480*a^17*b^2*ta
n(1/2*d*x + 1/2*c)^10 - 299880*a^15*b^4*tan(1/2*d*x + 1/2*c)^10 - 15680*a^1
3*b^6*tan(1/2*d*x + 1/2*c)^10 + 276430*a^11*b^8*tan(1/2*d*x + 1/2*c)^10 + 3
6960*a^9*b^10*tan(1/2*d*x + 1/2*c)^10 + 97440*a^7*b^12*tan(1/2*d*x + 1/2*c)
^10 - 166880*a^5*b^14*tan(1/2*d*x + 1/2*c)^10 + 67200*a^3*b^16*tan(1/2*d*x
+ 1/2*c)^10 + 121800*a^18*b*tan(1/2*d*x + 1/2*c)^9 + 238770*a^16*b^3*tan(1/
2*d*x + 1/2*c)^9 - 1067605*a^14*b^5*tan(1/2*d*x + 1/2*c)^9 + 656390*a^12*b^
7*tan(1/2*d*x + 1/2*c)^9 + 345156*a^10*b^9*tan(1/2*d*x + 1/2*c)^9 + 214032*
a^8*b^11*tan(1/2*d*x + 1/2*c)^9 - 87472*a^6*b^13*tan(1/2*d*x + 1/2*c)^9 - 1
```

26336*a^4*b^15*tan(1/2*d*x + 1/2*c)^9 + 80640*a^2*b^17*tan(1/2*d*x + 1/2*c)
^9 + 25200*a^19*tan(1/2*d*x + 1/2*c)^8 + 514360*a^17*b^2*tan(1/2*d*x + 1/2*
c)^8 - 490350*a^15*b^4*tan(1/2*d*x + 1/2*c)^8 - 1389885*a^13*b^6*tan(1/2*d*
x + 1/2*c)^8 + 1764630*a^11*b^8*tan(1/2*d*x + 1/2*c)^8 + 201544*a^9*b^10*ta
n(1/2*d*x + 1/2*c)^8 + 305088*a^7*b^12*tan(1/2*d*x + 1/2*c)^8 - 336448*a^5*
b^14*tan(1/2*d*x + 1/2*c)^8 + 27776*a^3*b^16*tan(1/2*d*x + 1/2*c)^8 + 53760
*a*b^18*tan(1/2*d*x + 1/2*c)^8 + 235200*a^18*b*tan(1/2*d*x + 1/2*c)^7 + 744
800*a^16*b^3*tan(1/2*d*x + 1/2*c)^7 - 2263800*a^14*b^5*tan(1/2*d*x + 1/2*c)
^7 + 382620*a^12*b^7*tan(1/2*d*x + 1/2*c)^7 + 1776432*a^10*b^9*tan(1/2*d*x
+ 1/2*c)^7 + 204848*a^8*b^11*tan(1/2*d*x + 1/2*c)^7 - 47616*a^6*b^13*tan(1/
2*d*x + 1/2*c)^7 - 258560*a^4*b^15*tan(1/2*d*x + 1/2*c)^7 + 111616*a^2*b^17
*tan(1/2*d*x + 1/2*c)^7 + 15360*b^19*tan(1/2*d*x + 1/2*c)^7 + 33600*a^19*ta
n(1/2*d*x + 1/2*c)^6 + 730240*a^17*b^2*tan(1/2*d*x + 1/2*c)^6 - 534240*a^15
*b^4*tan(1/2*d*x + 1/2*c)^6 - 2260440*a^13*b^6*tan(1/2*d*x + 1/2*c)^6 + 244
3980*a^11*b^8*tan(1/2*d*x + 1/2*c)^6 + 593824*a^9*b^10*tan(1/2*d*x + 1/2*c)
^6 + 148848*a^7*b^12*tan(1/2*d*x + 1/2*c)^6 - 336448*a^5*b^14*tan(1/2*d*x +
 1/2*c)^6 + 27776*a^3*b^16*tan(1/2*d*x + 1/2*c)^6 + 53760*a*b^18*tan(1/2*d*
x + 1/2*c)^6 + 231000*a^18*b*tan(1/2*d*x + 1/2*c)^5 + 643230*a^16*b^3*tan(1
/2*d*x + 1/2*c)^5 - 2226175*a^14*b^5*tan(1/2*d*x + 1/2*c)^5 + 749980*a^12*b
^7*tan(1/2*d*x + 1/2*c)^5 + 1482936*a^10*b^9*tan(1/2*d*x + 1/2*c)^5 - 72128
*a^8*b^11*tan(1/2*d*x + 1/2*c)^5 - 87472*a^6*b^13*tan(1/2*d*x + 1/2*c)^5 -
126336*a^4*b^15*tan(1/2*d*x + 1/2*c)^5 + 80640*a^2*b^17*tan(1/2*d*x + 1/2*c
)^5 + 25200*a^19*tan(1/2*d*x + 1/2*c)^4 + 461160*a^17*b^2*tan(1/2*d*x + 1/2
*c)^4 - 667674*a^15*b^4*tan(1/2*d*x + 1/2*c)^4 - 857003*a^13*b^6*tan(1/2*d*
x + 1/2*c)^4 + 1686188*a^11*b^8*tan(1/2*d*x + 1/2*c)^4 - 290976*a^9*b^10*ta
n(1/2*d*x + 1/2*c)^4 + 118160*a^7*b^12*tan(1/2*d*x + 1/2*c)^4 - 166880*a^5*
b^14*tan(1/2*d*x + 1/2*c)^4 + 67200*a^3*b^16*tan(1/2*d*x + 1/2*c)^4 + 11424
0*a^18*b*tan(1/2*d*x + 1/2*c)^3 + 89880*a^16*b^3*tan(1/2*d*x + 1/2*c)^3 - 8
81776*a^14*b^5*tan(1/2*d*x + 1/2*c)^3 + 996478*a^12*b^7*tan(1/2*d*x + 1/2*c
)^3 - 212688*a^10*b^9*tan(1/2*d*x + 1/2*c)^3 + 108976*a^8*b^11*tan(1/2*d*x
+ 1/2*c)^3 - 98560*a^6*b^13*tan(1/2*d*x + 1/2*c)^3 + 33600*a^4*b^15*tan(1/2
*d*x + 1/2*c)^3 + 10080*a^19*tan(1/2*d*x + 1/2*c)^2 + 101920*a^17*b^2*tan(1
/2*d*x + 1/2*c)^2 - 344568*a^15*b^4*tan(1/2*d*x + 1/2*c)^2 + 331128*a^13*b^
6*tan(1/2*d*x + 1/2*c)^2 - 79226*a^11*b^8*tan(1/2*d*x + 1/2*c)^2 + 44800*a^
9*b^10*tan(1/2*d*x + 1/2*c)^2 - 33264*a^7*b^12*tan(1/2*d*x + 1/2*c)^2 + 100
80*a^5*b^14*tan(1/2*d*x + 1/2*c)^2 + 22680*a^18*b*tan(1/2*d*x + 1/2*c) - 64
330*a^16*b^3*tan(1/2*d*x + 1/2*c) + 58569*a^14*b^5*tan(1/2*d*x + 1/2*c) - 1
4322*a^12*b^7*tan(1/2*d*x + 1/2*c) + 8372*a^10*b^9*tan(1/2*d*x + 1/2*c) - 5
824*a^8*b^11*tan(1/2*d*x + 1/2*c) + 1680*a^6*b^13*tan(1/2*d*x + 1/2*c) + 16
80*a^19 - 4760*a^17*b^2 + 4326*a^15*b^4 - 1143*a^13*b^6 + 958*a^11*b^8 - 77
6*a^9*b^10 + 240*a^7*b^12)/((a^13*b^7 - 3*a^11*b^9 + 3*a^9*b^11 - a^7*b^13)
*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^7) - 840*(d*x +
c)/b^8)/d

3.468 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=407

$$\frac{5a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{9/2}}-\frac{\cos(c+dx)\left(4a^2+10ab\sin(c+dx)+9b^2\right)}{42b^5d(a+b\sin(c+dx))^5}+\frac{\left(-38a^4b^2+87a^2b^4+8a^6+48b^6\right)\cos(c+dx)}{336b^5d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}$$

```
[Out] (5*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(9/2)
*d) - Cos[c + d*x]^5/(7*b*d*(a + b*Sin[c + d*x])^7) + (a*(4*a^2 - b^2)*Cos[
c + d*x])/(168*b^5*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^4) + ((4*a^4 - 9*a^2*
b^2 + 12*b^4)*Cos[c + d*x])/(168*b^5*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^3
) + (a*(8*a^4 - 30*a^2*b^2 + 57*b^4)*Cos[c + d*x])/(336*b^5*(a^2 - b^2)^3*d
*(a + b*Sin[c + d*x])^2) + ((8*a^6 - 38*a^4*b^2 + 87*a^2*b^4 + 48*b^6)*Cos[
c + d*x])/(336*b^5*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (5*Cos[c + d*x]^
3*(2*a + 3*b*Sin[c + d*x]))/(42*b^3*d*(a + b*Sin[c + d*x])^6) - (Cos[c + d*
x]*(4*a^2 + 9*b^2 + 10*a*b*Sin[c + d*x]))/(42*b^5*d*(a + b*Sin[c + d*x])^5)
```

Rubi [A] time = 0.79152, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2693, 2863, 2754, 12, 2660, 618, 204}

$$\frac{5a\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{9/2}}-\frac{\cos(c+dx)\left(4a^2+10ab\sin(c+dx)+9b^2\right)}{42b^5d(a+b\sin(c+dx))^5}+\frac{\left(-38a^4b^2+87a^2b^4+8a^6+48b^6\right)\cos(c+dx)}{336b^5d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^8,x]
```


```
[Out] (5*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(9/2)
*d) - Cos[c + d*x]^5/(7*b*d*(a + b*Sin[c + d*x])^7) + (a*(4*a^2 - b^2)*Cos[
c + d*x])/(168*b^5*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^4) + ((4*a^4 - 9*a^2*
b^2 + 12*b^4)*Cos[c + d*x])/(168*b^5*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^3
) + (a*(8*a^4 - 30*a^2*b^2 + 57*b^4)*Cos[c + d*x])/(336*b^5*(a^2 - b^2)^3*d
*(a + b*Sin[c + d*x])^2) + ((8*a^6 - 38*a^4*b^2 + 87*a^2*b^4 + 48*b^6)*Cos[
c + d*x])/(336*b^5*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (5*Cos[c + d*x]^
3*(2*a + 3*b*Sin[c + d*x]))/(42*b^3*d*(a + b*Sin[c + d*x])^6) - (Cos[c + d*
x]*(4*a^2 + 9*b^2 + 10*a*b*Sin[c + d*x]))/(42*b^5*d*(a + b*Sin[c + d*x])^5)
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^8}\,dx = -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{5\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^7}\,dx}{7b}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{5\cos^3(c+dx)(2a+3b\sin(c+dx))}{42b^3d(a+b\sin(c+dx))^6} - \frac{5\int \frac{\cos^2(c+dx)(-6b-4a\sin(c+}{(a+b\sin(c+dx))^6}}{28b^3}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{5\cos^3(c+dx)(2a+3b\sin(c+dx))}{42b^3d(a+b\sin(c+dx))^6} - \frac{\cos(c+dx)\left(4a^2+9b^2+\right.}{42b^5d(a+b\sin}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{5\cos^3(c+dx)(2a+3b}{42b^3d(a+b\sin(c}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{\left(4a^4-9a^2b^2+12b^4\right)}{168b^5\left(a^2-b^2\right)^2d(a+}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{\left(4a^4-9a^2b^2+12b^4\right)}{168b^5\left(a^2-b^2\right)^2d(a+}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{\left(4a^4-9a^2b^2+12b^4\right)}{168b^5\left(a^2-b^2\right)^2d(a+}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{\left(4a^4-9a^2b^2+12b^4\right)}{168b^5\left(a^2-b^2\right)^2d(a+}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{\left(4a^4-9a^2b^2+12b^4\right)}{168b^5\left(a^2-b^2\right)^2d(a+}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4} + \frac{\left(4a^4-9a^2b^2+12b^4\right)}{168b^5\left(a^2-b^2\right)^2d(a+}$$

$$= \frac{5a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{9/2}d} - \frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\left(4a^2-b^2\right)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))}$$

Mathematica [A] time = 6.04288, size = 552, normalized size = 1.36

$$a\cos(c+dx)\left(-\frac{(1-\sin(c+dx))^{5/2}(\sin(c+dx)+1)^{9/2}}{7(b-a)(a+b\sin(c+dx))^7}-\frac{5\left(-\frac{(1-\sin(c+dx))^{3/2}(\sin(c+dx)+1)^{9/2}}{6(b-a)(a+b\sin(c+dx))^6}-\frac{-\frac{\sqrt{1-\sin(c+dx)}(\sin(c+dx)}{5(b-a)(a+b\sin(c+dx)}}{}\right.}{}\right.$$

$\cos^7(c+dx)$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] Cos[c + d*x]^7/(7*(a - b)*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]*(-((1
 - Sin[c + d*x])^(5/2)*(1 + Sin[c + d*x])^(9/2))/(7*(-a + b)*(a + b*Sin[c +
 d*x])^7) - (5*(-((1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(9/2))/(6*(-a
 + b)*(a + b*Sin[c + d*x])^6) - (-(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x]
)^(9/2))/(5*(-a + b)*(a + b*Sin[c + d*x])^5) - (-(Sqrt[1 - Sin[c + d*x]]*(1
 + Sin[c + d*x])^(7/2))/(4*(a + b)*(a + b*Sin[c + d*x])^4) + (7*(-(Sqrt[1 -
 Sin[c + d*x]]*(1 + Sin[c + d*x])^(5/2))/(3*(a + b)*(a + b*Sin[c + d*x])^3)
 + (5*(-(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(3/2))/(2*(a + b)*(a + b
*Sin[c + d*x])^2) + (3*((-2*ArcTan[(Sqrt[-a + b]*Sqrt[1 - Sin[c + d*x]])/(S
qrt[-a - b]*Sqrt[1 + Sin[c + d*x]])])/((-a - b)^(3/2)*Sqrt[-a + b]) + (Sqrt
[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])/((-a - b)*(a + b*Sin[c + d*x])))
)/(2*(a + b))))/(3*(a + b))))/(4*(a + b)))/(5*(-a + b)))/(2*(-a + b))))/(7*
(-a + b))))/((a - b)*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])
```

Maple [B] time = 0.211, size = 6933, normalized size = 17.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^8,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.25552, size = 5125, normalized size = 12.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [1/672*(2*(8*a^8*b - 46*a^6*b^3 + 125*a^4*b^5 - 39*a^2*b^7 - 48*b^9)*cos(d*
x + c)^7 + 28*(7*a^8*b - 56*a^6*b^3 - 44*a^4*b^5 + 93*a^2*b^7)*cos(d*x + c)
^5 + 70*(7*a^8*b + 83*a^6*b^3 - 43*a^4*b^5 - 47*a^2*b^7)*cos(d*x + c)^3 - 1
05*(7*a^2*b^6*cos(d*x + c)^6 - a^8 - 21*a^6*b^2 - 35*a^4*b^4 - 7*a^2*b^6 -
7*(5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c)^4 + 7*(3*a^6*b^2 + 10*a^4*b^4 + 3*a^
2*b^6)*cos(d*x + c)^2 + (a*b^7*cos(d*x + c)^6 - 7*a^7*b - 35*a^5*b^3 - 21*a
^3*b^5 - a*b^7 - 3*(7*a^3*b^5 + a*b^7)*cos(d*x + c)^4 + (35*a^5*b^3 + 42*a^
3*b^5 + 3*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2
 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)
*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2)) - 420*(3*a^8*b + 7*a^6*b^3 - 7*a^4*b^5 - 3*a^
2*b^7)*cos(d*x + c) - 14*((8*a^9 - 46*a^7*b^2 + 125*a^5*b^4 - 54*a^3*b^6 -
33*a*b^8)*cos(d*x + c)^5 + 10*(a^9 - 11*a^7*b^2 - 25*a^5*b^4 + 31*a^3*b^6 +
 4*a*b^8)*cos(d*x + c)^3 + 15*(a^9 + 14*a^7*b^2 - 14*a^3*b^6 - a*b^8)*cos(d
*x + c))*sin(d*x + c))/(7*(a^11*b^6 - 5*a^9*b^8 + 10*a^7*b^10 - 10*a^5*b^12
 + 5*a^3*b^14 - a*b^16)*d*cos(d*x + c)^6 - 7*(5*a^13*b^4 - 22*a^11*b^6 + 35
*a^9*b^8 - 20*a^7*b^10 - 5*a^5*b^12 + 10*a^3*b^14 - 3*a*b^16)*d*cos(d*x + c
)^4 + 7*(3*a^15*b^2 - 5*a^13*b^4 - 17*a^11*b^6 + 55*a^9*b^8 - 55*a^7*b^10 +
 17*a^5*b^12 + 5*a^3*b^14 - 3*a*b^16)*d*cos(d*x + c)^2 - (a^17 + 16*a^15*b^
2 - 60*a^13*b^4 + 32*a^11*b^6 + 110*a^9*b^8 - 176*a^7*b^10 + 84*a^5*b^12 -
7*a*b^16)*d + ((a^10*b^7 - 5*a^8*b^9 + 10*a^6*b^11 - 10*a^4*b^13 + 5*a^2*b^
15 - b^17)*d*cos(d*x + c)^6 - 3*(7*a^12*b^5 - 34*a^10*b^7 + 65*a^8*b^9 - 60
*a^6*b^11 + 25*a^4*b^13 - 2*a^2*b^15 - b^17)*d*cos(d*x + c)^4 + (35*a^14*b^
3 - 133*a^12*b^5 + 143*a^10*b^7 + 55*a^8*b^9 - 215*a^6*b^11 + 145*a^4*b^13
- 27*a^2*b^15 - 3*b^17)*d*cos(d*x + c)^2 - (7*a^16*b - 84*a^12*b^5 + 176*a^
10*b^7 - 110*a^8*b^9 - 32*a^6*b^11 + 60*a^4*b^13 - 16*a^2*b^15 - b^17)*d)*s
in(d*x + c)), 1/336*((8*a^8*b - 46*a^6*b^3 + 125*a^4*b^5 - 39*a^2*b^7 - 48*
b^9)*cos(d*x + c)^7 + 14*(7*a^8*b - 56*a^6*b^3 - 44*a^4*b^5 + 93*a^2*b^7)*c
os(d*x + c)^5 + 35*(7*a^8*b + 83*a^6*b^3 - 43*a^4*b^5 - 47*a^2*b^7)*cos(d*x
 + c)^3 - 105*(7*a^2*b^6*cos(d*x + c)^6 - a^8 - 21*a^6*b^2 - 35*a^4*b^4 - 7
*a^2*b^6 - 7*(5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c)^4 + 7*(3*a^6*b^2 + 10*a^4
*b^4 + 3*a^2*b^6)*cos(d*x + c)^2 + (a*b^7*cos(d*x + c)^6 - 7*a^7*b - 35*a^5
*b^3 - 21*a^3*b^5 - a*b^7 - 3*(7*a^3*b^5 + a*b^7)*cos(d*x + c)^4 + (35*a^5*
b^3 + 42*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 210*(3*a^8*b
+ 7*a^6*b^3 - 7*a^4*b^5 - 3*a^2*b^7)*cos(d*x + c) - 7*((8*a^9 - 46*a^7*b^2
```

```
+ 125*a^5*b^4 - 54*a^3*b^6 - 33*a*b^8)*cos(d*x + c)^5 + 10*(a^9 - 11*a^7*b^
2 - 25*a^5*b^4 + 31*a^3*b^6 + 4*a*b^8)*cos(d*x + c)^3 + 15*(a^9 + 14*a^7*b^
2 - 14*a^3*b^6 - a*b^8)*cos(d*x + c))*sin(d*x + c))/(7*(a^11*b^6 - 5*a^9*b^
8 + 10*a^7*b^10 - 10*a^5*b^12 + 5*a^3*b^14 - a*b^16)*d*cos(d*x + c)^6 - 7*(
5*a^13*b^4 - 22*a^11*b^6 + 35*a^9*b^8 - 20*a^7*b^10 - 5*a^5*b^12 + 10*a^3*b
^14 - 3*a*b^16)*d*cos(d*x + c)^4 + 7*(3*a^15*b^2 - 5*a^13*b^4 - 17*a^11*b^6
 + 55*a^9*b^8 - 55*a^7*b^10 + 17*a^5*b^12 + 5*a^3*b^14 - 3*a*b^16)*d*cos(d*
x + c)^2 - (a^17 + 16*a^15*b^2 - 60*a^13*b^4 + 32*a^11*b^6 + 110*a^9*b^8 -
176*a^7*b^10 + 84*a^5*b^12 - 7*a*b^16)*d + ((a^10*b^7 - 5*a^8*b^9 + 10*a^6*
b^11 - 10*a^4*b^13 + 5*a^2*b^15 - b^17)*d*cos(d*x + c)^6 - 3*(7*a^12*b^5 -
34*a^10*b^7 + 65*a^8*b^9 - 60*a^6*b^11 + 25*a^4*b^13 - 2*a^2*b^15 - b^17)*d
*cos(d*x + c)^4 + (35*a^14*b^3 - 133*a^12*b^5 + 143*a^10*b^7 + 55*a^8*b^9 -
 215*a^6*b^11 + 145*a^4*b^13 - 27*a^2*b^15 - 3*b^17)*d*cos(d*x + c)^2 - (7*
a^16*b - 84*a^12*b^5 + 176*a^10*b^7 - 110*a^8*b^9 - 32*a^6*b^11 + 60*a^4*b^
13 - 16*a^2*b^15 - b^17)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.55771, size = 2228, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/168*(105*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
 + 1/2*c) + b)/sqrt(a^2 - b^2)))*a/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^
6 + b^8)*sqrt(a^2 - b^2)) - (231*a^14*tan(1/2*d*x + 1/2*c)^13 - 1344*a^12*b
^2*tan(1/2*d*x + 1/2*c)^13 + 2016*a^10*b^4*tan(1/2*d*x + 1/2*c)^13 - 1344*a
^8*b^6*tan(1/2*d*x + 1/2*c)^13 + 336*a^6*b^8*tan(1/2*d*x + 1/2*c)^13 + 651*
```

a^13*b*tan(1/2*d*x + 1/2*c)^12 - 8064*a^11*b^3*tan(1/2*d*x + 1/2*c)^12 + 12096*a^9*b^5*tan(1/2*d*x + 1/2*c)^12 - 8064*a^7*b^7*tan(1/2*d*x + 1/2*c)^12 + 2016*a^5*b^9*tan(1/2*d*x + 1/2*c)^12 + 196*a^14*tan(1/2*d*x + 1/2*c)^11 - 4354*a^12*b^2*tan(1/2*d*x + 1/2*c)^11 - 21504*a^10*b^4*tan(1/2*d*x + 1/2*c)^11 + 36736*a^8*b^6*tan(1/2*d*x + 1/2*c)^11 - 25984*a^6*b^8*tan(1/2*d*x + 1/2*c)^11 + 6720*a^4*b^10*tan(1/2*d*x + 1/2*c)^11 + 140*a^13*b*tan(1/2*d*x + 1/2*c)^10 - 40250*a^11*b^3*tan(1/2*d*x + 1/2*c)^10 - 6720*a^9*b^5*tan(1/2*d*x + 1/2*c)^10 + 49280*a^7*b^7*tan(1/2*d*x + 1/2*c)^10 - 45920*a^5*b^9*tan(1/2*d*x + 1/2*c)^10 + 13440*a^3*b^11*tan(1/2*d*x + 1/2*c)^10 + 595*a^14*tan(1/2*d*x + 1/2*c)^9 - 20650*a^12*b^2*tan(1/2*d*x + 1/2*c)^9 - 103740*a^10*b^4*tan(1/2*d*x + 1/2*c)^9 + 70336*a^8*b^6*tan(1/2*d*x + 1/2*c)^9 + 2576*a^6*b^8*tan(1/2*d*x + 1/2*c)^9 - 40320*a^4*b^10*tan(1/2*d*x + 1/2*c)^9 + 16128*a^2*b^12*tan(1/2*d*x + 1/2*c)^9 - 3045*a^13*b*tan(1/2*d*x + 1/2*c)^8 - 100450*a^11*b^3*tan(1/2*d*x + 1/2*c)^8 - 92120*a^9*b^5*tan(1/2*d*x + 1/2*c)^8 + 129024*a^7*b^7*tan(1/2*d*x + 1/2*c)^8 - 74816*a^5*b^9*tan(1/2*d*x + 1/2*c)^8 - 4480*a^3*b^11*tan(1/2*d*x + 1/2*c)^8 + 10752*a*b^13*tan(1/2*d*x + 1/2*c)^8 - 39060*a^12*b^2*tan(1/2*d*x + 1/2*c)^7 - 188720*a^10*b^4*tan(1/2*d*x + 1/2*c)^7 + 58352*a^8*b^6*tan(1/2*d*x + 1/2*c)^7 + 39936*a^6*b^8*tan(1/2*d*x + 1/2*c)^7 - 73216*a^4*b^10*tan(1/2*d*x + 1/2*c)^7 + 19456*a^2*b^12*tan(1/2*d*x + 1/2*c)^7 + 3072*b^14*tan(1/2*d*x + 1/2*c)^7 - 6720*a^13*b*tan(1/2*d*x + 1/2*c)^6 - 122500*a^11*b^3*tan(1/2*d*x + 1/2*c)^6 - 109760*a^9*b^5*tan(1/2*d*x + 1/2*c)^6 + 127344*a^7*b^7*tan(1/2*d*x + 1/2*c)^6 - 74816*a^5*b^9*tan(1/2*d*x + 1/2*c)^6 - 4480*a^3*b^11*tan(1/2*d*x + 1/2*c)^6 + 10752*a*b^13*tan(1/2*d*x + 1/2*c)^6 - 595*a^14*tan(1/2*d*x + 1/2*c)^5 - 37940*a^12*b^2*tan(1/2*d*x + 1/2*c)^5 - 140280*a^10*b^4*tan(1/2*d*x + 1/2*c)^5 + 65296*a^8*b^6*tan(1/2*d*x + 1/2*c)^5 + 2576*a^6*b^8*tan(1/2*d*x + 1/2*c)^5 - 40320*a^4*b^10*tan(1/2*d*x + 1/2*c)^5 + 16128*a^2*b^12*tan(1/2*d*x + 1/2*c)^5 - 5999*a^13*b*tan(1/2*d*x + 1/2*c)^4 - 70084*a^11*b^3*tan(1/2*d*x + 1/2*c)^4 - 16800*a^9*b^5*tan(1/2*d*x + 1/2*c)^4 + 50288*a^7*b^7*tan(1/2*d*x + 1/2*c)^4 - 45920*a^5*b^9*tan(1/2*d*x + 1/2*c)^4 + 13440*a^3*b^11*tan(1/2*d*x + 1/2*c)^4 - 196*a^14*tan(1/2*d*x + 1/2*c)^3 - 19082*a^12*b^2*tan(1/2*d*x + 1/2*c)^3 - 29232*a^10*b^4*tan(1/2*d*x + 1/2*c)^3 + 37744*a^8*b^6*tan(1/2*d*x + 1/2*c)^3 - 25984*a^6*b^8*tan(1/2*d*x + 1/2*c)^3 + 6720*a^4*b^10*tan(1/2*d*x + 1/2*c)^3 - 2604*a^13*b*tan(1/2*d*x + 1/2*c)^2 - 13090*a^11*b^3*tan(1/2*d*x + 1/2*c)^2 + 13888*a^9*b^5*tan(1/2*d*x + 1/2*c)^2 - 8400*a^7*b^7*tan(1/2*d*x + 1/2*c)^2 + 2016*a^5*b^9*tan(1/2*d*x + 1/2*c)^2 - 231*a^14*tan(1/2*d*x + 1/2*c) - 2562*a^12*b^2*tan(1/2*d*x + 1/2*c) + 2548*a^10*b^4*tan(1/2*d*x + 1/2*c) - 1456*a^8*b^6*tan(1/2*d*x + 1/2*c) + 336*a^6*b^8*tan(1/2*d*x + 1/2*c) - 279*a^13*b + 326*a^11*b^3 - 200*a^9*b^5 + 48*a^7*b^7)/((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^7))/d

3.469 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=411

$$\frac{3a\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{11/2}}-\frac{\left(-40a^4b^2-247a^2b^4+4a^6-32b^6\right)\cos(c+dx)}{560b^3d\left(a^2-b^2\right)^5(a+b\sin(c+dx))}-\frac{a\left(-36a^2b^2+4a^4-73b^4\right)\cos(}{560b^3d\left(a^2-b^2\right)^4(a+b\sin(c}$$

```
[Out] (3*a*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^
2 - b^2)^(11/2)*d) - Cos[c + d*x]^3/(7*b*d*(a + b*Sin[c + d*x])^7) - ((a^2
- 3*b^2)*Cos[c + d*x])/(140*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^5) - (a*
(2*a^2 - 11*b^2)*Cos[c + d*x])/(280*b^3*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]
)^4) - ((2*a^4 - 15*a^2*b^2 - 8*b^4)*Cos[c + d*x])/(280*b^3*(a^2 - b^2)^3*d
*(a + b*Sin[c + d*x])^3) - (a*(4*a^4 - 36*a^2*b^2 - 73*b^4)*Cos[c + d*x])/(
560*b^3*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^2) - ((4*a^6 - 40*a^4*b^2 - 24
7*a^2*b^4 - 32*b^6)*Cos[c + d*x])/(560*b^3*(a^2 - b^2)^5*d*(a + b*Sin[c + d
*x])) + (Cos[c + d*x]*(a + 3*b*Sin[c + d*x]))/(28*b^3*d*(a + b*Sin[c + d*x]
)^6)
```

Rubi [A] time = 0.790644, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2693, 2863, 2754, 12, 2660, 618, 204}

$$\frac{3a\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{11/2}}-\frac{\left(-40a^4b^2-247a^2b^4+4a^6-32b^6\right)\cos(c+dx)}{560b^3d\left(a^2-b^2\right)^5(a+b\sin(c+dx))}-\frac{a\left(-36a^2b^2+4a^4-73b^4\right)\cos(}{560b^3d\left(a^2-b^2\right)^4(a+b\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (3*a*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^
2 - b^2)^(11/2)*d) - Cos[c + d*x]^3/(7*b*d*(a + b*Sin[c + d*x])^7) - ((a^2
- 3*b^2)*Cos[c + d*x])/(140*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^5) - (a*
(2*a^2 - 11*b^2)*Cos[c + d*x])/(280*b^3*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]
)^4) - ((2*a^4 - 15*a^2*b^2 - 8*b^4)*Cos[c + d*x])/(280*b^3*(a^2 - b^2)^3*d
*(a + b*Sin[c + d*x])^3) - (a*(4*a^4 - 36*a^2*b^2 - 73*b^4)*Cos[c + d*x])/(
560*b^3*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^2) - ((4*a^6 - 40*a^4*b^2 - 24
7*a^2*b^4 - 32*b^6)*Cos[c + d*x])/(560*b^3*(a^2 - b^2)^5*d*(a + b*Sin[c + d
```

```
*x])) + (Cos[c + d*x]*(a + 3*b*Sin[c + d*x]))/(28*b^3*d*(a + b*Sin[c + d*x]
)^6)
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^8}\,dx = -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{3\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^7}\,dx}{7b}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{\cos(c+dx)(a+3b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))^6} - \frac{\int \frac{-6b-2a\sin(c+dx)}{(a+b\sin(c+dx))^6}\,dx}{56b^3}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} + \frac{\cos(c+dx)(a+3b\sin(c}{28b^3d(a+b\sin(c+d}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-11b^2\right)\cos(}{280b^3\left(a^2-b^2\right)^2 d(a+b\,\mathrm{s}}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-11b^2\right)\cos(}{280b^3\left(a^2-b^2\right)^2 d(a+b\,\mathrm{s}}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-11b^2\right)\cos(}{280b^3\left(a^2-b^2\right)^2 d(a+b\,\mathrm{s}}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-11b^2\right)\cos(}{280b^3\left(a^2-b^2\right)^2 d(a+b\,\mathrm{s}}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-11b^2\right)\cos(}{280b^3\left(a^2-b^2\right)^2 d(a+b\,\mathrm{s}}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-11b^2\right)\cos(}{280b^3\left(a^2-b^2\right)^2 d(a+b\,\mathrm{s}}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5} - \frac{a\left(2a^2-11b^2\right)\cos(}{280b^3\left(a^2-b^2\right)^2 d(a+b\,\mathrm{s}}$$

$$= \frac{3a\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{11/2}d} - \frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\left(a^2-3b^2\right)\cos(c+}{140b^3\left(a^2-b^2\right)d(a+b\sin}$$

<u>Mathematica</u> [B] time = 6.07593, size = 1167, normalized size = 2.84

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] Cos[c + d*x]^5/(5*(a - b)*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]*(-(b*
(1 - Sin[c + d*x])^(5/2)*(1 + Sin[c + d*x])^(7/2))/(7*(-a + b)*(a + b)*(a +
 b*Sin[c + d*x])^7) - (-((a*b + (7*a - b)*b)*(1 - Sin[c + d*x])^(5/2)*(1 +
Sin[c + d*x])^(7/2))/(6*(-a + b)*(a + b)*(a + b*Sin[c + d*x])^6) - (7*(6*a^
2 - 2*a*b + b^2)*(-((1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(7/2))/(5*(
-a + b)*(a + b*Sin[c + d*x])^5) - (3*(-(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c +
 d*x])^(7/2))/(4*(-a + b)*(a + b*Sin[c + d*x])^4) - (-(Sqrt[1 - Sin[c + d*x
]]*(1 + Sin[c + d*x])^(5/2))/(3*(a + b)*(a + b*Sin[c + d*x])^3) + (5*(-(Sqr
t[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(3/2))/(2*(a + b)*(a + b*Sin[c + d*x
])^2) + (3*((-2*ArcTan[(Sqrt[-a + b]*Sqrt[1 - Sin[c + d*x]])/(Sqrt[-a - b]*
Sqrt[1 + Sin[c + d*x]])])/((-a - b)^(3/2)*Sqrt[-a + b]) + (Sqrt[1 - Sin[c +
 d*x]]*Sqrt[1 + Sin[c + d*x]])/((-a - b)*(a + b*Sin[c + d*x]))))/(2*(a + b)
)))/(3*(a + b)))/(4*(-a + b))))/(5*(-a + b))))/(6*(-a + b)*(a + b)))/(7*(-a
 + b)*(a + b))))/((a - b)*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])
+ (2*b*(Cos[c + d*x]^7/(7*(a - b)*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*
x]*(-((1 - Sin[c + d*x])^(5/2)*(1 + Sin[c + d*x])^(9/2))/(7*(-a + b)*(a + b
*Sin[c + d*x])^7) - (5*(-((1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(9/2)
)/(6*(-a + b)*(a + b*Sin[c + d*x])^6) - (-(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[
c + d*x])^(9/2))/(5*(-a + b)*(a + b*Sin[c + d*x])^5) - (-(Sqrt[1 - Sin[c +
d*x]]*(1 + Sin[c + d*x])^(7/2))/(4*(a + b)*(a + b*Sin[c + d*x])^4) + (7*(-(
Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(5/2))/(3*(a + b)*(a + b*Sin[c +
d*x])^3) + (5*(-(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(3/2))/(2*(a + b
)*(a + b*Sin[c + d*x])^2) + (3*((-2*ArcTan[(Sqrt[-a + b]*Sqrt[1 - Sin[c + d
*x]])/(Sqrt[-a - b]*Sqrt[1 + Sin[c + d*x]])])/((-a - b)^(3/2)*Sqrt[-a + b])
 + (Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])/((-a - b)*(a + b*Sin[c +
 d*x]))))/(2*(a + b))))/(3*(a + b))))/(4*(a + b)))/(5*(-a + b)))/(2*(-a + b
))))/(7*(-a + b))))/((a - b)*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]
])))/(5*(a - b))
```

Maple [B] time = 0.208, size = 9171, normalized size = 22.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^8,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.72923, size = 6137, normalized size = 14.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [-1/1120*(2*(4*a^8*b^3 - 44*a^6*b^5 - 207*a^4*b^7 + 215*a^2*b^9 + 32*b^11)*
cos(d*x + c)^7 - 28*(6*a^10*b - 65*a^8*b^3 - 224*a^6*b^5 + 222*a^4*b^7 + 53
*a^2*b^9 + 8*b^11)*cos(d*x + c)^5 - 70*(14*a^10*b + 173*a^8*b^3 - 3*a^6*b^5
 - 137*a^4*b^7 - 47*a^2*b^9)*cos(d*x + c)^3 + 105*(2*a^10 + 43*a^8*b^2 + 91
*a^6*b^4 + 49*a^4*b^6 + 7*a^2*b^8 - 7*(2*a^4*b^6 + a^2*b^8)*cos(d*x + c)^6
+ 7*(10*a^6*b^4 + 11*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^4 - 7*(6*a^8*b^2 + 2
3*a^6*b^4 + 16*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^2 + (14*a^9*b + 77*a^7*b^3
 + 77*a^5*b^5 + 23*a^3*b^7 + a*b^9 - (2*a^3*b^7 + a*b^9)*cos(d*x + c)^6 + 3
*(14*a^5*b^5 + 9*a^3*b^7 + a*b^9)*cos(d*x + c)^4 - (70*a^7*b^3 + 119*a^5*b^
5 + 48*a^3*b^7 + 3*a*b^9)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*lo
g(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*co
s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 420*(6*a^10*b + 17*a^8*b^3 - 7*a^
6*b^5 - 13*a^4*b^7 - 3*a^2*b^9)*cos(d*x + c) - 14*((4*a^9*b^2 - 44*a^7*b^4
- 177*a^5*b^6 + 200*a^3*b^8 + 17*a*b^10)*cos(d*x + c)^5 - 10*(2*a^11 - 21*a
^9*b^2 - 61*a^7*b^4 + 37*a^5*b^6 + 39*a^3*b^8 + 4*a*b^10)*cos(d*x + c)^3 -
15*(2*a^11 + 29*a^9*b^2 + 14*a^7*b^4 - 28*a^5*b^6 - 16*a^3*b^8 - a*b^10)*co
s(d*x + c))*sin(d*x + c))/(7*(a^13*b^6 - 6*a^11*b^8 + 15*a^9*b^10 - 20*a^7*
b^12 + 15*a^5*b^14 - 6*a^3*b^16 + a*b^18)*d*cos(d*x + c)^6 - 7*(5*a^15*b^4
- 27*a^13*b^6 + 57*a^11*b^8 - 55*a^9*b^10 + 15*a^7*b^12 + 15*a^5*b^14 - 13*
```

a^3*b^16 + 3*a*b^18)*d*cos(d*x + c)^4 + 7*(3*a^17*b^2 - 8*a^15*b^4 - 12*a^1
3*b^6 + 72*a^11*b^8 - 110*a^9*b^10 + 72*a^7*b^12 - 12*a^5*b^14 - 8*a^3*b^16
 + 3*a*b^18)*d*cos(d*x + c)^2 - (a^19 + 15*a^17*b^2 - 76*a^15*b^4 + 92*a^13
*b^6 + 78*a^11*b^8 - 286*a^9*b^10 + 260*a^7*b^12 - 84*a^5*b^14 - 7*a^3*b^16
 + 7*a*b^18)*d + ((a^12*b^7 - 6*a^10*b^9 + 15*a^8*b^11 - 20*a^6*b^13 + 15*a
^4*b^15 - 6*a^2*b^17 + b^19)*d*cos(d*x + c)^6 - 3*(7*a^14*b^5 - 41*a^12*b^7
 + 99*a^10*b^9 - 125*a^8*b^11 + 85*a^6*b^13 - 27*a^4*b^15 + a^2*b^17 + b^19
)*d*cos(d*x + c)^4 + (35*a^16*b^3 - 168*a^14*b^5 + 276*a^12*b^7 - 88*a^10*b
^9 - 270*a^8*b^11 + 360*a^6*b^13 - 172*a^4*b^15 + 24*a^2*b^17 + 3*b^19)*d*c
os(d*x + c)^2 - (7*a^18*b - 7*a^16*b^3 - 84*a^14*b^5 + 260*a^12*b^7 - 286*a
^10*b^9 + 78*a^8*b^11 + 92*a^6*b^13 - 76*a^4*b^15 + 15*a^2*b^17 + b^19)*d)*
sin(d*x + c)), -1/560*((4*a^8*b^3 - 44*a^6*b^5 - 207*a^4*b^7 + 215*a^2*b^9
+ 32*b^11)*cos(d*x + c)^7 - 14*(6*a^10*b - 65*a^8*b^3 - 224*a^6*b^5 + 222*a
^4*b^7 + 53*a^2*b^9 + 8*b^11)*cos(d*x + c)^5 - 35*(14*a^10*b + 173*a^8*b^3
- 3*a^6*b^5 - 137*a^4*b^7 - 47*a^2*b^9)*cos(d*x + c)^3 - 105*(2*a^10 + 43*a
^8*b^2 + 91*a^6*b^4 + 49*a^4*b^6 + 7*a^2*b^8 - 7*(2*a^4*b^6 + a^2*b^8)*cos(
d*x + c)^6 + 7*(10*a^6*b^4 + 11*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^4 - 7*(6*
a^8*b^2 + 23*a^6*b^4 + 16*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^2 + (14*a^9*b +
 77*a^7*b^3 + 77*a^5*b^5 + 23*a^3*b^7 + a*b^9 - (2*a^3*b^7 + a*b^9)*cos(d*x
 + c)^6 + 3*(14*a^5*b^5 + 9*a^3*b^7 + a*b^9)*cos(d*x + c)^4 - (70*a^7*b^3 +
 119*a^5*b^5 + 48*a^3*b^7 + 3*a*b^9)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2
 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 210*
(6*a^10*b + 17*a^8*b^3 - 7*a^6*b^5 - 13*a^4*b^7 - 3*a^2*b^9)*cos(d*x + c) -
 7*((4*a^9*b^2 - 44*a^7*b^4 - 177*a^5*b^6 + 200*a^3*b^8 + 17*a*b^10)*cos(d*
x + c)^5 - 10*(2*a^11 - 21*a^9*b^2 - 61*a^7*b^4 + 37*a^5*b^6 + 39*a^3*b^8 +
 4*a*b^10)*cos(d*x + c)^3 - 15*(2*a^11 + 29*a^9*b^2 + 14*a^7*b^4 - 28*a^5*b
^6 - 16*a^3*b^8 - a*b^10)*cos(d*x + c))*sin(d*x + c))/(7*(a^13*b^6 - 6*a^11
*b^8 + 15*a^9*b^10 - 20*a^7*b^12 + 15*a^5*b^14 - 6*a^3*b^16 + a*b^18)*d*cos
(d*x + c)^6 - 7*(5*a^15*b^4 - 27*a^13*b^6 + 57*a^11*b^8 - 55*a^9*b^10 + 15*
a^7*b^12 + 15*a^5*b^14 - 13*a^3*b^16 + 3*a*b^18)*d*cos(d*x + c)^4 + 7*(3*a^
17*b^2 - 8*a^15*b^4 - 12*a^13*b^6 + 72*a^11*b^8 - 110*a^9*b^10 + 72*a^7*b^1
2 - 12*a^5*b^14 - 8*a^3*b^16 + 3*a*b^18)*d*cos(d*x + c)^2 - (a^19 + 15*a^17
*b^2 - 76*a^15*b^4 + 92*a^13*b^6 + 78*a^11*b^8 - 286*a^9*b^10 + 260*a^7*b^1
2 - 84*a^5*b^14 - 7*a^3*b^16 + 7*a*b^18)*d + ((a^12*b^7 - 6*a^10*b^9 + 15*a
^8*b^11 - 20*a^6*b^13 + 15*a^4*b^15 - 6*a^2*b^17 + b^19)*d*cos(d*x + c)^6 -
 3*(7*a^14*b^5 - 41*a^12*b^7 + 99*a^10*b^9 - 125*a^8*b^11 + 85*a^6*b^13 - 2
7*a^4*b^15 + a^2*b^17 + b^19)*d*cos(d*x + c)^4 + (35*a^16*b^3 - 168*a^14*b^
5 + 276*a^12*b^7 - 88*a^10*b^9 - 270*a^8*b^11 + 360*a^6*b^13 - 172*a^4*b^15
 + 24*a^2*b^17 + 3*b^19)*d*cos(d*x + c)^2 - (7*a^18*b - 7*a^16*b^3 - 84*a^1
4*b^5 + 260*a^12*b^7 - 286*a^10*b^9 + 78*a^8*b^11 + 92*a^6*b^13 - 76*a^4*b^
15 + 15*a^2*b^17 + b^19)*d)*sin(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.54474, size = 2608, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/280*(105*(2*a^3 + a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arcta
n((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^10 - 5*a^8*b^2 + 10*a^
6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*sqrt(a^2 - b^2)) - (350*a^16*tan(1/2
*d*x + 1/2*c)^13 - 2905*a^14*b^2*tan(1/2*d*x + 1/2*c)^13 + 5600*a^12*b^4*ta
n(1/2*d*x + 1/2*c)^13 - 5600*a^10*b^6*tan(1/2*d*x + 1/2*c)^13 + 2800*a^8*b^
8*tan(1/2*d*x + 1/2*c)^13 - 560*a^6*b^10*tan(1/2*d*x + 1/2*c)^13 + 630*a^15
*b*tan(1/2*d*x + 1/2*c)^12 - 18165*a^13*b^3*tan(1/2*d*x + 1/2*c)^12 + 33600
*a^11*b^5*tan(1/2*d*x + 1/2*c)^12 - 33600*a^9*b^7*tan(1/2*d*x + 1/2*c)^12 +
 16800*a^7*b^9*tan(1/2*d*x + 1/2*c)^12 - 3360*a^5*b^11*tan(1/2*d*x + 1/2*c)
^12 + 840*a^16*tan(1/2*d*x + 1/2*c)^11 - 15680*a^14*b^2*tan(1/2*d*x + 1/2*c
)^11 - 41090*a^12*b^4*tan(1/2*d*x + 1/2*c)^11 + 89600*a^10*b^6*tan(1/2*d*x
+ 1/2*c)^11 - 100800*a^8*b^8*tan(1/2*d*x + 1/2*c)^11 + 53760*a^6*b^10*tan(1
/2*d*x + 1/2*c)^11 - 11200*a^4*b^12*tan(1/2*d*x + 1/2*c)^11 - 840*a^15*b*ta
n(1/2*d*x + 1/2*c)^10 - 102760*a^13*b^3*tan(1/2*d*x + 1/2*c)^10 + 11270*a^1
1*b^5*tan(1/2*d*x + 1/2*c)^10 + 78400*a^9*b^7*tan(1/2*d*x + 1/2*c)^10 - 151
200*a^7*b^9*tan(1/2*d*x + 1/2*c)^10 + 97440*a^5*b^11*tan(1/2*d*x + 1/2*c)^1
0 - 22400*a^3*b^13*tan(1/2*d*x + 1/2*c)^10 + 630*a^16*tan(1/2*d*x + 1/2*c)^
9 - 51905*a^14*b^2*tan(1/2*d*x + 1/2*c)^9 - 249410*a^12*b^4*tan(1/2*d*x + 1
/2*c)^9 + 202244*a^10*b^6*tan(1/2*d*x + 1/2*c)^9 - 129360*a^8*b^8*tan(1/2*d
*x + 1/2*c)^9 - 62832*a^6*b^10*tan(1/2*d*x + 1/2*c)^9 + 92288*a^4*b^12*tan(
1/2*d*x + 1/2*c)^9 - 26880*a^2*b^14*tan(1/2*d*x + 1/2*c)^9 - 8330*a^15*b*ta
n(1/2*d*x + 1/2*c)^8 - 248745*a^13*b^3*tan(1/2*d*x + 1/2*c)^8 - 190610*a^11
*b^5*tan(1/2*d*x + 1/2*c)^8 + 253736*a^9*b^7*tan(1/2*d*x + 1/2*c)^8 - 33824
0*a^7*b^9*tan(1/2*d*x + 1/2*c)^8 + 120512*a^5*b^11*tan(1/2*d*x + 1/2*c)^8 +
```

24192*a^3*b^13*tan(1/2*d*x + 1/2*c)^8 - 17920*a*b^15*tan(1/2*d*x + 1/2*c)^8 - 96040*a^14*b^2*tan(1/2*d*x + 1/2*c)^7 - 452340*a^12*b^4*tan(1/2*d*x + 1/2*c)^7 + 164528*a^10*b^6*tan(1/2*d*x + 1/2*c)^7 - 99344*a^8*b^8*tan(1/2*d*x + 1/2*c)^7 - 177664*a^6*b^10*tan(1/2*d*x + 1/2*c)^7 + 153088*a^4*b^12*tan(1/2*d*x + 1/2*c)^7 - 27648*a^2*b^14*tan(1/2*d*x + 1/2*c)^7 - 5120*b^16*tan(1/2*d*x + 1/2*c)^7 - 15680*a^15*b*tan(1/2*d*x + 1/2*c)^6 - 296520*a^13*b^3*tan(1/2*d*x + 1/2*c)^6 - 247940*a^11*b^5*tan(1/2*d*x + 1/2*c)^6 + 232736*a^9*b^7*tan(1/2*d*x + 1/2*c)^6 - 339920*a^7*b^9*tan(1/2*d*x + 1/2*c)^6 + 120512*a^5*b^11*tan(1/2*d*x + 1/2*c)^6 + 24192*a^3*b^13*tan(1/2*d*x + 1/2*c)^6 - 17920*a*b^15*tan(1/2*d*x + 1/2*c)^6 - 630*a^16*tan(1/2*d*x + 1/2*c)^5 - 92155*a^14*b^2*tan(1/2*d*x + 1/2*c)^5 - 333060*a^12*b^4*tan(1/2*d*x + 1/2*c)^5 + 151144*a^10*b^6*tan(1/2*d*x + 1/2*c)^5 - 133280*a^8*b^8*tan(1/2*d*x + 1/2*c)^5 - 62832*a^6*b^10*tan(1/2*d*x + 1/2*c)^5 + 92288*a^4*b^12*tan(1/2*d*x + 1/2*c)^5 - 26880*a^2*b^14*tan(1/2*d*x + 1/2*c)^5 - 13566*a^15*b*tan(1/2*d*x + 1/2*c)^4 - 166775*a^13*b^3*tan(1/2*d*x + 1/2*c)^4 - 41412*a^11*b^5*tan(1/2*d*x + 1/2*c)^4 + 72128*a^9*b^7*tan(1/2*d*x + 1/2*c)^4 - 150640*a^7*b^9*tan(1/2*d*x + 1/2*c)^4 + 97440*a^5*b^11*tan(1/2*d*x + 1/2*c)^4 - 22400*a^3*b^13*tan(1/2*d*x + 1/2*c)^4 - 840*a^16*tan(1/2*d*x + 1/2*c)^3 - 41944*a^14*b^2*tan(1/2*d*x + 1/2*c)^3 - 76650*a^12*b^4*tan(1/2*d*x + 1/2*c)^3 + 87472*a^10*b^6*tan(1/2*d*x + 1/2*c)^3 - 100688*a^8*b^8*tan(1/2*d*x + 1/2*c)^3 + 53760*a^6*b^10*tan(1/2*d*x + 1/2*c)^3 - 11200*a^4*b^12*tan(1/2*d*x + 1/2*c)^3 - 5432*a^15*b*tan(1/2*d*x + 1/2*c)^2 - 33264*a^13*b^3*tan(1/2*d*x + 1/2*c)^2 + 34846*a^11*b^5*tan(1/2*d*x + 1/2*c)^2 - 34272*a^9*b^7*tan(1/2*d*x + 1/2*c)^2 + 16912*a^7*b^9*tan(1/2*d*x + 1/2*c)^2 - 3360*a^5*b^11*tan(1/2*d*x + 1/2*c)^2 - 350*a^16*tan(1/2*d*x + 1/2*c) - 6699*a^14*b^2*tan(1/2*d*x + 1/2*c) + 6790*a^12*b^4*tan(1/2*d*x + 1/2*c) - 6188*a^10*b^6*tan(1/2*d*x + 1/2*c) + 2912*a^8*b^8*tan(1/2*d*x + 1/2*c) - 560*a^6*b^10*tan(1/2*d*x + 1/2*c) - 686*a^15*b + 885*a^13*b^3 - 842*a^11*b^5 + 408*a^9*b^7 - 80*a^7*b^9)/((a^17 - 5*a^15*b^2 + 10*a^13*b^4 - 10*a^11*b^6 + 5*a^9*b^8 - a^7*b^10)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^7))/d

3.470 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=422

$$\frac{a\left(20a^2b^2+8a^4+5b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{13/2}}+\frac{\left(1518a^4b^2+1779a^2b^4+40a^6+128b^6\right)\cos(c+dx)}{1680bd\left(a^2-b^2\right)^6(a+b\sin(c+dx))}+\frac{a\,(718a^2b^2+40}{1680bd\,(a^2-b}$$

```
[Out] (a*(8*a^4 + 20*a^2*b^2 + 5*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(8*(a^2 - b^2)^(13/2)*d) - Cos[c + d*x]/(7*b*d*(a + b*Sin[c + d*x])^
7) + (a*Cos[c + d*x])/(42*b*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^6) + ((5*a^2
 + 6*b^2)*Cos[c + d*x])/(210*b*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^5) + (a
*(20*a^2 + 79*b^2)*Cos[c + d*x])/(840*b*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x]
)^4) + ((20*a^4 + 179*a^2*b^2 + 32*b^4)*Cos[c + d*x])/(840*b*(a^2 - b^2)^4*
d*(a + b*Sin[c + d*x])^3) + (a*(40*a^4 + 718*a^2*b^2 + 397*b^4)*Cos[c + d*x
])/(1680*b*(a^2 - b^2)^5*d*(a + b*Sin[c + d*x])^2) + ((40*a^6 + 1518*a^4*b^
2 + 1779*a^2*b^4 + 128*b^6)*Cos[c + d*x])/(1680*b*(a^2 - b^2)^6*d*(a + b*Si
n[c + d*x]))
```

Rubi [A] time = 0.745975, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2693, 2754, 12, 2660, 618, 204}

$$\frac{a\left(20a^2b^2+8a^4+5b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{13/2}}+\frac{\left(1518a^4b^2+1779a^2b^4+40a^6+128b^6\right)\cos(c+dx)}{1680bd\left(a^2-b^2\right)^6(a+b\sin(c+dx))}+\frac{a\,(718a^2b^2+40}{1680bd\,(a^2-b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a*(8*a^4 + 20*a^2*b^2 + 5*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(8*(a^2 - b^2)^(13/2)*d) - Cos[c + d*x]/(7*b*d*(a + b*Sin[c + d*x])^
7) + (a*Cos[c + d*x])/(42*b*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^6) + ((5*a^2
 + 6*b^2)*Cos[c + d*x])/(210*b*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^5) + (a
*(20*a^2 + 79*b^2)*Cos[c + d*x])/(840*b*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x]
)^4) + ((20*a^4 + 179*a^2*b^2 + 32*b^4)*Cos[c + d*x])/(840*b*(a^2 - b^2)^4*
d*(a + b*Sin[c + d*x])^3) + (a*(40*a^4 + 718*a^2*b^2 + 397*b^4)*Cos[c + d*x
])/(1680*b*(a^2 - b^2)^5*d*(a + b*Sin[c + d*x])^2) + ((40*a^6 + 1518*a^4*b^
```

```
2 + 1779*a^2*b^4 + 128*b^6)*Cos[c + d*x])/(1680*b*(a^2 - b^2)^6*d*(a + b*Si
n[c + d*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^8}\,dx = -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\int \frac{\sin(c+dx)}{(a+b\sin(c+dx))^7}\,dx}{7b}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\int \frac{6b-5a\sin(c+dx)}{(a+b\sin(c+dx))^6}\,dx}{42b\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cos(c}{210b\left(a^2-b^2\right)^2 d(a+b\,\mathrm{si}}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cos(c}{210b\left(a^2-b^2\right)^2 d(a+b\,\mathrm{si}}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cos(c}{210b\left(a^2-b^2\right)^2 d(a+b\,\mathrm{si}}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cos(c}{210b\left(a^2-b^2\right)^2 d(a+b\,\mathrm{si}}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cos(c}{210b\left(a^2-b^2\right)^2 d(a+b\,\mathrm{si}}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cos(c}{210b\left(a^2-b^2\right)^2 d(a+b\,\mathrm{si}}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cos(c}{210b\left(a^2-b^2\right)^2 d(a+b\,\mathrm{si}}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6} + \frac{\left(5a^2+6b^2\right)\cos(c}{210b\left(a^2-b^2\right)^2 d(a+b\,\mathrm{si}}$$

$$= \frac{a\left(8a^4+20a^2b^2+5b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{13/2}d} - \frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\,\mathrm{co}}{42b\left(a^2-b^2\right)d}$$

Mathematica [B] time = 6.19127, size = 1896, normalized size = 4.49

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] Cos[c + d*x]^3/(3*(a - b)*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]*(-(b*
(1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(5/2))/(7*(-a + b)*(a + b)*(a +
 b*Sin[c + d*x])^7) - (-((3*a*b + (7*a - b)*b)*(1 - Sin[c + d*x])^(3/2)*(1
+ Sin[c + d*x])^(5/2))/(6*(-a + b)*(a + b)*(a + b*Sin[c + d*x])^6) - (-((2*
a*(10*a - b)*b + b*(42*a^2 - 16*a*b + 19*b^2))*(1 - Sin[c + d*x])^(3/2)*(1
+ Sin[c + d*x])^(5/2))/(5*(-a + b)*(a + b)*(a + b*Sin[c + d*x])^5) - (-((a*
b*(62*a^2 - 18*a*b + 19*b^2) + b*(210*a^3 - 142*a^2*b + 213*a*b^2 - 29*b^3)
)*(1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(5/2))/(4*(-a + b)*(a + b)*(a
 + b*Sin[c + d*x])^4) - (105*(8*a^4 - 8*a^3*b + 12*a^2*b^2 - 4*a*b^3 + b^4)
*(-(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(5/2))/(3*(-a + b)*(a + b*Sin
[c + d*x])^3) - (-(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(3/2))/(2*(a +
 b)*(a + b*Sin[c + d*x])^2) + (3*((-2*ArcTan[(Sqrt[-a + b]*Sqrt[1 - Sin[c +
 d*x]])/(Sqrt[-a - b]*Sqrt[1 + Sin[c + d*x]])])/((-a - b)^(3/2)*Sqrt[-a + b
]) + (Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])/((-a - b)*(a + b*Sin[c
 + d*x]))))/(2*(a + b)))/(3*(-a + b))))/(4*(-a + b)*(a + b)))/(5*(-a + b)*(
a + b)))/(6*(-a + b)*(a + b)))/(7*(-a + b)*(a + b))))/((a - b)*d*Sqrt[1 - S
in[c + d*x]]*Sqrt[1 + Sin[c + d*x]]) + (4*b*(Cos[c + d*x]^5/(5*(a - b)*d*(a
 + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]*(-(b*(1 - Sin[c + d*x])^(5/2)*(1 +
Sin[c + d*x])^(7/2))/(7*(-a + b)*(a + b)*(a + b*Sin[c + d*x])^7) - (-((a*b
+ (7*a - b)*b)*(1 - Sin[c + d*x])^(5/2)*(1 + Sin[c + d*x])^(7/2))/(6*(-a +
b)*(a + b)*(a + b*Sin[c + d*x])^6) - (7*(6*a^2 - 2*a*b + b^2)*(-((1 - Sin[c
 + d*x])^(3/2)*(1 + Sin[c + d*x])^(7/2))/(5*(-a + b)*(a + b*Sin[c + d*x])^5
) - (3*(-(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(7/2))/(4*(-a + b)*(a +
 b*Sin[c + d*x])^4) - (-(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(5/2))/(
3*(a + b)*(a + b*Sin[c + d*x])^3) + (5*(-(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c
 + d*x])^(3/2))/(2*(a + b)*(a + b*Sin[c + d*x])^2) + (3*((-2*ArcTan[(Sqrt[-
a + b]*Sqrt[1 - Sin[c + d*x]])/(Sqrt[-a - b]*Sqrt[1 + Sin[c + d*x]])])/((-a
 - b)^(3/2)*Sqrt[-a + b]) + (Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])
/((-a - b)*(a + b*Sin[c + d*x]))))/(2*(a + b))))/(3*(a + b)))/(4*(-a + b)))
)/(5*(-a + b))))/(6*(-a + b)*(a + b)))/(7*(-a + b)*(a + b))))/((a - b)*d*Sq
rt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]]) + (2*b*(Cos[c + d*x]^7/(7*(a -
 b)*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]*(-((1 - Sin[c + d*x])^(5/2)
*(1 + Sin[c + d*x])^(9/2))/(7*(-a + b)*(a + b*Sin[c + d*x])^7) - (5*(-((1 -
 Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(9/2))/(6*(-a + b)*(a + b*Sin[c + d
*x])^6) - (-(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(9/2))/(5*(-a + b)*(
a + b*Sin[c + d*x])^5) - (-(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(7/2)
```

```
)/(4*(a + b)*(a + b*Sin[c + d*x])^4) + (7*(-(Sqrt[1 - Sin[c + d*x]]*(1 + Si
n[c + d*x])^(5/2))/(3*(a + b)*(a + b*Sin[c + d*x])^3) + (5*(-(Sqrt[1 - Sin[
c + d*x]]*(1 + Sin[c + d*x])^(3/2))/(2*(a + b)*(a + b*Sin[c + d*x])^2) + (3
*((-2*ArcTan[(Sqrt[-a + b]*Sqrt[1 - Sin[c + d*x]])/(Sqrt[-a - b]*Sqrt[1 + S
in[c + d*x]])])/((-a - b)^(3/2)*Sqrt[-a + b]) + (Sqrt[1 - Sin[c + d*x]]*Sqr
t[1 + Sin[c + d*x]])/((-a - b)*(a + b*Sin[c + d*x]))))/(2*(a + b))))/(3*(a
+ b))))/(4*(a + b)))/(5*(-a + b)))/(2*(-a + b))))/(7*(-a + b))))/((a - b)*d
*Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])))/(5*(a - b))))/(3*(a - b))
```

Maple [B] time = 0.206, size = 11250, normalized size = 26.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^8,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.42321, size = 7096, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

[Out] [1/3360*(2*(40*a^8*b^5 + 1478*a^6*b^7 + 261*a^4*b^9 - 1651*a^2*b^11 - 128*b^13)*cos(d*x + c)^7 - 28*(60*a^10*b^3 + 1837*a^8*b^5 + 176*a^6*b^7 - 1680*a^4*b^9 - 361*a^2*b^11 - 32*b^13)*cos(d*x + c)^5 + 70*(40*a^12*b + 900*a^10*b^3 + 1111*a^8*b^5 - 501*a^6*b^7 - 1395*a^4*b^9 - 139*a^2*b^11 - 16*b^13)*cos(d*x + c)^3 + 105*(8*a^12 + 188*a^10*b^2 + 705*a^8*b^4 + 861*a^6*b^6 + 315*a^4*b^8 + 35*a^2*b^10 - 7*(8*a^6*b^6 + 20*a^4*b^8 + 5*a^2*b^10)*cos(d*x + c)^6 + 7*(40*a^8*b^4 + 124*a^6*b^6 + 85*a^4*b^8 + 15*a^2*b^10)*cos(d*x + c)^4 - 7*(24*a^10*b^2 + 140*a^8*b^4 + 239*a^6*b^6 + 110*a^4*b^8 + 15*a^2*b^10)*cos(d*x + c)^2 + (56*a^11*b + 420*a^9*b^3 + 903*a^7*b^5 + 603*a^5*b^7 + 125*a^3*b^9 + 5*a*b^11 - (8*a^5*b^7 + 20*a^3*b^9 + 5*a*b^11)*cos(d*x + c)^6 + 3*(56*a^7*b^5 + 148*a^5*b^7 + 55*a^3*b^9 + 5*a*b^11)*cos(d*x + c)^4 - (280*a^9*b^3 + 1036*a^7*b^5 + 1039*a^5*b^7 + 270*a^3*b^9 + 15*a*b^11)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 420*(24*a^12*b + 116*a^10*b^3 + 99*a^8*b^5 - 129*a^6*b^7 - 95*a^4*b^9 - 15*a^2*b^11)*cos(d*x + c) - 14*((40*a^9*b^4 + 1358*a^7*b^6 + 81*a^5*b^8 - 1426*a^3*b^10 - 53*a*b^12)*cos(d*x + c)^5 - 10*(20*a^11*b^2 + 535*a^9*b^4 + 147*a^7*b^6 - 407*a^5*b^8 - 283*a^3*b^10 - 12*a*b^12)*cos(d*x + c)^3 + 15*(8*a^13 + 132*a^11*b^2 + 285*a^9*b^4 - 42*a^7*b^6 - 288*a^5*b^8 - 90*a^3*b^10 - 5*a*b^12)*cos(d*x + c))*sin(d*x + c))/(7*(a^15*b^6 - 7*a^13*b^8 + 21*a^11*b^10 - 35*a^9*b^12 + 35*a^7*b^14 - 21*a^5*b^16 + 7*a^3*b^18 - a*b^20)*d*cos(d*x + c)^6 - 7*(5*a^17*b^4 - 32*a^15*b^6 + 84*a^13*b^8 - 112*a^11*b^10 + 70*a^9*b^12 - 28*a^5*b^16 + 16*a^3*b^18 - 3*a*b^20)*d*cos(d*x + c)^4 + 7*(3*a^19*b^2 - 11*a^17*b^4 - 4*a^15*b^6 + 84*a^13*b^8 - 182*a^11*b^10 + 182*a^9*b^12 - 84*a^7*b^14 + 4*a^5*b^16 + 11*a^3*b^18 - 3*a*b^20)*d*cos(d*x + c)^2 - (a^21 + 14*a^19*b^2 - 91*a^17*b^4 + 168*a^15*b^6 - 14*a^13*b^8 - 364*a^11*b^10 + 546*a^9*b^12 - 344*a^7*b^14 + 77*a^5*b^16 + 14*a^3*b^18 - 7*a*b^20)*d + ((a^14*b^7 - 7*a^12*b^9 + 21*a^10*b^11 - 35*a^8*b^13 + 35*a^6*b^15 - 21*a^4*b^17 + 7*a^2*b^19 - b^21)*d*cos(d*x + c)^6 - 3*(7*a^16*b^5 - 48*a^14*b^7 + 140*a^12*b^9 - 224*a^10*b^11 + 210*a^8*b^13 - 112*a^6*b^15 + 28*a^4*b^17 - b^21)*d*cos(d*x + c)^4 + (35*a^18*b^3 - 203*a^16*b^5 + 444*a^14*b^7 - 364*a^12*b^9 - 182*a^10*b^11 + 630*a^8*b^13 - 532*a^6*b^15 + 196*a^4*b^17 - 21*a^2*b^19 - 3*b^21)*d*cos(d*x + c)^2 - (7*a^20*b - 14*a^18*b^3 - 77*a^16*b^5 + 344*a^14*b^7 - 546*a^12*b^9 + 364*a^10*b^11 + 14*a^8*b^13 - 168*a^6*b^15 + 91*a^4*b^17 - 14*a^2*b^19 - b^21)*d)*sin(d*x + c)), 1/1680*((40*a^8*b^5 + 1478*a^6*b^7 + 261*a^4*b^9 - 1651*a^2*b^11 - 128*b^13)*cos(d*x + c)^7 - 14*(60*a^10*b^3 + 1837*a^8*b^5 + 176*a^6*b^7 - 1680*a^4*b^9 - 361*a^2*b^11 - 32*b^13)*cos(d*x + c)^5 + 35*(40*a^12*b + 900*a^10*b^3 + 1111*a^8*b^5 - 501*a^6*b^7 - 1395*a^4*b^9 - 139*a^2*b^11 - 16*b^13)*cos(d*x + c)^3 + 105*(8*a^12 + 188*a^10*b^2 + 705*a^8*b^4 + 861*a^6*b^6 + 315*a^4*b^8 + 35*a^2*b^10 - 7*(8*a^6*b^6 + 20*a^4*b^8 + 5*a^2*b^10)*cos(d*x + c)^6 + 7*(40*a^8*b^4 + 124*a^6*b^6 + 85*a^4*b^8 + 15*a^2*b^10)*cos(d*x + c)^4 - 7*(24*a^10*b^2 + 140*a^8*b^4 + 239*a^6*b^6 + 110*a^4*b^8 + 15*a^2*b^10)*cos(d*x + c)^2 + (56*a^11*b + 420*a^9*b^3 + 903*a^7*b^5 + 603*a^5*b^7 + 125*a^3*b

```
^9 + 5*a*b^11 - (8*a^5*b^7 + 20*a^3*b^9 + 5*a*b^11)*cos(d*x + c)^6 + 3*(56*
a^7*b^5 + 148*a^5*b^7 + 55*a^3*b^9 + 5*a*b^11)*cos(d*x + c)^4 - (280*a^9*b^
3 + 1036*a^7*b^5 + 1039*a^5*b^7 + 270*a^3*b^9 + 15*a*b^11)*cos(d*x + c)^2)*
sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)
*cos(d*x + c))) - 210*(24*a^12*b + 116*a^10*b^3 + 99*a^8*b^5 - 129*a^6*b^7
- 95*a^4*b^9 - 15*a^2*b^11)*cos(d*x + c) - 7*((40*a^9*b^4 + 1358*a^7*b^6 +
81*a^5*b^8 - 1426*a^3*b^10 - 53*a*b^12)*cos(d*x + c)^5 - 10*(20*a^11*b^2 +
535*a^9*b^4 + 147*a^7*b^6 - 407*a^5*b^8 - 283*a^3*b^10 - 12*a*b^12)*cos(d*x
 + c)^3 + 15*(8*a^13 + 132*a^11*b^2 + 285*a^9*b^4 - 42*a^7*b^6 - 288*a^5*b^
8 - 90*a^3*b^10 - 5*a*b^12)*cos(d*x + c))*sin(d*x + c))/(7*(a^15*b^6 - 7*a^
13*b^8 + 21*a^11*b^10 - 35*a^9*b^12 + 35*a^7*b^14 - 21*a^5*b^16 + 7*a^3*b^1
8 - a*b^20)*d*cos(d*x + c)^6 - 7*(5*a^17*b^4 - 32*a^15*b^6 + 84*a^13*b^8 -
112*a^11*b^10 + 70*a^9*b^12 - 28*a^5*b^16 + 16*a^3*b^18 - 3*a*b^20)*d*cos(d
*x + c)^4 + 7*(3*a^19*b^2 - 11*a^17*b^4 - 4*a^15*b^6 + 84*a^13*b^8 - 182*a^
11*b^10 + 182*a^9*b^12 - 84*a^7*b^14 + 4*a^5*b^16 + 11*a^3*b^18 - 3*a*b^20)
*d*cos(d*x + c)^2 - (a^21 + 14*a^19*b^2 - 91*a^17*b^4 + 168*a^15*b^6 - 14*a
^13*b^8 - 364*a^11*b^10 + 546*a^9*b^12 - 344*a^7*b^14 + 77*a^5*b^16 + 14*a^
3*b^18 - 7*a*b^20)*d + ((a^14*b^7 - 7*a^12*b^9 + 21*a^10*b^11 - 35*a^8*b^13
 + 35*a^6*b^15 - 21*a^4*b^17 + 7*a^2*b^19 - b^21)*d*cos(d*x + c)^6 - 3*(7*a
^16*b^5 - 48*a^14*b^7 + 140*a^12*b^9 - 224*a^10*b^11 + 210*a^8*b^13 - 112*a
^6*b^15 + 28*a^4*b^17 - b^21)*d*cos(d*x + c)^4 + (35*a^18*b^3 - 203*a^16*b^
5 + 444*a^14*b^7 - 364*a^12*b^9 - 182*a^10*b^11 + 630*a^8*b^13 - 532*a^6*b^
15 + 196*a^4*b^17 - 21*a^2*b^19 - 3*b^21)*d*cos(d*x + c)^2 - (7*a^20*b - 14
*a^18*b^3 - 77*a^16*b^5 + 344*a^14*b^7 - 546*a^12*b^9 + 364*a^10*b^11 + 14*
a^8*b^13 - 168*a^6*b^15 + 91*a^4*b^17 - 14*a^2*b^19 - b^21)*d)*sin(d*x + c)
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.09701, size = 2979, normalized size = 7.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/840*(105*(8*a^5 + 20*a^3*b^2 + 5*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^12 - 6*
a^10*b^2 + 15*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^2*b^10 + b^12)*sqrt(a
^2 - b^2)) - (840*a^18*tan(1/2*d*x + 1/2*c)^13 - 12180*a^16*b^2*tan(1/2*d*x
 + 1/2*c)^13 + 24675*a^14*b^4*tan(1/2*d*x + 1/2*c)^13 - 33600*a^12*b^6*tan(
1/2*d*x + 1/2*c)^13 + 25200*a^10*b^8*tan(1/2*d*x + 1/2*c)^13 - 10080*a^8*b^
10*tan(1/2*d*x + 1/2*c)^13 + 1680*a^6*b^12*tan(1/2*d*x + 1/2*c)^13 - 840*a^
17*b*tan(1/2*d*x + 1/2*c)^12 - 87780*a^15*b^3*tan(1/2*d*x + 1/2*c)^12 + 144
375*a^13*b^5*tan(1/2*d*x + 1/2*c)^12 - 201600*a^11*b^7*tan(1/2*d*x + 1/2*c)
^12 + 151200*a^9*b^9*tan(1/2*d*x + 1/2*c)^12 - 60480*a^7*b^11*tan(1/2*d*x +
 1/2*c)^12 + 10080*a^5*b^13*tan(1/2*d*x + 1/2*c)^12 + 3360*a^18*tan(1/2*d*x
 + 1/2*c)^11 - 94080*a^16*b^2*tan(1/2*d*x + 1/2*c)^11 - 220500*a^14*b^4*tan
(1/2*d*x + 1/2*c)^11 + 287350*a^12*b^6*tan(1/2*d*x + 1/2*c)^11 - 537600*a^1
0*b^8*tan(1/2*d*x + 1/2*c)^11 + 450240*a^8*b^10*tan(1/2*d*x + 1/2*c)^11 - 1
92640*a^6*b^12*tan(1/2*d*x + 1/2*c)^11 + 33600*a^4*b^14*tan(1/2*d*x + 1/2*c
)^11 - 13440*a^17*b*tan(1/2*d*x + 1/2*c)^10 - 554400*a^15*b^3*tan(1/2*d*x +
 1/2*c)^10 - 165900*a^13*b^5*tan(1/2*d*x + 1/2*c)^10 - 66850*a^11*b^7*tan(1
/2*d*x + 1/2*c)^10 - 621600*a^9*b^9*tan(1/2*d*x + 1/2*c)^10 + 719040*a^7*b^
11*tan(1/2*d*x + 1/2*c)^10 - 355040*a^5*b^13*tan(1/2*d*x + 1/2*c)^10 + 6720
0*a^3*b^15*tan(1/2*d*x + 1/2*c)^10 + 4200*a^18*tan(1/2*d*x + 1/2*c)^9 - 304
500*a^16*b^2*tan(1/2*d*x + 1/2*c)^9 - 1418025*a^14*b^4*tan(1/2*d*x + 1/2*c)
^9 + 147070*a^12*b^6*tan(1/2*d*x + 1/2*c)^9 - 1316700*a^10*b^8*tan(1/2*d*x
+ 1/2*c)^9 + 242592*a^8*b^10*tan(1/2*d*x + 1/2*c)^9 + 439376*a^6*b^12*tan(1
/2*d*x + 1/2*c)^9 - 352128*a^4*b^14*tan(1/2*d*x + 1/2*c)^9 + 80640*a^2*b^16
*tan(1/2*d*x + 1/2*c)^9 - 49000*a^17*b*tan(1/2*d*x + 1/2*c)^8 - 1357300*a^1
5*b^3*tan(1/2*d*x + 1/2*c)^8 - 1726305*a^13*b^5*tan(1/2*d*x + 1/2*c)^8 - 34
6570*a^11*b^7*tan(1/2*d*x + 1/2*c)^8 - 1972600*a^9*b^9*tan(1/2*d*x + 1/2*c)
^8 + 1360128*a^7*b^11*tan(1/2*d*x + 1/2*c)^8 - 298816*a^5*b^13*tan(1/2*d*x
+ 1/2*c)^8 - 122752*a^3*b^15*tan(1/2*d*x + 1/2*c)^8 + 53760*a*b^17*tan(1/2*
d*x + 1/2*c)^8 - 509600*a^16*b^2*tan(1/2*d*x + 1/2*c)^7 - 2685200*a^14*b^4*
tan(1/2*d*x + 1/2*c)^7 - 900900*a^12*b^6*tan(1/2*d*x + 1/2*c)^7 - 2070320*a
^10*b^8*tan(1/2*d*x + 1/2*c)^7 - 278096*a^8*b^10*tan(1/2*d*x + 1/2*c)^7 + 9
52320*a^6*b^12*tan(1/2*d*x + 1/2*c)^7 - 538112*a^4*b^14*tan(1/2*d*x + 1/2*c
)^7 + 68608*a^2*b^16*tan(1/2*d*x + 1/2*c)^7 + 15360*b^18*tan(1/2*d*x + 1/2*
c)^7 - 78400*a^17*b*tan(1/2*d*x + 1/2*c)^6 - 1607200*a^15*b^3*tan(1/2*d*x +
 1/2*c)^6 - 2326800*a^13*b^5*tan(1/2*d*x + 1/2*c)^6 - 823060*a^11*b^7*tan(1
/2*d*x + 1/2*c)^6 - 2094400*a^9*b^9*tan(1/2*d*x + 1/2*c)^6 + 1351728*a^7*b^
11*tan(1/2*d*x + 1/2*c)^6 - 298816*a^5*b^13*tan(1/2*d*x + 1/2*c)^6 - 122752
*a^3*b^15*tan(1/2*d*x + 1/2*c)^6 + 53760*a*b^17*tan(1/2*d*x + 1/2*c)^6 - 42
00*a^18*tan(1/2*d*x + 1/2*c)^5 - 459900*a^16*b^2*tan(1/2*d*x + 1/2*c)^5 - 2
```

100175*a^14*b^4*tan(1/2*d*x + 1/2*c)^5 - 647780*a^12*b^6*tan(1/2*d*x + 1/2*c)^5 - 1643880*a^10*b^8*tan(1/2*d*x + 1/2*c)^5 + 228592*a^8*b^10*tan(1/2*d*x + 1/2*c)^5 + 439376*a^6*b^12*tan(1/2*d*x + 1/2*c)^5 - 352128*a^4*b^14*tan(1/2*d*x + 1/2*c)^5 + 80640*a^2*b^16*tan(1/2*d*x + 1/2*c)^5 - 63000*a^17*b*tan(1/2*d*x + 1/2*c)^4 - 918540*a^15*b^3*tan(1/2*d*x + 1/2*c)^4 - 858683*a^13*b^5*tan(1/2*d*x + 1/2*c)^4 - 434644*a^11*b^7*tan(1/2*d*x + 1/2*c)^4 - 634368*a^9*b^9*tan(1/2*d*x + 1/2*c)^4 + 719600*a^7*b^11*tan(1/2*d*x + 1/2*c)^4 - 355040*a^5*b^13*tan(1/2*d*x + 1/2*c)^4 + 67200*a^3*b^15*tan(1/2*d*x + 1/2*c)^4 - 3360*a^18*tan(1/2*d*x + 1/2*c)^3 - 211680*a^16*b^2*tan(1/2*d*x + 1/2*c)^3 - 575260*a^14*b^4*tan(1/2*d*x + 1/2*c)^3 + 43918*a^12*b^6*tan(1/2*d*x + 1/2*c)^3 - 534576*a^10*b^8*tan(1/2*d*x + 1/2*c)^3 + 449008*a^8*b^10*tan(1/2*d*x + 1/2*c)^3 - 192640*a^6*b^12*tan(1/2*d*x + 1/2*c)^3 + 33600*a^4*b^14*tan(1/2*d*x + 1/2*c)^3 - 24640*a^17*b*tan(1/2*d*x + 1/2*c)^2 - 199360*a^15*b^3*tan(1/2*d*x + 1/2*c)^2 + 44604*a^13*b^5*tan(1/2*d*x + 1/2*c)^2 - 186410*a^11*b^7*tan(1/2*d*x + 1/2*c)^2 + 144928*a^9*b^9*tan(1/2*d*x + 1/2*c)^2 - 59472*a^7*b^11*tan(1/2*d*x + 1/2*c)^2 + 10080*a^5*b^13*tan(1/2*d*x + 1/2*c)^2 - 840*a^18*tan(1/2*d*x + 1/2*c) - 38780*a^16*b^2*tan(1/2*d*x + 1/2*c) + 12565*a^14*b^4*tan(1/2*d*x + 1/2*c) - 35322*a^12*b^6*tan(1/2*d*x + 1/2*c) + 25844*a^10*b^8*tan(1/2*d*x + 1/2*c) - 10192*a^8*b^10*tan(1/2*d*x + 1/2*c) + 1680*a^6*b^12*tan(1/2*d*x + 1/2*c) - 3640*a^17*b + 2660*a^15*b^3 - 4923*a^13*b^5 + 3646*a^11*b^7 - 1448*a^9*b^9 + 240*a^7*b^11)/((a^19 - 6*a^17*b^2 + 15*a^15*b^4 - 20*a^13*b^6 + 15*a^11*b^8 - 6*a^9*b^10 + a^7*b^12)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^7))/d

3.471 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=529

$$-\frac{9ab^2\left(336a^4b^2+280a^2b^4+64a^6+35b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{17/2}}+\frac{b\left(41484a^4b^2+22767a^2b^4+9800a^6+1024b^6\right)\sec(}{560d\left(a^2-b^2\right)^7(a+b\sin(c+dx))}$$

```
[Out] (-9*a*b^2*(64*a^6 + 336*a^4*b^2 + 280*a^2*b^4 + 35*b^6)*ArcTan[(b + a*Tan[(
c + d*x)/2]])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(17/2)*d) + (b*Sec[c + d*x])/
(7*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) + (5*a*b*Sec[c + d*x])/(14*(a^2 -
b^2)^2*d*(a + b*Sin[c + d*x])^6) + (b*(49*a^2 + 16*b^2)*Sec[c + d*x])/(70*(
a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^5) + (13*a*b*(28*a^2 + 27*b^2)*Sec[c +
d*x])/(280*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^4) + (b*(700*a^4 + 1317*a^2
*b^2 + 128*b^4)*Sec[c + d*x])/(280*(a^2 - b^2)^5*d*(a + b*Sin[c + d*x])^3)
+ (11*a*b*(280*a^4 + 844*a^2*b^2 + 241*b^4)*Sec[c + d*x])/(560*(a^2 - b^2)^
6*d*(a + b*Sin[c + d*x])^2) + (b*(9800*a^6 + 41484*a^4*b^2 + 22767*a^2*b^4
+ 1024*b^6)*Sec[c + d*x])/(560*(a^2 - b^2)^7*d*(a + b*Sin[c + d*x])) - (Sec
[c + d*x]*(315*a*b*(64*a^6 + 336*a^4*b^2 + 280*a^2*b^4 + 35*b^6) - (560*a^8
 + 42472*a^6*b^2 + 125634*a^4*b^4 + 54511*a^2*b^6 + 2048*b^8)*Sin[c + d*x])
)/(560*(a^2 - b^2)^8*d)
```

Rubi [A] time = 1.76526, antiderivative size = 529, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2694, 2864, 2866, 12, 2660, 618, 204}

$$-\frac{9ab^2\left(336a^4b^2+280a^2b^4+64a^6+35b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{17/2}}+\frac{b\left(41484a^4b^2+22767a^2b^4+9800a^6+1024b^6\right)\sec(}{560d\left(a^2-b^2\right)^7(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (-9*a*b^2*(64*a^6 + 336*a^4*b^2 + 280*a^2*b^4 + 35*b^6)*ArcTan[(b + a*Tan[(
c + d*x)/2]])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(17/2)*d) + (b*Sec[c + d*x])/
(7*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) + (5*a*b*Sec[c + d*x])/(14*(a^2 -
b^2)^2*d*(a + b*Sin[c + d*x])^6) + (b*(49*a^2 + 16*b^2)*Sec[c + d*x])/(70*(
a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^5) + (13*a*b*(28*a^2 + 27*b^2)*Sec[c +
```

```
d*x])/(280*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^4) + (b*(700*a^4 + 1317*a^2
*b^2 + 128*b^4)*Sec[c + d*x])/(280*(a^2 - b^2)^5*d*(a + b*Sin[c + d*x])^3)
+ (11*a*b*(280*a^4 + 844*a^2*b^2 + 241*b^4)*Sec[c + d*x])/(560*(a^2 - b^2)^
6*d*(a + b*Sin[c + d*x])^2) + (b*(9800*a^6 + 41484*a^4*b^2 + 22767*a^2*b^4
+ 1024*b^6)*Sec[c + d*x])/(560*(a^2 - b^2)^7*d*(a + b*Sin[c + d*x])) - (Sec
[c + d*x]*(315*a*b*(64*a^6 + 336*a^4*b^2 + 280*a^2*b^4 + 35*b^6) - (560*a^8
 + 42472*a^6*b^2 + 125634*a^4*b^4 + 54511*a^2*b^6 + 2048*b^8)*Sin[c + d*x])
)/(560*(a^2 - b^2)^8*d)
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} - \frac{\int \frac{\sec^2(c+dx)(-7a+8b\sin(c+dx))}{(a+b\sin(c+dx))^7}\,dx}{7\left(a^2-b^2\right)}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{\int \frac{\sec^2(c+dx)\left(6\left(7a^2+8\right.\right.}{(a+b\sin}}{42\left(a^2\right.}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2+16b^2\right)}{70\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2+16b^2\right)}{70\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2+16b^2\right)}{70\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2+16b^2\right)}{70\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2+16b^2\right)}{70\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2+16b^2\right)}{70\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2+16b^2\right)}{70\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2+16b^2\right)}{70\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(49a^2+16b^2\right)}{70\left(a^2-b^2\right)^3 d(a+}$$

$$= -\frac{9ab^2\left(64a^6+336a^4b^2+280a^2b^4+35b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{17/2}d} + \frac{b\sec(c+dx}{7\left(a^2-b^2\right)d(a+b\sin}$$

Mathematica [A] time = 4.96575, size = 494, normalized size = 0.93

$$-\frac{\frac{630ab^2(336a^4b^2+280a^2b^4+64a^6+35b^6)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{17/2}}+\frac{b^3(86434a^4b^2+38831a^2b^4+26792a^6+1488b^6)\cos(c+dx)}{(a^2-b^2)^8(a+b\sin(c+dx))}+\frac{ab^3(23066a^2b^2+11112a^4+50}{(a^2-b^2)^7(a+b\sin(c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -((630*a*b^2*(64*a^6 + 336*a^4*b^2 + 280*a^2*b^4 + 35*b^6)*ArcTan[(b + a*Ta
n[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(17/2) + (80*b^3*Cos[c + d*x]
)/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^7) + (360*a*b^3*Cos[c + d*x])/((a^2 -
 b^2)^3*(a + b*Sin[c + d*x])^6) + (8*b^3*(129*a^2 + 26*b^2)*Cos[c + d*x])/(
(a^2 - b^2)^4*(a + b*Sin[c + d*x])^5) + (2*a*b^3*(1216*a^2 + 739*b^2)*Cos[c
 + d*x])/((a^2 - b^2)^5*(a + b*Sin[c + d*x])^4) + (2*b^3*(2616*a^4 + 3207*a
^2*b^2 + 232*b^4)*Cos[c + d*x])/((a^2 - b^2)^6*(a + b*Sin[c + d*x])^3) + (a
*b^3*(11112*a^4 + 23066*a^2*b^2 + 5057*b^4)*Cos[c + d*x])/((a^2 - b^2)^7*(a
 + b*Sin[c + d*x])^2) + (b^3*(26792*a^6 + 86434*a^4*b^2 + 38831*a^2*b^4 + 1
488*b^6)*Cos[c + d*x])/((a^2 - b^2)^8*(a + b*Sin[c + d*x])) - (560*Sec[c +
d*x]*(-8*a*b*(a^6 + 7*a^4*b^2 + 7*a^2*b^4 + b^6) + (a^8 + 28*a^6*b^2 + 70*a
^4*b^4 + 28*a^2*b^6 + b^8)*Sin[c + d*x]))/(a^2 - b^2)^8)/(560*d)
```

Maple [B] time = 0.216, size = 7675, normalized size = 14.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^8,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 10.0989, size = 9913, normalized size = 18.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [1/1120*(1120*a^16*b - 8960*a^14*b^3 + 31360*a^12*b^5 - 62720*a^10*b^7 + 78
400*a^8*b^9 - 62720*a^6*b^11 + 31360*a^4*b^13 - 8960*a^2*b^15 + 1120*b^17 -
 2*(560*a^10*b^7 + 41912*a^8*b^9 + 83162*a^6*b^11 - 71123*a^4*b^13 - 52463*
a^2*b^15 - 2048*b^17)*cos(d*x + c)^8 + 28*(840*a^12*b^5 + 53648*a^10*b^7 +
95441*a^8*b^9 - 77704*a^6*b^11 - 60644*a^4*b^13 - 11069*a^2*b^15 - 512*b^17
)*cos(d*x + c)^6 - 70*(560*a^14*b^3 + 27440*a^12*b^5 + 71064*a^10*b^7 + 299
27*a^8*b^9 - 81421*a^6*b^11 - 43131*a^4*b^13 - 4183*a^2*b^15 - 256*b^17)*co
s(d*x + c)^4 + 140*(56*a^16*b + 1400*a^14*b^3 + 13832*a^12*b^5 + 24080*a^10
*b^7 - 4591*a^8*b^9 - 23443*a^6*b^11 - 10717*a^4*b^13 - 553*a^2*b^15 - 64*b
^17)*cos(d*x + c)^2 - 315*(7*(64*a^8*b^8 + 336*a^6*b^10 + 280*a^4*b^12 + 35
*a^2*b^14)*cos(d*x + c)^7 - 7*(320*a^10*b^6 + 1872*a^8*b^8 + 2408*a^6*b^10
+ 1015*a^4*b^12 + 105*a^2*b^14)*cos(d*x + c)^5 + 7*(192*a^12*b^4 + 1648*a^1
0*b^6 + 4392*a^8*b^8 + 3913*a^6*b^10 + 1190*a^4*b^12 + 105*a^2*b^14)*cos(d*
x + c)^3 - (64*a^14*b^2 + 1680*a^12*b^4 + 9576*a^10*b^6 + 18123*a^8*b^8 + 1
2887*a^6*b^10 + 3185*a^4*b^12 + 245*a^2*b^14)*cos(d*x + c) + ((64*a^7*b^9 +
 336*a^5*b^11 + 280*a^3*b^13 + 35*a*b^15)*cos(d*x + c)^7 - 3*(448*a^9*b^7 +
 2416*a^7*b^9 + 2296*a^5*b^11 + 525*a^3*b^13 + 35*a*b^15)*cos(d*x + c)^5 +
(2240*a^11*b^5 + 14448*a^9*b^7 + 24104*a^7*b^9 + 13993*a^5*b^11 + 2310*a^3*
b^13 + 105*a*b^15)*cos(d*x + c)^3 - (448*a^13*b^3 + 4592*a^11*b^5 + 15064*a
^9*b^7 + 17165*a^7*b^9 + 7441*a^5*b^11 + 1015*a^3*b^13 + 35*a*b^15)*cos(d*x
 + c))*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2)) - 14*(80*a^17 - 640*a^15*b^2 + 2240*a^13*b^4 - 4480*a^11*b^6 + 5600*a^
9*b^8 - 4480*a^7*b^10 + 2240*a^5*b^12 - 640*a^3*b^14 + 80*a*b^16 - (560*a^1
1*b^6 + 39032*a^9*b^8 + 70922*a^7*b^10 - 68603*a^5*b^12 - 41438*a^3*b^14 -
473*a*b^16)*cos(d*x + c)^6 + 10*(280*a^13*b^4 + 15960*a^11*b^6 + 29463*a^9*
b^8 - 13541*a^7*b^10 - 23679*a^5*b^12 - 8391*a^3*b^14 - 92*a*b^16)*cos(d*x
+ c)^4 - 15*(112*a^15*b^2 + 4256*a^13*b^4 + 13272*a^11*b^6 + 11977*a^9*b^8
- 15634*a^7*b^10 - 11088*a^5*b^12 - 2870*a^3*b^14 - 25*a*b^16)*cos(d*x + c)
```

^2)*sin(d*x + c))/(7*(a^19*b^6 - 9*a^17*b^8 + 36*a^15*b^10 - 84*a^13*b^12 + 126*a^11*b^14 - 126*a^9*b^16 + 84*a^7*b^18 - 36*a^5*b^20 + 9*a^3*b^22 - a*b^24)*d*cos(d*x + c)^7 - 7*(5*a^21*b^4 - 42*a^19*b^6 + 153*a^17*b^8 - 312*a^15*b^10 + 378*a^13*b^12 - 252*a^11*b^14 + 42*a^9*b^16 + 72*a^7*b^18 - 63*a^5*b^20 + 22*a^3*b^22 - 3*a*b^24)*d*cos(d*x + c)^5 + 7*(3*a^23*b^2 - 17*a^21*b^4 + 21*a^19*b^6 + 81*a^17*b^8 - 354*a^15*b^10 + 630*a^13*b^12 - 630*a^11*b^14 + 354*a^9*b^16 - 81*a^7*b^18 - 21*a^5*b^20 + 17*a^3*b^22 - 3*a*b^24)*d*cos(d*x + c)^3 - (a^25 + 12*a^23*b^2 - 118*a^21*b^4 + 364*a^19*b^6 - 441*a^17*b^8 - 168*a^15*b^10 + 1260*a^13*b^12 - 1800*a^11*b^14 + 1311*a^9*b^16 - 484*a^7*b^18 + 42*a^5*b^20 + 28*a^3*b^22 - 7*a*b^24)*d*cos(d*x + c) + ((a^18*b^7 - 9*a^16*b^9 + 36*a^14*b^11 - 84*a^12*b^13 + 126*a^10*b^15 - 126*a^8*b^17 + 84*a^6*b^19 - 36*a^4*b^21 + 9*a^2*b^23 - b^25)*d*cos(d*x + c)^7 - 3*(7*a^20*b^5 - 62*a^18*b^7 + 243*a^16*b^9 - 552*a^14*b^11 + 798*a^12*b^13 - 756*a^10*b^15 + 462*a^8*b^17 - 168*a^6*b^19 + 27*a^4*b^21 + 2*a^2*b^23 - b^25)*d*cos(d*x + c)^5 + (35*a^22*b^3 - 273*a^20*b^5 + 885*a^18*b^7 - 1455*a^16*b^9 + 990*a^14*b^11 + 630*a^12*b^13 - 1974*a^10*b^15 + 1890*a^8*b^17 - 945*a^6*b^19 + 235*a^4*b^21 - 15*a^2*b^23 - 3*b^25)*d*cos(d*x + c)^3 - (7*a^24*b - 28*a^22*b^3 - 42*a^20*b^5 + 484*a^18*b^7 - 1311*a^16*b^9 + 1800*a^14*b^11 - 1260*a^12*b^13 + 168*a^10*b^15 + 441*a^8*b^17 - 364*a^6*b^19 + 118*a^4*b^21 - 12*a^2*b^23 - b^25)*d*cos(d*x + c))*sin(d*x + c)), 1/560*(560*a^16*b - 4480*a^14*b^3 + 15680*a^12*b^5 - 31360*a^10*b^7 + 39200*a^8*b^9 - 31360*a^6*b^11 + 15680*a^4*b^13 - 4480*a^2*b^15 + 560*b^17 - (560*a^10*b^7 + 41912*a^8*b^9 + 83162*a^6*b^11 - 71123*a^4*b^13 - 52463*a^2*b^15 - 2048*b^17)*cos(d*x + c)^8 + 14*(840*a^12*b^5 + 53648*a^10*b^7 + 95441*a^8*b^9 - 77704*a^6*b^11 - 60644*a^4*b^13 - 11069*a^2*b^15 - 512*b^17)*cos(d*x + c)^6 - 35*(560*a^14*b^3 + 27440*a^12*b^5 + 71064*a^10*b^7 + 29927*a^8*b^9 - 81421*a^6*b^11 - 43131*a^4*b^13 - 4183*a^2*b^15 - 256*b^17)*cos(d*x + c)^4 + 70*(56*a^16*b + 1400*a^14*b^3 + 13832*a^12*b^5 + 24080*a^10*b^7 - 4591*a^8*b^9 - 23443*a^6*b^11 - 10717*a^4*b^13 - 553*a^2*b^15 - 64*b^17)*cos(d*x + c)^2 + 315*(7*(64*a^8*b^8 + 336*a^6*b^10 + 280*a^4*b^12 + 35*a^2*b^14)*cos(d*x + c)^7 - 7*(320*a^10*b^6 + 1872*a^8*b^8 + 2408*a^6*b^10 + 1015*a^4*b^12 + 105*a^2*b^14)*cos(d*x + c)^5 + 7*(192*a^12*b^4 + 1648*a^10*b^6 + 4392*a^8*b^8 + 3913*a^6*b^10 + 1190*a^4*b^12 + 105*a^2*b^14)*cos(d*x + c)^3 - (64*a^14*b^2 + 1680*a^12*b^4 + 9576*a^10*b^6 + 18123*a^8*b^8 + 12887*a^6*b^10 + 3185*a^4*b^12 + 245*a^2*b^14)*cos(d*x + c) + ((64*a^7*b^9 + 336*a^5*b^11 + 280*a^3*b^13 + 35*a*b^15)*cos(d*x + c)^7 - 3*(448*a^9*b^7 + 2416*a^7*b^9 + 2296*a^5*b^11 + 525*a^3*b^13 + 35*a*b^15)*cos(d*x + c)^5 + (2240*a^11*b^5 + 14448*a^9*b^7 + 24104*a^7*b^9 + 13993*a^5*b^11 + 2310*a^3*b^13 + 105*a*b^15)*cos(d*x + c)^3 - (448*a^13*b^3 + 4592*a^11*b^5 + 15064*a^9*b^7 + 17165*a^7*b^9 + 7441*a^5*b^11 + 1015*a^3*b^13 + 35*a*b^15)*cos(d*x + c))*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 7*(80*a^17 - 640*a^15*b^2 + 2240*a^13*b^4 - 4480*a^11*b^6 + 5600*a^9*b^8 - 4480*a^7*b^10 + 2240*a^5*b^12 - 640*a^3*b^14 + 80*a*b^16 - (560*a^11*b^6 + 39032*a^9*b^8 + 70922*a^7*b^10 - 68603*a^5*b^12 - 41438*a^3*b^14 - 473*a*b^16)*cos(d*x + c)^6 + 10*(280*a^13*b^4 + 15960*a^11*b^6 + 29463*a^9*

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b^8 - 13541*a^7*b^10 - 23679*a^5*b^12 - 8391*a^3*b^14 - 92*a*b^16)*cos(d*x
+ c)^4 - 15*(112*a^15*b^2 + 4256*a^13*b^4 + 13272*a^11*b^6 + 11977*a^9*b^8
- 15634*a^7*b^10 - 11088*a^5*b^12 - 2870*a^3*b^14 - 25*a*b^16)*cos(d*x + c)
^2)*sin(d*x + c))/(7*(a^19*b^6 - 9*a^17*b^8 + 36*a^15*b^10 - 84*a^13*b^12 +
 126*a^11*b^14 - 126*a^9*b^16 + 84*a^7*b^18 - 36*a^5*b^20 + 9*a^3*b^22 - a*
b^24)*d*cos(d*x + c)^7 - 7*(5*a^21*b^4 - 42*a^19*b^6 + 153*a^17*b^8 - 312*a
^15*b^10 + 378*a^13*b^12 - 252*a^11*b^14 + 42*a^9*b^16 + 72*a^7*b^18 - 63*a
^5*b^20 + 22*a^3*b^22 - 3*a*b^24)*d*cos(d*x + c)^5 + 7*(3*a^23*b^2 - 17*a^2
1*b^4 + 21*a^19*b^6 + 81*a^17*b^8 - 354*a^15*b^10 + 630*a^13*b^12 - 630*a^1
1*b^14 + 354*a^9*b^16 - 81*a^7*b^18 - 21*a^5*b^20 + 17*a^3*b^22 - 3*a*b^24)
*d*cos(d*x + c)^3 - (a^25 + 12*a^23*b^2 - 118*a^21*b^4 + 364*a^19*b^6 - 441
*a^17*b^8 - 168*a^15*b^10 + 1260*a^13*b^12 - 1800*a^11*b^14 + 1311*a^9*b^16
 - 484*a^7*b^18 + 42*a^5*b^20 + 28*a^3*b^22 - 7*a*b^24)*d*cos(d*x + c) + ((
a^18*b^7 - 9*a^16*b^9 + 36*a^14*b^11 - 84*a^12*b^13 + 126*a^10*b^15 - 126*a
^8*b^17 + 84*a^6*b^19 - 36*a^4*b^21 + 9*a^2*b^23 - b^25)*d*cos(d*x + c)^7 -
 3*(7*a^20*b^5 - 62*a^18*b^7 + 243*a^16*b^9 - 552*a^14*b^11 + 798*a^12*b^13
 - 756*a^10*b^15 + 462*a^8*b^17 - 168*a^6*b^19 + 27*a^4*b^21 + 2*a^2*b^23 -
 b^25)*d*cos(d*x + c)^5 + (35*a^22*b^3 - 273*a^20*b^5 + 885*a^18*b^7 - 1455
*a^16*b^9 + 990*a^14*b^11 + 630*a^12*b^13 - 1974*a^10*b^15 + 1890*a^8*b^17
- 945*a^6*b^19 + 235*a^4*b^21 - 15*a^2*b^23 - 3*b^25)*d*cos(d*x + c)^3 - (7
*a^24*b - 28*a^22*b^3 - 42*a^20*b^5 + 484*a^18*b^7 - 1311*a^16*b^9 + 1800*a
^14*b^11 - 1260*a^12*b^13 + 168*a^10*b^15 + 441*a^8*b^17 - 364*a^6*b^19 + 1
18*a^4*b^21 - 12*a^2*b^23 - b^25)*d*cos(d*x + c))*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.85505, size = 3524, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/280*(315*(64*a^7*b^2 + 336*a^5*b^4 + 280*a^3*b^6 + 35*a*b^8)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^
2 - b^2)))/((a^16 - 8*a^14*b^2 + 28*a^12*b^4 - 56*a^10*b^6 + 70*a^8*b^8 - 5
6*a^6*b^10 + 28*a^4*b^12 - 8*a^2*b^14 + b^16)*sqrt(a^2 - b^2)) + 560*(a^8*t
an(1/2*d*x + 1/2*c) + 28*a^6*b^2*tan(1/2*d*x + 1/2*c) + 70*a^4*b^4*tan(1/2*
d*x + 1/2*c) + 28*a^2*b^6*tan(1/2*d*x + 1/2*c) + b^8*tan(1/2*d*x + 1/2*c) -
 8*a^7*b - 56*a^5*b^3 - 56*a^3*b^5 - 8*a*b^7)/((a^16 - 8*a^14*b^2 + 28*a^12
*b^4 - 56*a^10*b^6 + 70*a^8*b^8 - 56*a^6*b^10 + 28*a^4*b^12 - 8*a^2*b^14 +
b^16)*(tan(1/2*d*x + 1/2*c)^2 - 1)) + (82320*a^18*b^4*tan(1/2*d*x + 1/2*c)^
13 + 41160*a^16*b^6*tan(1/2*d*x + 1/2*c)^13 + 49665*a^14*b^8*tan(1/2*d*x +
1/2*c)^13 - 31360*a^12*b^10*tan(1/2*d*x + 1/2*c)^13 + 15680*a^10*b^12*tan(1
/2*d*x + 1/2*c)^13 - 4480*a^8*b^14*tan(1/2*d*x + 1/2*c)^13 + 560*a^6*b^16*t
an(1/2*d*x + 1/2*c)^13 + 47040*a^19*b^3*tan(1/2*d*x + 1/2*c)^12 + 952560*a^
17*b^5*tan(1/2*d*x + 1/2*c)^12 + 743400*a^15*b^7*tan(1/2*d*x + 1/2*c)^12 +
370685*a^13*b^9*tan(1/2*d*x + 1/2*c)^12 - 188160*a^11*b^11*tan(1/2*d*x + 1/
2*c)^12 + 94080*a^9*b^13*tan(1/2*d*x + 1/2*c)^12 - 26880*a^7*b^15*tan(1/2*d
*x + 1/2*c)^12 + 3360*a^5*b^17*tan(1/2*d*x + 1/2*c)^12 + 987840*a^18*b^4*ta
n(1/2*d*x + 1/2*c)^11 + 5221440*a^16*b^6*tan(1/2*d*x + 1/2*c)^11 + 4792620*
a^14*b^8*tan(1/2*d*x + 1/2*c)^11 + 1272530*a^12*b^10*tan(1/2*d*x + 1/2*c)^1
1 - 501760*a^10*b^12*tan(1/2*d*x + 1/2*c)^11 + 277760*a^8*b^14*tan(1/2*d*x
+ 1/2*c)^11 - 85120*a^6*b^16*tan(1/2*d*x + 1/2*c)^11 + 11200*a^4*b^18*tan(1
/2*d*x + 1/2*c)^11 + 282240*a^19*b^3*tan(1/2*d*x + 1/2*c)^10 + 7056000*a^17
*b^5*tan(1/2*d*x + 1/2*c)^10 + 18695040*a^15*b^7*tan(1/2*d*x + 1/2*c)^10 +
15575140*a^13*b^9*tan(1/2*d*x + 1/2*c)^10 + 2689610*a^11*b^11*tan(1/2*d*x +
 1/2*c)^10 - 721280*a^9*b^13*tan(1/2*d*x + 1/2*c)^10 + 474880*a^7*b^15*tan(
1/2*d*x + 1/2*c)^10 - 160160*a^5*b^17*tan(1/2*d*x + 1/2*c)^10 + 22400*a^3*b
^19*tan(1/2*d*x + 1/2*c)^10 + 3704400*a^18*b^4*tan(1/2*d*x + 1/2*c)^9 + 269
48040*a^16*b^6*tan(1/2*d*x + 1/2*c)^9 + 46663365*a^14*b^8*tan(1/2*d*x + 1/2
*c)^9 + 29114330*a^12*b^10*tan(1/2*d*x + 1/2*c)^9 + 3411772*a^10*b^12*tan(1
/2*d*x + 1/2*c)^9 - 305536*a^8*b^14*tan(1/2*d*x + 1/2*c)^9 + 388976*a^6*b^1
6*tan(1/2*d*x + 1/2*c)^9 - 167552*a^4*b^18*tan(1/2*d*x + 1/2*c)^9 + 26880*a
^2*b^20*tan(1/2*d*x + 1/2*c)^9 + 705600*a^19*b^3*tan(1/2*d*x + 1/2*c)^8 + 1
8780720*a^17*b^5*tan(1/2*d*x + 1/2*c)^8 + 65305800*a^15*b^7*tan(1/2*d*x + 1
/2*c)^8 + 77673085*a^13*b^9*tan(1/2*d*x + 1/2*c)^8 + 32483570*a^11*b^11*tan
(1/2*d*x + 1/2*c)^8 + 2139928*a^9*b^13*tan(1/2*d*x + 1/2*c)^8 + 587776*a^7*
b^15*tan(1/2*d*x + 1/2*c)^8 - 7616*a^5*b^17*tan(1/2*d*x + 1/2*c)^8 - 74368*
a^3*b^19*tan(1/2*d*x + 1/2*c)^8 + 17920*a*b^21*tan(1/2*d*x + 1/2*c)^8 + 658
5600*a^18*b^4*tan(1/2*d*x + 1/2*c)^7 + 51038400*a^16*b^6*tan(1/2*d*x + 1/2*
c)^7 + 104499360*a^14*b^8*tan(1/2*d*x + 1/2*c)^7 + 80185140*a^12*b^10*tan(1
/2*d*x + 1/2*c)^7 + 20029744*a^10*b^12*tan(1/2*d*x + 1/2*c)^7 + 661136*a^8*
b^14*tan(1/2*d*x + 1/2*c)^7 + 683008*a^6*b^16*tan(1/2*d*x + 1/2*c)^7 - 2176
00*a^4*b^18*tan(1/2*d*x + 1/2*c)^7 + 13312*a^2*b^20*tan(1/2*d*x + 1/2*c)^7
```

```
+ 5120*b^22*tan(1/2*d*x + 1/2*c)^7 + 940800*a^19*b^3*tan(1/2*d*x + 1/2*c)^6
 + 23614080*a^17*b^5*tan(1/2*d*x + 1/2*c)^6 + 83805120*a^15*b^7*tan(1/2*d*x
 + 1/2*c)^6 + 103990880*a^13*b^9*tan(1/2*d*x + 1/2*c)^6 + 45853220*a^11*b^1
1*tan(1/2*d*x + 1/2*c)^6 + 4650688*a^9*b^13*tan(1/2*d*x + 1/2*c)^6 + 692496
*a^7*b^15*tan(1/2*d*x + 1/2*c)^6 - 7616*a^5*b^17*tan(1/2*d*x + 1/2*c)^6 - 7
4368*a^3*b^19*tan(1/2*d*x + 1/2*c)^6 + 17920*a*b^21*tan(1/2*d*x + 1/2*c)^6
+ 6174000*a^18*b^4*tan(1/2*d*x + 1/2*c)^5 + 43023960*a^16*b^6*tan(1/2*d*x +
 1/2*c)^5 + 82755435*a^14*b^8*tan(1/2*d*x + 1/2*c)^5 + 55248340*a^12*b^10*t
an(1/2*d*x + 1/2*c)^5 + 10337432*a^10*b^12*tan(1/2*d*x + 1/2*c)^5 - 175056*
a^8*b^14*tan(1/2*d*x + 1/2*c)^5 + 388976*a^6*b^16*tan(1/2*d*x + 1/2*c)^5 -
167552*a^4*b^18*tan(1/2*d*x + 1/2*c)^5 + 26880*a^2*b^20*tan(1/2*d*x + 1/2*c
)^5 + 705600*a^19*b^3*tan(1/2*d*x + 1/2*c)^4 + 14429520*a^17*b^5*tan(1/2*d*
x + 1/2*c)^4 + 42782712*a^15*b^7*tan(1/2*d*x + 1/2*c)^4 + 41655719*a^13*b^9
*tan(1/2*d*x + 1/2*c)^4 + 10567396*a^11*b^11*tan(1/2*d*x + 1/2*c)^4 - 70403
2*a^9*b^13*tan(1/2*d*x + 1/2*c)^4 + 485520*a^7*b^15*tan(1/2*d*x + 1/2*c)^4
- 160160*a^5*b^17*tan(1/2*d*x + 1/2*c)^4 + 22400*a^3*b^19*tan(1/2*d*x + 1/2
*c)^4 + 2963520*a^18*b^4*tan(1/2*d*x + 1/2*c)^3 + 14864640*a^16*b^6*tan(1/2
*d*x + 1/2*c)^3 + 20500788*a^14*b^8*tan(1/2*d*x + 1/2*c)^3 + 5857306*a^12*b
^10*tan(1/2*d*x + 1/2*c)^3 - 479696*a^10*b^12*tan(1/2*d*x + 1/2*c)^3 + 2812
32*a^8*b^14*tan(1/2*d*x + 1/2*c)^3 - 85120*a^6*b^16*tan(1/2*d*x + 1/2*c)^3
+ 11200*a^4*b^18*tan(1/2*d*x + 1/2*c)^3 + 282240*a^19*b^3*tan(1/2*d*x + 1/2
*c)^2 + 3575040*a^17*b^5*tan(1/2*d*x + 1/2*c)^2 + 6358464*a^15*b^7*tan(1/2*
d*x + 1/2*c)^2 + 1843996*a^13*b^9*tan(1/2*d*x + 1/2*c)^2 - 146062*a^11*b^11
*tan(1/2*d*x + 1/2*c)^2 + 85120*a^9*b^13*tan(1/2*d*x + 1/2*c)^2 - 25648*a^7
*b^15*tan(1/2*d*x + 1/2*c)^2 + 3360*a^5*b^17*tan(1/2*d*x + 1/2*c)^2 + 57624
0*a^18*b^4*tan(1/2*d*x + 1/2*c) + 1111320*a^16*b^6*tan(1/2*d*x + 1/2*c) + 3
24303*a^14*b^8*tan(1/2*d*x + 1/2*c) - 26894*a^12*b^10*tan(1/2*d*x + 1/2*c)
+ 14924*a^10*b^12*tan(1/2*d*x + 1/2*c) - 4368*a^8*b^14*tan(1/2*d*x + 1/2*c)
 + 560*a^6*b^16*tan(1/2*d*x + 1/2*c) + 47040*a^19*b^3 + 82320*a^17*b^5 + 26
712*a^15*b^7 - 4161*a^13*b^9 + 2186*a^11*b^11 - 632*a^9*b^13 + 80*a^7*b^15)
/((a^23 - 8*a^21*b^2 + 28*a^19*b^4 - 56*a^17*b^6 + 70*a^15*b^8 - 56*a^13*b^
10 + 28*a^11*b^12 - 8*a^9*b^14 + a^7*b^16)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*
tan(1/2*d*x + 1/2*c) + a)^7))/d
```

3.472 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=653

$$\frac{165ab^4\left(112a^4b^2+70a^2b^4+32a^6+7b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{19/2}}+\frac{b\left(28420a^4b^2+12907a^2b^4+9212a^6+512b^6\right)\sec^3(c\ldots}{112d\left(a^2-b^2\right)^7(a+b\sin(c+dx))}$$

```
[Out] (165*a*b^4*(32*a^6 + 112*a^4*b^2 + 70*a^2*b^4 + 7*b^6)*ArcTan[(b + a*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(19/2)*d) + (b*Sec[c + d*x]^3)
/(7*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) + (17*a*b*Sec[c + d*x]^3)/(42*(a^
2 - b^2)^2*d*(a + b*Sin[c + d*x])^6) + (b*(13*a^2 + 4*b^2)*Sec[c + d*x]^3)/
(14*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^5) + (a*b*(118*a^2 + 103*b^2)*Sec[
c + d*x]^3)/(56*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^4) + (b*(882*a^4 + 142
1*a^2*b^2 + 128*b^4)*Sec[c + d*x]^3)/(168*(a^2 - b^2)^5*d*(a + b*Sin[c + d*
x])^3) + (13*a*b*(140*a^4 + 336*a^2*b^2 + 85*b^4)*Sec[c + d*x]^3)/(112*(a^2
 - b^2)^6*d*(a + b*Sin[c + d*x])^2) + (b*(9212*a^6 + 28420*a^4*b^2 + 12907*
a^2*b^4 + 512*b^6)*Sec[c + d*x]^3)/(112*(a^2 - b^2)^7*d*(a + b*Sin[c + d*x]
)) - (Sec[c + d*x]^3*(1155*a*b*(32*a^6 + 112*a^4*b^2 + 70*a^2*b^4 + 7*b^6)
- (112*a^8 + 52528*a^6*b^2 + 142902*a^4*b^4 + 57665*a^2*b^6 + 2048*b^8)*Sin
[c + d*x]))/(336*(a^2 - b^2)^8*d) + (Sec[c + d*x]*(3465*a*b^3*(32*a^6 + 112
*a^4*b^2 + 70*a^2*b^4 + 7*b^6) + (224*a^10 - 6048*a^8*b^2 - 207332*a^6*b^4
- 413024*a^4*b^6 - 135489*a^2*b^8 - 4096*b^10)*Sin[c + d*x]))/(336*(a^2 - b
^2)^9*d)
```

Rubi [A] time = 2.137, antiderivative size = 653, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2694, 2864, 2866, 12, 2660, 618, 204}

$$\frac{165ab^4\left(112a^4b^2+70a^2b^4+32a^6+7b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{19/2}}+\frac{b\left(28420a^4b^2+12907a^2b^4+9212a^6+512b^6\right)\sec^3(c\ldots}{112d\left(a^2-b^2\right)^7(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (165*a*b^4*(32*a^6 + 112*a^4*b^2 + 70*a^2*b^4 + 7*b^6)*ArcTan[(b + a*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(19/2)*d) + (b*Sec[c + d*x]^3)
```

```
/(7*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) + (17*a*b*Sec[c + d*x]^3)/(42*(a^
2 - b^2)^2*d*(a + b*Sin[c + d*x])^6) + (b*(13*a^2 + 4*b^2)*Sec[c + d*x]^3)/
(14*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^5) + (a*b*(118*a^2 + 103*b^2)*Sec[
c + d*x]^3)/(56*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^4) + (b*(882*a^4 + 142
1*a^2*b^2 + 128*b^4)*Sec[c + d*x]^3)/(168*(a^2 - b^2)^5*d*(a + b*Sin[c + d*
x])^3) + (13*a*b*(140*a^4 + 336*a^2*b^2 + 85*b^4)*Sec[c + d*x]^3)/(112*(a^2
 - b^2)^6*d*(a + b*Sin[c + d*x])^2) + (b*(9212*a^6 + 28420*a^4*b^2 + 12907*
a^2*b^4 + 512*b^6)*Sec[c + d*x]^3)/(112*(a^2 - b^2)^7*d*(a + b*Sin[c + d*x]
)) - (Sec[c + d*x]^3*(1155*a*b*(32*a^6 + 112*a^4*b^2 + 70*a^2*b^4 + 7*b^6)
- (112*a^8 + 52528*a^6*b^2 + 142902*a^4*b^4 + 57665*a^2*b^6 + 2048*b^8)*Sin
[c + d*x]))/(336*(a^2 - b^2)^8*d) + (Sec[c + d*x]*(3465*a*b^3*(32*a^6 + 112
*a^4*b^2 + 70*a^2*b^4 + 7*b^6) + (224*a^10 - 6048*a^8*b^2 - 207332*a^6*b^4
- 413024*a^4*b^6 - 135489*a^2*b^8 - 4096*b^10)*Sin[c + d*x]))/(336*(a^2 - b
^2)^9*d)
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} - \frac{\int \frac{\sec^4(c+dx)(-7a+10b\sin(c+dx))}{(a+b\sin(c+dx))^7}\,dx}{7\left(a^2-b^2\right)}$$

$$= \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{\int \frac{\sec^4(c+dx)\left(6\left(7a^2+1\right.\right.}{(a+b\sin}}{42\left(a\right.}$$

$$= \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(13a^2+4b^2\right)}{14\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(13a^2+4b^2\right)}{14\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(13a^2+4b^2\right)}{14\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(13a^2+4b^2\right)}{14\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(13a^2+4b^2\right)}{14\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(13a^2+4b^2\right)}{14\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(13a^2+4b^2\right)}{14\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(13a^2+4b^2\right)}{14\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(13a^2+4b^2\right)}{14\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(13a^2+4b^2\right)}{14\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^6} + \frac{b\left(13a^2+4b^2\right)}{14\left(a^2-b^2\right)^3 d(a+}$$

$$= \frac{165ab^4\left(32a^6+112a^4b^2+70a^2b^4+7b^6\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{19/2}d} + \frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(\right.}$$

Mathematica [A] time = 5.45203, size = 597, normalized size = 0.91

$$\frac{6930ab^4\left(112a^4b^2+70a^2b^4+32a^6+7b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{19/2}}+\frac{b^5\left(234272a^4b^2+81057a^2b^4+103844a^6+2528b^6\right)\cos(c+dx)}{\left(a^2-b^2\right)^9(a+b\sin(c+dx))}+\frac{ab^5\left(48820a^2b^2+33284a^4+828\right.}{\left(a^2-b^2\right)^8(a+b\sin(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((6930*a*b^4*(32*a^6 + 112*a^4*b^2 + 70*a^2*b^4 + 7*b^6)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(19/2) + (48*b^5*Cos[c + d*x])/
((a^2 - b^2)^3*(a + b*Sin[c + d*x])^7) + (328*a*b^5*Cos[c + d*x])/((a^2 - b
^2)^4*(a + b*Sin[c + d*x])^6) + (8*b^5*(167*a^2 + 24*b^2)*Cos[c + d*x])/((a
^2 - b^2)^5*(a + b*Sin[c + d*x])^5) + (2*a*b^5*(2138*a^2 + 925*b^2)*Cos[c +
 d*x])/((a^2 - b^2)^6*(a + b*Sin[c + d*x])^4) + (2*b^5*(6058*a^4 + 5273*a^2
*b^2 + 296*b^4)*Cos[c + d*x])/((a^2 - b^2)^7*(a + b*Sin[c + d*x])^3) + (a*b
^5*(33284*a^4 + 48820*a^2*b^2 + 8287*b^4)*Cos[c + d*x])/((a^2 - b^2)^8*(a +
 b*Sin[c + d*x])^2) + (b^5*(103844*a^6 + 234272*a^4*b^2 + 81057*a^2*b^4 + 2
528*b^6)*Cos[c + d*x])/((a^2 - b^2)^9*(a + b*Sin[c + d*x])) + (112*Sec[c +
d*x]^3*(-8*a*b*(a^6 + 7*a^4*b^2 + 7*a^2*b^4 + b^6) + (a^8 + 28*a^6*b^2 + 70
*a^4*b^4 + 28*a^2*b^6 + b^8)*Sin[c + d*x]))/(a^2 - b^2)^8 + (224*Sec[c + d*
x]*(12*(15*a^7*b^3 + 63*a^5*b^5 + 45*a^3*b^7 + 5*a*b^9) + (a^10 - 27*a^8*b^
2 - 462*a^6*b^4 - 798*a^4*b^6 - 243*a^2*b^8 - 7*b^10)*Sin[c + d*x]))/(a^2 -
 b^2)^9)/(336*d)
```

Maple [B] time = 0.354, size = 7823, normalized size = 12.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^8,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 13.9975, size = 11641, normalized size = 17.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [1/672*(224*a^18*b - 2016*a^16*b^3 + 8064*a^14*b^5 - 18816*a^12*b^7 + 28224
*a^10*b^9 - 28224*a^8*b^11 + 18816*a^6*b^13 - 8064*a^4*b^15 + 2016*a^2*b^17
 - 224*b^19 - 2*(224*a^12*b^7 - 6272*a^10*b^9 - 201284*a^8*b^11 - 205692*a^
6*b^13 + 277535*a^4*b^15 + 131393*a^2*b^17 + 4096*b^19)*cos(d*x + c)^10 + 2
8*(336*a^14*b^5 - 9352*a^12*b^7 - 252014*a^10*b^9 - 230159*a^8*b^11 + 29731
2*a^6*b^13 + 165122*a^4*b^15 + 27731*a^2*b^17 + 1024*b^19)*cos(d*x + c)^8 -
 70*(224*a^16*b^3 - 5936*a^14*b^5 - 126448*a^12*b^7 - 243082*a^10*b^9 - 297
47*a^8*b^11 + 284285*a^6*b^13 + 109607*a^4*b^15 + 10585*a^2*b^17 + 512*b^19
)*cos(d*x + c)^6 + 28*(112*a^18*b - 2296*a^16*b^3 - 35224*a^14*b^5 - 308392
*a^12*b^7 - 337750*a^10*b^9 + 149783*a^8*b^11 + 394751*a^6*b^13 + 130949*a^
4*b^15 + 7427*a^2*b^17 + 640*b^19)*cos(d*x + c)^4 - 224*(7*a^18*b - 46*a^16
*b^3 + 116*a^14*b^5 - 112*a^12*b^7 - 70*a^10*b^9 + 308*a^8*b^11 - 364*a^6*b
^13 + 224*a^4*b^15 - 73*a^2*b^17 + 10*b^19)*cos(d*x + c)^2 + 3465*(7*(32*a^
8*b^10 + 112*a^6*b^12 + 70*a^4*b^14 + 7*a^2*b^16)*cos(d*x + c)^9 - 7*(160*a
^10*b^8 + 656*a^8*b^10 + 686*a^6*b^12 + 245*a^4*b^14 + 21*a^2*b^16)*cos(d*x
 + c)^7 + 7*(96*a^12*b^6 + 656*a^10*b^8 + 1426*a^8*b^10 + 1057*a^6*b^12 + 2
80*a^4*b^14 + 21*a^2*b^16)*cos(d*x + c)^5 - (32*a^14*b^4 + 784*a^12*b^6 + 3
542*a^10*b^8 + 5621*a^8*b^10 + 3381*a^6*b^12 + 735*a^4*b^14 + 49*a^2*b^16)*
cos(d*x + c)^3 + ((32*a^7*b^11 + 112*a^5*b^13 + 70*a^3*b^15 + 7*a*b^17)*cos
(d*x + c)^9 - 3*(224*a^9*b^9 + 816*a^7*b^11 + 602*a^5*b^13 + 119*a^3*b^15 +
 7*a*b^17)*cos(d*x + c)^7 + (1120*a^11*b^7 + 5264*a^9*b^9 + 7250*a^7*b^11 +
 3521*a^5*b^13 + 504*a^3*b^15 + 21*a*b^17)*cos(d*x + c)^5 - (224*a^13*b^5 +
 1904*a^11*b^7 + 5082*a^9*b^9 + 4883*a^7*b^11 + 1827*a^5*b^13 + 217*a^3*b^1
5 + 7*a*b^17)*cos(d*x + c)^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*s
in(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2)) - 14*(16*a^19 - 144*a^17*b^2 + 576*a^15*b^4 - 1
344*a^13*b^6 + 2016*a^11*b^8 - 2016*a^9*b^10 + 1344*a^7*b^12 - 576*a^5*b^14
```

+ 144*a^3*b^16 - 16*a*b^18 - (224*a^13*b^6 - 6272*a^11*b^8 - 185444*a^9*b^10 - 166092*a^7*b^12 + 256745*a^5*b^14 + 100208*a^3*b^16 + 631*a*b^18)*cos(d*x + c)^8 + 10*(112*a^15*b^4 - 3080*a^13*b^6 - 73962*a^11*b^8 - 78323*a^9*b^10 + 60829*a^7*b^12 + 73923*a^5*b^14 + 20401*a^3*b^16 + 100*a*b^18)*cos(d*x + c)^6 - 3*(224*a^17*b^2 - 5712*a^15*b^4 - 95648*a^13*b^6 - 254254*a^11*b^8 - 120855*a^9*b^10 + 282886*a^7*b^12 + 157892*a^5*b^14 + 35448*a^3*b^16 + 19*a*b^18)*cos(d*x + c)^4 + 16*(2*a^19 - 35*a^17*b^2 + 208*a^15*b^4 - 644*a^13*b^6 + 1204*a^11*b^8 - 1442*a^9*b^10 + 1120*a^7*b^12 - 548*a^5*b^14 + 154*a^3*b^16 - 19*a*b^18)*cos(d*x + c)^2)*sin(d*x + c))/(7*(a^21*b^6 - 10*a^19*b^8 + 45*a^17*b^10 - 120*a^15*b^12 + 210*a^13*b^14 - 252*a^11*b^16 + 210*a^9*b^18 - 120*a^7*b^20 + 45*a^5*b^22 - 10*a^3*b^24 + a*b^26)*d*cos(d*x + c)^9 - 7*(5*a^23*b^4 - 47*a^21*b^6 + 195*a^19*b^8 - 465*a^17*b^10 + 690*a^15*b^12 - 630*a^13*b^14 + 294*a^11*b^16 + 30*a^9*b^18 - 135*a^7*b^20 + 85*a^5*b^22 - 25*a^3*b^24 + 3*a*b^26)*d*cos(d*x + c)^7 + 7*(3*a^25*b^2 - 20*a^23*b^4 + 38*a^21*b^6 + 60*a^19*b^8 - 435*a^17*b^10 + 984*a^15*b^12 - 1260*a^13*b^14 + 984*a^11*b^16 - 435*a^9*b^18 + 60*a^7*b^20 + 38*a^5*b^22 - 20*a^3*b^24 + 3*a*b^26)*d*cos(d*x + c)^5 - (a^27 + 11*a^25*b^2 - 130*a^23*b^4 + 482*a^21*b^6 - 805*a^19*b^8 + 273*a^17*b^10 + 1428*a^15*b^12 - 3060*a^13*b^14 + 3111*a^11*b^16 - 1795*a^9*b^18 + 526*a^7*b^20 - 14*a^5*b^22 - 35*a^3*b^24 + 7*a*b^26)*d*cos(d*x + c)^3 + ((a^20*b^7 - 10*a^18*b^9 + 45*a^16*b^11 - 120*a^14*b^13 + 210*a^12*b^15 - 252*a^10*b^17 + 210*a^8*b^19 - 120*a^6*b^21 + 45*a^4*b^23 - 10*a^2*b^25 + b^27)*d*cos(d*x + c)^9 - 3*(7*a^22*b^5 - 69*a^20*b^7 + 305*a^18*b^9 - 795*a^16*b^11 + 1350*a^14*b^13 - 1554*a^12*b^15 + 1218*a^10*b^17 - 630*a^8*b^19 + 195*a^6*b^21 - 25*a^4*b^23 - 3*a^2*b^25 + b^27)*d*cos(d*x + c)^7 + (35*a^24*b^3 - 308*a^22*b^5 + 1158*a^20*b^7 - 2340*a^18*b^9 + 2445*a^16*b^11 - 360*a^14*b^13 - 2604*a^12*b^15 + 3864*a^10*b^17 - 2835*a^8*b^19 + 1180*a^6*b^21 - 250*a^4*b^23 + 12*a^2*b^25 + 3*b^27)*d*cos(d*x + c)^5 - (7*a^26*b - 35*a^24*b^3 - 14*a^22*b^5 + 526*a^20*b^7 - 1795*a^18*b^9 + 3111*a^16*b^11 - 3060*a^14*b^13 + 1428*a^12*b^15 + 273*a^10*b^17 - 805*a^8*b^19 + 482*a^6*b^21 - 130*a^4*b^23 + 11*a^2*b^25 + b^27)*d*cos(d*x + c)^3)*sin(d*x + c)), 1/336*(112*a^18*b - 1008*a^16*b^3 + 4032*a^14*b^5 - 9408*a^12*b^7 + 14112*a^10*b^9 - 14112*a^8*b^11 + 9408*a^6*b^13 - 4032*a^4*b^15 + 1008*a^2*b^17 - 112*b^19 - (224*a^12*b^7 - 6272*a^10*b^9 - 201284*a^8*b^11 - 205692*a^6*b^13 + 277535*a^4*b^15 + 131393*a^2*b^17 + 4096*b^19)*cos(d*x + c)^10 + 14*(336*a^14*b^5 - 9352*a^12*b^7 - 252014*a^10*b^9 - 230159*a^8*b^11 + 297312*a^6*b^13 + 165122*a^4*b^15 + 27731*a^2*b^17 + 1024*b^19)*cos(d*x + c)^8 - 35*(224*a^16*b^3 - 5936*a^14*b^5 - 126448*a^12*b^7 - 243082*a^10*b^9 - 29747*a^8*b^11 + 284285*a^6*b^13 + 109607*a^4*b^15 + 10585*a^2*b^17 + 512*b^19)*cos(d*x + c)^6 + 14*(112*a^18*b - 2296*a^16*b^3 - 35224*a^14*b^5 - 308392*a^12*b^7 - 337750*a^10*b^9 + 149783*a^8*b^11 + 394751*a^6*b^13 + 130949*a^4*b^15 + 7427*a^2*b^17 + 640*b^19)*cos(d*x + c)^4 - 112*(7*a^18*b - 46*a^16*b^3 + 116*a^14*b^5 - 112*a^12*b^7 - 70*a^10*b^9 + 308*a^8*b^11 - 364*a^6*b^13 + 224*a^4*b^15 - 73*a^2*b^17 + 10*b^19)*cos(d*x + c)^2 - 3465*(7*(32*a^8*b^10 + 112*a^6*b^12 + 70*a^4*b^14 + 7*a^2*b^16)*cos(d*x + c)^9 - 7*(160*a^10*b^8 + 656*a^8*b^10 + 686*a^6*b^12 + 245*a^4*b^1

4 + 21*a^2*b^16)*cos(d*x + c)^7 + 7*(96*a^12*b^6 + 656*a^10*b^8 + 1426*a^8*
b^10 + 1057*a^6*b^12 + 280*a^4*b^14 + 21*a^2*b^16)*cos(d*x + c)^5 - (32*a^1
4*b^4 + 784*a^12*b^6 + 3542*a^10*b^8 + 5621*a^8*b^10 + 3381*a^6*b^12 + 735*
a^4*b^14 + 49*a^2*b^16)*cos(d*x + c)^3 + ((32*a^7*b^11 + 112*a^5*b^13 + 70*
a^3*b^15 + 7*a*b^17)*cos(d*x + c)^9 - 3*(224*a^9*b^9 + 816*a^7*b^11 + 602*a
^5*b^13 + 119*a^3*b^15 + 7*a*b^17)*cos(d*x + c)^7 + (1120*a^11*b^7 + 5264*a
^9*b^9 + 7250*a^7*b^11 + 3521*a^5*b^13 + 504*a^3*b^15 + 21*a*b^17)*cos(d*x
+ c)^5 - (224*a^13*b^5 + 1904*a^11*b^7 + 5082*a^9*b^9 + 4883*a^7*b^11 + 182
7*a^5*b^13 + 217*a^3*b^15 + 7*a*b^17)*cos(d*x + c)^3)*sin(d*x + c))*sqrt(a^
2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 7*(
16*a^19 - 144*a^17*b^2 + 576*a^15*b^4 - 1344*a^13*b^6 + 2016*a^11*b^8 - 201
6*a^9*b^10 + 1344*a^7*b^12 - 576*a^5*b^14 + 144*a^3*b^16 - 16*a*b^18 - (224
*a^13*b^6 - 6272*a^11*b^8 - 185444*a^9*b^10 - 166092*a^7*b^12 + 256745*a^5*
b^14 + 100208*a^3*b^16 + 631*a*b^18)*cos(d*x + c)^8 + 10*(112*a^15*b^4 - 30
80*a^13*b^6 - 73962*a^11*b^8 - 78323*a^9*b^10 + 60829*a^7*b^12 + 73923*a^5*
b^14 + 20401*a^3*b^16 + 100*a*b^18)*cos(d*x + c)^6 - 3*(224*a^17*b^2 - 5712
*a^15*b^4 - 95648*a^13*b^6 - 254254*a^11*b^8 - 120855*a^9*b^10 + 282886*a^7
*b^12 + 157892*a^5*b^14 + 35448*a^3*b^16 + 19*a*b^18)*cos(d*x + c)^4 + 16*(
2*a^19 - 35*a^17*b^2 + 208*a^15*b^4 - 644*a^13*b^6 + 1204*a^11*b^8 - 1442*a
^9*b^10 + 1120*a^7*b^12 - 548*a^5*b^14 + 154*a^3*b^16 - 19*a*b^18)*cos(d*x
+ c)^2)*sin(d*x + c))/(7*(a^21*b^6 - 10*a^19*b^8 + 45*a^17*b^10 - 120*a^15*
b^12 + 210*a^13*b^14 - 252*a^11*b^16 + 210*a^9*b^18 - 120*a^7*b^20 + 45*a^5
*b^22 - 10*a^3*b^24 + a*b^26)*d*cos(d*x + c)^9 - 7*(5*a^23*b^4 - 47*a^21*b^
6 + 195*a^19*b^8 - 465*a^17*b^10 + 690*a^15*b^12 - 630*a^13*b^14 + 294*a^11
*b^16 + 30*a^9*b^18 - 135*a^7*b^20 + 85*a^5*b^22 - 25*a^3*b^24 + 3*a*b^26)*
d*cos(d*x + c)^7 + 7*(3*a^25*b^2 - 20*a^23*b^4 + 38*a^21*b^6 + 60*a^19*b^8
- 435*a^17*b^10 + 984*a^15*b^12 - 1260*a^13*b^14 + 984*a^11*b^16 - 435*a^9*
b^18 + 60*a^7*b^20 + 38*a^5*b^22 - 20*a^3*b^24 + 3*a*b^26)*d*cos(d*x + c)^5
- (a^27 + 11*a^25*b^2 - 130*a^23*b^4 + 482*a^21*b^6 - 805*a^19*b^8 + 273*a
^17*b^10 + 1428*a^15*b^12 - 3060*a^13*b^14 + 3111*a^11*b^16 - 1795*a^9*b^18
+ 526*a^7*b^20 - 14*a^5*b^22 - 35*a^3*b^24 + 7*a*b^26)*d*cos(d*x + c)^3 +
((a^20*b^7 - 10*a^18*b^9 + 45*a^16*b^11 - 120*a^14*b^13 + 210*a^12*b^15 - 2
52*a^10*b^17 + 210*a^8*b^19 - 120*a^6*b^21 + 45*a^4*b^23 - 10*a^2*b^25 + b^
27)*d*cos(d*x + c)^9 - 3*(7*a^22*b^5 - 69*a^20*b^7 + 305*a^18*b^9 - 795*a^1
6*b^11 + 1350*a^14*b^13 - 1554*a^12*b^15 + 1218*a^10*b^17 - 630*a^8*b^19 +
195*a^6*b^21 - 25*a^4*b^23 - 3*a^2*b^25 + b^27)*d*cos(d*x + c)^7 + (35*a^24
*b^3 - 308*a^22*b^5 + 1158*a^20*b^7 - 2340*a^18*b^9 + 2445*a^16*b^11 - 360*
a^14*b^13 - 2604*a^12*b^15 + 3864*a^10*b^17 - 2835*a^8*b^19 + 1180*a^6*b^21
- 250*a^4*b^23 + 12*a^2*b^25 + 3*b^27)*d*cos(d*x + c)^5 - (7*a^26*b - 35*a
^24*b^3 - 14*a^22*b^5 + 526*a^20*b^7 - 1795*a^18*b^9 + 3111*a^16*b^11 - 306
0*a^14*b^13 + 1428*a^12*b^15 + 273*a^10*b^17 - 805*a^8*b^19 + 482*a^6*b^21
- 130*a^4*b^23 + 11*a^2*b^25 + b^27)*d*cos(d*x + c)^3)*sin(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.7795, size = 4113, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/168*(3465*(32*a^7*b^4 + 112*a^5*b^6 + 70*a^3*b^8 + 7*a*b^10)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2
 - b^2)))/((a^18 - 9*a^16*b^2 + 36*a^14*b^4 - 84*a^12*b^6 + 126*a^10*b^8 -
126*a^8*b^10 + 84*a^6*b^12 - 36*a^4*b^14 + 9*a^2*b^16 - b^18)*sqrt(a^2 - b^
2)) - 112*(3*a^10*tan(1/2*d*x + 1/2*c)^5 - 27*a^8*b^2*tan(1/2*d*x + 1/2*c)^
5 - 882*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 1638*a^4*b^6*tan(1/2*d*x + 1/2*c)^
5 - 513*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 - 15*b^10*tan(1/2*d*x + 1/2*c)^5 - 2
4*a^9*b*tan(1/2*d*x + 1/2*c)^4 + 216*a^7*b^3*tan(1/2*d*x + 1/2*c)^4 + 1512*
a^5*b^5*tan(1/2*d*x + 1/2*c)^4 + 1224*a^3*b^7*tan(1/2*d*x + 1/2*c)^4 + 144*
a*b^9*tan(1/2*d*x + 1/2*c)^4 - 2*a^10*tan(1/2*d*x + 1/2*c)^3 + 162*a^8*b^2*
tan(1/2*d*x + 1/2*c)^3 + 1932*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 + 3108*a^4*b^6
*tan(1/2*d*x + 1/2*c)^3 + 918*a^2*b^8*tan(1/2*d*x + 1/2*c)^3 + 26*b^10*tan(
1/2*d*x + 1/2*c)^3 - 720*a^7*b^3*tan(1/2*d*x + 1/2*c)^2 - 3024*a^5*b^5*tan(
1/2*d*x + 1/2*c)^2 - 2160*a^3*b^7*tan(1/2*d*x + 1/2*c)^2 - 240*a*b^9*tan(1/
2*d*x + 1/2*c)^2 + 3*a^10*tan(1/2*d*x + 1/2*c) - 27*a^8*b^2*tan(1/2*d*x + 1
/2*c) - 882*a^6*b^4*tan(1/2*d*x + 1/2*c) - 1638*a^4*b^6*tan(1/2*d*x + 1/2*c
) - 513*a^2*b^8*tan(1/2*d*x + 1/2*c) - 15*b^10*tan(1/2*d*x + 1/2*c) - 8*a^9
*b + 312*a^7*b^3 + 1512*a^5*b^5 + 1128*a^3*b^7 + 128*a*b^9)/((a^18 - 9*a^16
*b^2 + 36*a^14*b^4 - 84*a^12*b^6 + 126*a^10*b^8 - 126*a^8*b^10 + 84*a^6*b^1
2 - 36*a^4*b^14 + 9*a^2*b^16 - b^18)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3) + (232
848*a^18*b^6*tan(1/2*d*x + 1/2*c)^13 + 142758*a^16*b^8*tan(1/2*d*x + 1/2*c)
^13 + 64911*a^14*b^10*tan(1/2*d*x + 1/2*c)^13 - 28224*a^12*b^12*tan(1/2*d*x
 + 1/2*c)^13 + 12096*a^10*b^14*tan(1/2*d*x + 1/2*c)^13 - 3024*a^8*b^16*tan(
1/2*d*x + 1/2*c)^13 + 336*a^6*b^18*tan(1/2*d*x + 1/2*c)^13 + 155232*a^19*b^
```

5*tan(1/2*d*x + 1/2*c)^12 + 2783088*a^17*b^7*tan(1/2*d*x + 1/2*c)^12 + 2110
878*a^15*b^9*tan(1/2*d*x + 1/2*c)^12 + 545811*a^13*b^11*tan(1/2*d*x + 1/2*c
)^12 - 169344*a^11*b^13*tan(1/2*d*x + 1/2*c)^12 + 72576*a^9*b^15*tan(1/2*d*
x + 1/2*c)^12 - 18144*a^7*b^17*tan(1/2*d*x + 1/2*c)^12 + 2016*a^5*b^19*tan(
1/2*d*x + 1/2*c)^12 + 3104640*a^18*b^6*tan(1/2*d*x + 1/2*c)^11 + 15506568*a
^16*b^8*tan(1/2*d*x + 1/2*c)^11 + 12397616*a^14*b^10*tan(1/2*d*x + 1/2*c)^1
1 + 2172366*a^12*b^12*tan(1/2*d*x + 1/2*c)^11 - 451584*a^10*b^14*tan(1/2*d*
x + 1/2*c)^11 + 213696*a^8*b^16*tan(1/2*d*x + 1/2*c)^11 - 57344*a^6*b^18*ta
n(1/2*d*x + 1/2*c)^11 + 6720*a^4*b^20*tan(1/2*d*x + 1/2*c)^11 + 931392*a^19
*b^5*tan(1/2*d*x + 1/2*c)^10 + 22042944*a^17*b^7*tan(1/2*d*x + 1/2*c)^10 +
54377400*a^15*b^9*tan(1/2*d*x + 1/2*c)^10 + 38316040*a^13*b^11*tan(1/2*d*x
+ 1/2*c)^10 + 5346390*a^11*b^13*tan(1/2*d*x + 1/2*c)^10 - 685440*a^9*b^15*t
an(1/2*d*x + 1/2*c)^10 + 372960*a^7*b^17*tan(1/2*d*x + 1/2*c)^10 - 108640*a
^5*b^19*tan(1/2*d*x + 1/2*c)^10 + 13440*a^3*b^21*tan(1/2*d*x + 1/2*c)^10 +
12030480*a^18*b^6*tan(1/2*d*x + 1/2*c)^9 + 83208510*a^16*b^8*tan(1/2*d*x +
1/2*c)^9 + 129442775*a^14*b^10*tan(1/2*d*x + 1/2*c)^9 + 68997390*a^12*b^12*
tan(1/2*d*x + 1/2*c)^9 + 8026116*a^10*b^14*tan(1/2*d*x + 1/2*c)^9 - 418320*
a^8*b^16*tan(1/2*d*x + 1/2*c)^9 + 328720*a^6*b^18*tan(1/2*d*x + 1/2*c)^9 -
115584*a^4*b^20*tan(1/2*d*x + 1/2*c)^9 + 16128*a^2*b^22*tan(1/2*d*x + 1/2*c
)^9 + 2328480*a^19*b^5*tan(1/2*d*x + 1/2*c)^8 + 60558960*a^17*b^7*tan(1/2*d
*x + 1/2*c)^8 + 194655230*a^15*b^9*tan(1/2*d*x + 1/2*c)^8 + 204067311*a^13*
b^11*tan(1/2*d*x + 1/2*c)^8 + 74359166*a^11*b^13*tan(1/2*d*x + 1/2*c)^8 + 6
423144*a^9*b^15*tan(1/2*d*x + 1/2*c)^8 + 342720*a^7*b^17*tan(1/2*d*x + 1/2*
c)^8 + 38080*a^5*b^19*tan(1/2*d*x + 1/2*c)^8 - 54656*a^3*b^21*tan(1/2*d*x +
 1/2*c)^8 + 10752*a*b^23*tan(1/2*d*x + 1/2*c)^8 + 21732480*a^18*b^6*tan(1/2
*d*x + 1/2*c)^7 + 160923840*a^16*b^8*tan(1/2*d*x + 1/2*c)^7 + 294582904*a^1
4*b^10*tan(1/2*d*x + 1/2*c)^7 + 198535596*a^12*b^12*tan(1/2*d*x + 1/2*c)^7
+ 45251248*a^10*b^14*tan(1/2*d*x + 1/2*c)^7 + 2197104*a^8*b^16*tan(1/2*d*x
+ 1/2*c)^7 + 545280*a^6*b^18*tan(1/2*d*x + 1/2*c)^7 - 137728*a^4*b^20*tan(1
/2*d*x + 1/2*c)^7 + 5120*a^2*b^22*tan(1/2*d*x + 1/2*c)^7 + 3072*b^24*tan(1/
2*d*x + 1/2*c)^7 + 3104640*a^19*b^5*tan(1/2*d*x + 1/2*c)^6 + 77468160*a^17*
b^7*tan(1/2*d*x + 1/2*c)^6 + 251081600*a^15*b^9*tan(1/2*d*x + 1/2*c)^6 + 27
4259160*a^13*b^11*tan(1/2*d*x + 1/2*c)^6 + 105524636*a^11*b^13*tan(1/2*d*x
+ 1/2*c)^6 + 11690784*a^9*b^15*tan(1/2*d*x + 1/2*c)^6 + 515760*a^7*b^17*tan
(1/2*d*x + 1/2*c)^6 + 38080*a^5*b^19*tan(1/2*d*x + 1/2*c)^6 - 54656*a^3*b^2
1*tan(1/2*d*x + 1/2*c)^6 + 10752*a*b^23*tan(1/2*d*x + 1/2*c)^6 + 20568240*a
^18*b^6*tan(1/2*d*x + 1/2*c)^5 + 136444770*a^16*b^8*tan(1/2*d*x + 1/2*c)^5
+ 229744669*a^14*b^10*tan(1/2*d*x + 1/2*c)^5 + 133540988*a^12*b^12*tan(1/2*
d*x + 1/2*c)^5 + 22390536*a^10*b^14*tan(1/2*d*x + 1/2*c)^5 - 189280*a^8*b^1
6*tan(1/2*d*x + 1/2*c)^5 + 328720*a^6*b^18*tan(1/2*d*x + 1/2*c)^5 - 115584*
a^4*b^20*tan(1/2*d*x + 1/2*c)^5 + 16128*a^2*b^22*tan(1/2*d*x + 1/2*c)^5 + 2
328480*a^19*b^5*tan(1/2*d*x + 1/2*c)^4 + 47733840*a^17*b^7*tan(1/2*d*x + 1/
2*c)^4 + 125203386*a^15*b^9*tan(1/2*d*x + 1/2*c)^4 + 105004865*a^13*b^11*ta
n(1/2*d*x + 1/2*c)^4 + 21568540*a^11*b^13*tan(1/2*d*x + 1/2*c)^4 - 612864*a
^9*b^15*tan(1/2*d*x + 1/2*c)^4 + 385168*a^7*b^17*tan(1/2*d*x + 1/2*c)^4 - 1

```
08640*a^5*b^19*tan(1/2*d*x + 1/2*c)^4 + 13440*a^3*b^21*tan(1/2*d*x + 1/2*c)
^4 + 9934848*a^18*b^6*tan(1/2*d*x + 1/2*c)^3 + 46275768*a^16*b^8*tan(1/2*d*
x + 1/2*c)^3 + 52916248*a^14*b^10*tan(1/2*d*x + 1/2*c)^3 + 11715494*a^12*b^
12*tan(1/2*d*x + 1/2*c)^3 - 403536*a^10*b^14*tan(1/2*d*x + 1/2*c)^3 + 21828
8*a^8*b^16*tan(1/2*d*x + 1/2*c)^3 - 57344*a^6*b^18*tan(1/2*d*x + 1/2*c)^3 +
 6720*a^4*b^20*tan(1/2*d*x + 1/2*c)^3 + 931392*a^19*b^5*tan(1/2*d*x + 1/2*c
)^2 + 11782848*a^17*b^7*tan(1/2*d*x + 1/2*c)^2 + 16561160*a^15*b^9*tan(1/2*
d*x + 1/2*c)^2 + 3685248*a^13*b^11*tan(1/2*d*x + 1/2*c)^2 - 117586*a^11*b^1
3*tan(1/2*d*x + 1/2*c)^2 + 64736*a^9*b^15*tan(1/2*d*x + 1/2*c)^2 - 17136*a^
7*b^17*tan(1/2*d*x + 1/2*c)^2 + 2016*a^5*b^19*tan(1/2*d*x + 1/2*c)^2 + 1940
400*a^18*b^6*tan(1/2*d*x + 1/2*c) + 2910138*a^16*b^8*tan(1/2*d*x + 1/2*c) +
 644413*a^14*b^10*tan(1/2*d*x + 1/2*c) - 21546*a^12*b^12*tan(1/2*d*x + 1/2*
c) + 11284*a^10*b^14*tan(1/2*d*x + 1/2*c) - 2912*a^8*b^16*tan(1/2*d*x + 1/2
*c) + 336*a^6*b^18*tan(1/2*d*x + 1/2*c) + 155232*a^19*b^5 + 218064*a^17*b^7
 + 50666*a^15*b^9 - 3555*a^13*b^11 + 1670*a^11*b^13 - 424*a^9*b^15 + 48*a^7
*b^17)/((a^25 - 9*a^23*b^2 + 36*a^21*b^4 - 84*a^19*b^6 + 126*a^17*b^8 - 126
*a^15*b^10 + 84*a^13*b^12 - 36*a^11*b^14 + 9*a^9*b^16 - a^7*b^18)*(a*tan(1/
2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^7))/d
```

3.473 $\int \cos^5(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{7/2}}{7b^5d}-\frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^5d}+\frac{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{3/2}}{3b^5d}+\frac{2(a+b\sin}{11}$$

```
[Out] (2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^(3/2))/(3*b^5*d) - (8*a*(a^2 - b^2)*(
a + b*Sin[c + d*x])^(5/2))/(5*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x]
)^(7/2))/(7*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(9/2))/(9*b^5*d) + (2*(a + b
*Sin[c + d*x])^(11/2))/(11*b^5*d)
```

Rubi [A] time = 0.11659, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{7/2}}{7b^5d}-\frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^5d}+\frac{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{3/2}}{3b^5d}+\frac{2(a+b\sin}{11}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^(3/2))/(3*b^5*d) - (8*a*(a^2 - b^2)*(
a + b*Sin[c + d*x])^(5/2))/(5*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x]
)^(7/2))/(7*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(9/2))/(9*b^5*d) + (2*(a + b
*Sin[c + d*x])^(11/2))/(11*b^5*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+x}\left(b^2-x^2\right)^2\,dx,x,b\sin(c+dx)\right)}{b^5d}$$

$$= \frac{\text{Subst}\left(\int\left(\left(a^2-b^2\right)^2\sqrt{a+x}-4\left(a^3-ab^2\right)(a+x)^{3/2}+2\left(3a^2-b^2\right)(a+x)^{5/2}-4\right.\right.}{b^5d}$$

$$= \frac{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{3/2}}{3b^5d} - \frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^5d} + \frac{4\left(3a^2-\right.}{}$$

Mathematica [A] time = 0.324085, size = 117, normalized size = 0.76

$$\frac{2(a+b\sin(c+dx))^{3/2}\left(8\left(15b^2\left(2a^2-3b^2\right)\sin^2(c+dx)+\left(99ab^3-24a^3b\right)\sin(c+dx)-66a^2b^2+16a^4-35ab^3\sin^3(c+\right.\right.}{3465b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(3/2)*(315*b^4*Cos[c + d*x]^4 + 8*(16*a^4 - 66*a^2*
b^2 + 105*b^4 + (-24*a^3*b + 99*a*b^3)*Sin[c + d*x] + 15*b^2*(2*a^2 - 3*b^2
)*Sin[c + d*x]^2 - 35*a*b^3*Sin[c + d*x]^3)))/(3465*b^5*d)
```

Maple [A] time = 0.346, size = 126, normalized size = 0.8

$$\frac{630\,b^4(\cos(dx+c))^4+560\,ab^3(\cos(dx+c))^2\sin(dx+c)-480\,a^2b^2(\cos(dx+c))^2+720\,b^4(\cos(dx+c))^2-384\,a^3b\,\text{si}}{3465\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3465/b^5*(a+b*sin(d*x+c))^(3/2)*(315*b^4*cos(d*x+c)^4+280*a*b^3*cos(d*x+c
)^2*sin(d*x+c)-240*a^2*b^2*cos(d*x+c)^2+360*b^4*cos(d*x+c)^2-192*a^3*b*sin(
d*x+c)+512*a*b^3*sin(d*x+c)+128*a^4-288*a^2*b^2+480*b^4)/d
```

Maxima [A] time = 0.969357, size = 157, normalized size = 1.02

$$\frac{2\left(315\,(b\sin\left(dx+c\right)+a)^{\frac{11}{2}}-1540\,(b\sin\left(dx+c\right)+a)^{\frac{9}{2}}a+990\left(3\,a^2-b^2\right)(b\sin\left(dx+c\right)+a)^{\frac{7}{2}}-2772\left(a^3-ab^2\right)(b\sin\left(dx+c\right)+a)^{\frac{5}{2}}+1155\left(a^4-2\,a^2b^2+b^4\right)(b\sin\left(dx+c\right)+a)^{\frac{3}{2}}\right)}{3465\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(b*sin(d*x + c) + a)^(11/2) - 1540*(b*sin(d*x + c) + a)^(9/2)*a
 + 990*(3*a^2 - b^2)*(b*sin(d*x + c) + a)^(7/2) - 2772*(a^3 - a*b^2)*(b*sin
(d*x + c) + a)^(5/2) + 1155*(a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c) + a)^(3
/2))/(b^5*d)
```

Fricas [A] time = 3.58679, size = 348, normalized size = 2.26

$$\frac{2\left(35\,ab^4\cos\left(dx+c\right)^4+128\,a^5-480\,a^3b^2+992\,ab^4-16\left(3\,a^3b^2-8\,ab^4\right)\cos\left(dx+c\right)^2+\left(315\,b^5\cos\left(dx+c\right)^4-64\,a^4b+224\,a^2b^3+480\,b^5+40\left(a^2b^3+9\,b^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{3465\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(35*a*b^4*cos(d*x + c)^4 + 128*a^5 - 480*a^3*b^2 + 992*a*b^4 - 16*(3
*a^3*b^2 - 8*a*b^4)*cos(d*x + c)^2 + (315*b^5*cos(d*x + c)^4 - 64*a^4*b + 2
24*a^2*b^3 + 480*b^5 + 40*(a^2*b^3 + 9*b^5)*cos(d*x + c)^2)*sin(d*x + c))*s
qrt(b*sin(d*x + c) + a)/(b^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45228, size = 234, normalized size = 1.52

$$\frac{2\left(1155\,(b\sin\left(dx+c\right)+a)^{\frac{3}{2}}+\frac{315\,(b\sin(dx+c)+a)^{\frac{11}{2}}}{b^4}-\frac{1540\,(b\sin(dx+c)+a)^{\frac{9}{2}}a}{b^4}+\frac{2970\,(b\sin(dx+c)+a)^{\frac{7}{2}}a^2}{b^4}-\frac{2772\,(b\sin(dx+c)+a)^{\frac{5}{2}}a^3}{b^4}+\frac{1155}{}\right.}{3465\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(1155*(b*sin(d*x + c) + a)^(3/2) + 315*(b*sin(d*x + c) + a)^(11/2)/b
^4 - 1540*(b*sin(d*x + c) + a)^(9/2)*a/b^4 + 2970*(b*sin(d*x + c) + a)^(7/2
)*a^2/b^4 - 2772*(b*sin(d*x + c) + a)^(5/2)*a^3/b^4 + 1155*(b*sin(d*x + c)
+ a)^(3/2)*a^4/b^4 - 990*(b*sin(d*x + c) + a)^(7/2)/b^2 + 2772*(b*sin(d*x +
 c) + a)^(5/2)*a/b^2 - 2310*(b*sin(d*x + c) + a)^(3/2)*a^2/b^2)/(b*d)
```

3.474 $\int \cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=83

$$-\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^3d}-\frac{2(a+b\sin(c+dx))^{7/2}}{7b^3d}+\frac{4a(a+b\sin(c+dx))^{5/2}}{5b^3d}$$

```
[Out] (-2*(a^2 - b^2)*(a + b*Sin[c + d*x])^(3/2))/(3*b^3*d) + (4*a*(a + b*Sin[c +
 d*x])^(5/2))/(5*b^3*d) - (2*(a + b*Sin[c + d*x])^(7/2))/(7*b^3*d)
```

Rubi [A] time = 0.0819508, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$-\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^3d}-\frac{2(a+b\sin(c+dx))^{7/2}}{7b^3d}+\frac{4a(a+b\sin(c+dx))^{5/2}}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] (-2*(a^2 - b^2)*(a + b*Sin[c + d*x])^(3/2))/(3*b^3*d) + (4*a*(a + b*Sin[c +
 d*x])^(5/2))/(5*b^3*d) - (2*(a + b*Sin[c + d*x])^(7/2))/(7*b^3*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \sqrt{a+x}\left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\
&= \frac{\operatorname{Subst}\left(\int \left(\left(-a^2+b^2\right)\sqrt{a+x}+2a(a+x)^{3/2}-(a+x)^{5/2}\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\
&= -\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^3d}+\frac{4a(a+b\sin(c+dx))^{5/2}}{5b^3d}-\frac{2(a+b\sin(c+dx}{7b^3d}
\end{aligned}
$$

Mathematica [A] time = 0.122831, size = 58, normalized size = 0.7

$$
\frac{(a+b\sin(c+dx))^{3/2}\left(-16a^2+24ab\sin(c+dx)+15b^2\cos(2(c+dx))+55b^2\right)}{105b^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((a + b*Sin[c + d*x])^(3/2)*(-16*a^2 + 55*b^2 + 15*b^2*Cos[2*(c + d*x)] + 2
4*a*b*Sin[c + d*x]))/(105*b^3*d)
```

Maple [A] time = 0.295, size = 55, normalized size = 0.7

$$
-\frac{-30\,b^2\left(\cos\left(dx+c\right)\right)^2-24\,ab\sin\left(dx+c\right)+16\,a^2-40\,b^2}{105\,b^3d}\left(a+b\sin\left(dx+c\right)\right)^{\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/105/b^3*(a+b*sin(d*x+c))^(3/2)*(-15*b^2*cos(d*x+c)^2-12*a*b*sin(d*x+c)+8
*a^2-20*b^2)/d
```

Maxima [A] time = 0.950959, size = 82, normalized size = 0.99

$$
-\frac{2\left(15\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}-42\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a+35\left(a^2-b^2\right)\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\right)}{105\,b^3d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/105*(15*(b*sin(d*x + c) + a)^(7/2) - 42*(b*sin(d*x + c) + a)^(5/2)*a + 3
5*(a^2 - b^2)*(b*sin(d*x + c) + a)^(3/2))/(b^3*d)
```

Fricas [A] time = 3.13987, size = 192, normalized size = 2.31

$$\frac{2\left(3\,ab^{2}\cos\left(dx+c\right)^{2}-8\,a^{3}+32\,ab^{2}+\left(15\,b^{3}\cos\left(dx+c\right)^{2}+4\,a^{2}b+20\,b^{3}\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{105\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(3*a*b^2*cos(d*x + c)^2 - 8*a^3 + 32*a*b^2 + (15*b^3*cos(d*x + c)^2 +
 4*a^2*b + 20*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.6574, size = 105, normalized size = 1.27

$$\frac{2\left(35\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}-\frac{15\left(b\sin(dx+c)+a\right)^{\frac{7}{2}}}{b^{2}}+\frac{42\left(b\sin(dx+c)+a\right)^{\frac{5}{2}}a}{b^{2}}-\frac{35\left(b\sin(dx+c)+a\right)^{\frac{3}{2}}a^{2}}{b^{2}}\right)}{105\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(35*(b*sin(d*x + c) + a)^(3/2) - 15*(b*sin(d*x + c) + a)^(7/2)/b^2 +
42*(b*sin(d*x + c) + a)^(5/2)*a/b^2 - 35*(b*sin(d*x + c) + a)^(3/2)*a^2/b^2
)/(b*d)
```

3.475 $\int \cos(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=24

$$\frac{2(a+b\sin(c+dx))^{3/2}}{3bd}$$

```
[Out] (2*(a + b*Sin[c + d*x])^(3/2))/(3*b*d)
```

Rubi [A] time = 0.0357776, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 32}

$$\frac{2(a+b\sin(c+dx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(3/2))/(3*b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+x}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{2(a+b\sin(c+dx))^{3/2}}{3bd}\end{aligned}$$

Mathematica [A] time = 0.0158773, size = 24, normalized size = 1.

$$\frac{2(a+b\sin(c+dx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(3/2))/(3*b*d)
```

Maple [A] time = 0.007, size = 21, normalized size = 0.9

$$\frac{2}{3\,bd}\left(a+b\sin\left(dx+c\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3*(a+b*sin(d*x+c))^(3/2)/b/d
```

Maxima [A] time = 0.942176, size = 27, normalized size = 1.12

$$\frac{2\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*sin(d*x + c) + a)^(3/2)/(b*d)
```

Fricas [A] time = 2.75918, size = 51, normalized size = 2.12

$$\frac{2\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*sin(d*x + c) + a)^(3/2)/(b*d)
```

Sympy [A] time = 0.517719, size = 83, normalized size = 3.46

$$\begin{cases} \sqrt{a}x\cos{(c)} & \text{for } b = 0 \wedge d = 0 \\ x\sqrt{a + b\sin{(c)}}\cos{(c)} & \text{for } d = 0 \\ \frac{\sqrt{a}\sin{(c+dx)}}{d} & \text{for } b = 0 \\ \frac{2a\sqrt{a+b\sin{(c+dx)}}}{3bd} + \frac{2\sqrt{a+b\sin{(c+dx)}}\sin{(c+dx)}}{3d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((sqrt(a)*x*cos(c), Eq(b, 0) & Eq(d, 0)), (x*sqrt(a + b*sin(c))*co
s(c), Eq(d, 0)), (sqrt(a)*sin(c + d*x)/d, Eq(b, 0)), (2*a*sqrt(a + b*sin(c
+ d*x))/(3*b*d) + 2*sqrt(a + b*sin(c + d*x))*sin(c + d*x)/(3*d), True))
```

Giac [A] time = 1.07427, size = 27, normalized size = 1.12

$$\frac{2\,(b\sin{(dx+c)}+a)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*sin(d*x + c) + a)^(3/2)/(b*d)
```

3.476 $\int \sec(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
+ b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d
```

Rubi [A] time = 0.11644, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2668, 700, 1130, 206}

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
+ b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
```

```
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sec(c+dx)\sqrt{a+b\sin(c+dx)}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d} \\
&= \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d} \\
&= -\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx \right.}{d} \\
&= -\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.0534467, size = 74, normalized size = 1.

$$
\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d} - \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
+ b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d
```

Maple [A] time = 0.291, size = 63, normalized size = 0.9

$$
-\frac{1}{d}\sqrt{-a+b}\arctan\left(\sqrt{a+b\sin(dx+c)}\frac{1}{\sqrt{-a+b}}\right) + \frac{1}{d}\text{Artanh}\left(\sqrt{a+b\sin(dx+c)}\frac{1}{\sqrt{a+b}}\right)\sqrt{a+b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))+arctanh((a+b*
sin(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.07131, size = 4132, normalized size = 55.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*b
^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2
+ 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)
^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(
b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14
*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*
cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + sqrt(a - b)*lo
g((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*
b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2
*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x +
c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*
sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*
b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(
cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/d, -1/8*(2*sqrt(-a - b)*arctan(-1/4
```

```
*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x +
c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3
- (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c))) -
 sqrt(a - b)*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 -
256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(
16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 -
(b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin
(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4
- (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/d, -1/8*(2*sqrt(-a +
 b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b
^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b +
2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*si
n(d*x + c))) - sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^3*b +
320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*
x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos
(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(d*x +
c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2 + 64*a
*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x +
c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/d, -1/
4*(sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*
(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3
- 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b
^2 + b^3)*sin(d*x + c))) + sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8
*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c)
+ a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x
+ c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c))))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sin{(c + dx)}} \sec{(c + dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c), x)
```

3.477 $\int \sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}+\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d\sqrt{a+b}}+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\sin(c+dx)}}{2d}$$

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*Sqrt[a - b]*d
) + ((2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*Sqrt[a + b
]*d) + (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.166941, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2668, 737, 827, 1166, 206}

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}+\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d\sqrt{a+b}}+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\sin(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*Sqrt[a - b]*d
) + ((2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*Sqrt[a + b
]*d) + (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/(2*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
```

```
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{b^3\,\mathrm{Subst}\left(\int \frac{\sqrt{a+x}}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\tan(c+dx)}{2d} - \frac{b\,\mathrm{Subst}\left(\int \frac{-a-\frac{x}{2}}{\sqrt{a+x}\left(b^2-x^2\right)}\,dx, x, b\sin(c$$

$$= \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\tan(c+dx)}{2d} - \frac{b\,\mathrm{Subst}\left(\int \frac{-\frac{a}{2}-\frac{x^2}{2}}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a}\right.}{d}$$

$$= \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\tan(c+dx)}{2d} - \frac{(2a-b)\,\mathrm{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+}\right.}{4d}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4\sqrt{a-b}d} + \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4\sqrt{a+b}d} + \frac{\sec(c+dx)}{}$$

Mathematica [A] time = 0.666654, size = 143, normalized size = 1.15

$$\frac{(a-b)\left(\sqrt{a+b}(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+2(a+b)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sin(c+dx)}\right)-\sqrt{a-b}\left(2a^2+a\right.}{4d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-(Sqrt[a - b]*(2*a^2 + a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a
- b]]) + (a - b)*(Sqrt[a + b]*(2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sq
rt[a + b]] + 2*(a + b)*Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x]))
/(4*(a^2 - b^2)*d)
```

Maple [A] time = 0.408, size = 185, normalized size = 1.5

$$\frac{1}{4\left(\cos(dx+c)\right)^2 d}\left(2\sqrt{a+b\sin(dx+c)}\sqrt{-a+b}\sqrt{a+b}\sin(dx+c)-\left(-2\,\mathrm{Artanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)a\sqrt{-a+b}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/4*(2*(a+b*sin(d*x+c))^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(d*x+c)-(-2*arcta
nh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a*(-a+b)^(1/2)-arctanh((a+b*sin(d*x+
c))^(1/2)/(a+b)^(1/2))*b*(-a+b)^(1/2)-2*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b
)^(1/2))*a*(a+b)^(1/2)+arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*b*(a+b)^
(1/2))*cos(d*x+c)^2)/(-a+b)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^2/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.34974, size = 4968, normalized size = 40.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((2*a^2 - a*b - b^2)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)
^4 + 128*a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2
 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b
^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 2
8*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64
*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2
)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)
*sin(d*x + c) + 8)) - (2*a^2 + a*b - b^2)*sqrt(a - b)*cos(d*x + c)^2*log((b
^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4
- 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 - 24*a^2*b +
 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2
 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt
(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)
*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(
d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a^2 - b^2)*sqrt(b*sin(d*x + c) + a
)*sin(d*x + c))/((a^2 - b^2)*d*cos(d*x + c)^2), -1/32*(2*(2*a^2 - a*b - b^2
)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*
(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3
+ 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b
^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^2 + a*b - b^2)*sqrt(a - b)*c
os(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 -
 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*
(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 -
 (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*si
n(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4
 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(
d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(a^2 - b^2)*sqr
t(b*sin(d*x + c) + a)*sin(d*x + c))/((a^2 - b^2)*d*cos(d*x + c)^2), -1/32*(
2*(2*a^2 + a*b - b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 +
 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*s
qrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2
 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 - (2*a^2 - a*b -
 b^2)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^
3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*
cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^
```

```
3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(
d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2
+ 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(
d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) -
 16*(a^2 - b^2)*sqrt(b*sin(d*x + c) + a)*sin(d*x + c))/((a^2 - b^2)*d*cos(d
*x + c)^2), -1/16*((2*a^2 + a*b - b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x
 + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*si
n(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^
3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2
 + (2*a^2 - a*b - b^2)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2
 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)
*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)
^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 - 8*(a^2 - b^2
)*sqrt(b*sin(d*x + c) + a)*sin(d*x + c))/((a^2 - b^2)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sin{(c+dx)}}\sec^3{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\sec(dx+c)^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^3, x)
```

3.478 $\int \sec^5(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=207

$$-\frac{\left(12a^2-18ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{3/2}}+\frac{\left(12a^2+18ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{3/2}}-\frac{\sec^2(c+dx)\sqrt{a+b\sin(c+}{16}$$

```
[Out] -((12*a^2 - 18*a*b + 5*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/
(32*(a - b)^(3/2)*d) + ((12*a^2 + 18*a*b + 5*b^2)*ArcTanh[Sqrt[a + b*Sin[c
+ d*x]]/Sqrt[a + b]])/(32*(a + b)^(3/2)*d) - (Sec[c + d*x]^2*Sqrt[a + b*Sin
[c + d*x]]*(a*b - (6*a^2 - 5*b^2)*Sin[c + d*x]))/(16*(a^2 - b^2)*d) + (Sec[
c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.323333, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2668, 737, 823, 827, 1166, 206}

$$-\frac{\left(12a^2-18ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{3/2}}+\frac{\left(12a^2+18ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{3/2}}-\frac{\sec^2(c+dx)\sqrt{a+b\sin(c+}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] -((12*a^2 - 18*a*b + 5*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/
(32*(a - b)^(3/2)*d) + ((12*a^2 + 18*a*b + 5*b^2)*ArcTanh[Sqrt[a + b*Sin[c
+ d*x]]/Sqrt[a + b]])/(32*(a + b)^(3/2)*d) - (Sec[c + d*x]^2*Sqrt[a + b*Sin
[c + d*x]]*(a*b - (6*a^2 - 5*b^2)*Sin[c + d*x]))/(16*(a^2 - b^2)*d) + (Sec[
c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/(4*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
```

```
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{b^5\,\mathrm{Subst}\left(\int \frac{\sqrt{a+x}}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\tan(c+dx)}{4d} - \frac{b^3\,\mathrm{Subst}\left(\int \frac{-3a-\frac{5x}{2}}{\sqrt{a+x}\left(b^2-x^2\right)^2}\,dx, x, b\sin\right.}{4d}$$

$$= -\frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\left(ab-\left(6a^2-5b^2\right)\sin(c+dx)\right)}{16\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\sqrt{a}}{}$$

$$= -\frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\left(ab-\left(6a^2-5b^2\right)\sin(c+dx)\right)}{16\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\sqrt{a}}{}$$

$$= -\frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\left(ab-\left(6a^2-5b^2\right)\sin(c+dx)\right)}{16\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\sqrt{a}}{}$$

$$= -\frac{\left(12a^2-18ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{3/2}d} + \frac{\left(12a^2+18ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+}}{}\right.}{32(a+b)^{3/2}d}$$

Mathematica [A] time = 1.48296, size = 224, normalized size = 1.08

$$\frac{-\sqrt{a-b}(a+b)^2\left(12a^2-18ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)+(a-b)^2\sqrt{a+b}\left(12a^2+18ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-(Sqrt[a - b]*(a + b)^2*(12*a^2 - 18*a*b + 5*b^2)*ArcTanh[Sqrt[a + b*Sin[c
 + d*x]]/Sqrt[a - b]]) + (a - b)^2*Sqrt[a + b]*(12*a^2 + 18*a*b + 5*b^2)*Ar
cTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + ((a^2 - b^2)*Sec[c + d*x]^4*S
qrt[a + b*Sin[c + d*x]]*(-2*a*b - 2*a*b*Cos[2*(c + d*x)] + (22*a^2 - 21*b^2
)*Sin[c + d*x] + 6*a^2*Sin[3*(c + d*x)] - 5*b^2*Sin[3*(c + d*x)]))/2)/(32*(
a^2 - b^2)^2*d)
```

Maple [B] time = 0.717, size = 509, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -3/16/d/(b*sin(d*x+c)+b)^2*b/(a-b)*(a+b*sin(d*x+c))^(3/2)*a+5/32/d/(b*sin(d
*x+c)+b)^2*b^2/(a-b)*(a+b*sin(d*x+c))^(3/2)+3/16/d/(b*sin(d*x+c)+b)^2*b*(a+
b*sin(d*x+c))^(1/2)*a-7/32/d/(b*sin(d*x+c)+b)^2*b^2*(a+b*sin(d*x+c))^(1/2)+
3/8/d/(a-b)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2-9/
16/d/(a-b)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a*b+5/3
2/d/(a-b)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*b^2-3/16
/d/(b*sin(d*x+c)-b)^2*b/(a+b)*(a+b*sin(d*x+c))^(3/2)*a-5/32/d/(b*sin(d*x+c)
-b)^2*b^2/(a+b)*(a+b*sin(d*x+c))^(3/2)+3/16/d/(b*sin(d*x+c)-b)^2*b*(a+b*sin
(d*x+c))^(1/2)*a+7/32/d/(b*sin(d*x+c)-b)^2*b^2*(a+b*sin(d*x+c))^(1/2)+3/8/d
/(a+b)^(3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2+9/16/d/(a+b)^(
3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a*b+5/32/d/(a+b)^(3/2)*arc
tanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sin\left(dx+c\right)+a}\sec\left(dx+c\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^5, x)
```

3.479 $\int \cos^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=298

$$-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a\left(a^2-3b^2\right)-3b\left(a^2+7b^2\right)\sin(c+dx)\right)}{315b^3d}+\frac{32a\left(-4a^2b^2+a^4+3b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F}{315b^4d\sqrt{a+b\sin(c+}}$$

```
[Out] (-4*a*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(21*b*d) + (2*Cos[c + d*x]^3
*(a + b*Sin[c + d*x])^(3/2))/(9*b*d) - (8*(4*a^4 - 15*a^2*b^2 - 21*b^4)*Ell
ipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^
4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (32*a*(a^4 - 4*a^2*b^2 + 3*b^4)*E
llipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)])/(315*b^4*d*Sqrt[a + b*Sin[c + d*x]]) - (4*Cos[c + d*x]*Sqrt[a + b*Sin[
c + d*x]]*(4*a*(a^2 - 3*b^2) - 3*b*(a^2 + 7*b^2)*Sin[c + d*x]))/(315*b^3*d)
```

Rubi [A] time = 0.565301, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2695, 2862, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a\left(a^2-3b^2\right)-3b\left(a^2+7b^2\right)\sin(c+dx)\right)}{315b^3d}+\frac{32a\left(-4a^2b^2+a^4+3b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F}{315b^4d\sqrt{a+b\sin(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] (-4*a*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(21*b*d) + (2*Cos[c + d*x]^3
*(a + b*Sin[c + d*x])^(3/2))/(9*b*d) - (8*(4*a^4 - 15*a^2*b^2 - 21*b^4)*Ell
ipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^
4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (32*a*(a^4 - 4*a^2*b^2 + 3*b^4)*E
llipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)])/(315*b^4*d*Sqrt[a + b*Sin[c + d*x]]) - (4*Cos[c + d*x]*Sqrt[a + b*Sin[
c + d*x]]*(4*a*(a^2 - 3*b^2) - 3*b*(a^2 + 7*b^2)*Sin[c + d*x]))/(315*b^3*d)
```


Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{2\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9bd} + \frac{2\int \cos^2(c+dx)(b+a\sin(c+dx))\sqrt{a+}}{3b}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9bd} + \frac{4}{}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9bd} - \frac{4}{}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9bd} - \frac{4}{}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9bd} - \frac{4}{}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9bd} - \frac{8(}{}$$

Mathematica [A] time = 0.838995, size = 233, normalized size = 0.78

$$\frac{2b\cos(c+dx)(a+b\sin(c+dx))\left(b\left(24a^2+203b^2\right)\sin(c+dx)-32a^3+10ab^2\cos(2(c+dx))+106ab^2+35b^3\sin(3(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (32*(a*b^2*(a^2 - 33*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] +
 (4*a^4 - 15*a^2*b^2 - 21*b^4)*((a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]))*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)] + 2*b*Cos[c + d*x]*(a + b*Sin[c + d*x])*(-32*a^3
 + 106*a*b^2 + 10*a*b^2*Cos[2*(c + d*x)] + b*(24*a^2 + 203*b^2)*Sin[c + d*x
] + 35*b^3*Sin[3*(c + d*x)]))/(1260*b^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.542, size = 1189, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/315*(-35*b^6*sin(d*x+c)^6-40*a*b^5*sin(d*x+c)^5+16*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-12*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^4*b^2-64*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^3*b^3-72*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+48*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5+84*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6
-16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+
b))^(1/2))*a^6+76*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a^4*b^2+24*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-84*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6+a^2*b^4*si
```

```
n(d*x+c)^4+112*b^6*sin(d*x+c)^4-2*a^3*b^3*sin(d*x+c)^3+146*a*b^5*sin(d*x+c)
^3-8*a^4*b^2*sin(d*x+c)^2+28*a^2*b^4*sin(d*x+c)^2-77*b^6*sin(d*x+c)^2+2*a^3
*b^3*sin(d*x+c)-106*a*b^5*sin(d*x+c)+8*a^4*b^2-29*a^2*b^4)/b^5/cos(d*x+c)/(
a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\cos(dx+c)^4\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{b\sin(dx+c)+a}\cos(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sin(c+dx)}\cos^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*cos(c + d*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4, x)
```

3.480 $\int \cos^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=215

$$-\frac{4a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\sin(c+dx)}}+\frac{4\left(a^2+3b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2\cos(c+\cdots}{\cdots}$$

```
[Out] (-4*a*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(15*b*d) + (2*Cos[c + d*x]*(a
+ b*Sin[c + d*x])^(3/2))/(5*b*d) + (4*(a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15*b^2*d*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)]) - (4*a*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(
a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15*b^2*d*Sqrt[a + b*Sin[c + d*
x]])
```

Rubi [A] time = 0.260553, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2695, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\sin(c+dx)}}+\frac{4\left(a^2+3b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2\cos(c+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] (-4*a*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(15*b*d) + (2*Cos[c + d*x]*(a
+ b*Sin[c + d*x])^(3/2))/(5*b*d) + (4*(a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15*b^2*d*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)]) - (4*a*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(
a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15*b^2*d*Sqrt[a + b*Sin[c + d*
x]])
```


Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
```

```
0] && IntegersQ[2*m, 2*p]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd} + \frac{2\int(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}\,d}{5b}$$

$$= -\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd} + \frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd} + \frac{4\int}{}$$

$$= -\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd} + \frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd} - \frac{(2a}{}$$

$$= -\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd} + \frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd} + \frac{(2(}{}$$

$$= -\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd} + \frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd} + \frac{4(a}{}$$

Mathematica [A] time = 0.798058, size = 185, normalized size = 0.86

$$\frac{b\cos(c+dx)\left(2a^2+8ab\sin(c+dx)-3b^2\cos(2(c+dx))+3b^2\right)+4a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-}{15b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-4*(a^3 + a^2*b + 3*a*b^2 + 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/
(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 4*a*(a^2 - b^2)*EllipticF[(-2
*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*C
os[c + d*x]*(2*a^2 + 3*b^2 - 3*b^2*Cos[2*(c + d*x)] + 8*a*b*Sin[c + d*x]))/
(15*b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.389, size = 792, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/15*(2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^3*b+6*a^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^2-2*a*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^3-6*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4-2*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
a^4-4*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^2*b^2+6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*b^4-3*b^4*sin(d*x+c)^4-4*a*b^3*sin(d*x+c)^3-a
^2*b^2*sin(d*x+c)^2+3*b^4*sin(d*x+c)^2+4*a*b^3*sin(d*x+c)+a^2*b^2)/b^3/cos(
d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin (dx+c)+a} \cos (dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sin (dx+c)+a} \cos (dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sin{(c + dx)}}\cos^2{(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*cos(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx + c) + a}\cos(dx + c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2, x)
```

3.481 $\int \sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=149

$$\frac{\tan(c+dx)\sqrt{a+b\sin(c+dx)}}{d}+\frac{a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}-\frac{\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] -((EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(
d*Sqrt[(a + b*Sin[c + d*x])/(a + b)])) + (a*EllipticF[(c - Pi/2 + d*x)/2, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x
]]) + (Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.170638, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2690, 12, 2752, 2663, 2661, 2655, 2653}

$$\frac{\tan(c+dx)\sqrt{a+b\sin(c+dx)}}{d}+\frac{a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}-\frac{\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] -((EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(
d*Sqrt[(a + b*Sin[c + d*x])/(a + b)])) + (a*EllipticF[(c - Pi/2 + d*x)/2, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x
]]) + (Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/d
```


Rule 2690

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^m*Sin[e + f*x])/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1)*(a*(p + 2) + b*(m + p + 2)*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[0
, m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{\sqrt{a+b\sin(c+dx)}\tan(c+dx)}{d} - \int \frac{b\sin(c+dx)}{2\sqrt{a+b\sin(c+dx)}}\,dx$$
$$= \frac{\sqrt{a+b\sin(c+dx)}\tan(c+dx)}{d} - \frac{1}{2}b\int \frac{\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$
$$= \frac{\sqrt{a+b\sin(c+dx)}\tan(c+dx)}{d} - \frac{1}{2}\int \sqrt{a+b\sin(c+dx)}\,dx + \frac{1}{2}a\int \frac{1}{\sqrt{a+b\sin(}}$$
$$= \frac{\sqrt{a+b\sin(c+dx)}\tan(c+dx)}{d} - \frac{\sqrt{a+b\sin(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}\,dx}{2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{(a}{}$$
$$= -\frac{E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c}{a+b}}}{d\sqrt{a+b\sin(c+dx)}}$$

Mathematica [A] time = 2.78513, size = 127, normalized size = 0.85

$$\frac{\tan(c+dx)(a+b\sin(c+dx)) - a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right) + (a+b)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right.}{d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(
a + b*Sin[c + d*x])/(a + b)] + (a + b*Sin[c + d*x])*Tan[c + d*x])/(d*Sqrt[a
 + b*Sin[c + d*x]])
```

Maple [B] time = 0.486, size = 617, normalized size = 4.1

$$-\frac{1}{bd\cos(dx+c)}\sqrt{(\cos(dx+c))^2\sin(dx+c)\,b+a(\cos(dx+c))^2}\left(\mathrm{EllipticF}\left(\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)\sqrt{-\frac{b\,\mathrm{s}}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/b*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*(EllipticF((b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b
))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2)*a*b-EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/
2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)
*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b^2-(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/
2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2+(-
b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2),((a-b)/(a+b))^(1/2))*b^2+b^2*cos(d*x+c)^2-a*b*sin(d*x+c)-b^2)/(-(a+b*si
n(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/cos(d*x+c)/(a+b*sin(d*x+c))^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sin(dx+c)+a}\sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sin(c+dx)}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^2, x)
```

3.482 $\int \sec^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=248

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(ab-\left(4a^2-3b^2\right)\sin(c+dx)\right)}{6d\left(a^2-b^2\right)}-\frac{\left(4a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{6d\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] -((4*a^2 - 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(6*(a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (2*a*E
llipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)])/(3*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]
]*(a*b - (4*a^2 - 3*b^2)*Sin[c + d*x]))/(6*(a^2 - b^2)*d) + (Sec[c + d*x]^2
*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.37143, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2690, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(ab-\left(4a^2-3b^2\right)\sin(c+dx)\right)}{6d\left(a^2-b^2\right)}-\frac{\left(4a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{6d\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] -((4*a^2 - 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(6*(a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (2*a*E
llipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)])/(3*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]
]*(a*b - (4*a^2 - 3*b^2)*Sin[c + d*x]))/(6*(a^2 - b^2)*d) + (Sec[c + d*x]^2
*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/(3*d)
```


Rule 2690

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^m*Sin[e + f*x])/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1)*(a*(p + 2) + b*(m + p + 2)*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[0
```

```
, m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\tan(c+dx)}{3d} - \frac{1}{3}\int \frac{\sec^2(c+dx)\left(-2a-\frac{3}{2}b\sin(c\right.}{\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(ab-\left(4a^2-3b^2\right)\sin(c+dx)\right)}{6\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx)\sqrt{}}{}$$

$$= -\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(ab-\left(4a^2-3b^2\right)\sin(c+dx)\right)}{6\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx)\sqrt{}}{}$$

$$= -\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(ab-\left(4a^2-3b^2\right)\sin(c+dx)\right)}{6\left(a^2-b^2\right)d} + \frac{\sec^2(c+dx)\sqrt{}}{}$$

$$= -\frac{\left(4a^2-3b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{6\left(a^2-b^2\right)d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\right.}{3d\sqrt{a+b\,\mathrm{si}}}$$

Mathematica [A] time = 3.29538, size = 270, normalized size = 1.09

$$\frac{-4a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\left(4a^2b+4a^3-3ab^2-3b^3\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)
/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 4*a*(a^2 - b^2)*EllipticF[(-
2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + (S
ec[c + d*x]^3*(8*a^2*b - 11*b^3 + (-12*a^2*b + 8*b^3)*Cos[2*(c + d*x)] + (-
4*a^2*b + 3*b^3)*Cos[4*(c + d*x)] + 24*a^3*Sin[c + d*x] - 24*a*b^2*Sin[c +
d*x] + 8*a^3*Sin[3*(c + d*x)] - 8*a*b^2*Sin[3*(c + d*x)]))/8)/(6*(a - b)*(a
 + b)*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.571, size = 1259, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/6*(-4*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b*(a^2-b^2)*sin(
d*x+c)*cos(d*x+c)^2-2*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b*
(a^2-b^2)*sin(d*x+c)+(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*b^2*(
4*a^2-3*b^2)*cos(d*x+c)^4+(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*
(4*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/
(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b
)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b-3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b
)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^2-4*
EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-
b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2)*a*b^3+3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*s
in(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b^4-4*(-b/(a-
b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),(
(a-b)/(a+b))^(1/2))*a^4+7*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin
(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^2-3*(-b/(a-b)*si
n(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*
x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)
/(a+b))^(1/2))*b^4-a^2*b^2+b^4)*cos(d*x+c)^2-2*(cos(d*x+c)^2*sin(d*x+c)*b+a
*cos(d*x+c)^2)^(1/2)*a^2*b^2+2*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(
1/2)*b^4)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(a+b)/(a-
b)/(1+sin(d*x+c))/(sin(d*x+c)-1)/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sin\left(dx+c\right)+a}\sec\left(dx+c\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sin\left(dx+c\right)+a}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^4, x)
```

3.483 $\int \cos^5(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=154

$$\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{9/2}}{9b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{7/2}}{7b^5d}+\frac{2(a^2-b^2)^2(a+b\sin(c+dx))^{5/2}}{5b^5d}+\frac{2(a+b\sin(\ldots}{13b\ldots}$$

```
[Out] (2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^(5/2))/(5*b^5*d) - (8*a*(a^2 - b^2)*(
a + b*Sin[c + d*x])^(7/2))/(7*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x]
)^(9/2))/(9*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(11/2))/(11*b^5*d) + (2*(a +
 b*Sin[c + d*x])^(13/2))/(13*b^5*d)
```

Rubi [A] time = 0.122571, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{9/2}}{9b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{7/2}}{7b^5d}+\frac{2(a^2-b^2)^2(a+b\sin(c+dx))^{5/2}}{5b^5d}+\frac{2(a+b\sin(\ldots}{13b\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^(5/2))/(5*b^5*d) - (8*a*(a^2 - b^2)*(
a + b*Sin[c + d*x])^(7/2))/(7*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x]
)^(9/2))/(9*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(11/2))/(11*b^5*d) + (2*(a +
 b*Sin[c + d*x])^(13/2))/(13*b^5*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{\operatorname{Subst}\left(\int (a+x)^{3/2}\left(b^2-x^2\right)^2\,dx,x,b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\left(a^2-b^2\right)^2(a+x)^{3/2}-4\left(a^3-ab^2\right)(a+x)^{5/2}+2\left(3a^2-b^2\right)(a+x)\right.\right.}{b^5d}$$
$$= \frac{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{5/2}}{5b^5d}-\frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{7/2}}{7b^5d}+\frac{4\left(3a\right.}{}$$

Mathematica [A] time = 0.70725, size = 131, normalized size = 0.85

$$\frac{2\left(\frac{2}{9}\left(3a^2-b^2\right)(a+b\sin(c+dx))^{9/2}+\frac{1}{5}\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{5/2}+\frac{1}{13}(a+b\sin(c+dx))^{13/2}-\frac{4}{11}a(a+b\sin(c+\right.}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(((a^2 - b^2)^2*(a + b*Sin[c + d*x])^(5/2))/5 - (4*a*(a - b)*(a + b)*(a
+ b*Sin[c + d*x])^(7/2))/7 + (2*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^(9/2))/9
 - (4*a*(a + b*Sin[c + d*x])^(11/2))/11 + (a + b*Sin[c + d*x])^(13/2)/13))/
(b^5*d)
```

Maple [A] time = 0.462, size = 126, normalized size = 0.8

$$\frac{6930\,b^4(\cos(dx+c))^4+5040\,ab^3(\cos(dx+c))^2\sin(dx+c)-3360\,a^2b^2(\cos(dx+c))^2+6160\,b^4(\cos(dx+c))^2-192}{45045\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/45045/b^5*(a+b*sin(d*x+c))^(5/2)*(3465*b^4*cos(d*x+c)^4+2520*a*b^3*cos(d*
x+c)^2*sin(d*x+c)-1680*a^2*b^2*cos(d*x+c)^2+3080*b^4*cos(d*x+c)^2-960*a^3*b
*sin(d*x+c)+3200*a*b^3*sin(d*x+c)+384*a^4-608*a^2*b^2+2464*b^4)/d
```

Maxima [A] time = 0.950118, size = 157, normalized size = 1.02

$$\frac{2\left(3465\,(b\sin(dx+c)+a)^{\frac{13}{2}}-16380\,(b\sin(dx+c)+a)^{\frac{11}{2}}a+10010\left(3\,a^2-b^2\right)(b\sin(dx+c)+a)^{\frac{9}{2}}-25740\left(a^3-ab^2\right)(\right.}{45045\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3465*(b*sin(d*x + c) + a)^(13/2) - 16380*(b*sin(d*x + c) + a)^(11/
2)*a + 10010*(3*a^2 - b^2)*(b*sin(d*x + c) + a)^(9/2) - 25740*(a^3 - a*b^2)
*(b*sin(d*x + c) + a)^(7/2) + 9009*(a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c)
+ a)^(5/2))/(b^5*d)
```

Fricas [A] time = 3.30025, size = 462, normalized size = 3.

$$-\frac{2\left(3465\,b^6\cos(dx+c)^6-384\,a^6+2144\,a^4b^2-8256\,a^2b^4-2464\,b^6-35\left(3\,a^2b^4+11\,b^6\right)\cos(dx+c)^4+8\left(18\,a^4b^2-81\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45045*(3465*b^6*cos(d*x + c)^6 - 384*a^6 + 2144*a^4*b^2 - 8256*a^2*b^4 -
 2464*b^6 - 35*(3*a^2*b^4 + 11*b^6)*cos(d*x + c)^4 + 8*(18*a^4*b^2 - 81*a^2
*b^4 - 77*b^6)*cos(d*x + c)^2 - 2*(2205*a*b^5*cos(d*x + c)^4 - 96*a^5*b + 5
12*a^3*b^3 + 4064*a*b^5 + 20*(3*a^3*b^3 + 137*a*b^5)*cos(d*x + c)^2)*sin(d*
x + c))*sqrt(b*sin(d*x + c) + a)/(b^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^5, x)
```

3.484 $\int \cos^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=83

$$-\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^3d}-\frac{2(a+b\sin(c+dx))^{9/2}}{9b^3d}+\frac{4a(a+b\sin(c+dx))^{7/2}}{7b^3d}$$

```
[Out] (-2*(a^2 - b^2)*(a + b*Sin[c + d*x])^(5/2))/(5*b^3*d) + (4*a*(a + b*Sin[c +
 d*x])^(7/2))/(7*b^3*d) - (2*(a + b*Sin[c + d*x])^(9/2))/(9*b^3*d)
```

Rubi [A] time = 0.0913597, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$-\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^3d}-\frac{2(a+b\sin(c+dx))^{9/2}}{9b^3d}+\frac{4a(a+b\sin(c+dx))^{7/2}}{7b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)*(a + b*Sin[c + d*x])^(5/2))/(5*b^3*d) + (4*a*(a + b*Sin[c +
 d*x])^(7/2))/(7*b^3*d) - (2*(a + b*Sin[c + d*x])^(9/2))/(9*b^3*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{\text{Subst}\left(\int (a+x)^{3/2}\left(b^2-x^2\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^{3/2}+2a(a+x)^{5/2}-(a+x)^{7/2}\right)dx, x, b\sin(c+dx\right.}{b^3d}$$
$$= -\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^3d}+\frac{4a(a+b\sin(c+dx))^{7/2}}{7b^3d}-\frac{2(a+b\sin(c\,-}{9b^3d}$$

Mathematica [A] time = 0.181559, size = 58, normalized size = 0.7

$$\frac{(a+b\sin(c+dx))^{5/2}\left(-16a^2+40ab\sin(c+dx)+35b^2\cos(2(c+dx))+91b^2\right)}{315b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a + b*Sin[c + d*x])^(5/2)*(-16*a^2 + 91*b^2 + 35*b^2*Cos[2*(c + d*x)] + 4
0*a*b*Sin[c + d*x]))/(315*b^3*d)
```

Maple [A] time = 0.22, size = 55, normalized size = 0.7

$$-\frac{-70\,b^2(\cos(dx+c))^2-40\,ab\sin(dx+c)+16\,a^2-56\,b^2}{315\,b^3d}(a+b\sin(dx+c))^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/315/b^3*(a+b*sin(d*x+c))^(5/2)*(-35*b^2*cos(d*x+c)^2-20*a*b*sin(d*x+c)+8
*a^2-28*b^2)/d
```

Maxima [A] time = 0.961138, size = 82, normalized size = 0.99

$$-\frac{2\left(35\,(b\sin(dx+c)+a)^{\frac{9}{2}}-90\,(b\sin(dx+c)+a)^{\frac{7}{2}}a+63\left(a^2-b^2\right)(b\sin(dx+c)+a)^{\frac{5}{2}}\right)}{315\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*(b*sin(d*x + c) + a)^(9/2) - 90*(b*sin(d*x + c) + a)^(7/2)*a + 6
3*(a^2 - b^2)*(b*sin(d*x + c) + a)^(5/2))/(b^3*d)
```

Fricas [A] time = 2.49276, size = 265, normalized size = 3.19

$$-\frac{2\left(35\,b^{4}\cos\left(dx+c\right)^{4}+8\,a^{4}-60\,a^{2}b^{2}-28\,b^{4}-\left(3\,a^{2}b^{2}+7\,b^{4}\right)\cos\left(dx+c\right)^{2}-2\left(25\,ab^{3}\cos\left(dx+c\right)^{2}+2\,a^{3}b+38\,ab^{3}\right)s}{315\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*b^4*cos(d*x + c)^4 + 8*a^4 - 60*a^2*b^2 - 28*b^4 - (3*a^2*b^2 +
7*b^4)*cos(d*x + c)^2 - 2*(25*a*b^3*cos(d*x + c)^2 + 2*a^3*b + 38*a*b^3)*si
n(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^3*d)
```

Sympy [A] time = 134.232, size = 314, normalized size = 3.78

$$\begin{cases} a^{\frac{3}{2}} x \cos^{3}\left(c\right) \\ a^{\frac{3}{2}} \left(\frac{2\sin^{3}\left(c+dx\right)}{3d}+\frac{\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}\right) \\ x\left(a+b\sin\left(c\right)\right)^{\frac{3}{2}}\cos^{3}\left(c\right) \\ -\frac{16a^{4}\sqrt{a+b\sin\left(c+dx\right)}}{315b^{3}d}+\frac{8a^{3}\sqrt{a+b\sin\left(c+dx\right)}\sin\left(c+dx\right)}{315b^{2}d}+\frac{8a^{2}\sqrt{a+b\sin\left(c+dx\right)}\sin^{2}\left(c+dx\right)}{21bd}+\frac{2a^{2}\sqrt{a+b\sin\left(c+dx\right)}\cos^{2}\left(c+dx\right)}{5bd}+\frac{152a\sqrt{a+b\sin\left(c+dx\right)}\mathrm{si}}{315d} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((a**(3/2)*x*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (a**(3/2)*(2*sin(c +
 d*x)**3/(3*d) + sin(c + d*x)*cos(c + d*x)**2/d), Eq(b, 0)), (x*(a + b*sin(
c))**(3/2)*cos(c)**3, Eq(d, 0)), (-16*a**4*sqrt(a + b*sin(c + d*x))/(315*b*
*3*d) + 8*a**3*sqrt(a + b*sin(c + d*x))*sin(c + d*x)/(315*b**2*d) + 8*a**2*
sqrt(a + b*sin(c + d*x))*sin(c + d*x)**2/(21*b*d) + 2*a**2*sqrt(a + b*sin(c
 + d*x))*cos(c + d*x)**2/(5*b*d) + 152*a*sqrt(a + b*sin(c + d*x))*sin(c + d
```

```
*x)**3/(315*d) + 4*a*sqrt(a + b*sin(c + d*x))*sin(c + d*x)*cos(c + d*x)**2/
(5*d) + 8*b*sqrt(a + b*sin(c + d*x))*sin(c + d*x)**4/(45*d) + 2*b*sqrt(a +
b*sin(c + d*x))*sin(c + d*x)**2*cos(c + d*x)**2/(5*d), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

3.485 $\int \cos(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a+b\sin(c+dx))^{5/2}}{5bd}$$

```
[Out] (2*(a + b*Sin[c + d*x])^(5/2))/(5*b*d)
```

Rubi [A] time = 0.0387544, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 32}

$$\frac{2(a+b\sin(c+dx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(5/2))/(5*b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{3/2}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{2(a+b\sin(c+dx))^{5/2}}{5bd}\end{aligned}$$

Mathematica [A] time = 0.0212864, size = 24, normalized size = 1.

$$\frac{2(a+b\sin(c+dx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(5/2))/(5*b*d)
```

Maple [A] time = 0.006, size = 21, normalized size = 0.9

$$\frac{2}{5\,bd}\left(a+b\sin\left(dx+c\right)\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5*(a+b*sin(d*x+c))^(5/2)/b/d
```

Maxima [A] time = 0.949259, size = 27, normalized size = 1.12

$$\frac{2\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*sin(d*x + c) + a)^(5/2)/(b*d)
```

Fricas [B] time = 2.35848, size = 123, normalized size = 5.12

$$-\frac{2\left(b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}\right)\sqrt{b\sin\left(dx+c\right)+a}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(b*sin(d*x +
 c) + a)/(b*d)
```

Sympy [A] time = 27.4081, size = 116, normalized size = 4.83

$$\begin{cases} a^{\frac{3}{2}} x \cos{(c)} & \text{for } b = 0 \wedge d = 0 \\ x (a + b \sin{(c)})^{\frac{3}{2}} \cos{(c)} & \text{for } d = 0 \\ \frac{a^{\frac{3}{2}} \sin{(c+dx)}}{d} & \text{for } b = 0 \\ \frac{2a^2\sqrt{a+b\sin{(c+dx)}}}{5bd} + \frac{4a\sqrt{a+b\sin{(c+dx)}}\sin{(c+dx)}}{5d} + \frac{2b\sqrt{a+b\sin{(c+dx)}}\sin^2{(c+dx)}}{5d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((a**(3/2)*x*cos(c), Eq(b, 0) & Eq(d, 0)), (x*(a + b*sin(c))**(3/2
)*cos(c), Eq(d, 0)), (a**(3/2)*sin(c + d*x)/d, Eq(b, 0)), (2*a**2*sqrt(a +
b*sin(c + d*x))/(5*b*d) + 4*a*sqrt(a + b*sin(c + d*x))*sin(c + d*x)/(5*d) +
 2*b*sqrt(a + b*sin(c + d*x))*sin(c + d*x)**2/(5*d), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sin{(dx+c)} + a)^{\frac{3}{2}} \cos{(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

3.486 $\int \sec(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=94

$$-\frac{2b\sqrt{a+b\sin(c+dx)}}{d}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + ((a +
b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d - (2*b*Sqrt[a + b
*Sin[c + d*x]])/d
```

Rubi [A] time = 0.16744, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2668, 704, 827, 1166, 206}

$$-\frac{2b\sqrt{a+b\sin(c+dx)}}{d}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + ((a +
b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d - (2*b*Sqrt[a + b
*Sin[c + d*x]])/d
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 704

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*
d^2 - a*e^2 + 2*c*d*e*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x]
&& NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{(a+x)^{3/2}}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{2b\sqrt{a+b\sin(c+dx)}}{d} - \frac{b\,\mathrm{Subst}\left(\int \frac{-a^2-b^2-2ax}{\sqrt{a+x}\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{2b\sqrt{a+b\sin(c+dx)}}{d} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{a^2-b^2-2ax^2}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d}$$

$$= -\frac{2b\sqrt{a+b\sin(c+dx)}}{d} - \frac{(a-b)^2\,\mathrm{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d} + \cdots$$

$$= -\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d} - \frac{2b\sqrt{a+\cdots}}{}$$

Mathematica [A] time = 0.0962952, size = 89, normalized size = 0.95

$$\frac{-2b\sqrt{a+b\sin(c+dx)}+(a-b)^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]) + (a + b)^(
3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] - 2*b*Sqrt[a + b*Sin[c +
 d*x]])/d
```

Maple [B] time = 0.378, size = 218, normalized size = 2.3

$$-2\,\frac{b\sqrt{a+b\sin(dx+c)}}{d}+\frac{a^2}{d}\arctan\left(\sqrt{a+b\sin(dx+c)}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}-2\,\frac{ab}{d\sqrt{-a+b}}\arctan\left(\frac{\sqrt{a+b\sin(dx+c}}{\sqrt{-a+b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2*b*(a+b*sin(d*x+c))^(1/2)/d+1/d/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2
)/(-a+b)^(1/2))*a^2-2/d*b/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)
^(1/2))*a+1/d*b^2/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))+
1/d/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2+2/d*b/(a+b)
^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+1/d*b^2/(a+b)^(1/2)*ar
ctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sec(dx+c)\sin(dx+c)+a\sec(dx+c))\sqrt{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)*sin(d*x + c) + a*sec(d*x + c))*sqrt(b*sin(d*x + c)
 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.487 $\int \sec^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{a-b}(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d}+\frac{(2a-b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+b)\sqrt{a+}}{2d}$$

```
[Out] -(Sqrt[a - b]*(2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*d
) + ((2*a - b)*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(
4*d) + (Sec[c + d*x]^2*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(2*d)
```

Rubi [A] time = 0.274222, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2668, 739, 827, 1166, 206}

$$-\frac{\sqrt{a-b}(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d}+\frac{(2a-b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+b)\sqrt{a+}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] -(Sqrt[a - b]*(2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*d
) + ((2*a - b)*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(
4*d) + (Sec[c + d*x]^2*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(2*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
```

```
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{(a+x)^{3/2}}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d} - \frac{b\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(-2a^2+b^2\right)-\frac{ax}{2}}{\sqrt{a+x}\left(b^2-x^2\right)}\,d \cdots\right.}{2d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d} - \frac{b\,\text{Subst}\left(\int \frac{\frac{a^2}{2}+\frac{1}{2}\left(-2a^2+b^2\right)-\cdots}{-a^2+b^2+2ax^2-x^4 \cdots}\right.}{\cdots}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d} + \frac{((2a-b)(a+b))\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{a-b}(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d} + \frac{(2a-b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}} \cdots\right.}{4d}$$

Mathematica [A] time = 0.68235, size = 121, normalized size = 0.93

$$\frac{-\sqrt{a-b}(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)+(2a-b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+2\sec^2(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-(Sqrt[a - b]*(2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]) + (
2*a - b)*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + 2*Sec[
c + d*x]^2*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(4*d)
```

Maple [B] time = 0.385, size = 279, normalized size = 2.2

$$\frac{1}{4\left(\cos(dx+c)\right)^2 d}\left(2a\sqrt{a+b\sin(dx+c)}\sqrt{-a+b}\sqrt{a+b}\sin(dx+c)-\left(-2\,\mathrm{Artanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)a^2\sqrt{-a+b}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/4*(2*a*(a+b*sin(d*x+c))^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(d*x+c)-(-2*arc
tanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2*(-a+b)^(1/2)-b*arctanh((a+b*si
n(d*x+c))^(1/2)/(a+b)^(1/2))*a*(-a+b)^(1/2)+b^2*arctanh((a+b*sin(d*x+c))^(1
/2)/(a+b)^(1/2))*(-a+b)^(1/2)-2*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))
*a^2*(a+b)^(1/2)+b*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a*(a+b)^(1/2
)+b^2*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*(a+b)^(1/2))*cos(d*x+c)^2
+2*(a+b*sin(d*x+c))^(1/2)*b*(-a+b)^(1/2)*(a+b)^(1/2))/(-a+b)^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^2/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.45827, size = 4775, normalized size = 36.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((2*a - b)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*
a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b
^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*
a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 -
 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b +
112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x
 + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x
+ c) + 8)) - (2*a + b)*sqrt(a - b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 +
 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 2
8*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 -
 (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*
b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3
*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*si
n(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin
(d*x + c) + 8)) - 16*(a*sin(d*x + c) + b)*sqrt(b*sin(d*x + c) + a))/(d*cos(
d*x + c)^2), -1/32*(2*(2*a - b)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^
2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x
+ c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos
(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 - (2*
a + b)*sqrt(a - b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a
^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)
*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b
^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin
(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2
 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos
(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))
- 16*(a*sin(d*x + c) + b)*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^2), -1/
32*(2*(2*a + b)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b
 - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a
 + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*
a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a - b)*sqrt(a + b
)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*b^
```

```
2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 -
 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)^
2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b
*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14*
b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*c
os(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(a*sin(d*x +
 c) + b)*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^2), -1/16*((2*a + b)*sqr
t(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b
 - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^
2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b
^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a - b)*sqrt(-a - b)*arctan(-1/4*(b^2
*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*s
qrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*
b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*
x + c)^2 - 8*(a*sin(d*x + c) + b)*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)
^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.488 $\int \sec^5(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=188

$$-\frac{3\left(4a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d\sqrt{a-b}}+\frac{3\left(4a^2+2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d\sqrt{a+b}}+\frac{\sec^4(c+dx)(a\sin(c+dx)+b}{4d}$$

```
[Out] (-3*(4*a^2 - 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(3
2*Sqrt[a - b]*d) + (3*(4*a^2 + 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]
]/Sqrt[a + b]])/(32*Sqrt[a + b]*d) - (Sec[c + d*x]^2*(b - 6*a*Sin[c + d*x])
*Sqrt[a + b*Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x])*Sq
rt[a + b*Sin[c + d*x]])/(4*d)
```

Rubi [A] time = 0.318461, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2668, 739, 823, 827, 1166, 206}

$$-\frac{3\left(4a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d\sqrt{a-b}}+\frac{3\left(4a^2+2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d\sqrt{a+b}}+\frac{\sec^4(c+dx)(a\sin(c+dx)+b}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-3*(4*a^2 - 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(3
2*Sqrt[a - b]*d) + (3*(4*a^2 + 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]
]/Sqrt[a + b]])/(32*Sqrt[a + b]*d) - (Sec[c + d*x]^2*(b - 6*a*Sin[c + d*x])
*Sqrt[a + b*Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x])*Sq
rt[a + b*Sin[c + d*x]])/(4*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
```

```
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b^5\,\mathrm{Subst}\left(\int \frac{(a+x)^{3/2}}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4d} - \frac{b^3\,\mathrm{Subst}\left(\int \frac{\frac{1}{2}\left(-6a^2+b^2\right)-\frac{5ax}{2}}{\sqrt{a+x}\left(b^2-x^2\right)^2} \cdots\right.}{4d}$$

$$= -\frac{\sec^2(c+dx)(b-6a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{16d} + \frac{\sec^4(c+dx)(b+a\sin(c \cdots}{\cdots}$$

$$= -\frac{\sec^2(c+dx)(b-6a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{16d} + \frac{\sec^4(c+dx)(b+a\sin(c \cdots}{\cdots}$$

$$= -\frac{\sec^2(c+dx)(b-6a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{16d} + \frac{\sec^4(c+dx)(b+a\sin(c \cdots}{\cdots}$$

$$= -\frac{3\left(4a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32\sqrt{a-b}\,d} + \frac{3\left(4a^2+2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{s}\cdots}}{\sqrt{\cdots}}\right.}{32\sqrt{a+b}\,d}$$

Mathematica [A] time = 2.48678, size = 297, normalized size = 1.58

$$-\frac{-2b\sqrt{a+b\sin(c+dx)}\left(\left(6a^3b-4ab^3\right)\sin(c+dx)-13a^2b^2+12a^4+3b^4\right)+3\sqrt{a-b}(a+b)^2\left(-6a^2b+4a^3+ab^2+b^3\right)\mathrm{t}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(3*Sqrt[a - b]*(a + b)^2*(4*a^3 - 6*a^2*b + a*b^2 + b^3)*ArcTanh[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[a - b]] - 3*(a - b)^2*Sqrt[a + b]*(4*a^3 + 6*a^2*b + a
*b^2 - b^3)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + 8*(-a^2 + b^2)*
Sec[c + d*x]^4*(-b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(5/2) + 2*Sec[c +
 d*x]^2*(a + b*Sin[c + d*x])^(5/2)*(5*a^2*b - 3*b^3 + (-6*a^3 + 4*a*b^2)*Si
n[c + d*x]) - 2*b*Sqrt[a + b*Sin[c + d*x]]*(12*a^4 - 13*a^2*b^2 + 3*b^4 + (
6*a^3*b - 4*a*b^3)*Sin[c + d*x]))/(32*(a^2 - b^2)^2*d)
```

Maple [B] time = 0.474, size = 409, normalized size = 2.2

$$\frac{1}{32\,b\,(\cos(dx+c))^4\,d}\left(4\sqrt{-a+b}\sqrt{a+b}\sqrt{a+b\sin(dx+c)}\,b\left(b(\cos(dx+c))^2+8\,a\sin(dx+c)-b\right)+3\,b\left(4\,\mathrm{Artanh}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/32*(4*(-a+b)^(1/2)*(a+b)^(1/2)*(a+b*sin(d*x+c))^(1/2)*b*(b*cos(d*x+c)^2+8
*a*sin(d*x+c)-b)+3*b*(4*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2*(-a
+b)^(1/2)+2*b*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a*(-a+b)^(1/2)-b^
2*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*(-a+b)^(1/2)+4*arctan((a+b*si
n(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2*(a+b)^(1/2)-2*b*arctan((a+b*sin(d*x+c))^(
1/2)/(-a+b)^(1/2))*a*(a+b)^(1/2)-b^2*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(
1/2))*(a+b)^(1/2))*cos(d*x+c)^4+6*(-a+b)^(1/2)*(a+b)^(1/2)*(a+b*sin(d*x+c))
^(1/2)*b*(2*a*sin(d*x+c)-b)*cos(d*x+c)^2-24*(a+b*sin(d*x+c))^(3/2)*a*(-a+b)
^(1/2)*(a+b)^(1/2)+24*(a+b*sin(d*x+c))^(1/2)*a^2*(-a+b)^(1/2)*(a+b)^(1/2)+1
2*(a+b*sin(d*x+c))^(1/2)*b^2*(-a+b)^(1/2)*(a+b)^(1/2))/(-a+b)^(1/2)/(a+b)^(
1/2)/b/cos(d*x+c)^4/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec(dx+c)^5\sin(dx+c)+a\sec(dx+c)^5\right)\sqrt{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^5*sin(d*x + c) + a*sec(d*x + c)^5)*sqrt(b*sin(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.489 $\int \cos^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=329

$$\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(a^2+28ab\sin(c+dx)+3b^2\right)}{231bd}-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(-3ab\left(a^2+31b^2\right)\sin\right.}{1155b^3d}$$

```
[Out] (-2*b*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(11*d) - (32*a*(a^4 - 6*a^2*
b^2 - 27*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c
 + d*x]])/(1155*b^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(4*a^6 - 25*
a^4*b^2 + 6*a^2*b^4 + 15*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(1155*b^4*d*Sqrt[a + b*Sin[c + d*x]]) +
 (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(a^2 + 3*b^2 + 28*a*b*Sin[c + d
*x]))/(231*b*d) - (4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(4*a^4 - 21*a^2*
b^2 - 15*b^4 - 3*a*b*(a^2 + 31*b^2)*Sin[c + d*x]))/(1155*b^3*d)
```

Rubi [A] time = 0.691636, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2692, 2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(a^2+28ab\sin(c+dx)+3b^2\right)}{231bd}-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(-3ab\left(a^2+31b^2\right)\sin\right.}{1155b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-2*b*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(11*d) - (32*a*(a^4 - 6*a^2*
b^2 - 27*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c
 + d*x]])/(1155*b^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(4*a^6 - 25*
a^4*b^2 + 6*a^2*b^4 + 15*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(1155*b^4*d*Sqrt[a + b*Sin[c + d*x]]) +
 (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(a^2 + 3*b^2 + 28*a*b*Sin[c + d
*x]))/(231*b*d) - (4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(4*a^4 - 21*a^2*
b^2 - 15*b^4 - 3*a*b*(a^2 + 31*b^2)*Sin[c + d*x]))/(1155*b^3*d)
```


Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d}+\frac{2}{11}\int\frac{\cos^4(c+dx)\left(\frac{11a^2}{2}+\frac{b^2}{2}+6ab\sin\right.}{\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d}+\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(a^2+\right.}{231bd}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d}+\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(a^2+\right.}{231bd}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d}+\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(a^2+\right.}{231bd}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d}+\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(a^2+\right.}{231bd}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d}-\frac{32a\left(a^4-6a^2b^2-27b^4\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+\right.\right.}{1155b^4d\sqrt{\frac{a+b\sin}{a+}}}$$

Mathematica [A] time = 1.05822, size = 278, normalized size = 0.84

$$\frac{64\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\left(b^2\left(-114a^2b^2+a^4-15b^4\right)F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+4\left(-6a^3b^2+a^5-27ab^4\right)\left((a+b)E\left(\frac{1}{4}(-2c-2d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (64*(b^2*(a^4 - 114*a^2*b^2 - 15*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)] + 4*(a^5 - 6*a^3*b^2 - 27*a*b^4)*((a + b)*EllipticE[(-2*c + Pi -
 2*d*x)/4, (2*b)/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b
)]))*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - b*(a + b*Sin[c + d*x])*(2*(64*a^4
```

```
 - 366*a^2*b^2 + 195*b^4)*Cos[c + d*x] + 5*b^2*(-4*a^2 + 93*b^2)*Cos[3*(c +
 d*x)] + 105*b^4*Cos[5*(c + d*x)] - 16*a*b*(3*a^2 + 128*b^2)*Sin[2*(c + d*x
)] - 280*a*b^3*Sin[4*(c + d*x)]))/(9240*b^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.607, size = 1355, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/1155*(-245*a*b^6*sin(d*x+c)^6+24*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5+60*a*b^6-47*a^3*b^4+8*a^
5*b^2-105*b^7*sin(d*x+c)^7+300*b^7*sin(d*x+c)^5-255*b^7*sin(d*x+c)^3+60*b^7
*sin(d*x+c)-145*a^2*b^5*sin(d*x+c)^5+2*a^4*b^3*sin(d*x+c)-581*a*b^6*sin(d*x
+c)^2-16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a^7+60*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*b^7+16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b-12*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-100*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^4*b^3-360*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a^3*b^4+336*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4-432*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6-8*a^5*b^
2*sin(d*x+c)^2+a^3*b^4*sin(d*x+c)^4+46*a^3*b^4*sin(d*x+c)^2-2*a^4*b^3*sin(d
*x+c)^3+518*a^2*b^5*sin(d*x+c)^3+766*a*b^6*sin(d*x+c)^4-373*a^2*b^5*sin(d*x
+c)+372*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a*b^6+112*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2)/b^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1
```

```
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\cos(dx+c)^4\sin(dx+c)+a\cos(dx+c)^4\right)\sqrt{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4*sin(d*x + c) + a*cos(d*x + c)^4)*sqrt(b*sin(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4, x)
```

3.490 $\int \cos^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=247

$$\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(3a^2+24ab\sin(c+dx)+5b^2\right)}{105bd}-\frac{4\left(2a^2b^2+3a^4-5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\right.}{105b^2d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] (-2*b*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(7*d) + (4*a*(3*a^2 + 29*b^2
)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(1
05*b^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (4*(3*a^4 + 2*a^2*b^2 - 5*b^
4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(105*b^2*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]]*(3*a^2 + 5*b^2 + 24*a*b*Sin[c + d*x]))/(105*b*d)
```

Rubi [A] time = 0.462135, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2692, 2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(3a^2+24ab\sin(c+dx)+5b^2\right)}{105bd}-\frac{4\left(2a^2b^2+3a^4-5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\right.}{105b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(7*d) + (4*a*(3*a^2 + 29*b^2
)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(1
05*b^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (4*(3*a^4 + 2*a^2*b^2 - 5*b^
4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(105*b^2*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]]*(3*a^2 + 5*b^2 + 24*a*b*Sin[c + d*x]))/(105*b*d)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
```

```
GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{2}{7}\int \frac{\cos^2(c+dx)\left(\frac{7a^2}{2}+\frac{b^2}{2}+4ab\sin(\right.}{\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(3a^2+\right.}{105bd}$$

$$= -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(3a^2+\right.}{105bd}$$

$$= -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(3a^2+\right.}{105bd}$$

$$= -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{4a\left(29+\frac{3a^2}{b^2}\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{}}{105d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.01005, size = 222, normalized size = 0.9

$$\frac{b\cos(c+dx)\left(b\left(108a^2+5b^2\right)\sin(c+dx)+12a^3-78ab^2\cos(2(c+dx))+38ab^2-15b^3\sin(3(c+dx))\right)+8\left(2a^2b^2+3a\right.}{210b^2d\sqrt{a-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a*(3*a^3 + 3*a^2*b + 29*a*b^2 + 29*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4
, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 8*(3*a^4 + 2*a^2*b^2
- 5*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)] + b*Cos[c + d*x]*(12*a^3 + 38*a*b^2 - 78*a*b^2*Cos[2*(c +
d*x)] + b*(108*a^2 + 5*b^2)*Sin[c + d*x] - 15*b^3*Sin[3*(c + d*x)]))/(210*b
^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.432, size = 943, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/105*(-15*b^5*sin(d*x+c)^5+6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b+48*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+4*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b
^3-48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a*b^4-10*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*b^5-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5-52*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+58*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a
*b^4-39*a*b^4*sin(d*x+c)^4-27*a^2*b^3*sin(d*x+c)^3+25*b^5*sin(d*x+c)^3-3*a^
3*b^2*sin(d*x+c)^2+49*a*b^4*sin(d*x+c)^2+27*a^2*b^3*sin(d*x+c)-10*b^5*sin(d
*x+c)+3*a^3*b^2-10*a*b^4)/b^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+a\cos\left(dx+c\right)^{2}\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^2*sin(d*x + c) + a*cos(d*x + c)^2)*sqrt(b*sin(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

3.491 $\int \sec^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=168

$$\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}+\frac{\sec(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{d}-\frac{a\sqrt{a+b\sin(c+dx)}}{d\sqrt{a+b}}$$

```
[Out] (Sec[c + d*x]*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/d - (a*Ellipti
cE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]) + ((a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]
])
```

Rubi [A] time = 0.203109, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2691, 2752, 2663, 2661, 2655, 2653}

$$\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}+\frac{\sec(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{d}-\frac{a\sqrt{a+b\sin(c+dx)}}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/d - (a*Ellipti
cE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]) + ((a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]
])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
```

```
|| IntegerQ[m])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{\sec(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d} - \int \frac{\frac{b^2}{2}+\frac{1}{2}ab\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

$$= \frac{\sec(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d} - \frac{1}{2}a\int \sqrt{a+b\sin(c+dx)}\,dx$$

$$= \frac{\sec(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d} - \frac{\left(a\sqrt{a+b\sin(c+dx)}\right)\int\sqrt{\cdots}}{2\sqrt{\frac{a+b\sin(c+\cdots}{a+b}}}$$

$$= \frac{\sec(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d} - \frac{aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a\cdots}}{d\sqrt{\frac{a+b\sin(c+d\cdots}{a+b}}}$$

Mathematica [A] time = 0.634687, size = 163, normalized size = 0.97

$$\frac{-\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+a^2\tan(c+dx)+ab\sec(c+dx)+ab\sin(c+dx)\tan(c+dx)+a(a+\cdots}{d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*b*Sec[c + d*x] + a*(a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b
)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - (a^2 - b^2)*EllipticF[(-2*c + Pi -
2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + a^2*Tan[c + d
*x] + b^2*Tan[c + d*x] + a*b*Sin[c + d*x]*Tan[c + d*x])/(d*Sqrt[a + b*Sin[c
 + d*x]])
```

Maple [B] time = 0.614, size = 633, normalized size = 3.8

$$\frac{1}{bd\cos(dx+c)}\sqrt{(\cos(dx+c))^2\sin(dx+c)\,b+a(\cos(dx+c))^2}\left(\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\sqrt{-\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/b*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*((b/(a-b)*sin(d*x+c)+1
/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/
(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(
1/2))*a^3-(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b)
)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1
/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2-(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(
1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2
)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b
+(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(
-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2),((a-b)/(a+b))^(1/2))*b^3-a*b^2*cos(d*x+c)^2+a^2*b*sin(d*x+c)+b^3*sin
(d*x+c)+2*a*b^2)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/co
s(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin (dx+c)+a)^{\frac{3}{2}} \sec (dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec (dx+c)^2 \sin (dx+c)+a\sec (dx+c)^2\right)\sqrt{b\sin (dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^2*sin(d*x + c) + a*sec(d*x + c)^2)*sqrt(b*sin(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.492 $\int \sec^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=218

$$\frac{\left(4a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{6d\sqrt{a+b\sin(c+dx)}}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{3d}-\sec(c+dx)(b-4$$

```
[Out] -(Sec[c + d*x]*(b - 4*a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(6*d) + (Se
c[c + d*x]^3*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(3*d) - (2*a*El
lipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*d*S
qrt[(a + b*Sin[c + d*x])/(a + b)]) + ((4*a^2 - b^2)*EllipticF[(c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(6*d*Sqrt[a + b*S
in[c + d*x]])
```

Rubi [A] time = 0.438024, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2691, 2866, 2752, 2663, 2661, 2655, 2653}

$$\frac{\left(4a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{6d\sqrt{a+b\sin(c+dx)}}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{3d}-\sec(c+dx)(b-4$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] -(Sec[c + d*x]*(b - 4*a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(6*d) + (Se
c[c + d*x]^3*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(3*d) - (2*a*El
lipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*d*S
qrt[(a + b*Sin[c + d*x])/(a + b)]) + ((4*a^2 - b^2)*EllipticF[(c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(6*d*Sqrt[a + b*S
in[c + d*x]])
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
```

```
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{\sec^3(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3d} - \frac{1}{3}\int \frac{\sec^2(c+dx)\left(-2a^2\right.}{\sqrt{a+b}}$$

$$= -\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d} + \frac{\sec^3(c+dx)(b+a\sin(}{}$$

$$= -\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d} + \frac{\sec^3(c+dx)(b+a\sin(}{}$$

$$= -\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d} + \frac{\sec^3(c+dx)(b+a\sin(}{}$$

$$= -\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d} + \frac{\sec^3(c+dx)(b+a\sin(}{}$$

Mathematica [A] time = 2.33819, size = 211, normalized size = 0.97

$$\frac{-4\left(4a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\sec^3(c+dx)\left(12a^2\sin(c+dx)+4a^2\sin(3(c+dx))-6ab\cos(2(\right.}{24d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*a*(a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*S
in[c + d*x])/(a + b)] - 4*(4*a^2 - b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + Sec[c + d*x]^3*(12*a*b -
6*a*b*Cos[2*(c + d*x)] - 2*a*b*Cos[4*(c + d*x)] + 12*a^2*Sin[c + d*x] + 7*b
^2*Sin[c + d*x] + 4*a^2*Sin[3*(c + d*x)] - b^2*Sin[3*(c + d*x)]))/(24*d*Sqr
t[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.568, size = 937, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/6*(-(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*b*(4*a^2-b^2)*sin(d*
x+c)*cos(d*x+c)^2-2*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*b*(a^2
+b^2)*sin(d*x+c)+4*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b^2*c
os(d*x+c)^4-(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*(4*(b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin
(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b
)/(a+b))^(1/2))*a^3-4*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*
x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*
sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2-4*(b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)
-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b)
)^(1/2))*a^2*b+3*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+
b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d
*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2+(b/(a-b)*sin(d*x+c)+1/(a-
b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b
))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2)
)*b^3+a*b^2)*cos(d*x+c)^2-4*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2
)*a*b^2)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(1+sin(d*x
+c))/(sin(d*x+c)-1)/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec\left(dx+c\right)^{4} \sin\left(dx+c\right) + a \sec\left(dx+c\right)^{4}\right)\sqrt{b \sin\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^4*sin(d*x + c) + a*sec(d*x + c)^4)*sqrt(b*sin(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.493 $\int \sec^6(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=330

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(-13a^2b^2+8a^4+5b^4\right)-a\left(-61a^2b^2+32a^4+29b^4\right)\sin(c+dx)\right)}{60d\left(a^2-b^2\right)^2}+\frac{\left(32a^2-5b^2\right)\sqrt{\frac{a+b}{}}}{60d\sqrt{}}$$

```
[Out] -(Sec[c + d*x]^3*(b - 8*a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(30*d) +
(Sec[c + d*x]^5*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(5*d) - (a*(
32*a^2 - 29*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Si
n[c + d*x]])/(60*(a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((32*a
^2 - 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/(60*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a +
b*Sin[c + d*x]]*(b*(8*a^4 - 13*a^2*b^2 + 5*b^4) - a*(32*a^4 - 61*a^2*b^2 +
29*b^4)*Sin[c + d*x]))/(60*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.697539, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2691, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(-13a^2b^2+8a^4+5b^4\right)-a\left(-61a^2b^2+32a^4+29b^4\right)\sin(c+dx)\right)}{60d\left(a^2-b^2\right)^2}+\frac{\left(32a^2-5b^2\right)\sqrt{\frac{a+b}{}}}{60d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(Sec[c + d*x]^3*(b - 8*a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(30*d) +
(Sec[c + d*x]^5*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(5*d) - (a*(
32*a^2 - 29*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Si
n[c + d*x]])/(60*(a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((32*a
^2 - 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/(60*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a +
b*Sin[c + d*x]]*(b*(8*a^4 - 13*a^2*b^2 + 5*b^4) - a*(32*a^4 - 61*a^2*b^2 +
29*b^4)*Sin[c + d*x]))/(60*(a^2 - b^2)^2*d)
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
```

```
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{\sec^5(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5d} - \frac{1}{5}\int \frac{\sec^4(c+dx)\left(-4a^2+\right.}{\sqrt{a+b\,\mathrm{si}}}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)(b+a\sin(c}{}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)(b+a\sin(c}{}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)(b+a\sin(c}{}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)(b+a\sin(c}{}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)(b+a\sin(c}{}$$

Mathematica [A] time = 6.24963, size = 364, normalized size = 1.1

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{\sec(c+dx)\left(-8a^2b+32a^3\sin(c+dx)-29ab^2\sin(c+dx)+5b^3\right)}{60\left(a^2-b^2\right)} + \frac{1}{5}\sec^5(c+dx)(a\sin(c+dx)+b) + \frac{1}{30}\sec^3(c+dx)(8a\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*((Sec[c + d*x]^5*(b + a*Sin[c + d*x]))/5 + (Sec[c
 + d*x]^3*(-b + 8*a*Sin[c + d*x]))/30 + (Sec[c + d*x]*(-8*a^2*b + 5*b^3 + 3
2*a^3*Sin[c + d*x] - 29*a*b^2*Sin[c + d*x]))/(60*(a^2 - b^2))))/d - (b*((-2
*(8*a^2*b - 5*b^3)*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a +
b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((32*a^3 - 29*a*b^2)*(
(2*(a + b)*EllipticE[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*a*EllipticF[(-c + Pi/2 - d*
x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c +
 d*x]]))/b))/(120*(a - b)*(a + b)*d)
```

Maple [B] time = 0.843, size = 1519, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/120*(2*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*b*(32*a^4-37*a^2*
b^2+5*b^4)*sin(d*x+c)*cos(d*x+c)^4+4*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c
)^2)^(1/2)*b*(8*a^4-9*a^2*b^2+b^4)*cos(d*x+c)^2*sin(d*x+c)+24*(cos(d*x+c)^2
*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*b*(a^4-b^4)*sin(d*x+c)-2*(cos(d*x+c)^2*
sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b^2*(32*a^2-29*b^2)*cos(d*x+c)^6-2*(co
s(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*(32*(-b/(a-b)*sin(d*x+c)-b/(a
-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/
2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2
)*a^4*b-24*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2
)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^3*b^2-37*(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2
))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)
*a^2*b^3+24*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+
c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a*b^4+5*(-b/(a-b)*sin(d*x+c)-b/(a-b)
)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))
*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b
^5-32*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b
/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^5+61*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)
*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^3*b^2-
29*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
```

```
*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a
+b)*sin(d*x+c)+b/(a+b))^(1/2)*a*b^4-8*a^3*b^2+8*a*b^4)*cos(d*x+c)^4+4*(cos(
d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b^2*(a^2-b^2)*cos(d*x+c)^2+48
*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a^3*b^2-48*(cos(d*x+c)^2*
sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b^4)/(1+sin(d*x+c))^2/(a-b)/(-(a+b*sin
(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(a+b)/(sin(d*x+c)-1)^2/b/cos(
d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*sec(d*x + c)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec(dx+c)^6\sin(dx+c)+a\sec(dx+c)^6\right)\sqrt{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^6*sin(d*x + c) + a*sec(d*x + c)^6)*sqrt(b*sin(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.494 $\int \cos^5(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=154

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{11/2}}{11b^5d}-\frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{9/2}}{9b^5d}+\frac{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{7/2}}{7b^5d}+\frac{2(a+b\sin}{15}$$

```
[Out] (2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^(7/2))/(7*b^5*d) - (8*a*(a^2 - b^2)*(
a + b*Sin[c + d*x])^(9/2))/(9*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x]
)^(11/2))/(11*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(13/2))/(13*b^5*d) + (2*(a
 + b*Sin[c + d*x])^(15/2))/(15*b^5*d)
```

Rubi [A] time = 0.119945, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{11/2}}{11b^5d}-\frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{9/2}}{9b^5d}+\frac{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{7/2}}{7b^5d}+\frac{2(a+b\sin}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] (2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^(7/2))/(7*b^5*d) - (8*a*(a^2 - b^2)*(
a + b*Sin[c + d*x])^(9/2))/(9*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x]
)^(11/2))/(11*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(13/2))/(13*b^5*d) + (2*(a
 + b*Sin[c + d*x])^(15/2))/(15*b^5*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\text{Subst}\left(\int (a+x)^{5/2}\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2 (a+x)^{5/2} - 4\left(a^3-ab^2\right)(a+x)^{7/2} + 2\left(3a^2-b^2\right)(a+x)\right.\right.}{b^5 d}$$
$$= \frac{2\left(a^2-b^2\right)^2 (a+b\sin(c+dx))^{7/2}}{7b^5 d} - \frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{9/2}}{9b^5 d} + \frac{4\left(3a\right.}{}$$

Mathematica [A] time = 0.572357, size = 113, normalized size = 0.73

$$\frac{2(a+b\sin(c+dx))^{7/2}\left(8190\left(3a^2-b^2\right)(a+b\sin(c+dx))^2+6435\left(a^2-b^2\right)^2+3003(a+b\sin(c+dx))^4-13860a(a+\right.}{45045b^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(7/2)*(6435*(a^2 - b^2)^2 - 20020*a*(a - b)*(a + b)
*(a + b*Sin[c + d*x]) + 8190*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^2 - 13860*a
*(a + b*Sin[c + d*x])^3 + 3003*(a + b*Sin[c + d*x])^4))/(45045*b^5*d)
```

Maple [A] time = 0.454, size = 126, normalized size = 0.8

$$\frac{6006\,b^4(\cos(dx+c))^4+3696\,ab^3(\cos(dx+c))^2\sin(dx+c)-2016\,a^2b^2(\cos(dx+c))^2+4368\,b^4(\cos(dx+c))^2-89}{45045\,b^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/45045/b^5*(a+b*sin(d*x+c))^(7/2)*(3003*b^4*cos(d*x+c)^4+1848*a*b^3*cos(d*
x+c)^2*sin(d*x+c)-1008*a^2*b^2*cos(d*x+c)^2+2184*b^4*cos(d*x+c)^2-448*a^3*b
*sin(d*x+c)+1792*a*b^3*sin(d*x+c)+128*a^4-32*a^2*b^2+1248*b^4)/d
```

Maxima [A] time = 0.954272, size = 157, normalized size = 1.02

$$\frac{2\left(3003\,(b\sin(dx+c)+a)^{\frac{15}{2}}-13860\,(b\sin(dx+c)+a)^{\frac{13}{2}}a+8190\left(3\,a^2-b^2\right)(b\sin(dx+c)+a)^{\frac{11}{2}}-20020\left(a^3-ab^2\right)(b}{45045\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(b*sin(d*x + c) + a)^(15/2) - 13860*(b*sin(d*x + c) + a)^(13/
2)*a + 8190*(3*a^2 - b^2)*(b*sin(d*x + c) + a)^(11/2) - 20020*(a^3 - a*b^2)
*(b*sin(d*x + c) + a)^(9/2) + 6435*(a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c)
+ a)^(7/2))/(b^5*d)
```

Fricas [A] time = 2.61458, size = 554, normalized size = 3.6

$$-\frac{2\left(7161\,ab^6\cos(dx+c)^6-128\,a^7+992\,a^5b^2-6080\,a^3b^4-5536\,ab^6-7\left(5\,a^3b^4+79\,ab^6\right)\cos(dx+c)^4+16\left(3\,a^5b^2-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/45045*(7161*a*b^6*cos(d*x + c)^6 - 128*a^7 + 992*a^5*b^2 - 6080*a^3*b^4
- 5536*a*b^6 - 7*(5*a^3*b^4 + 79*a*b^6)*cos(d*x + c)^4 + 16*(3*a^5*b^2 - 20
*a^3*b^4 - 67*a*b^6)*cos(d*x + c)^2 + (3003*b^7*cos(d*x + c)^6 + 64*a^6*b -
 480*a^4*b^3 - 9088*a^2*b^5 - 1248*b^7 - 63*(71*a^2*b^5 + 13*b^7)*cos(d*x +
 c)^4 - 8*(5*a^4*b^3 + 718*a^2*b^5 + 117*b^7)*cos(d*x + c)^2)*sin(d*x + c))
*sqrt(b*sin(d*x + c) + a)/(b^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^5, x)
```

3.495 $\int \cos^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=83

$$-\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{7/2}}{7b^3d}-\frac{2(a+b\sin(c+dx))^{11/2}}{11b^3d}+\frac{4a(a+b\sin(c+dx))^{9/2}}{9b^3d}$$

```
[Out] (-2*(a^2 - b^2)*(a + b*Sin[c + d*x])^(7/2))/(7*b^3*d) + (4*a*(a + b*Sin[c +
 d*x])^(9/2))/(9*b^3*d) - (2*(a + b*Sin[c + d*x])^(11/2))/(11*b^3*d)
```

Rubi [A] time = 0.0910357, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$-\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{7/2}}{7b^3d}-\frac{2(a+b\sin(c+dx))^{11/2}}{11b^3d}+\frac{4a(a+b\sin(c+dx))^{9/2}}{9b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)*(a + b*Sin[c + d*x])^(7/2))/(7*b^3*d) + (4*a*(a + b*Sin[c +
 d*x])^(9/2))/(9*b^3*d) - (2*(a + b*Sin[c + d*x])^(11/2))/(11*b^3*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\text{Subst}\left(\int (a+x)^{5/2}\left(b^2-x^2\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^{5/2}+2a(a+x)^{7/2}-(a+x)^{9/2}\right)dx, x, b\sin(c+dx\right.}{b^3d}$$
$$= -\frac{2\left(a^2-b^2\right)(a+b\sin(c+dx))^{7/2}}{7b^3d}+\frac{4a(a+b\sin(c+dx))^{9/2}}{9b^3d}-\frac{2(a+b\sin(c\right.}{11b^3d}$$

Mathematica [A] time = 0.0858061, size = 58, normalized size = 0.7

$$-\frac{2(a+b\sin(c+dx))^{7/2}\left(8a^2-28ab\sin(c+dx)+63b^2\sin^2(c+dx)-99b^2\right)}{693b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a + b*Sin[c + d*x])^(7/2)*(8*a^2 - 99*b^2 - 28*a*b*Sin[c + d*x] + 63*b
^2*Sin[c + d*x]^2))/(693*b^3*d)
```

Maple [A] time = 0.224, size = 55, normalized size = 0.7

$$-\frac{-126\,b^2\left(\cos\left(dx+c\right)\right)^2-56\,ab\sin\left(dx+c\right)+16\,a^2-72\,b^2}{693\,b^3d}\left(a+b\sin\left(dx+c\right)\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/693/b^3*(a+b*sin(d*x+c))^(7/2)*(-63*b^2*cos(d*x+c)^2-28*a*b*sin(d*x+c)+8
*a^2-36*b^2)/d
```

Maxima [A] time = 0.951021, size = 82, normalized size = 0.99

$$-\frac{2\left(63\left(b\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}-154\left(b\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a+99\left(a^2-b^2\right)\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}\right)}{693\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/693*(63*(b*sin(d*x + c) + a)^(11/2) - 154*(b*sin(d*x + c) + a)^(9/2)*a +
 99*(a^2 - b^2)*(b*sin(d*x + c) + a)^(7/2))/(b^3*d)
```

Fricas [B] time = 2.25444, size = 342, normalized size = 4.12

$$-\frac{2\left(161\,ab^4\cos\left(dx+c\right)^4+8\,a^5-96\,a^3b^2-136\,ab^4-\left(3\,a^3b^2+25\,ab^4\right)\cos\left(dx+c\right)^2+\left(63\,b^5\cos\left(dx+c\right)^4-4\,a^4b-184\right.\right.}{693\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/693*(161*a*b^4*cos(d*x + c)^4 + 8*a^5 - 96*a^3*b^2 - 136*a*b^4 - (3*a^3*
b^2 + 25*a*b^4)*cos(d*x + c)^2 + (63*b^5*cos(d*x + c)^4 - 4*a^4*b - 184*a^2
*b^3 - 36*b^5 - (113*a^2*b^3 + 27*b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(b
*sin(d*x + c) + a)/(b^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

3.496 $\int \cos(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a+b\sin(c+dx))^{7/2}}{7bd}$$

```
[Out] (2*(a + b*Sin[c + d*x])^(7/2))/(7*b*d)
```

Rubi [A] time = 0.0374452, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 32}

$$\frac{2(a+b\sin(c+dx))^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(7/2))/(7*b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{5/2}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{2(a+b\sin(c+dx))^{7/2}}{7bd}\end{aligned}$$

Mathematica [A] time = 0.0283785, size = 24, normalized size = 1.

$$\frac{2(a+b\sin(c+dx))^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(7/2))/(7*b*d)
```

Maple [A] time = 0.007, size = 21, normalized size = 0.9

$$\frac{2}{7\,bd}\left(a+b\sin\left(dx+c\right)\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/7*(a+b*sin(d*x+c))^(7/2)/b/d
```

Maxima [A] time = 0.933294, size = 27, normalized size = 1.12

$$\frac{2\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*sin(d*x + c) + a)^(7/2)/(b*d)
```

Fricas [B] time = 2.11921, size = 176, normalized size = 7.33

$$-\frac{2\left(3\,ab^{2}\cos\left(dx+c\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(dx+c\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/7*(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*
b - b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

3.497 $\int \sec(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=117

$$-\frac{2b(a+b\sin(c+dx))^{3/2}}{3d}-\frac{4ab\sqrt{a+b\sin(c+dx)}}{d}-\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] -(((a - b)^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + ((a +
b)^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d - (4*a*b*Sqrt[a +
 b*Sin[c + d*x]])/d - (2*b*(a + b*Sin[c + d*x])^(3/2))/(3*d)
```

Rubi [A] time = 0.232252, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2668, 704, 825, 827, 1166, 206}

$$-\frac{2b(a+b\sin(c+dx))^{3/2}}{3d}-\frac{4ab\sqrt{a+b\sin(c+dx)}}{d}-\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] -(((a - b)^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + ((a +
b)^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d - (4*a*b*Sqrt[a +
 b*Sin[c + d*x]])/d - (2*b*(a + b*Sin[c + d*x])^(3/2))/(3*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 704

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*
d^2 - a*e^2 + 2*c*d*e*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x]
&& NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 1]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b\,\text{Subst}\left(\int \frac{(a+x)^{5/2}}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{2b(a+b\sin(c+dx))^{3/2}}{3d} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a+x}\left(-a^2-b^2-2ax\right)}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{4ab\sqrt{a+b\sin(c+dx)}}{d} - \frac{2b(a+b\sin(c+dx))^{3/2}}{3d} + \frac{b\,\text{Subst}\left(\int \frac{a\left(a^2+3b^2\right)+\left(3a^2\right.}{\sqrt{a+x}\left(b^2-x^2\right.}\right.}{d}$$

$$= -\frac{4ab\sqrt{a+b\sin(c+dx)}}{d} - \frac{2b(a+b\sin(c+dx))^{3/2}}{3d} + \frac{(2b)\,\text{Subst}\left(\int \frac{-a\left(3a^2+b^2\right)+}{-a^2}\right.}{}$$

$$= -\frac{4ab\sqrt{a+b\sin(c+dx)}}{d} - \frac{2b(a+b\sin(c+dx))^{3/2}}{3d} - \frac{(a-b)^3\,\text{Subst}\left(\int \frac{1}{a-b-x^2}\right.}{d}$$

$$= -\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d} - \frac{4ab\sqrt{a}}{}$$

Mathematica [A] time = 0.155927, size = 105, normalized size = 0.9

$$\frac{-2b\sqrt{a+b\sin(c+dx)}(7a+b\sin(c+dx)) - 3(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right) + 3(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-3*(a - b)^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]] + 3*(a + b)
^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] - 2*b*Sqrt[a + b*Sin[c
 + d*x]]*(7*a + b*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 0.368, size = 312, normalized size = 2.7

$$-\frac{2\,b}{3\,d}\left(a+b\sin\left(dx+c\right)\right)^{\frac{3}{2}} - 4\,\frac{ab\sqrt{a+b\sin\left(dx+c\right)}}{d} + \frac{a^3}{d}\arctan\left(\sqrt{a+b\sin\left(dx+c\right)}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}} - 3\,\frac{a^2b}{d\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3*b*(a+b*sin(d*x+c))^(3/2)/d-4*a*b*(a+b*sin(d*x+c))^(1/2)/d+1/d/(-a+b)^(
1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^3-3/d*b/(-a+b)^(1/2)*arc
tan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2+3/d*b^2/(-a+b)^(1/2)*arctan((a
+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a-1/d*b^3/(-a+b)^(1/2)*arctan((a+b*sin(d
*x+c))^(1/2)/(-a+b)^(1/2))+1/d/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(
a+b)^(1/2))*a^3+3/d*b/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2
))*a^2+3/d*b^2/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+1/
d*b^3/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 12.4489, size = 4676, normalized size = 39.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2 + 2*a*b + b^2)*sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^4
+ 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 +
 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^
2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b
^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*
a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c
))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c)
 + 8)) + 3*(a^2 - 2*a*b + b^2)*sqrt(a - b)*log((b^4*cos(d*x + c)^4 + 128*a^
4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3
 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*
```

```
b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8
*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 11
2*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x +
 c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x +
c) + 8)) - 16*(b^2*sin(d*x + c) + 7*a*b)*sqrt(b*sin(d*x + c) + a))/d, -1/24
*(6*(a^2 + 2*a*b + b^2)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^
2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a
)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c
)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c))) - 3*(a^2 - 2*a*b + b^2)*sqrt
(a - b)*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a
*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^
3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*
cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x
+ c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*
a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x +
c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(b^2*sin(d*x + c) + 7
*a*b)*sqrt(b*sin(d*x + c) + a))/d, -1/24*(6*(a^2 - 2*a*b + b^2)*sqrt(-a + b
)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2
)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*
a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(
d*x + c))) - 3*(a^2 + 2*a*b + b^2)*sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 12
8*a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a
*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (1
0*a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2
 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b
+ 112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d
*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*
x + c) + 8)) + 16*(b^2*sin(d*x + c) + 7*a*b)*sqrt(b*sin(d*x + c) + a))/d, -
1/12*(3*(a^2 - 2*a*b + b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8
*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c)
+ a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x
+ c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c))) + 3*(a^2 + 2*a*b + b^2)*s
qrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*
a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3
*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2
+ b^3)*sin(d*x + c))) + 8*(b^2*sin(d*x + c) + 7*a*b)*sqrt(b*sin(d*x + c) +
a))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.498 $\int \sec^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=155

$$\frac{ab\sqrt{a+b\sin(c+dx)}}{2d}-\frac{(a-b)^{3/2}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d}+\frac{(2a-3b)(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d}+\frac{\sec^2(c}{}$$

```
[Out] -((a - b)^(3/2)*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/
(4*d) + ((2*a - 3*b)*(a + b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a
+ b]])/(4*d) + (a*b*Sqrt[a + b*Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(b +
a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(2*d)
```

Rubi [A] time = 0.269716, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2668, 739, 825, 827, 1166, 206}

$$\frac{ab\sqrt{a+b\sin(c+dx)}}{2d}-\frac{(a-b)^{3/2}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d}+\frac{(2a-3b)(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d}+\frac{\sec^2(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a - b)^(3/2)*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/
(4*d) + ((2*a - 3*b)*(a + b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a
+ b]])/(4*d) + (a*b*Sqrt[a + b*Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(b +
a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(2*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b^3\,\mathrm{Subst}\left(\int \frac{(a+x)^{5/2}}{\left(b^2-x^2\right)^2}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{2d} - \frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt{a+x}\left(\frac{1}{2}\left(-2a^2\right.\right.}{b^2-}\right.}{}$$

$$= \frac{ab\sqrt{a+b\sin(c+dx)}}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{2d}$$

$$= \frac{ab\sqrt{a+b\sin(c+dx)}}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{2d}$$

$$= \frac{ab\sqrt{a+b\sin(c+dx)}}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{2d}$$

$$= -\frac{(a-b)^{3/2}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d} + \frac{(2a-3b)(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+}}{}\right.}{4d}$$

Mathematica [A] time = 0.824758, size = 147, normalized size = 0.95

$$\frac{-\sqrt{a-b}\left(2a^2+ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)+\sqrt{a+b}\left(2a^2-ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+2\sec^2(c+dx)\sqrt{a-}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-(Sqrt[a - b]*(2*a^2 + a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[
a - b]]) + Sqrt[a + b]*(2*a^2 - a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x
]]/Sqrt[a + b]] + 2*Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]]*(2*a*b + (a^2 +
 b^2)*Sin[c + d*x]))/(4*d)
```

Maple [B] time = 0.446, size = 356, normalized size = 2.3

$$\frac{1}{4\left(\cos(dx+c)\right)^2 d}\left(2\sin(dx+c)\sqrt{a+b\sin(dx+c)}\sqrt{-a+b}\sqrt{a+b}\left(a^2+b^2\right)-\left(-2\,\mathrm{Artanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)a^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/4*(2*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*(a^2+b^2)
-(-2*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^3*(-a+b)^(1/2)-b*arctanh
((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2*(-a+b)^(1/2)+4*b^2*arctanh((a+b*si
n(d*x+c))^(1/2)/(a+b)^(1/2))*a*(-a+b)^(1/2)+3*b^3*arctanh((a+b*sin(d*x+c))^
(1/2)/(a+b)^(1/2))*(-a+b)^(1/2)-2*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2
))*a^3*(a+b)^(1/2)+b*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2*(a+b)^
(1/2)+4*b^2*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a*(a+b)^(1/2)-3*b^3
*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*(a+b)^(1/2))*cos(d*x+c)^2+4*(a
+b*sin(d*x+c))^(1/2)*b*a*(-a+b)^(1/2)*(a+b)^(1/2))/(-a+b)^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^2/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.17225, size = 4980, normalized size = 32.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((2*a^2 - a*b - 3*b^2)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x +
 c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*
b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2 +
8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b
- 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*
(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 -
 2)*sin(d*x + c) + 8)) + (2*a^2 + a*b - 3*b^2)*sqrt(a - b)*cos(d*x + c)^2*l
```

og((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72
b^4 - 8(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 - 24*a^
2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x +
 c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)
sqrt(a - b) + 4(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7
*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*
(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(2*a*b + (a^2 + b^2)*sin(d*x +
 c))*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^2), -1/32*(2*(2*a^2 - a*b -
3*b^2)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2
 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2
*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b +
4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^2 + a*b - 3*b^2)*sqrt(a
 - b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^
2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)
^2 + 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x +
 c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sq
rt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 -
 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 -
 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(2*a*b +
 (a^2 + b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^2), -1
/32*(2*(2*a^2 + a*b - 3*b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 -
8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c)
 + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x
 + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^2
- a*b - 3*b^2)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4
 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3
+ 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b
^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*
b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112
*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x +
c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c
) + 8)) - 16*(2*a*b + (a^2 + b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(
d*cos(d*x + c)^2), -1/16*((2*a^2 + a*b - 3*b^2)*sqrt(-a + b)*arctan(1/4*(b^
2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*
sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a
*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d
*x + c)^2 + (2*a^2 - a*b - 3*b^2)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c
)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*
x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*c
os(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 - 8
*(2*a*b + (a^2 + b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x +
c)^2)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.499 $\int \sec^5(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=199

$$-\frac{3\sqrt{a-b}\left(4a^2+2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d}+\frac{3\sqrt{a+b}\left(4a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d}+3\sec^2(c+dx)$$

```
[Out] (-3*Sqrt[a - b]*(4*a^2 + 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt
[a - b]])/(32*d) + (3*Sqrt[a + b]*(4*a^2 - 2*a*b - b^2)*ArcTanh[Sqrt[a + b*
Sin[c + d*x]]/Sqrt[a + b]])/(32*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x])*(
a + b*Sin[c + d*x])^(3/2))/(4*d) + (3*Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x
]]*(a*b + (2*a^2 - b^2)*Sin[c + d*x]))/(16*d)
```

Rubi [A] time = 0.282019, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2668, 739, 821, 827, 1166, 206}

$$-\frac{3\sqrt{a-b}\left(4a^2+2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d}+\frac{3\sqrt{a+b}\left(4a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d}+3\sec^2(c+dx)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-3*Sqrt[a - b]*(4*a^2 + 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt
[a - b]])/(32*d) + (3*Sqrt[a + b]*(4*a^2 - 2*a*b - b^2)*ArcTanh[Sqrt[a + b*
Sin[c + d*x]]/Sqrt[a + b]])/(32*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x])*(
a + b*Sin[c + d*x])^(3/2))/(4*d) + (3*Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x
]]*(a*b + (2*a^2 - b^2)*Sin[c + d*x]))/(16*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
```

```
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b^5\,\mathrm{Subst}\left(\int \frac{(a+x)^{5/2}}{\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{4d} - \frac{b^3\,\mathrm{Subst}\left(\int \frac{\sqrt{a+x}\left(-\frac{3}{2}(2a\right.}{\left(b^2-\right.}\right.}{}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{4d} + \frac{3\sec^2(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{4d} + \frac{3\sec^2(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{4d} + \frac{3\sec^2(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= -\frac{3\sqrt{a-b}\left(4a^2+2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d} + \frac{3\sqrt{a+b}\left(4a^2-2ab-b^2\right.}{32}$$

<u>Mathematica</u> [A] time = 3.26178, size = 307, normalized size = 1.54

$$-\frac{3\sqrt{a-b}\left(a^2-b^2\right)^2\left(4a^2+2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)-3\sqrt{a+b}\left(a^2-b^2\right)^2\left(4a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx}}{\sqrt{a+b}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(3*Sqrt[a - b]*(a^2 - b^2)^2*(4*a^2 + 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[
c + d*x]]/Sqrt[a - b]] - 3*Sqrt[a + b]*(a^2 - b^2)^2*(4*a^2 - 2*a*b - b^2)*
ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + 8*(-a^2 + b^2)*Sec[c + d*x]
^4*(-b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(7/2) - 2*Sec[c + d*x]^2*(-7*
a^2*b + b^3 + 6*a^3*Sin[c + d*x])*(a + b*Sin[c + d*x])^(7/2) - 2*b*Sqrt[a +
 b*Sin[c + d*x]]*(18*a^5 - 16*a^3*b^2 + 7*a*b^4 - 3*a^3*b^2*Cos[2*(c + d*x)
] + b*(18*a^4 - 7*a^2*b^2 + b^4)*Sin[c + d*x]))/(32*(a^2 - b^2)^2*d)
```

Maple [B] time = 0.809, size = 538, normalized size = 2.7

$$\frac{1}{32\,b\,(\cos\,(dx+c))^4\,d}\left(4\,\sqrt{a+b\sin\,(dx+c)}\sqrt{-a+b}\sqrt{a+b}b\left(3\,ab\,(\cos\,(dx+c))^2+8\,a^2\sin\,(dx+c)-b^2\sin\,(dx+c)-3\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/32*(4*(a+b*sin(d*x+c))^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*b*(3*a*b*cos(d*x+c)
^2+8*a^2*sin(d*x+c)-b^2*sin(d*x+c)-3*a*b)+3*b*(4*arctanh((a+b*sin(d*x+c))^(
1/2)/(a+b)^(1/2))*a^3*(-a+b)^(1/2)+2*b*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)
^(1/2))*a^2*(-a+b)^(1/2)-3*b^2*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*
a*(-a+b)^(1/2)-b^3*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*(-a+b)^(1/2)
+4*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^3*(a+b)^(1/2)-2*b*arctan((
a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2*(a+b)^(1/2)-3*b^2*arctan((a+b*sin(d
*x+c))^(1/2)/(-a+b)^(1/2))*a*(a+b)^(1/2)+b^3*arctan((a+b*sin(d*x+c))^(1/2)/
(-a+b)^(1/2))*(a+b)^(1/2))*cos(d*x+c)^4+2*(a+b*sin(d*x+c))^(1/2)*(-a+b)^(1/
2)*(a+b)^(1/2)*b*(6*a^2*sin(d*x+c)-3*b^2*sin(d*x+c)-7*a*b)*cos(d*x+c)^2-24*
(a+b*sin(d*x+c))^(3/2)*a^2*(-a+b)^(1/2)*(a+b)^(1/2)+12*(a+b*sin(d*x+c))^(3/
2)*b^2*(-a+b)^(1/2)*(a+b)^(1/2)+24*(a+b*sin(d*x+c))^(1/2)*a^3*(-a+b)^(1/2)*
(a+b)^(1/2)+16*a*(a+b*sin(d*x+c))^(1/2)*b^2*(-a+b)^(1/2)*(a+b)^(1/2))/(-a+b
)^(1/2)/(a+b)^(1/2)/b/cos(d*x+c)^4/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.74365, size = 5315, normalized size = 26.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/256*(3*(4*a^2 - 2*a*b - b^2)*sqrt(a + b)*cos(d*x + c)^4*log((b^4*cos(d*
x + c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a
^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2
 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2
*b - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) +
 4*(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x
+ c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^
2 - 2)*sin(d*x + c) + 8)) + 3*(4*a^2 + 2*a*b - b^2)*sqrt(a - b)*cos(d*x + c
)^4*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3
 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 -
24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(
d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c)
 + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^
3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2
 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a*b*cos(d*x + c)^2 - 8*a
*b - (3*(2*a^2 - b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^2)*sin(d*x + c))*sqrt(b*
sin(d*x + c) + a))/(d*cos(d*x + c)^4), -1/256*(6*(4*a^2 - 2*a*b - b^2)*sqrt
(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b
 + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^
2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b
^3)*sin(d*x + c)))*cos(d*x + c)^4 + 3*(4*a^2 + 2*a*b - b^2)*sqrt(a - b)*cos
(d*x + c)^4*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 2
56*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(1
6*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (
b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(
d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 -
 (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*
x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a*b*cos(d*x + c)
^2 - 8*a*b - (3*(2*a^2 - b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^2)*sin(d*x + c))
*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^4), -1/256*(6*(4*a^2 + 2*a*b - b
^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2
*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3
 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*
b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^4 + 3*(4*a^2 - 2*a*b - b^2)*sqrt(a +
 b)*cos(d*x + c)^4*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*
b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2
 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c
)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt
(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 1
4*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8
*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a*b*cos(d
*x + c)^2 - 8*a*b - (3*(2*a^2 - b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^2)*sin(d*
```

```
x + c))*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^4), -1/128*(3*(4*a^2 + 2*
a*b - b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*
b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)
/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b
 - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^4 + 3*(4*a^2 - 2*a*b - b^2)*s
qrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*
a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3
*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2
+ b^3)*sin(d*x + c)))*cos(d*x + c)^4 + 8*(a*b*cos(d*x + c)^2 - 8*a*b - (3*(
2*a^2 - b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^2)*sin(d*x + c))*sqrt(b*sin(d*x +
 c) + a))/(d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**(5/2),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out
```

3.500 $\int \cos^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=398

$$\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(7b\left(53a^2+11b^2\right)\sin(c+dx)+a\left(5a^2+59b^2\right)\right)}{3003bd}-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(4a$$

```
[Out] (-32*a*b*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(143*d) - (2*b*Cos[c + d*
x]^5*(a + b*Sin[c + d*x])^(3/2))/(13*d) - (8*(20*a^6 - 175*a^4*b^2 - 1662*a
^2*b^4 - 231*b^6)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(15015*b^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (32*a*(5*a
^6 - 45*a^4*b^2 - 53*a^2*b^4 + 93*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/
(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15015*b^4*d*Sqrt[a + b*Sin[c
+ d*x]]) + (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(a*(5*a^2 + 59*b^2) +
 7*b*(53*a^2 + 11*b^2)*Sin[c + d*x]))/(3003*b*d) - (4*Cos[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]]*(4*a*(5*a^4 - 40*a^2*b^2 - 93*b^4) - 3*b*(5*a^4 + 430*a^2*
b^2 + 77*b^4)*Sin[c + d*x]))/(15015*b^3*d)
```

Rubi [A] time = 0.9365, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2692, 2862, 2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(7b\left(53a^2+11b^2\right)\sin(c+dx)+a\left(5a^2+59b^2\right)\right)}{3003bd}-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(4a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-32*a*b*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(143*d) - (2*b*Cos[c + d*
x]^5*(a + b*Sin[c + d*x])^(3/2))/(13*d) - (8*(20*a^6 - 175*a^4*b^2 - 1662*a
^2*b^4 - 231*b^6)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(15015*b^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (32*a*(5*a
^6 - 45*a^4*b^2 - 53*a^2*b^4 + 93*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/
(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15015*b^4*d*Sqrt[a + b*Sin[c
+ d*x]]) + (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(a*(5*a^2 + 59*b^2) +
 7*b*(53*a^2 + 11*b^2)*Sin[c + d*x]))/(3003*b*d) - (4*Cos[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]]*(4*a*(5*a^4 - 40*a^2*b^2 - 93*b^4) - 3*b*(5*a^4 + 430*a^2*
b^2 + 77*b^4)*Sin[c + d*x]))/(15015*b^3*d)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{2b\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d} + \frac{2}{13}\int \cos^4(c+dx)\sqrt{a+b\sin(c+dx)}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d}$$

Mathematica [A] time = 1.2141, size = 321, normalized size = 0.81

$$128\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\left(b\left(-1450a^3b^3+5a^5b-603ab^5\right)F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\left(-175a^4b^2-1662a^2b^4+20a^6-231b^6\right)\left((a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (128*(b*(5*a^5*b - 1450*a^3*b^3 - 603*a*b^5)*EllipticF[(-2*c + Pi - 2*d*x)/
4, (2*b)/(a + b)] + (20*a^6 - 175*a^4*b^2 - 1662*a^2*b^4 - 231*b^6)*((a + b
)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - a*EllipticF[(-2*c + Pi
- 2*d*x)/4, (2*b)/(a + b)]))*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - b*(a + b*
Sin[c + d*x])*(4*a*(320*a^4 - 2710*a^2*b^2 + 6453*b^4)*Cos[c + d*x] - 10*a*
b^2*(20*a^2 - 2599*b^2)*Cos[3*(c + d*x)] + 5670*a*b^4*Cos[5*(c + d*x)] - b*
(480*a^4 + 56120*a^2*b^2 + 4697*b^4)*Sin[2*(c + d*x)] + 140*b^3*(-53*a^2 +
22*b^2)*Sin[4*(c + d*x)] + 1155*b^5*Sin[6*(c + d*x)]))/(240240*b^4*d*Sqrt[a
```

```
 + b*Sin[c + d*x]])
```

Maple [B] time = 0.595, size = 1619, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/15015*(-345*a^4*b^4+1796*a^2*b^6+40*a^6*b^2+4236*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6+924*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*b^8-80*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^8-924*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*b^8+3080*b^8*sin(d*x+c)^6-2233*b^8*sin(d*x+
c)^4+308*b^8*sin(d*x+c)^2-1155*b^8*sin(d*x+c)^8-4690*a^2*b^6*sin(d*x+c)^6-1
880*a^3*b^5*sin(d*x+c)^5+11290*a*b^7*sin(d*x+c)^5+5*a^4*b^4*sin(d*x+c)^4+14
500*a^2*b^6*sin(d*x+c)^4-10*a^5*b^3*sin(d*x+c)^3+6660*a^3*b^5*sin(d*x+c)^3-
9404*a*b^7*sin(d*x+c)^3-40*a^6*b^2*sin(d*x+c)^2+340*a^4*b^4*sin(d*x+c)^2-11
606*a^2*b^6*sin(d*x+c)^2+10*a^5*b^3*sin(d*x+c)-4780*a^3*b^5*sin(d*x+c)+2104
*a*b^7*sin(d*x+c)-3990*a*b^7*sin(d*x+c)^7+1488*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^7+780*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*
b^2+5948*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a^4*b^4-5724*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6+80*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*b-60*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b
^2-720*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a^5*b^3-5100*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
```

```
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4-848*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^5)/b^5/cos(d*x+c
)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2\cos(dx+c)^6-2\,ab\cos(dx+c)^4\sin(dx+c)-\left(a^2+b^2\right)\cos(dx+c)^4\right)\sqrt{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^6 - 2*a*b*cos(d*x + c)^4*sin(d*x + c) - (a^2 +
b^2)*cos(d*x + c)^4)*sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

3.501 $\int \cos^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=299

$$\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(3b\left(25a^2+7b^2\right)\sin(c+dx)+a\left(5a^2+27b^2\right)\right)}{315bd}-\frac{4a\left(22a^2b^2+5a^4-27b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{315b^2d\sqrt{a+b\sin(c-}}$$

```
[Out] (-8*a*b*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(21*d) - (2*b*Cos[c + d*x]
^3*(a + b*Sin[c + d*x])^(3/2))/(9*d) + (4*(5*a^4 + 102*a^2*b^2 + 21*b^4)*El
lipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b
^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (4*a*(5*a^4 + 22*a^2*b^2 - 27*b^
4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(315*b^2*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]]*(a*(5*a^2 + 27*b^2) + 3*b*(25*a^2 + 7*b^2)*Sin[c + d*x]))/(31
5*b*d)
```

Rubi [A] time = 0.669294, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2692, 2862, 2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(3b\left(25a^2+7b^2\right)\sin(c+dx)+a\left(5a^2+27b^2\right)\right)}{315bd}-\frac{4a\left(22a^2b^2+5a^4-27b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{315b^2d\sqrt{a+b\sin(c-}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-8*a*b*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(21*d) - (2*b*Cos[c + d*x]
^3*(a + b*Sin[c + d*x])^(3/2))/(9*d) + (4*(5*a^4 + 102*a^2*b^2 + 21*b^4)*El
lipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b
^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (4*a*(5*a^4 + 22*a^2*b^2 - 27*b^
4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(315*b^2*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]]*(a*(5*a^2 + 27*b^2) + 3*b*(25*a^2 + 7*b^2)*Sin[c + d*x]))/(31
5*b*d)
```


Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
```

```
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{2b\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9d} + \frac{2}{9}\int \cos^2(c+dx)\sqrt{a+b\sin(c+dx)}\,\Big($$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9d} +$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9d} +$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9d} +$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9d} +$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9d} +$$

Mathematica [A] time = 0.991456, size = 239, normalized size = 0.8

$$b(a+b\sin(c+dx))\left(\left(40a^3-354ab^2\right)\cos(c+dx)+2b\left(\sin(2(c+dx))\left(150a^2-35b^2\cos(2(c+dx))+7b^2\right)-95ab\cos(3($$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-16*(16*b*(5*a^3*b + 3*a*b^3)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a +
b)] + (5*a^4 + 102*a^2*b^2 + 21*b^4)*((a + b)*EllipticE[(-2*c + Pi - 2*d*x)
/4, (2*b)/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]))*Sq
rt[(a + b*Sin[c + d*x])/(a + b)] + b*(a + b*Sin[c + d*x])*((40*a^3 - 354*a*
b^2)*Cos[c + d*x] + 2*b*(-95*a*b*Cos[3*(c + d*x)] + (150*a^2 + 7*b^2 - 35*b
^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])))/(1260*b^2*d*Sqrt[a + b*Sin[c + d*x
]])
```

Maple [B] time = 0.488, size = 1190, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/315*(-35*b^6*sin(d*x+c)^6-130*a*b^5*sin(d*x+c)^5+10*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b+150*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*a^4*b^2+44*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^3*b^3-108*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-54*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5-42*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b
^6-10*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^6-194*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2+162*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+42*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
```

```
2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6-170*a^
2*b^4*sin(d*x+c)^4+49*b^6*sin(d*x+c)^4-80*a^3*b^3*sin(d*x+c)^3+212*a*b^5*si
n(d*x+c)^3-5*a^4*b^2*sin(d*x+c)^2+238*a^2*b^4*sin(d*x+c)^2-14*b^6*sin(d*x+c
)^2+80*a^3*b^3*sin(d*x+c)-82*a*b^5*sin(d*x+c)+5*a^4*b^2-68*a^2*b^4)/b^3/cos
(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b^2\cos(dx+c)^4-2\,ab\cos(dx+c)^2\sin(dx+c)-\left(a^2+b^2\right)\cos(dx+c)^2\right)\sqrt{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2*sin(d*x + c) - (a^2 +
b^2)*cos(d*x + c)^2)*sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

3.502 $\int \sec^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=203

$$\frac{a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}-\frac{\left(a^2+3b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{ab\cos(c+dx)}{}$$

```
[Out] (a*b*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/d + (Sec[c + d*x]*(b + a*Sin[c
+ d*x])*(a + b*Sin[c + d*x])^(3/2))/d - ((a^2 + 3*b^2)*EllipticE[(c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[(a + b*Sin[c + d
*x])/(a + b)]) + (a*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.280267, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2691, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}-\frac{\left(a^2+3b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{ab\cos(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a*b*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/d + (Sec[c + d*x]*(b + a*Sin[c
+ d*x])*(a + b*Sin[c + d*x])^(3/2))/d - ((a^2 + 3*b^2)*EllipticE[(c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[(a + b*Sin[c + d
*x])/(a + b)]) + (a*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
```

```
|| IntegerQ[m])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{d} - \int \sqrt{a+b\sin(c+dx)}\left(\frac{3b}{2}\right.$$

$$= \frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+}{d}$$

$$= \frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+}{d}$$

$$= \frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+}{d}$$

$$= \frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+}{d}$$

Mathematica [A] time = 0.867851, size = 203, normalized size = 1.

$$\frac{-a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\left(a^2b+a^3+3ab^2+3b^3\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+}{d\sqrt{a+b\sin(c+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*b*Sec[c + d*x] + (a^3 + a^2*b + 3*a*b^2 + 3*b^3)*EllipticE[(-2*c + P
i - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - a*(a^2 -
b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*
x])/(a + b)] + a^3*Tan[c + d*x] + 3*a*b^2*Tan[c + d*x] + a^2*b*Sin[c + d*x]
*Tan[c + d*x] + b^3*Sin[c + d*x]*Tan[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.685, size = 1039, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/b*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*((-b/(a-b)*sin(d*x+c)-
b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a
-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(
1/2))*a^4+2*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b
))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)
+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^2-3*(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))
*b^4-EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-
b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b-3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-
b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a
+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^2+
EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-
b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2)*a*b^3+3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*s
in(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b^4-a^2*b^2*c
os(d*x+c)^2-b^4*cos(d*x+c)^2+a^3*b*sin(d*x+c)+3*a*b^3*sin(d*x+c)+3*a^2*b^2+
b^4)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/cos(d*x+c)/(a+
b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(2\,ab\sec(dx+c)^2\sin(dx+c)-\left(b^2\cos(dx+c)^2-a^2-b^2\right)\sec(dx+c)^2\right)\sqrt{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*sec(d*x + c)^2*sin(d*x + c) - (b^2*cos(d*x + c)^2 - a^2 - b
^2)*sec(d*x + c)^2)*sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.503 $\int \sec^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=238

$$\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(\left(4a^2-3b^2\right)\sin(c+dx)+ab\right)}{6d}+\frac{2a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\sin(c+dx)}}-(4a$$

```
[Out] (Sec[c + d*x]^3*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(3*d) - ((
4*a^2 - 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[
c + d*x]])/(6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (2*a*(a^2 - b^2)*Elli
pticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]
)/(3*d*Sqrt[a + b*Sin[c + d*x]]) + (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(
a*b + (4*a^2 - 3*b^2)*Sin[c + d*x]))/(6*d)
```

Rubi [A] time = 0.393696, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2691, 2861, 2752, 2663, 2661, 2655, 2653}

$$\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(\left(4a^2-3b^2\right)\sin(c+dx)+ab\right)}{6d}+\frac{2a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\sin(c+dx)}}-(4a$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^3*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(3*d) - ((
4*a^2 - 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[
c + d*x]])/(6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (2*a*(a^2 - b^2)*Elli
pticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]
)/(3*d*Sqrt[a + b*Sin[c + d*x]]) + (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(
a*b + (4*a^2 - 3*b^2)*Sin[c + d*x]))/(6*d)
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
```

```
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} - \frac{1}{3}\int \sec^2(c+dx)\sqrt{a+$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} + \frac{\sec(c+dx)\sqrt{a+b\sin}}{}$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} + \frac{\sec(c+dx)\sqrt{a+b\sin}}{}$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} + \frac{\sec(c+dx)\sqrt{a+b\sin}}{}$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} - \frac{(4a^2-3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{6d\sqrt{}}$$

Mathematica [A] time = 3.42777, size = 259, normalized size = 1.09

$$\frac{-4a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\left(4a^2b+4a^3-3ab^2-3b^3\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)
/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 4*a*(a^2 - b^2)*EllipticF[(-
2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + (S
ec[c + d*x]^3*(40*a^2*b + 5*b^3 - 4*(3*a^2*b + 2*b^3)*Cos[2*(c + d*x)] + (-
4*a^2*b + 3*b^3)*Cos[4*(c + d*x)] + 24*a^3*Sin[c + d*x] + 40*a*b^2*Sin[c +
d*x] + 8*a^3*Sin[3*(c + d*x)] - 8*a*b^2*Sin[3*(c + d*x)]))/8)/(6*d*Sqrt[a +
 b*Sin[c + d*x]])
```

Maple [B] time = 0.605, size = 1249, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/6*(-4*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b*(a^2-b^2)*sin(
d*x+c)*cos(d*x+c)^2-2*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b*
(a^2+3*b^2)*sin(d*x+c)+(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*b^2
*(4*a^2-3*b^2)*cos(d*x+c)^4+(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2
)*(4*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-
b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b-3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-
b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a
+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^2-
4*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(
a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)
*sin(d*x+c)+1/(a-b)*a)^(1/2)*a*b^3+3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*
a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b^4-4*(-b/(
a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)
*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
,((a-b)/(a+b))^(1/2))*a^4+7*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*s
in(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/
(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^2-3*(-b/(a-b)*
sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-
b)/(a+b))^(1/2))*b^4-a^2*b^2+5*b^4)*cos(d*x+c)^2-6*(cos(d*x+c)^2*sin(d*x+c)
*b+a*cos(d*x+c)^2)^(1/2)*a^2*b^2-2*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^
2)^(1/2)*b^4)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(1+si
n(d*x+c))/(sin(d*x+c)-1)/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(2\,ab\sec\left(dx+c\right)^{4}\sin\left(dx+c\right)-\left(b^{2}\cos\left(dx+c\right)^{2}-a^{2}-b^{2}\right)\sec\left(dx+c\right)^{4}\right)\sqrt{b\sin\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*sec(d*x + c)^4*sin(d*x + c) - (b^2*cos(d*x + c)^2 - a^2 - b
^2)*sec(d*x + c)^4)*sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.504 $\int \sec^6(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=322

$$\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(\left(8a^2-3b^2\right)\sin(c+dx)+5ab\right)}{30d}-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(8ab\left(a^2-b^2\right)-\left(-41a^2b\right.\right.}{60d\left(a^2-b^2\right)}$$

```
[Out] (Sec[c + d*x]^5*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(5*d) - ((
32*a^2 - 9*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin
[c + d*x]])/(60*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (a*(32*a^2 - 17*b^2
)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/(60*d*Sqrt[a + b*Sin[c + d*x]]) + (Sec[c + d*x]^3*Sqrt[a + b*Sin[c
+ d*x]]*(5*a*b + (8*a^2 - 3*b^2)*Sin[c + d*x]))/(30*d) - (Sec[c + d*x]*Sqrt
[a + b*Sin[c + d*x]]*(8*a*b*(a^2 - b^2) - (32*a^4 - 41*a^2*b^2 + 9*b^4)*Sin
[c + d*x]))/(60*(a^2 - b^2)*d)
```

Rubi [A] time = 0.672409, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2691, 2861, 2866, 2752, 2663, 2661, 2655, 2653}

$$\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(\left(8a^2-3b^2\right)\sin(c+dx)+5ab\right)}{30d}-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(8ab\left(a^2-b^2\right)-\left(-41a^2b\right.\right.}{60d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^(5/2),x]
```



```
[Out] (Sec[c + d*x]^5*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(5*d) - ((
32*a^2 - 9*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin
[c + d*x]])/(60*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (a*(32*a^2 - 17*b^2
)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/(60*d*Sqrt[a + b*Sin[c + d*x]]) + (Sec[c + d*x]^3*Sqrt[a + b*Sin[c
+ d*x]]*(5*a*b + (8*a^2 - 3*b^2)*Sin[c + d*x]))/(30*d) - (Sec[c + d*x]*Sqrt
[a + b*Sin[c + d*x]]*(8*a*b*(a^2 - b^2) - (32*a^4 - 41*a^2*b^2 + 9*b^4)*Sin
[c + d*x]))/(60*(a^2 - b^2)*d)
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
```

```
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
```

```
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} - \frac{1}{5}\int \sec^4(c+dx)\sqrt{a+b}$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} + \frac{\sec^3(c+dx)\sqrt{a+b\sin(}}{}$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} + \frac{\sec^3(c+dx)\sqrt{a+b\sin(}}{}$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} + \frac{\sec^3(c+dx)\sqrt{a+b\sin(}}{}$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} + \frac{\sec^3(c+dx)\sqrt{a+b\sin(}}{}$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} - \frac{\left(32a^2-9b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{60d\sqrt{}}$$

Mathematica [A] time = 6.25912, size = 351, normalized size = 1.09

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{1}{5}\sec^5(c+dx)\left(a^2\sin(c+dx)+2ab+b^2\sin(c+dx)\right)+\frac{1}{30}\sec^3(c+dx)\left(8a^2\sin(c+dx)-ab-3b^2\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*((Sec[c + d*x]*(-8*a*b + 32*a^2*Sin[c + d*x] - 9*
b^2*Sin[c + d*x]))/60 + (Sec[c + d*x]^3*(-(a*b) + 8*a^2*Sin[c + d*x] - 3*b^
2*Sin[c + d*x]))/30 + (Sec[c + d*x]^5*(2*a*b + a^2*Sin[c + d*x] + b^2*Sin[c
 + d*x]))/5))/d - (b*((-16*a*b*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((32*a^2 -
 9*b^2)*((2*(a + b)*EllipticE[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a +
 b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*a*EllipticF[(-c +
Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a +
b*Sin[c + d*x]]))/b))/(120*d)
```

Maple [B] time = 0.66, size = 1360, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/60*((cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b*(32*a^2-17*b^2)*
sin(d*x+c)*cos(d*x+c)^4+8*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*
a*b*(2*a^2-b^2)*cos(d*x+c)^2*sin(d*x+c)+12*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos
(d*x+c)^2)^(1/2)*a*b*(a^2+3*b^2)*sin(d*x+c)-(cos(d*x+c)^2*sin(d*x+c)*b+a*co
s(d*x+c)^2)^(1/2)*b^2*(32*a^2-9*b^2)*cos(d*x+c)^6+(cos(d*x+c)^2*sin(d*x+c)*
b+a*cos(d*x+c)^2)^(1/2)*(32*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*s
in(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/
(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4-41*(-b/(a-b)*sin
(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x
+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/
(a+b))^(1/2))*a^2*b^2+9*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d
```

```
*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b
)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^4-32*EllipticF((b/(a-b
)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(
a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2)*a^3*b+24*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a
+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b
))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^2+17*EllipticF((b/(a-b)
*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a
-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*
a)^(1/2)*a*b^3-9*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b
))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))
^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b^4+8*a^2*b^2-3*b^4)*cos(d*x+c)
^4+2*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*b^2*(a^2-9*b^2)*cos(d
*x+c)^2+36*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a^2*b^2+12*(cos
(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*b^4)/(1+sin(d*x+c))^2/(-(a+b*s
in(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(sin(d*x+c)-1)^2/b/cos(d*x+
c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*sec(d*x + c)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(2\,ab\sec(dx+c)^6\sin(dx+c)-\left(b^2\cos(dx+c)^2-a^2-b^2\right)\sec(dx+c)^6\right)\sqrt{b\sin(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*sec(d*x + c)^6*sin(d*x + c) - (b^2*cos(d*x + c)^2 - a^2 - b
^2)*sec(d*x + c)^6)*sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.505 $\int \sec^8(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=439

$$\frac{3\sec^5(c+dx)\sqrt{a+b\sin(c+dx)}\left(\left(4a^2-b^2\right)\sin(c+dx)+3ab\right)}{70d}-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab\left(a^2-b^2\right)-\left(-39a\right.\right.}{140d\left(a^2-b^2\right)}$$

```
[Out] (Sec[c + d*x]^7*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(7*d) - ((
128*a^4 - 144*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[a + b*Sin[c + d*x]])/(280*(a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)]) + (2*a*(8*a^2 - 3*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]
*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(35*d*Sqrt[a + b*Sin[c + d*x]]) + (3*S
ec[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]]*(3*a*b + (4*a^2 - b^2)*Sin[c + d*x])
)/(70*d) - (Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(4*a*b*(a^2 - b^2) - (3
2*a^4 - 39*a^2*b^2 + 7*b^4)*Sin[c + d*x]))/(140*(a^2 - b^2)*d) - (Sec[c + d
*x]*Sqrt[a + b*Sin[c + d*x]]*(a*b*(32*a^4 - 59*a^2*b^2 + 27*b^4) - (128*a^6
 - 272*a^4*b^2 + 165*a^2*b^4 - 21*b^6)*Sin[c + d*x]))/(280*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.943791, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2691, 2861, 2866, 2752, 2663, 2661, 2655, 2653}

$$\frac{3\sec^5(c+dx)\sqrt{a+b\sin(c+dx)}\left(\left(4a^2-b^2\right)\sin(c+dx)+3ab\right)}{70d}-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab\left(a^2-b^2\right)-\left(-39a\right.\right.}{140d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^7*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(7*d) - ((
128*a^4 - 144*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[a + b*Sin[c + d*x]])/(280*(a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)]) + (2*a*(8*a^2 - 3*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]
*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(35*d*Sqrt[a + b*Sin[c + d*x]]) + (3*S
ec[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]]*(3*a*b + (4*a^2 - b^2)*Sin[c + d*x])
)/(70*d) - (Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(4*a*b*(a^2 - b^2) - (3
2*a^4 - 39*a^2*b^2 + 7*b^4)*Sin[c + d*x]))/(140*(a^2 - b^2)*d) - (Sec[c + d
*x]*Sqrt[a + b*Sin[c + d*x]]*(a*b*(32*a^4 - 59*a^2*b^2 + 27*b^4) - (128*a^6
 - 272*a^4*b^2 + 165*a^2*b^4 - 21*b^6)*Sin[c + d*x]))/(280*(a^2 - b^2)^2*d)
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} - \frac{1}{7}\int \sec^6(c+dx)\sqrt{a+$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} - \frac{(128a^4-144a^2b^2+21}{28}$$

Mathematica [A] time = 4.45949, size = 338, normalized size = 0.77

$$\frac{\frac{\sec(c+dx)(a+b\sin(c+dx))\left(-144a^2b^2\sin(c+dx)+40\left(a^2-b^2\right)\sec^6(c+dx)\left(\left(a^2+b^2\right)\sin(c+dx)+2ab\right)-4\left(a^2-b^2\right)\sec^4(c+dx)\left(3\left(b^2-4a^2\right)\sin(c+dx)+ab\right)+2\left(a^2-b^2\right)\mathrm{sec}}{a^2-b^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((((128*a^4 - 144*a^2*b^2 + 21*b^4)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/
(a + b)] - 16*a*(8*a^3 - 8*a^2*b - 3*a*b^2 + 3*b^3)*EllipticF[(-2*c + Pi -
2*d*x)/4, (2*b)/(a + b)])*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a - b) + (Se
c[c + d*x]*(a + b*Sin[c + d*x])*(-32*a^3*b + 27*a*b^3 + 128*a^4*Sin[c + d*x
] - 144*a^2*b^2*Sin[c + d*x] + 21*b^4*Sin[c + d*x] + 2*(a^2 - b^2)*Sec[c +
d*x]^2*(-4*a*b + (32*a^2 - 7*b^2)*Sin[c + d*x]) - 4*(a^2 - b^2)*Sec[c + d*x
]^4*(a*b + 3*(-4*a^2 + b^2)*Sin[c + d*x]) + 40*(a^2 - b^2)*Sec[c + d*x]^6*(
```

```
2*a*b + (a^2 + b^2)*Sin[c + d*x]))/(a^2 - b^2))/(280*d*Sqrt[a + b*Sin[c +
d*x]])
```

Maple [B] time = 6.417, size = 1888, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/280*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)/cos(d*x+c)^9/(a+b*sin(d*x+c))
^(3/2)/b/(a^2-b^2)*(2*cos(d*x+c)^4*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^
2)^(1/2)*b^2*(4*a^4-5*a^2*b^2+b^4)+40*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+
c)^2)^(1/2)*b^2*(3*a^4-2*a^2*b^2-b^4)+4*cos(d*x+c)^2*(cos(d*x+c)^2*sin(d*x+
c)*b+a*cos(d*x+c)^2)^(1/2)*b^2*(a^4-14*a^2*b^2+13*b^4)-cos(d*x+c)^8*(cos(d*
x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*b^2*(128*a^4-144*a^2*b^2+21*b^4)+
16*cos(d*x+c)^2*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b*(3*a^4
-4*a^2*b^2+b^4)*sin(d*x+c)+40*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1
/2)*a*b*(a^4+2*a^2*b^2-3*b^4)*sin(d*x+c)+16*cos(d*x+c)^6*(cos(d*x+c)^2*sin(
d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b*(8*a^4-11*a^2*b^2+3*b^4)*sin(d*x+c)+2*co
s(d*x+c)^4*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b*(32*a^4-43*
a^2*b^2+11*b^4)*sin(d*x+c)-cos(d*x+c)^6*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*
x+c)^2)^(1/2)*(128*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c
)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b
))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^5*b-96*(b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))
^(1/2)*a^4*b^2-176*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c
)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b
))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^3*b^3+117*(b/(a-b)*sin(d*x+
c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*
sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*a^2*b^4+48*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x
+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a
+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a*b^5-21*(b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b
))^(1/2)*b^6-128*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+
b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d
*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^6+272*(b/(a-b)*sin(d*x+c)+1/(
a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a
```

```
-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/
2))*a^4*b^2-165*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b
/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*
x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+21*(b/(a-b)*sin(d*x+c)+1
/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/
(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(
1/2))*b^6-32*a^4*b^2+39*a^2*b^4-7*b^6))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sin (dx+c)+a)^{\frac{5}{2}} \sec (dx+c)^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*sec(d*x + c)^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(2\,ab \sec (dx+c)^8 \sin (dx+c) - \left(b^2 \cos (dx+c)^2 - a^2 - b^2\right) \sec (dx+c)^8\right) \sqrt{b \sin (dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((2*a*b*sec(d*x + c)^8*sin(d*x + c) - (b^2*cos(d*x + c)^2 - a^2 - b
^2)*sec(d*x + c)^8)*sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.506 $\int \frac{\cos^5(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=152

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^5d}-\frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^5d}+\frac{2\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}{b^5d}+\frac{2(a+b\sin(c}{9b^5}$$

```
[Out] (2*(a^2 - b^2)^2*Sqrt[a + b*Sin[c + d*x]])/(b^5*d) - (8*a*(a^2 - b^2)*(a +
b*Sin[c + d*x])^(3/2))/(3*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^(5
/2))/(5*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(7/2))/(7*b^5*d) + (2*(a + b*Sin
[c + d*x])^(9/2))/(9*b^5*d)
```

Rubi [A] time = 0.114031, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{5/2}}{5b^5d}-\frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^5d}+\frac{2\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}{b^5d}+\frac{2(a+b\sin(c}{9b^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a^2 - b^2)^2*Sqrt[a + b*Sin[c + d*x]])/(b^5*d) - (8*a*(a^2 - b^2)*(a +
b*Sin[c + d*x])^(3/2))/(3*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^(5
/2))/(5*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(7/2))/(7*b^5*d) + (2*(a + b*Sin
[c + d*x])^(9/2))/(9*b^5*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{\mathrm{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{\sqrt{a+x}}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\mathrm{Subst}\left(\int \left(\frac{\left(a^2-b^2\right)^2}{\sqrt{a+x}} - 4\left(a^3-ab^2\right)\sqrt{a+x} + 2\left(3a^2-b^2\right)(a+x)^{3/2} - 4a(a+x)^{5/2} + (a+x)^{7/2}\right)\right.}{b^5 d}$$

$$= \frac{2\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}{b^5 d} - \frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^5 d} + \frac{4\left(3a^2-b^2\right)(a+b\sin(\right.}{5b^5 d}$$

Mathematica [A] time = 0.288291, size = 118, normalized size = 0.78

$$\frac{\sqrt{a+b\sin(c+dx)}\left(-4\left(48a^2b^2-91b^4\right)\cos(2(c+dx)) - 2496a^2b^2 - 512a^3b\sin(c+dx) + 1024a^4 + 1104ab^3\sin(c+dx) +\right.}{1260b^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*(1024*a^4 - 2496*a^2*b^2 + 2121*b^4 - 4*(48*a^2*b
^2 - 91*b^4)*Cos[2*(c + d*x)] + 35*b^4*Cos[4*(c + d*x)] - 512*a^3*b*Sin[c +
 d*x] + 1104*a*b^3*Sin[c + d*x] + 80*a*b^3*Sin[3*(c + d*x)]))/(1260*b^5*d)
```

Maple [A] time = 0.241, size = 126, normalized size = 0.8

$$\frac{70\,b^4(\cos(dx+c))^4 + 80\,ab^3(\cos(dx+c))^2\sin(dx+c) - 96\,a^2b^2(\cos(dx+c))^2 + 112\,b^4(\cos(dx+c))^2 - 128\,a^3b\sin(d}{315\,b^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/315/b^5*(a+b*sin(d*x+c))^(1/2)*(35*b^4*cos(d*x+c)^4+40*a*b^3*cos(d*x+c)^2
*sin(d*x+c)-48*a^2*b^2*cos(d*x+c)^2+56*b^4*cos(d*x+c)^2-64*a^3*b*sin(d*x+c)
+128*a*b^3*sin(d*x+c)+128*a^4-288*a^2*b^2+224*b^4)/d
```

Maxima [A] time = 0.981467, size = 216, normalized size = 1.42

$$\frac{2\left(315\sqrt{b\sin(dx+c)+a} - \frac{42\left(3\,(b\sin(dx+c)+a)^{\frac{5}{2}} - 10\,(b\sin(dx+c)+a)^{\frac{3}{2}}a + 15\sqrt{b\sin(dx+c)+a}\,a^2\right)}{b^2} + \frac{35\,(b\sin(dx+c)+a)^{\frac{9}{2}} - 180\,(b\sin(dx+c)+a)^{\frac{7}{2}}a + \cdots}{\cdots}\right.}{315\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(b*sin(d*x + c) + a) - 42*(3*(b*sin(d*x + c) + a)^(5/2) - 10
*(b*sin(d*x + c) + a)^(3/2)*a + 15*sqrt(b*sin(d*x + c) + a)*a^2)/b^2 + (35*
(b*sin(d*x + c) + a)^(9/2) - 180*(b*sin(d*x + c) + a)^(7/2)*a + 378*(b*sin(
d*x + c) + a)^(5/2)*a^2 - 420*(b*sin(d*x + c) + a)^(3/2)*a^3 + 315*sqrt(b*s
in(d*x + c) + a)*a^4)/b^4)/(b*d)
```

Fricas [A] time = 2.13668, size = 270, normalized size = 1.78

$$\frac{2\left(35\,b^4\cos(dx+c)^4 + 128\,a^4 - 288\,a^2b^2 + 224\,b^4 - 8\left(6\,a^2b^2 - 7\,b^4\right)\cos(dx+c)^2 + 8\left(5\,ab^3\cos(dx+c)^2 - 8\,a^3b + 16\cdots\right.\right.}{315\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*cos(d*x + c)^4 + 128*a^4 - 288*a^2*b^2 + 224*b^4 - 8*(6*a^2*b
^2 - 7*b^4)*cos(d*x + c)^2 + 8*(5*a*b^3*cos(d*x + c)^2 - 8*a^3*b + 16*a*b^3
)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2114, size = 217, normalized size = 1.43

$$2\left(35\left(b\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}-180\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a+378\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a^2-420\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a^3+315\sqrt{\vphantom{x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(b*sin(d*x + c) + a)^(9/2) - 180*(b*sin(d*x + c) + a)^(7/2)*a + 3
78*(b*sin(d*x + c) + a)^(5/2)*a^2 - 420*(b*sin(d*x + c) + a)^(3/2)*a^3 + 31
5*sqrt(b*sin(d*x + c) + a)*a^4 - 126*(b*sin(d*x + c) + a)^(5/2)*b^2 + 420*(
b*sin(d*x + c) + a)^(3/2)*a*b^2 - 630*sqrt(b*sin(d*x + c) + a)*a^2*b^2 + 31
5*sqrt(b*sin(d*x + c) + a)*b^4)/(b^5*d)
```

3.507 $\int \frac{\cos^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=81

$$-\frac{2\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^3d}-\frac{2(a+b\sin(c+dx))^{5/2}}{5b^3d}+\frac{4a(a+b\sin(c+dx))^{3/2}}{3b^3d}$$

```
[Out] (-2*(a^2 - b^2)*Sqrt[a + b*Sin[c + d*x]])/(b^3*d) + (4*a*(a + b*Sin[c + d*x
])^(3/2))/(3*b^3*d) - (2*(a + b*Sin[c + d*x])^(5/2))/(5*b^3*d)
```

Rubi [A] time = 0.0848283, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$-\frac{2\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^3d}-\frac{2(a+b\sin(c+dx))^{5/2}}{5b^3d}+\frac{4a(a+b\sin(c+dx))^{3/2}}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*(a^2 - b^2)*Sqrt[a + b*Sin[c + d*x]])/(b^3*d) + (4*a*(a + b*Sin[c + d*x
])^(3/2))/(3*b^3*d) - (2*(a + b*Sin[c + d*x])^(5/2))/(5*b^3*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{\sqrt{a+x}}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{\sqrt{a+x}}+2a\sqrt{a+x}-(a+x)^{3/2}\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= -\frac{2\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^3d}+\frac{4a(a+b\sin(c+dx))^{3/2}}{3b^3d}-\frac{2(a+b\sin(c+dx))^{5/2}}{5b^3d}$$

Mathematica [A] time = 0.0756197, size = 58, normalized size = 0.72

$$\frac{2\sqrt{a+b\sin(c+dx)}\left(-8a^2+4ab\sin(c+dx)-3b^2\sin^2(c+dx)+15b^2\right)}{15b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Sin[c + d*x]]*(-8*a^2 + 15*b^2 + 4*a*b*Sin[c + d*x] - 3*b^2*S
in[c + d*x]^2))/(15*b^3*d)
```

Maple [A] time = 0.169, size = 55, normalized size = 0.7

$$-\frac{-6\,b^2\left(\cos(dx+c)\right)^2-8\,ab\sin(dx+c)+16\,a^2-24\,b^2}{15\,b^3d}\sqrt{a+b\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/15/b^3*(a+b*sin(d*x+c))^(1/2)*(-3*b^2*cos(d*x+c)^2-4*a*b*sin(d*x+c)+8*a^
2-12*b^2)/d
```

Maxima [A] time = 0.966539, size = 101, normalized size = 1.25

$$\frac{2\left(15\sqrt{b\sin(dx+c)+a}-\frac{3\,(b\sin(dx+c)+a)^{\frac{5}{2}}-10\,(b\sin(dx+c)+a)^{\frac{3}{2}}a+15\,\sqrt{b\sin(dx+c)+a}a^2}{b^2}\right)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(b*sin(d*x + c) + a) - (3*(b*sin(d*x + c) + a)^(5/2) - 10*(b*s
in(d*x + c) + a)^(3/2)*a + 15*sqrt(b*sin(d*x + c) + a)*a^2)/b^2)/(b*d)
```

Fricas [A] time = 1.98215, size = 135, normalized size = 1.67

$$\frac{2\left(3\,b^2\cos\left(dx+c\right)^2+4\,ab\sin\left(dx+c\right)-8\,a^2+12\,b^2\right)\sqrt{b\sin\left(dx+c\right)+a}}{15\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*cos(d*x + c)^2 + 4*a*b*sin(d*x + c) - 8*a^2 + 12*b^2)*sqrt(b*si
n(d*x + c) + a)/(b^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0996, size = 97, normalized size = 1.2

$$-\frac{2\left(3\,(b\sin\left(dx+c\right)+a)^{\frac{5}{2}}-10\,(b\sin\left(dx+c\right)+a)^{\frac{3}{2}}a+15\,\sqrt{b\sin\left(dx+c\right)+a}a^2-15\,\sqrt{b\sin\left(dx+c\right)+a}b^2\right)}{15\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(3*(b*sin(d*x + c) + a)^(5/2) - 10*(b*sin(d*x + c) + a)^(3/2)*a + 15*
sqrt(b*sin(d*x + c) + a)*a^2 - 15*sqrt(b*sin(d*x + c) + a)*b^2)/(b^3*d)
```

3.508 $\int \frac{\cos(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=22

$$\frac{2\sqrt{a+b\sin(c+dx)}}{bd}$$

```
[Out] (2*Sqrt[a + b*Sin[c + d*x]])/(b*d)
```

Rubi [A] time = 0.0353723, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 32}

$$\frac{2\sqrt{a+b\sin(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Sin[c + d*x]])/(b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+x}}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{2\sqrt{a+b\sin(c+dx)}}{bd}$$

Mathematica [A] time = 0.0134846, size = 22, normalized size = 1.

$$\frac{2\sqrt{a+b\sin(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Sin[c + d*x]])/(b*d)
```

Maple [A] time = 0.005, size = 21, normalized size = 1.

$$2\,\frac{\sqrt{a+b\sin(dx+c)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2*(a+b*sin(d*x+c))^(1/2)/b/d
```

Maxima [A] time = 0.964226, size = 27, normalized size = 1.23

$$\frac{2\sqrt{b\sin(dx+c)+a}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*sin(d*x + c) + a)/(b*d)
```

Fricas [A] time = 1.95857, size = 46, normalized size = 2.09

$$\frac{2\sqrt{b\sin\left(dx+c\right)+a}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*sin(d*x + c) + a)/(b*d)
```

Sympy [A] time = 1.02259, size = 54, normalized size = 2.45

$$\begin{cases} \frac{x\cos\left(c\right)}{\sqrt{a}} & \text{for}\ b=0 \wedge d=0 \\ \frac{x\cos\left(c\right)}{\sqrt{a+b\sin\left(c\right)}} & \text{for}\ d=0 \\ \frac{\sin\left(c+dx\right)}{\sqrt{a}d} & \text{for}\ b=0 \\ \frac{2\sqrt{a+b\sin\left(c+dx\right)}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((x*cos(c)/sqrt(a), Eq(b, 0) & Eq(d, 0)), (x*cos(c)/sqrt(a + b*sin
(c)), Eq(d, 0)), (sin(c + d*x)/(sqrt(a)*d), Eq(b, 0)), (2*sqrt(a + b*sin(c
+ d*x))/(b*d), True))
```

Giac [A] time = 1.09504, size = 27, normalized size = 1.23

$$\frac{2\sqrt{b\sin\left(dx+c\right)+a}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*sin(d*x + c) + a)/(b*d)
```

3.509 $\int \frac{\sec(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh[
Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rubi [A] time = 0.0942083, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2668, 708, 1093, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh[
Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+x}\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}\\
&= \frac{(2b)\,\text{Subst}\left(\int \frac{1}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d}+\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}
\end{aligned}$$

Mathematica [A] time = 0.0544054, size = 74, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh[
Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Maple [A] time = 0.263, size = 62, normalized size = 0.8

$$\frac{1}{d}\arctan\left(\sqrt{a+b\sin(dx+c)}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}+\frac{1}{d}\mathrm{Artanh}\left(\sqrt{a+b\sin(dx+c)}\frac{1}{\sqrt{a+b}}\right)\frac{1}{\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/d/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))+arctanh((a+b*s
in(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec(dx+c)}{\sqrt{b\sin(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.08022, size = 101, normalized size = 1.36

$$\frac{b\left(\frac{\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}b}-\frac{\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] b*(arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a + b))/(sqrt(-a + b)*b) - arctan(
sqrt(b*sin(d*x + c) + a)/sqrt(-a - b))/(sqrt(-a - b)*b))/d
```

3.510 $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=144

$$-\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d\left(a^2-b^2\right)}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d(a+b)^{3/2}}$$

```
[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(3/
2)*d) + ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*(a +
 b)^(3/2)*d) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]
])/(2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.305468, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2668, 741, 827, 1166, 206}

$$-\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d\left(a^2-b^2\right)}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(3/
2)*d) + ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*(a +
 b)^(3/2)*d) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]
])/(2*(a^2 - b^2)*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
```

```
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{1}{\sqrt{a+x}\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2\left(a^2-b^2\right)d} + \frac{b\,\text{Subst}\left(\int \frac{\frac{1}{2}\left(2a^2-3b^2\right)+\frac{ax}{2}}{\sqrt{a+x}\left(b^2-x^2\right)}\,dx, x, b\sin(c\right.}{2\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2\left(a^2-b^2\right)d} + \frac{b\,\text{Subst}\left(\int \frac{-\frac{a^2}{2}+\frac{1}{2}\left(2a^2-3b^2\right)+\frac{ax^2}{2}}{-a^2+b^2+2ax^2-x^4}\,dx, x, \right.}{\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2\left(a^2-b^2\right)d} - \frac{(2a-3b)\,\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+}\right.}{4(a-b)d}$$

$$= -\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d} + \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{3/2}d} - \frac{\sec^2(c+dx)(b-a\,s}{2}$$

Mathematica [A] time = 0.493575, size = 176, normalized size = 1.22

$$\frac{\sqrt{a+b}\left(2a^2-ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)-\sqrt{a-b}\left(\left(2a^2+ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+2\sqrt{a+b}\sec^2(c+d\right.}{4d\sqrt{a-b}\sqrt{a+b}\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a + b]*(2*a^2 - a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a
- b]] - Sqrt[a - b]*((2*a^2 + a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]
/Sqrt[a + b]] + 2*Sqrt[a + b]*Sec[c + d*x]^2*(-b + a*Sin[c + d*x])*Sqrt[a +
 b*Sin[c + d*x]]))/(4*Sqrt[a - b]*Sqrt[a + b]*(-a^2 + b^2)*d)
```

Maple [A] time = 0.487, size = 218, normalized size = 1.5

$$-\frac{b}{4\,d\,(a-b)\,(b\sin(dx+c)+b)}\sqrt{a+b\sin(dx+c)}+\frac{a}{2\,d\,(a-b)}\arctan\left(\sqrt{a+b\sin(dx+c)}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}-\frac{3}{4\,d\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d*b/(a-b)*(a+b*sin(d*x+c))^(1/2)/(b*sin(d*x+c)+b)+1/2/d/(a-b)/(-a+b)^(
1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a-3/4/d/(a-b)/(-a+b)^(1/2)
*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*b-1/4/d*b/(a+b)*(a+b*sin(d*x+c
))^(1/2)/(b*sin(d*x+c)-b)+1/2/d/(a+b)^(3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/
(a+b)^(1/2))*a+3/4/d/(a+b)^(3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2)
)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{3}(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.10636, size = 286, normalized size = 1.99

$$\frac{b^3\left(\frac{(2a-3b)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{(ab^3-b^4)\sqrt{-a+b}}-\frac{(2a+3b)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{(ab^3+b^4)\sqrt{-a-b}}-\frac{2\left((b\sin(dx+c)+a)^{\frac{3}{2}}a-\sqrt{b\sin(dx+c)+a}\,a^2-\sqrt{b\sin(dx+c)+a}\,b^2\right)}{(a^2b^2-b^4)\left((b\sin(dx+c)+a)^2-2\,(b\sin(dx+c)+a)a+a^2-b^2\right)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b^3*((2*a - 3*b)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a + b))/((a*b^3
- b^4)*sqrt(-a + b)) - (2*a + 3*b)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a
- b))/((a*b^3 + b^4)*sqrt(-a - b)) - 2*((b*sin(d*x + c) + a)^(3/2)*a - sqrt
(b*sin(d*x + c) + a)*a^2 - sqrt(b*sin(d*x + c) + a)*b^2)/((a^2*b^2 - b^4)*(
(b*sin(d*x + c) + a)^2 - 2*(b*sin(d*x + c) + a)*a + a^2 - b^2)))/d
```

3.511 $\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=230

$$-\frac{3\left(4a^2-10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{5/2}}+\frac{3\left(4a^2+10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{5/2}}-\frac{\sec^4(c+dx)(b-a\sin(c+}{4d\left(a^2\right.}$$

```
[Out] (-3*(4*a^2 - 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])
/(32*(a - b)^(5/2)*d) + (3*(4*a^2 + 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b*Sin[
c + d*x]]/Sqrt[a + b]])/(32*(a + b)^(5/2)*d) - (Sec[c + d*x]^4*(b - a*Sin[c
 + d*x])*Sqrt[a + b*Sin[c + d*x]])/(4*(a^2 - b^2)*d) - (Sec[c + d*x]^2*Sqrt
[a + b*Sin[c + d*x]]*(b*(a^2 - 7*b^2) - 6*a*(a^2 - 2*b^2)*Sin[c + d*x]))/(1
6*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.389479, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2668, 741, 823, 827, 1166, 206}

$$-\frac{3\left(4a^2-10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{5/2}}+\frac{3\left(4a^2+10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{5/2}}-\frac{\sec^4(c+dx)(b-a\sin(c+}{4d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-3*(4*a^2 - 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])
/(32*(a - b)^(5/2)*d) + (3*(4*a^2 + 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b*Sin[
c + d*x]]/Sqrt[a + b]])/(32*(a + b)^(5/2)*d) - (Sec[c + d*x]^4*(b - a*Sin[c
 + d*x])*Sqrt[a + b*Sin[c + d*x]])/(4*(a^2 - b^2)*d) - (Sec[c + d*x]^2*Sqrt
[a + b*Sin[c + d*x]]*(b*(a^2 - 7*b^2) - 6*a*(a^2 - 2*b^2)*Sin[c + d*x]))/(1
6*(a^2 - b^2)^2*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{b^5\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+x}\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4\left(a^2-b^2\right)d} + \frac{b^3\operatorname{Subst}\left(\int \frac{\frac{1}{2}\left(6a^2-7b^2\right)+\frac{5ax}{2}}{\sqrt{a+x}\left(b^2-x^2\right)^2}\,dx, x, b\sin(c\right.}{4\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(a^2-\right.\right.}{16\left(a^2-b\right.}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(a^2-\right.\right.}{16\left(a^2-b\right.}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(a^2-\right.\right.}{16\left(a^2-b\right.}$$

$$= -\frac{3\left(4a^2-10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{5/2}d} + \frac{3\left(4a^2+10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32(a+b)^{5/2}d}$$

Mathematica [A] time = 1.86918, size = 244, normalized size = 1.06

$$\frac{\sqrt{a-b}\left(3(a-b)^2\left(4a^2+10ab+7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+\sqrt{a+b}\sec^4(c+dx)\sqrt{a+b\sin(c+dx)}\left(a\left(11a^2-14b^2\right)\sin\right.\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-3*(a + b)^(5/2)*(4*a^2 - 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]
/Sqrt[a - b]] + Sqrt[a - b]*(3*(a - b)^2*(4*a^2 + 10*a*b + 7*b^2)*ArcTanh[S
qrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + Sqrt[a + b]*Sec[c + d*x]^4*Sqrt[a +
b*Sin[c + d*x]]*(-9*a^2*b + 15*b^3 + (-(a^2*b) + 7*b^3)*Cos[2*(c + d*x)] +
a*(11*a^2 - 14*b^2)*Sin[c + d*x] + 3*(a^3 - 2*a*b^2)*Sin[3*(c + d*x)])))/(3
2*Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)^2*d)
```

Maple [B] time = 0.816, size = 618, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -3/16/d/(b*sin(d*x+c)+b)^2*b/(a^2-2*a*b+b^2)*(a+b*sin(d*x+c))^(3/2)*a+9/32/
d/(b*sin(d*x+c)+b)^2*b^2/(a^2-2*a*b+b^2)*(a+b*sin(d*x+c))^(3/2)+3/16/d/(b*s
in(d*x+c)+b)^2*b/(a-b)*(a+b*sin(d*x+c))^(1/2)*a-11/32/d/(b*sin(d*x+c)+b)^2*
b^2/(a-b)*(a+b*sin(d*x+c))^(1/2)+3/8/d/(a^2-2*a*b+b^2)/(-a+b)^(1/2)*arctan(
(a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2-15/16/d/(a^2-2*a*b+b^2)/(-a+b)^(1/
2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a*b+21/32/d/(a^2-2*a*b+b^2)/
(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*b^2-3/16/d/(b*sin(
d*x+c)-b)^2*b/(a^2+2*a*b+b^2)*(a+b*sin(d*x+c))^(3/2)*a-9/32/d/(b*sin(d*x+c)
-b)^2*b^2/(a^2+2*a*b+b^2)*(a+b*sin(d*x+c))^(3/2)+3/16/d/(b*sin(d*x+c)-b)^2*
b/(a+b)*(a+b*sin(d*x+c))^(1/2)*a+11/32/d/(b*sin(d*x+c)-b)^2*b^2/(a+b)*(a+b*
sin(d*x+c))^(1/2)+3/8/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c)
)^(1/2)/(a+b)^(1/2))*a^2+15/16/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctanh((a+b*s
in(d*x+c))^(1/2)/(a+b)^(1/2))*a*b+21/32/d/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arcta
nh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13278, size = 566, normalized size = 2.46

$$\frac{b^5\left(\frac{3\left(4a^2-10ab+7b^2\right)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{\left(a^2b^5-2ab^6+b^7\right)\sqrt{-a+b}}-\frac{3\left(4a^2+10ab+7b^2\right)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{\left(a^2b^5+2ab^6+b^7\right)\sqrt{-a-b}}-\frac{2\left(6\left(b\sin(dx+c)+a\right)^{\frac{7}{2}}a^3-18\left(b\sin(dx+c)+a\right)^{\frac{5}{2}}a^4+18\left(b\sin(d\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*b^5*(3*(4*a^2 - 10*a*b + 7*b^2)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-
a + b))/((a^2*b^5 - 2*a*b^6 + b^7)*sqrt(-a + b)) - 3*(4*a^2 + 10*a*b + 7*b^
2)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a - b))/((a^2*b^5 + 2*a*b^6 + b^7)
*sqrt(-a - b)) - 2*(6*(b*sin(d*x + c) + a)^(7/2)*a^3 - 18*(b*sin(d*x + c) +
 a)^(5/2)*a^4 + 18*(b*sin(d*x + c) + a)^(3/2)*a^5 - 6*sqrt(b*sin(d*x + c) +
 a)*a^6 - 12*(b*sin(d*x + c) + a)^(7/2)*a*b^2 + 35*(b*sin(d*x + c) + a)^(5/
2)*a^2*b^2 - 44*(b*sin(d*x + c) + a)^(3/2)*a^3*b^2 + 21*sqrt(b*sin(d*x + c)
 + a)*a^4*b^2 + 7*(b*sin(d*x + c) + a)^(5/2)*b^4 + 2*(b*sin(d*x + c) + a)^(
3/2)*a*b^4 - 4*sqrt(b*sin(d*x + c) + a)*a^2*b^4 - 11*sqrt(b*sin(d*x + c) +
a)*b^6)/((a^4*b^4 - 2*a^2*b^6 + b^8)*((b*sin(d*x + c) + a)^2 - 2*(b*sin(d*x
 + c) + a)*a + a^2 - b^2)^2))/d
```

3.512 $\int \frac{\cos^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=247

$$-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a^2-3ab\sin(c+dx)-5b^2\right)}{35b^3d}+\frac{8\left(-9a^2b^2+4a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right.\right.}{35b^4d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(7*b*d) - (32*a*(a^2 - 2*b^2)*E
llipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(35*b
^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(4*a^4 - 9*a^2*b^2 + 5*b^4)*E
llipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)])/(35*b^4*d*Sqrt[a + b*Sin[c + d*x]]) - (4*Cos[c + d*x]*Sqrt[a + b*Sin[c
 + d*x]]*(4*a^2 - 5*b^2 - 3*a*b*Sin[c + d*x]))/(35*b^3*d)
```

Rubi [A] time = 0.36545, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2695, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a^2-3ab\sin(c+dx)-5b^2\right)}{35b^3d}+\frac{8\left(-9a^2b^2+4a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right.\right.}{35b^4d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(7*b*d) - (32*a*(a^2 - 2*b^2)*E
llipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(35*b
^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(4*a^4 - 9*a^2*b^2 + 5*b^4)*E
llipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)])/(35*b^4*d*Sqrt[a + b*Sin[c + d*x]]) - (4*Cos[c + d*x]*Sqrt[a + b*Sin[c
 + d*x]]*(4*a^2 - 5*b^2 - 3*a*b*Sin[c + d*x]))/(35*b^3*d)
```


Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
```

```
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} + \frac{6\int \frac{\cos^2(c+dx)(b+a\sin(c+dx))}{\sqrt{a+b\sin(c+dx)}}\,dx}{7b}$$

$$= \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a^2-5b^2-3ab\sin(c\right.}{35b^3d}$$

$$= \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a^2-5b^2-3ab\sin(c\right.}{35b^3d}$$

$$= \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(4a^2-5b^2-3ab\sin(c\right.}{35b^3d}$$

$$= \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} - \frac{32a\left(a^2-2b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx}}{35b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.03655, size = 219, normalized size = 0.89

$$\frac{b\cos(c+dx)\left(\left(45b^3-8a^2b\right)\sin(c+dx)-32a^3-2ab^2\cos(2(c+dx))+62ab^2+5b^3\sin(3(c+dx))\right)-16\left(-9a^2b^2+4a^4\right.}{70b^4d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (64*a*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 16*(4*a^4 - 9*a^2*b^2 + 5*b
^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)] + b*Cos[c + d*x]*(-32*a^3 + 62*a*b^2 - 2*a*b^2*Cos[2*(c + d*x)]
 + (-8*a^2*b + 45*b^3)*Sin[c + d*x] + 5*b^3*Sin[3*(c + d*x)]))/(70*b^4*d*Sq
rt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.547, size = 942, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -2/35*(-5*b^5*sin(d*x+c)^5+16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-12*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-36*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*
b^3+12*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a*b^4+20*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*b^5-16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5+48*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-32*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a*b^4+a*b^4*sin(d*x+c)^4-2*a^2*b^3*sin(d*x+c)^3+20*b^5*sin(d*x+c)^3-8*a^3*
b^2*sin(d*x+c)^2+14*a*b^4*sin(d*x+c)^2+2*a^2*b^3*sin(d*x+c)-15*b^5*sin(d*x+
c)+8*a^3*b^2-15*a*b^4)/b^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{4}}{\sqrt{b\sin\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos^{4}\left(c+dx\right)}{\sqrt{a+b\sin\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**4/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(dx+c\right)^{4}}{\sqrt{b\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

3.513 $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=175

$$-\frac{4\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\sin(c+dx)}}+\frac{4a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd}$$

```
[Out] (2*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(3*b*d) + (4*a*EllipticE[(c - Pi/
2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*b^2*d*Sqrt[(a + b*S
in[c + d*x])/(a + b)]) - (4*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)
/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d
*x]])
```

Rubi [A] time = 0.187962, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2695, 2752, 2663, 2661, 2655, 2653}

$$-\frac{4\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\sin(c+dx)}}+\frac{4a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] (2*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(3*b*d) + (4*a*EllipticE[(c - Pi/
2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*b^2*d*Sqrt[(a + b*S
in[c + d*x])/(a + b)]) - (4*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)
/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d
*x]])
```


Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} + \frac{2\int \frac{b+a\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{3b}$$
$$= \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} + \frac{1}{3}\left(2\left(1-\frac{a^2}{b^2}\right)\right)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx + \frac{(2a)\int \sqrt{a+b\,\mathrm{s}}}{3b}$$
$$= \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} + \frac{\left(2a\sqrt{a+b\sin(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}\,dx}{3b^2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{\left(2\left(1-\right.\right.}{}$$
$$= \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} + \frac{4aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{3b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{4\left(1-\frac{a^2}{b^2}\right)}{}$$

Mathematica [A] time = 0.749753, size = 145, normalized size = 0.83

$$\frac{4\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+2b\cos(c+dx)(a+b\sin(c+dx))-4a(a+b)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}E\left(\frac{1}{4}(-2c\right.}{3b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*b*Cos[c + d*x]*(a + b*Sin[c + d*x]) - 4*a*(a + b)*EllipticE[(-2*c + Pi -
 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 4*(a^2 - b^2
)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.419, size = 462, normalized size = 2.6

$$\frac{2}{3\,b^3\cos(dx+c)\,d}\left(2\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)\,b}{a+b}}\sqrt{-\frac{(1+\sin(dx+c))\,b}{a-b}}\mathrm{EllipticF}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 2/3*(2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a^2*b-2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*b^3-2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3+2*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^2-sin(d*x+c)^3*b
^3-sin(d*x+c)^2*a*b^2+b^3*sin(d*x+c)+a*b^2)/b^3/cos(d*x+c)/(a+b*sin(d*x+c))
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos(dx+c)^2}{\sqrt{b\sin(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

3.514 $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=183

$$-\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d\left(a^2-b^2\right)}-\frac{a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c\right.\right.}{d\sqrt{a+b\sin}}$$

```
[Out] -((Sec[c + d*x]*(b - a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/((a^2 - b^2)
*d)) - (a*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d
*x]])/((a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (EllipticF[(c -
Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a
 + b*Sin[c + d*x]])
```

Rubi [A] time = 0.203331, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2696, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d\left(a^2-b^2\right)}-\frac{a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c\right.\right.}{d\sqrt{a+b\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] -((Sec[c + d*x]*(b - a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/((a^2 - b^2)
*d)) - (a*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d
*x]])/((a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (EllipticF[(c -
Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a
 + b*Sin[c + d*x]])
```


Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
```

```
2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{(a^2-b^2)\,d} + \frac{\int \frac{\frac{b^2}{2}+\frac{1}{2}ab\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{-a^2+b^2}$$

$$= -\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{(a^2-b^2)\,d} + \frac{1}{2}\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx - \frac{a\int \sqrt{a+}}{2}$$

$$= -\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{(a^2-b^2)\,d} - \frac{\left(a\sqrt{a+b\sin(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c}{a+}}}{2\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

$$= -\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{(a^2-b^2)\,d} - \frac{aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c}}{\left(a^2-b^2\right)d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.637521, size = 177, normalized size = 0.97

$$\frac{-\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+a^2\tan(c+dx)-ab\sec(c+dx)+ab\sin(c+dx)\tan(c+dx)+a(a}{d(a-b)(a+b)\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-(a*b*Sec[c + d*x]) + a*(a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - (a^2 - b^2)*EllipticF[(-2*c + Pi
 - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + a^2*Tan[c
+ d*x] - b^2*Tan[c + d*x] + a*b*Sin[c + d*x]*Tan[c + d*x])/((a - b)*(a + b)
*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.585, size = 640, normalized size = 3.5

$$-\frac{1}{(a+b)\,b\,(a-b)\cos(dx+c)\,d}\sqrt{(\cos(dx+c))^2\sin(dx+c)\,b+a(\cos(dx+c))^2}\left(\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\sqrt{-\frac{b\sin(dx}{a+b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/b*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*((b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b
/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^
(1/2))*a^2*b-(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a
+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^3-(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/
2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3+
(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-
b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2),((a-b)/(a+b))^(1/2))*a*b^2+a*b^2*cos(d*x+c)^2-a^2*b*sin(d*x+c)+b^3*si
n(d*x+c))/(a+b)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(a-
b)/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^2}{\sqrt{b\sin\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^2}{\sqrt{b\sin\left(dx+c\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

3.515 $\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=291

$$-\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3d\left(a^2-b^2\right)}-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(a^2-5b^2\right)-4a\left(a^2-2b^2\right)\sin(c+\right.}{6d\left(a^2-b^2\right)^2}$$

```
[Out] -(Sec[c + d*x]^3*(b - a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(3*(a^2 - b
^2)*d) - (2*a*(a^2 - 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sq
rt[a + b*Sin[c + d*x]])/(3*(a^2 - b^2)^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b
)]) + ((4*a^2 - 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)])/(6*(a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) - (
Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(b*(a^2 - 5*b^2) - 4*a*(a^2 - 2*b^2)*
Sin[c + d*x]))/(6*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.439049, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2696, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3d\left(a^2-b^2\right)}-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(a^2-5b^2\right)-4a\left(a^2-2b^2\right)\sin(c+\right.}{6d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] -(Sec[c + d*x]^3*(b - a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(3*(a^2 - b
^2)*d) - (2*a*(a^2 - 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sq
rt[a + b*Sin[c + d*x]])/(3*(a^2 - b^2)^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b
)]) + ((4*a^2 - 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)])/(6*(a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) - (
Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(b*(a^2 - 5*b^2) - 4*a*(a^2 - 2*b^2)*
Sin[c + d*x]))/(6*(a^2 - b^2)^2*d)
```


Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
```

```
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{\int \frac{\sec^2(c+dx)\left(-2a^2+\frac{5b^2}{2}-\frac{3}{2}ab\sin(c+dx)\right)}{\sqrt{a+b\sin(c+dx)}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(a^2-\right.\right.}{6\left(a^2-b^2\right.}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(a^2-\right.\right.}{6\left(a^2-b^2\right.}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(a^2-\right.\right.}{6\left(a^2-b^2\right.}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{2a\left(a^2-2b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{}}{3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\sin(c+}{a+b}}}$$

Mathematica [A] time = 4.03752, size = 306, normalized size = 1.05

$$\frac{-4\left(-9a^2b^2+4a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+16a\left(a^2b+a^3-2ab^2-2b^3\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}E\left(\frac{1}{4}(-2c-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (16*a*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 4*(4*a^4 - 9*a^2*b^2 + 5*b^
4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x]
)/(a + b)] + Sec[c + d*x]^3*(-4*a^3*b + 10*a*b^3 + (-6*a^3*b + 14*a*b^3)*Co
```

```
s[2*(c + d*x)] + (-2*a^3*b + 4*a*b^3)*Cos[4*(c + d*x)] + 12*a^4*Sin[c + d*x
] - 25*a^2*b^2*Sin[c + d*x] + 13*b^4*Sin[c + d*x] + 4*a^4*Sin[3*(c + d*x)]
- 9*a^2*b^2*Sin[3*(c + d*x)] + 5*b^4*Sin[3*(c + d*x)]))/(24*(a - b)^2*(a +
b)^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 1.633, size = 1314, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/6*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)/cos(d*x+c)^5/(a+b*sin(d*x+c))^(
3/2)/b/(a^4-2*a^2*b^2+b^4)*(-4*cos(d*x+c)^4*(cos(d*x+c)^2*sin(d*x+c)*b+a*co
s(d*x+c)^2)^(1/2)*a*b^2*(a^2-2*b^2)+cos(d*x+c)^2*(cos(d*x+c)^2*sin(d*x+c)*b
+a*cos(d*x+c)^2)^(1/2)*b*(4*a^4-9*a^2*b^2+5*b^4)*sin(d*x+c)+2*(cos(d*x+c)^2
*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*b*(a^4-2*a^2*b^2+b^4)*sin(d*x+c)-cos(d*
x+c)^2*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*(4*(-b/(a-b)*sin(d*
x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(
a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a
+b))^(1/2)*a^4*b-3*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^3*b^2-9*(-b/(a-b)*sin(d*x+c
)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b
))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b)
)^(1/2)*a^2*b^3+3*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*
a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a*b^4+5*(-b/(a-b)*sin(d*x+c)-b
/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^
(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(
1/2)*b^5-4*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*
a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2)
)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^5+12*(-b/(a-b)*sin(d*x+c)-b/(a-b))^
(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/
(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a^3
*b^2-8*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(
1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-
b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a*b^4-a^3*b^2+a*b^4))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(dx+c)^4}{\sqrt{b\sin(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

3.516 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^5d}-\frac{8a\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^5d}-\frac{2\left(a^2-b^2\right)^2}{b^5d\sqrt{a+b\sin(c+dx)}}+\frac{2(a+b\sin(c+dx))^{7/2}}{7b^5d}-$$

```
[Out] (-2*(a^2 - b^2)^2)/(b^5*d*Sqrt[a + b*Sin[c + d*x]]) - (8*a*(a^2 - b^2)*Sqrt
[a + b*Sin[c + d*x]])/(b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^(3/2)
)/(3*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(5/2))/(5*b^5*d) + (2*(a + b*Sin[c
+ d*x])^(7/2))/(7*b^5*d)
```

Rubi [A] time = 0.120683, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$\frac{4\left(3a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}{3b^5d}-\frac{8a\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^5d}-\frac{2\left(a^2-b^2\right)^2}{b^5d\sqrt{a+b\sin(c+dx)}}+\frac{2(a+b\sin(c+dx))^{7/2}}{7b^5d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)^2)/(b^5*d*Sqrt[a + b*Sin[c + d*x]]) - (8*a*(a^2 - b^2)*Sqrt
[a + b*Sin[c + d*x]])/(b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^(3/2)
)/(3*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(5/2))/(5*b^5*d) + (2*(a + b*Sin[c
+ d*x])^(7/2))/(7*b^5*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{(a+x)^{3/2}}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a^2-b^2)^2}{(a+x)^{3/2}} - \frac{4(a^3-ab^2)}{\sqrt{a+x}} + 2\left(3a^2-b^2\right)\sqrt{a+x} - 4a(a+x)^{3/2} + (a+x)^{5/2}\right) dx, x, b\sin$$

$$= -\frac{2\left(a^2-b^2\right)^2}{b^5 d\sqrt{a+b\sin(c+dx)}} - \frac{8a\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^5 d} + \frac{4\left(3a^2-b^2\right)(a+b\sin(c+d}{3b^5 d}$$

Mathematica [A] time = 0.25319, size = 116, normalized size = 0.77

$$\frac{30b^4\cos^4(c+dx) - 16\left(\left(5b^4-6a^2b^2\right)\sin^2(c+dx) + ab\left(24a^2-35b^2\right)\sin(c+dx) - 70a^2b^2 + 48a^4 + 3ab^3\sin^3(c+dx)\right.}{105b^5 d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (30*b^4*Cos[c + d*x]^4 - 16*(48*a^4 - 70*a^2*b^2 + 15*b^4 + a*b*(24*a^2 - 3
5*b^2)*Sin[c + d*x] + (-6*a^2*b^2 + 5*b^4)*Sin[c + d*x]^2 + 3*a*b^3*Sin[c +
 d*x]^3))/(105*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [A] time = 0.224, size = 116, normalized size = 0.8

$$\frac{48\,ab^3\left(\cos(dx+c)\right)^2\sin(dx+c) + 2\left(-192\,a^3b + 256\,ab^3\right)\sin(dx+c) + 30\,b^4\left(\cos(dx+c)\right)^4 + 2\left(-48\,a^2b^2 + 40\,b^4\right)(}{105\,b^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/105/b^5*(24*a*b^3*cos(d*x+c)^2*sin(d*x+c)+(-192*a^3*b+256*a*b^3)*sin(d*x+
c)+15*b^4*cos(d*x+c)^4+(-48*a^2*b^2+40*b^4)*cos(d*x+c)^2-384*a^4+608*a^2*b^
```

```
2-160*b^4)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [A] time = 0.975225, size = 167, normalized size = 1.11

$$\frac{2\left(\frac{15\,(b\sin(dx+c)+a)^{\frac{7}{2}}-84\,(b\sin(dx+c)+a)^{\frac{5}{2}}a+70\left(3\,a^2-b^2\right)(b\sin(dx+c)+a)^{\frac{3}{2}}-420\left(a^3-ab^2\right)\sqrt{b\sin(dx+c)+a}}{b^4}-\frac{105\left(a^4-2\,a^2b^2+b^4\right)}{\sqrt{b\sin(dx+c)+a}\,b^4}\right)}{105\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(b*sin(d*x + c) + a)^(7/2) - 84*(b*sin(d*x + c) + a)^(5/2)*a + 7
0*(3*a^2 - b^2)*(b*sin(d*x + c) + a)^(3/2) - 420*(a^3 - a*b^2)*sqrt(b*sin(d
*x + c) + a))/b^4 - 105*(a^4 - 2*a^2*b^2 + b^4)/(sqrt(b*sin(d*x + c) + a)*b
^4))/(b*d)
```

Fricas [A] time = 2.35217, size = 302, normalized size = 2.01

$$\frac{2\left(15\,b^4\cos(dx+c)^4-384\,a^4+608\,a^2b^2-160\,b^4-8\left(6\,a^2b^2-5\,b^4\right)\cos(dx+c)^2+8\left(3\,ab^3\cos(dx+c)^2-24\,a^3b+32\,a\right.\right.}{105\left(b^6d\sin(dx+c)+ab^5d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^4*cos(d*x + c)^4 - 384*a^4 + 608*a^2*b^2 - 160*b^4 - 8*(6*a^2*b
^2 - 5*b^4)*cos(d*x + c)^2 + 8*(3*a*b^3*cos(d*x + c)^2 - 24*a^3*b + 32*a*b^
3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^6*d*sin(d*x + c) + a*b^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10681, size = 184, normalized size = 1.23

$$\frac{2\left(15\,(b\sin\left(dx+c\right)+a)^{\frac{7}{2}}-84\,(b\sin\left(dx+c\right)+a)^{\frac{5}{2}}a+210\,(b\sin\left(dx+c\right)+a)^{\frac{3}{2}}a^{2}-420\,\sqrt{b\sin\left(dx+c\right)+a}a^{3}-70\,(b\,\mathrm{si}\right.}{105\,b^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(15*(b*sin(d*x + c) + a)^(7/2) - 84*(b*sin(d*x + c) + a)^(5/2)*a + 21
0*(b*sin(d*x + c) + a)^(3/2)*a^2 - 420*sqrt(b*sin(d*x + c) + a)*a^3 - 70*(b
*sin(d*x + c) + a)^(3/2)*b^2 + 420*sqrt(b*sin(d*x + c) + a)*a*b^2 - 105*(a^
4 - 2*a^2*b^2 + b^4)/sqrt(b*sin(d*x + c) + a))/(b^5*d)
```

3.517 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2\left(a^2-b^2\right)}{b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2(a+b\sin(c+dx))^{3/2}}{3b^3d}+\frac{4a\sqrt{a+b\sin(c+dx)}}{b^3d}$$

```
[Out] (2*(a^2 - b^2))/(b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (4*a*Sqrt[a + b*Sin[c + d*x]])/(b^3*d) - (2*(a + b*Sin[c + d*x])^(3/2))/(3*b^3*d)
```

Rubi [A] time = 0.0935693, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$\frac{2\left(a^2-b^2\right)}{b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2(a+b\sin(c+dx))^{3/2}}{3b^3d}+\frac{4a\sqrt{a+b\sin(c+dx)}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 - b^2))/(b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (4*a*Sqrt[a + b*Sin[c + d*x]])/(b^3*d) - (2*(a + b*Sin[c + d*x])^(3/2))/(3*b^3*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^{3/2}}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{(a+x)^{3/2}} + \frac{2a}{\sqrt{a+x}} - \sqrt{a+x}\right) dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= \frac{2\left(a^2-b^2\right)}{b^3 d\sqrt{a+b\sin(c+dx)}} + \frac{4a\sqrt{a+b\sin(c+dx)}}{b^3 d} - \frac{2(a+b\sin(c+dx))^{3/2}}{3b^3 d}$$

Mathematica [A] time = 0.0670612, size = 57, normalized size = 0.72

$$\frac{16a^2+8ab\sin(c+dx)+b^2\cos(2(c+dx))-7b^2}{3b^3 d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*a^2 - 7*b^2 + b^2*Cos[2*(c + d*x)] + 8*a*b*Sin[c + d*x])/(3*b^3*d*Sqrt[
a + b*Sin[c + d*x]])
```

Maple [A] time = 0.24, size = 54, normalized size = 0.7

$$\frac{2\,b^2\left(\cos\left(dx+c\right)\right)^2+8\,ab\sin\left(dx+c\right)+16\,a^2-8\,b^2}{3\,b^3 d}\frac{1}{\sqrt{a+b\sin\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/3/b^3/(a+b*sin(d*x+c))^(1/2)*(b^2*cos(d*x+c)^2+4*a*b*sin(d*x+c)+8*a^2-4*b
^2)/d
```

Maxima [A] time = 0.971005, size = 90, normalized size = 1.14

$$-\frac{2\left(\frac{(b\sin(dx+c)+a)^{\frac{3}{2}}-6\sqrt{b\sin(dx+c)+a}\,a}{b^2}-\frac{3\left(a^2-b^2\right)}{\sqrt{b\sin(dx+c)+a}\,b^2}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*(((b*sin(d*x + c) + a)^(3/2) - 6*sqrt(b*sin(d*x + c) + a)*a)/b^2 - 3*(
a^2 - b^2)/(sqrt(b*sin(d*x + c) + a)*b^2))/(b*d)
```

Fricas [A] time = 2.02746, size = 161, normalized size = 2.04

$$\frac{2\left(b^2\cos\left(dx+c\right)^2+4\,ab\sin\left(dx+c\right)+8\,a^2-4\,b^2\right)\sqrt{b\sin\left(dx+c\right)+a}}{3\left(b^4d\sin\left(dx+c\right)+ab^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*cos(d*x + c)^2 + 4*a*b*sin(d*x + c) + 8*a^2 - 4*b^2)*sqrt(b*sin(d*
x + c) + a)/(b^4*d*sin(d*x + c) + a*b^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09855, size = 80, normalized size = 1.01

$$-\frac{2\left(\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}-6\sqrt{b\sin\left(dx+c\right)+a}\,a-\frac{3\left(a^2-b^2\right)}{\sqrt{b\sin\left(dx+c\right)+a}}\right)}{3\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*((b*sin(d*x + c) + a)^(3/2) - 6*sqrt(b*sin(d*x + c) + a)*a - 3*(a^2 -
b^2)/sqrt(b*sin(d*x + c) + a))/(b^3*d)
```

3.518 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2}{bd\sqrt{a+b\sin(c+dx)}}$$

```
[Out] -2/(b*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.0377284, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 32}

$$-\frac{2}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= -\frac{2}{bd\sqrt{a+b\sin(c+dx)}}$$

Mathematica [A] time = 0.0140742, size = 22, normalized size = 1.

$$-\frac{2}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [A] time = 0.004, size = 21, normalized size = 1.

$$-2\,\frac{1}{bd\sqrt{a+b\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/b/d/(a+b*sin(d*x+c))^(1/2)
```

Maxima [A] time = 0.945956, size = 27, normalized size = 1.23

$$-\frac{2}{\sqrt{b\sin(dx+c)+a}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*sin(d*x + c) + a)*b*d)
```

Fricas [A] time = 1.93253, size = 78, normalized size = 3.55

$$-\frac{2\sqrt{b\sin(dx+c)+a}}{b^2d\sin(dx+c)+abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*sin(d*x + c) + a)/(b^2*d*sin(d*x + c) + a*b*d)
```

Sympy [A] time = 3.5954, size = 56, normalized size = 2.55

$$\begin{cases} \frac{x\cos(c)}{a^{\frac{3}{2}}} & \text{for } b=0 \wedge d=0 \\ \frac{x\cos(c)}{(a+b\sin(c))^{\frac{3}{2}}} & \text{for } d=0 \\ \frac{\sin(c+dx)}{a^{\frac{3}{2}}d} & \text{for } b=0 \\ -\frac{2}{bd\sqrt{a+b\sin(c+dx)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((x*cos(c)/a**(3/2), Eq(b, 0) & Eq(d, 0)), (x*cos(c)/(a + b*sin(c)
)**(3/2), Eq(d, 0)), (sin(c + d*x)/(a**(3/2)*d), Eq(b, 0)), (-2/(b*d*sqrt(a
 + b*sin(c + d*x))), True))
```

Giac [A] time = 1.08248, size = 27, normalized size = 1.23

$$-\frac{2}{\sqrt{b\sin(dx+c)+a}\,abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(b*sin(d*x + c) + a)*b*d)
```

3.519 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=105

$$\frac{2b}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d)) + ArcTan
h[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b)/((a^2 - b
^2)*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.157626, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2668, 710, 827, 1166, 206}

$$\frac{2b}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d)) + ArcTan
h[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b)/((a^2 - b
^2)*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{b\,\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{2b}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{b\,\text{Subst}\left(\int \frac{a-x}{\sqrt{a+x}\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{\left(a^2-b^2\right)d}$$

$$= \frac{2b}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{(2b)\,\text{Subst}\left(\int \frac{2a-x^2}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{\left(a^2-b^2\right)d}$$

$$= \frac{2b}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{(a-b)d} + \text{Subst}\left(\int \right.$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{2b}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}}$$

Mathematica [C] time = 0.0792309, size = 91, normalized size = 0.87

$$\frac{(a+b)\,_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(b-a)\,_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)}{d(a-b)(a+b)\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a - b)] + (-
a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a + b)])/((a -
 b)*(a + b)*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [A] time = 0.442, size = 99, normalized size = 0.9

$$2\,\frac{b}{d\,(a+b)\,(a-b)\,\sqrt{a+b\sin\,(dx+c)}}+\frac{1}{d\,(a-b)}\arctan\left(\sqrt{a+b\sin\,(dx+c)}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}+\frac{1}{d}\mathrm{Artanh}\left(\sqrt{a+b\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/d*b/(a+b)/(a-b)/(a+b*sin(d*x+c))^(1/2)+1/d/(a-b)/(-a+b)^(1/2)*arctan((a+b
*sin(d*x+c))^(1/2)/(-a+b)^(1/2))+arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2)
)/(a+b)^(3/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\sec\left(dx+c\right)}{b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)/(b^2*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [A] time = 1.08793, size = 161, normalized size = 1.53

$$b\left(\frac{\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{\left(abd-b^2d\right)\sqrt{-a+b}}-\frac{\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{\left(abd+b^2d\right)\sqrt{-a-b}}+\frac{2}{\left(a^2d-b^2d\right)\sqrt{b\sin\left(dx+c\right)+a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] b*(arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a + b))/((a*b*d - b^2*d)*sqrt(-a +
 b)) - arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a - b))/((a*b*d + b^2*d)*sqrt(
-a - b)) + 2/((a^2*d - b^2*d)*sqrt(b*sin(d*x + c) + a)))
```

3.520 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{3/2}} \, dx$

Optimal. Leaf size=186

$$-\frac{b\left(a^2+5b^2\right)}{2d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}-\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{5/2}}+\frac{(2a+5b)\,\mathrm{tanh}}{4d(a}$$

```
[Out] -((2*a - 5*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/
2)*d) + ((2*a + 5*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*(a +
 b)^(5/2)*d) - (b*(a^2 + 5*b^2))/(2*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]
]) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*Sqrt[a + b*Sin[
c + d*x]])
```

Rubi [A] time = 0.3343, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2668, 741, 829, 827, 1166, 206}

$$-\frac{b\left(a^2+5b^2\right)}{2d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}-\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{5/2}}+\frac{(2a+5b)\,\mathrm{tanh}}{4d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((2*a - 5*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/
2)*d) + ((2*a + 5*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*(a +
 b)^(5/2)*d) - (b*(a^2 + 5*b^2))/(2*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]
]) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*Sqrt[a + b*Sin[
c + d*x]])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{b\,\text{Subst}\left(\int \frac{\frac{1}{2}(2a^2-5b^2)+\frac{3ax}{2}}{(a+x)^{3/2}(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{2(a^2-b^2)\,d}$$

$$= -\frac{b(a^2+5b^2)}{2(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{b\,\text{Subst}\left(\int \frac{-a(a^2-4b^2}{\sqrt{a+}}\right.}{2}$$

$$= -\frac{b(a^2+5b^2)}{2(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{b\,\text{Subst}\left(\int \frac{-\frac{1}{2}a(-a^2-}{}\right.}{}$$

$$= -\frac{b(a^2+5b^2)}{2(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{(2a-5b)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{5/2}d} + \frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{5/2}d} - \frac{b(a^2+5}{2(a^2-b^2)^2\,d\sqrt{a+}}$$

Mathematica [C] time = 1.1061, size = 221, normalized size = 1.19

$$\frac{\frac{(a^2+5b^2)\left((a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(b-a)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)\right)}{(a-b)(a+b)\sqrt{a+b\sin(c+dx)}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{3a\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + \frac{2\sec^2(c+dx)(b-}{\sqrt{a+b\sin(}}}{4d(b^2-a^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((3*a*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/Sqrt[a - b] - (3*a*Arc
Tanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/Sqrt[a + b] + ((a^2 + 5*b^2)*((
a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a - b)] + (-a
+ b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a + b)]))/((a -
b)*(a + b)*Sqrt[a + b*Sin[c + d*x]]) + (2*Sec[c + d*x]^2*(b - a*Sin[c + d*x
]))/Sqrt[a + b*Sin[c + d*x]])/(4*(-a^2 + b^2)*d)
```

Maple [A] time = 0.545, size = 250, normalized size = 1.3

$$-\frac{b}{4\,d\,(a-b)^{2}\,(b\sin(dx+c)+b)}\sqrt{a+b\sin(dx+c)}+\frac{a}{2\,d\,(a-b)^{2}}\arctan\left(\sqrt{a+b\sin(dx+c)}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}-\frac{}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*b/(a-b)^2*(a+b*sin(d*x+c))^(1/2)/(b*sin(d*x+c)+b)+1/2/d/(a-b)^2/(-a+
b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a-5/4/d*b/(a-b)^2/(-a+
b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))-2/d*b^3/(a+b)^2/(a-b)^
2/(a+b*sin(d*x+c))^(1/2)-1/4/d*b/(a+b)^2*(a+b*sin(d*x+c))^(1/2)/(b*sin(d*x+
c)-b)+1/2/d/(a+b)^(5/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+5/4/d
*b/(a+b)^(5/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [A] time = 1.11553, size = 401, normalized size = 2.16

$$\frac{1}{4}\,b^3\left(\frac{(2\,a-5\,b)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{\left(a^2b^3d-2\,ab^4d+b^5d\right)\sqrt{-a+b}}-\frac{(2\,a+5\,b)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{\left(a^2b^3d+2\,ab^4d+b^5d\right)\sqrt{-a-b}}-\frac{2\left((b\sin(dx+c)+a)^2a^2-(b\sin(dx+c)+\right.}{\left(a^4b^2d-2\,a^2b^4d+b^6d\right)\left((b\sin(dx+c)+a)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*b^3*((2*a - 5*b)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a + b))/((a^2*b^
3*d - 2*a*b^4*d + b^5*d)*sqrt(-a + b)) - (2*a + 5*b)*arctan(sqrt(b*sin(d*x
+ c) + a)/sqrt(-a - b))/((a^2*b^3*d + 2*a*b^4*d + b^5*d)*sqrt(-a - b)) - 2*
((b*sin(d*x + c) + a)^2*a^2 - (b*sin(d*x + c) + a)*a^3 + 5*(b*sin(d*x + c)
+ a)^2*b^2 - 11*(b*sin(d*x + c) + a)*a*b^2 + 4*a^2*b^2 - 4*b^4)/((a^4*b^2*d
 - 2*a^2*b^4*d + b^6*d)*((b*sin(d*x + c) + a)^(5/2) - 2*(b*sin(d*x + c) + a
)^(3/2)*a + sqrt(b*sin(d*x + c) + a)*a^2 - sqrt(b*sin(d*x + c) + a)*b^2)))
```

3.521 $\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{3b\left(-7a^2b^2+2a^4-15b^4\right)}{16d\left(a^2-b^2\right)^3\sqrt{a+b\sin(c+dx)}}-\frac{3\left(4a^2-14ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{7/2}}+\frac{3\left(4a^2+14ab+15b^2\right)\tanh^{-1}\left(\sqrt{a}\right.}{32d(a+b)^{7/2}}$$

```
[Out] (-3*(4*a^2 - 14*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]
)/(32*(a - b)^(7/2)*d) + (3*(4*a^2 + 14*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Si
n[c + d*x]]/Sqrt[a + b]])/(32*(a + b)^(7/2)*d) - (3*b*(2*a^4 - 7*a^2*b^2 -
15*b^4))/(16*(a^2 - b^2)^3*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]^4*(b
 - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) + (Sec[c + d
*x]^2*(b*(a^2 + 9*b^2) + 2*a*(3*a^2 - 8*b^2)*Sin[c + d*x]))/(16*(a^2 - b^2)
^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.520548, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2668, 741, 823, 829, 827, 1166, 206}

$$-\frac{3b\left(-7a^2b^2+2a^4-15b^4\right)}{16d\left(a^2-b^2\right)^3\sqrt{a+b\sin(c+dx)}}-\frac{3\left(4a^2-14ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{7/2}}+\frac{3\left(4a^2+14ab+15b^2\right)\tanh^{-1}\left(\sqrt{a}\right.}{32d(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-3*(4*a^2 - 14*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]
)/(32*(a - b)^(7/2)*d) + (3*(4*a^2 + 14*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Si
n[c + d*x]]/Sqrt[a + b]])/(32*(a + b)^(7/2)*d) - (3*b*(2*a^4 - 7*a^2*b^2 -
15*b^4))/(16*(a^2 - b^2)^3*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]^4*(b
 - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) + (Sec[c + d
*x]^2*(b*(a^2 + 9*b^2) + 2*a*(3*a^2 - 8*b^2)*Sin[c + d*x]))/(16*(a^2 - b^2)
^2*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
```

```
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{b^3\,\text{Subst}\left(\int \frac{\frac{3}{2}\left(2a^2-3b^2\right)+\frac{7ax}{2}}{(a+x)^{3/2}\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{\sec^2(c+dx)\left(b\left(a^2+9b^2\right)+2a\left(3a^2-8b^2\right)\sin(c+dx\right.}{16\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{3b\left(2a^4-7a^2b^2-15b^4\right)}{16\left(a^2-b^2\right)^3 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{\sec^2(c+dx)\left(b\left(a\right.\right.}{16\left(a^2\right.}$$

$$= -\frac{3b\left(2a^4-7a^2b^2-15b^4\right)}{16\left(a^2-b^2\right)^3 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{\sec^2(c+dx)\left(b\left(a\right.\right.}{16\left(a^2\right.}$$

$$= -\frac{3b\left(2a^4-7a^2b^2-15b^4\right)}{16\left(a^2-b^2\right)^3 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{\sec^2(c+dx)\left(b\left(a\right.\right.}{16\left(a^2\right.}$$

$$= -\frac{3\left(4a^2-14ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{7/2}d} + \frac{3\left(4a^2+14ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c}}{\sqrt{a+b}}\right.}{32(a+b)^{7/2}d}$$

Mathematica [C] time = 2.19539, size = 324, normalized size = 1.14

$$\frac{\frac{3}{2}\left(-7a^2b^2+2a^4-15b^4\right)\left((a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(b-a)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)\right)+3a\sqrt{a-b}\sqrt{a+b}\left(3a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((3*(2*a^4 - 7*a^2*b^2 - 15*b^4)*((a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (
a + b*Sin[c + d*x])/(a - b)] + (-a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a
+ b*Sin[c + d*x])/(a + b)]))/2 - 4*(a - b)^2*(a + b)^2*Sec[c + d*x]^4*(-b +
 a*Sin[c + d*x]) + 3*a*Sqrt[a - b]*Sqrt[a + b]*(3*a^2 - 8*b^2)*(Sqrt[a + b]
*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]] - Sqrt[a - b]*ArcTanh[Sqrt[a
 + b*Sin[c + d*x]]/Sqrt[a + b]])*Sqrt[a + b*Sin[c + d*x]] - (a - b)*(a + b)
*Sec[c + d*x]^2*(b*(a^2 + 9*b^2) + 2*a*(3*a^2 - 8*b^2)*Sin[c + d*x]))/(16*(
a^2 - b^2)^2*(-a^2 + b^2)*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.749, size = 649, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -3/16/d*b/(a-b)^3/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(3/2)*a+13/32/d*b^2/(
a-b)^3/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(3/2)+3/16/d*b/(a-b)^3/(b*sin(d*
x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)*a^2-21/32/d*b^2/(a-b)^3/(b*sin(d*x+c)+b)^2
*(a+b*sin(d*x+c))^(1/2)*a+15/32/d*b^3/(a-b)^3/(b*sin(d*x+c)+b)^2*(a+b*sin(d
*x+c))^(1/2)+3/8/d/(a-b)^3/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b
)^(1/2))*a^2-21/16/d*b/(a-b)^3/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(
-a+b)^(1/2))*a+45/32/d*b^2/(a-b)^3/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/
2)/(-a+b)^(1/2))+2/d*b^5/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c))^(1/2)-3/16/d*b/(a
+b)^3/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(3/2)*a-13/32/d*b^2/(a+b)^3/(b*si
n(d*x+c)-b)^2*(a+b*sin(d*x+c))^(3/2)+3/16/d*b/(a+b)^3/(b*sin(d*x+c)-b)^2*(a
+b*sin(d*x+c))^(1/2)*a^2+21/32/d*b^2/(a+b)^3/(b*sin(d*x+c)-b)^2*(a+b*sin(d*
x+c))^(1/2)*a+15/32/d*b^3/(a+b)^3/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(1/2)
+3/8/d/(a+b)^(7/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2+21/16/d*
b/(a+b)^(7/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+45/32/d*b^2/(a+
b)^(7/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\sec\left(dx+c\right)^{5}}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^5/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19322, size = 733, normalized size = 2.58

$$\frac{1}{32}\,b^{5}\left(\frac{3\left(4\,a^{2}-14\,ab+15\,b^{2}\right)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{\left(a^{3}b^{5}d-3\,a^{2}b^{6}d+3\,ab^{7}d-b^{8}d\right)\sqrt{-a+b}}-\frac{3\left(4\,a^{2}+14\,ab+15\,b^{2}\right)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{\left(a^{3}b^{5}d+3\,a^{2}b^{6}d+3\,ab^{7}d+b^{8}d\right)\sqrt{-a-b}}+\frac{}{\left(a^{6}d-3\,a^{4}b^{2}d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*b^5*(3*(4*a^2 - 14*a*b + 15*b^2)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(
-a + b))/((a^3*b^5*d - 3*a^2*b^6*d + 3*a*b^7*d - b^8*d)*sqrt(-a + b)) - 3*(
4*a^2 + 14*a*b + 15*b^2)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a - b))/((a^
3*b^5*d + 3*a^2*b^6*d + 3*a*b^7*d + b^8*d)*sqrt(-a - b)) + 64/((a^6*d - 3*a
^4*b^2*d + 3*a^2*b^4*d - b^6*d)*sqrt(b*sin(d*x + c) + a)) - 2*(6*(b*sin(d*x
 + c) + a)^(7/2)*a^4 - 18*(b*sin(d*x + c) + a)^(5/2)*a^5 + 18*(b*sin(d*x +
c) + a)^(3/2)*a^6 - 6*sqrt(b*sin(d*x + c) + a)*a^7 - 21*(b*sin(d*x + c) + a
)^(7/2)*a^2*b^2 + 62*(b*sin(d*x + c) + a)^(5/2)*a^3*b^2 - 71*(b*sin(d*x + c
) + a)^(3/2)*a^4*b^2 + 30*sqrt(b*sin(d*x + c) + a)*a^5*b^2 - 13*(b*sin(d*x
+ c) + a)^(7/2)*b^4 + 68*(b*sin(d*x + c) + a)^(5/2)*a*b^4 - 76*(b*sin(d*x +
 c) + a)^(3/2)*a^2*b^4 + 30*sqrt(b*sin(d*x + c) + a)*a^3*b^4 + 17*(b*sin(d*
x + c) + a)^(3/2)*b^6 - 54*sqrt(b*sin(d*x + c) + a)*a*b^6)/((a^6*b^4*d - 3*
a^4*b^6*d + 3*a^2*b^8*d - b^10*d)*((b*sin(d*x + c) + a)^2 - 2*(b*sin(d*x +
c) + a)*a + a^2 - b^2)^2))
```

3.522 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=313

$$-\frac{8\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(a\left(32a^2-33b^2\right)-3b\left(8a^2-7b^2\right)\sin(c+dx)\right)}{63b^5d}+\frac{16a\left(-65a^2b^2+32a^4+33b^4\right)\sqrt{\frac{a+b\,\mathrm{s}}{}}}{63b^6d\sqrt{a+b\,\mathrm{si}}}$$

```
[Out] (-2*Cos[c + d*x]^5)/(b*d*Sqrt[a + b*Sin[c + d*x]]) + (20*Cos[c + d*x]^3*(8*
a - 7*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(63*b^3*d) - (16*(32*a^4 -
57*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a +
b*Sin[c + d*x]])/(63*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (16*a*(32*
a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)])/(63*b^6*d*Sqrt[a + b*Sin[c + d*x]]) - (8*C
os[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(a*(32*a^2 - 33*b^2) - 3*b*(8*a^2 - 7*
b^2)*Sin[c + d*x]))/(63*b^5*d)
```

Rubi [A] time = 0.539254, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2693, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{8\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(a\left(32a^2-33b^2\right)-3b\left(8a^2-7b^2\right)\sin(c+dx)\right)}{63b^5d}+\frac{16a\left(-65a^2b^2+32a^4+33b^4\right)\sqrt{\frac{a+b\,\mathrm{s}}{}}}{63b^6d\sqrt{a+b\,\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5)/(b*d*Sqrt[a + b*Sin[c + d*x]]) + (20*Cos[c + d*x]^3*(8*
a - 7*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(63*b^3*d) - (16*(32*a^4 -
57*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a +
b*Sin[c + d*x]])/(63*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (16*a*(32*
a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)])/(63*b^6*d*Sqrt[a + b*Sin[c + d*x]]) - (8*C
os[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(a*(32*a^2 - 33*b^2) - 3*b*(8*a^2 - 7*
b^2)*Sin[c + d*x]))/(63*b^5*d)
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{10\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{b}$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^3d} - \frac{40\int$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^3d} - \frac{8\cos$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^3d} - \frac{8\cos$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^3d} - \frac{8\cos$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^3d} - \frac{16\,(3$$

Mathematica [A] time = 1.4752, size = 273, normalized size = 0.87

$$\frac{b\cos(c+dx)\left(\left(84b^4-64a^2b^2\right)\cos(2(c+dx))+1760a^2b^2-256a^3b\sin(c+dx)-1024a^4+404ab^3\sin(c+dx)+20ab^3\,\mathrm{s}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (64*(32*a^5 + 32*a^4*b - 57*a^3*b^2 - 57*a^2*b^3 + 21*a*b^4 + 21*b^5)*Ellip
ticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b
)] - 64*a*(32*a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*Cos[c + d*x]*(-1024*a^
4 + 1760*a^2*b^2 - 595*b^4 + (-64*a^2*b^2 + 84*b^4)*Cos[2*(c + d*x)] + 7*b^
4*Cos[4*(c + d*x)] - 256*a^3*b*Sin[c + d*x] + 404*a*b^3*Sin[c + d*x] + 20*a
*b^3*Sin[3*(c + d*x)]))/(252*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.601, size = 1195, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] -2/63*(7*b^6*sin(d*x+c)^6-10*a*b^5*sin(d*x+c)^5+256*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-192*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^4*b^2-520*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^3*b^3+360*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+264*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5-168*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
b^6-256*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^6+712*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-624*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+168*((a+b*sin(d*x+
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6+16*
a^2*b^4*sin(d*x+c)^4-35*b^6*sin(d*x+c)^4-32*a^3*b^3*sin(d*x+c)^3+68*a*b^5*s
in(d*x+c)^3-128*a^4*b^2*sin(d*x+c)^2+196*a^2*b^4*sin(d*x+c)^2-35*b^6*sin(d*
x+c)^2+32*a^3*b^3*sin(d*x+c)-58*a*b^5*sin(d*x+c)+128*a^4*b^2-212*a^2*b^4+63
```

```
*b^6)/b^7/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^6}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\cos(dx+c)^6}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^6/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^6}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/(b*sin(d*x + c) + a)^(3/2), x)
```

3.523 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{32a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{5b^4d\sqrt{a+b\sin(c+dx)}}+\frac{8\left(4a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{5b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{4\cos(}{}$$

```
[Out] (-2*Cos[c + d*x]^3)/(b*d*Sqrt[a + b*Sin[c + d*x]]) + (4*Cos[c + d*x]*(4*a -
 3*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(5*b^3*d) + (8*(4*a^2 - 3*b^2)
*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(5*
b^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (32*a*(a^2 - b^2)*EllipticF[(c
- Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(5*b^4*
d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.332658, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2693, 2865, 2752, 2663, 2661, 2655, 2653}

$$-\frac{32a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{5b^4d\sqrt{a+b\sin(c+dx)}}+\frac{8\left(4a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{5b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{4\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-2*Cos[c + d*x]^3)/(b*d*Sqrt[a + b*Sin[c + d*x]]) + (4*Cos[c + d*x]*(4*a -
 3*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(5*b^3*d) + (8*(4*a^2 - 3*b^2)
*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(5*
b^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (32*a*(a^2 - b^2)*EllipticF[(c
- Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(5*b^4*
d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
```

```
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{6\int \frac{\cos^2(c+dx)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{b}$$

$$= -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5b^3d} - \frac{8\int \frac{-\frac{ab}{2}}{}}{}$$

$$= -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5b^3d} + \frac{\left(4\left(4a^2\right.\right.}{}$$

$$= -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5b^3d} + \frac{\left(4\left(4a^2\right.\right.}{}$$

$$= -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5b^3d} + \frac{8\left(4a^2\right.}{}$$

Mathematica [A] time = 1.05006, size = 187, normalized size = 0.82

$$\frac{b\cos(c+dx)\left(16a^2+4ab\sin(c+dx)+b^2\cos(2(c+dx))-11b^2\right)+32a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)}{5b^4d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*(4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 32*a*(a^2 - b^2)*Elliptic
F[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]
+ b*Cos[c + d*x]*(16*a^2 - 11*b^2 + b^2*Cos[2*(c + d*x)] + 4*a*b*Sin[c + d*
x]))/(5*b^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.471, size = 797, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2/5*(16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^3*b-12*a^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^2-16*a*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^3+12*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4-16*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^4+28*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^2*b^2-12*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4+b^4*sin(d*x+c)^4-2*a*b^3*sin(d*x+c)
^3-8*a^2*b^2*sin(d*x+c)^2+4*b^4*sin(d*x+c)^2+2*a*b^3*sin(d*x+c)+8*a^2*b^2-5
*b^4)/b^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\cos\left(dx+c\right)^{4}}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{4}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c) + a)^(3/2), x)
```

3.524 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{4a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{b^2d\sqrt{a+b\sin(c+dx)}}-\frac{4\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}}$$

```
[Out] (-2*Cos[c + d*x])/(b*d*Sqrt[a + b*Sin[c + d*x]]) - (4*EllipticE[(c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(b^2*d*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)]) + (4*a*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[
(a + b*Sin[c + d*x])/(a + b)])/(b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.187307, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2693, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{b^2d\sqrt{a+b\sin(c+dx)}}-\frac{4\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x])/(b*d*Sqrt[a + b*Sin[c + d*x]]) - (4*EllipticE[(c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(b^2*d*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)]) + (4*a*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[
(a + b*Sin[c + d*x])/(a + b)])/(b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{2\int \frac{\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{b}$$

$$= -\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{2\int \sqrt{a+b\sin(c+dx)}\,dx}{b^2} + \frac{(2a)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx}{b^2}$$

$$= -\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{\left(2\sqrt{a+b\sin(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}\,dx}{b^2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{\left(2a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\right.}{b^2\sqrt{a+}}$$

$$= -\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{4aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right.\right.}{b^2d\sqrt{a+b}}$$

Mathematica [A] time = 2.73182, size = 125, normalized size = 0.78

$$\frac{4(a+b)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-2\left(2a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+b\cos(c+dx)\right)}{b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (4*(a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[
c + d*x])/(a + b)] - 2*(b*Cos[c + d*x] + 2*a*EllipticF[(-2*c + Pi - 2*d*x)/
4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]))/(b^2*d*Sqrt[a + b*Si
n[c + d*x]])
```

Maple [B] time = 0.549, size = 434, normalized size = 2.7

$$2\frac{1}{b^3\cos(dx+c)\sqrt{a+b\sin(dx+c)}d}\left(2\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\mathrm{EllipticE}\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 2*(2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^2-2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*b^2-2*a*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b+2*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^2+sin(d*x+c)^2*b^2-b^2
)/b^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\cos(dx+c)^2}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

3.525 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab-\left(a^2+3b^2\right)\sin(c+dx)\right)}{d\left(a^2-b^2\right)^2}+\frac{2b\sec(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}+\frac{a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c\right.\right.}{d\left(a^2-b^2\right)\sqrt{a+}}$$

```
[Out] (2*b*Sec[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) - ((a^2 + 3*b^2
)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/((
a^2 - b^2)^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (a*EllipticF[(c - Pi/2
 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/((a^2 - b^2)*
d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(4*a*b
 - (a^2 + 3*b^2)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Rubi [A] time = 0.365161, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2694, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab-\left(a^2+3b^2\right)\sin(c+dx)\right)}{d\left(a^2-b^2\right)^2}+\frac{2b\sec(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}+\frac{a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c\right.\right.}{d\left(a^2-b^2\right)\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] (2*b*Sec[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) - ((a^2 + 3*b^2
)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/((
a^2 - b^2)^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (a*EllipticF[(c - Pi/2
 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/((a^2 - b^2)*
d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(4*a*b
 - (a^2 + 3*b^2)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```


Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
```

```
2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{2b\sec(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{2\int \frac{\sec^2(c+dx)\left(-\frac{a}{2}+\frac{3}{2}b\sin(c+dx)\right)}{\sqrt{a+b\sin(c+dx)}}\,dx}{a^2-b^2}$$

$$= \frac{2b\sec(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab-\left(a^2+3b^2\right)\sin(c+\right.}{\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab-\left(a^2+3b^2\right)\sin(c+\right.}{\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab-\left(a^2+3b^2\right)\sin(c+\right.}{\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\left(a^2+3b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \cdots$$

Mathematica [A] time = 1.65651, size = 205, normalized size = 0.82

$$\frac{-a\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\left(a^2b+a^3+3ab^2+3b^3\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-\cdots}{d(a-b)^2(a+b)^2\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a^3 + a^2*b + 3*a*b^2 + 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - a*(a^2 - b^2)*EllipticF[(-2*c +
Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - (Sec[c +
 d*x]*(3*a^2*b + b^3 + b*(a^2 + 3*b^2)*Cos[2*(c + d*x)] - 2*a*(a^2 - b^2)*S
in[c + d*x]))/2)/((a - b)^2*(a + b)^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 0.788, size = 1062, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/b*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*((-b/(a-b)*sin(d*x+c)-
b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a
-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(
1/2))*a^4+2*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b
))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)
+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^2-3*(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))
*b^4-EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-
b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b-3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-
b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a
+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^2+
EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-
b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2)*a*b^3+3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*s
in(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b^4-a^2*b^2*c
os(d*x+c)^2-3*b^4*cos(d*x+c)^2+a^3*b*sin(d*x+c)-a*b^3*sin(d*x+c)-a^2*b^2+b^
4)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(a+b)/(a-b)/(a^2
-b^2)/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^2}{(b\sin (dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\sec\left(dx+c\right)^{2}}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^2/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{2}\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)^{2}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

3.526 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=359

$$-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(8ab-\left(a^2+7b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^2}+\frac{2b\sec^3(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}-\frac{\sec(c+dx)\sqrt{a+b\sin}}{}$$

```
[Out] (2*b*Sec[c + d*x]^3)/((a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) - ((4*a^4 - 1
5*a^2*b^2 - 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b
*Sin[c + d*x]])/(6*(a^2 - b^2)^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (2
*a*(a^2 - 3*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*S
in[c + d*x])/(a + b)])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[
c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(8*a*b - (a^2 + 7*b^2)*Sin[c + d*x]))/(
3*(a^2 - b^2)^2*d) - (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(a*b*(a^2 - 33*
b^2) - (4*a^4 - 15*a^2*b^2 - 21*b^4)*Sin[c + d*x]))/(6*(a^2 - b^2)^3*d)
```

Rubi [A] time = 0.61347, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2694, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(8ab-\left(a^2+7b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^2}+\frac{2b\sec^3(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}-\frac{\sec(c+dx)\sqrt{a+b\sin}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sec[c + d*x]^3)/((a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) - ((4*a^4 - 1
5*a^2*b^2 - 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b
*Sin[c + d*x]])/(6*(a^2 - b^2)^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (2
*a*(a^2 - 3*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*S
in[c + d*x])/(a + b)])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[
c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(8*a*b - (a^2 + 7*b^2)*Sin[c + d*x]))/(
3*(a^2 - b^2)^2*d) - (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(a*b*(a^2 - 33*
b^2) - (4*a^4 - 15*a^2*b^2 - 21*b^4)*Sin[c + d*x]))/(6*(a^2 - b^2)^3*d)
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{2b\sec^3(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{2\int \frac{\sec^4(c+dx)\left(-\frac{a}{2}+\frac{7}{2}b\sin(c+dx)\right)}{\sqrt{a+b\sin(c+dx)}}\,dx}{a^2-b^2}$$

$$= \frac{2b\sec^3(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(8ab-\left(a^2+7b^2\right)\sin(c+d\right.}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec^3(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(8ab-\left(a^2+7b^2\right)\sin(c+d\right.}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec^3(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(8ab-\left(a^2+7b^2\right)\sin(c+d\right.}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec^3(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(8ab-\left(a^2+7b^2\right)\sin(c+d\right.}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b\sec^3(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{\left(4a^4-15a^2b^2-21b^4\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c}}{6\left(a^2-b^2\right)^3 d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A] time = 2.9746, size = 348, normalized size = 0.97

$$\frac{-4a\left(-4a^2b^2+a^4+3b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\left(-15a^3b^2-15a^2b^3+4a^4b+4a^5-21ab^4-21b^5\right)\sqrt{\frac{a+b}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((4*a^5 + 4*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 - 21*a*b^4 - 21*b^5)*EllipticE[
(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] -
```

```
4*a*(a^4 - 4*a^2*b^2 + 3*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b
)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + (Sec[c + d*x]^3*(-24*a^4*b + 101*a^
2*b^3 + 19*b^5 + (-12*a^4*b + 84*a^2*b^3 + 56*b^5)*Cos[2*(c + d*x)] + (-4*a
^4*b + 15*a^2*b^3 + 21*b^5)*Cos[4*(c + d*x)] + 24*a^5*Sin[c + d*x] - 64*a^3
*b^2*Sin[c + d*x] + 40*a*b^4*Sin[c + d*x] + 8*a^5*Sin[3*(c + d*x)] - 32*a^3
*b^2*Sin[3*(c + d*x)] + 24*a*b^4*Sin[3*(c + d*x)]))/8)/(6*(a - b)^3*(a + b)
^3*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] time = 3.27, size = 1646, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/6*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)/cos(d*x+c)^5/(a+b*sin(d*x+c))^(
3/2)/b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*(-2*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d
*x+c)^2)^(1/2)*b^2*(a^4-2*a^2*b^2+b^4)-cos(d*x+c)^4*(cos(d*x+c)^2*sin(d*x+c
)*b+a*cos(d*x+c)^2)^(1/2)*b^2*(4*a^4-15*a^2*b^2-21*b^4)+4*cos(d*x+c)^2*(cos
(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b*(a^4-4*a^2*b^2+3*b^4)*sin(
d*x+c)+2*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(1/2)*a*b*(a^4-2*a^2*b^
2+b^4)*sin(d*x+c)+cos(d*x+c)^2*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos(d*x+c)^2)^(
1/2)*(4*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^
(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(
a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^6-19*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)
*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^
2-6*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2
)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+21*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)
*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^6-4*
(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*El
lipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)
*sin(d*x+c)-b/(a-b))^(1/2)*a^5*b+3*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b
/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(
1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^4*b^2+16*(b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*Elli
pticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*s
in(d*x+c)-b/(a-b))^(1/2)*a^3*b^3+18*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-
b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
```

```
(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^2*b^4-12*(
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*Ell
ipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*
sin(d*x+c)-b/(a-b))^(1/2)*a*b^5-21*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b
/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(
1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b^6+a^4*b^2+6
*a^2*b^4-7*b^6))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{b \sin\left(dx+c\right)+a}\sec\left(dx+c\right)^{4}}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^4/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx + c)^4}{(b \sin (dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sin(d*x + c) + a)^(3/2), x)
```

3.527 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=150

$$\frac{4(3a^2-b^2)\sqrt{a+b\sin(c+dx)}}{b^5d}+\frac{8a(a^2-b^2)}{b^5d\sqrt{a+b\sin(c+dx)}}-\frac{2(a^2-b^2)^2}{3b^5d(a+b\sin(c+dx))^{3/2}}+\frac{2(a+b\sin(c+dx))^{5/2}}{5b^5d}-8a(a+$$

```
[Out] (-2*(a^2 - b^2)^2)/(3*b^5*d*(a + b*Sin[c + d*x])^(3/2)) + (8*a*(a^2 - b^2))
/(b^5*d*Sqrt[a + b*Sin[c + d*x]]) + (4*(3*a^2 - b^2)*Sqrt[a + b*Sin[c + d*x
]])/(b^5*d) - (8*a*(a + b*Sin[c + d*x])^(3/2))/(3*b^5*d) + (2*(a + b*Sin[c
+ d*x])^(5/2))/(5*b^5*d)
```

Rubi [A] time = 0.12241, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$\frac{4(3a^2-b^2)\sqrt{a+b\sin(c+dx)}}{b^5d}+\frac{8a(a^2-b^2)}{b^5d\sqrt{a+b\sin(c+dx)}}-\frac{2(a^2-b^2)^2}{3b^5d(a+b\sin(c+dx))^{3/2}}+\frac{2(a+b\sin(c+dx))^{5/2}}{5b^5d}-8a(a+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-2*(a^2 - b^2)^2)/(3*b^5*d*(a + b*Sin[c + d*x])^(3/2)) + (8*a*(a^2 - b^2))
/(b^5*d*Sqrt[a + b*Sin[c + d*x]]) + (4*(3*a^2 - b^2)*Sqrt[a + b*Sin[c + d*x
]])/(b^5*d) - (8*a*(a + b*Sin[c + d*x])^(3/2))/(3*b^5*d) + (2*(a + b*Sin[c
+ d*x])^(5/2))/(5*b^5*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{(a+x)^{5/2}}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\left(a^2-b^2\right)^2}{(a+x)^{5/2}} - \frac{4\left(a^3-ab^2\right)}{(a+x)^{3/2}} + \frac{2\left(3a^2-b^2\right)}{\sqrt{a+x}} - 4a\sqrt{a+x} + (a+x)^{3/2}\right) dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= -\frac{2\left(a^2-b^2\right)^2}{3b^5 d(a+b\sin(c+dx))^{3/2}} + \frac{8a\left(a^2-b^2\right)}{b^5 d\sqrt{a+b\sin(c+dx)}} + \frac{4\left(3a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}{b^5 d} -$$

Mathematica [A] time = 0.29451, size = 117, normalized size = 0.78

$$\frac{16\left(\left(6a^2b^2-3b^4\right)\sin^2(c+dx)+3ab\left(8a^2-5b^2\right)\sin(c+dx)-10a^2b^2+16a^4-ab^3\sin^3(c+dx)-b^4\right)+6b^4\cos^4(c+dx}{15b^5 d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (6*b^4*Cos[c + d*x]^4 + 16*(16*a^4 - 10*a^2*b^2 - b^4 + 3*a*b*(8*a^2 - 5*b^
2)*Sin[c + d*x] + (6*a^2*b^2 - 3*b^4)*Sin[c + d*x]^2 - a*b^3*Sin[c + d*x]^3
))/(15*b^5*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [A] time = 0.332, size = 116, normalized size = 0.8

$$\frac{16\,ab^3\left(\cos(dx+c)\right)^2\sin(dx+c)+2\left(192\,a^3b-128\,ab^3\right)\sin(dx+c)+6\,b^4\left(\cos(dx+c)\right)^4+2\left(-48\,a^2b^2+24\,b^4\right)(\cos}{15\,b^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/15/b^5*(8*a*b^3*cos(d*x+c)^2*sin(d*x+c)+(192*a^3*b-128*a*b^3)*sin(d*x+c)+
3*b^4*cos(d*x+c)^4+(-48*a^2*b^2+24*b^4)*cos(d*x+c)^2+128*a^4-32*a^2*b^2-32*
```

```
b^4)/(a+b*sin(d*x+c))^(3/2)/d
```

Maxima [A] time = 0.953407, size = 165, normalized size = 1.1

$$\frac{2\left(\frac{3\,(b\sin(dx+c)+a)^{\frac{5}{2}}-20\,(b\sin(dx+c)+a)^{\frac{3}{2}}a+30\left(3\,a^2-b^2\right)\sqrt{b\sin(dx+c)+a}}{b^4}-\frac{5\left(a^4-2\,a^2b^2+b^4-12\left(a^3-ab^2\right)(b\sin(dx+c)+a)\right)}{(b\sin(dx+c)+a)^{\frac{3}{2}}b^4}\right)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(b*sin(d*x + c) + a)^(5/2) - 20*(b*sin(d*x + c) + a)^(3/2)*a + 30*
(3*a^2 - b^2)*sqrt(b*sin(d*x + c) + a))/b^4 - 5*(a^4 - 2*a^2*b^2 + b^4 - 12
*(a^3 - a*b^2)*(b*sin(d*x + c) + a))/((b*sin(d*x + c) + a)^(3/2)*b^4))/(b*d
)
```

Fricas [A] time = 2.95455, size = 344, normalized size = 2.29

$$-\frac{2\left(3\,b^4\cos\left(dx+c\right)^4+128\,a^4-32\,a^2b^2-32\,b^4-24\left(2\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2+8\left(ab^3\cos\left(dx+c\right)^2+24\,a^3b-16\,ab^3\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{15\left(b^7d\cos\left(dx+c\right)^2-2\,ab^6d\sin\left(dx+c\right)-\left(a^2b^5+b^7\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*b^4*cos(d*x + c)^4 + 128*a^4 - 32*a^2*b^2 - 32*b^4 - 24*(2*a^2*b^2
 - b^4)*cos(d*x + c)^2 + 8*(a*b^3*cos(d*x + c)^2 + 24*a^3*b - 16*a*b^3)*sin
(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^7*d*cos(d*x + c)^2 - 2*a*b^6*d*sin(d
*x + c) - (a^2*b^5 + b^7)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11167, size = 184, normalized size = 1.23

$$\frac{2\left(3\,(b\sin\left(dx+c\right)+a)^{\frac{5}{2}}-20\,(b\sin\left(dx+c\right)+a)^{\frac{3}{2}}a+90\,\sqrt{b\sin\left(dx+c\right)+a}a^{2}-30\,\sqrt{b\sin\left(dx+c\right)+a}b^{2}+\frac{5\left(12\,(b\sin(dx\cdots\right.}{\cdots}\right.}{15\,b^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*sin(d*x + c) + a)^(5/2) - 20*(b*sin(d*x + c) + a)^(3/2)*a + 90*s
qrt(b*sin(d*x + c) + a)*a^2 - 30*sqrt(b*sin(d*x + c) + a)*b^2 + 5*(12*(b*si
n(d*x + c) + a)*a^3 - a^4 - 12*(b*sin(d*x + c) + a)*a*b^2 + 2*a^2*b^2 - b^4
)/(b*sin(d*x + c) + a)^(3/2))/(b^5*d)
```

3.528 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2(a^2-b^2)}{3b^3d(a+b\sin(c+dx))^{3/2}}-\frac{4a}{b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\sqrt{a+b\sin(c+dx)}}{b^3d}$$

```
[Out] (2*(a^2 - b^2))/(3*b^3*d*(a + b*Sin[c + d*x])^(3/2)) - (4*a)/(b^3*d*Sqrt[a
+ b*Sin[c + d*x]]) - (2*Sqrt[a + b*Sin[c + d*x]])/(b^3*d)
```

Rubi [A] time = 0.0939744, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2668, 697}

$$\frac{2(a^2-b^2)}{3b^3d(a+b\sin(c+dx))^{3/2}}-\frac{4a}{b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\sqrt{a+b\sin(c+dx)}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a^2 - b^2))/(3*b^3*d*(a + b*Sin[c + d*x])^(3/2)) - (4*a)/(b^3*d*Sqrt[a
+ b*Sin[c + d*x]]) - (2*Sqrt[a + b*Sin[c + d*x]])/(b^3*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule steps

$$\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^{5/2}}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{(a+x)^{5/2}} + \frac{2a}{(a+x)^{3/2}} - \frac{1}{\sqrt{a+x}}\right)dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{2\left(a^2-b^2\right)}{3b^3d(a+b\sin(c+dx))^{3/2}} - \frac{4a}{b^3d\sqrt{a+b\sin(c+dx)}} - \frac{2\sqrt{a+b\sin(c+dx)}}{b^3d}$$

Mathematica [A] time = 0.0541289, size = 56, normalized size = 0.71

$$-\frac{2\left(8a^2+12ab\sin(c+dx)+3b^2\sin^2(c+dx)+b^2\right)}{3b^3d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(8*a^2 + b^2 + 12*a*b*Sin[c + d*x] + 3*b^2*Sin[c + d*x]^2))/(3*b^3*d*(a
 + b*Sin[c + d*x])^(3/2))
```

Maple [A] time = 0.241, size = 55, normalized size = 0.7

$$-\frac{-6\,b^2\left(\cos\left(dx+c\right)\right)^2+24\,ab\sin\left(dx+c\right)+16\,a^2+8\,b^2}{3\,b^3d}\left(a+b\sin\left(dx+c\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3/b^3*(-3*b^2*cos(d*x+c)^2+12*a*b*sin(d*x+c)+8*a^2+4*b^2)/(a+b*sin(d*x+c
))^(3/2)/d
```

Maxima [A] time = 0.94406, size = 86, normalized size = 1.09

$$-\frac{2\left(\frac{3\sqrt{b\sin(dx+c)+a}}{b^2} + \frac{6\,(b\sin(dx+c)+a)a-a^2+b^2}{(b\sin(dx+c)+a)^{\frac{3}{2}}\,b^2}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(b*sin(d*x + c) + a)/b^2 + (6*(b*sin(d*x + c) + a)*a - a^2 + b^
2)/((b*sin(d*x + c) + a)^(3/2)*b^2))/(b*d)
```

Fricas [A] time = 2.36386, size = 216, normalized size = 2.73

$$-\frac{2\left(3\,b^2\cos\left(dx+c\right)^2-12\,ab\sin\left(dx+c\right)-8\,a^2-4\,b^2\right)\sqrt{b\sin\left(dx+c\right)+a}}{3\left(b^5d\cos\left(dx+c\right)^2-2\,ab^4d\sin\left(dx+c\right)-\left(a^2b^3+b^5\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b^2*cos(d*x + c)^2 - 12*a*b*sin(d*x + c) - 8*a^2 - 4*b^2)*sqrt(b*si
n(d*x + c) + a)/(b^5*d*cos(d*x + c)^2 - 2*a*b^4*d*sin(d*x + c) - (a^2*b^3 +
 b^5)*d)
```

Sympy [A] time = 30.6728, size = 304, normalized size = 3.85

$$\begin{cases}\frac{x\cos^3\left(c\right)}{a^{\frac{5}{2}}}\\ \frac{\frac{2\sin^3\left(c+dx\right)}{3d}+\frac{\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}}{a^{\frac{5}{2}}}\\ \frac{x\cos^3\left(c\right)}{\left(a+b\sin\left(c\right)\right)^{\frac{5}{2}}}\\ -\frac{16a^2}{3ab^3d\sqrt{a+b\sin\left(c+dx\right)}+3b^4d\sqrt{a+b\sin\left(c+dx\right)}\sin\left(c+dx\right)}-\frac{24ab\sin\left(c+dx\right)}{3ab^3d\sqrt{a+b\sin\left(c+dx\right)}+3b^4d\sqrt{a+b\sin\left(c+dx\right)}\sin\left(c+dx\right)}-\frac{8b^2\sin^2\left(c+dx\right)}{3ab^3d\sqrt{a+b\sin\left(c+dx\right)}+3b^4d\sqrt{a+b\sin}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((x*cos(c)**3/a**(5/2), Eq(b, 0) & Eq(d, 0)), ((2*sin(c + d*x)**3/
(3*d) + sin(c + d*x)*cos(c + d*x)**2/d)/a**(5/2), Eq(b, 0)), (x*cos(c)**3/(
a + b*sin(c))**(5/2), Eq(d, 0)), (-16*a**2/(3*a*b**3*d*sqrt(a + b*sin(c + d
*x)) + 3*b**4*d*sqrt(a + b*sin(c + d*x))*sin(c + d*x)) - 24*a*b*sin(c + d*x
```

```
)/(3*a*b**3*d*sqrt(a + b*sin(c + d*x)) + 3*b**4*d*sqrt(a + b*sin(c + d*x))*
sin(c + d*x)) - 8*b**2*sin(c + d*x)**2/(3*a*b**3*d*sqrt(a + b*sin(c + d*x))
 + 3*b**4*d*sqrt(a + b*sin(c + d*x))*sin(c + d*x)) - 2*b**2*cos(c + d*x)**2
/(3*a*b**3*d*sqrt(a + b*sin(c + d*x)) + 3*b**4*d*sqrt(a + b*sin(c + d*x))*s
in(c + d*x)), True))
```

Giac [A] time = 1.11739, size = 78, normalized size = 0.99

$$-\frac{2\left(3\sqrt{b\sin\left(dx+c\right)+a}+\frac{6\,(b\sin(dx+c)+a)a-a^2+b^2}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\right)}{3\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*sqrt(b*sin(d*x + c) + a) + (6*(b*sin(d*x + c) + a)*a - a^2 + b^2)/(
b*sin(d*x + c) + a)^(3/2))/(b^3*d)
```

3.529 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=24

$$-\frac{2}{3bd(a+b\sin(c+dx))^{3/2}}$$

```
[Out] -2/(3*b*d*(a + b*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.0422203, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 32}

$$-\frac{2}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -2/(3*b*d*(a + b*Sin[c + d*x])^(3/2))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}}\,dx, x, b\sin(c+dx)\right)}{bd}\\
&= -\frac{2}{3bd(a+b\sin(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0187543, size = 24, normalized size = 1.

$$-\frac{2}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -2/(3*b*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [A] time = 0.004, size = 21, normalized size = 0.9

$$-\frac{2}{3\,bd}\left(a+b\sin\left(dx+c\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3/b/d/(a+b*sin(d*x+c))^(3/2)
```

Maxima [A] time = 0.943273, size = 27, normalized size = 1.12

$$-\frac{2}{3\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((b*sin(d*x + c) + a)^(3/2)*b*d)
```

Fricas [B] time = 2.26043, size = 130, normalized size = 5.42

$$\frac{2\sqrt{b\sin\left(dx+c\right)+a}}{3\left(b^{3}d\cos\left(dx+c\right)^{2}-2\,ab^{2}d\sin\left(dx+c\right)-\left(a^{2}b+b^{3}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*sin(d*x + c) + a)/(b^3*d*cos(d*x + c)^2 - 2*a*b^2*d*sin(d*x + c)
 - (a^2*b + b^3)*d)
```

Sympy [A] time = 29.4815, size = 87, normalized size = 3.62

$$\begin{cases} \frac{x\cos(c)}{a^{\frac{5}{2}}} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sin(c+dx)}{a^{\frac{5}{2}}d} & \text{for } b = 0 \\ \frac{x\cos(c)}{(a+b\sin(c))^{\frac{5}{2}}} & \text{for } d = 0 \\ -\frac{2}{3abd\sqrt{a+b\sin(c+dx)}+3b^2d\sqrt{a+b\sin(c+dx)}\sin(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((x*cos(c)/a**(5/2), Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**(5/2)
*d), Eq(b, 0)), (x*cos(c)/(a + b*sin(c))**(5/2), Eq(d, 0)), (-2/(3*a*b*d*sq
rt(a + b*sin(c + d*x)) + 3*b**2*d*sqrt(a + b*sin(c + d*x))*sin(c + d*x)), T
rue))
```

Giac [A] time = 1.09457, size = 27, normalized size = 1.12

$$-\frac{2}{3\,(b\sin(dx+c)+a)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/((b*sin(d*x + c) + a)^(3/2)*b*d)
```

3.530 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=139

$$\frac{4ab}{d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}+\frac{2b}{3d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/((a - b)^(5/2)*d)) + ArcTan
h[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/((a + b)^(5/2)*d) + (2*b)/(3*(a^2 -
 b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + (4*a*b)/((a^2 - b^2)^2*d*Sqrt[a + b*S
in[c + d*x]])
```

Rubi [A] time = 0.242626, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2668, 710, 829, 827, 1166, 206}

$$\frac{4ab}{d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}+\frac{2b}{3d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/((a - b)^(5/2)*d)) + ArcTan
h[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/((a + b)^(5/2)*d) + (2*b)/(3*(a^2 -
 b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + (4*a*b)/((a^2 - b^2)^2*d*Sqrt[a + b*S
in[c + d*x]])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
```

```
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{b\,\mathrm{Subst}\left(\int \frac{1}{(a+x)^{5/2}(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{2b}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{b\,\mathrm{Subst}\left(\int \frac{a-x}{(a+x)^{3/2}(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{(a^2-b^2)\,d}$$

$$= \frac{2b}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{4ab}{(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{b\,\mathrm{Subst}\left(\int \frac{-a^2-b^2+}{\sqrt{a+x}(b^2}\right.}{(a^2}$$

$$= \frac{2b}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{4ab}{(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{-3a^2}{-a^2+}\right.}{}$$

$$= \frac{2b}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{4ab}{(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{\mathrm{Subst}\left(\int \frac{1}{a-b-x^2}\,dx\right.}{(a}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d} + \frac{2b}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} +$$

Mathematica [C] time = 0.0717091, size = 94, normalized size = 0.68

$$\frac{(a+b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right) + (b-a)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)}{3d(a-b)(a+b)(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((a + b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[c + d*x])/(a - b)] + (
-a + b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[c + d*x])/(a + b)])/(3*
(a - b)*(a + b)*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [A] time = 0.437, size = 130, normalized size = 0.9

$$\frac{1}{d(a-b)^2}\arctan\left(\sqrt{a+b\sin(dx+c)}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}} + \frac{2b}{3d(a+b)(a-b)}(a+b\sin(dx+c))^{-\frac{3}{2}} + 4\frac{}{d(a+b)^2(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/d/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))+2/3/d*
b/(a+b)/(a-b)/(a+b*sin(d*x+c))^(3/2)+4/d*b*a/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c
))^(1/2)+arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.07383, size = 7309, normalized size = 52.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a^5 - 3*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2
 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*sin(d*x + c))*sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^4 +
 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 +
9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2
 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^
3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a
^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c)
)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c)
+ 8)) + 3*(a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b^2
 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2
*b^3 + a*b^4)*sin(d*x + c))*sqrt(a - b)*log((b^4*cos(d*x + c)^4 + 128*a^4 -
 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 +
```

```
9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2
 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^
3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a
^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c)
)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c)
+ 8)) + 16*(7*a^4*b - 8*a^2*b^3 + b^5 + 6*(a^3*b^2 - a*b^4)*sin(d*x + c))*s
qrt(b*sin(d*x + c) + a))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x
 + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^8 -
 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d), 1/24*(6*(a^5 - 3*a^4*b + 4*a^3*b^2 - 4*a^
2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(d*x + c)^
2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*sin(d*x + c))*sqrt(-a - b)*ar
ctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*s
in(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b
^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x
 + c))) - 3*(a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b
^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a
^2*b^3 + a*b^4)*sin(d*x + c))*sqrt(a - b)*log((b^4*cos(d*x + c)^4 + 128*a^4
 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3
+ 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b
^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*
b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112
*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x +
c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c
) + 8)) - 16*(7*a^4*b - 8*a^2*b^3 + b^5 + 6*(a^3*b^2 - a*b^4)*sin(d*x + c))
*sqrt(b*sin(d*x + c) + a))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d
*x + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^8
 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d), 1/24*(6*(a^5 + 3*a^4*b + 4*a^3*b^2 + 4*
a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c
)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*sin(d*x + c))*sqrt(-a + b)*
arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*
sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*
b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*
x + c))) - 3*(a^5 - 3*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*
b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*
a^2*b^3 - a*b^4)*sin(d*x + c))*sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^
4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3
 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*
b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8
*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 11
2*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x +
 c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x +
c) + 8)) - 16*(7*a^4*b - 8*a^2*b^3 + b^5 + 6*(a^3*b^2 - a*b^4)*sin(d*x + c)
)*sqrt(b*sin(d*x + c) + a))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(
d*x + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^
8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d), 1/12*(3*(a^5 + 3*a^4*b + 4*a^3*b^2 + 4
```

```
*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x +
c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*sin(d*x + c))*sqrt(-a + b)
*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)
*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a
*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d
*x + c))) + 3*(a^5 - 3*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3
*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3
*a^2*b^3 - a*b^4)*sin(d*x + c))*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^
2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x
+ c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos
(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c))) - 8*(7*a^4*b - 8*a^2
*b^3 + b^5 + 6*(a^3*b^2 - a*b^4)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/((
a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(a^7*b - 3*a^5*
b^3 + 3*a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^
8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0873, size = 227, normalized size = 1.63

$$\frac{1}{3}\,b\left(\frac{3\,\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{\left(a^2bd-2\,ab^2d+b^3d\right)\sqrt{-a+b}}-\frac{3\,\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{\left(a^2bd+2\,ab^2d+b^3d\right)\sqrt{-a-b}}+\frac{2\left(6\,(b\sin\left(dx+c\right)+a)a+a^2-b^2\right)}{\left(a^4d-2\,a^2b^2d+b^4d\right)\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*b*(3*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a + b))/((a^2*b*d - 2*a*b^2*
d + b^3*d)*sqrt(-a + b)) - 3*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a - b))/
((a^2*b*d + 2*a*b^2*d + b^3*d)*sqrt(-a - b)) + 2*(6*(b*sin(d*x + c) + a)*a
+ a^2 - b^2)/((a^4*d - 2*a^2*b^2*d + b^4*d)*(b*sin(d*x + c) + a)^(3/2)))
```

3.531 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=231

$$-\frac{ab\left(a^2+19b^2\right)}{2d\left(a^2-b^2\right)^3\sqrt{a+b\sin(c+dx)}}-\frac{b\left(3a^2+7b^2\right)}{6d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{3/2}}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}-\frac{(2a-7b)}{}$$

```
[Out] -((2*a - 7*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(7/
2)*d) + ((2*a + 7*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*(a +
 b)^(7/2)*d) - (b*(3*a^2 + 7*b^2))/(6*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^
(3/2)) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[
c + d*x])^(3/2)) - (a*b*(a^2 + 19*b^2))/(2*(a^2 - b^2)^3*d*Sqrt[a + b*Sin[c
 + d*x]])
```

Rubi [A] time = 0.41596, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2668, 741, 829, 827, 1166, 206}

$$-\frac{ab\left(a^2+19b^2\right)}{2d\left(a^2-b^2\right)^3\sqrt{a+b\sin(c+dx)}}-\frac{b\left(3a^2+7b^2\right)}{6d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{3/2}}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}-\frac{(2a-7b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] -((2*a - 7*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(7/
2)*d) + ((2*a + 7*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*(a +
 b)^(7/2)*d) - (b*(3*a^2 + 7*b^2))/(6*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^
(3/2)) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[
c + d*x])^(3/2)) - (a*b*(a^2 + 19*b^2))/(2*(a^2 - b^2)^3*d*Sqrt[a + b*Sin[c
 + d*x]])
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{b\,\text{Subst}\left(\int \frac{\frac{1}{2}(2a^2-7b^2)+\frac{5ax}{2}}{(a+x)^{5/2}(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{2(a^2-b^2)\,d}$$

$$= -\frac{b(3a^2+7b^2)}{6(a^2-b^2)^2\,d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} - \frac{b\,\text{Subst}\left(\int \frac{-a}{}\right.}{}$$

$$= -\frac{b(3a^2+7b^2)}{6(a^2-b^2)^2\,d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} - \frac{ab\,(a}{2(a^2-b^2)^3\,d}$$

$$= -\frac{b(3a^2+7b^2)}{6(a^2-b^2)^2\,d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} - \frac{ab\,(a}{2(a^2-b^2)^3\,d}$$

$$= -\frac{b(3a^2+7b^2)}{6(a^2-b^2)^2\,d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} - \frac{ab\,(a}{2(a^2-b^2)^3\,d}$$

$$= -\frac{(2a-7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{7/2}d} + \frac{(2a+7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{7/2}d} - \frac{b\,(3a^2}{6(a^2-b^2)^2\,d(a}$$

Mathematica [C] time = 0.855788, size = 245, normalized size = 1.06

$$\frac{-(3a^2b+3a^3+7ab^2+7b^3)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(-3a^2b+3a^3+7ab^2-7b^3)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)+1}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-((3*a^3 + 3*a^2*b + 7*a*b^2 + 7*b^3)*Hypergeometric2F1[-3/2, 1, -1/2, (a
+ b*Sin[c + d*x])/(a - b)]) + (3*a^3 - 3*a^2*b + 7*a*b^2 - 7*b^3)*Hypergeom
etric2F1[-3/2, 1, -1/2, (a + b*Sin[c + d*x])/(a + b)] + 15*a*(a + b)*Hyperg
eometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a - b)]*(a + b*Sin[c + d*x]
) - 3*(a - b)*(-2*(a + b)*Sec[c + d*x]^2*(-b + a*Sin[c + d*x]) + 5*a*Hyperg
```

```
eometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d*x]
)))/(12*(a - b)^2*(a + b)^2*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [A] time = 0.784, size = 283, normalized size = 1.2

$$-\frac{b}{4\,d\,(a-b)^3\,(b\sin(dx+c)+b)}\sqrt{a+b\sin(dx+c)}+\frac{a}{2\,d\,(a-b)^3}\arctan\left(\sqrt{a+b\sin(dx+c)}\frac{1}{\sqrt{-a+b}}\right)\frac{1}{\sqrt{-a+b}}-\frac{7}{4\,d\,(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -1/4/d*b/(a-b)^3*(a+b*sin(d*x+c))^(1/2)/(b*sin(d*x+c)+b)+1/2/d/(a-b)^3/(-a+
b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a-7/4/d*b/(a-b)^3/(-a+
b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))-2/3/d*b^3/(a+b)^2/(a-b
)^2/(a+b*sin(d*x+c))^(3/2)-8/d*b^3*a/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))^(1/2)
-1/4/d*b/(a+b)^3*(a+b*sin(d*x+c))^(1/2)/(b*sin(d*x+c)-b)+1/2/d/(a+b)^(7/2)*
arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+7/4/d*b/(a+b)^(7/2)*arctanh((
a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\sec(dx+c)^3}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^3/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15749, size = 512, normalized size = 2.22

$$\frac{1}{12}b^3\left(\frac{3(2a-7b)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{\left(a^3b^3d-3a^2b^4d+3ab^5d-b^6d\right)\sqrt{-a+b}}-\frac{3(2a+7b)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{\left(a^3b^3d+3a^2b^4d+3ab^5d+b^6d\right)\sqrt{-a-b}}-\frac{6\left((b\sin(dx+c)+a)^{\frac{3}{2}}a\right.}{\left(a^6b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*b^3*(3*(2*a - 7*b)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a + b))/((a^3
*b^3*d - 3*a^2*b^4*d + 3*a*b^5*d - b^6*d)*sqrt(-a + b)) - 3*(2*a + 7*b)*arc
tan(sqrt(b*sin(d*x + c) + a)/sqrt(-a - b))/((a^3*b^3*d + 3*a^2*b^4*d + 3*a*
b^5*d + b^6*d)*sqrt(-a - b)) - 6*((b*sin(d*x + c) + a)^(3/2)*a^3 - sqrt(b*s
in(d*x + c) + a)*a^4 + 3*(b*sin(d*x + c) + a)^(3/2)*a*b^2 - 6*sqrt(b*sin(d*
x + c) + a)*a^2*b^2 - sqrt(b*sin(d*x + c) + a)*b^4)/((a^6*b^2*d - 3*a^4*b^4
*d + 3*a^2*b^6*d - b^8*d)*((b*sin(d*x + c) + a)^2 - 2*(b*sin(d*x + c) + a)*
a + a^2 - b^2)) - 8*(12*(b*sin(d*x + c) + a)*a + a^2 - b^2)/((a^6*d - 3*a^4
*b^2*d + 3*a^2*b^4*d - b^6*d)*(b*sin(d*x + c) + a)^(3/2)))
```

3.532 $\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=339

$$-\frac{ab\left(-16a^2b^2+3a^4-127b^4\right)}{8d\left(a^2-b^2\right)^4\sqrt{a+b\sin(c+dx)}}-\frac{b\left(-81a^2b^2+18a^4-77b^4\right)}{48d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^{3/2}}-\frac{\left(12a^2-54ab+77b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{9/2}}$$

```
[Out] -((12*a^2 - 54*a*b + 77*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])
/(32*(a - b)^(9/2)*d) + ((12*a^2 + 54*a*b + 77*b^2)*ArcTanh[Sqrt[a + b*Sin[
c + d*x]]/Sqrt[a + b]])/(32*(a + b)^(9/2)*d) - (b*(18*a^4 - 81*a^2*b^2 - 77
*b^4))/(48*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^(3/2)) - (Sec[c + d*x]^4*(b
 - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) - (a*b*(3*
a^4 - 16*a^2*b^2 - 127*b^4))/(8*(a^2 - b^2)^4*d*Sqrt[a + b*Sin[c + d*x]]) +
 (Sec[c + d*x]^2*(b*(3*a^2 + 11*b^2) + 2*a*(3*a^2 - 10*b^2)*Sin[c + d*x]))/
(16*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.644966, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2668, 741, 823, 829, 827, 1166, 206}

$$-\frac{ab\left(-16a^2b^2+3a^4-127b^4\right)}{8d\left(a^2-b^2\right)^4\sqrt{a+b\sin(c+dx)}}-\frac{b\left(-81a^2b^2+18a^4-77b^4\right)}{48d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^{3/2}}-\frac{\left(12a^2-54ab+77b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((12*a^2 - 54*a*b + 77*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])
/(32*(a - b)^(9/2)*d) + ((12*a^2 + 54*a*b + 77*b^2)*ArcTanh[Sqrt[a + b*Sin[
c + d*x]]/Sqrt[a + b]])/(32*(a + b)^(9/2)*d) - (b*(18*a^4 - 81*a^2*b^2 - 77
*b^4))/(48*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^(3/2)) - (Sec[c + d*x]^4*(b
 - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) - (a*b*(3*
a^4 - 16*a^2*b^2 - 127*b^4))/(8*(a^2 - b^2)^4*d*Sqrt[a + b*Sin[c + d*x]]) +
 (Sec[c + d*x]^2*(b*(3*a^2 + 11*b^2) + 2*a*(3*a^2 - 10*b^2)*Sin[c + d*x]))/
(16*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^(3/2))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 829

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)
), x] + Dist[1/(c*d^2 + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{b^5\operatorname{Subst}\left(\int \frac{1}{(a+x)^{5/2}\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{b^3\operatorname{Subst}\left(\int \frac{\frac{1}{2}\left(6a^2-11b^2\right)+\frac{9ax}{2}}{(a+x)^{5/2}\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{\sec^2(c+dx)\left(b\left(3a^2+11b^2\right)+2a\left(3a^2-10b^2\right)\sin(c+\right.}{16\left(a^2-b^2\right)^2 d(a+b\sin(c+dx))^{3/2}}$$

$$= -\frac{b\left(18a^4-81a^2b^2-77b^4\right)}{48\left(a^2-b^2\right)^3 d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{\sec^2(c+dx)\left(\right.}{16}$$

$$= -\frac{b\left(18a^4-81a^2b^2-77b^4\right)}{48\left(a^2-b^2\right)^3 d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} - \frac{ab\left(3a^4-1\right.}{8\left(a^2-b^2\right)^4 d}$$

$$= -\frac{b\left(18a^4-81a^2b^2-77b^4\right)}{48\left(a^2-b^2\right)^3 d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} - \frac{ab\left(3a^4-1\right.}{8\left(a^2-b^2\right)^4 d}$$

$$= -\frac{b\left(18a^4-81a^2b^2-77b^4\right)}{48\left(a^2-b^2\right)^3 d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} - \frac{ab\left(3a^4-1\right.}{8\left(a^2-b^2\right)^4 d}$$

$$= -\frac{\left(12a^2-54ab+77b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{9/2}d} + \frac{\left(12a^2+54ab+77b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+}}{\sqrt{a+b}}\right.}{32(a+b)^{9/2}d}$$

Mathematica [C] time = 3.36129, size = 296, normalized size = 0.87

$$\frac{\frac{1}{2}\left(-81a^2b^2+18a^4-77b^4\right)\left((a+b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(b-a)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)\right)-15a\left(3a^2-10b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((18*a^4 - 81*a^2*b^2 - 77*b^4)*((a + b)*Hypergeometric2F1[-3/2, 1, -1/2,
(a + b*Sin[c + d*x])/(a - b)] + (-a + b)*Hypergeometric2F1[-3/2, 1, -1/2, (
a + b*Sin[c + d*x])/(a + b)]))/2 - 12*(a - b)^2*(a + b)^2*Sec[c + d*x]^4*(-
b + a*Sin[c + d*x]) - 15*a*(3*a^2 - 10*b^2)*((a + b)*Hypergeometric2F1[-1/2
, 1, 1/2, (a + b*Sin[c + d*x])/(a - b)] + (-a + b)*Hypergeometric2F1[-1/2,
1, 1/2, (a + b*Sin[c + d*x])/(a + b)])*(a + b*Sin[c + d*x]) - 3*(a - b)*(a
+ b)*Sec[c + d*x]^2*(3*a^2*b + 11*b^3 + (6*a^3 - 20*a*b^2)*Sin[c + d*x]))/(
48*(a^2 - b^2)^2*(-a^2 + b^2)*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [B] time = 0.795, size = 682, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -3/16/d*b/(a-b)^4/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(3/2)*a+17/32/d*b^2/(
a-b)^4/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(3/2)+3/16/d*b/(a-b)^4/(b*sin(d*
x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)*a^2-25/32/d*b^2/(a-b)^4/(b*sin(d*x+c)+b)^2
*(a+b*sin(d*x+c))^(1/2)*a+19/32/d*b^3/(a-b)^4/(b*sin(d*x+c)+b)^2*(a+b*sin(d
*x+c))^(1/2)+3/8/d/(a-b)^4/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b
)^(1/2))*a^2-27/16/d*b/(a-b)^4/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(
-a+b)^(1/2))*a+77/32/d*b^2/(a-b)^4/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/
2)/(-a+b)^(1/2))+2/3/d*b^5/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c))^(3/2)+12/d*b^5*
a/(a-b)^4/(a+b)^4/(a+b*sin(d*x+c))^(1/2)-3/16/d*b/(a+b)^4/(b*sin(d*x+c)-b)^
2*(a+b*sin(d*x+c))^(3/2)*a-17/32/d*b^2/(a+b)^4/(b*sin(d*x+c)-b)^2*(a+b*sin(
d*x+c))^(3/2)+3/16/d*b/(a+b)^4/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(1/2)*a^
2+25/32/d*b^2/(a+b)^4/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(1/2)*a+19/32/d*b
^3/(a+b)^4/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(1/2)+3/8/d/(a+b)^(9/2)*arct
anh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2+27/16/d*b/(a+b)^(9/2)*arctanh((
```

```
a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+77/32/d*b^2/(a+b)^(9/2)*arctanh((a+b*s
in(d*x+c))^(1/2)/(a+b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\sec\left(dx+c\right)^{5}}{3\,ab^{2}\cos\left(dx+c\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(dx+c\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^5/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21919, size = 861, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/96*b^5*(3*(12*a^2 - 54*a*b + 77*b^2)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt
(-a + b))/((a^4*b^5*d - 4*a^3*b^6*d + 6*a^2*b^7*d - 4*a*b^8*d + b^9*d)*sqrt
(-a + b)) - 3*(12*a^2 + 54*a*b + 77*b^2)*arctan(sqrt(b*sin(d*x + c) + a)/sq
rt(-a - b))/((a^4*b^5*d + 4*a^3*b^6*d + 6*a^2*b^7*d + 4*a*b^8*d + b^9*d)*sq
rt(-a - b)) + 64*(18*(b*sin(d*x + c) + a)*a + a^2 - b^2)/((a^8*d - 4*a^6*b^
2*d + 6*a^4*b^4*d - 4*a^2*b^6*d + b^8*d)*(b*sin(d*x + c) + a)^(3/2)) - 6*(6
*(b*sin(d*x + c) + a)^(7/2)*a^5 - 18*(b*sin(d*x + c) + a)^(5/2)*a^6 + 18*(b
*sin(d*x + c) + a)^(3/2)*a^7 - 6*sqrt(b*sin(d*x + c) + a)*a^8 - 32*(b*sin(d
*x + c) + a)^(7/2)*a^3*b^2 + 95*(b*sin(d*x + c) + a)^(5/2)*a^4*b^2 - 104*(b
*sin(d*x + c) + a)^(3/2)*a^5*b^2 + 41*sqrt(b*sin(d*x + c) + a)*a^6*b^2 - 62
*(b*sin(d*x + c) + a)^(7/2)*a*b^4 + 260*(b*sin(d*x + c) + a)^(5/2)*a^2*b^4
- 310*(b*sin(d*x + c) + a)^(3/2)*a^3*b^4 + 125*sqrt(b*sin(d*x + c) + a)*a^4
*b^4 + 15*(b*sin(d*x + c) + a)^(5/2)*b^6 + 44*(b*sin(d*x + c) + a)^(3/2)*a*
b^6 - 141*sqrt(b*sin(d*x + c) + a)*a^2*b^6 - 19*sqrt(b*sin(d*x + c) + a)*b^
8)/((a^8*b^4*d - 4*a^6*b^6*d + 6*a^4*b^8*d - 4*a^2*b^10*d + b^12*d)*((b*sin
(d*x + c) + a)^2 - 2*(b*sin(d*x + c) + a)*a + a^2 - b^2)^2))
```

3.533 $\int \frac{\cos^8(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=384

$$\frac{40\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(32a^2-28ab\sin(c+dx)-3b^2\right)}{99b^5d}-\frac{16\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(-3ab\left(32a^2-31b\right.\right.}{99b^7d}$$

```
[Out] (-2*Cos[c + d*x]^7)/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) - (128*a*(8*a^2 - 9*
b^2)*(4*a^2 - 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a +
b*Sin[c + d*x]])/(99*b^8*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (32*(128*a
^6 - 272*a^4*b^2 + 159*a^2*b^4 - 15*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(99*b^8*d*Sqrt[a + b*Sin[c +
 d*x]]) - (28*Cos[c + d*x]^5*(12*a + b*Sin[c + d*x]))/(33*b^3*d*Sqrt[a + b*
Sin[c + d*x]]) + (40*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(32*a^2 - 3*b^
2 - 28*a*b*Sin[c + d*x]))/(99*b^5*d) - (16*Cos[c + d*x]*Sqrt[a + b*Sin[c +
d*x]]*(128*a^4 - 144*a^2*b^2 + 15*b^4 - 3*a*b*(32*a^2 - 31*b^2)*Sin[c + d*x
]))/(99*b^7*d)
```

Rubi [A] time = 0.761083, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2693, 2863, 2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{40\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(32a^2-28ab\sin(c+dx)-3b^2\right)}{99b^5d}-\frac{16\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(-3ab\left(32a^2-31b\right.\right.}{99b^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-2*Cos[c + d*x]^7)/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) - (128*a*(8*a^2 - 9*
b^2)*(4*a^2 - 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a +
b*Sin[c + d*x]])/(99*b^8*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (32*(128*a
^6 - 272*a^4*b^2 + 159*a^2*b^4 - 15*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(99*b^8*d*Sqrt[a + b*Sin[c +
 d*x]]) - (28*Cos[c + d*x]^5*(12*a + b*Sin[c + d*x]))/(33*b^3*d*Sqrt[a + b*
Sin[c + d*x]]) + (40*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(32*a^2 - 3*b^
2 - 28*a*b*Sin[c + d*x]))/(99*b^5*d) - (16*Cos[c + d*x]*Sqrt[a + b*Sin[c +
d*x]]*(128*a^4 - 144*a^2*b^2 + 15*b^4 - 3*a*b*(32*a^2 - 31*b^2)*Sin[c + d*x
```

```
]))/(99*b^7*d)
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{14\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{280\int \frac{\cos^4(c+dx)\left(-\frac{b}{2}-6\right.}{\sqrt{a+b\sin(c}}}{33b^3}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\cos^3(c+dx)\sqrt{a+}}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\cos^3(c+dx)\sqrt{a+}}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\cos^3(c+dx)\sqrt{a+}}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\cos^3(c+dx)\sqrt{a+}}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{128a\left(8a^2-9b^2\right)\left(4a^2-3b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b}}{99b^8d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.76154, size = 356, normalized size = 0.93

$$\frac{\frac{1}{2}b\cos(c+dx)\left(74112a^3b^3\sin(c+dx)-384a^3b^3\sin(3(c+dx))+\left(-3648a^2b^4+2048a^4b^2+1383b^6\right)\cos(2(c+dx))+\right(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (256*(a + b)*(b*(32*a^4*b - 51*a^2*b^3 + 15*b^5)*EllipticF[(-2*c + Pi - 2*d
*x)/4, (2*b)/(a + b)] + 4*(32*a^5 - 60*a^3*b^2 + 27*a*b^4)*((a + b)*Ellipti
cE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/
4, (2*b)/(a + b)]))*((a + b*Sin[c + d*x])/(a + b))^(3/2) + (b*Cos[c + d*x]*
(-32768*a^6 + 55296*a^4*b^2 - 18144*a^2*b^4 - 2574*b^6 + (2048*a^4*b^2 - 36
48*a^2*b^4 + 1383*b^6)*Cos[2*(c + d*x)] + (-96*a^2*b^4 + 126*b^6)*Cos[4*(c
+ d*x)] + 9*b^6*Cos[6*(c + d*x)] - 40960*a^5*b*Sin[c + d*x] + 74112*a^3*b^3
```

```
*Sin[c + d*x] - 30920*a*b^5*Sin[c + d*x] - 384*a^3*b^3*Sin[3*(c + d*x)] + 5
96*a*b^5*Sin[3*(c + d*x)] + 28*a*b^5*Sin[5*(c + d*x)]))/2)/(792*b^8*d*(a +
b*Sin[c + d*x])^(3/2))
```

Maple [B] time = 0.596, size = 2253, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/99*(-14*a*b^7*sin(d*x+c)*cos(d*x+c)^6+(48*a^3*b^5-64*a*b^7)*cos(d*x+c)^4
*sin(d*x+c)+(1280*a^5*b^3-2328*a^3*b^5+984*a*b^7)*cos(d*x+c)^2*sin(d*x+c)+1
6*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b*(128*EllipticF((b/(a-b)*sin(d*x+c)+1/
(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^6*b-96*EllipticF((b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-272*EllipticF((b/(a-b)*sin(d*
x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3+189*EllipticF((b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+159*EllipticF((b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5-93*EllipticF((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^6-15*EllipticF
((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^7-128*Elliptic
E((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^7+368*Ellipti
cE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-348*El
lipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+1
08*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^
6)*sin(d*x+c)-9*b^8*cos(d*x+c)^8+(24*a^2*b^6-18*b^8)*cos(d*x+c)^6+(-128*a^4
*b^4+204*a^2*b^6-60*b^8)*cos(d*x+c)^4+(1024*a^6*b^2-1664*a^4*b^4+456*a^2*b^
6+120*b^8)*cos(d*x+c)^2-2048*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^8+5888*(b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*si
n(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-
b)/(a+b))^(1/2))*a^6*b^2-5568*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b
)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+1728*(b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-
b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
,((a-b)/(a+b))^(1/2))*a^2*b^6+2048*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b
/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*Ellipt
icF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^7*b-1536*(b
```

```
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/
(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2),((a-b)/(a+b))^(1/2))*a^6*b^2-4352*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*
(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*Ell
ipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^3+30
24*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)
*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*
a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+2544*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(
1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2
)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b
^5-1488*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^
(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(
a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6-240*(b/(a-b)*sin(d*x+c)+1/(a-b)*
a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^
(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a
*b^7)/(a+b*sin(d*x+c))^(3/2)/b^9/cos(d*x+c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^8}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^8/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\cos(dx+c)^8}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^8/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^8}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^8/(b*sin(d*x + c) + a)^(5/2), x)
```

3.534 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=293

$$\frac{8\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(32a^2-24ab\sin(c+dx)-5b^2\right)}{21b^5d}-\frac{16\left(-37a^2b^2+32a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx\right.\right.}{21b^6d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] (-2*Cos[c + d*x]^5)/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) + (16*a*(32*a^2 - 29
*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]]
)/(21*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (16*(32*a^4 - 37*a^2*b^2
+ 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/(21*b^6*d*Sqrt[a + b*Sin[c + d*x]]) - (20*Cos[c + d*x]^3*(8*
a + b*Sin[c + d*x]))/(21*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (8*Cos[c + d*x]*
Sqrt[a + b*Sin[c + d*x]]*(32*a^2 - 5*b^2 - 24*a*b*Sin[c + d*x]))/(21*b^5*d)
```

Rubi [A] time = 0.512548, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2693, 2863, 2865, 2752, 2663, 2661, 2655, 2653}

$$\frac{8\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(32a^2-24ab\sin(c+dx)-5b^2\right)}{21b^5d}-\frac{16\left(-37a^2b^2+32a^4+5b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx\right.\right.}{21b^6d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^(5/2),x]
```



```
[Out] (-2*Cos[c + d*x]^5)/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) + (16*a*(32*a^2 - 29
*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]]
)/(21*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (16*(32*a^4 - 37*a^2*b^2
+ 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/(21*b^6*d*Sqrt[a + b*Sin[c + d*x]]) - (20*Cos[c + d*x]^3*(8*
a + b*Sin[c + d*x]))/(21*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (8*Cos[c + d*x]*
Sqrt[a + b*Sin[c + d*x]]*(32*a^2 - 5*b^2 - 24*a*b*Sin[c + d*x]))/(21*b^5*d)
```


Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
```

```
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
```

```
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{10\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{20\cos^3(c+dx)(8a+b\sin(c+dx))}{21b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\int \frac{\cos^2(c+dx)\left(-\frac{b}{2}-4a\sin\right.}{\sqrt{a+b\sin(c+d}}}{7b^3}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{20\cos^3(c+dx)(8a+b\sin(c+dx))}{21b^3d\sqrt{a+b\sin(c+dx)}} + \frac{8\cos(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{20\cos^3(c+dx)(8a+b\sin(c+dx))}{21b^3d\sqrt{a+b\sin(c+dx)}} + \frac{8\cos(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{20\cos^3(c+dx)(8a+b\sin(c+dx))}{21b^3d\sqrt{a+b\sin(c+dx)}} + \frac{8\cos(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{16a\left(32a^2-29b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx}}{21b^6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.18673, size = 244, normalized size = 0.83

$$\frac{\frac{1}{2}b\cos(c+dx)\left(\left(52b^4-64a^2b^2\right)\cos(2(c+dx))-736a^2b^2+1280a^3b\sin(c+dx)+1024a^4-1076ab^3\sin(c+dx)+12ab^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-32*(a + b)*(a*(32*a^3 + 32*a^2*b - 29*a*b^2 - 29*b^3)*EllipticE[(-2*c + P
i - 2*d*x)/4, (2*b)/(a + b)] + (-32*a^4 + 37*a^2*b^2 - 5*b^4)*EllipticF[(-2
*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((a + b*Sin[c + d*x])/(a + b))^(3/2) +
(b*Cos[c + d*x]*(1024*a^4 - 736*a^2*b^2 - 111*b^4 + (-64*a^2*b^2 + 52*b^4)*
Cos[2*(c + d*x)] + 3*b^4*Cos[4*(c + d*x)] + 1280*a^3*b*Sin[c + d*x] - 1076*
a*b^3*Sin[c + d*x] + 12*a*b^3*Sin[3*(c + d*x)]))/2)/(42*b^6*d*(a + b*Sin[c
+ d*x])^(3/2))
```

Maple [B] time = 0.592, size = 1642, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/21*(6*a*b^5*sin(d*x+c)*cos(d*x+c)^4+(160*a^3*b^3-136*a*b^5)*cos(d*x+c)^2*
sin(d*x+c)-8*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b
)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b*(32*EllipticE((b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5-61*EllipticE((b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+29*EllipticE((b/(a-b)
*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-32*EllipticF((b/(a-
b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b+24*EllipticF((b/(
a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+37*EllipticF(
(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3-24*Ellipt
icF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-5*Ellip
ticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^5)*sin(d*x
+c)+3*b^6*cos(d*x+c)^6+(-16*a^2*b^4+10*b^6)*cos(d*x+c)^4+(128*a^4*b^2-84*a^
2*b^4-20*b^6)*cos(d*x+c)^2+256*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+
b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
,((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^5*b-192*(b/(a-b
)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF
```

```
((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*
x+c)-b/(a-b))^(1/2)*a^4*b^2-296*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a
+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2
),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^3*b^3+192*(b/(
a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*Ellipt
icF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin
(d*x+c)-b/(a-b))^(1/2)*a^2*b^4+40*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/
(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a*b^5-256*(b/(
a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a
-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2
),((a-b)/(a+b))^(1/2))*a^6+488*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+
b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE(
(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-232*(b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-
b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
,((a-b)/(a+b))^(1/2))*a^2*b^4)/(a+b*sin(d*x+c))^(3/2)/b^7/cos(d*x+c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^6}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\cos(dx+c)^6}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^6/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^6}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/(b*sin(d*x + c) + a)^(5/2), x)
```

3.535 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=221

$$\frac{8\left(4a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^4d\sqrt{a+b\sin(c+dx)}}-\frac{4\cos(c+dx)(4a+b\sin(c+dx))}{3b^3d\sqrt{a+b\sin(c+dx)}}-\frac{32a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+\right.\right.}{3b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] (-2*Cos[c + d*x]^3)/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) - (32*a*EllipticE[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*b^4*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) + (8*(4*a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^4*d*Sqrt[a + b*S
in[c + d*x]]) - (4*Cos[c + d*x]*(4*a + b*Sin[c + d*x]))/(3*b^3*d*Sqrt[a + b
*Sin[c + d*x]])
```

Rubi [A] time = 0.319578, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2693, 2863, 2752, 2663, 2661, 2655, 2653}

$$\frac{8\left(4a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^4d\sqrt{a+b\sin(c+dx)}}-\frac{4\cos(c+dx)(4a+b\sin(c+dx))}{3b^3d\sqrt{a+b\sin(c+dx)}}-\frac{32a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+\right.\right.}{3b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] (-2*Cos[c + d*x]^3)/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) - (32*a*EllipticE[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*b^4*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) + (8*(4*a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^4*d*Sqrt[a + b*S
in[c + d*x]]) - (4*Cos[c + d*x]*(4*a + b*Sin[c + d*x]))/(3*b^3*d*Sqrt[a + b
*Sin[c + d*x]])
```


Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
```

```
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{2\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx}{b}$$

$$= -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx)(4a+b\sin(c+dx))}{3b^3d\sqrt{a+b\sin(c+dx)}} + \frac{8\int \frac{-\frac{b}{2}-2a\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{3b^3}$$

$$= -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx)(4a+b\sin(c+dx))}{3b^3d\sqrt{a+b\sin(c+dx)}} - \frac{(16a)\int \sqrt{a+b\sin(c+a}}{3b^4}$$

$$= -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx)(4a+b\sin(c+dx))}{3b^3d\sqrt{a+b\sin(c+dx)}} - \frac{\left(16a\sqrt{a+b\sin(c+dx)}\right)}{3b^4\sqrt{\frac{a+b}{}}}$$

$$= -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{32aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{3b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{8\left(4a^2-b^2\right.}{}$$

Mathematica [A] time = 1.02733, size = 174, normalized size = 0.79

$$\frac{b\cos(c+dx)\left(-16a^2-20ab\sin(c+dx)+b^2\cos(2(c+dx))-3b^2\right)-8\left(4a^2-b^2\right)(a+b)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}F\left(\frac{1}{4}(-2c-2d\right.}{3b^4d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (32*a*(a + b)^2*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin
[c + d*x])/(a + b))^(3/2) - 8*(a + b)*(4*a^2 - b^2)*EllipticF[(-2*c + Pi -
2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin[c + d*x])/(a + b))^(3/2) + b*Cos[c + d
*x]*(-16*a^2 - 3*b^2 + b^2*Cos[2*(c + d*x)] - 20*a*b*Sin[c + d*x]))/(3*b^4*
d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [B] time = 0.497, size = 1047, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3*(10*a*b^3*cos(d*x+c)^2*sin(d*x+c)+4*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2
)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*
b*(4*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^
2*b-3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a
*b^2-EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^
3-4*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3
+4*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^
2)*sin(d*x+c)-b^4*cos(d*x+c)^4+(8*a^2*b^2+2*b^4)*cos(d*x+c)^2+16*EllipticF(
(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x
+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2)*a^3*b-12*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((
a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)
+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^2-4*EllipticF((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c
)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/
(a-b)*a)^(1/2)*a*b^3-16*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d
*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b
)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4+16*(-b/(a-b)*sin(d*x
+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b
))^(1/2))*a^2*b^2)/(a+b*sin(d*x+c))^(3/2)/b^5/cos(d*x+c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\cos\left(dx+c\right)^{4}}{3\,ab^{2}\cos\left(dx+c\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(dx+c\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{4}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c) + a)^(5/2), x)
```

3.536 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=219

$$\frac{4a\cos(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}+\frac{4a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] (-2*Cos[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) + (4*a*Cos[c + d*x])/(
3*b*(a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) + (4*a*EllipticE[(c - Pi/2 + d*
x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*b^2*(a^2 - b^2)*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) - (4*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.262706, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2693, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a\cos(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}+\frac{4a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) + (4*a*Cos[c + d*x])/(
3*b*(a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) + (4*a*EllipticE[(c - Pi/2 + d*
x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*b^2*(a^2 - b^2)*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) - (4*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{2\int \frac{\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{4a\cos(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{4\int \frac{\frac{b}{2}+\frac{1}{2}a\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{4a\cos(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} - \frac{2\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx}{3b^2} + (2a$$

$$= -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{4a\cos(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{\left(2a\sqrt{a+b\sin(c+dx)}\right)\int}{3b^2\left(a^2-b^2\right)\sqrt{}}$$

$$= -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{4a\cos(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}} + \frac{4aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)}{3b^2\left(a^2-b^2\right)d\sqrt{}}$$

Mathematica [A] time = 1.00898, size = 167, normalized size = 0.76

$$\frac{2b\cos(c+dx)\left(a^2+2ab\sin(c+dx)+b^2\right)+4(a-b)(a+b)^2\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)-4a(a+b)^2\left(\frac{a+b\sin}{}\right.}{3b^2d(a-b)(a+b)(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*a*(a + b)^2*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin
[c + d*x])/(a + b))^(3/2) + 4*(a - b)*(a + b)^2*EllipticF[(-2*c + Pi - 2*d*
x)/4, (2*b)/(a + b)]*((a + b*Sin[c + d*x])/(a + b))^(3/2) + 2*b*Cos[c + d*x
]*(a^2 + b^2 + 2*a*b*Sin[c + d*x]))/(3*(a - b)*b^2*(a + b)*d*(a + b*Sin[c +
 d*x])^(3/2))
```

Maple [B] time = 0.535, size = 864, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/3*(2*a*b^3*cos(d*x+c)^2*sin(d*x+c)-2*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*
(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b*
(EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3-El
lipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2-Ell
ipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b+Elli
pticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^3)*sin(d*
x+c)+(a^2*b^2+b^4)*cos(d*x+c)^2+2*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*si
n(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b-2*Ellipt
icF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin
(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x
+c)+1/(a-b)*a)^(1/2)*a*b^3-2*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*
sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4+2*(-b/(a-b)*sin
(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x
+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/
(a+b))^(1/2))*a^2*b^2)/(a^2-b^2)/(a+b*sin(d*x+c))^(3/2)/b^3/cos(d*x+c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\cos(dx+c)^2}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^2}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

3.537 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=325

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(27a^2+5b^2\right)-a\left(3a^2+29b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^3}+\frac{16ab\sec(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}+\frac{\cdots}{3d\left(\cdots\right.}$$

```
[Out] (2*b*Sec[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + (16*a*b*S
ec[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (a*(3*a^2 + 29*
b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])
/(3*(a^2 - b^2)^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((3*a^2 + 5*b^2)*
EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
 b)])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]]*(b*(27*a^2 + 5*b^2) - a*(3*a^2 + 29*b^2)*Sin[c + d*x]))/(3
*(a^2 - b^2)^3*d)
```

Rubi [A] time = 0.609488, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2694, 2864, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(27a^2+5b^2\right)-a\left(3a^2+29b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^3}+\frac{16ab\sec(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}+\frac{\cdots}{3d\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^(5/2),x]
```


```
[Out] (2*b*Sec[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + (16*a*b*S
ec[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (a*(3*a^2 + 29*
b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])
/(3*(a^2 - b^2)^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((3*a^2 + 5*b^2)*
EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
 b)])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]]*(b*(27*a^2 + 5*b^2) - a*(3*a^2 + 29*b^2)*Sin[c + d*x]))/(3
*(a^2 - b^2)^3*d)
```


Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
```

```
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} - \frac{2\int \frac{\sec^2(c+dx)\left(-\frac{3a}{2}+\frac{5}{2}b\sin(c+dx)\right)}{(a+b\sin(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} + \frac{4\int \frac{\sec^2(c+dx)\left(\frac{1}{4}(3a\cdots\right.}{\sqrt{a+b\cdots}}}{3\left(a^{\cdots}\right.}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+dx)\sqrt{a+\cdots}}{\cdots}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+dx)\sqrt{a+\cdots}}{\cdots}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+dx)\sqrt{a+\cdots}}{\cdots}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{a\left(3a^2+29b^2\right)E\left(\cdots\right.}{3\left(\cdots\right.}$$

Mathematica [A] time = 1.84628, size = 241, normalized size = 0.74

$$\frac{\frac{\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}\left(\left(3a^2b-3a^3-5ab^2+5b^3\right)F\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)+\left(3a^3+29ab^2\right)E\left(\frac{1}{4}(-2c-2dx+\pi)|\frac{2b}{a+b}\right)\right)}{(a-b)^3(a+b)}-\frac{2b^3\left(a^2-b^2\right)\cos(c+dx)+3\sec(c+dx)(a+b\sin(c+dx))^2(}{}}{3d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((((3*a^3 + 29*a*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] + (-3
*a^3 + 3*a^2*b - 5*a*b^2 + 5*b^3)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
 + b)])*((a + b*Sin[c + d*x])/(a + b))^(3/2))/((a - b)^3*(a + b)) - (2*b^3*
(a^2 - b^2)*Cos[c + d*x] + 20*a*b^3*Cos[c + d*x]*(a + b*Sin[c + d*x]) + 3*S
ec[c + d*x]*(a + b*Sin[c + d*x])^2*(3*a^2*b + b^3 - a*(a^2 + 3*b^2)*Sin[c +
 d*x]))/(a^2 - b^2)^3)/(3*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [B] time = 3.105, size = 1653, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] (-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*(-2*a*b^2/(a+b)^2/(a-b)^2*(2*b*cos(
d*x+c)^2/(a^2-b^2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)+2*a/(a^2-b^2)*(a
/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(
d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*EllipticF(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+2/(a^2-b^2)*b*(a/b-1)*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-
1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-a/b-1)*Ellipti
cE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))-b^2/(a+b)/(a-b)*(2/3/(a^2-b^2)/
b*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)/(sin(d*x+c)+a/b)^2+8/3*b*cos(d*x+
c)^2/(a^2-b^2)^2*a/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)+2*(3*a^2+b^2)/(3
*a^4-6*a^2*b^2+3*b^4)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+
c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*
x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
+8/3*a*b/(a^2-b^2)^2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c
))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x
```

```
+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))+1
/2/(a+b)^3/b/cos(d*x+c)^2/(a+b*sin(d*x+c))*(cos(d*x+c)^2*sin(d*x+c)*b+a*cos
(d*x+c)^2)^(1/2)*((-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+
b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2-(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))
*b^2-b^2*cos(d*x+c)^2+a*b*sin(d*x+c)+b^2*sin(d*x+c)+a*b+b^2)+1/2/(a-b)^2*(-
(-sin(d*x+c)^2*b-a*sin(d*x+c)+b*sin(d*x+c)+a)/(a-b)/((-b*sin(d*x+c)-a)*(sin
(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)-2*b/(2*a-2*b)*(a/b-1)*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(
-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))-b/(a-b)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b
*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+
c)-a)*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2)))))/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^2}{(b\sin (dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin (dx+c)+a}\sec (dx+c)^2}{3\,ab^2\cos (dx+c)^2-a^3-3\,ab^2+\left(b^3\cos (dx+c)^2-3\,a^2b-b^3\right)\sin (dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^2/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

3.538 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=425

$$-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(29a^2+3b^2\right)-a\left(a^2+31b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^3}+\frac{8ab\sec^3(c+dx)}{d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}+\frac{}{3d\left(a\right.}$$

```
[Out] (2*b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + (8*a*b*
Sec[c + d*x]^3)/((a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (2*a*(a^4 - 6*
a^2*b^2 - 27*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(3*(a^2 - b^2)^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((4*
a^4 - 21*a^2*b^2 - 15*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)])/(6*(a^2 - b^2)^3*d*Sqrt[a + b*Sin[c + d*x]
]) - (Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(b*(29*a^2 + 3*b^2) - a*(a^2
+ 31*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^3*d) - (Sec[c + d*x]*Sqrt[a + b*Sin
[c + d*x]]*(b*(a^4 - 114*a^2*b^2 - 15*b^4) - 4*a*(a^4 - 6*a^2*b^2 - 27*b^4)
*Sin[c + d*x]))/(6*(a^2 - b^2)^4*d)
```

Rubi [A] time = 0.87617, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2694, 2864, 2866, 2752, 2663, 2661, 2655, 2653}

$$-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(29a^2+3b^2\right)-a\left(a^2+31b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^3}+\frac{8ab\sec^3(c+dx)}{d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}+\frac{}{3d\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```



```
[Out] (2*b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + (8*a*b*
Sec[c + d*x]^3)/((a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (2*a*(a^4 - 6*
a^2*b^2 - 27*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(3*(a^2 - b^2)^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((4*
a^4 - 21*a^2*b^2 - 15*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)])/(6*(a^2 - b^2)^3*d*Sqrt[a + b*Sin[c + d*x]
]) - (Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(b*(29*a^2 + 3*b^2) - a*(a^2
+ 31*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^3*d) - (Sec[c + d*x]*Sqrt[a + b*Sin
[c + d*x]]*(b*(a^4 - 114*a^2*b^2 - 15*b^4) - 4*a*(a^4 - 6*a^2*b^2 - 27*b^4)
```

```
*Sin[c + d*x]))/(6*(a^2 - b^2)^4*d)
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{2b\sec^3(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} - \frac{2\int \frac{\sec^4(c+dx)\left(-\frac{3a}{2}+\frac{9}{2}b\sin(c+dx)\right)}{(a+b\sin(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2b\sec^3(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} + \frac{4\int \frac{\sec^4(c+dx)\left(\frac{3}{4}\left(a^2+3b^2 \right.\right.}{\sqrt{a+b\sin}}}{3\left(a^2- \right.}$$

$$= \frac{2b\sec^3(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{2b\sec^3(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{2b\sec^3(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{2b\sec^3(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{2b\sec^3(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{2a\left(a^4-6a^2b^2-27b\right.}{3}$$

<u>Mathematica</u> [A] time = 2.4375, size = 341, normalized size = 0.8

$$\frac{\frac{4b^5\left(a^2-b^2\right)\cos(c+dx)+2\left(a^2-b^2\right)\sec^3(c+dx)(a+b\sin(c+dx))^2\left(a\left(a^2+3b^2\right)\sin(c+dx)-b\left(3a^2+b^2\right)\right)+\sec(c+dx)(a+b\sin(c+dx))^2\left(4a\left(-6a^2b^2+a^4-11b^4\right)\sin(c+dx)+5\right.}{\left(a^2-b^2\right)^4}}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((4*(a^5 - 6*a^3*b^2 - 27*a*b^4)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
 + b)] + (-4*a^5 + 4*a^4*b + 21*a^3*b^2 - 21*a^2*b^3 + 15*a*b^4 - 15*b^5)*E
llipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((a + b*Sin[c + d*x])/(a +
b))^(3/2))/((a - b)^4*(a + b)^2) + (4*b^5*(a^2 - b^2)*Cos[c + d*x] + 64*a*b
```

```
^5*Cos[c + d*x]*(a + b*Sin[c + d*x]) + 2*(a^2 - b^2)*Sec[c + d*x]^3*(a + b*
Sin[c + d*x])^2*(-(b*(3*a^2 + b^2)) + a*(a^2 + 3*b^2)*Sin[c + d*x]) + Sec[c
 + d*x]*(a + b*Sin[c + d*x])^2*(-(a^4*b) + 54*a^2*b^3 + 11*b^5 + 4*a*(a^4 -
 6*a^2*b^2 - 11*b^4)*Sin[c + d*x]))/(a^2 - b^2)^4)/(6*d*(a + b*Sin[c + d*x]
)^(3/2))
```

Maple [B] time = 4.317, size = 2585, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] (-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*(4*a*b^4/(a+b)^3/(a-b)^3*(2*b*cos(d
*x+c)^2/(a^2-b^2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)+2*a/(a^2-b^2)*(a/
b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d
*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+2/(a^2-b^2)*b*(a/b-1)*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1
)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-a/b-1)*Elliptic
E(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))+b^4/(a+b)^2/(a-b)^2*(2/3/(a^2-b^
2)/b*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)/(sin(d*x+c)+a/b)^2+8/3*b*cos(d
*x+c)^2/(a^2-b^2)^2*a/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)+2*(3*a^2+b^2)
/(3*a^4-6*a^2*b^2+3*b^4)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d
*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos
(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))+8/3*a*b/(a^2-b^2)^2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*
x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(
d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)))
)-1/4*(-a-3*b)/(a+b)^4/b/cos(d*x+c)^2/(a+b*sin(d*x+c))*(cos(d*x+c)^2*sin(d*
x+c)*b+a*cos(d*x+c)^2)^(1/2)*((-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticE((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2-(-b/(a-b)*sin(
d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+
c)+1/(a-b)*a)^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(
a+b))^(1/2))*b^2-b^2*cos(d*x+c)^2+a*b*sin(d*x+c)+b^2*sin(d*x+c)+a*b+b^2)+1/
4/(a-b)^2*(-1/3/(a-b)*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)/(1+sin(d*x+c)
)^2-1/3*(-sin(d*x+c)^2*b-a*sin(d*x+c)+b*sin(d*x+c)+a)/(a-b)^2*(a-3*b)/((-b*
sin(d*x+c)-a)*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)+2*b^2/(3*a^2-6*a*b+3*b^2
```

```
)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-
sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*Ellipt
icF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))-1/3*b*(a-3*b)/(a-b)
^2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((
-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-a/
b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))+1/4*(a-3*b)/(a-b)^3
*(-(-sin(d*x+c)^2*b-a*sin(d*x+c)+b*sin(d*x+c)+a)/(a-b)/((-b*sin(d*x+c)-a)*(
sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)-2*b/(2*a-2*b)*(a/b-1)*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2
)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))-b/(a-b)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d
*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))))+1/4/(a+b)^2*(1/3/(a+b)*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/
2)/(sin(d*x+c)-1)^2-1/3*(-sin(d*x+c)^2*b-a*sin(d*x+c)-b*sin(d*x+c)-a)/(a+b)
^2*(a+3*b)/((-b*sin(d*x+c)-a)*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)+2*b^2/(3
*a^2+6*a*b+3*b^2)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/
(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)
^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))-1/3
*b*(a+3*b)/(a+b)^2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))
/(a+b))^(1/2)*((-sin(d*x+c)-1)*b/(a-b))^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c
)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)))))/co
s(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b\sin\left(dx+c\right)+a}\sec\left(dx+c\right)^{4}}{3\,ab^{2}\cos\left(dx+c\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(dx+c\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^4/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{4}}{\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sin(d*x + c) + a)^(5/2), x)
```

3.539 $\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=124

$$\frac{10ae^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}+\frac{10ae^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{5/2}}{7d}-\frac{2b(e\cos(c+}{9de}$$

```
[Out] (-2*b*(e*Cos[c + d*x])^(9/2))/(9*d*e) + (10*a*e^4*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (10*a*e^3*Sqrt[e*Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*a*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(
7*d)
```

Rubi [A] time = 0.0921037, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2635, 2642, 2641}

$$\frac{10ae^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}+\frac{10ae^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{5/2}}{7d}-\frac{2b(e\cos(c+}{9de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x]),x]
```



```
[Out] (-2*b*(e*Cos[c + d*x])^(9/2))/(9*d*e) + (10*a*e^4*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (10*a*e^3*Sqrt[e*Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*a*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(
7*d)
```


Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))\,dx = -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + a\int (e\cos(c+dx))^{7/2}\,dx$$
$$= -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + \frac{2ae(e\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}\left(5ae^2\right)\int (e\,\mathrm{co}$$
$$= -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2ae(e\cos(c+dx}{7}$$
$$= -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2ae(e\cos(c+dx}{7}$$
$$= -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}} + \frac{10ae^3\sqrt{e\cos(}{}$$

Mathematica [A] time = 0.824756, size = 104, normalized size = 0.84

$$\frac{e^3\sqrt{e\cos(c+dx)}\left(\sqrt{\cos(c+dx)}(138a\sin(c+dx)+18a\sin(3(c+dx))-28b\cos(2(c+dx))-7b\cos(4(c+dx))-21b)+\right.}{252d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] (e^3*Sqrt[e*Cos[c + d*x]]*(120*a*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d
*x]]*(-21*b - 28*b*Cos[2*(c + d*x)] - 7*b*Cos[4*(c + d*x)] + 138*a*Sin[c +
d*x] + 18*a*Sin[3*(c + d*x)])))/(252*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.99, size = 259, normalized size = 2.1

$$-\frac{2e^4}{63d}\left(-224b(\sin(1/2dx+c/2))^{11}+144a\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^8+560b(\sin(1/2dx+c/2))^9-216a(\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] -2/63/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(-224*b*si
n(1/2*d*x+1/2*c)^11+144*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+560*b*sin
(1/2*d*x+1/2*c)^9-216*a*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-560*b*sin(1
/2*d*x+1/2*c)^7+168*a*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+280*b*sin(1/2
*d*x+1/2*c)^5+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-48*a*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-70*b*sin(1/2*d*x+1/2*c)^3+7*b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{7}{2}}(b\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(be^3\cos(dx+c)^3\sin(dx+c)+ae^3\cos(dx+c)^3\right)\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*e^3*cos(d*x + c)^3*sin(d*x + c) + a*e^3*cos(d*x + c)^3)*sqrt(e*
cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos (dx+c))^{\frac{7}{2}} (b\sin (dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a), x)
```

3.540 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=95

$$\frac{6ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{3/2}}{5d}-\frac{2b(e\cos(c+dx))^{7/2}}{7de}$$

```
[Out] (-2*b*(e*Cos[c + d*x])^(7/2))/(7*d*e) + (6*a*e^2*Sqrt[e*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a*e*(e*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0730842, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2635, 2640, 2639}

$$\frac{6ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{3/2}}{5d}-\frac{2b(e\cos(c+dx))^{7/2}}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(e*Cos[c + d*x])^(7/2))/(7*d*e) + (6*a*e^2*Sqrt[e*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a*e*(e*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(5*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))\,dx &= -\frac{2b(e\cos(c+dx))^{7/2}}{7de} + a\int (e\cos(c+dx))^{5/2}\,dx \\
&= -\frac{2b(e\cos(c+dx))^{7/2}}{7de} + \frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\left(3ae^2\right)\int \sqrt{e\,\mathrm{c} \\
&= -\frac{2b(e\cos(c+dx))^{7/2}}{7de} + \frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\left(3ae^2\sqrt{e\cos(c+}\right.}{5\sqrt{}} \\
&= -\frac{2b(e\cos(c+dx))^{7/2}}{7de} + \frac{6ae^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2ae(e\cos(c+}{}
\end{aligned}
$$

Mathematica [A] time = 0.482203, size = 79, normalized size = 0.83

$$
\frac{(e\cos(c+dx))^{5/2}\left(\cos^{\frac{3}{2}}(c+dx)(14a\sin(c+dx)-5b\cos(2(c+dx))-5b)+42aE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{35d\cos^{\frac{5}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(42*a*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^(3/2
)*(-5*b - 5*b*Cos[2*(c + d*x)] + 14*a*Sin[c + d*x])))/(35*d*Cos[c + d*x]^(5
/2))
```

Maple [B] time = 1.102, size = 222, normalized size = 2.3

$$\frac{2\,e^{3}}{35\,d}\left(-80\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{9}+56\,a\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}\cos\left(1/2\,dx+c/2\right)+160\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{7}-56\,a\left(\sin\left(1/2\,\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] 2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(-80*b*sin(
1/2*d*x+1/2*c)^9+56*a*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+160*b*sin(1/2
*d*x+1/2*c)^7-56*a*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-120*b*sin(1/2*d*
x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+14*a*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c)+40*b*sin(1/2*d*x+1/2*c)^3-5*b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}\,(b\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(be^{2}\cos(dx+c)^{2}\sin(dx+c)+ae^{2}\cos(dx+c)^{2}\right)\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*e^2*cos(d*x + c)^2*sin(d*x + c) + a*e^2*cos(d*x + c)^2)*sqrt(e*
cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a), x)
```

3.541 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=95

$$\frac{2ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}+\frac{2ae\sin(c+dx)\sqrt{e\cos(c+dx)}}{3d}-\frac{2b(e\cos(c+dx))^{5/2}}{5de}$$

```
[Out] (-2*b*(e*Cos[c + d*x])^(5/2))/(5*d*e) + (2*a*e^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) + (2*a*e*Sqrt[e*Cos[c + d*x]
]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.071392, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2635, 2642, 2641}

$$\frac{2ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}+\frac{2ae\sin(c+dx)\sqrt{e\cos(c+dx)}}{3d}-\frac{2b(e\cos(c+dx))^{5/2}}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(e*Cos[c + d*x])^(5/2))/(5*d*e) + (2*a*e^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) + (2*a*e*Sqrt[e*Cos[c + d*x]
]*Sin[c + d*x])/(3*d)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))\,dx &= -\frac{2b(e\cos(c+dx))^{5/2}}{5de}+a\int (e\cos(c+dx))^{3/2}\,dx\\ &= -\frac{2b(e\cos(c+dx))^{5/2}}{5de}+\frac{2ae\sqrt{e\cos(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}\left(ae^2\right)\int \frac{1}{\sqrt{e\cos(}}\\ &= -\frac{2b(e\cos(c+dx))^{5/2}}{5de}+\frac{2ae\sqrt{e\cos(c+dx)}\sin(c+dx)}{3d}+\frac{\left(ae^2\sqrt{\cos(c+dx)}\right)}{3\sqrt{e\cos(}}\\ &= -\frac{2b(e\cos(c+dx))^{5/2}}{5de}+\frac{2ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}+\frac{2ae\sqrt{e\cos(c+}}{3}\end{aligned}$$

Mathematica [A] time = 0.463928, size = 79, normalized size = 0.83

$$\frac{(e\cos(c+dx))^{3/2}\left(\sqrt{\cos(c+dx)}(10a\sin(c+dx)-3b\cos(2(c+dx))-3b)+10aF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(10*a*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]
]*(-3*b - 3*b*Cos[2*(c + d*x)] + 10*a*Sin[c + d*x])))/(15*d*Cos[c + d*x]^(3
/2))
```

Maple [A] time = 1.019, size = 185, normalized size = 2.

$$-\frac{2e^2}{15d}\left(-24b(\sin(1/2dx+c/2))^7+20a(\sin(1/2dx+c/2))^4\cos(1/2dx+c/2)+36b(\sin(1/2dx+c/2))^5+5\sqrt{(\sin(1/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] -2/15/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(-24*b*sin
(1/2*d*x+1/2*c)^7+20*a*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+36*b*sin(1/2
*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-10*a*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)-18*b*sin(1/2*d*x+1/2*c)^3+3*b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((be\cos(dx+c)\sin(dx+c)+ae\cos(dx+c))\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*e*cos(d*x + c)*sin(d*x + c) + a*e*cos(d*x + c))*sqrt(e*cos(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a), x)
```

3.542 $\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=63

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2b(e\cos(c+dx))^{3/2}}{3de}$$

```
[Out] (-2*b*(e*Cos[c + d*x])^(3/2))/(3*d*e) + (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0508737, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2669, 2640, 2639}

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2b(e\cos(c+dx))^{3/2}}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]),x]
```


```
[Out] (-2*b*(e*Cos[c + d*x])^(3/2))/(3*d*e) + (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```


Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))\,dx &= -\frac{2b(e\cos(c+dx))^{3/2}}{3de}+a\int \sqrt{e\cos(c+dx)}\,dx \\
&= -\frac{2b(e\cos(c+dx))^{3/2}}{3de}+\frac{\left(a\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2b(e\cos(c+dx))^{3/2}}{3de}+\frac{2a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.101375, size = 56, normalized size = 0.89

$$
-\frac{2\sqrt{e\cos(c+dx)}\left(b\cos^{\frac{3}{2}}(c+dx)-3aE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*(b*Cos[c + d*x]^(3/2) - 3*a*EllipticE[(c + d*x)/2,
 2]))/(3*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.753, size = 123, normalized size = 2.

$$
\frac{2e}{3d}\left(-4b\left(\sin\left(1/2\,dx+c/2\right)\right)^{5}+3\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\mathrm{EllipticE}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)a
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(-4*b*sin(1/2*
d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
```

```
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+4*b*sin(1/2*d*x+1/2*c)^3-b*sin(1/
2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a), x)
```

3.543 $\int \frac{a+b\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2b\sqrt{e\cos(c+dx)}}{de}$$

```
[Out] (-2*b*Sqrt[e*Cos[c + d*x]])/(d*e) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.0517239, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2669, 2642, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2b\sqrt{e\cos(c+dx)}}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*b*Sqrt[e*Cos[c + d*x]])/(d*e) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx &= -\frac{2b\sqrt{e\cos(c+dx)}}{de}+a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx \\
&= -\frac{2b\sqrt{e\cos(c+dx)}}{de}+\frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{e\cos(c+dx)}} \\
&= -\frac{2b\sqrt{e\cos(c+dx)}}{de}+\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.195861, size = 50, normalized size = 0.82

$$
\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-2b\cos(c+dx)}{d\sqrt{e\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*b*Cos[c + d*x] + 2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*S
qrt[e*Cos[c + d*x]])
```

Maple [A] time = 0.594, size = 106, normalized size = 1.7

$$
-2\,\frac{\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)a-2\,b\,(\sin(1/2\,dx+c/2))^3+b\,s}{\sin(1/2\,dx+c/2)\,\sqrt{-2\,(\sin(1/2\,dx+c/2))^2\,e+ed}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*((sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
```

```
2^(1/2))*a-2*b*sin(1/2*d*x+1/2*c)^3+b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin(dx+c)+a}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)}{e\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)/(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin(dx+c)+a}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

3.544 $\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}+\frac{2b}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] (2*b)/(d*e*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(d*e*Sqrt[e*Co
s[c + d*x]])
```

Rubi [A] time = 0.0736677, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2636, 2640, 2639}

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}+\frac{2b}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*b)/(d*e*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(d*e*Sqrt[e*Co
s[c + d*x]])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{2b}{de\sqrt{e\cos(c+dx)}}+a\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2b}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}-\frac{a\int \sqrt{e\cos(c+dx)}\,dx}{e^2}\\ &= \frac{2b}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}-\frac{\left(a\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{e^2\sqrt{\cos(c+dx)}}\\ &= \frac{2b}{de\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.116442, size = 54, normalized size = 0.59

$$\frac{2\left(a\sin(c+dx)-a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\right)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(b - a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + a*Sin[c + d*x]))/(
d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 1.112, size = 119, normalized size = 1.3

$$-2\,\frac{\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)a-2\,a\,(\sin(1/2\,dx+c/2))^2\cos}{e\sqrt{-2\,(\sin(1/2\,dx+c/2))^2e+e}\sin(1/2\,dx+c/2)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*((sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*a-2*a*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-b*sin(1/2*d*x+1/2*
c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin(dx+c)+a}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)}{e^2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)/(e^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \sin(dx + c) + a}{(e \cos(dx + c))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

3.545 $\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}+\frac{2b}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] (2*b)/(3*d*e*(e*Cos[c + d*x])^(3/2)) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(3*d*e*(
e*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0716748, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2636, 2642, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}+\frac{2b}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```


```
[Out] (2*b)/(3*d*e*(e*Cos[c + d*x])^(3/2)) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(3*d*e*(
e*Cos[c + d*x])^(3/2))
```


Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx &= \frac{2b}{3de(e\cos(c+dx))^{3/2}}+a\int \frac{1}{(e\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2b}{3de(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}+\frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2} \\
&= \frac{2b}{3de(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}+\frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3e^2\sqrt{e\cos(c+dx)}} \\
&= \frac{2b}{3de(e\cos(c+dx))^{3/2}}+\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.145656, size = 55, normalized size = 0.57

$$\frac{2\left(a\sin(c+dx)+a\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+b\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + a*Sin[c + d*x]))/(
3*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [A] time = 1.668, size = 193, normalized size = 2.

$$-\frac{2}{3\,d e^{2}}\left(2\sqrt{(\sin(1/2\,dx+c/2))^{2}}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2)\,,\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^{2}-1}a\,(\sin(1/2\,dx+c/2))^{2}-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*sin(1/2*d*x+1/2*c)^2-(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*a+2*a*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+b*sin(1/2*d*
x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\sin(dx+c)+a}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)}{e^{3}\cos(dx+c)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)/(e^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

3.546 $\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=126

$$\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{2b}{5de(e\cos(c+dx))^{5/2}}$$

```
[Out] (2*b)/(5*d*e*(e*Cos[c + d*x])^(5/2)) - (6*a*Sqrt[e*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(5*d*e*(
e*Cos[c + d*x])^(5/2)) + (6*a*Sin[c + d*x])/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.0914813, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2669, 2636, 2640, 2639}

$$\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{2b}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```



```
[Out] (2*b)/(5*d*e*(e*Cos[c + d*x])^(5/2)) - (6*a*Sqrt[e*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(5*d*e*(
e*Cos[c + d*x])^(5/2)) + (6*a*Sin[c + d*x])/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```


Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx &= \frac{2b}{5de(e\cos(c+dx))^{5/2}}+a\int \frac{1}{(e\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2b}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{(3a)\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx}{5e^2} \\
&= \frac{2b}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{(3a)\int \sqrt{e\cos(c+dx)}\,d}{5e^4} \\
&= \frac{2b}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{\left(3a\sqrt{e\cos(c+dx)}\right)\int \sqrt{}}{5e^4\sqrt{\cos(c+}} \\
&= \frac{2b}{5de(e\cos(c+dx))^{5/2}}-\frac{6a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{6a\sin}{5de^3\sqrt{e\,c}}
\end{aligned}
$$

Mathematica [A] time = 0.325333, size = 70, normalized size = 0.56

$$
\frac{7a\sin(c+dx)+3a\sin(3(c+dx))-12a\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+4b}{10de(e\cos(c+dx))^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (4*b - 12*a*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 7*a*Sin[c + d*x]
 + 3*a*Sin[3*(c + d*x)])/(10*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [B] time = 2.526, size = 310, normalized size = 2.5

$$-\frac{2}{5\,d e^{3}}\left(12\sqrt{2\,(\sin(1/2\,dx+c/2))^{2}-1}\sqrt{(\sin(1/2\,dx+c/2))^{2}}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2)\,,\sqrt{2}\right)a\,(\sin(1/2\,dx+c/2))^{4}-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*sin(1
/2*d*x+1/2*c)^4-24*a*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a*sin(1/2*d*x+1/2*c)^2+24*a*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-8*a*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)-b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\sin(dx+c)+a}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)}{e^{4}\cos(dx+c)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)/(e^4*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \sin (dx + c) + a}{(e \cos (dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

3.547 $\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=188

$$\frac{10e^3\left(11a^2+2b^2\right)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{10e^4\left(11a^2+2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{2e\left(11a^2+2b^2\right)\sin(c+\right.}{77d}$$

```
[Out] (-26*a*b*(e*Cos[c + d*x])^(9/2))/(99*d*e) + (10*(11*a^2 + 2*b^2)*e^4*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (10*
(11*a^2 + 2*b^2)*e^3*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(11*a^
2 + 2*b^2)*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(77*d) - (2*b*(e*Cos[c +
d*x])^(9/2)*(a + b*Sin[c + d*x]))/(11*d*e)
```

Rubi [A] time = 0.191299, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2692, 2669, 2635, 2642, 2641}

$$\frac{10e^3\left(11a^2+2b^2\right)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{10e^4\left(11a^2+2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{2e\left(11a^2+2b^2\right)\sin(c+\right.}{77d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-26*a*b*(e*Cos[c + d*x])^(9/2))/(99*d*e) + (10*(11*a^2 + 2*b^2)*e^4*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (10*
(11*a^2 + 2*b^2)*e^3*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(11*a^
2 + 2*b^2)*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(77*d) - (2*b*(e*Cos[c +
d*x])^(9/2)*(a + b*Sin[c + d*x]))/(11*d*e)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2\,dx = -\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{11de}+\frac{2}{11}\int (e\cos(c+dx))^{7/2}\left(\frac{11a^2}{2}+\right.$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}-\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{11de}+\frac{1}{11}\left(11a\right.$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}+\frac{2\left(11a^2+2b^2\right)e(e\cos(c+dx))^{5/2}\sin(c+dx)}{77d}-$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}+\frac{10\left(11a^2+2b^2\right)e^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{231d}+$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}+\frac{10\left(11a^2+2b^2\right)e^3\sqrt{e\cos(c+dx)}\sin(c+dx)}{231d}+$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}+\frac{10\left(11a^2+2b^2\right)e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 1.90765, size = 160, normalized size = 0.85

$$\frac{(e\cos(c+dx))^{7/2}\left(40\left(11a^2+2b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{1}{6}\sqrt{\cos(c+dx)}\left(6\left(572a^2+41b^2\right)\sin(c+dx)+8\cos(2(c+dx))\left(9\left(\right.\right.\right.\right.}{924d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*(-154*a*b*Sqrt[Cos[c + d*x]] + 40*(11*a^2 + 2*b^2)*
EllipticF[(c + d*x)/2, 2] + (Sqrt[Cos[c + d*x]]*(6*(572*a^2 + 41*b^2)*Sin[c
 + d*x] - 14*b*Cos[4*(c + d*x)]*(22*a + 9*b*Sin[c + d*x]) + 8*Cos[2*(c + d*
x)]*(-154*a*b + 9*(11*a^2 - 5*b^2)*Sin[c + d*x])))/6))/(924*d*Cos[c + d*x]^
(7/2))
```

Maple [B] time = 1.214, size = 473, normalized size = 2.5

$$-\frac{2\,e^4}{693\,d}\left(-4032\,b^2\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^{12}-4928\,ab\,(\sin(1/2\,dx+c/2))^{11}+10080\,b^2\,(\sin(1/2\,dx+c/2))^{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -2/693/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(-4032*b^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-4928*a*b*sin(1/2*d*x+1/2*c)^11+1
0080*b^2*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+1584*a^2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^8+12320*a*b*sin(1/2*d*x+1/2*c)^9-9792*b^2*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^8-2376*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^6-12320*a*b*sin(1/2*d*x+1/2*c)^7+4608*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^6+1848*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6160*a*b*sin(1/2*d
*x+1/2*c)^5-924*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+165*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a^2+30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-528*a^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2-1540*a*b*sin(1/2*d*x+1/2*c)^3+30*b^2*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2+154*sin(1/2*d*x+1/2*c)*a*b)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{7}{2}} (b\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2e^3\cos(dx+c)^5 - 2\,abe^3\cos(dx+c)^3\sin(dx+c) - \left(a^2+b^2\right)e^3\cos(dx+c)^3\right)\sqrt{e\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*e^3*cos(d*x + c)^5 - 2*a*b*e^3*cos(d*x + c)^3*sin(d*x + c) -
 (a^2 + b^2)*e^3*cos(d*x + c)^3)*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{7}{2}} (b\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^2, x)
```

3.548 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=149

$$\frac{2e^2\left(9a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2e\left(9a^2+2b^2\right)\sin(c+dx)(e\cos(c+dx))^{3/2}}{45d}-\frac{22ab(e\cos(c+dx))^{7/2}}{63de}$$

```
[Out] (-22*a*b*(e*Cos[c + d*x])^(7/2))/(63*d*e) + (2*(9*a^2 + 2*b^2)*e^2*Sqrt[e*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(9*a
^2 + 2*b^2)*e*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) - (2*b*(e*Cos[c +
 d*x])^(7/2)*(a + b*Sin[c + d*x]))/(9*d*e)
```

Rubi [A] time = 0.166829, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2692, 2669, 2635, 2640, 2639}

$$\frac{2e^2\left(9a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2e\left(9a^2+2b^2\right)\sin(c+dx)(e\cos(c+dx))^{3/2}}{45d}-\frac{22ab(e\cos(c+dx))^{7/2}}{63de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-22*a*b*(e*Cos[c + d*x])^(7/2))/(63*d*e) + (2*(9*a^2 + 2*b^2)*e^2*Sqrt[e*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(9*a
^2 + 2*b^2)*e*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) - (2*b*(e*Cos[c +
 d*x])^(7/2)*(a + b*Sin[c + d*x]))/(9*d*e)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2\,dx &= -\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{9de}+\frac{2}{9}\int (e\cos(c+dx))^{5/2}\left(\frac{9a^2}{2}+b^2+\right.\\ &= -\frac{22ab(e\cos(c+dx))^{7/2}}{63de}-\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{9de}+\frac{1}{9}\left(9a^2+\right.\\ &= -\frac{22ab(e\cos(c+dx))^{7/2}}{63de}+\frac{2\left(9a^2+2b^2\right)e(e\cos(c+dx))^{3/2}\sin(c+dx)}{45d}-\frac{2b(}{}\\ &= -\frac{22ab(e\cos(c+dx))^{7/2}}{63de}+\frac{2\left(9a^2+2b^2\right)e(e\cos(c+dx))^{3/2}\sin(c+dx)}{45d}-\frac{2b(}{}\\ &= -\frac{22ab(e\cos(c+dx))^{7/2}}{63de}+\frac{2\left(9a^2+2b^2\right)e^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\end{aligned}$$

Mathematica [A] time = 0.88886, size = 113, normalized size = 0.76

$$\frac{(e\cos(c+dx))^{5/2}\left(84\left(9a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^{\frac{3}{2}}(c+dx)\left(21\left(12a^2+b^2\right)\sin(c+dx)-5b(36a+7b\sin(3(c+dx)\right.\right.}{630d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(84*(9*a^2 + 2*b^2)*EllipticE[(c + d*x)/2, 2] + Cos
[c + d*x]^(3/2)*(-180*a*b*Cos[2*(c + d*x)] + 21*(12*a^2 + b^2)*Sin[c + d*x]
 - 5*b*(36*a + 7*b*Sin[3*(c + d*x)]))))/(630*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 1.271, size = 408, normalized size = 2.7

$$\frac{2e^3}{315d}\left(-1120b^2(\sin(1/2dx+c/2))^{10}\cos(1/2dx+c/2)-1440ab(\sin(1/2dx+c/2))^9+2240b^2\cos(1/2dx+c/2)(\sin(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] 2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(-1120*b^2
*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-1440*a*b*sin(1/2*d*x+1/2*c)^9+224
0*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+504*a^2*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^6+2880*a*b*sin(1/2*d*x+1/2*c)^7-1568*b^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^6-504*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2160
*a*b*sin(1/2*d*x+1/2*c)^5+448*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+1
89*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*a^2+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+126*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+720*a*b*sin(1/2*d*x+1/2*c)^3-42*b^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-90*sin(1/2*d*x+1/2*c)*a*b)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2e^2\cos(dx+c)^4-2\,abe^2\cos(dx+c)^2\sin(dx+c)-\left(a^2+b^2\right)e^2\cos(dx+c)^2\right)\sqrt{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*e^2*cos(d*x + c)^4 - 2*a*b*e^2*cos(d*x + c)^2*sin(d*x + c) -
 (a^2 + b^2)*e^2*cos(d*x + c)^2)*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^2, x)
```

3.549 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=149

$$\frac{2e^2\left(7a^2+2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{2e\left(7a^2+2b^2\right)\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}-\frac{18ab(e\cos(c+dx))^{5/2}}{35de}-2b$$

```
[Out] (-18*a*b*(e*Cos[c + d*x])^(5/2))/(35*d*e) + (2*(7*a^2 + 2*b^2)*e^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (2*(7*a
^2 + 2*b^2)*e*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(21*d) - (2*b*(e*Cos[c + d
*x])^(5/2)*(a + b*Sin[c + d*x]))/(7*d*e)
```

Rubi [A] time = 0.164014, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2692, 2669, 2635, 2642, 2641}

$$\frac{2e^2\left(7a^2+2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{2e\left(7a^2+2b^2\right)\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}-\frac{18ab(e\cos(c+dx))^{5/2}}{35de}-2b$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-18*a*b*(e*Cos[c + d*x])^(5/2))/(35*d*e) + (2*(7*a^2 + 2*b^2)*e^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (2*(7*a
^2 + 2*b^2)*e*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(21*d) - (2*b*(e*Cos[c + d
*x])^(5/2)*(a + b*Sin[c + d*x]))/(7*d*e)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2\,dx &= -\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{7de}+\frac{2}{7}\int (e\cos(c+dx))^{3/2}\left(\frac{7a^2}{2}+b^2+\right.\\ &= -\frac{18ab(e\cos(c+dx))^{5/2}}{35de}-\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{7de}+\frac{1}{7}\left(7a^2+\right.\\ &= -\frac{18ab(e\cos(c+dx))^{5/2}}{35de}+\frac{2\left(7a^2+2b^2\right)e\sqrt{e\cos(c+dx)}\sin(c+dx)}{21d}-\frac{2b(e\,c}{}\\ &= -\frac{18ab(e\cos(c+dx))^{5/2}}{35de}+\frac{2\left(7a^2+2b^2\right)e\sqrt{e\cos(c+dx)}\sin(c+dx)}{21d}-\frac{2b(e\,c}{}\\ &= -\frac{18ab(e\cos(c+dx))^{5/2}}{35de}+\frac{2\left(7a^2+2b^2\right)e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{2}{}\end{aligned}$$

Mathematica [A] time = 1.14132, size = 115, normalized size = 0.77

$$\frac{(e\cos(c+dx))^{3/2}\left(20\left(7a^2+2b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\left(5\left(28a^2+5b^2\right)\sin(c+dx)-3b(28a+5b\sin(3(c+dx\right.\right.}{210d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(20*(7*a^2 + 2*b^2)*EllipticF[(c + d*x)/2, 2] + Sqr
t[Cos[c + d*x]]*(-84*a*b*Cos[2*(c + d*x)] + 5*(28*a^2 + 5*b^2)*Sin[c + d*x]
 - 3*b*(28*a + 5*b*Sin[3*(c + d*x)]))))/(210*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 1.028, size = 343, normalized size = 2.3

$$-\frac{2\,e^{2}}{105\,d}\left(-240\,b^{2}\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}-336\,ab\left(\sin\left(1/2\,dx+c/2\right)\right)^{7}+360\,b^{2}\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -2/105/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(-240*b^2
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-336*a*b*sin(1/2*d*x+1/2*c)^7+360*b
^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+140*a^2*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^4+504*a*b*sin(1/2*d*x+1/2*c)^5-140*b^2*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^4+35*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+10*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*b^2-70*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-252*a*b*sin(1/2*d*x+1
/2*c)^3+10*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+42*sin(1/2*d*x+1/2*c
)*a*b)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\sin\left(dx+c\right)+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b^{2}e\cos\left(dx+c\right)^{3}-2\,abe\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(a^{2}+b^{2}\right)e\cos\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*e*cos(d*x + c)^3 - 2*a*b*e*cos(d*x + c)*sin(d*x + c) - (a^2
+ b^2)*e*cos(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\sin\left(dx+c\right)+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^2, x)
```

3.550 $\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=109

$$\frac{2\left(5a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{14ab(e\cos(c+dx))^{3/2}}{15de}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de}$$

```
[Out] (-14*a*b*(e*Cos[c + d*x])^(3/2))/(15*d*e) + (2*(5*a^2 + 2*b^2)*Sqrt[e*Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) - (2*b*(e*Cos[
c + d*x])^(3/2)*(a + b*Sin[c + d*x]))/(5*d*e)
```

Rubi [A] time = 0.129601, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2692, 2669, 2640, 2639}

$$\frac{2\left(5a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{14ab(e\cos(c+dx))^{3/2}}{15de}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-14*a*b*(e*Cos[c + d*x])^(3/2))/(15*d*e) + (2*(5*a^2 + 2*b^2)*Sqrt[e*Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) - (2*b*(e*Cos[
c + d*x])^(3/2)*(a + b*Sin[c + d*x]))/(5*d*e)
```


Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2\,dx &= -\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de}+\frac{2}{5}\int \sqrt{e\cos(c+dx)}\left(\frac{5a^2}{2}+b^2+\frac{7}{2}a\right.\\ &= -\frac{14ab(e\cos(c+dx))^{3/2}}{15de}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de}+\frac{1}{5}\left(5a^2+2\right.\\ &= -\frac{14ab(e\cos(c+dx))^{3/2}}{15de}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de}+\frac{\left(\left(5a^2+2b\right.\right.}{}\\ &= -\frac{14ab(e\cos(c+dx))^{3/2}}{15de}+\frac{2\left(5a^2+2b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}-\frac{2b(}{}\end{aligned}$$

Mathematica [A] time = 0.312287, size = 80, normalized size = 0.73

$$\frac{\sqrt{e\cos(c+dx)}\left(6\left(5a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)-2b\cos^{\frac{3}{2}}(c+dx)(10a+3b\sin(c+dx))\right)}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*(6*(5*a^2 + 2*b^2)*EllipticE[(c + d*x)/2, 2] - 2*b*Co
s[c + d*x]^(3/2)*(10*a + 3*b*Sin[c + d*x])))/(15*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 1.132, size = 251, normalized size = 2.3

$$\frac{2e}{15d}\left(-24\,b^2\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^6-40\,ab\left(\sin\left(1/2\,dx+c/2\right)\right)^5+24\,b^2\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(-24*b^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-40*a*b*sin(1/2*d*x+1/2*c)^5+24*b^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+6*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*b^2+40*a*b*sin(1/2*d*x+1/2*c)^3-6*b^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2-10*sin(1/2*d*x+1/2*c)*a*b)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos\left(dx+c\right)}\left(b\sin\left(dx+c\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2\right)\sqrt{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(e*cos(
d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^2, x)
```

3.551 $\int \frac{(a+b\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=109

$$\frac{2\left(3a^2+2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{10ab\sqrt{e\cos(c+dx)}}{3de}-\frac{2b\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de}$$

```
[Out] (-10*a*b*Sqrt[e*Cos[c + d*x]])/(3*d*e) + (2*(3*a^2 + 2*b^2)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) - (2*b*Sqrt[e*Cos
[c + d*x]]*(a + b*Sin[c + d*x]))/(3*d*e)
```

Rubi [A] time = 0.128876, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2692, 2669, 2642, 2641}

$$\frac{2\left(3a^2+2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{10ab\sqrt{e\cos(c+dx)}}{3de}-\frac{2b\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2/Sqrt[e*Cos[c + d*x]],x]
```


```
[Out] (-10*a*b*Sqrt[e*Cos[c + d*x]])/(3*d*e) + (2*(3*a^2 + 2*b^2)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) - (2*b*Sqrt[e*Cos
[c + d*x]]*(a + b*Sin[c + d*x]))/(3*d*e)
```


Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}\,dx &= -\frac{2b\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de}+\frac{2}{3}\int \frac{\frac{3a^2}{2}+b^2+\frac{5}{2}ab\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx\\ &= -\frac{10ab\sqrt{e\cos(c+dx)}}{3de}-\frac{2b\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de}+\frac{1}{3}\left(3a^2+2b^2\right)\int \frac{1}{\sqrt{e\cos(c+}}\\ &= -\frac{10ab\sqrt{e\cos(c+dx)}}{3de}-\frac{2b\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de}+\frac{\left(\left(3a^2+2b^2\right)\sqrt{\cos(c+dx)}\right)\int}{3\sqrt{e\cos(c+dx)}}\\ &= -\frac{10ab\sqrt{e\cos(c+dx)}}{3de}+\frac{2\left(3a^2+2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{2b\sqrt{e\cos(c+dx)}(a+}{3de}\end{aligned}$$

Mathematica [A] time = 0.392046, size = 75, normalized size = 0.69

$$\frac{2\left(3a^2+2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-2b\cos(c+dx)(6a+b\sin(c+dx))}{3d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (2*(3*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - 2*b*Cos[c
 + d*x]*(6*a + b*Sin[c + d*x]))/(3*d*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 0.777, size = 210, normalized size = 1.9

$$-\frac{2}{3d}\left(-4b^2\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^4+3\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\mathrm{EllipticF}\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-4*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*b^2-12*a*b*sin(1/2*d*x+1/2*c)^3+2*b^2*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2+6*sin(1/2*d*x+1/2*c)*a*b)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^2}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/sqrt(e*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2\right)\sqrt{e\cos(dx+c)}}{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(e*cos(
d*x + c))/(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^2}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/sqrt(e*cos(d*x + c)), x)
```

3.552 $\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{2\left(a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2ab(e\cos(c+dx))^{3/2}}{de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] (2*a*b*(e*Cos[c + d*x])^(3/2))/(d*e^3) - (2*(a^2 + 2*b^2)*Sqrt[e*Cos[c + d*
x]]*EllipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (2*(b + a*Sin[c
 + d*x])*(a + b*Sin[c + d*x]))/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.135237, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2691, 2669, 2640, 2639}

$$-\frac{2\left(a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2ab(e\cos(c+dx))^{3/2}}{de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(3/2),x]
```



```
[Out] (2*a*b*(e*Cos[c + d*x])^(3/2))/(d*e^3) - (2*(a^2 + 2*b^2)*Sqrt[e*Cos[c + d*
x]]*EllipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (2*(b + a*Sin[c
 + d*x])*(a + b*Sin[c + d*x]))/(d*e*Sqrt[e*Cos[c + d*x]])
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}} - \frac{2\int \sqrt{e\cos(c+dx)}\left(\frac{a^2}{2}+b^2+\frac{3}{2}ab\sin(c+dx)\right)dx}{e^2}\\ &= \frac{2ab(e\cos(c+dx))^{3/2}}{de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}} - \frac{\left(a^2+2b^2\right)\int \sqrt{e\cos(c+dx}}{e^2}\\ &= \frac{2ab(e\cos(c+dx))^{3/2}}{de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}} - \frac{\left(\left(a^2+2b^2\right)\sqrt{e\cos(c+dx)}\right.}{e^2\sqrt{\cos(c+}}\\ &= \frac{2ab(e\cos(c+dx))^{3/2}}{de^3} - \frac{2\left(a^2+2b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}} + \frac{2(b+a\sin(c+dx))(a}{de\sqrt{e\cos(c}}\end{aligned}$$

Mathematica [A] time = 0.233122, size = 71, normalized size = 0.63

$$\frac{2\left(a^2+b^2\right)\sin(c+dx)-2\left(a^2+2b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+4ab}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (4*a*b - 2*(a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(
a^2 + b^2)*Sin[c + d*x])/(d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 1.283, size = 197, normalized size = 1.7

$$-2\frac{\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)a^2+2\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*((sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*a^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-2*a^2*cos(1/2*d*x+1/2*c)*sin(
1/2*d*x+1/2*c)^2-2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*
x+1/2*c)*a*b)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2\right)\sqrt{e\cos(dx+c)}}{e^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(e*cos(
d*x + c))/(e^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(3/2), x)
```

3.553 $\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=119

$$\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2ab\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] (2*a*b*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) + (2*(a^2 - 2*b^2)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (2*(b + a*Sin
[c + d*x])*(a + b*Sin[c + d*x]))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.139395, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2691, 2669, 2642, 2641}

$$\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2ab\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(5/2),x]
```


```
[Out] (2*a*b*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) + (2*(a^2 - 2*b^2)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (2*(b + a*Sin
[c + d*x])*(a + b*Sin[c + d*x]))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a^2}{2}+b^2+\frac{1}{2}ab\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}} + \frac{\left(a^2-2b^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}} + \frac{\left(\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\right)\int}{3e^2\sqrt{e\cos(c+dx}}$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{2(b+a\sin(c+dx))(a+b}{3de(e\cos(c+dx}$$

Mathematica [A] time = 0.255964, size = 72, normalized size = 0.61

$$\frac{2\left(\left(a^2+b^2\right)\sin(c+dx)+\left(a^2-2b^2\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2ab\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(2*a*b + (a^2 - 2*b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + (a
^2 + b^2)*Sin[c + d*x]))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [B] time = 1.688, size = 333, normalized size = 2.8

$$-\frac{2}{3\,d e^{2}}\left(2\sqrt{2\,(\sin(1/2\,dx+c/2))^{2}-1}\sqrt{(\sin(1/2\,dx+c/2))^{2}}\,\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2)\,,\sqrt{2}\right)a^{2}\,(\sin(1/2\,dx+c/2))^{2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*sin(1/2*d*x+1/2*c)^2-4*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*b^2*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*b^2+2*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*sin(1/2*d*x+1/2*c)*a*b)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^{2}}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(b^{2}\cos(dx+c)^{2}-2\,ab\sin(dx+c)-a^{2}-b^{2}\right)\sqrt{e\cos(dx+c)}}{e^{3}\cos(dx+c)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(e*cos(
d*x + c))/(e^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sin(dx+c)+a)^2}{(e \cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(5/2), x)
```

3.554 $\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=160

$$\frac{2\left(3a^2-2b^2\right)\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{2\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2ab}{5de^3\sqrt{e\cos(c+dx)}}+\frac{2(a\sin(c+dx)+b)(}{5de(e\cos(c}$$

```
[Out] (2*a*b)/(5*d*e^3*Sqrt[e*Cos[c + d*x]]) - (2*(3*a^2 - 2*b^2)*Sqrt[e*Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]]) + (2*(3*a^2 -
 2*b^2)*Sin[c + d*x])/(5*d*e^3*Sqrt[e*Cos[c + d*x]]) + (2*(b + a*Sin[c + d*
x])*(a + b*Sin[c + d*x]))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Rubi [A] time = 0.172351, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2691, 2669, 2636, 2640, 2639}

$$\frac{2\left(3a^2-2b^2\right)\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{2\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2ab}{5de^3\sqrt{e\cos(c+dx)}}+\frac{2(a\sin(c+dx)+b)(}{5de(e\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(7/2),x]
```


```
[Out] (2*a*b)/(5*d*e^3*Sqrt[e*Cos[c + d*x]]) - (2*(3*a^2 - 2*b^2)*Sqrt[e*Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]]) + (2*(3*a^2 -
 2*b^2)*Sin[c + d*x])/(5*d*e^3*Sqrt[e*Cos[c + d*x]]) + (2*(b + a*Sin[c + d*
x])*(a + b*Sin[c + d*x]))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{5de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+b^2-\frac{1}{2}ab\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx}{5e^2}$$

$$= \frac{2ab}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{5de(e\cos(c+dx))^{5/2}} + \frac{\left(3a^2-2b^2\right)\int \frac{1}{(e\cos(c+dx))^{3/2}}\,d}{5e^2}$$

$$= \frac{2ab}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2\left(3a^2-2b^2\right)\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{5de(e\cos(c+dx))^{5/2}} -$$

$$= \frac{2ab}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2\left(3a^2-2b^2\right)\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{5de(e\cos(c+dx))^{5/2}} -$$

$$= \frac{2ab}{5de^3\sqrt{e\cos(c+dx)}} - \frac{2\left(3a^2-2b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{2\left(3a^2-2b^2\right)\sin(c+}{5de^3\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 0.535524, size = 105, normalized size = 0.66

$$\frac{(7a^2+2b^2)\sin(c+dx)-4(3a^2-2b^2)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+3a^2\sin(3(c+dx))+8ab-2b^2\sin(3(c+dx))}{10de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (8*a*b - 4*(3*a^2 - 2*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + (
7*a^2 + 2*b^2)*Sin[c + d*x] + 3*a^2*Sin[3*(c + d*x)] - 2*b^2*Sin[3*(c + d*x
)])/(10*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [B] time = 3.381, size = 564, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*sin
(1/2*d*x+1/2*c)^4-8*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2*sin(1/2*d*x+1/2*c)^4-24*a^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+16*b^2*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*sin(1/2*d*x+1/2*c)^2+8*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*b^2*sin(1/2*d*x+1/2*c)^2+24*a^2*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^4-16*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-8*a^2*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*sin(1/
2*d*x+1/2*c)*a*b)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2\right)\sqrt{e\cos(dx+c)}}{e^4\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(e*cos(
d*x + c))/(e^4*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^2}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2/(e*cos(d*x + c))^(7/2), x)
```

3.555 $\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=237

$$\frac{10ae^3\left(11a^2+6b^2\right)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{10ae^4\left(11a^2+6b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{2b\left(177a^2+44b^2\right)(e}{1287de}$$

```
[Out] (-2*b*(177*a^2 + 44*b^2)*(e*Cos[c + d*x])^(9/2))/(1287*d*e) + (10*a*(11*a^2
 + 6*b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*C
os[c + d*x]]) + (10*a*(11*a^2 + 6*b^2)*e^3*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x
])/(231*d) + (2*a*(11*a^2 + 6*b^2)*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(
77*d) - (34*a*b*(e*Cos[c + d*x])^(9/2)*(a + b*Sin[c + d*x]))/(143*d*e) - (2
*b*(e*Cos[c + d*x])^(9/2)*(a + b*Sin[c + d*x])^2)/(13*d*e)
```

Rubi [A] time = 0.307995, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2692, 2862, 2669, 2635, 2642, 2641}

$$\frac{10ae^3\left(11a^2+6b^2\right)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{10ae^4\left(11a^2+6b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{2b\left(177a^2+44b^2\right)(e}{1287de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(177*a^2 + 44*b^2)*(e*Cos[c + d*x])^(9/2))/(1287*d*e) + (10*a*(11*a^2
 + 6*b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*C
os[c + d*x]]) + (10*a*(11*a^2 + 6*b^2)*e^3*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x
])/(231*d) + (2*a*(11*a^2 + 6*b^2)*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(
77*d) - (34*a*b*(e*Cos[c + d*x])^(9/2)*(a + b*Sin[c + d*x]))/(143*d*e) - (2
*b*(e*Cos[c + d*x])^(9/2)*(a + b*Sin[c + d*x])^2)/(13*d*e)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3\,dx = -\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))^2}{13de} + \frac{2}{13}\int (e\cos(c+dx))^{7/2}(a+b\sin($$

$$= -\frac{34ab(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{143de} - \frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c}{13de}$$

$$= -\frac{2b\left(177a^2+44b^2\right)(e\cos(c+dx))^{9/2}}{1287de} - \frac{34ab(e\cos(c+dx))^{9/2}(a+b\sin(c+}{143de}$$

$$= -\frac{2b\left(177a^2+44b^2\right)(e\cos(c+dx))^{9/2}}{1287de} + \frac{2a\left(11a^2+6b^2\right)e(e\cos(c+dx))^{5/2}\,\mathrm{si}}{77d}$$

$$= -\frac{2b\left(177a^2+44b^2\right)(e\cos(c+dx))^{9/2}}{1287de} + \frac{10a\left(11a^2+6b^2\right)e^3\sqrt{e\cos(c+dx)}\,\mathrm{si}}{231d}$$

$$= -\frac{2b\left(177a^2+44b^2\right)(e\cos(c+dx))^{9/2}}{1287de} + \frac{10a\left(11a^2+6b^2\right)e^3\sqrt{e\cos(c+dx)}\,\mathrm{si}}{231d}$$

$$= -\frac{2b\left(177a^2+44b^2\right)(e\cos(c+dx))^{9/2}}{1287de} + \frac{10a\left(11a^2+6b^2\right)e^4\sqrt{\cos(c+dx)}F\Big(}{231d\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 2.01581, size = 205, normalized size = 0.86

$$\frac{(e\cos(c+dx))^{7/2}\left(-154b\left(78a^2+11b^2\right)\sqrt{\cos(c+dx)}+2080\left(11a^3+6ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{1}{3}\sqrt{\cos(c+dx)}\left(156a\left(506a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*(-154*b*(78*a^2 + 11*b^2)*Sqrt[Cos[c + d*x]] + 2080
*(11*a^3 + 6*a*b^2)*EllipticF[(c + d*x)/2, 2] + (Sqrt[Cos[c + d*x]]*(-77*b*
(624*a^2 + 73*b^2)*Cos[2*(c + d*x)] + 154*b*(-78*a^2 + b^2)*Cos[4*(c + d*x)
] + 693*b^3*Cos[6*(c + d*x)] + 156*a*(506*a^2 + 213*b^2)*Sin[c + d*x] + 234
*a*(44*a^2 - 39*b^2)*Sin[3*(c + d*x)] - 4914*a*b^2*Sin[5*(c + d*x)]))/3))/(
48048*d*Cos[c + d*x]^(7/2))
```

Maple [B] time = 2.566, size = 618, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -2/9009/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(1170*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*a*b^2+20592*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^8-308*sin(1/2*d*x+1/2*c)^3*b^3-18172*b^3*sin(1/2*d*x+1/2*c)^5-310464*b^3*
sin(1/2*d*x+1/2*c)^13+433664*b^3*sin(1/2*d*x+1/2*c)^11+88704*b^3*sin(1/2*d*
x+1/2*c)^15-308000*b^3*sin(1/2*d*x+1/2*c)^9+113960*b^3*sin(1/2*d*x+1/2*c)^7
+308*b^3*sin(1/2*d*x+1/2*c)-30888*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^6+24024*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-6864*a^3*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2+240240*a^2*b*sin(1/2*d*x+1/2*c)^9-240240*a^2*b*s
in(1/2*d*x+1/2*c)^7+120120*a^2*b*sin(1/2*d*x+1/2*c)^5-30030*a^2*b*sin(1/2*d
*x+1/2*c)^3-96096*a^2*b*sin(1/2*d*x+1/2*c)^11+3003*a^2*b*sin(1/2*d*x+1/2*c)
-157248*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+393120*a*b^2*sin(1/2
*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-381888*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^8+179712*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36036*a*b^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+1170*a*b^2*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2+2145*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{7}{2}} (b\sin(dx+c)+a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\text{integral}\left(-\left(3\,ab^2e^3\cos(dx+c)^5-\left(a^3+3\,ab^2\right)e^3\cos(dx+c)^3+\left(b^3e^3\cos(dx+c)^5-\left(3\,a^2b+b^3\right)e^3\cos(dx+c)^3\right)\sin\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*e^3*cos(d*x + c)^5 - (a^3 + 3*a*b^2)*e^3*cos(d*x + c)^3
+ (b^3*e^3*cos(d*x + c)^5 - (3*a^2*b + b^3)*e^3*cos(d*x + c)^3)*sin(d*x + c
))*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{7}{2}} (b\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^3, x)
```

3.556 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=197

$$\frac{2ae^2\left(3a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2b\left(43a^2+12b^2\right)(e\cos(c+dx))^{7/2}}{231de}+\frac{2ae\left(3a^2+2b^2\right)\sin(c+dx)(e\,c}{15d}$$

```
[Out] (-2*b*(43*a^2 + 12*b^2)*(e*Cos[c + d*x])^(7/2))/(231*d*e) + (2*a*(3*a^2 + 2
*b^2)*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
 d*x]]) + (2*a*(3*a^2 + 2*b^2)*e*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*d
) - (10*a*b*(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x]))/(33*d*e) - (2*b*(e
*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2)/(11*d*e)
```

Rubi [A] time = 0.286019, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2692, 2862, 2669, 2635, 2640, 2639}

$$\frac{2ae^2\left(3a^2+2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2b\left(43a^2+12b^2\right)(e\cos(c+dx))^{7/2}}{231de}+\frac{2ae\left(3a^2+2b^2\right)\sin(c+dx)(e\,c}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (-2*b*(43*a^2 + 12*b^2)*(e*Cos[c + d*x])^(7/2))/(231*d*e) + (2*a*(3*a^2 + 2
*b^2)*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
 d*x]]) + (2*a*(3*a^2 + 2*b^2)*e*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*d
) - (10*a*b*(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x]))/(33*d*e) - (2*b*(e
*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2)/(11*d*e)
```


Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3\,dx = -\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}{11de} + \frac{2}{11}\int (e\cos(c+dx))^{5/2}(a+b\sin$$

$$= -\frac{10ab(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{33de} - \frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(}{11de}$$

$$= -\frac{2b\left(43a^2+12b^2\right)(e\cos(c+dx))^{7/2}}{231de} - \frac{10ab(e\cos(c+dx))^{7/2}(a+b\sin(c+}{33de}$$

$$= -\frac{2b\left(43a^2+12b^2\right)(e\cos(c+dx))^{7/2}}{231de} + \frac{2a\left(3a^2+2b^2\right)e(e\cos(c+dx))^{3/2}\sin}{15d}$$

$$= -\frac{2b\left(43a^2+12b^2\right)(e\cos(c+dx))^{7/2}}{231de} + \frac{2a\left(3a^2+2b^2\right)e(e\cos(c+dx))^{3/2}\sin}{15d}$$

$$= -\frac{2b\left(43a^2+12b^2\right)(e\cos(c+dx))^{7/2}}{231de} + \frac{2a\left(3a^2+2b^2\right)e^2\sqrt{e\cos(c+dx)}E\left(\right.}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 1.38534, size = 150, normalized size = 0.76

$$\frac{(e\cos(c+dx))^{5/2}\left(1848\left(3a^3+2ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^{\frac{3}{2}}(c+dx)\left(-60\left(33a^2b+4b^3\right)\cos(2(c+dx))-1980a^2b+18\right.\right.}{4620d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(1848*(3*a^3 + 2*a*b^2)*EllipticE[(c + d*x)/2, 2] +
 Cos[c + d*x]^(3/2)*(-1980*a^2*b - 345*b^3 - 60*(33*a^2*b + 4*b^3)*Cos[2*(c
 + d*x)] + 105*b^3*Cos[4*(c + d*x)] + 1848*a^3*Sin[c + d*x] + 462*a*b^2*Sin
[c + d*x] - 770*a*b^2*Sin[3*(c + d*x)])))/(4620*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 2.238, size = 534, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] 2/1155/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(6720*b^3
*sin(1/2*d*x+1/2*c)^13-12320*a*b^2*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)
-20160*b^3*sin(1/2*d*x+1/2*c)^11-7920*a^2*b*sin(1/2*d*x+1/2*c)^9+24640*a*b^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+22560*b^3*sin(1/2*d*x+1/2*c)^9+18
48*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+15840*a^2*b*sin(1/2*d*x+1/2*
c)^7-17248*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-11520*b^3*sin(1/2*
d*x+1/2*c)^7-1848*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-11880*a^2*b*s
in(1/2*d*x+1/2*c)^5+4928*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+2340
*b^3*sin(1/2*d*x+1/2*c)^5+693*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+462*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*a*b^2+462*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3960*a^2
*b*sin(1/2*d*x+1/2*c)^3-462*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6
0*sin(1/2*d*x+1/2*c)^3*b^3-495*a^2*b*sin(1/2*d*x+1/2*c)-60*b^3*sin(1/2*d*x+
1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,ab^2e^2\cos(dx+c)^4-\left(a^3+3\,ab^2\right)e^2\cos(dx+c)^2+\left(b^3e^2\cos(dx+c)^4-\left(3\,a^2b+b^3\right)e^2\cos(dx+c)^2\right)\sin(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*e^2*cos(d*x + c)^4 - (a^3 + 3*a*b^2)*e^2*cos(d*x + c)^2
+ (b^3*e^2*cos(d*x + c)^4 - (3*a^2*b + b^3)*e^2*cos(d*x + c)^2)*sin(d*x + c
))*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^3, x)
```

3.557 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=197

$$\frac{2ae^2\left(7a^2+6b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}-\frac{2b\left(89a^2+28b^2\right)(e\cos(c+dx))^{5/2}}{315de}+\frac{2ae\left(7a^2+6b^2\right)\sin(c+dx)\sqrt{e\cos}}{21d}$$

```
[Out] (-2*b*(89*a^2 + 28*b^2)*(e*Cos[c + d*x])^(5/2))/(315*d*e) + (2*a*(7*a^2 + 6
*b^2)*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c
+ d*x]]) + (2*a*(7*a^2 + 6*b^2)*e*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(21*d)
 - (26*a*b*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x]))/(63*d*e) - (2*b*(e*
Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2)/(9*d*e)
```

Rubi [A] time = 0.288608, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2692, 2862, 2669, 2635, 2642, 2641}

$$\frac{2ae^2\left(7a^2+6b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}-\frac{2b\left(89a^2+28b^2\right)(e\cos(c+dx))^{5/2}}{315de}+\frac{2ae\left(7a^2+6b^2\right)\sin(c+dx)\sqrt{e\cos}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(89*a^2 + 28*b^2)*(e*Cos[c + d*x])^(5/2))/(315*d*e) + (2*a*(7*a^2 + 6
*b^2)*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c
+ d*x]]) + (2*a*(7*a^2 + 6*b^2)*e*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(21*d)
 - (26*a*b*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x]))/(63*d*e) - (2*b*(e*
Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2)/(9*d*e)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3\,dx = -\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}{9de} + \frac{2}{9}\int (e\cos(c+dx))^{3/2}(a+b\sin(c$$

$$= -\frac{26ab(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{63de} - \frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c}{9de}$$

$$= -\frac{2b\left(89a^2+28b^2\right)(e\cos(c+dx))^{5/2}}{315de} - \frac{26ab(e\cos(c+dx))^{5/2}(a+b\sin(c+d}{63de}$$

$$= -\frac{2b\left(89a^2+28b^2\right)(e\cos(c+dx))^{5/2}}{315de} + \frac{2a\left(7a^2+6b^2\right)e\sqrt{e\cos(c+dx)}\sin(c}{21d}$$

$$= -\frac{2b\left(89a^2+28b^2\right)(e\cos(c+dx))^{5/2}}{315de} + \frac{2a\left(7a^2+6b^2\right)e\sqrt{e\cos(c+dx)}\sin(c}{21d}$$

$$= -\frac{2b\left(89a^2+28b^2\right)(e\cos(c+dx))^{5/2}}{315de} + \frac{2a\left(7a^2+6b^2\right)e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{21d\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 1.35052, size = 153, normalized size = 0.78

$$\frac{(e\cos(c+dx))^{3/2}\left(80\left(7a^3+6ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{2}{3}\sqrt{\cos(c+dx)}\left(-28\left(27a^2b+4b^3\right)\cos(2(c+dx))-756a^2b+840a^3\right.\right.}{840d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(80*(7*a^3 + 6*a*b^2)*EllipticF[(c + d*x)/2, 2] + (
2*Sqrt[Cos[c + d*x]]*(-756*a^2*b - 147*b^3 - 28*(27*a^2*b + 4*b^3)*Cos[2*(c
 + d*x)] + 35*b^3*Cos[4*(c + d*x)] + 840*a^3*Sin[c + d*x] + 450*a*b^2*Sin[c
 + d*x] - 270*a*b^2*Sin[3*(c + d*x)]))/3))/(840*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 1.996, size = 450, normalized size = 2.3

$$-\frac{2e^2}{315d}\left(1120b^3(\sin(1/2dx+c/2))^{11} - 2160ab^2\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^8 - 2800b^3(\sin(1/2dx+c/2))^9 - 15\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(1120*b^3
*sin(1/2*d*x+1/2*c)^11-2160*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-2
800*b^3*sin(1/2*d*x+1/2*c)^9-1512*a^2*b*sin(1/2*d*x+1/2*c)^7+3240*a*b^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2296*b^3*sin(1/2*d*x+1/2*c)^7+420*a^3*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+2268*a^2*b*sin(1/2*d*x+1/2*c)^5-126
0*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-644*b^3*sin(1/2*d*x+1/2*c)^
5+105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+90*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-210*a^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-1134*a^2*b*sin(1/2*d*x+1/2*c)^3+9
0*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-28*sin(1/2*d*x+1/2*c)^3*b^3
+189*a^2*b*sin(1/2*d*x+1/2*c)+28*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,ab^2e\cos(dx+c)^3-\left(a^3+3\,ab^2\right)e\cos(dx+c)+\left(b^3e\cos(dx+c)^3-\left(3\,a^2b+b^3\right)e\cos(dx+c)\right)\sin(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*e*cos(d*x + c)^3 - (a^3 + 3*a*b^2)*e*cos(d*x + c) + (b^3
*e*cos(d*x + c)^3 - (3*a^2*b + b^3)*e*cos(d*x + c))*sin(d*x + c))*sqrt(e*co
s(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^3, x)
```

3.558 $\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=156

$$-\frac{2b\left(57a^2+20b^2\right)(e\cos(c+dx))^{3/2}}{105de}+\frac{2a\left(5a^2+6b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c}{7de}$$

```
[Out] (-2*b*(57*a^2 + 20*b^2)*(e*Cos[c + d*x])^(3/2))/(105*d*e) + (2*a*(5*a^2 + 6
*b^2)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x
]]) - (22*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]))/(35*d*e) - (2*b*
(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2)/(7*d*e)
```

Rubi [A] time = 0.240711, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2692, 2862, 2669, 2640, 2639}

$$-\frac{2b\left(57a^2+20b^2\right)(e\cos(c+dx))^{3/2}}{105de}+\frac{2a\left(5a^2+6b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(57*a^2 + 20*b^2)*(e*Cos[c + d*x])^(3/2))/(105*d*e) + (2*a*(5*a^2 + 6
*b^2)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x
]]) - (22*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]))/(35*d*e) - (2*b*
(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2)/(7*d*e)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
```

```
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3\,dx &= -\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}{7de}+\frac{2}{7}\int \sqrt{e\cos(c+dx)}(a+b\sin(c+
\\
&= -\frac{22ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{35de}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+}{7de}
\\
&= -\frac{2b\left(57a^2+20b^2\right)(e\cos(c+dx))^{3/2}}{105de}-\frac{22ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx}{35de}
\\
&= -\frac{2b\left(57a^2+20b^2\right)(e\cos(c+dx))^{3/2}}{105de}-\frac{22ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx}{35de}
\\
&= -\frac{2b\left(57a^2+20b^2\right)(e\cos(c+dx))^{3/2}}{105de}+\frac{2a\left(5a^2+6b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.605945, size = 101, normalized size = 0.65

$$\frac{\sqrt{e\cos(c+dx)}\left(42\left(5a^3+6ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\cos^{\frac{3}{2}}(c+dx)\left(-210a^2-126ab\sin(c+dx)+15b^2\cos(2(c+dx))-\right.\right.}{105d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*(42*(5*a^3 + 6*a*b^2)*EllipticE[(c + d*x)/2, 2] + b*C
os[c + d*x]^(3/2)*(-210*a^2 - 55*b^2 + 15*b^2*Cos[2*(c + d*x)] - 126*a*b*Si
n[c + d*x])))/(105*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 1.639, size = 339, normalized size = 2.2

$$\frac{2e}{105d}\left(240b^3(\sin(1/2dx+c/2))^9-504ab^2\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^6-480b^3(\sin(1/2dx+c/2))^7-420a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/105/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(240*b^3*sin
(1/2*d*x+1/2*c)^9-504*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-480*b^3
*sin(1/2*d*x+1/2*c)^7-420*a^2*b*sin(1/2*d*x+1/2*c)^5+504*a*b^2*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4+220*b^3*sin(1/2*d*x+1/2*c)^5+105*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*a^3+126*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+420*a^2*b*sin(1/2*d*x+1/2
*c)^3-126*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+20*sin(1/2*d*x+1/2*
c)^3*b^3-105*a^2*b*sin(1/2*d*x+1/2*c)-20*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,ab^2\cos\left(dx+c\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.559 $\int \frac{(a+b\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=152

$$-\frac{2b\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{5de}+\frac{2a\left(a^2+2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2b\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2}{5de}$$

```
[Out] (-2*b*(11*a^2 + 4*b^2)*Sqrt[e*Cos[c + d*x]])/(5*d*e) + (2*a*(a^2 + 2*b^2)*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]]) - (6*
a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]))/(5*d*e) - (2*b*Sqrt[e*Cos[c
+ d*x]]*(a + b*Sin[c + d*x])^2)/(5*d*e)
```

Rubi [A] time = 0.24042, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2692, 2862, 2669, 2642, 2641}

$$-\frac{2b\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{5de}+\frac{2a\left(a^2+2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2b\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*b*(11*a^2 + 4*b^2)*Sqrt[e*Cos[c + d*x]])/(5*d*e) + (2*a*(a^2 + 2*b^2)*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]]) - (6*
a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]))/(5*d*e) - (2*b*Sqrt[e*Cos[c
+ d*x]]*(a + b*Sin[c + d*x])^2)/(5*d*e)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2b\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2}{5de} + \frac{2}{5}\int \frac{(a+b\sin(c+dx))\left(\frac{5a^2}{2}+2b^2+\frac{9}{2}ab\sin(c+\right.}{\sqrt{e\cos(c+dx)}}$$

$$= -\frac{6ab\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{5de} - \frac{2b\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2}{5de} + \frac{4}{15}\int \frac{\frac{15}{4}}{}$$

$$= -\frac{2b\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{5de} - \frac{6ab\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{5de} - \frac{2b\sqrt{e\cos(c+d}}{}$$

$$= -\frac{2b\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{5de} - \frac{6ab\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{5de} - \frac{2b\sqrt{e\cos(c+d}}{}$$

$$= -\frac{2b\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{5de} + \frac{2a\left(a^2+2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}} - \frac{6ab\sqrt{e\,\mathrm{co}}}{}$$

Mathematica [A] time = 0.771315, size = 94, normalized size = 0.62

$$\frac{b\cos(c+dx)\left(-30a^2-10ab\sin(c+dx)+b^2\cos(2(c+dx))-9b^2\right)+10a\left(a^2+2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (10*a*(a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*Cos[c
+ d*x]*(-30*a^2 - 9*b^2 + b^2*Cos[2*(c + d*x)] - 10*a*b*Sin[c + d*x]))/(5*d
*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 1.382, size = 279, normalized size = 1.8

$$-\frac{2}{5d}\left(8b^3\left(\sin\left(1/2\,dx+c/2\right)\right)^7-20\,ab^2\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^4-12\,b^3\left(\sin\left(1/2\,dx+c/2\right)\right)^5+5\sqrt{\left(\sin\left(1/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/5/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(8*b^3*sin(1/2*
d*x+1/2*c)^7-20*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-12*b^3*sin(1/
2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+10*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a
*b^2-30*a^2*b*sin(1/2*d*x+1/2*c)^3+10*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2-4*sin(1/2*d*x+1/2*c)^3*b^3+15*a^2*b*sin(1/2*d*x+1/2*c)+4*b^3*sin(1
/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^3}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/sqrt(e*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^3}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/sqrt(e*cos(d*x + c)), x)
```

3.560 $\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{2b\left(3a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{3de^3}-\frac{2a\left(a^2+6b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{de^3}$$

```
[Out] (2*b*(3*a^2 + 4*b^2)*(e*Cos[c + d*x])^(3/2))/(3*d*e^3) - (2*a*(a^2 + 6*b^2)
*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]])
 + (2*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]))/(d*e^3) + (2*(b + a*
Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.241024, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2691, 2862, 2669, 2640, 2639}

$$\frac{2b\left(3a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{3de^3}-\frac{2a\left(a^2+6b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*b*(3*a^2 + 4*b^2)*(e*Cos[c + d*x])^(3/2))/(3*d*e^3) - (2*a*(a^2 + 6*b^2)
*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]])
 + (2*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]))/(d*e^3) + (2*(b + a*
Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{3/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{de\sqrt{e\cos(c+dx)}} - \frac{2\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))\left(\frac{a^2}{2}+2b^2+\right.}{e^2}$$

$$= \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{de\sqrt{e\cos(c+dx)}} - \frac{4\int \sqrt{e}}{}$$

$$= \frac{2b\left(3a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{3de^3} + \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{de^3} + \frac{2(b+a\sin(c+}{de\sqrt{}}$$

$$= \frac{2b\left(3a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{3de^3} + \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{de^3} + \frac{2(b+a\sin(c+}{de\sqrt{}}$$

$$= \frac{2b\left(3a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{3de^3} - \frac{2a\left(a^2+6b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}} + \frac{2ab(e\cos}{}$$

Mathematica [A] time = 0.396142, size = 98, normalized size = 0.61

$$\frac{6\left(a\left(a^2+3b^2\right)\sin(c+dx)+3a^2b+b^3\right)-6a\left(a^2+6b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2b^3\cos^2(c+dx)}{3de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*b^3*Cos[c + d*x]^2 - 6*a*(a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2] + 6*(3*a^2*b + b^3 + a*(a^2 + 3*b^2)*Sin[c + d*x]))/(3*d*e*Sqrt
[e*Cos[c + d*x]])
```

Maple [A] time = 1.937, size = 248, normalized size = 1.6

$$-\frac{2}{3\,de}\left(-4\,b^3\left(\sin\left(1/2\,dx+c/2\right)\right)^5+3\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\mathrm{EllipticE}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-4*b^3*sin(1
/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+18*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a*b^2-6*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-18*a*b^2*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+4*sin(1/2*d*x+1/2*c)^3*b^3-9*a^2*b*sin(1/2*d*x+1/
2*c)-4*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(3/2), x)
```

3.561 $\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=164

$$\frac{2b\left(a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2a\left(a^2-6b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2ab\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de^3}+\frac{2}{}$$

```
[Out] (2*b*(a^2 + 4*b^2)*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) + (2*a*(a^2 - 6*b^2)*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) +
 (2*a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]))/(3*d*e^3) + (2*(b + a*Si
n[c + d*x])*(a + b*Sin[c + d*x])^2)/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.249331, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2691, 2862, 2669, 2642, 2641}

$$\frac{2b\left(a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2a\left(a^2-6b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2ab\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de^3}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*b*(a^2 + 4*b^2)*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) + (2*a*(a^2 - 6*b^2)*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) +
 (2*a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]))/(3*d*e^3) + (2*(b + a*Si
n[c + d*x])*(a + b*Sin[c + d*x])^2)/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3de(e\cos(c+dx))^{3/2}} - \frac{2\int \frac{(a+b\sin(c+dx))\left(-\frac{a^2}{2}+2b^2+\frac{3}{2}ab\sin(c+dx)\right)}{\sqrt{e\cos(c+dx)}}dx}{3e^2}$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3de(e\cos(c+dx))^{3/2}} - \frac{4\int \frac{-\frac{3}{4}a(}{}}{}$$

$$= \frac{2b\left(a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2ab\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de^3} + \frac{2(b+a\sin(c+dx)}{3de(e\cos}$$

$$= \frac{2b\left(a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2ab\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de^3} + \frac{2(b+a\sin(c+dx)}{3de(e\cos}$$

$$= \frac{2b\left(a^2+4b^2\right)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2a\left(a^2-6b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{2ab\sqrt{e\cos(c}}{}$$

Mathematica [A] time = 0.684329, size = 103, normalized size = 0.63

$$\frac{2a\left(a^2-6b^2\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+6a^2b+2a^3\sin(c+dx)+6ab^2\sin(c+dx)+3b^3\cos(2(c+dx))+5b^3}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*a^2*b + 5*b^3 + 3*b^3*Cos[2*(c + d*x)] + 2*a*(a^2 - 6*b^2)*Cos[c + d*x]^
(3/2)*EllipticF[(c + d*x)/2, 2] + 2*a^3*Sin[c + d*x] + 6*a*b^2*Sin[c + d*x]
)/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [B] time = 1.881, size = 384, normalized size = 2.3

$$-\frac{2}{3\,de^2}\left(2\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)a^3(\sin(1/2\,dx+c/2))^2 - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*sin(1/2*d*x+1/2*c)^2-12*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b^2*sin(1/2*d*x+1/2*c)^2+12*b^3*sin(1/2*d*x+1/2*c)^
5-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*a^3+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+2*a^3*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2-12*sin(1/2*d*x+1/2*c)^3*b^3+3*a^2*b*sin(1/2*d*x+1/2*c)+4*b^3*sin(1/
2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(5/2), x)
```

3.562 $\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=187

$$\frac{2b\left(3a^2-4b^2\right)(e\cos(c+dx))^{3/2}}{5de^5}-\frac{2\left(ab-\left(3a^2-4b^2\right)\sin(c+dx)\right)(a+b\sin(c+dx))}{5de^3\sqrt{e\cos(c+dx)}}-\frac{6a\left(a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e}}{5de^4\sqrt{\cos(c+dx)}}$$

```
[Out] (2*b*(3*a^2 - 4*b^2)*(e*Cos[c + d*x])^(3/2))/(5*d*e^5) - (6*a*(a^2 - 2*b^2)
*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]
]) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(5*d*e*(e*Cos[c + d*x]
)^(5/2)) - (2*(a + b*Sin[c + d*x])*(a*b - (3*a^2 - 4*b^2)*Sin[c + d*x]))/(5
*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.264079, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2691, 2861, 2669, 2640, 2639}

$$\frac{2b\left(3a^2-4b^2\right)(e\cos(c+dx))^{3/2}}{5de^5}-\frac{2\left(ab-\left(3a^2-4b^2\right)\sin(c+dx)\right)(a+b\sin(c+dx))}{5de^3\sqrt{e\cos(c+dx)}}-\frac{6a\left(a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e}}{5de^4\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(7/2),x]
```


```
[Out] (2*b*(3*a^2 - 4*b^2)*(e*Cos[c + d*x])^(3/2))/(5*d*e^5) - (6*a*(a^2 - 2*b^2)
*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]
]) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(5*d*e*(e*Cos[c + d*x]
)^(5/2)) - (2*(a + b*Sin[c + d*x])*(a*b - (3*a^2 - 4*b^2)*Sin[c + d*x]))/(5
*d*e^3*Sqrt[e*Cos[c + d*x]])
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{(a+b\sin(c+dx))\left(-\frac{3a^2}{2}+2b^2+\frac{1}{2}ab\sin(c+dx)\right)}{(e\cos(c+dx))^{3/2}}dx}{5e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5de(e\cos(c+dx))^{5/2}} - \frac{2(a+b\sin(c+dx))\left(ab-\left(3a^2-4b^2\right)\sin(c+dx)\right)}{5de^3\sqrt{e\cos(c+dx)}}$$

$$= \frac{2b\left(3a^2-4b^2\right)(e\cos(c+dx))^{3/2}}{5de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5de(e\cos(c+dx))^{5/2}} - \frac{2(a+b\sin(c+$$

$$= \frac{2b\left(3a^2-4b^2\right)(e\cos(c+dx))^{3/2}}{5de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5de(e\cos(c+dx))^{5/2}} - \frac{2(a+b\sin(c+$$

$$= \frac{2b\left(3a^2-4b^2\right)(e\cos(c+dx))^{3/2}}{5de^5} - \frac{6a\left(a^2-2b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{2(b+a\sin$$

Mathematica [A] time = 0.704941, size = 126, normalized size = 0.67

$$\frac{2\left(b\left(3a^2+b^2\right)\sec^2(c+dx)-3a\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(a^2+3b^2\right)\tan(c+dx)\sec(c+dx)+3a^3\sin$$
$$5de^3\sqrt{e\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(-5*b^3 - 3*a*(a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
 + b*(3*a^2 + b^2)*Sec[c + d*x]^2 + 3*a^3*Sin[c + d*x] - 6*a*b^2*Sin[c + d*
x] + a*(a^2 + 3*b^2)*Sec[c + d*x]*Tan[c + d*x]))/(5*d*e^3*Sqrt[e*Cos[c + d*
x]])
```

Maple [B] time = 3.974, size = 618, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin
(1/2*d*x+1/2*c)^4-24*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^4-24
*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+48*a*b^2*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2+24*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2+24*a^3*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4-48*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20
*b^3*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-6*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*a*b^2-8*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*a*b^2*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-20*sin(1/2*d*x+1/2*c)^3*b^3-3*a^2*b*sin(1
/2*d*x+1/2*c)+4*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)\right)\sqrt{e\cos(dx+c)}}{e^4\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^4*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(7/2), x)
```

3.563 $\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=188

$$\frac{2b\left(5a^2-4b^2\right)\sqrt{e\cos(c+dx)}}{21de^5}+\frac{2\left(\left(5a^2-4b^2\right)\sin(c+dx)+ab\right)(a+b\sin(c+dx))}{21de^3(e\cos(c+dx))^{3/2}}+\frac{2a\left(5a^2-6b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.}{21de^4\sqrt{e\cos(c+dx)}}$$

```
[Out] (2*b*(5*a^2 - 4*b^2)*Sqrt[e*Cos[c + d*x]])/(21*d*e^5) + (2*a*(5*a^2 - 6*b^2
)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt[e*Cos[c + d*
x]]) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(7*d*e*(e*Cos[c + d*
x])^(7/2)) + (2*(a + b*Sin[c + d*x])*(a*b + (5*a^2 - 4*b^2)*Sin[c + d*x]))/
(21*d*e^3*(e*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.267374, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2691, 2861, 2669, 2642, 2641}

$$\frac{2b\left(5a^2-4b^2\right)\sqrt{e\cos(c+dx)}}{21de^5}+\frac{2\left(\left(5a^2-4b^2\right)\sin(c+dx)+ab\right)(a+b\sin(c+dx))}{21de^3(e\cos(c+dx))^{3/2}}+\frac{2a\left(5a^2-6b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.}{21de^4\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(9/2),x]
```


```
[Out] (2*b*(5*a^2 - 4*b^2)*Sqrt[e*Cos[c + d*x]])/(21*d*e^5) + (2*a*(5*a^2 - 6*b^2
)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt[e*Cos[c + d*
x]]) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(7*d*e*(e*Cos[c + d*
x])^(7/2)) + (2*(a + b*Sin[c + d*x])*(a*b + (5*a^2 - 4*b^2)*Sin[c + d*x]))/
(21*d*e^3*(e*Cos[c + d*x])^(3/2))
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{9/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}} - \frac{2\int \frac{(a+b\sin(c+dx))\left(-\frac{5a^2}{2}+2b^2-\frac{1}{2}ab\sin(c+dx)\right)}{(e\cos(c+dx))^{5/2}}dx}{7e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}} + \frac{2(a+b\sin(c+dx))\left(ab+\left(5a^2-4b^2\right)\sin(c+dx\right.}{21de^3(e\cos(c+dx))^{3/2}}$$

$$= \frac{2b\left(5a^2-4b^2\right)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}} + \frac{2(a+b\sin(c+}{2}$$

$$= \frac{2b\left(5a^2-4b^2\right)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}} + \frac{2(a+b\sin(c+}{2}$$

$$= \frac{2b\left(5a^2-4b^2\right)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2a\left(5a^2-6b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}} + \frac{2(b+a\sin}{7}$$

Mathematica [A] time = 0.638152, size = 140, normalized size = 0.74

$$\frac{\sec^4(c+dx)\sqrt{e\cos(c+dx)}\left(4a\left(5a^2-6b^2\right)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+36a^2b+17a^3\sin(c+dx)+5a^3\sin(3(c+dx\right.}{42de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^4*(36*a^2*b - 2*b^3 - 14*b^3*Cos[2*(c +
d*x)] + 4*a*(5*a^2 - 6*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] +
17*a^3*Sin[c + d*x] + 30*a*b^2*Sin[c + d*x] + 5*a^3*Sin[3*(c + d*x)] - 6*a*
b^2*Sin[3*(c + d*x)]))/(42*d*e^5)
```

Maple [B] time = 4.631, size = 750, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(40*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^6-48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^
2*sin(1/2*d*x+1/2*c)^6-60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^4
+72*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^4+40*a^3*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^6-48*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
6+30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*sin(1/2*d*x+1/2*c)^2-36*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*a*b^2*sin(1/2*d*x+1/2*c)^2-40*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^4+48*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-28*b^3*sin(1/2*d*x+
1/2*c)^5-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+16
*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*a*b^2*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2+28*sin(1/2*d*x+1/2*c)^3*b^3+9*a^2*b*sin(1/2*d*x+1/2*c)-4*
b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \sin\left(dx+c\right)+a\right)^{3}}{\left(e \cos\left(dx+c\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(3\,ab^{2}\cos\left(dx+c\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(dx+c\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(dx+c\right)\right)\sqrt{e\cos\left(dx+c\right)}}{e^{5}\cos\left(dx+c\right)^{5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*sqrt(e*cos(d*x + c))/(e^5*cos(d*x + c)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^3}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3/(e*cos(d*x + c))^(9/2), x)
```

3.564 $\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=305

$$\frac{2e^3\left(60a^2b^2+55a^4+4b^4\right)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{2e^4\left(60a^2b^2+55a^4+4b^4\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{34ab\left(\right.}{}$$

```
[Out] (-34*a*b*(53*a^2 + 38*b^2)*(e*Cos[c + d*x])^(9/2))/(6435*d*e) + (2*(55*a^4
+ 60*a^2*b^2 + 4*b^4)*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(23
1*d*Sqrt[e*Cos[c + d*x]]) + (2*(55*a^4 + 60*a^2*b^2 + 4*b^4)*e^3*Sqrt[e*Cos
[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(55*a^4 + 60*a^2*b^2 + 4*b^4)*e*(e*Co
s[c + d*x])^(5/2)*Sin[c + d*x])/(385*d) - (2*b*(93*a^2 + 26*b^2)*(e*Cos[c +
 d*x])^(9/2)*(a + b*Sin[c + d*x]))/(715*d*e) - (14*a*b*(e*Cos[c + d*x])^(9/
2)*(a + b*Sin[c + d*x])^2)/(65*d*e) - (2*b*(e*Cos[c + d*x])^(9/2)*(a + b*Si
n[c + d*x])^3)/(15*d*e)
```

Rubi [A] time = 0.551403, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2692, 2862, 2669, 2635, 2642, 2641}

$$\frac{2e^3\left(60a^2b^2+55a^4+4b^4\right)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{2e^4\left(60a^2b^2+55a^4+4b^4\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{34ab\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^4,x]
```


```
[Out] (-34*a*b*(53*a^2 + 38*b^2)*(e*Cos[c + d*x])^(9/2))/(6435*d*e) + (2*(55*a^4
+ 60*a^2*b^2 + 4*b^4)*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(23
1*d*Sqrt[e*Cos[c + d*x]]) + (2*(55*a^4 + 60*a^2*b^2 + 4*b^4)*e^3*Sqrt[e*Cos
[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(55*a^4 + 60*a^2*b^2 + 4*b^4)*e*(e*Co
s[c + d*x])^(5/2)*Sin[c + d*x])/(385*d) - (2*b*(93*a^2 + 26*b^2)*(e*Cos[c +
 d*x])^(9/2)*(a + b*Sin[c + d*x]))/(715*d*e) - (14*a*b*(e*Cos[c + d*x])^(9/
2)*(a + b*Sin[c + d*x])^2)/(65*d*e) - (2*b*(e*Cos[c + d*x])^(9/2)*(a + b*Si
n[c + d*x])^3)/(15*d*e)
```


Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^4\,dx = -\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))^3}{15de} + \frac{2}{15}\int (e\cos(c+dx))^{7/2}(a+b\sin$$

$$= -\frac{14ab(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))^2}{65de} - \frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(}{15de}$$

$$= -\frac{2b\left(93a^2+26b^2\right)(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{715de} - \frac{14ab(e\cos(c+dx)}{}$$

$$= -\frac{34ab\left(53a^2+38b^2\right)(e\cos(c+dx))^{9/2}}{6435de} - \frac{2b\left(93a^2+26b^2\right)(e\cos(c+dx))^{9/2}}{715de}$$

$$= -\frac{34ab\left(53a^2+38b^2\right)(e\cos(c+dx))^{9/2}}{6435de} + \frac{2\left(55a^4+60a^2b^2+4b^4\right)e(e\cos(c}{385d}$$

$$= -\frac{34ab\left(53a^2+38b^2\right)(e\cos(c+dx))^{9/2}}{6435de} + \frac{2\left(55a^4+60a^2b^2+4b^4\right)e^3\sqrt{e\cos(}}{231d}$$

$$= -\frac{34ab\left(53a^2+38b^2\right)(e\cos(c+dx))^{9/2}}{6435de} + \frac{2\left(55a^4+60a^2b^2+4b^4\right)e^3\sqrt{e\cos(}}{231d}$$

$$= -\frac{34ab\left(53a^2+38b^2\right)(e\cos(c+dx))^{9/2}}{6435de} + \frac{2\left(55a^4+60a^2b^2+4b^4\right)e^4\sqrt{\cos(c}}{231d\sqrt{e\cos(c+}}$$

Mathematica [A] time = 4.81313, size = 251, normalized size = 0.82

$$\frac{(e\cos(c+dx))^{7/2}\left(-154ab\left(26a^2+11b^2\right)\sqrt{\cos(c+dx)}+104\left(60a^2b^2+55a^4+4b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{1}{120}\sqrt{\cos(c+dx)}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*(-154*a*b*(26*a^2 + 11*b^2)*Sqrt[Cos[c + d*x]] + 10
4*(55*a^4 + 60*a^2*b^2 + 4*b^4)*EllipticF[(c + d*x)/2, 2] + (Sqrt[Cos[c + d
*x]]*(156*(5720*a^4 + 2460*a^2*b^2 + 87*b^4)*Sin[c + d*x] + 462*b^3*Cos[6*(
c + d*x)]*(60*a + 13*b*Sin[c + d*x]) - 28*b*Cos[4*(c + d*x)]*(220*a*(26*a^2
 - b^2) + 39*b*(180*a^2 + b^2)*Sin[c + d*x]) + Cos[2*(c + d*x)]*(-3080*(208
*a^3*b + 73*a*b^3) + 78*(2640*a^4 - 7200*a^2*b^2 - 557*b^4)*Sin[c + d*x]))
/120))/(12012*d*Cos[c + d*x]^(7/2))
```

Maple [B] time = 2.684, size = 863, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^4,x)
```

```
[Out] -2/45045/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(-26906
88*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14-1572480*a^2*b^2*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^12+3931200*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^10-3818880*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+1797120
*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-360360*a^2*b^2*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4+11700*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2+10725*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+780*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4
+768768*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^16+3739008*b^4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^12-2620800*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^10+102960*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+946608*b^4*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-154440*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^6-144456*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+120120*a^4*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-34320*a^4*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2+780*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-6209280*a*b^3
*sin(1/2*d*x+1/2*c)^13+8673280*a*b^3*sin(1/2*d*x+1/2*c)^11+1774080*a*b^3*si
n(1/2*d*x+1/2*c)^15-640640*a^3*b*sin(1/2*d*x+1/2*c)^11+1601600*a^3*b*sin(1/
2*d*x+1/2*c)^9-6160000*a*b^3*sin(1/2*d*x+1/2*c)^9-1601600*a^3*b*sin(1/2*d*x
+1/2*c)^7+2279200*a*b^3*sin(1/2*d*x+1/2*c)^7+800800*a^3*b*sin(1/2*d*x+1/2*c
)^5-363440*a*b^3*sin(1/2*d*x+1/2*c)^5-200200*a^3*b*sin(1/2*d*x+1/2*c)^3-616
0*a*b^3*sin(1/2*d*x+1/2*c)^3+20020*a^3*b*sin(1/2*d*x+1/2*c)+6160*a*b^3*sin(
1/2*d*x+1/2*c)+11700*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{7}{2}} (b\sin(dx+c)+a)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^4e^3\cos(dx+c)^7 - 2\left(3a^2b^2 + b^4\right)e^3\cos(dx+c)^5 + \left(a^4 + 6a^2b^2 + b^4\right)e^3\cos(dx+c)^3 - 4\left(ab^3e^3\cos(dx+c)^5\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*e^3*cos(d*x + c)^7 - 2*(3*a^2*b^2 + b^4)*e^3*cos(d*x + c)^5 +
 (a^4 + 6*a^2*b^2 + b^4)*e^3*cos(d*x + c)^3 - 4*(a*b^3*e^3*cos(d*x + c)^5 -
 (a^3*b + a*b^3)*e^3*cos(d*x + c)^3)*sin(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{7}{2}}(b\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^4, x)
```

3.565 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=258

$$\frac{2e^2\left(52a^2b^2+39a^4+4b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{65d\sqrt{\cos(c+dx)}}-\frac{10ab\left(115a^2+94b^2\right)(e\cos(c+dx))^{7/2}}{3003de}-\frac{2b\left(73a^2+22b^2\right)}{}$$

```
[Out] (-10*a*b*(115*a^2 + 94*b^2)*(e*Cos[c + d*x])^(7/2))/(3003*d*e) + (2*(39*a^4
 + 52*a^2*b^2 + 4*b^4)*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/
(65*d*Sqrt[Cos[c + d*x]]) + (2*(39*a^4 + 52*a^2*b^2 + 4*b^4)*e*(e*Cos[c + d
*x])^(3/2)*Sin[c + d*x])/(195*d) - (2*b*(73*a^2 + 22*b^2)*(e*Cos[c + d*x])^
(7/2)*(a + b*Sin[c + d*x]))/(429*d*e) - (38*a*b*(e*Cos[c + d*x])^(7/2)*(a +
 b*Sin[c + d*x])^2)/(143*d*e) - (2*b*(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c +
d*x])^3)/(13*d*e)
```

Rubi [A] time = 0.508205, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2692, 2862, 2669, 2635, 2640, 2639}

$$\frac{2e^2\left(52a^2b^2+39a^4+4b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{65d\sqrt{\cos(c+dx)}}-\frac{10ab\left(115a^2+94b^2\right)(e\cos(c+dx))^{7/2}}{3003de}-\frac{2b\left(73a^2+22b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^4,x]
```


```
[Out] (-10*a*b*(115*a^2 + 94*b^2)*(e*Cos[c + d*x])^(7/2))/(3003*d*e) + (2*(39*a^4
 + 52*a^2*b^2 + 4*b^4)*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/
(65*d*Sqrt[Cos[c + d*x]]) + (2*(39*a^4 + 52*a^2*b^2 + 4*b^4)*e*(e*Cos[c + d
*x])^(3/2)*Sin[c + d*x])/(195*d) - (2*b*(73*a^2 + 22*b^2)*(e*Cos[c + d*x])^
(7/2)*(a + b*Sin[c + d*x]))/(429*d*e) - (38*a*b*(e*Cos[c + d*x])^(7/2)*(a +
 b*Sin[c + d*x])^2)/(143*d*e) - (2*b*(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c +
d*x])^3)/(13*d*e)
```


Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
```

```
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^4\,dx = -\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3}{13de} + \frac{2}{13}\int (e\cos(c+dx))^{5/2}(a+b\,\mathrm{si}$$

$$= -\frac{38ab(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}{143de} - \frac{2b(e\cos(c+dx))^{7/2}(a+b\,\mathrm{sin}}{13de}$$

$$= -\frac{2b\left(73a^2+22b^2\right)(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{429de} - \frac{38ab(e\cos(c+d}{}$$

$$= -\frac{10ab\left(115a^2+94b^2\right)(e\cos(c+dx))^{7/2}}{3003de} - \frac{2b\left(73a^2+22b^2\right)(e\cos(c+dx))}{429de}$$

$$= -\frac{10ab\left(115a^2+94b^2\right)(e\cos(c+dx))^{7/2}}{3003de} + \frac{2\left(39a^4+52a^2b^2+4b^4\right)e(e\cos(}{195d}$$

$$= -\frac{10ab\left(115a^2+94b^2\right)(e\cos(c+dx))^{7/2}}{3003de} + \frac{2\left(39a^4+52a^2b^2+4b^4\right)e(e\cos(}{195d}$$

$$= -\frac{10ab\left(115a^2+94b^2\right)(e\cos(c+dx))^{7/2}}{3003de} + \frac{2\left(39a^4+52a^2b^2+4b^4\right)e^2\sqrt{e\,\mathrm{co}}}{65d\sqrt{\cos(c}}$$

Mathematica [A] time = 2.10344, size = 209, normalized size = 0.81

$$\frac{(e\cos(c+dx))^{5/2}\left(2\left(52a^2b^2+39a^4+4b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+65\sqrt{\cos(c+dx)}\left(-\frac{1}{78}b^2\left(13a^2+b^2\right)\sin(4(c+dx))+\frac{(-208}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(2*(39*a^4 + 52*a^2*b^2 + 4*b^4)*EllipticE[(c + d*x
)/2, 2] + 65*Sqrt[Cos[c + d*x]]*(-(a*b*(66*a^2 + 31*b^2)*Cos[c + d*x])/77 -
 (a*b*(44*a^2 + 9*b^2)*Cos[3*(c + d*x)])/154 + (a*b^3*Cos[5*(c + d*x)])/22
+ ((624*a^4 - 208*a^2*b^2 - 61*b^4)*Sin[2*(c + d*x)])/3120 - (b^2*(13*a^2 +
 b^2)*Sin[4*(c + d*x)])/78 + (b^4*Sin[6*(c + d*x)])/208)))/(65*d*Cos[c + d*
x]^(5/2))
```

Maple [B] time = 2.417, size = 776, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^4,x)
```

```
[Out] 2/15015/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(12012*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+616*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+147840*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^14-320320*a^2*b^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+640640*a^2*b^2*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^8-448448*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+12
8128*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-12012*a^2*b^2*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+9009*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+924*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*b^4-443520*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12
+492800*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-246400*b^4*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^8+24024*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6+48664*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-24024*a^4*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4+6006*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2-924*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+349440*a*b^3*sin(1/2*d*x
+1/2*c)^13-1048320*a*b^3*sin(1/2*d*x+1/2*c)^11-137280*a^3*b*sin(1/2*d*x+1/2
*c)^9+1173120*a*b^3*sin(1/2*d*x+1/2*c)^9+274560*a^3*b*sin(1/2*d*x+1/2*c)^7-
599040*a*b^3*sin(1/2*d*x+1/2*c)^7-205920*a^3*b*sin(1/2*d*x+1/2*c)^5+121680*
a*b^3*sin(1/2*d*x+1/2*c)^5+68640*a^3*b*sin(1/2*d*x+1/2*c)^3+3120*a*b^3*sin(
1/2*d*x+1/2*c)^3-8580*a^3*b*sin(1/2*d*x+1/2*c)-3120*a*b^3*sin(1/2*d*x+1/2*c
))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^4e^2\cos(dx+c)^6 - 2\left(3a^2b^2+b^4\right)e^2\cos(dx+c)^4 + \left(a^4+6a^2b^2+b^4\right)e^2\cos(dx+c)^2 - 4\left(ab^3e^2\cos(dx+c)^4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*e^2*cos(d*x + c)^6 - 2*(3*a^2*b^2 + b^4)*e^2*cos(d*x + c)^4 +
 (a^4 + 6*a^2*b^2 + b^4)*e^2*cos(d*x + c)^2 - 4*(a*b^3*e^2*cos(d*x + c)^4 -
 (a^3*b + a*b^3)*e^2*cos(d*x + c)^2)*sin(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}} (b\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^4, x)
```

3.566 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=258

$$\frac{2e^2\left(132a^2b^2+77a^4+12b^4\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{26ab\left(79a^2+74b^2\right)(e\cos(c+dx))^{5/2}}{3465de}-\frac{2b\left(167a^2+54b^2\right)(e$$

```
[Out] (-26*a*b*(79*a^2 + 74*b^2)*(e*Cos[c + d*x])^(5/2))/(3465*d*e) + (2*(77*a^4
+ 132*a^2*b^2 + 12*b^4)*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(
231*d*Sqrt[e*Cos[c + d*x]]) + (2*(77*a^4 + 132*a^2*b^2 + 12*b^4)*e*Sqrt[e*C
os[c + d*x]]*Sin[c + d*x])/(231*d) - (2*b*(167*a^2 + 54*b^2)*(e*Cos[c + d*x
])^(5/2)*(a + b*Sin[c + d*x]))/(693*d*e) - (34*a*b*(e*Cos[c + d*x])^(5/2)*(
a + b*Sin[c + d*x])^2)/(99*d*e) - (2*b*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c
+ d*x])^3)/(11*d*e)
```

Rubi [A] time = 0.509408, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2692, 2862, 2669, 2635, 2642, 2641}

$$\frac{2e^2\left(132a^2b^2+77a^4+12b^4\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{26ab\left(79a^2+74b^2\right)(e\cos(c+dx))^{5/2}}{3465de}-\frac{2b\left(167a^2+54b^2\right)(e$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^4,x]
```


```
[Out] (-26*a*b*(79*a^2 + 74*b^2)*(e*Cos[c + d*x])^(5/2))/(3465*d*e) + (2*(77*a^4
+ 132*a^2*b^2 + 12*b^4)*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(
231*d*Sqrt[e*Cos[c + d*x]]) + (2*(77*a^4 + 132*a^2*b^2 + 12*b^4)*e*Sqrt[e*C
os[c + d*x]]*Sin[c + d*x])/(231*d) - (2*b*(167*a^2 + 54*b^2)*(e*Cos[c + d*x
])^(5/2)*(a + b*Sin[c + d*x]))/(693*d*e) - (34*a*b*(e*Cos[c + d*x])^(5/2)*(
a + b*Sin[c + d*x])^2)/(99*d*e) - (2*b*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c
+ d*x])^3)/(11*d*e)
```


Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
```

```
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^4\,dx = -\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3}{11de} + \frac{2}{11}\int (e\cos(c+dx))^{3/2}(a+b\sin($$

$$= -\frac{34ab(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}{99de} - \frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(}{11de}$$

$$= -\frac{2b\left(167a^2+54b^2\right)(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{693de} - \frac{34ab(e\cos(c+dx}{}$$

$$= -\frac{26ab\left(79a^2+74b^2\right)(e\cos(c+dx))^{5/2}}{3465de} - \frac{2b\left(167a^2+54b^2\right)(e\cos(c+dx))^{5/}}{693de}$$

$$= -\frac{26ab\left(79a^2+74b^2\right)(e\cos(c+dx))^{5/2}}{3465de} + \frac{2\left(77a^4+132a^2b^2+12b^4\right)e\sqrt{e\cos}}{231d}$$

$$= -\frac{26ab\left(79a^2+74b^2\right)(e\cos(c+dx))^{5/2}}{3465de} + \frac{2\left(77a^4+132a^2b^2+12b^4\right)e\sqrt{e\cos}}{231d}$$

$$= -\frac{26ab\left(79a^2+74b^2\right)(e\cos(c+dx))^{5/2}}{3465de} + \frac{2\left(77a^4+132a^2b^2+12b^4\right)e^2\sqrt{\cos}}{231d\sqrt{e\cos(c+}}$$

Mathematica [A] time = 2.72029, size = 189, normalized size = 0.73

$$\frac{(e\cos(c+dx))^{3/2}\left(240\left(132a^2b^2+77a^4+12b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\left(-45b\left(264a^2b+31b^3\right)\sin(3(c+dx))+3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(240*(77*a^4 + 132*a^2*b^2 + 12*b^4)*EllipticF[(c +
 d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(-1848*b*(12*a^3 + 7*a*b^2) - 2464*(9*a^3*
b + 4*a*b^3)*Cos[2*(c + d*x)] + 3080*a*b^3*Cos[4*(c + d*x)] + 30*(616*a^4 +
 660*a^2*b^2 + 39*b^4)*Sin[c + d*x] - 45*b*(264*a^2*b + 31*b^3)*Sin[3*(c +
d*x)] + 315*b^4*Sin[5*(c + d*x)])))/(27720*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 2.106, size = 639, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^4,x)
```

```
[Out] -2/3465/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(20160*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+49280*a*b^3*sin(1/2*d*x+1/2*c)^
11-50400*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-47520*a^2*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-123200*a*b^3*sin(1/2*d*x+1/2*c)^9+41040*b^
4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-22176*a^3*b*sin(1/2*d*x+1/2*c)^7+
71280*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+101024*a*b^3*sin(1/2*
d*x+1/2*c)^7-11160*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4620*a^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+33264*a^3*b*sin(1/2*d*x+1/2*c)^5-27720
*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-28336*a*b^3*sin(1/2*d*x+1/
2*c)^5+1155*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+1980*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*
b^2+180*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-2310*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2-16632*a^3*b*sin(1/2*d*x+1/2*c)^3+1980*a^2*b^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2-1232*a*b^3*sin(1/2*d*x+1/2*c)^3+180*b^4*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2+2772*a^3*b*sin(1/2*d*x+1/2*c)+1232*a*b^3*sin(1/2
*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\mathrm{integral}\left(\left(b^4 e\cos(dx+c)^5 - 2\left(3a^2b^2+b^4\right)e\cos(dx+c)^3 + \left(a^4+6a^2b^2+b^4\right)e\cos(dx+c) - 4\left(ab^3e\cos(dx+c)^3 - \left(\right.\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*e*cos(d*x + c)^5 - 2*(3*a^2*b^2 + b^4)*e*cos(d*x + c)^3 + (a^
4 + 6*a^2*b^2 + b^4)*e*cos(d*x + c) - 4*(a*b^3*e*cos(d*x + c)^3 - (a^3*b +
a*b^3)*e*cos(d*x + c))*sin(d*x + c))*sqrt(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^4, x)
```

3.567 $\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=210

$$-\frac{22ab\left(17a^2+18b^2\right)(e\cos(c+dx))^{3/2}}{315de}-\frac{2b\left(41a^2+14b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{105de}+\frac{2\left(36a^2b^2+15a^4+4b\right.}{15a}$$

```
[Out] (-22*a*b*(17*a^2 + 18*b^2)*(e*Cos[c + d*x])^(3/2))/(315*d*e) + (2*(15*a^4 +
 36*a^2*b^2 + 4*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*
Sqrt[Cos[c + d*x]]) - (2*b*(41*a^2 + 14*b^2)*(e*Cos[c + d*x])^(3/2)*(a + b*
Sin[c + d*x]))/(105*d*e) - (10*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*
x])^2)/(21*d*e) - (2*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3)/(9*d*
e)
```

Rubi [A] time = 0.441776, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2692, 2862, 2669, 2640, 2639}

$$-\frac{22ab\left(17a^2+18b^2\right)(e\cos(c+dx))^{3/2}}{315de}-\frac{2b\left(41a^2+14b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{105de}+\frac{2\left(36a^2b^2+15a^4+4b\right.}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-22*a*b*(17*a^2 + 18*b^2)*(e*Cos[c + d*x])^(3/2))/(315*d*e) + (2*(15*a^4 +
 36*a^2*b^2 + 4*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*
Sqrt[Cos[c + d*x]]) - (2*b*(41*a^2 + 14*b^2)*(e*Cos[c + d*x])^(3/2)*(a + b*
Sin[c + d*x]))/(105*d*e) - (10*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*
x])^2)/(21*d*e) - (2*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3)/(9*d*
e)
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^4\,dx = -\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3}{9de} + \frac{2}{9}\int \sqrt{e\cos(c+dx)}(a+b\sin(c+$$

$$= -\frac{10ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}{21de} - \frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c}{9de}$$

$$= -\frac{2b\left(41a^2+14b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{105de} - \frac{10ab(e\cos(c+dx))}{2}$$

$$= -\frac{22ab\left(17a^2+18b^2\right)(e\cos(c+dx))^{3/2}}{315de} - \frac{2b\left(41a^2+14b^2\right)(e\cos(c+dx))^{3/2}(}{105de}$$

$$= -\frac{22ab\left(17a^2+18b^2\right)(e\cos(c+dx))^{3/2}}{315de} - \frac{2b\left(41a^2+14b^2\right)(e\cos(c+dx))^{3/2}(}{105de}$$

$$= -\frac{22ab\left(17a^2+18b^2\right)(e\cos(c+dx))^{3/2}}{315de} + \frac{2\left(15a^4+36a^2b^2+4b^4\right)\sqrt{e\cos(c+}}{15d\sqrt{\cos(c+dx}}$$

Mathematica [A] time = 1.0896, size = 137, normalized size = 0.65

$$\frac{\sqrt{e\cos(c+dx)}\left(84\left(36a^2b^2+15a^4+4b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right) - b\cos^{\frac{3}{2}}(c+dx)\left(21b\left(72a^2+13b^2\right)\sin(c+dx)+5\left(336a^3+\right.\right.\right.}{630d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*(84*(15*a^4 + 36*a^2*b^2 + 4*b^4)*EllipticE[(c + d*x)
/2, 2] - b*Cos[c + d*x]^(3/2)*(-360*a*b^2*Cos[2*(c + d*x)] + 21*b*(72*a^2 +
 13*b^2)*Sin[c + d*x] + 5*(336*a^3 + 264*a*b^2 - 7*b^3*Sin[3*(c + d*x)]))))
/(630*d*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 1.846, size = 525, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(1120*b^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+2880*a*b^3*sin(1/2*d*x+1/2*c)^9-2240
*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-3024*a^2*b^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^6-5760*a*b^3*sin(1/2*d*x+1/2*c)^7+1064*b^4*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^6-1680*a^3*b*sin(1/2*d*x+1/2*c)^5+3024*a^2*b^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+2640*a*b^3*sin(1/2*d*x+1/2*c)^5+56*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+315*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4
+756*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+84*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+1680*
a^3*b*sin(1/2*d*x+1/2*c)^3-756*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2+240*a*b^3*sin(1/2*d*x+1/2*c)^3-84*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2-420*a^3*b*sin(1/2*d*x+1/2*c)-240*a*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^4\cos(dx+c)^4+a^4+6\,a^2b^2+b^4-2\left(3\,a^2b^2+b^4\right)\cos(dx+c)^2-4\left(ab^3\cos(dx+c)^2-a^3b-ab^3\right)\sin(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^4, x)
```

3.568 $\int \frac{(a+b\sin(c+dx))^4}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=210

$$-\frac{6ab\left(31a^2+34b^2\right)\sqrt{e\cos(c+dx)}}{35de}-\frac{2b\left(29a^2+10b^2\right)\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{35de}+\frac{2\left(28a^2b^2+7a^4+4b^4\right)\sqrt{\cos(}}{7d\sqrt{e\cos(c}}$$

```
[Out] (-6*a*b*(31*a^2 + 34*b^2)*Sqrt[e*Cos[c + d*x]])/(35*d*e) + (2*(7*a^4 + 28*a
^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*d*Sqrt[e*C
os[c + d*x]]) - (2*b*(29*a^2 + 10*b^2)*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c +
d*x]))/(35*d*e) - (26*a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2)/(35*
d*e) - (2*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3)/(7*d*e)
```

Rubi [A] time = 0.445017, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2692, 2862, 2669, 2642, 2641}

$$-\frac{6ab\left(31a^2+34b^2\right)\sqrt{e\cos(c+dx)}}{35de}-\frac{2b\left(29a^2+10b^2\right)\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{35de}+\frac{2\left(28a^2b^2+7a^4+4b^4\right)\sqrt{\cos(}}{7d\sqrt{e\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/Sqrt[e*Cos[c + d*x]],x]
```


```
[Out] (-6*a*b*(31*a^2 + 34*b^2)*Sqrt[e*Cos[c + d*x]])/(35*d*e) + (2*(7*a^4 + 28*a
^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*d*Sqrt[e*C
os[c + d*x]]) - (2*b*(29*a^2 + 10*b^2)*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c +
d*x]))/(35*d*e) - (26*a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2)/(35*
d*e) - (2*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3)/(7*d*e)
```


Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2b\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3}{7de} + \frac{2}{7}\int \frac{(a+b\sin(c+dx))^2\left(\frac{7a^2}{2}+3b^2+\frac{13}{2}ab\sin(c+\right.}{\sqrt{e\cos(c+dx)}}$$

$$= -\frac{26ab\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2}{35de} - \frac{2b\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3}{7de} + \frac{4}{35}\int \frac{(a}{}$$

$$= -\frac{2b\left(29a^2+10b^2\right)\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{35de} - \frac{26ab\sqrt{e\cos(c+dx)}(a+b\sin(c+dx)}{35de}$$

$$= -\frac{6ab\left(31a^2+34b^2\right)\sqrt{e\cos(c+dx)}}{35de} - \frac{2b\left(29a^2+10b^2\right)\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{35de} - $$

$$= -\frac{6ab\left(31a^2+34b^2\right)\sqrt{e\cos(c+dx)}}{35de} - \frac{2b\left(29a^2+10b^2\right)\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{35de} - $$

$$= -\frac{6ab\left(31a^2+34b^2\right)\sqrt{e\cos(c+dx)}}{35de} + \frac{2\left(7a^4+28a^2b^2+4b^4\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}$$

Mathematica [A] time = 1.09054, size = 130, normalized size = 0.62

$$\frac{20\left(28a^2b^2+7a^4+4b^4\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right) - b\cos(c+dx)\left(5b\left(56a^2+11b^2\right)\sin(c+dx)+560a^3-56ab^2\cos\right.}{70d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (20*(7*a^4 + 28*a^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2] - b*Cos[c + d*x]*(560*a^3 + 504*a*b^2 - 56*a*b^2*Cos[2*(c + d*x)] + 5*b*
(56*a^2 + 11*b^2)*Sin[c + d*x] - 5*b^3*Sin[3*(c + d*x)]))/(70*d*Sqrt[e*Cos[
c + d*x]])
```

Maple [A] time = 1.437, size = 412, normalized size = 2.

$$-\frac{2}{35\,d}\left(80\,b^4\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^8+224\,ab^3(\sin(1/2\,dx+c/2))^7-120\,b^4\cos(1/2\,dx+c/2)(\sin(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(80*b^4*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+224*a*b^3*sin(1/2*d*x+1/2*c)^7-120*b^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-280*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^4-336*a*b^3*sin(1/2*d*x+1/2*c)^5+35*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a
^4+140*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-280
*a^3*b*sin(1/2*d*x+1/2*c)^3+140*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2-112*a*b^3*sin(1/2*d*x+1/2*c)^3+20*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2+140*a^3*b*sin(1/2*d*x+1/2*c)+112*a*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^4}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/sqrt(e*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^4\cos(dx+c)^4+a^4+6\,a^2b^2+b^4-2\left(3\,a^2b^2+b^4\right)\cos(dx+c)^2-4\left(ab^3\cos(dx+c)^2-a^3b-ab^3\right)\sin(dx+\right.}{e\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c))/(e*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^4}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/sqrt(e*cos(d*x + c)), x)
```

3.569 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=218

$$\frac{2ab\left(15a^2+62b^2\right)(e\cos(c+dx))^{3/2}}{15de^3}+\frac{2b\left(5a^2+6b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^3}-\frac{2\left(60a^2b^2+5a^4+12b^4\right)E\left(\right.}{5de^2\sqrt{\text{co}}}$$

```
[Out] (2*a*b*(15*a^2 + 62*b^2)*(e*Cos[c + d*x])^(3/2))/(15*d*e^3) - (2*(5*a^4 + 6
0*a^2*b^2 + 12*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^
2*Sqrt[Cos[c + d*x]]) + (2*b*(5*a^2 + 6*b^2)*(e*Cos[c + d*x])^(3/2)*(a + b*
Sin[c + d*x]))/(5*d*e^3) + (2*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x
])^2)/(d*e^3) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(d*e*Sqrt[e
*Cos[c + d*x]])
```

Rubi [A] time = 0.438102, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2691, 2862, 2669, 2640, 2639}

$$\frac{2ab\left(15a^2+62b^2\right)(e\cos(c+dx))^{3/2}}{15de^3}+\frac{2b\left(5a^2+6b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^3}-\frac{2\left(60a^2b^2+5a^4+12b^4\right)E\left(\right.}{5de^2\sqrt{\text{co}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(3/2),x]
```


```
[Out] (2*a*b*(15*a^2 + 62*b^2)*(e*Cos[c + d*x])^(3/2))/(15*d*e^3) - (2*(5*a^4 + 6
0*a^2*b^2 + 12*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^
2*Sqrt[Cos[c + d*x]]) + (2*b*(5*a^2 + 6*b^2)*(e*Cos[c + d*x])^(3/2)*(a + b*
Sin[c + d*x]))/(5*d*e^3) + (2*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x
])^2)/(d*e^3) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(d*e*Sqrt[e
*Cos[c + d*x]])
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
```

```
|| IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{3/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{de\sqrt{e\cos(c+dx)}} - \frac{2\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2\left(\frac{a^2}{2}+3b^2\right.}{e^2}$$

$$= \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}{de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{de\sqrt{e\cos(c+dx)}} - \frac{4\int\sqrt{}}{}$$

$$= \frac{2b\left(5a^2+6b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^3} + \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx$$

$$= \frac{2ab\left(15a^2+62b^2\right)(e\cos(c+dx))^{3/2}}{15de^3} + \frac{2b\left(5a^2+6b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^3}$$

$$= \frac{2ab\left(15a^2+62b^2\right)(e\cos(c+dx))^{3/2}}{15de^3} + \frac{2b\left(5a^2+6b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^3}$$

$$= \frac{2ab\left(15a^2+62b^2\right)(e\cos(c+dx))^{3/2}}{15de^3} - \frac{2\left(5a^4+60a^2b^2+12b^4\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{5de^2\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.57541, size = 135, normalized size = 0.62

$$\frac{\frac{1}{2}\left(\left(360a^2b^2+60a^4+63b^4\right)\sin(c+dx)+240a^3b+40ab^3\cos(2(c+dx))+280ab^3+3b^4\sin(3(c+dx))\right)-6\left(60a^2b^2+\right.}{15de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-6*(5*a^4 + 60*a^2*b^2 + 12*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2] + (240*a^3*b + 280*a*b^3 + 40*a*b^3*Cos[2*(c + d*x)] + (60*a^4 + 360*a^
2*b^2 + 63*b^4)*Sin[c + d*x] + 3*b^4*Sin[3*(c + d*x)])/2)/(15*d*e*Sqrt[e*Co
s[c + d*x]])
```

Maple [A] time = 2.007, size = 378, normalized size = 1.7

$$-\frac{2}{15\,de}\left(-24\,b^4\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^6-80\,ab^3(\sin(1/2\,dx+c/2))^5+24\,b^4\cos(1/2\,dx+c/2)(\sin(1/2\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/15/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-24*b^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-80*a*b^3*sin(1/2*d*x+1/2*c)^5+24*b^4*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+180*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+36*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-30*a^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-180*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2+80*a*b^3*sin(1/2*d*x+1/2*c)^3-36*b^4*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2-60*a^3*b*sin(1/2*d*x+1/2*c)-80*a*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^4\cos(dx+c)^4+a^4+6a^2b^2+b^4-2\left(3a^2b^2+b^4\right)\cos(dx+c)^2-4\left(ab^3\cos(dx+c)^2-a^3b-ab^3\right)\sin(dx+\right.}{e^2\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c))/(e^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(3/2), x)
```

3.570 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=216

$$\frac{2ab\left(a^2+14b^2\right)\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2b\left(a^2+2b^2\right)\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de^3}+\frac{2\left(-12a^2b^2+a^4-4b^4\right)\sqrt{\cos(c+dx)}}{3de^2\sqrt{e\cos(c+dx)}}$$

```
[Out] (2*a*b*(a^2 + 14*b^2)*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) + (2*(a^4 - 12*a^2*b^
2 - 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Co
s[c + d*x]]) + (2*b*(a^2 + 2*b^2)*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])
)/(3*d*e^3) + (2*a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2)/(3*d*e^3)
 + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(3*d*e*(e*Cos[c + d*x])^
(3/2))
```

Rubi [A] time = 0.445278, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2691, 2862, 2669, 2642, 2641}

$$\frac{2ab\left(a^2+14b^2\right)\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2b\left(a^2+2b^2\right)\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de^3}+\frac{2\left(-12a^2b^2+a^4-4b^4\right)\sqrt{\cos(c+dx)}}{3de^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a*b*(a^2 + 14*b^2)*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) + (2*(a^4 - 12*a^2*b^
2 - 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Co
s[c + d*x]]) + (2*b*(a^2 + 2*b^2)*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])
)/(3*d*e^3) + (2*a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2)/(3*d*e^3)
 + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(3*d*e*(e*Cos[c + d*x])^
(3/2))
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
```

```
|| IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{3de(e\cos(c+dx))^{3/2}} - \frac{2\int \frac{(a+b\sin(c+dx))^2\left(-\frac{a^2}{2}+3b^2+\frac{5}{2}ab\sin(c+dx)\right)}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2}{3de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{3de(e\cos(c+dx))^{3/2}} - \frac{4\int \frac{(a+b\,\mathrm{s}}{}}{}$$

$$= \frac{2b\left(a^2+2b^2\right)\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de^3} + \frac{2ab\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2}{3de^3} + \frac{2}{}$$

$$= \frac{2ab\left(a^2+14b^2\right)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2b\left(a^2+2b^2\right)\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de^3} + \frac{2ab\sqrt{e\,\mathrm{c}}}{}$$

$$= \frac{2ab\left(a^2+14b^2\right)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2b\left(a^2+2b^2\right)\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{3de^3} + \frac{2ab\sqrt{e\,\mathrm{c}}}{}$$

$$= \frac{2ab\left(a^2+14b^2\right)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2\left(a^4-12a^2b^2-4b^4\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{2b\,(}{}$$

Mathematica [A] time = 1.15919, size = 137, normalized size = 0.63

$$\frac{24a^2b^2\sin(c+dx) + 4\left(-12a^2b^2+a^4-4b^4\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right) + 16a^3b + 4a^4\sin(c+dx) + 24ab^3\cos(2(c+dx}{6de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (16*a^3*b + 40*a*b^3 + 24*a*b^3*Cos[2*(c + d*x)] + 4*(a^4 - 12*a^2*b^2 - 4*
b^4)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 4*a^4*Sin[c + d*x] + 24
*a^2*b^2*Sin[c + d*x] + 5*b^4*Sin[c + d*x] + b^4*Sin[3*(c + d*x)])/(6*d*e*(
e*Cos[c + d*x])^(3/2))
```

Maple [B] time = 2.206, size = 575, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(8*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*a^4*sin(1/2*d*x+1/2*c)^2-24*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2
*sin(1/2*d*x+1/2*c)^2-8*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4*sin(1/2*d*x+1/2*c)^2+4
8*a*b^3*sin(1/2*d*x+1/2*c)^5-8*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*a^4+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+4*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*b^4+2*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+12*a^2*b
^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-48*a*b^3*sin(1/2*d*x+1/2*c)^3+4*
b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+4*a^3*b*sin(1/2*d*x+1/2*c)+16*a
*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^4\cos(dx+c)^4+a^4+6\,a^2b^2+b^4-2\left(3\,a^2b^2+b^4\right)\cos(dx+c)^2-4\left(ab^3\cos(dx+c)^2-a^3b-ab^3\right)\sin(dx\right.}{e^3\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c))/(e^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(5/2), x)
```

3.571 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=237

$$\frac{2ab\left(3a^2-10b^2\right)(e\cos(c+dx))^{3/2}}{5de^5}-\frac{6\left(ab-\left(a^2-2b^2\right)\sin(c+dx)\right)(a+b\sin(c+dx))^2}{5de^3\sqrt{e\cos(c+dx)}}+\frac{6b\left(a^2-2b^2\right)(e\cos(c+dx))^{3}}{5de^5}$$

```
[Out] (2*a*b*(3*a^2 - 10*b^2)*(e*Cos[c + d*x])^(3/2))/(5*d*e^5) - (6*(a^4 - 4*a^2
*b^2 - 4*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt
[Cos[c + d*x]]) + (6*b*(a^2 - 2*b^2)*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c +
d*x]))/(5*d*e^5) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(5*d*e*(
e*Cos[c + d*x])^(5/2)) - (6*(a + b*Sin[c + d*x])^2*(a*b - (a^2 - 2*b^2)*Sin
[c + d*x]))/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.465307, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2691, 2861, 2862, 2669, 2640, 2639}

$$\frac{2ab\left(3a^2-10b^2\right)(e\cos(c+dx))^{3/2}}{5de^5}-\frac{6\left(ab-\left(a^2-2b^2\right)\sin(c+dx)\right)(a+b\sin(c+dx))^2}{5de^3\sqrt{e\cos(c+dx)}}+\frac{6b\left(a^2-2b^2\right)(e\cos(c+dx))^{3}}{5de^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(7/2),x]
```


```
[Out] (2*a*b*(3*a^2 - 10*b^2)*(e*Cos[c + d*x])^(3/2))/(5*d*e^5) - (6*(a^4 - 4*a^2
*b^2 - 4*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt
[Cos[c + d*x]]) + (6*b*(a^2 - 2*b^2)*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c +
d*x]))/(5*d*e^5) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(5*d*e*(
e*Cos[c + d*x])^(5/2)) - (6*(a + b*Sin[c + d*x])^2*(a*b - (a^2 - 2*b^2)*Sin
[c + d*x]))/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
```

```
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{5de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{(a+b\sin(c+dx))^2\left(-\frac{3a^2}{2}+3b^2+\frac{3}{2}ab\sin(c+dx)\right)}{(e\cos(c+dx))^{3/2}}dx}{5e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{5de(e\cos(c+dx))^{5/2}} - \frac{6(a+b\sin(c+dx))^2\left(ab-\left(a^2-2b^2\right)\sin(c+dx\right.}{5de^3\sqrt{e\cos(c+dx)}}$$

$$= \frac{6b\left(a^2-2b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{5de(e\cos(c+dx))^{5/2}}$$

$$= \frac{2ab\left(3a^2-10b^2\right)(e\cos(c+dx))^{3/2}}{5de^5} + \frac{6b\left(a^2-2b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^5} + \frac{2}{}$$

$$= \frac{2ab\left(3a^2-10b^2\right)(e\cos(c+dx))^{3/2}}{5de^5} + \frac{6b\left(a^2-2b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^5} + \frac{2}{}$$

$$= \frac{2ab\left(3a^2-10b^2\right)(e\cos(c+dx))^{3/2}}{5de^5} - \frac{6\left(a^4-4a^2b^2-4b^4\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} +$$

Mathematica [A] time = 0.568214, size = 152, normalized size = 0.64

$$\frac{2\left(-12a^2b^2\sin(c+dx)+4ab\left(a^2+b^2\right)\sec^2(c+dx)-3\left(-4a^2b^2+a^4-4b^4\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\left(6a^2b^2+a\right.\right.}{5de^3\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(-20*a*b^3 - 3*(a^4 - 4*a^2*b^2 - 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2] + 4*a*b*(a^2 + b^2)*Sec[c + d*x]^2 + 3*a^4*Sin[c + d*x] - 12*
a^2*b^2*Sin[c + d*x] - 7*b^4*Sin[c + d*x] + (a^4 + 6*a^2*b^2 + b^4)*Sec[c +
 d*x]*Tan[c + d*x]))/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Maple [B] time = 4.376, size = 874, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^4*sin
(1/2*d*x+1/2*c)^4-48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^4-
48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^4-24*a^4*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^6+96*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6
+56*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*a^4*sin(1/2*d*x+1/2*c)^2+48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1
/2*c)^2+48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+24*a^4*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-96*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^4+80*a*b^3*sin(1/2*d*x+1/2*c)^5-56*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b
^2-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-8*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2+12*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-80*a*b^3*sin(1/2*d
*x+1/2*c)^3+12*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-4*a^3*b*sin(1/2*
d*x+1/2*c)+16*a*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^4\cos\left(dx+c\right)^4+a^4+6\,a^2b^2+b^4-2\left(3\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2-4\left(ab^3\cos\left(dx+c\right)^2-a^3b-ab^3\right)\sin\left(dx+\right.\right.}{e^4\cos\left(dx+c\right)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c))/(e^4*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(dx+c\right)+a\right)^4}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(7/2), x)
```

3.572 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=241

$$\frac{10ab\left(a^2-2b^2\right)\sqrt{e\cos(c+dx)}}{21de^5}-\frac{2\left(ab-\left(5a^2-6b^2\right)\sin(c+dx)\right)(a+b\sin(c+dx))^2}{21de^3(e\cos(c+dx))^{3/2}}+\frac{2b\left(5a^2-6b^2\right)\sqrt{e\cos(c+dx)}(a}{21de^5}$$

```
[Out] (10*a*b*(a^2 - 2*b^2)*Sqrt[e*Cos[c + d*x]])/(21*d*e^5) + (2*(5*a^4 - 12*a^2
*b^2 + 12*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt
[e*Cos[c + d*x]]) + (2*b*(5*a^2 - 6*b^2)*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c
+ d*x]))/(21*d*e^5) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(7*d*
e*(e*Cos[c + d*x])^(7/2)) - (2*(a + b*Sin[c + d*x])^2*(a*b - (5*a^2 - 6*b^2
)*Sin[c + d*x]))/(21*d*e^3*(e*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.462852, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2691, 2861, 2862, 2669, 2642, 2641}

$$\frac{10ab\left(a^2-2b^2\right)\sqrt{e\cos(c+dx)}}{21de^5}-\frac{2\left(ab-\left(5a^2-6b^2\right)\sin(c+dx)\right)(a+b\sin(c+dx))^2}{21de^3(e\cos(c+dx))^{3/2}}+\frac{2b\left(5a^2-6b^2\right)\sqrt{e\cos(c+dx)}(a}{21de^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (10*a*b*(a^2 - 2*b^2)*Sqrt[e*Cos[c + d*x]])/(21*d*e^5) + (2*(5*a^4 - 12*a^2
*b^2 + 12*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt
[e*Cos[c + d*x]]) + (2*b*(5*a^2 - 6*b^2)*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c
+ d*x]))/(21*d*e^5) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(7*d*
e*(e*Cos[c + d*x])^(7/2)) - (2*(a + b*Sin[c + d*x])^2*(a*b - (5*a^2 - 6*b^2
)*Sin[c + d*x]))/(21*d*e^3*(e*Cos[c + d*x])^(3/2))
```

Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
```

```
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{9/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{7de(e\cos(c+dx))^{7/2}} - \frac{2\int \frac{(a+b\sin(c+dx))^2\left(-\frac{5a^2}{2}+3b^2+\frac{1}{2}ab\sin(c+dx)\right)}{(e\cos(c+dx))^{5/2}}dx}{7e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{7de(e\cos(c+dx))^{7/2}} - \frac{2(a+b\sin(c+dx))^2\left(ab-\left(5a^2-6b^2\right)\sin(c+dx\right.}{21de^3(e\cos(c+dx))^{3/2}}$$

$$= \frac{2b\left(5a^2-6b^2\right)\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{21de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{7de(e\cos(c+dx))^{7/2}} -$$

$$= \frac{10ab\left(a^2-2b^2\right)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2b\left(5a^2-6b^2\right)\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{21de^5} + \frac{2(b+a}{}$$

$$= \frac{10ab\left(a^2-2b^2\right)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2b\left(5a^2-6b^2\right)\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}{21de^5} + \frac{2(b+a}{}$$

$$= \frac{10ab\left(a^2-2b^2\right)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2\left(5a^4-12a^2b^2+12b^4\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}} + \frac{2}{}$$

Mathematica [A] time = 0.879653, size = 177, normalized size = 0.73

$$\frac{\sec^4(c+dx)\sqrt{e\cos(c+dx)}\left(60a^2b^2\sin(c+dx)-12a^2b^2\sin(3(c+dx))+4\left(-12a^2b^2+5a^4+12b^4\right)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+\right.\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^4*(48*a^3*b - 8*a*b^3 - 56*a*b^3*Cos[2*(
c + d*x)] + 4*(5*a^4 - 12*a^2*b^2 + 12*b^4)*Cos[c + d*x]^(7/2)*EllipticF[(c
 + d*x)/2, 2] + 17*a^4*Sin[c + d*x] + 60*a^2*b^2*Sin[c + d*x] + 3*b^4*Sin[c
 + d*x] + 5*a^4*Sin[3*(c + d*x)] - 12*a^2*b^2*Sin[3*(c + d*x)] - 9*b^4*Sin[
3*(c + d*x)]))/(42*d*e^5)
```

Maple [B] time = 5.161, size = 1067, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(72*b^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-96*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^6+96*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+12*a^2*b^2*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-12*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*b^4+40*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4*sin(1/2*d*x+1/2
*c)^6+96*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4*sin(1/2*d*x+1/2*c)^6-60*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a^4*sin(1/2*d*x+1/2*c)^4-144*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4*sin(1
/2*d*x+1/2*c)^4+144*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2*sin(1/2*d*x+1/2*c)^4-9
6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2*sin(1/2*d*x+1/2*c)^6+30*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a^4*sin(1/2*d*x+1/2*c)^2+72*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4*sin(1/2*d
*x+1/2*c)^2+40*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-72*b^4*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-40*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4+16*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-12*b^4*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2-112*a*b^3*sin(1/2*d*x+1/2*c)^5+112*a*b^3*sin(1/2*d*x+
1/2*c)^3+12*a^3*b*sin(1/2*d*x+1/2*c)-16*a*b^3*sin(1/2*d*x+1/2*c)-72*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2*b^2*sin(1/2*d*x+1/2*c)^2+12*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
a^2*b^2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^4\cos\left(dx+c\right)^4+a^4+6\,a^2b^2+b^4-2\left(3\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2-4\left(ab^3\cos\left(dx+c\right)^2-a^3b-ab^3\right)\sin\left(dx+\right.\right.}{e^5\cos\left(dx+c\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c))/(e^5*cos(d*x + c)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sin\left(dx+c\right)+a\right)^4}{\left(e\cos\left(dx+c\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(9/2), x)
```

3.573 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=264

$$\frac{2ab\left(21a^2-22b^2\right)(e\cos(c+dx))^{3/2}}{45de^7}+\frac{2\left(\left(7a^2-6b^2\right)\sin(c+dx)+ab\right)(a+b\sin(c+dx))^2}{45de^3(e\cos(c+dx))^{5/2}}-\frac{2\left(b\left(7a^2-6b^2\right)-a\left(21a^2\right.\right.}{45d}$$

```
[Out] (2*a*b*(21*a^2 - 22*b^2)*(e*Cos[c + d*x])^(3/2))/(45*d*e^7) - (2*(7*a^4 - 1
2*a^2*b^2 + 4*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*e^
6*Sqrt[Cos[c + d*x]]) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(9*
d*e*(e*Cos[c + d*x])^(9/2)) - (2*(a + b*Sin[c + d*x])*(b*(7*a^2 - 6*b^2) -
a*(21*a^2 - 22*b^2)*Sin[c + d*x]))/(45*d*e^5*Sqrt[e*Cos[c + d*x]]) + (2*(a
+ b*Sin[c + d*x])^2*(a*b + (7*a^2 - 6*b^2)*Sin[c + d*x]))/(45*d*e^3*(e*Cos[
c + d*x])^(5/2))
```

Rubi [A] time = 0.479028, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2691, 2861, 2669, 2640, 2639}

$$\frac{2ab\left(21a^2-22b^2\right)(e\cos(c+dx))^{3/2}}{45de^7}+\frac{2\left(\left(7a^2-6b^2\right)\sin(c+dx)+ab\right)(a+b\sin(c+dx))^2}{45de^3(e\cos(c+dx))^{5/2}}-\frac{2\left(b\left(7a^2-6b^2\right)-a\left(21a^2\right.\right.}{45d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(11/2),x]
```


```
[Out] (2*a*b*(21*a^2 - 22*b^2)*(e*Cos[c + d*x])^(3/2))/(45*d*e^7) - (2*(7*a^4 - 1
2*a^2*b^2 + 4*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*e^
6*Sqrt[Cos[c + d*x]]) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(9*
d*e*(e*Cos[c + d*x])^(9/2)) - (2*(a + b*Sin[c + d*x])*(b*(7*a^2 - 6*b^2) -
a*(21*a^2 - 22*b^2)*Sin[c + d*x]))/(45*d*e^5*Sqrt[e*Cos[c + d*x]]) + (2*(a
+ b*Sin[c + d*x])^2*(a*b + (7*a^2 - 6*b^2)*Sin[c + d*x]))/(45*d*e^3*(e*Cos[
c + d*x])^(5/2))
```


Rule 2691

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x]
)^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), I
nt[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2
```

```
*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g},
x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p]
|| IntegerQ[m])
```

Rule 2861

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((g*
Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p
+ 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e
 + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &&
SimplerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{11/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} - \frac{2\int \frac{(a+b\sin(c+dx))^2\left(-\frac{7a^2}{2}+3b^2-\frac{1}{2}ab\sin(c+dx)\right)}{(e\cos(c+dx))^{7/2}}dx}{9e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} + \frac{2(a+b\sin(c+dx))^2\left(ab+\left(7a^2-6b^2\right)\sin(c+d\right.}{45de^3(e\cos(c+dx))^{5/2}}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} - \frac{2(a+b\sin(c+dx))\left(b\left(7a^2-6b^2\right)-a\left(21a^2-22\right.\right.}{45de^5\sqrt{e\cos(c+dx)}}$$

$$= \frac{2ab\left(21a^2-22b^2\right)(e\cos(c+dx))^{3/2}}{45de^7} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} - \frac{2(a+b\sin}{}$$

$$= \frac{2ab\left(21a^2-22b^2\right)(e\cos(c+dx))^{3/2}}{45de^7} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} - \frac{2(a+b\sin}{}$$

$$= \frac{2ab\left(21a^2-22b^2\right)(e\cos(c+dx))^{3/2}}{45de^7} - \frac{2\left(7a^4-12a^2b^2+4b^4\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\right|2}{15de^6\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 1.59691, size = 219, normalized size = 0.83

$$\frac{\sec^5(c+dx)\sqrt{e\cos(c+dx)}\left(360a^2b^2\sin(c+dx)-156a^2b^2\sin(3(c+dx))-36a^2b^2\sin(5(c+dx))-48\left(-12a^2b^2+7a^4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^5*(320*a^3*b + 32*a*b^3 - 288*a*b^3*Cos[
2*(c + d*x)] - 48*(7*a^4 - 12*a^2*b^2 + 4*b^4)*Cos[c + d*x]^(9/2)*EllipticE
[(c + d*x)/2, 2] + 150*a^4*Sin[c + d*x] + 360*a^2*b^2*Sin[c + d*x] + 60*b^4
*Sin[c + d*x] + 91*a^4*Sin[3*(c + d*x)] - 156*a^2*b^2*Sin[3*(c + d*x)] - 8*
b^4*Sin[3*(c + d*x)] + 21*a^4*Sin[5*(c + d*x)] - 36*a^2*b^2*Sin[5*(c + d*x)
] + 12*b^4*Sin[5*(c + d*x)]))/(360*d*e^6)
```

Maple [B] time = 7.477, size = 1416, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x)
```

```
[Out] -2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c
)^4-8*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e
+e)^(1/2)/e^5*(-36*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+104*b^4*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+1152*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^10-2304*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+1824*a^2*b^2*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-672*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^4+36*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+504*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^4+288*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*
sin(1/2*d*x+1/2*c)^4-168*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^2-
96*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+21*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*a^4+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-384*b^4*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^10+1344*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+768*b^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-1064*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^6-488*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+392*a^4*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-66*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2-12*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+144*a*b^3*sin(1/2*d*x+1
/2*c)^5-144*a*b^3*sin(1/2*d*x+1/2*c)^3-20*a^3*b*sin(1/2*d*x+1/2*c)+16*a*b^3
*sin(1/2*d*x+1/2*c)-576*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)
^8+1152*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^6-672*a^4*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+336*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*sin(1/
2*d*x+1/2*c)^8+192*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^8-672*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^6-384*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*b^4*sin(1/2*d*x+1/2*c)^6-864*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x
+1/2*c)^4+288*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^4\cos(dx+c)^4+a^4+6a^2b^2+b^4-2\left(3a^2b^2+b^4\right)\cos(dx+c)^2-4\left(ab^3\cos(dx+c)^2-a^3b-ab^3\right)\sin(dx}{e^6\cos(dx+c)^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*
cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))*sqr
t(e*cos(d*x + c))/(e^6*cos(d*x + c)^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^4}{(e\cos(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4/(e*cos(d*x + c))^(11/2), x)
```

3.574 $\int \frac{(e\cos(c+dx))^{11/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=531

$$\frac{2e^5\sqrt{e\cos(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\sin(c+dx)\right)}{21b^5d}-\frac{2e^3(e\cos(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\sin(c+dx)\right)}{35b^3d}-$$

```
[Out] -(((-a^2 + b^2)^(9/4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(b^(11/2)*d)) - ((-a^2 + b^2)^(9/4)*e^(11/2)*ArcTa
nh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(11/2)*
d) + (2*e*(e*Cos[c + d*x])^(9/2))/(9*b*d) + (2*a*(21*a^4 - 49*a^2*b^2 + 33*
b^4)*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^6*d*Sqrt[e*Cos
[c + d*x]]) - (a*(a^2 - b^2)^3*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b -
 Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*
d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)^3*e^6*Sqrt[Cos[c + d*x]]*EllipticP
i[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^6*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (2*e^3*(e*Cos[c + d*x])^(5/2)*(7*(a^2
 - b^2) - 5*a*b*Sin[c + d*x]))/(35*b^3*d) + (2*e^5*Sqrt[e*Cos[c + d*x]]*(21
*(a^2 - b^2)^2 - a*b*(7*a^2 - 12*b^2)*Sin[c + d*x]))/(21*b^5*d)
```

Rubi [A] time = 1.90796, antiderivative size = 531, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2695, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{2e^5\sqrt{e\cos(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\sin(c+dx)\right)}{21b^5d}-\frac{2e^3(e\cos(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\sin(c+dx)\right)}{35b^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] -(((-a^2 + b^2)^(9/4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(b^(11/2)*d)) - ((-a^2 + b^2)^(9/4)*e^(11/2)*ArcTa
nh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(11/2)*
d) + (2*e*(e*Cos[c + d*x])^(9/2))/(9*b*d) + (2*a*(21*a^4 - 49*a^2*b^2 + 33*
b^4)*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^6*d*Sqrt[e*Cos
[c + d*x]]) - (a*(a^2 - b^2)^3*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b -
 Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*
```

```
d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)^3*e^6*Sqrt[Cos[c + d*x]]*EllipticP
i[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^6*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (2*e^3*(e*Cos[c + d*x])^(5/2)*(7*(a^2
 - b^2) - 5*a*b*Sin[c + d*x]))/(35*b^3*d) + (2*e^5*Sqrt[e*Cos[c + d*x]]*(21
*(a^2 - b^2)^2 - a*b*(7*a^2 - 12*b^2)*Sin[c + d*x]))/(21*b^5*d)
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{a+b\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{9/2}}{9bd} + \frac{e^2\int \frac{(e\cos(c+dx))^{7/2}(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} - \frac{2e^3(e\cos(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\sin(c+dx)\right)}{35b^3d} + \frac{\left(2e^4\right)\int \frac{(e\cos(c+dx}{}}{}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} - \frac{2e^3(e\cos(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\sin(c+dx)\right)}{35b^3d} + \frac{2e^5\sqrt{e\cos(c+dx}}{}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} - \frac{2e^3(e\cos(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\sin(c+dx)\right)}{35b^3d} + \frac{2e^5\sqrt{e\cos(c+dx}}{}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} - \frac{2e^3(e\cos(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\sin(c+dx)\right)}{35b^3d} + \frac{2e^5\sqrt{e\cos(c+dx}}{}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} + \frac{2a\left(21a^4-49a^2b^2+33b^4\right)e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^6d\sqrt{e\cos(c+dx)}} - \frac{2e^3(e\cos(}{}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} + \frac{2a\left(21a^4-49a^2b^2+33b^4\right)e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^6d\sqrt{e\cos(c+dx)}} + \frac{a\left(-a^2+b\right.}{}$$

$$= -\frac{\left(-a^2+b^2\right)^{9/4}e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{11/2}d} - \frac{\left(-a^2+b^2\right)^{9/4}e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{11/2}d} + \frac{2e(e}{}$$

Mathematica [C] time = 27.9987, size = 2035, normalized size = 3.83

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(11/2)*((-2*(280*a^4 - 636*a^2*b^2 + 721*b^4)*(a + b*Sqrt
[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c +
d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Co
s[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c
 + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4,
3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*
x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1
 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 +
((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 +
b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c +
d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 -
 Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) + ((840*a^4 - 1764*a^2*b^2 + 959*b^4
)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 +
 b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/
(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^
(3/4)) + (4*Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c +
 d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b
^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(
5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^
2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2
*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Co
s[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^
2*(-1 + Cos[c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2
] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x
]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^
2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[
c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*
x]^2]*(-1 + 2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2*(-392*a^3*b + 722*
a*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/
2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c +
d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4
, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/
4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2
- b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^
2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcT
an[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1
 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 -
 b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*
x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c +
d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*
x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(1680*b^4*d*Cos[c + d*x
```

```
]^(11/2)) + ((e*Cos[c + d*x])^(11/2)*Sec[c + d*x]^5*(((-9*a^2 + 14*b^2)*Cos
[2*(c + d*x)])/(45*b^3) + Cos[4*(c + d*x)]/(36*b) - (a*(28*a^2 - 51*b^2)*Si
n[c + d*x])/(42*b^4) + (a*Sin[3*(c + d*x)])/(14*b^2)))/d
```

Maple [C] time = 3.802, size = 3711, normalized size = 7.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] 8/5/d*e^5/b^3*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)*a^2+2/d*e^5/b^5*(e*(2*cos(
1/2*d*x+1/2*c)^2-1))^(1/2)*a^4-6/d*e^5/b^3*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(
1/2)*a^2-2/d*e^7/b^5*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^
2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)
*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2
-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^6+6/d*e^7/b^3*sum((_R^4+_R^2*e
)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*s
in(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=Ro
otOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*
e^4))*a^4-6/d*e^7/b*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2
-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*
cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-
10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^2+32/9/d*e^5/b*cos(1/2*d*x+1/2*
c)^8*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-8/5/d*e^5/b^3*cos(1/2*d*x+1/2*c)^4*
(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)*a^2+8/5/d*e^5/b^3*cos(1/2*d*x+1/2*c)^2*(
2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)*a^2+3/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*s
in(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a^3/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2
*c)^2-1))^(1/2)/b^4*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2
+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/
2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(
1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-s
in(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*
sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_a
lpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/8/d*(e*(2*c
os(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a/sin(1/2*d*x+1/2*c)
/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2*sum(1/_alpha/(2*_alpha^2-1)*(2^(1
/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(
4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_al
pha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*
```

sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha
^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(
1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b
^2+a^2))+1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^
6*a^7/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^8*sum(1/_al
pha/(2*_alpha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctan
h(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(
1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2
b^2)/b^2)^(1/2)/(-e(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+
8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=Ro
otOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-3/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*e^6*a^5/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2
-1))^(1/2)/b^6*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-
2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x
+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/
(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/
2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1
/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^
2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-32/7/d*(e*(2*cos(1
/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a*sin(1/2*d*x+1/2*c)^7/(
e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*
d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)+48/7/d*(e*(2*cos(1/2*d*x+1/2*c)^2-
1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a*sin(1/2*d*x+1/2*c)^5/(e*(2*cos(1/2*d*x
+1/2*c)^2-1))^(1/2)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^
(1/2)*cos(1/2*d*x+1/2*c)+8/3/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*e^6*a^3*sin(1/2*d*x+1/2*c)^3/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(
1/2)/b^4/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d
*x+1/2*c)-8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6
*a*sin(1/2*d*x+1/2*c)^3/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2/(-2*sin(1/
2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)-4/3/d*(e*
(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a^3*sin(1/2*d*x+
1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^4/(-2*sin(1/2*d*x+1/2*c)^4*e+
sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)+20/7/d*(e*(2*cos(1/2*d*x+1
/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a*sin(1/2*d*x+1/2*c)/(e*(2*cos(1
/2*d*x+1/2*c)^2-1))^(1/2)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)
^2*e)^(1/2)*cos(1/2*d*x+1/2*c)-22/7/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2
*d*x+1/2*c)^2)^(1/2)*e^6*a/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1)
)^(1/2)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-2/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*e^6*a^5/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^6/(

```
-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+14/3/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*
a^3/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^4/(-2*sin(1/2
*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-152/
45/d*e^5/b*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+6/d*e^5/b*(e*(2*cos(1/2*d*x+1
/2*c)^2-1))^(1/2)+2/d*e^7*b*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3
*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-
e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*
a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-64/9/d*e^5/b*cos(1/2*d*x+
1/2*c)^6*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+104/15/d*e^5/b*cos(1/2*d*x+1/2*
c)^4*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-152/45/d*e^5/b*cos(1/2*d*x+1/2*c)^2
*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{11}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(11/2)/(b*sin(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.575 $\int \frac{(e\cos(c+dx))^{9/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=446

$$-\frac{2e^3(e\cos(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{15b^3d}+\frac{e^{9/2}\left(b^2-a^2\right)^{7/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}d}-\frac{e^{9/2}\left(b^2-a^2\right)^{7/4}\tanh^{-1}\left(\cdots\right.}{b^{9/2}d}$$

```
[Out] ((-a^2 + b^2)^(7/4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(9/2)*d) - ((-a^2 + b^2)^(7/4)*e^(9/2)*ArcTanh[(Sq
rt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(9/2)*d) + (2
*e*(e*Cos[c + d*x])^(7/2))/(7*b*d) - (2*a*(5*a^2 - 8*b^2)*e^4*Sqrt[e*Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sqrt[Cos[c + d*x]]) + (a*(a^2 -
 b^2)^2*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c
+ d*x)/2, 2])/(b^5*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (a*(a^2
 - b^2)^2*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (
c + d*x)/2, 2])/(b^5*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (2*e^
3*(e*Cos[c + d*x])^(3/2)*(5*(a^2 - b^2) - 3*a*b*Sin[c + d*x]))/(15*b^3*d)
```

Rubi [A] time = 1.28578, antiderivative size = 446, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2695, 2865, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{2e^3(e\cos(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{15b^3d}+\frac{e^{9/2}\left(b^2-a^2\right)^{7/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}d}-\frac{e^{9/2}\left(b^2-a^2\right)^{7/4}\tanh^{-1}\left(\cdots\right.}{b^{9/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a^2 + b^2)^(7/4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(9/2)*d) - ((-a^2 + b^2)^(7/4)*e^(9/2)*ArcTanh[(Sq
rt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(9/2)*d) + (2
*e*(e*Cos[c + d*x])^(7/2))/(7*b*d) - (2*a*(5*a^2 - 8*b^2)*e^4*Sqrt[e*Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sqrt[Cos[c + d*x]]) + (a*(a^2 -
 b^2)^2*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c
+ d*x)/2, 2])/(b^5*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (a*(a^2
 - b^2)^2*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (
c + d*x)/2, 2])/(b^5*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (2*e^
```

```
3*(e*Cos[c + d*x])^(3/2)*(5*(a^2 - b^2) - 3*a*b*Sin[c + d*x]))/(15*b^3*d)
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{a+b\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{7/2}}{7bd} + \frac{e^2\int \frac{(e\cos(c+dx))^{5/2}(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2e^3(e\cos(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{15b^3d} + \frac{\left(2e^4\right)\int \frac{\sqrt{e\cos(c+}}{}}{}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2e^3(e\cos(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{15b^3d} - \frac{\left(a\left(5a^2-8b^2\right)e^4\right.}{}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2e^3(e\cos(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\sin(c+dx)\right)}{15b^3d} - \frac{\left(a\left(a^2-b^2\right)^2e^5\right)}{}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2a\left(5a^2-8b^2\right)e^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} - \frac{2e^3(e\cos(c+dx))^{3/2}}{}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2a\left(5a^2-8b^2\right)e^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{a\left(a^2-b^2\right)^2e^5\sqrt{\cos}}{b^5\left(b-\sqrt{-}\right.}$$

$$= \frac{\left(-a^2+b^2\right)^{7/4}e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{9/2}d} - \frac{\left(-a^2+b^2\right)^{7/4}e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{9/2}d} + \frac{2e(e\cos}{}$$

Mathematica [C] time = 27.1263, size = 834, normalized size = 1.87

$$\frac{(e\cos(c+dx))^{9/2}\sec^4(c+dx)\left(\frac{\left(37b^2-28a^2\right)\cos(c+dx)}{42b^3}+\frac{\cos(3(c+dx))}{14b}+\frac{a\sin(2(c+dx))}{5b^2}\right)}{d} - \frac{(e\cos(c+dx))^{9/2}\left(-\frac{\left(5a^3-8ab^2\right)\left(a+b\sqrt{1-\cos}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((e*Cos[c + d*x])^(9/2)*((-2*(2*a^2*b - 5*b^3)*(a + b*Sqrt[1 - Cos[c + d*x
]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(
-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1
 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 +
((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 +
b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c +
d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])
/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((5*a^3 - 8*a*b^2)*(a +
b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c +
d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a
*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2
- b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^
2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Co
s[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2
- b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(
3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(5*b^3*d*Cos
[c + d*x]^(9/2)) + ((e*Cos[c + d*x])^(9/2)*Sec[c + d*x]^4*(((-28*a^2 + 37*b
^2)*Cos[c + d*x])/(42*b^3) + Cos[3*(c + d*x)]/(14*b) + (a*Sin[2*(c + d*x)])
/(5*b^2)))/d
```

Maple [C] time = 2.586, size = 2126, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] 16/7/d*e^4/b*cos(1/2*d*x+1/2*c)^6*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-24/7/d
*e^4/b*cos(1/2*d*x+1/2*c)^4*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+64/21/d*e^4/
b*cos(1/2*d*x+1/2*c)^2*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+64/21/d*e^4/b*(2*
cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-4/3/d*e^4/b^3*cos(1/2*d*x+1/2*c)^2*(2*cos(1
/2*d*x+1/2*c)^2*e-e)^(1/2)*a^2-4/3/d*e^4/b^3*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(
1/2)*a^2+2/d*e^4/b^3*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*a^2-4/d*e^4/b*(e*
(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)+1/2/d*e^5/b^3*sum((_R^6-_R^4*e-_R^2*e^2+e
^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2
*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=
RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^
2*e^4))*a^4-1/d*e^5/b*sum((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e
```

```
+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)
^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z
^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^2+1/2/d*e^5*b*su
m((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b
^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d
*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^
2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-16/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*e^5*a/b^2/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*cos(
1/2*d*x+1/2*c)^7+32/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*e^5*a/b^2/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/s
in(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*cos(1/2*d*x+1/2*c)^5
-4/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^5*a/b^2/(-
e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(
e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*cos(1/2*d*x+1/2*c)^3-2/d*(e*(2*cos(1/2*
d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^5*a^3/b^4/(-e*(2*sin(1/2*d*x+
1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+
1/2*c)^2-1))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)+16/5/d*(e*(2*cos(1/2*d*x+1/2*c
)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^5*a/b^2/(-e*(2*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)+4/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*e^5*a/b^2/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*cos(
1/2*d*x+1/2*c)-1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*e^5/a/b^6/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*sum((
a^4-2*a^2*b^2+b^4)/_alpha*(8*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(e*(2*_alpha^2*b^2+a^2-2*b^
2)/b^2)^(1/2)+a^2*2^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos
(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*
2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^
4-sin(1/2*d*x+1/2*c)^2))^(1/2))*(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2
*c)^2-1))^(1/2))/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*d*x+1/2
*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b
^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(b*sin(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(b*sin(d*x + c) + a), x)
```

3.576 $\int \frac{(e\cos(c+dx))^{7/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=461

$$-\frac{2e^3\sqrt{e\cos(c+dx)}\left(3\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{3b^3d}-\frac{e^{7/2}\left(b^2-a^2\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}d}-\frac{e^{7/2}\left(b^2-a^2\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{}}{\sqrt{}}\right.}{b^{7/2}d}$$

```
[Out] -(((-a^2 + b^2)^(5/4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(b^(7/2)*d)) - ((-a^2 + b^2)^(5/4)*e^(7/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(7/2)*d) +
 (2*e*(e*Cos[c + d*x])^(5/2))/(5*b*d) - (2*a*(3*a^2 - 4*b^2)*e^4*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^4*d*Sqrt[e*Cos[c + d*x]]) + (a*(a^
2 - b^2)^2*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
(c + d*x)/2, 2])/(b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x
]]) + (a*(a^2 - b^2)^2*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a
^2 + b^2]), (c + d*x)/2, 2])/(b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e
*Cos[c + d*x]]) - (2*e^3*Sqrt[e*Cos[c + d*x]]*(3*(a^2 - b^2) - a*b*Sin[c +
d*x]))/(3*b^3*d)
```

Rubi [A] time = 1.32294, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2695, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{2e^3\sqrt{e\cos(c+dx)}\left(3\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{3b^3d}-\frac{e^{7/2}\left(b^2-a^2\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}d}-\frac{e^{7/2}\left(b^2-a^2\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{}}{\sqrt{}}\right.}{b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x]),x]
```


```
[Out] -(((-a^2 + b^2)^(5/4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(b^(7/2)*d)) - ((-a^2 + b^2)^(5/4)*e^(7/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(7/2)*d) +
 (2*e*(e*Cos[c + d*x])^(5/2))/(5*b*d) - (2*a*(3*a^2 - 4*b^2)*e^4*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^4*d*Sqrt[e*Cos[c + d*x]]) + (a*(a^
2 - b^2)^2*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
(c + d*x)/2, 2])/(b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x
]]) + (a*(a^2 - b^2)^2*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a
```

```
^2 + b^2]), (c + d*x)/2, 2])/(b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e
*Cos[c + d*x]]) - (2*e^3*Sqrt[e*Cos[c + d*x]]*(3*(a^2 - b^2) - a*b*Sin[c +
d*x]))/(3*b^3*d)
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{a+b\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{5/2}}{5bd} + \frac{e^2\int \frac{(e\cos(c+dx))^{3/2}(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2e^3\sqrt{e\cos(c+dx)}\left(3\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{3b^3d} + \frac{\left(2e^4\right)\int \frac{-\frac{1}{2}b\left(2a^2-3b^2\right)\cdots}{\sqrt{e\cos(c+\cdots}}}{3\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2e^3\sqrt{e\cos(c+dx)}\left(3\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{3b^3d} - \frac{\left(a\left(3a^2-4b^2\right)e^4\right)\int\cdots}{3b^4}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2e^3\sqrt{e\cos(c+dx)}\left(3\left(a^2-b^2\right)-ab\sin(c+dx)\right)}{3b^3d} - \frac{\left(a\left(-a^2+b^2\right)^{3/2}e^4\right)\cdots}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2a\left(3a^2-4b^2\right)e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\sqrt{e\cos(c+dx)}} - \frac{2e^3\sqrt{e\cos(c+dx)}\left(3\cdots\right.}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2a\left(3a^2-4b^2\right)e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\sqrt{e\cos(c+dx)}} + \frac{a\left(-a^2+b^2\right)^{3/2}e^4\sqrt{\cos\cdots}}{b^4\left(b-\sqrt{\cdots}\right.}$$

$$= -\frac{\left(-a^2+b^2\right)^{5/4}e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{7/2}d} - \frac{\left(-a^2+b^2\right)^{5/4}e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{7/2}d} + \frac{2e(e\cos\cdots}{\cdots}$$

Mathematica [C] time = 29.0857, size = 1955, normalized size = 4.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^3*(Cos[2*(c + d*x)]/(5*b) + (2*a*Sin[c
 + d*x])/(3*b^2)))/d - ((e*Cos[c + d*x])^(7/2)*((-2*(10*a^2 - 27*b^2)*(a +
b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Co
s[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[
```

1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) + ((30*a^2 - 33*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) + (28*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(60*b^2*d*Cos[c + d*x]^(7/2))

Maple [C] time = 3.155, size = 2329, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] 8/5/d*e^3/b*cos(1/2*d*x+1/2*c)^4*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-8/5/d*e
^3/b*cos(1/2*d*x+1/2*c)^2*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-8/5/d*e^3/b*(2
*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-2/d*e^3/b^3*(e*(2*cos(1/2*d*x+1/2*c)^2-1))
^(1/2)*a^2+4/d*e^3/b*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)+2/d*e^5/b^3*sum((
_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^
3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2
)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^
3*_Z^2+b^2*e^4))*a^4-4/d*e^5/b*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_
R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/
2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(
16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^2+2/d*e^5*b*sum((_R^
4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*
ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_
R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_
Z^2+b^2*e^4))-8/3/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*e^4*a*sin(1/2*d*x+1/2*c)^3/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2/(-2*
sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)+4/3
/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*a*sin(1/2*
d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2/(-2*sin(1/2*d*x+1/2*c)^
4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)+2/d*(e*(2*cos(1/2*d*x+
1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*a^3/sin(1/2*d*x+1/2*c)/(e*(2*co
s(1/2*d*x+1/2*c)^2-1))^(1/2)/b^4/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2
*c)^2*e)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-8/3/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*a/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2
*c)^2-1))^(1/2)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*e^4*a^5/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(
1/2)/b^6*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)
/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c
)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*
_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+
1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x
+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2
```

```
^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+1/4/d*(e*(2*cos(1/2*d*x+
1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*a^3/sin(1/2*d*x+1/2*c)/(e*(2*co
s(1/2*d*x+1/2*c)^2-1))^(1/2)/b^4*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2
*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b
^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a
^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1
/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-
1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*a/sin(1
/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2*sum(1/_alpha/(2*_alp
ha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*
_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2
*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^
(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_
alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4
*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(b*sin(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(b*sin(d*x + c) + a), x)
```

3.577 $\int \frac{(e\cos(c+dx))^{5/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=384

$$\frac{e^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d}-\frac{e^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d}-\frac{ae^3\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\right.}{b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+}}$$

```
[Out] ((-a^2 + b^2)^(3/4)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(5/2)*d) - ((-a^2 + b^2)^(3/4)*e^(5/2)*ArcTanh[(Sq
rt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(5/2)*d) + (2
*e*(e*Cos[c + d*x])^(3/2))/(3*b*d) + (2*a*e^2*Sqrt[e*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) - (a*(a^2 - b^2)*e^3*Sqrt[Cos
[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^3*(
b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)*e^3*Sqrt[Cos
[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^3*(
b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.869977, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2695, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{e^{5/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d}-\frac{e^{5/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d}-\frac{ae^3\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\right.}{b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a^2 + b^2)^(3/4)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(5/2)*d) - ((-a^2 + b^2)^(3/4)*e^(5/2)*ArcTanh[(Sq
rt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(5/2)*d) + (2
*e*(e*Cos[c + d*x])^(3/2))/(3*b*d) + (2*a*e^2*Sqrt[e*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) - (a*(a^2 - b^2)*e^3*Sqrt[Cos
[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^3*(
b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)*e^3*Sqrt[Cos
[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^3*(
b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]])
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{a+b\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{(ae^2)\int\sqrt{e\cos(c+dx)}\,dx}{b^2} + \frac{\left(\left(-a^2+b^2\right)e^2\right)\int\frac{\sqrt{e\cos(c+dx)}}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{\left(a\left(a^2-b^2\right)e^3\right)\int\frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{2b^3} - \frac{\left(a\left(a^2-b^2\right)e^3\right)\int}{}$$

$$= \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{2ae^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} - \frac{\left(2\left(a^2-b^2\right)e^3\right)\mathrm{Subst}\left(\int\frac{x}{\left(a^2-b^2\right)}\right.}{bd}$$

$$= \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{2ae^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} - \frac{a\left(a^2-b^2\right)e^3\sqrt{\cos(c+dx)}\Pi\left(\frac{}{b-}\right.}{b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{}}$$

$$= \frac{\left(-a^2+b^2\right)^{3/4}e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d} - \frac{\left(-a^2+b^2\right)^{3/4}e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d} + \frac{2e(e\cos}{}$$

Mathematica [C] time = 21.4337, size = 709, normalized size = 1.85

$$(e\cos(c+dx))^{5/2}\left(-\frac{a\left(a+b\sqrt{\sin^2(c+dx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)+3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(-\log\left(-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(c+dx)}\right.\right.\right.}{4b^{3/2}\left(b^2-\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(2*Cos[c + d*x]^(3/2) - (a*(8*b^(5/2)*AppellF1[3/4,
 -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d
*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sq
rt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[C
os[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a
^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2]
```

```
+ Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*
(a + b*Sqrt[Sin[c + d*x]^2]))/(4*b^(3/2)*(-a^2 + b^2)*(a + b*Sin[c + d*x]))
 - (6*b*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)
/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan
[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1
+ ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2
+ b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c
+ d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Co
s[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x
]*(a + b*Sqrt[Sin[c + d*x]^2]))/(Sqrt[Sin[c + d*x]^2]*(a + b*Sin[c + d*x]))
))/(3*b*d*Cos[c + d*x]^(5/2))
```

Maple [C] time = 2.72, size = 1131, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] 4/3/d*e^2/b*cos(1/2*d*x+1/2*c)^2*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+4/3/d*e
^2/b*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-2/d*e^2/b*(e*(2*cos(1/2*d*x+1/2*c)^
2-1))^(1/2)-1/2/d*e^3/b*sum((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2
*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+
e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*
_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^2+1/2/d*e^3*b*
sum((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3
*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2
*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*
e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))+2/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*e^3*a/b^2/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2
*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*e^3/a/b^4/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)
*sum((a^2-b^2)/_alpha*(8*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)+a^2*2^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2
*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1
```

```
/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2))^(1/2))*(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^
2-1))^(1/2))/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*d*x+1/2*c)^
2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a
^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(b*sin(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(b*sin(d*x + c) + a), x)
```

3.578 $\int \frac{(e\cos(c+dx))^{3/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=397

$$-\frac{e^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}d}-\frac{e^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}d}-\frac{ae^2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(\right.}{b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c}}$$

```
[Out] -(((-a^2 + b^2)^(1/4)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*d)) - ((-a^2 + b^2)^(1/4)*e^(3/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*d) +
 (2*e*Sqrt[e*Cos[c + d*x]])/(b*d) + (2*a*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2])/(b^2*d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)*e^2*Sqrt[Cos[
c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^2*(a
^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)*e^2
*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2
])/(b^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.881962, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2695, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{e^{3/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}d}-\frac{e^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}d}-\frac{ae^2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(\right.}{b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((-a^2 + b^2)^(1/4)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*d)) - ((-a^2 + b^2)^(1/4)*e^(3/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*d) +
 (2*e*Sqrt[e*Cos[c + d*x]])/(b*d) + (2*a*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2])/(b^2*d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)*e^2*Sqrt[Cos[
c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^2*(a
^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)*e^2
*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2
])/(b^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]])
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{a+b\sin(c+dx)}\,dx = \frac{2e\sqrt{e\cos(c+dx)}}{bd} + \frac{e^2\int \frac{b+a\sin(c+dx)}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}\,dx}{b}$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{bd} + \frac{\left(ae^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{b^2} + \frac{\left(\left(-a^2+b^2\right)e^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}\,dx}{b^2}$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{bd} - \frac{\left(a\sqrt{-a^2+b^2}e^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{2b^2} - \frac{\left(a\sqrt{-a^2+b^2}e^2\right)\int -$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{bd} + \frac{2ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{e\cos(c+dx)}} - \frac{\left(2\left(a^2-b^2\right)e^3\right)\text{Subst}\left(\int \frac{1}{\left(a^2-b^2\right)e^2+b^2x}\right.}{bd}$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{bd} + \frac{2ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{e\cos(c+dx)}} + \frac{a\sqrt{-a^2+b^2}e^2\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2}}\right.}{b^2\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos}}$$

$$= -\frac{\sqrt[4]{-a^2+b^2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{3/2}d} - \frac{\sqrt[4]{-a^2+b^2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{3/2}d} + \frac{2e\sqrt{e\cos(c+}}{bd}$$

Mathematica [C] time = 4.70315, size = 233, normalized size = 0.59

$$\frac{\sec^4(c+dx)(e\cos(c+dx))^{3/2}\left(a^2+b^2\cos^2(c+dx)-b^2\right)\left(\frac{2b\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\left(1-\frac{a^2}{b^2}\right)\sec^2(c+dx)\right)}{\sqrt[4]{\sec^2(c+dx)}} + \frac{a\tan(c+dx)\left(\Pi\left(-\frac{\sqrt{b^2-a^2}}{b};-\sin^{-1}\left(\sqrt[4]{\sec^2(c\right.\right.\right.}{\sqrt{-}}\right.}{b^2d\sec^2(c+dx)^{3/4}(a+b\sin(c+dx))\left(a\sqrt{\sec^2(c+dx)}-b\tan(c+dx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(a^2 - b^2 + b^2*Cos[c + d*x]^2)*Sec[c + d*x]^4*((2
*b*Hypergeometric2F1[-1/4, 1, 3/4, (1 - a^2/b^2)*Sec[c + d*x]^2])/(Sec[c +
d*x]^2)^(1/4) + (a*(EllipticPi[-(Sqrt[-a^2 + b^2]/b), -ArcSin[(Sec[c + d*x]
^2)^(1/4)], -1] + EllipticPi[Sqrt[-a^2 + b^2]/b, -ArcSin[(Sec[c + d*x]^2)^(
1/4)], -1])*Tan[c + d*x])/Sqrt[-Tan[c + d*x]^2]))/(b^2*d*(Sec[c + d*x]^2)^(
3/4)*(a + b*Sin[c + d*x])*(a*Sqrt[Sec[c + d*x]^2] - b*Tan[c + d*x]))
```

Maple [C] time = 2.537, size = 1266, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] 2/d*e/b*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)-2/d*e^3/b*sum((_R^4+_R^2*e)/(_
R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1
/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf
(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4)
)*a^2+2/d*e^3*b*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_
R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(
1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b
^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-2/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*e^2*a/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^
2-1))^(1/2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*e^2*a^3/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2
)/b^4*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^
2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2
*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_al
pha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2
*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/
2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1
/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/8/d*(e*(2*cos(1/2*d*x+1/2
*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^2*a/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2
*d*x+1/2*c)^2-1))^(1/2)/b^2*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2*_alp
ha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(
4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*
b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/
2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)
^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2
/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(b*sin(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(b*sin(d*x + c) + a), x)
```

3.579 $\int \frac{\sqrt{e\cos(c+dx)}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=292

$$\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}d\sqrt[4]{b^2-a^2}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}d\sqrt[4]{b^2-a^2}}+\frac{ae\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{ae\sqrt{\cos(c+dx)}\Pi}{bd\left(\sqrt{b^2-a^2}\right.}$$

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/(Sqrt[b]*(-a^2 + b^2)^(1/4)*d) - (Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(Sqrt[b]*(-a^2 + b^2)^(1/4)*d) + (a
*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2,
 2])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (a*e*Sqrt[Cos[c +
d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b*(b + Sqr
t[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.582375, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}d\sqrt[4]{b^2-a^2}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}d\sqrt[4]{b^2-a^2}}+\frac{ae\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{ae\sqrt{\cos(c+dx)}\Pi}{bd\left(\sqrt{b^2-a^2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/(Sqrt[b]*(-a^2 + b^2)^(1/4)*d) - (Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(Sqrt[b]*(-a^2 + b^2)^(1/4)*d) + (a
*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2,
 2])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (a*e*Sqrt[Cos[c +
d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b*(b + Sqr
t[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]])
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{a+b\sin(c+dx)}\,dx = -\frac{(ae)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{2b} + \frac{(ae)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}+b\cos(c+dx)\right)}\,dx}{2b} + \frac{(be)\,\mathrm{Sub}}{}$$

$$= \frac{(2be)\,\mathrm{Subst}\left(\int \frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\cos(c+dx)}\right)}{d} - \frac{\left(ae\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\right.}}{2b\sqrt{e\cos(c+dx)}}$$

$$= \frac{ae\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+dx)}} + \frac{ae\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+dx)}} - \frac{e}{}$$

$$= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}d} - \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}d} + \frac{ae\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+\right.}{b\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+}}$$

Mathematica [C] time = 15.9687, size = 361, normalized size = 1.24

$$-\frac{2\sin(c+dx)\sqrt{e\cos(c+dx)}\left(a+b\sqrt{\sin^2(c+dx)}\right)\left(\frac{a\cos^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)}{3(a^2-b^2)} + \frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(-\log\left(-(1+i)\sqrt{b}\sqrt[4]{b^2-a^2}\right.\right.}{}\right.}{d\sqrt{\sin^2(c+dx)}\sqrt{\cos(}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8
 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/
4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d
*x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b
^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1
/4)))*Sin[c + d*x]*(a + b*Sqrt[Sin[c + d*x]^2]))/(d*Sqrt[Cos[c + d*x]]*Sqrt
[Sin[c + d*x]^2]*(a + b*Sin[c + d*x]))
```

Maple [C] time = 1.838, size = 682, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d*e*b*sum((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*
e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/
2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e
^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-1/8/d*(e*(2*cos(1/2*d*x+1/2*c)
^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e/a/b^2*sum(1/_alpha*(8*(e*(2*_alpha^2*b^
2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*
_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2)
)*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+a^2*2^(1/2)*arctanh(1/2*e*(4*_al
pha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)
^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/
2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))*(-sin(1/2*d*x+
1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2))/(e*(2*_alpha^2*b^2+a^2-2*b^2)
/b^2)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2),_alp
ha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+
1/2*c)^2-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{b\sin\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a), x)
```

3.580 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\left(b^2-a^2\right)^{3/4}}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\left(b^2-a^2\right)^{3/4}}+\frac{a\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{a\sqrt{\cos(c+dx)}\Pi\left(\right.}{d\left(a^2-b\left(\sqrt{b^2-a\right.\right.}}$$

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/((-a^2 + b^2)^(3/4)*d*Sqrt[e])) - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(3/4)*d*Sqrt[e]) +
 (a*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2
, 2])/((a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (a*Sqrt[C
os[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^
2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.57428, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\left(b^2-a^2\right)^{3/4}}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\left(b^2-a^2\right)^{3/4}}+\frac{a\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{a\sqrt{\cos(c+dx)}\Pi\left(\right.}{d\left(a^2-b\left(\sqrt{b^2-a\right.\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/((-a^2 + b^2)^(3/4)*d*Sqrt[e])) - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(3/4)*d*Sqrt[e]) +
 (a*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2
, 2])/((a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (a*Sqrt[C
os[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^
2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]])
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}\,dx = -\frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{2\sqrt{-a^2+b^2}} - \frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}+b\cos(c+dx)\right)}\,dx}{2\sqrt{-a^2+b^2}} +$$

$$= \frac{(2be)\,\mathrm{Subst}\left(\int \frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\cos(c+dx)}\right)}{d} - \frac{\left(a\sqrt{\cos(c+dx)}\right)\int}{2\sqrt{-a^2+b^2}}$$

$$= \frac{a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\cos(c+dx)}} + \frac{a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{-a^2+b^2}};\frac{1}{2}(c\right.}{\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\cos}}$$

$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{3/4}d\sqrt{e}} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{3/4}d\sqrt{e}} + \frac{a\sqrt{\cos(c+dx)}\,\Pi\left(\right.}{\left(a^2-b\left(b-\sqrt{-a^2\right.\right.}}$$

Mathematica [C] time = 16.7561, size = 558, normalized size = 1.87

$$-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(a+b\sqrt{\sin^2(c+dx)}\right)\left(\frac{5a\left(a^2-b^2\right)\sqrt{\cos(c+dx)}F_1\left(\right.}{\sqrt{\sin^2(c+dx)}\left(a^2+b^2\cos^2(c+dx)-b^2\right)\left(5\left(a^2-b^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)-2\cos^2\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((
1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2]
- (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]
 - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d
*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1
/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Co
s[c + d*x]])/((a^2 - b^2 + b^2*Cos[c + d*x]^2)*(5*(a^2 - b^2)*AppellF1[1/4,
 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2
*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*Sqrt[Sin[c + d*x]^2]))*(a + b*Sqrt[S
in[c + d*x]^2]))/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2]*(a + b*Sin[c
```

```
+ d*x]))
```

Maple [C] time = 2.056, size = 678, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*b*e*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*
e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+
1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*
_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/a/b^2*sum(1/_alpha/(2*_alpha^2-1)*(8*(e*(2*_alpha^2*b^2+
a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_a
lpha^3*b^2-8*b^2*_alpha*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*
(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+a^2*2^(1/2)*arctanh(1/2*e*(4*_alph
a^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2
+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)
/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))*(-sin(1/2*d*x+1/
2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2))/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2),_alpha
=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/
2*c)^2-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)), x)
```

3.581 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}\,dx$

Optimal. Leaf size=411

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{2(b-a\sin(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\cos(c+}}$$

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/((-a^2 + b^2)^(5/4)*d*e^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(5/4)*d*e^(3/2)) - (2
*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/((a^2 - b^2)*d*e^2*Sqrt[
Cos[c + d*x]]) - (a*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
b^2]), (c + d*x)/2, 2])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[
c + d*x]]) - (a*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]
), (c + d*x)/2, 2])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c +
d*x]]) - (2*(b - a*Sin[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.928426, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2696, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{2(b-a\sin(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])),x]
```


```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/((-a^2 + b^2)^(5/4)*d*e^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(5/4)*d*e^(3/2)) - (2
*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/((a^2 - b^2)*d*e^2*Sqrt[
Cos[c + d*x]]) - (a*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
b^2]), (c + d*x)/2, 2])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[
c + d*x]]) - (a*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]
), (c + d*x)/2, 2])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c +
d*x]]) - (2*(b - a*Sin[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}\,dx = -\frac{2(b-a\sin(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}} - \frac{2\int \frac{\sqrt{e\cos(c+dx)}\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{1}{2}ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(b-a\sin(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}} - \frac{a\int\sqrt{e\cos(c+dx)}\,dx}{\left(a^2-b^2\right)e^2} - \frac{b^2\int\frac{\sqrt{e\cos(c+dx)}}{a+b\sin(c+dx)}\,dx}{\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(b-a\sin(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}} + \frac{(ab)\int\frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{2\left(a^2-b^2\right)e} - \text{—} (ab$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)de^2\sqrt{\cos(c+dx)}} - \frac{2(b-a\sin(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}} - \frac{\left(2b^3\right)\text{Su}}{}$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)de^2\sqrt{\cos(c+dx)}} - \frac{ab\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx\right.}{\left(a^2-b^2\right)\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\cos(c}}$$

$$= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{5/4}de^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{5/4}de^{3/2}} - \frac{2a\sqrt{e\cos(c+dx)}}{\left(a^2-b^2\right)de^2\sqrt{}}$$

Mathematica [C] time = 21.7726, size = 791, normalized size = 1.92

$$\frac{2\cos(c+dx)(a\sin(c+dx)-b)}{d\left(a^2-b^2\right)(e\cos(c+dx))^{3/2}} - \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{a\sin^2(c+dx)\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)+\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] (2*Cos[c + d*x]*(-b + a*Sin[c + d*x]))/((a^2 - b^2)*d*(e*Cos[c + d*x])^(3/2
)) - (Cos[c + d*x]^(3/2)*((-2*(a^2 + b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*
((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
 b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1
```

```
 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 +
I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] -
 (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]
+ Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*
x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt
[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (a*(a + b*Sqrt[1 - Cos[c + d*x
]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d
*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*
ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcT
an[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a
^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c
+ d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 -
 Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/((a - b)*(a + b)*d*(e*Cos[c + d*x]
)^(3/2))
```

Maple [C] time = 3.057, size = 1103, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2/d/e^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))*(-2*sin(1/2
*d*x+1/2*c)^2*e+e)^(1/2)+1/2/d/e^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*d*x+1/2*c)+
1/2*2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-1/2/d/e*b^3/(a-b)/(a+b)*su
m((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b
^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d
*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^
2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-4/d/e*a/(a+b)/(a-b)/sin(1/2*d*x+1/2*c)/(e*
(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*cos(1/2*d*x+1/2*c)^3-2/d/e*a/(a+b)/(a-b)/
(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)
/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+4/d/e*a/(a+b)/(a-b)/sin(1/2*
d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*cos(1/2*d*x+1/2*c)+1/8/d/e/
a/(a+b)/(a-b)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*sum(1
/_alpha*(8*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^
2/a^2*(_alpha^2-1),2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*
```

```
b^2/a^2*(_alpha^2-1),2^(1/2))*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+a^2*
2^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*
a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alp
ha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*
c)^2))^(1/2))*(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2))/(
e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2
*d*x+1/2*c)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(e*(2*cos
(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\sin\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\sin\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)), x)
```

3.582 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}\,dx$

Optimal. Leaf size=434

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}}-\frac{ab^2\sqrt{\cos(c+dx)}\Pi\left(\cdots\right.}{de^2\left(a^2-b^2\right)\left(a^2-b\left(b-\cdots\right.\right.}$$

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/((-a^2 + b^2)^(7/4)*d*e^(5/2))) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(7/4)*d*e^(5/2)) +
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*(a^2 - b^2)*d*e^2*Sq
rt[e*Cos[c + d*x]]) - (a*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[
-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2])
)*d*e^2*Sqrt[e*Cos[c + d*x]]) - (a*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/
(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) - (2*(b - a*Sin[c + d*x]))/(3*(a^2
- b^2)*d*e*(e*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.998902, antiderivative size = 434, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2696, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}}-\frac{ab^2\sqrt{\cos(c+dx)}\Pi\left(\cdots\right.}{de^2\left(a^2-b^2\right)\left(a^2-b\left(b-\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])),x]
```



```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/((-a^2 + b^2)^(7/4)*d*e^(5/2))) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(7/4)*d*e^(5/2)) +
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*(a^2 - b^2)*d*e^2*Sq
rt[e*Cos[c + d*x]]) - (a*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[
-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2])
)*d*e^2*Sqrt[e*Cos[c + d*x]]) - (a*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/
(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) - (2*(b - a*Sin[c + d*x]))/(3*(a^2
```

```
- b^2)*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}\,dx = -\frac{2(b-a\sin(c+dx))}{3\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a^2}{2}+\frac{3b^2}{2}-\frac{1}{2}ab\sin(c+dx)}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}\,dx}{3\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(b-a\sin(c+dx))}{3\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3\left(a^2-b^2\right)e^2} - \frac{b^2\int \frac{1}{\sqrt{e\cos(c+dx)}(a+b}}{\left(a^2-b^2\right)}$$

$$= -\frac{2(b-a\sin(c+dx))}{3\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}} - \frac{\left(ab^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{2\left(-a^2+b^2\right)^{3/2}e^2}$$

$$= \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3\left(a^2-b^2\right)de^2\sqrt{e\cos(c+dx)}} - \frac{2(b-a\sin(c+dx))}{3\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}} - \frac{\left(2b^3\right)}{}$$

$$= \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3\left(a^2-b^2\right)de^2\sqrt{e\cos(c+dx)}} + \frac{ab^2\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+\right.}{\left(-a^2+b^2\right)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)de^2\sqrt{e\cos}}$$

$$= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{7/4}de^{5/2}} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{7/4}de^{5/2}} + \frac{2a\sqrt{\cos(c+d}}{3\left(a^2-b^2\right)de}$$

Mathematica [C] time = 24.7858, size = 1192, normalized size = 2.75

$$\left(-\frac{2ab\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(\frac{5b\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{1-\cos^2(c+dx)}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)}{\left(2\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)b^2+\left(a^2-b^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)\right)\cos^2(c+dx)-5\left(a^2-b^2\right)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\cos^2(c+dx),\right.}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] (2*Cos[c + d*x]*(-b + a*Sin[c + d*x]))/(3*(a^2 - b^2)*d*(e*Cos[c + d*x])^(5
/2)) + (Cos[c + d*x]^(5/2)*((-2*(a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^
2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c
 + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(
```

```
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c
 + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(
-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b
]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*S
qrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqr
t[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-
a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Co
s[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(
a + b*Sin[c + d*x])) - (2*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 -
 b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^
2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*App
ellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]
 + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2
)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b
^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c +
d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b
^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^
(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c
 + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/
(3*(a - b)*(a + b)*d*(e*Cos[c + d*x])^(5/2))
```

Maple [C] time = 3.79, size = 1083, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/12/d/e^3*b/(a^2-b^2)/(cos(1/2*d*x+1/2*c)+1/2*2^(1/2))^2*(-2*sin(1/2*d*x+
1/2*c)^2*e+e)^(1/2)-1/12/d/e^3*b*2^(1/2)/(a^2-b^2)/(cos(1/2*d*x+1/2*c)+1/2*
2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-1/12/d/e^3*b/(a^2-b^2)/(cos(1/
2*d*x+1/2*c)-1/2*2^(1/2))^2*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)+1/12/d/e^3*
b*2^(1/2)/(a^2-b^2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))*(-2*sin(1/2*d*x+1/2*c)
^2*e+e)^(1/2)-2/d/e*b^3/(a-b)/(a+b)*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*
e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e
)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_
Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))+1/3/d*(e*(2*cos(1
```

```
/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^3*a/sin(1/2*d*x+1/2*c)/(e*
(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-e*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-2
/3/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^2*a/sin(1/
2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4
-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/8/d*(
e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^2*a/sin(1/2*d*x+
1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a-b)/(a+b)*sum(1/_alpha/(2*_al
pha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4
*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/
2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)
^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*
_alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^
4*b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(b\sin\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)), x)
```

3.583 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}\,dx$

Optimal. Leaf size=486

$$\frac{2\left(a\left(3a^2-8b^2\right)\sin(c+dx)+5b^3\right)}{5de^3\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{7/2}\left(b^2-a^2\right)^{9/4}}-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{7/2}\left(b^2-a^2\right)^{9/4}}-\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}(c+\right.}{5de^4\left(a^2-b^2\right)}$$

```
[Out] (b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/((-a^2 + b^2)^(9/4)*d*e^(7/2)) - (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(9/4)*d*e^(7/2)) - (2
*a*(3*a^2 - 8*b^2)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*(a^2
- b^2)^2*d*e^4*Sqrt[Cos[c + d*x]]) + (a*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(
2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)^2*(b - Sqrt[-a^2
 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + (a*b^3*Sqrt[Cos[c + d*x]]*EllipticPi
[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)^2*(b + Sqrt[-a
^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) - (2*(b - a*Sin[c + d*x]))/(5*(a^2 -
 b^2)*d*e*(e*Cos[c + d*x])^(5/2)) + (2*(5*b^3 + a*(3*a^2 - 8*b^2)*Sin[c + d
*x]))/(5*(a^2 - b^2)^2*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 1.32848, antiderivative size = 486, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2696, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{2\left(a\left(3a^2-8b^2\right)\sin(c+dx)+5b^3\right)}{5de^3\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{7/2}\left(b^2-a^2\right)^{9/4}}-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{7/2}\left(b^2-a^2\right)^{9/4}}-\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}(c+\right.}{5de^4\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])),x]
```



```
[Out] (b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/((-a^2 + b^2)^(9/4)*d*e^(7/2)) - (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(9/4)*d*e^(7/2)) - (2
*a*(3*a^2 - 8*b^2)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*(a^2
- b^2)^2*d*e^4*Sqrt[Cos[c + d*x]]) + (a*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(
2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)^2*(b - Sqrt[-a^2
 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + (a*b^3*Sqrt[Cos[c + d*x]]*EllipticPi
[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)^2*(b + Sqrt[-a
```

```
^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) - (2*(b - a*Sin[c + d*x]))/(5*(a^2 -
 b^2)*d*e*(e*Cos[c + d*x])^(5/2)) + (2*(5*b^3 + a*(3*a^2 - 8*b^2)*Sin[c + d
*x]))/(5*(a^2 - b^2)^2*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}\,dx = -\frac{2(b-a\sin(c+dx))}{5\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+\frac{5b^2}{2}-\frac{3}{2}ab\sin(c+dx)}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}\,dx}{5\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(b-a\sin(c+dx))}{5\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}} + \frac{2\left(5b^3+a\left(3a^2-8b^2\right)\sin(c+dx)\right)}{5\left(a^2-b^2\right)^2de^3\sqrt{e\cos(c+dx)}} + \frac{4\int}{}$$

$$= -\frac{2(b-a\sin(c+dx))}{5\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}} + \frac{2\left(5b^3+a\left(3a^2-8b^2\right)\sin(c+dx)\right)}{5\left(a^2-b^2\right)^2de^3\sqrt{e\cos(c+dx)}} + \frac{b^4}{}$$

$$= -\frac{2(b-a\sin(c+dx))}{5\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}} + \frac{2\left(5b^3+a\left(3a^2-8b^2\right)\sin(c+dx)\right)}{5\left(a^2-b^2\right)^2de^3\sqrt{e\cos(c+dx)}} - \frac{(ab}{}$$

$$= -\frac{2a\left(3a^2-8b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5\left(a^2-b^2\right)^2de^4\sqrt{\cos(c+dx)}} - \frac{2(b-a\sin(c+dx))}{5\left(a^2-b^2\right)de(e\cos(c+dx}$$

$$= -\frac{2a\left(3a^2-8b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5\left(a^2-b^2\right)^2de^4\sqrt{\cos(c+dx)}} + \frac{ab^3\sqrt{\cos(c+dx)}\Pi\left(\frac{2}{b-\sqrt{-}}\right.}{\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b^2}\right.}$$

$$= \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{9/4}de^{7/2}} - \frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{9/4}de^{7/2}} - \frac{2a\left(3a^2-8b^2\right)\sqrt{e}}{5\left(a^2-b\right.}$$

Mathematica [C] time = 6.75686, size = 881, normalized size = 1.81

$$\frac{\cos^4(c+dx)\left(\frac{2(a\sin(c+dx)-b)\sec^3(c+dx)}{5\left(a^2-b^2\right)}+\frac{2\left(3\sin(c+dx)a^3-8b^2\sin(c+dx)a+5b^3\right)\sec(c+dx)}{5\left(a^2-b^2\right)^2}\right)}{d(e\cos(c+dx))^{7/2}} - \frac{\cos^{\frac{7}{2}}(c+dx)\left(-\frac{\left(3a^3b-8ab^3\right)\left(a+b\sqrt{1-\cos^2(c+dx}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] -(Cos[c + d*x]^(7/2)*((-2*(3*a^4 - 8*a^2*b^2 - 5*b^4)*(a + b*Sqrt[1 - Cos[c
 + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x
]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*Ar
cTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTa
n[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-
a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Co
s[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqr
t[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c +
 d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((3*a^3*b - 8*a*b^
3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4,
Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*S
qrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]
])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(
a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[
b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)
/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(5*(
a - b)^2*(a + b)^2*d*(e*Cos[c + d*x])^(7/2)) + (Cos[c + d*x]^4*((2*Sec[c +
d*x]^3*(-b + a*Sin[c + d*x]))/(5*(a^2 - b^2)) + (2*Sec[c + d*x]*(5*b^3 + 3*
a^3*Sin[c + d*x] - 8*a*b^2*Sin[c + d*x]))/(5*(a^2 - b^2)^2)))/(d*(e*Cos[c +
 d*x])^(7/2))
```

Maple [C] time = 5.316, size = 2399, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/e^4*b^3/(a^2-b^2)^2*2^(1/2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))*(-2*sin(
1/2*d*x+1/2*c)^2*e+e)^(1/2)+3/80/d/e^4*b/(a^2-b^2)/(cos(1/2*d*x+1/2*c)+1/2*
2^(1/2))^2*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)+3/80/d/e^4*b*2^(1/2)/(a^2-b^
2)/(cos(1/2*d*x+1/2*c)+1/2*2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-1/2
/d/e^4*b^3/(a^2-b^2)^2*2^(1/2)/(cos(1/2*d*x+1/2*c)+1/2*2^(1/2))*(-2*sin(1/2
*d*x+1/2*c)^2*e+e)^(1/2)+3/80/d/e^4*b/(a^2-b^2)/(cos(1/2*d*x+1/2*c)-1/2*2^(
1/2))^2*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-3/80/d/e^4*b*2^(1/2)/(a^2-b^2)/
(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-1/80/d
/e^4*b/(a^2-b^2)*2^(1/2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))^3*(-2*sin(1/2*d*x
+1/2*c)^2*e+e)^(1/2)+1/80/d/e^4*b/(a^2-b^2)*2^(1/2)/(cos(1/2*d*x+1/2*c)+1/2
```

2^(1/2))^3(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)+1/2/d/e^3*b^5/(a-b)^2/(a+b)^2*sum((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))+4/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)^2*b^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)-2/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a/sin(1/2*d*x+1/2*c)^3/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)^2*b^2/(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a*sin(1/2*d*x+1/2*c)^3/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)+24/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a*sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+48/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a*sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)-24/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-16/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)+6/5/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4*a/sin(1/2*d*x+1/2*c)^3/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/(a^2-b^2)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/8/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^3*a/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*b^2/(a-b)^2/(a+b)^2*sum(1/_alpha*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^

```
(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*
b^2-4*_Z^2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}(b\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}(b\sin(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)), x)
```

3.584 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=543

$$-\frac{3e^5\sqrt{e\cos(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\sin(c+dx)\right)}{7b^5d}-\frac{9ae^{11/2}\left(b^2-a^2\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{11/2}d}-\frac{9ae^{11/2}\left(b^2\right.}{}$$

```
[Out] (-9*a*(-a^2 + b^2)^(5/4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-
a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(11/2)*d) - (9*a*(-a^2 + b^2)^(5/4)*e^(11/
2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2
*b^(11/2)*d) - (3*(21*a^4 - 28*a^2*b^2 + 5*b^4)*e^6*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2])/(7*b^6*d*Sqrt[e*Cos[c + d*x]]) + (9*a^2*(a^2 - b^2)^
2*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)
/2, 2])/(2*b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (
9*a^2*(a^2 - b^2)^2*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
+ b^2]), (c + d*x)/2, 2])/(2*b^6*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*
Cos[c + d*x]]) + (9*e^3*(e*Cos[c + d*x])^(5/2)*(7*a - 5*b*Sin[c + d*x]))/(3
5*b^3*d) - (e*(e*Cos[c + d*x])^(9/2))/(b*d*(a + b*Sin[c + d*x])) - (3*e^5*S
qrt[e*Cos[c + d*x]]*(21*a*(a^2 - b^2) - b*(7*a^2 - 5*b^2)*Sin[c + d*x]))/(7
*b^5*d)
```

Rubi [A] time = 1.5189, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2693, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{3e^5\sqrt{e\cos(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\sin(c+dx)\right)}{7b^5d}-\frac{9ae^{11/2}\left(b^2-a^2\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{11/2}d}-\frac{9ae^{11/2}\left(b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-9*a*(-a^2 + b^2)^(5/4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-
a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(11/2)*d) - (9*a*(-a^2 + b^2)^(5/4)*e^(11/
2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2
*b^(11/2)*d) - (3*(21*a^4 - 28*a^2*b^2 + 5*b^4)*e^6*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2])/(7*b^6*d*Sqrt[e*Cos[c + d*x]]) + (9*a^2*(a^2 - b^2)^
2*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)
```

```
/2, 2])/(2*b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (
9*a^2*(a^2 - b^2)^2*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
+ b^2]), (c + d*x)/2, 2])/(2*b^6*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*
Cos[c + d*x]]) + (9*e^3*(e*Cos[c + d*x])^(5/2)*(7*a - 5*b*Sin[c + d*x]))/(3
5*b^3*d) - (e*(e*Cos[c + d*x])^(9/2))/(b*d*(a + b*Sin[c + d*x])) - (3*e^5*S
qrt[e*Cos[c + d*x]]*(21*a*(a^2 - b^2) - b*(7*a^2 - 5*b^2)*Sin[c + d*x]))/(7
*b^5*d)
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{\left(9e^2\right)\int \frac{(e\cos(c+dx))^{7/2}\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$

$$= \frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin(c+dx))}{35b^3d} - \frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{\left(9e^4\right)\int \frac{(e\cos(c+dx))^{3/2}\left(-ab-\right.}{a+b\,\mathrm{s}}}{7b^3}$$

$$= \frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin(c+dx))}{35b^3d} - \frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{3e^5\sqrt{e\cos(c+dx)}\left(21a\right.}{}$$

$$= \frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin(c+dx))}{35b^3d} - \frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{3e^5\sqrt{e\cos(c+dx)}\left(21a\right.}{}$$

$$= \frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin(c+dx))}{35b^3d} - \frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{3e^5\sqrt{e\cos(c+dx)}\left(21a\right.}{}$$

$$= -\frac{3\left(21a^4-28a^2b^2+5b^4\right)e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7b^6d\sqrt{e\cos(c+dx)}} + \frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin}{35b^3d}$$

$$= -\frac{3\left(21a^4-28a^2b^2+5b^4\right)e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7b^6d\sqrt{e\cos(c+dx)}} + \frac{9a^2\left(-a^2+b^2\right)^{3/2}e^6\sqrt{\cos(c+dx}}{2b^6\left(b-\sqrt{-a^2+b^2}\right)}$$

$$= -\frac{9a\left(-a^2+b^2\right)^{5/4}e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{11/2}d} - \frac{9a\left(-a^2+b^2\right)^{5/4}e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{11/2}d}$$

Mathematica [C] time = 27.7261, size = 2030, normalized size = 3.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^2,x]

[Out] -((e*Cos[c + d*x])^(11/2)*((-2*(70*a^3*b - 93*a*b^3)*(a + b*Sqrt[1 - Cos[c
+ d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^
2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d
*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2,
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4
, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2
+ b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)
*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sq
rt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log
[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
 I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*
x]^2]*(a + b*Sin[c + d*x])) + ((140*a^3*b - 147*a*b^3)*(a + b*Sqrt[1 - Cos[
c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^
(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c
 + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[c
 + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^
2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1
[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(
2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos
[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2
))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-
a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b
^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqr
t[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-
a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c +
d*x]^2)*(a + b*Sin[c + d*x])) - (2*(35*a^4 - 126*a^2*b^2 + 75*b^4)*(a + b*S
qrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[
c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 -
Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^
2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4
, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1
[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[
c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]
*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]
*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[
```

```
a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c
 + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos
[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(70*b^5*d*Cos[c + d*x]^(11/2)) + ((e*C
os[c + d*x])^(11/2)*Sec[c + d*x]^5*((2*a*Cos[2*(c + d*x)])/(5*b^3) - ((-28*
a^2 + 17*b^2)*Sin[c + d*x])/(14*b^4) - (-a^2 + b^2)^2/(b^5*(a + b*Sin[c + d
*x])) - Sin[3*(c + d*x)]/(14*b^2)))/d
```

Maple [C] time = 10.645, size = 19829, normalized size = 36.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{11}{2}}}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(11/2)/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.585 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=459

$$\frac{7ae^{9/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}-\frac{7ae^{9/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}+\frac{7e^4\left(5a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{5b^4d\sqrt{\cos(c+dx)}}$$

```
[Out] (7*a*(-a^2 + b^2)^(3/4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(9/2)*d) - (7*a*(-a^2 + b^2)^(3/4)*e^(9/2)*A
rcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(
9/2)*d) + (7*(5*a^2 - 3*b^2)*e^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2])/(5*b^4*d*Sqrt[Cos[c + d*x]]) - (7*a^2*(a^2 - b^2)*e^5*Sqrt[Cos[c + d*
x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^5*(b - S
qrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (7*a^2*(a^2 - b^2)*e^5*Sqrt[Cos[
c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^5*
(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (7*e^3*(e*Cos[c + d*x])^(3
/2)*(5*a - 3*b*Sin[c + d*x]))/(15*b^3*d) - (e*(e*Cos[c + d*x])^(7/2))/(b*d*
(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.12032, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2693, 2865, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{7ae^{9/2}\left(b^2-a^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}-\frac{7ae^{9/2}\left(b^2-a^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}+\frac{7e^4\left(5a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{5b^4d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (7*a*(-a^2 + b^2)^(3/4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(9/2)*d) - (7*a*(-a^2 + b^2)^(3/4)*e^(9/2)*A
rcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(
9/2)*d) + (7*(5*a^2 - 3*b^2)*e^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2])/(5*b^4*d*Sqrt[Cos[c + d*x]]) - (7*a^2*(a^2 - b^2)*e^5*Sqrt[Cos[c + d*
x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^5*(b - S
qrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (7*a^2*(a^2 - b^2)*e^5*Sqrt[Cos[
c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^5*
```

```
(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (7*e^3*(e*Cos[c + d*x])^(3
/2)*(5*a - 3*b*Sin[c + d*x]))/(15*b^3*d) - (e*(e*Cos[c + d*x])^(7/2))/(b*d*
(a + b*Sin[c + d*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e(e\cos(c+dx))^{7/2}}{bd(a+b\sin(c+dx))} - \frac{\left(7e^2\right)\int \frac{(e\cos(c+dx))^{5/2}\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$

$$= \frac{7e^3(e\cos(c+dx))^{3/2}(5a-3b\sin(c+dx))}{15b^3d} - \frac{e(e\cos(c+dx))^{7/2}}{bd(a+b\sin(c+dx))} - \frac{\left(7e^4\right)\int \frac{\sqrt{e\cos(c+dx)}\left(-ab-\right.}{a+b\,\mathrm{s}}}{5b}$$

$$= \frac{7e^3(e\cos(c+dx))^{3/2}(5a-3b\sin(c+dx))}{15b^3d} - \frac{e(e\cos(c+dx))^{7/2}}{bd(a+b\sin(c+dx))} + \frac{\left(7\left(5a^2-3b^2\right)e^4\right)\int \sqrt{}}{10b^4}$$

$$= \frac{7e^3(e\cos(c+dx))^{3/2}(5a-3b\sin(c+dx))}{15b^3d} - \frac{e(e\cos(c+dx))^{7/2}}{bd(a+b\sin(c+dx))} + \frac{\left(7a^2\left(a^2-b^2\right)e^5\right)\int \frac{}{\sqrt{e}}}{}$$

$$= \frac{7\left(5a^2-3b^2\right)e^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{7e^3(e\cos(c+dx))^{3/2}(5a-3b\sin(c+dx))}{15b^3d}$$

$$= \frac{7\left(5a^2-3b^2\right)e^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} - \frac{7a^2\left(a^2-b^2\right)e^5\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}}\right.}{2b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c}}$$

$$= \frac{7a\left(-a^2+b^2\right)^{3/4}e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{9/2}d} - \frac{7a\left(-a^2+b^2\right)^{3/4}e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{9/2}d} +$$

Mathematica [C] time = 26.893, size = 835, normalized size = 1.82

$$7\left(-\frac{\left(5a^2-3b^2\right)\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)\cos^{\frac{3}{2}}(c+dx)b^{5/2}+3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{}}{}\right.\right.\right.}{12b^{3/2}\left(b^2-a^2\right)\left(1-\cos^2(c+dx)\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (7*(e*Cos[c + d*x])^(9/2)*((-4*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((a*App
ellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*
Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*S
qrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[S
qrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I
*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1 - Co
s[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((5*a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[
c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3
/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] -
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log
[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b
*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^
2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(10*b^3*d*Cos[c + d*x]^(9/2
)) + ((e*Cos[c + d*x])^(9/2)*Sec[c + d*x]^4*((4*a*Cos[c + d*x])/(3*b^3) + (
a^2*Cos[c + d*x] - b^2*Cos[c + d*x])/(b^3*(a + b*Sin[c + d*x])) - Sin[2*(c
+ d*x)]/(5*b^2)))/d
```

Maple [C] time = 8.373, size = 20346, normalized size = 44.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{9}{2}}}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(9/2)/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.586 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=473

$$-\frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d}-\frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d}+\frac{5e^4\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d\right.}{3b^4d\sqrt{e\cos(c+dx)}}$$

```
[Out] (-5*a*(-a^2 + b^2)^(1/4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a
^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(7/2)*d) - (5*a*(-a^2 + b^2)^(1/4)*e^(7/2)*
ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^
(7/2)*d) + (5*(3*a^2 - b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
])/(3*b^4*d*Sqrt[e*Cos[c + d*x]]) - (5*a^2*(a^2 - b^2)*e^4*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^4*(a^2 -
b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (5*a^2*(a^2 - b^2)*e^4*
Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2]
)/(2*b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (5*e^3*
Sqrt[e*Cos[c + d*x]]*(3*a - b*Sin[c + d*x]))/(3*b^3*d) - (e*(e*Cos[c + d*x]
)^(5/2))/(b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.11844, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2693, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d}-\frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d}+\frac{5e^4\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d\right.}{3b^4d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-5*a*(-a^2 + b^2)^(1/4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a
^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(7/2)*d) - (5*a*(-a^2 + b^2)^(1/4)*e^(7/2)*
ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^
(7/2)*d) + (5*(3*a^2 - b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
])/(3*b^4*d*Sqrt[e*Cos[c + d*x]]) - (5*a^2*(a^2 - b^2)*e^4*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^4*(a^2 -
b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (5*a^2*(a^2 - b^2)*e^4*
Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2]
```

```
)/(2*b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (5*e^3*
Sqrt[e*Cos[c + d*x]]*(3*a - b*Sin[c + d*x]))/(3*b^3*d) - (e*(e*Cos[c + d*x]
)^(5/2))/(b*d*(a + b*Sin[c + d*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e(e\cos(c+dx))^{5/2}}{bd(a+b\sin(c+dx))} - \frac{\left(5e^2\right)\int \frac{(e\cos(c+dx))^{3/2}\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$

$$= \frac{5e^3\sqrt{e\cos(c+dx)}(3a-b\sin(c+dx))}{3b^3d} - \frac{e(e\cos(c+dx))^{5/2}}{bd(a+b\sin(c+dx))} - \frac{\left(5e^4\right)\int \frac{-ab-\frac{1}{2}\left(3a^2-b^2\right)\sin(c}{\sqrt{e\cos(c+dx)}(a+b\sin(c}}{3b^3}$$

$$= \frac{5e^3\sqrt{e\cos(c+dx)}(3a-b\sin(c+dx))}{3b^3d} - \frac{e(e\cos(c+dx))^{5/2}}{bd(a+b\sin(c+dx))} - \frac{\left(5a\left(a^2-b^2\right)e^4\right)\int \frac{}{\sqrt{e\cos(c}}}{2b^4}$$

$$= \frac{5e^3\sqrt{e\cos(c+dx)}(3a-b\sin(c+dx))}{3b^3d} - \frac{e(e\cos(c+dx))^{5/2}}{bd(a+b\sin(c+dx))} - \frac{\left(5a^2\sqrt{-a^2+b^2}e^4\right)\int \frac{}{\sqrt{e\cos}}}{4}$$

$$= \frac{5\left(3a^2-b^2\right)e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\sqrt{e\cos(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}(3a-b\sin(c+dx))}{3b^3d} - \frac{e(}{bd(}$$

$$= \frac{5\left(3a^2-b^2\right)e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\sqrt{e\cos(c+dx)}} + \frac{5a^2\sqrt{-a^2+b^2}e^4\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\right.}{2b^4\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+}}$$

$$= -\frac{5a\sqrt[4]{-a^2+b^2}e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{7/2}d} - \frac{5a\sqrt[4]{-a^2+b^2}e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{7/2}d} + \frac{5\left(3a\right.}{}$$

Mathematica [C] time = 27.3326, size = 1956, normalized size = 4.14

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^3*((-2*Sin[c + d*x])/(3*b^2) + (a^2 -
b^2)/(b^3*(a + b*Sin[c + d*x]))))/d + ((e*Cos[c + d*x])^(7/2)*((-8*a*b*(a +
 b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, C
os[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt
```

```
[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]
^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4
, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF
1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos
[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*A
rcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcT
an[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[
-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*C
os[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sq
rt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(S
qrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) + (6*a*b*(a + b*Sqrt[1 - Cos[
c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^
(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c
 + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[c
 + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^
2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1
[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(
2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos
[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2
))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-
a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b
^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqr
t[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-
a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c +
d*x]^2)*(a + b*Sin[c + d*x])) - (2*(3*a^2 - 5*b^2)*(a + b*Sqrt[1 - Cos[c +
d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2
*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])
/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/
4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2
 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Co
s[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c
+ d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sq
rt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a
+ b*Sin[c + d*x]))))/(6*b^3*d*Cos[c + d*x]^(7/2))
```

Maple [C] time = 8.384, size = 14392, normalized size = 30.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.587 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=390

$$\frac{3ae^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}}-\frac{3ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}}+\frac{3a^2e^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{3a^2e^3\sqrt{\cdots}}{2b^3\cdots}$$

```
[Out] (3*a*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*b^(5/2)*(-a^2 + b^2)^(1/4)*d) - (3*a*e^(5/2)*ArcTanh[(Sqrt[b]*Sqr
t[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(5/2)*(-a^2 + b^2)^(
1/4)*d) - (3*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqr
t[Cos[c + d*x]]) + (3*a^2*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt
[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^3*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[
c + d*x]]) + (3*a^2*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
+ b^2]), (c + d*x)/2, 2])/(2*b^3*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*
x]]) - (e*(e*Cos[c + d*x])^(3/2))/(b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.819143, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2693, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{3ae^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}}-\frac{3ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}}+\frac{3a^2e^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{3a^2e^3\sqrt{\cdots}}{2b^3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*b^(5/2)*(-a^2 + b^2)^(1/4)*d) - (3*a*e^(5/2)*ArcTanh[(Sqrt[b]*Sqr
t[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(5/2)*(-a^2 + b^2)^(
1/4)*d) - (3*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqr
t[Cos[c + d*x]]) + (3*a^2*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt
[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^3*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[
c + d*x]]) + (3*a^2*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
+ b^2]), (c + d*x)/2, 2])/(2*b^3*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*
x]]) - (e*(e*Cos[c + d*x])^(3/2))/(b*d*(a + b*Sin[c + d*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e(e\cos(c+dx))^{3/2}}{bd(a+b\sin(c+dx))} - \frac{\left(3e^2\right)\int \frac{\sqrt{e\cos(c+dx)}\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{bd(a+b\sin(c+dx))} - \frac{\left(3e^2\right)\int \sqrt{e\cos(c+dx)}\,dx}{2b^2} + \frac{\left(3ae^2\right)\int \frac{\sqrt{e\cos(c+dx)}}{a+b\sin(c+dx)}\,dx}{2b^2}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{bd(a+b\sin(c+dx))} - \frac{\left(3a^2e^3\right)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{4b^3} + \frac{\left(3a^2e^3\right)\int \frac{}{\sqrt{e\cos(c+dx)}(}}{4l}$$

$$= -\frac{3e^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{3/2}}{bd(a+b\sin(c+dx))} + \frac{\left(3ae^3\right)\text{Subst}\left(\int \frac{x^2}{(a^2-b^2)e^2+b^2x^4}\right.}{bd}$$

$$= -\frac{3e^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{3a^2e^3\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+dx)}} + \frac{3a^2e}{}$$

$$= \frac{3ae^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}d} - \frac{3ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}d} - \frac{3e^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+d\right.}{b^2d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 38.8137, size = 371, normalized size = 0.95

$$\frac{(e\cos(c+dx))^{5/2}\left(-\frac{\left(a+b\sqrt{\sin^2(c+dx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)+3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(-\log\left(-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(c+dx)}+\sqrt{a}\right.\right.\right)}{a}\right.}{8b^{5/2}d\cos^{\frac{5}{2}}(c+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(-8*b^(3/2)*Cos[c + d*x]^(3/2) - ((8*b^(5/2)*Appell
F1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Co
s[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqr
t[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2
- b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d
*x]]))*(a + b*Sqrt[Sin[c + d*x]^2]))/(a^2 - b^2)))/(8*b^(5/2)*d*Cos[c + d*x
```

```
]^(5/2)*(a + b*Sin[c + d*x]))
```

Maple [C] time = 6.415, size = 13221, normalized size = 33.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.588 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=404

$$-\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}d\left(b^2-a^2\right)^{3/4}}-\frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}d\left(b^2-a^2\right)^{3/4}}+\frac{a^2e^2\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{a^2e^2\sqrt{\cos\cdots}}{2b^2d\left(a^2-\cdots\right.}$$

```
[Out] -(a*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
e])])/(2*b^(3/2)*(-a^2 + b^2)^(3/4)*d) - (a*e^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e
*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(3/2)*(-a^2 + b^2)^(3/4
)*d) - (e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[e*Cos
[c + d*x]]) + (a^2*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
 b^2]), (c + d*x)/2, 2])/(2*b^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*C
os[c + d*x]]) + (a^2*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(2*b^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e
*Cos[c + d*x]]) - (e*Sqrt[e*Cos[c + d*x]])/(b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.892248, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2693, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}d\left(b^2-a^2\right)^{3/4}}-\frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}d\left(b^2-a^2\right)^{3/4}}+\frac{a^2e^2\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{a^2e^2\sqrt{\cos\cdots}}{2b^2d\left(a^2-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
e])])/(2*b^(3/2)*(-a^2 + b^2)^(3/4)*d) - (a*e^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e
*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(3/2)*(-a^2 + b^2)^(3/4
)*d) - (e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[e*Cos
[c + d*x]]) + (a^2*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
 b^2]), (c + d*x)/2, 2])/(2*b^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*C
os[c + d*x]]) + (a^2*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(2*b^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e
*Cos[c + d*x]]) - (e*Sqrt[e*Cos[c + d*x]])/(b*d*(a + b*Sin[c + d*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e\sqrt{e\cos(c+dx)}}{bd(a+b\sin(c+dx))} - \frac{e^2\int \frac{\sin(c+dx)}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}\,dx}{2b}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{bd(a+b\sin(c+dx))} - \frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{2b^2} + \frac{(ae^2)\int \frac{1}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}\,dx}{2b^2}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{bd(a+b\sin(c+dx))} - \frac{(a^2e^2)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{4b^2\sqrt{-a^2+b^2}} - \frac{(a^2e^2)\int \frac{}{\sqrt{e\cos(c+dx)}\left(\sqrt{-\right.}}}{4b^2\sqrt{-a^2}}$$

$$= -\frac{e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{e\cos(c+dx)}} - \frac{e\sqrt{e\cos(c+dx)}}{bd(a+b\sin(c+dx))} + \frac{(ae^3)\,\mathrm{Subst}\left(\int \frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx,\right.}{bd}$$

$$= -\frac{e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{e\cos(c+dx)}} + \frac{a^2e^2\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\cos(c+dx)}} - \frac{a^2e^2\sqrt{}}{2b^2\sqrt{-a}}$$

$$= -\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right.}{b^2d\sqrt{e\cos(c+dx)}}$$

Mathematica [C] time = 12.4934, size = 614, normalized size = 1.52

$$\frac{\sin^2(c+dx)(e\cos(c+dx))^{3/2}\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(\frac{5b\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{1-\cos^2(c}{\left(a^2+b^2\left(\cos^2(c+dx)-1\right)\right)\left(2\cos^2(c+dx)\left(2b^2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)+(a\right.\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((e*Cos[c + d*x])^(3/2)*Sec[c + d*x])/(b*d*(a + b*Sin[c + d*x]))) + ((e*C
os[c + d*x])^(3/2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*Appel
lF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*S
qrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4,
-1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2
*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) +
 (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)]
```

```
+ 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Lo
g[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*S
qrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4))
)*Sin[c + d*x]^2)/(b*d*Cos[c + d*x]^(3/2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c
 + d*x]))
```

Maple [C] time = 6.523, size = 9301, normalized size = 23.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((e*cos(d*x + c))^(3/2)/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\left(b\sin\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(b*sin(d*x + c) + a)^2, x)
```

3.589 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=422

$$\frac{b(e\cos(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}d\left(b^2-a^2\right)^{5/4}}+\frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}d\left(b^2-a^2\right)^{5/4}}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c}}{d\left(a^2-b^2\right)\sqrt{\cos(c+d}}$$

```
[Out] -(a*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
e])])/(2*Sqrt[b]*(-a^2 + b^2)^(5/4)*d) + (a*Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e
*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*Sqrt[b]*(-a^2 + b^2)^(5/4
)*d) + (Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/((a^2 - b^2)*d*Sqrt
[Cos[c + d*x]]) + (a^2*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(2*b*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e
*Cos[c + d*x]]) + (a^2*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(2*b*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e
*Cos[c + d*x]]) + (b*(e*Cos[c + d*x])^(3/2))/((a^2 - b^2)*d*e*(a + b*Sin[c
+ d*x]))
```

Rubi [A] time = 0.868491, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2694, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{b(e\cos(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}d\left(b^2-a^2\right)^{5/4}}+\frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}d\left(b^2-a^2\right)^{5/4}}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c}}{d\left(a^2-b^2\right)\sqrt{\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -(a*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
e])])/(2*Sqrt[b]*(-a^2 + b^2)^(5/4)*d) + (a*Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e
*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*Sqrt[b]*(-a^2 + b^2)^(5/4
)*d) + (Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/((a^2 - b^2)*d*Sqrt
[Cos[c + d*x]]) + (a^2*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(2*b*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e
*Cos[c + d*x]]) + (a^2*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(2*b*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e
*Cos[c + d*x]]) + (b*(e*Cos[c + d*x])^(3/2))/((a^2 - b^2)*d*e*(a + b*Sin[c
```

```
+ d*x]))
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^2}\,dx = \frac{b(e\cos(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\sin(c+dx))} + \frac{\int \frac{\sqrt{e\cos(c+dx)}\left(-a-\frac{1}{2}b\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\sin(c+dx))} + \frac{\int \sqrt{e\cos(c+dx)}\,dx}{2(a^2-b^2)} + \frac{a\int \frac{\sqrt{e\cos(c+dx)}}{a+b\sin(c+dx)}\,dx}{2(a^2-b^2)}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\sin(c+dx))} - \frac{(a^2e)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{4b(a^2-b^2)} + \frac{(a^2e)\int \frac{}{\sqrt{e\cos(c+}}}{4b}$$

$$= \frac{\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}} + \frac{b(e\cos(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\sin(c+dx))} + \frac{(abe)\,\mathrm{Subst}\left(\int \frac{x^2}{(a^2-b^2)e^2+}\right.}{(a^2-}$$

$$= \frac{\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}} + \frac{a^2e\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+dx)}} + \frac{a^2e\sqrt{\cos}}{2b(a^2-b}$$

$$= -\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\left(-a^2+b^2\right)^{5/4}d} + \frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\left(-a^2+b^2\right)^{5/4}d} + \frac{\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 16.3372, size = 787, normalized size = 1.86

$$-\frac{b\cos(c+dx)\sqrt{e\cos(c+dx)}}{d\left(b^2-a^2\right)(a+b\sin(c+dx))} + \frac{\sqrt{e\cos(c+dx)}\left(-\frac{\sin^2(c+dx)\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((b*Cos[c + d*x]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)*d*(a + b*Sin[c + d*x]
))) + (Sqrt[e*Cos[c + d*x]]*((-4*a*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((a*App
ellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*
Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*S
qrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
```

```
[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[S
qrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I
*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1 - Co
s[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*
b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-
a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1
- (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (S
qrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2]
 - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] +
 Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 - Cos[c +
d*x]^2)*(a + b*Sin[c + d*x]))))/(2*(a - b)*(a + b)*d*Sqrt[Cos[c + d*x]])
```

Maple [C] time = 6.78, size = 7033, normalized size = 16.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos(dx+c)}}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos(dx+c)}}{(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a)^2, x)
```

3.590 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=429

$$\frac{b\sqrt{e\cos(c+dx)}}{de\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{3a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\left(b^2-a^2\right)^{7/4}}+\frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\left(b^2-a^2\right)^{7/4}}-\frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{d\left(a^2-b^2\right)\sqrt{e\cos(c+}}$$

```
[Out] (3*a*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*(-a^2 + b^2)^(7/4)*d*Sqrt[e]) + (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqr
t[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(7/4)*d*S
qrt[e]) - (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/((a^2 - b^2)*d*Sqr
t[e*Cos[c + d*x]]) + (3*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-
a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]
))*d*Sqrt[e*Cos[c + d*x]]) + (3*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b
+ Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^
2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (b*Sqrt[e*Cos[c + d*x]])/((a^2 - b^2)*
d*e*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.898004, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2694, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{b\sqrt{e\cos(c+dx)}}{de\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{3a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\left(b^2-a^2\right)^{7/4}}+\frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\left(b^2-a^2\right)^{7/4}}-\frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{d\left(a^2-b^2\right)\sqrt{e\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (3*a*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*(-a^2 + b^2)^(7/4)*d*Sqrt[e]) + (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqr
t[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(7/4)*d*S
qrt[e]) - (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/((a^2 - b^2)*d*Sqr
t[e*Cos[c + d*x]]) + (3*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-
a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]
))*d*Sqrt[e*Cos[c + d*x]]) + (3*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b
+ Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^
2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (b*Sqrt[e*Cos[c + d*x]])/((a^2 - b^2)*
```

```
d*e*(a + b*Sin[c + d*x]))
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2}\,dx = \frac{b\sqrt{e\cos(c+dx)}}{(a^2-b^2)\,de(a+b\sin(c+dx))} + \frac{\int \frac{-a+\frac{1}{2}b\sin(c+dx)}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}\,dx}{-a^2+b^2}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{(a^2-b^2)\,de(a+b\sin(c+dx))} - \frac{\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{2(a^2-b^2)} + \frac{(3a)\int \frac{1}{\sqrt{e\cos(c+dx)}(a+b\sin(c\cdots}}{2(a^2-b^2)}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{(a^2-b^2)\,de(a+b\sin(c+dx))} + \frac{(3a^2)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{4(-a^2+b^2)^{3/2}} + \cdots$$

$$= -\frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{(a^2-b^2)\,d\sqrt{e\cos(c+dx)}} + \frac{b\sqrt{e\cos(c+dx)}}{(a^2-b^2)\,de(a+b\sin(c+dx))} + \frac{(3abe)\,\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{(a^2-b^2)\,d\sqrt{e\cos(c+dx)}} - \frac{3a^2\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|\cdots\right.}{2(-a^2+b^2)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c\cdots}}$$

$$= \frac{3a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2(-a^2+b^2)^{7/4}d\sqrt{e}} + \frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2(-a^2+b^2)^{7/4}d\sqrt{e}} - \frac{\sqrt{\cos(c+dx)}F\cdots}{(a^2-b^2)\,d\sqrt{\cdots}}$$

Mathematica [C] time = 24.19, size = 1181, normalized size = 2.75

$$\frac{b\cos(c+dx)}{(a^2-b^2)\,d\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))} + \frac{\left(\frac{2b\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(\frac{5b(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{1-\cdots}}{\left(2\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)b^2+(a^2-b^2)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\cos^2(c+dx\cdots\right.\right.\right.}\right.\right.}{\cdots}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (b*Cos[c + d*x])/((a^2 - b^2)*d*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]))
+ (Sqrt[Cos[c + d*x]]*((-4*a*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 -
b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
+ b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*Appell
```

```
F1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2
*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a
^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*C
os[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]
^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*
x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]]
)/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)
^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a
^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x
])) + (2*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4,
 -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[
c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1,
 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF
1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (
a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*
ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcT
an[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a
^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c
+ d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c
+ d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(2*(a - b)*(a + b)*
d*Sqrt[e*Cos[c + d*x]])
```

Maple [C] time = 6.516, size = 4457, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -512/d*a*b*e^(7/2)/(1024*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2
)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256
*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*a^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2+768*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/
2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2
*c)^8-192*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*a^2*cos(1/2*d
*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^
4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b
```

```
^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*2^(1/2)*cos(1/2*d*x+1/2*
c)^5+160/d*a*b*e^3/(1024*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2
)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256
*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*a^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2+768*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/
2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2
*c)^8-192*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*a^2*cos(1/2*d
*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^
4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b
^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*(e*(2*cos(1/2*d*x+1/2*c)
^2-1))^(1/2)*cos(1/2*d*x+1/2*c)^4+384/d*a*b*e^(7/2)/(1024*(-2*sin(1/2*d*x+1
/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6-1792*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/
2)*2^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+768*(-2*sin(1/2*d*x+
1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-192*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1
/2)*e^(7/2)*2^(1/2)*a^2*cos(1/2*d*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^
6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^
2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(
a^2-b^2)*2^(1/2)*cos(1/2*d*x+1/2*c)^3+160/d*a*b*e^2/(1024*(-2*sin(1/2*d*x+1
/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6-1792*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/
2)*2^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+768*(-2*sin(1/2*d*x+
1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-192*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1
/2)*e^(7/2)*2^(1/2)*a^2*cos(1/2*d*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^
6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^
2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(
a^2-b^2)*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(3/2)*cos(1/2*d*x+1/2*c)^2-64/d*a*b
*e^(7/2)/(1024*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2
)*e^(7/2)*2^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256*(-2*sin(1
/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2+768*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-192*
(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*a^2*cos(1/2*d*x+1/2*c)-
5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*b^2
*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*sin
(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*2^(1/2)*cos(1/2*d*x+1/2*c)+8/d*a*b
*e/(1024*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7
/2)*2^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256*(-2*sin(1/2*d*x
```

+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
c)^2+768(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-192*(-2*si
n(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*a^2*cos(1/2*d*x+1/2*c)-5120*b
^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*b^2*e^4*s
in(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*sin(1/2*d
*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(5/2)-48/
d*a*b*e^3/(1024*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/
2)*e^(7/2)*2^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256*(-2*sin(
1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2+768*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-192
*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*a^2*cos(1/2*d*x+1/2*c)
-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*b^
2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*si
n(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1
/2)*cos(1/2*d*x+1/2*c)^2-8/d*a*b*e^2/(1024*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1
/2)*e^(7/2)*2^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*(-2*si
n(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+256*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(1/2)*a^2*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+768*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(
1/2)*e^(7/2)*2^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e
^4*sin(1/2*d*x+1/2*c)^8-192*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^(7/2)*2^(
1/2)*a^2*cos(1/2*d*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*s
in(1/2*d*x+1/2*c)^4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d
*x+1/2*c)^2-1088*b^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*(e*(2*
cos(1/2*d*x+1/2*c)^2-1))^(3/2)+3/d*a*b*e/(a^2-b^2)*sum((_R^4+_R^2*e)/(_R^7*
b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d
*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2
*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-2/
d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2
c)/(e(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*b^2/(a^2-b^2)/e*cos(1/2*d*x+1/2*c)
(-e(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(4*b^2*cos(1/2*d*
x+1/2*c)^4-4*b^2*cos(1/2*d*x+1/2*c)^2+a^2)+1/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1
)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2
-1))^(1/2)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+1/16/d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/
b^2*sum((-5*a^2+2*b^2)/(a+b)/(a-b)/(2*_alpha^2-1)/_alpha*(2^(1/2)/(e*(2*_al
pha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*
(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2
*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1
/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/

```
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c
)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^
2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+1/8/
d*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2
*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/b^2*sum(1/_alpha/(2*_alpha^2-1)*(2
^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3
)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*
_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*
(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_al
pha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-s
in(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^
2*b^2+a^2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^2), x)
```

3.591 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=492

$$-\frac{5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{3/2}\left(b^2-a^2\right)^{9/4}}+\frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{3/2}\left(b^2-a^2\right)^{9/4}}-\frac{\left(2a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}-\frac{5ab-(2a}{de\left(a^2-\right.}$$

```
[Out] (-5*a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*(-a^2 + b^2)^(9/4)*d*e^(3/2)) + (5*a*b^(3/2)*ArcTanh[(Sqrt[b]*Sq
rt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(9/4)*d*
e^(3/2)) - ((2*a^2 + 3*b^2)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])
/((a^2 - b^2)^2*d*e^2*Sqrt[Cos[c + d*x]]) - (5*a^2*b*Sqrt[Cos[c + d*x]]*Ell
ipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)^2*(b
- Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) - (5*a^2*b*Sqrt[Cos[c + d*x]]
*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)^2
*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) + b/((a^2 - b^2)*d*e*Sqrt
[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])) - (5*a*b - (2*a^2 + 3*b^2)*Sin[c + d
*x])/((a^2 - b^2)^2*d*e*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 1.21853, antiderivative size = 492, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2694, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{3/2}\left(b^2-a^2\right)^{9/4}}+\frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{3/2}\left(b^2-a^2\right)^{9/4}}-\frac{\left(2a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}-\frac{5ab-(2a}{de\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (-5*a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*(-a^2 + b^2)^(9/4)*d*e^(3/2)) + (5*a*b^(3/2)*ArcTanh[(Sqrt[b]*Sq
rt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(9/4)*d*
e^(3/2)) - ((2*a^2 + 3*b^2)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])
/((a^2 - b^2)^2*d*e^2*Sqrt[Cos[c + d*x]]) - (5*a^2*b*Sqrt[Cos[c + d*x]]*Ell
ipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)^2*(b
- Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) - (5*a^2*b*Sqrt[Cos[c + d*x]]
*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)^2
```

```
*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) + b/((a^2 - b^2)*d*e*Sqrt
[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])) - (5*a*b - (2*a^2 + 3*b^2)*Sin[c + d
*x])/((a^2 - b^2)^2*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}\,dx = \frac{b}{\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))} + \frac{\int \frac{-a+\frac{3}{2}b\sin(c+dx)}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx)}}{-a^2+b^2}$$

$$= \frac{b}{\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))} - \frac{5ab-\left(2a^2+3b^2\right)\sin(c+}{\left(a^2-b^2\right)^2 de\sqrt{e\cos(c+}}$$

$$= \frac{b}{\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))} - \frac{5ab-\left(2a^2+3b^2\right)\sin(c+}{\left(a^2-b^2\right)^2 de\sqrt{e\cos(c+}}$$

$$= \frac{b}{\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))} - \frac{5ab-\left(2a^2+3b^2\right)\sin(c+}{\left(a^2-b^2\right)^2 de\sqrt{e\cos(c+}}$$

$$= -\frac{\left(2a^2+3b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)^2 de^2\sqrt{\cos(c+dx)}} + \frac{b}{\left(a^2-b^2\right)de\sqrt{e\cos(c+dx}}$$

$$= -\frac{\left(2a^2+3b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)^2 de^2\sqrt{\cos(c+dx)}} - \frac{5a^2b\sqrt{\cos(c+dx)}\Pi\left(\frac{}{b-\sqrt{}}\right.}{2\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b}\right.}$$

$$= -\frac{5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}} + \frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}} - \frac{\left(2a^2+3\right.}{(}$$

Mathematica [C] time = 6.63914, size = 777, normalized size = 1.58

$$\cos^{\frac{3}{2}}(c+dx)\left(\frac{12\left(4a\left(a^2-b^2\right)\sin(c+dx)-\left(2a^2b+3b^3\right)\cos(2(c+dx))-6a^2b+b^3\right)}{\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)\left(a+b\sqrt{\sin^2(c+dx)}\right)\left(-\frac{\left(2a^2+3b^2\right)\csc(c+dx)\left(8b^{5/2}\cos^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};-\frac{1}{2},1\cdots\right.\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*((12*(-6*a^2*b + b^3 - (2*a^2*b + 3*b^3)*Cos[2*(c + d*x
)] + 4*a*(a^2 - b^2)*Sin[c + d*x]))/((a^2 - b^2)^2*Sqrt[Cos[c + d*x]]) - (S
in[c + d*x]*(-(((2*a^2 + 3*b^2)*Csc[c + d*x]*(8*b^(5/2)*AppellF1[3/4, -1/2,
 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3
/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos
[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c +
 d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]])))/(Sqrt
[b]*(-a^2 + b^2))) - (48*a*(a^2 + 4*b^2)*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos
[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2
 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/
(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a
^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4
)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*S
qrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]
*(-a^2 + b^2)^(1/4))))/Sqrt[Sin[c + d*x]^2])*(a + b*Sqrt[Sin[c + d*x]^2]))/
((a - b)^2*(a + b)^2)))/(24*d*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]))
```

Maple [C] time = 10.079, size = 8216, normalized size = 16.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^2), x)
```

3.592 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=514

$$\frac{7ab^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{5/2}\left(b^2-a^2\right)^{11/4}}+\frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{5/2}\left(b^2-a^2\right)^{11/4}}+\frac{\left(2a^2+5b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}}-\frac{7a^2b^2}{2de^2\left(a^2-\right.}$$

```
[Out] (7*a*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*(-a^2 + b^2)^(11/4)*d*e^(5/2)) + (7*a*b^(5/2)*ArcTanh[(Sqrt[b]*Sq
rt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(11/4)*d
*e^(5/2)) + ((2*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/
(3*(a^2 - b^2)^2*d*e^2*Sqrt[e*Cos[c + d*x]]) - (7*a^2*b^2*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)^
2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) - (7*a^2*b^2
*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2
])/(2*(a^2 - b^2)^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d
*x]]) + b/((a^2 - b^2)*d*e*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])) - (
7*a*b - (2*a^2 + 5*b^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])
^(3/2))
```

Rubi [A] time = 1.3092, antiderivative size = 514, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2694, 2866, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{7ab^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{5/2}\left(b^2-a^2\right)^{11/4}}+\frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{5/2}\left(b^2-a^2\right)^{11/4}}+\frac{\left(2a^2+5b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}}-\frac{7a^2b^2}{2de^2\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2),x]
```



```
[Out] (7*a*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*(-a^2 + b^2)^(11/4)*d*e^(5/2)) + (7*a*b^(5/2)*ArcTanh[(Sqrt[b]*Sq
rt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(11/4)*d
*e^(5/2)) + ((2*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/
(3*(a^2 - b^2)^2*d*e^2*Sqrt[e*Cos[c + d*x]]) - (7*a^2*b^2*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)^
2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) - (7*a^2*b^2
```

```
*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2
])/(2*(a^2 - b^2)^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d
*x]]) + b/((a^2 - b^2)*d*e*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])) - (
7*a*b - (2*a^2 + 5*b^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])
^(3/2))
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}\,dx = \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))} + \frac{\int \frac{-a+\frac{5}{2}b\sin(c+dx)}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx}}{-a^2+b^2}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))} - \frac{7ab-\left(2a^2+5b^2\right)\sin(c}{3\left(a^2-b^2\right)^2 de(e\cos(c+}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))} - \frac{7ab-\left(2a^2+5b^2\right)\sin(c}{3\left(a^2-b^2\right)^2 de(e\cos(c+}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))} - \frac{7ab-\left(2a^2+5b^2\right)\sin(c}{3\left(a^2-b^2\right)^2 de(e\cos(c+}$$

$$= \frac{\left(2a^2+5b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3\left(a^2-b^2\right)^2 de^2\sqrt{e\cos(c+dx)}} + \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a}$$

$$= \frac{\left(2a^2+5b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3\left(a^2-b^2\right)^2 de^2\sqrt{e\cos(c+dx)}} - \frac{7a^2b^2\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2}}\right.}{2\left(-a^2+b^2\right)^{5/2}\left(b-\sqrt{-a^2+b^2}\right.}$$

$$= \frac{7ab^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}} + \frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}} + \frac{\left(2a^2+5b^2\right)}{3\left(a^2\right.}$$

Mathematica [C] time = 25.3386, size = 1258, normalized size = 2.45

$$\frac{\left(\frac{2\sec^2(c+dx)\left(\sin(c+dx)a^2-2ba+b^2\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2} - \frac{b^3}{\left(a^2-b^2\right)^2(a+b\sin(c+dx))}\right)\cos^3(c+dx)}{d(e\cos(c+dx))^{5/2}} + \frac{\left(-\frac{2\left(5b^3+2a^2b\right)\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(\frac{}{\left(2\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2\right.\right.\right.}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*((-2*(2*a^3 - 16*a*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2]
)*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^
2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-
a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[
c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c +
 d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1
 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*
Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqr
t[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^
2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[
c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a
+ b*Sin[c + d*x])) - (2*(2*a^2*b + 5*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*
((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^
2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(
-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c
+ d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-
1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x
]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/
(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4
)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqr
t[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c
 + d*x]))))/(6*(a - b)^2*(a + b)^2*d*(e*Cos[c + d*x])^(5/2)) + (Cos[c + d*x
]^3*(-(b^3/((a^2 - b^2)^2*(a + b*Sin[c + d*x]))) + (2*Sec[c + d*x]^2*(-2*a*
b + a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(a^2 - b^2)^2)))/(d*(e*Cos[c +
 d*x])^(5/2))
```

Maple [C] time = 13.628, size = 6022, normalized size = 11.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(b\sin\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^2), x)
```

3.593 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=574

$$\frac{3\left(\left(-10a^2b^2+2a^4-7b^4\right)\sin(c+dx)+15ab^3\right)}{5de^3\left(a^2-b^2\right)^3\sqrt{e\cos(c+dx)}}-\frac{9ab^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{7/2}\left(b^2-a^2\right)^{13/4}}+\frac{9ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{7/2}\left(b^2-a^2\right)^{13/4}}-\frac{3(-10a}{}$$

```
[Out] (-9*a*b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*(-a^2 + b^2)^(13/4)*d*e^(7/2)) + (9*a*b^(7/2)*ArcTanh[(Sqrt[b]*S
qrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(13/4)*
d*e^(7/2)) - (3*(2*a^4 - 10*a^2*b^2 - 7*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2])/(5*(a^2 - b^2)^3*d*e^4*Sqrt[Cos[c + d*x]]) + (9*a^2*b^3*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])
/(2*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + (9*a
^2*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x
)/2, 2])/(2*(a^2 - b^2)^3*(b + Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]
) + b/((a^2 - b^2)*d*e*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])) - (9*a*
b - (2*a^2 + 7*b^2)*Sin[c + d*x])/(5*(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])^(5/
2)) + (3*(15*a*b^3 + (2*a^4 - 10*a^2*b^2 - 7*b^4)*Sin[c + d*x]))/(5*(a^2 -
b^2)^3*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 1.6233, antiderivative size = 574, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2694, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{3\left(\left(-10a^2b^2+2a^4-7b^4\right)\sin(c+dx)+15ab^3\right)}{5de^3\left(a^2-b^2\right)^3\sqrt{e\cos(c+dx)}}-\frac{9ab^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{7/2}\left(b^2-a^2\right)^{13/4}}+\frac{9ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{7/2}\left(b^2-a^2\right)^{13/4}}-\frac{3(-10a}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (-9*a*b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*(-a^2 + b^2)^(13/4)*d*e^(7/2)) + (9*a*b^(7/2)*ArcTanh[(Sqrt[b]*S
qrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(13/4)*
d*e^(7/2)) - (3*(2*a^4 - 10*a^2*b^2 - 7*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2])/(5*(a^2 - b^2)^3*d*e^4*Sqrt[Cos[c + d*x]]) + (9*a^2*b^3*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])
```

```
/(2*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + (9*a
^2*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x
)/2, 2])/(2*(a^2 - b^2)^3*(b + Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]
) + b/((a^2 - b^2)*d*e*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])) - (9*a*
b - (2*a^2 + 7*b^2)*Sin[c + d*x])/(5*(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])^(5/
2)) + (3*(15*a*b^3 + (2*a^4 - 10*a^2*b^2 - 7*b^4)*Sin[c + d*x]))/(5*(a^2 -
b^2)^3*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
```

```
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}\,dx = \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} + \frac{\int \frac{-a+\frac{7}{2}b\sin(c+dx)}{(e\cos(c+dx))^{7/2}(a+b\sin(c+}}{-a^2+b^2}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} - \frac{9ab-\left(2a^2+7b^2\right)\sin(}{5\left(a^2-b^2\right)^2 de(e\cos(c+}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} - \frac{9ab-\left(2a^2+7b^2\right)\sin(}{5\left(a^2-b^2\right)^2 de(e\cos(c+}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} - \frac{9ab-\left(2a^2+7b^2\right)\sin(}{5\left(a^2-b^2\right)^2 de(e\cos(c+}$$

$$= \frac{b}{\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} - \frac{9ab-\left(2a^2+7b^2\right)\sin(}{5\left(a^2-b^2\right)^2 de(e\cos(c+}$$

$$= -\frac{3\left(2a^4-10a^2b^2-7b^4\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5\left(a^2-b^2\right)^3 de^4\sqrt{\cos(c+dx)}} + \frac{}{\left(a^2-b^2\right)de(e\,c}$$

$$= -\frac{3\left(2a^4-10a^2b^2-7b^4\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5\left(a^2-b^2\right)^3 de^4\sqrt{\cos(c+dx)}} + \frac{9a^2b^3\sqrt{\cos(c+}}{2\left(a^2-b^2\right)^3\left(b\right.}$$

$$= -\frac{9ab^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{13/4}de^{7/2}} + \frac{9ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{13/4}de^{7/2}} - \frac{3\left(2a^4-\right.}{}$$

Mathematica [C] time = 6.87459, size = 949, normalized size = 1.65

$$\frac{\cos^4(c+dx)\left(\frac{\cos(c+dx)b^5}{(a^2-b^2)^3(a+b\sin(c+dx))}+\frac{2\sec^3(c+dx)\left(\sin(c+dx)a^2-2ba+b^2\sin(c+dx)\right)}{5(a^2-b^2)^2}+\frac{2\sec(c+dx)\left(3\sin(c+dx)a^4-15b^2\sin(c+dx)a^2+20b^3a-8b^4\sin(\cdots\right)}{5(a^2-b^2)^3}\right.}{d(e\cos(c+dx))^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(7/2)*((-2*(2*a^5 - 10*a^3*b^2 - 22*a*b^4)*(a + b*Sqrt[1 -
 Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c
 + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)
*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2
*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[
Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*S
in[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((2*a^4*b -
10*a^2*b^3 - 7*b^5)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/
4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c +
 d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*
Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt
[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*
(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2
] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]])
)*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[
c + d*x]))))/(10*(a - b)^3*(a + b)^3*d*(e*Cos[c + d*x])^(7/2)) + (Cos[c + d
*x]^4*((b^5*Cos[c + d*x])/((a^2 - b^2)^3*(a + b*Sin[c + d*x])) + (2*Sec[c +
 d*x]^3*(-2*a*b + a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(5*(a^2 - b^2)^2) +
 (2*Sec[c + d*x]*(20*a*b^3 + 3*a^4*Sin[c + d*x] - 15*a^2*b^2*Sin[c + d*x] -
 8*b^4*Sin[c + d*x]))/(5*(a^2 - b^2)^3)))/(d*(e*Cos[c + d*x])^(7/2))
```

Maple [C] time = 21.724, size = 10743, normalized size = 18.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\left(b\sin\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^2), x)
```

3.594 $\int \frac{(e\cos(c+dx))^{13/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=575

$$\frac{11e^5(e\cos(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\sin(c+dx)\right)}{60b^5d}-\frac{11e^{13/2}\left(-11a^2b^2+9a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{13/2}d\sqrt[4]{b^2-a^2}}+\frac{11e^{13/2}\cdots}{\cdots}$$

```
[Out] (-11*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^(13/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d
*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(13/2)*(-a^2 + b^2)^(1/4)*d) + (1
1*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^(13/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x
]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(13/2)*(-a^2 + b^2)^(1/4)*d) + (11*
a*(45*a^2 - 37*b^2)*e^6*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(20
*b^6*d*Sqrt[Cos[c + d*x]]) - (11*a*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^7*Sqrt[Co
s[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*b^
7*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (11*a*(9*a^4 - 11*a^2*b^
2 + 2*b^4)*e^7*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
(c + d*x)/2, 2])/(8*b^7*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e
*(e*Cos[c + d*x])^(11/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (11*e^3*(e*Cos[c
 + d*x])^(7/2)*(9*a + 2*b*Sin[c + d*x]))/(28*b^3*d*(a + b*Sin[c + d*x])) +
(11*e^5*(e*Cos[c + d*x])^(3/2)*(5*(9*a^2 - 2*b^2) - 27*a*b*Sin[c + d*x]))/(
60*b^5*d)
```

Rubi [A] time = 1.41475, antiderivative size = 575, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {2693, 2863, 2865, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{11e^5(e\cos(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\sin(c+dx)\right)}{60b^5d}-\frac{11e^{13/2}\left(-11a^2b^2+9a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{13/2}d\sqrt[4]{b^2-a^2}}+\frac{11e^{13/2}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(13/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-11*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^(13/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d
*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(13/2)*(-a^2 + b^2)^(1/4)*d) + (1
1*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^(13/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x
]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(13/2)*(-a^2 + b^2)^(1/4)*d) + (11*
```

```
a*(45*a^2 - 37*b^2)*e^6*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(20
*b^6*d*Sqrt[Cos[c + d*x]]) - (11*a*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^7*Sqrt[Co
s[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*b^
7*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (11*a*(9*a^4 - 11*a^2*b^
2 + 2*b^4)*e^7*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
(c + d*x)/2, 2])/(8*b^7*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e
*(e*Cos[c + d*x])^(11/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (11*e^3*(e*Cos[c
 + d*x])^(7/2)*(9*a + 2*b*Sin[c + d*x]))/(28*b^3*d*(a + b*Sin[c + d*x])) +
(11*e^5*(e*Cos[c + d*x])^(3/2)*(5*(9*a^2 - 2*b^2) - 27*a*b*Sin[c + d*x]))/(
60*b^5*d)
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{13/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{\left(11e^2\right)\int \frac{(e\cos(c+dx))^{9/2}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+2b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))} + \frac{\left(11e^4\right)\int \frac{(e\cos(c+dx)}{}}{}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+2b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))} + \frac{11e^5(e\cos(c+dx)}{}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+2b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))} + \frac{11e^5(e\cos(c+dx)}{}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+2b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))} + \frac{11e^5(e\cos(c+dx)}{}$$

$$= \frac{11a\left(45a^2-37b^2\right)e^6\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{20b^6d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{11e^3(e\cos}{2}$$

$$= \frac{11a\left(45a^2-37b^2\right)e^6\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{20b^6d\sqrt{\cos(c+dx)}} - \frac{11a\left(9a^4-11a^2b^2+2b^4\right)e^7\sqrt{\cos(c+}}{8b^7\left(b-\sqrt{-a^2+b^2}\right)}$$

$$= -\frac{11\left(9a^4-11a^2b^2+2b^4\right)e^{13/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}d} + \frac{11\left(9a^4-11a^2b^2+2b^4\right)e^{13/2}\tanh^{-1}}{8b^{13/2}\sqrt[4]{-a^2+b^2}d}$$

Mathematica [C] time = 25.5633, size = 932, normalized size = 1.62

$$11\left(-\frac{\left(45a^3-37ab^2\right)\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)\cos^{\frac{3}{2}}(c+dx)b^{5/2}+3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\tan^{-1}\left(\right.\right.\right.}{12b^{3/2}\left(b^2-a^2\right)\left(1-\cos^2(c+\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(13/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (11*(e*Cos[c + d*x])^(13/2)*((-2*(18*a^2*b - 10*b^3)*(a + b*Sqrt[1 - Cos[c
+ d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]
^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*Arc
Tan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan
[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a
^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos
[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt
[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c +
d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((45*a^3 - 37*a*b^2
)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, C
os[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sq
rt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]]
)/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a
^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*
Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/
(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(40*b
^5*d*Cos[c + d*x]^(13/2)) + ((e*Cos[c + d*x])^(13/2)*Sec[c + d*x]^6*(-((-16
8*a^2 + 65*b^2)*Cos[c + d*x])/(42*b^5) - Cos[3*(c + d*x)]/(14*b^3) + (-(a^4
*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*Cos[c + d*x])/(2*b^5*(a + b*S
in[c + d*x])^2) + (19*(a^3*Cos[c + d*x] - a*b^2*Cos[c + d*x]))/(4*b^5*(a +
b*Sin[c + d*x])) - (3*a*Sin[2*(c + d*x)])/(5*b^4)))/d
```

Maple [C] time = 29.625, size = 111631, normalized size = 194.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(13/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.595 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=589

$$\frac{3e^5\sqrt{e\cos(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\sin(c+dx)\right)}{4b^5d}+\frac{9e^{11/2}\left(-9a^2b^2+7a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{11/2}d\left(b^2-a^2\right)^{3/4}}+\frac{9e^{11/2}\left(-9a^2b^2\right.}{}$$

```
[Out] (9*(7*a^4 - 9*a^2*b^2 + 2*b^4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]
])/((-a^2 + b^2)^(1/4)*Sqrt[e])]])/(8*b^(11/2)*(-a^2 + b^2)^(3/4)*d) + (9*(7
*a^4 - 9*a^2*b^2 + 2*b^4)*e^(11/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/(
(-a^2 + b^2)^(1/4)*Sqrt[e])]])/(8*b^(11/2)*(-a^2 + b^2)^(3/4)*d) + (3*a*(21*
a^2 - 13*b^2)*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(4*b^6*d*Sq
rt[e*Cos[c + d*x]]) - (9*a*(7*a^4 - 9*a^2*b^2 + 2*b^4)*e^6*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*b^6*(a^2 -
b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (9*a*(7*a^4 - 9*a^2*b^2
 + 2*b^4)*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (
c + d*x)/2, 2])/(8*b^6*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*
x]]) - (e*(e*Cos[c + d*x])^(9/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (9*e^3*(
e*Cos[c + d*x])^(5/2)*(7*a + 2*b*Sin[c + d*x]))/(20*b^3*d*(a + b*Sin[c + d*
x])) + (3*e^5*Sqrt[e*Cos[c + d*x]]*(3*(7*a^2 - 2*b^2) - 7*a*b*Sin[c + d*x])
)/(4*b^5*d)
```

Rubi [A] time = 1.48683, antiderivative size = 589, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {2693, 2863, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{3e^5\sqrt{e\cos(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\sin(c+dx)\right)}{4b^5d}+\frac{9e^{11/2}\left(-9a^2b^2+7a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{11/2}d\left(b^2-a^2\right)^{3/4}}+\frac{9e^{11/2}\left(-9a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] (9*(7*a^4 - 9*a^2*b^2 + 2*b^4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]
])/((-a^2 + b^2)^(1/4)*Sqrt[e])]])/(8*b^(11/2)*(-a^2 + b^2)^(3/4)*d) + (9*(7
*a^4 - 9*a^2*b^2 + 2*b^4)*e^(11/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/(
(-a^2 + b^2)^(1/4)*Sqrt[e])]])/(8*b^(11/2)*(-a^2 + b^2)^(3/4)*d) + (3*a*(21*
```

```
a^2 - 13*b^2)*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(4*b^6*d*Sq
rt[e*Cos[c + d*x]]) - (9*a*(7*a^4 - 9*a^2*b^2 + 2*b^4)*e^6*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*b^6*(a^2 -
b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (9*a*(7*a^4 - 9*a^2*b^2
 + 2*b^4)*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (
c + d*x)/2, 2])/(8*b^6*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*
x]]) - (e*(e*Cos[c + d*x])^(9/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (9*e^3*(
e*Cos[c + d*x])^(5/2)*(7*a + 2*b*Sin[c + d*x]))/(20*b^3*d*(a + b*Sin[c + d*
x])) + (3*e^5*Sqrt[e*Cos[c + d*x]]*(3*(7*a^2 - 2*b^2) - 7*a*b*Sin[c + d*x])
)/(4*b^5*d)
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{\left(9e^2\right)\int\frac{(e\cos(c+dx))^{7/2}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{9e^3(e\cos(c+dx))^{5/2}(7a+2b\sin(c+dx))}{20b^3d(a+b\sin(c+dx))} + \frac{\left(9e^4\right)\int\frac{(e\cos(c+dx))^{3/2}}{a+b\,\mathrm{s}}}{4b^3}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{9e^3(e\cos(c+dx))^{5/2}(7a+2b\sin(c+dx))}{20b^3d(a+b\sin(c+dx))} + \frac{3e^5\sqrt{e\cos(c+dx)}\,(3}{}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{9e^3(e\cos(c+dx))^{5/2}(7a+2b\sin(c+dx))}{20b^3d(a+b\sin(c+dx))} + \frac{3e^5\sqrt{e\cos(c+dx)}\,(3}{}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{9e^3(e\cos(c+dx))^{5/2}(7a+2b\sin(c+dx))}{20b^3d(a+b\sin(c+dx))} + \frac{3e^5\sqrt{e\cos(c+dx)}\,(3}{}$$

$$= \frac{3a\left(21a^2-13b^2\right)e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^6d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{9e^3(e\cos(c+a}{20b^3d}$$

$$= \frac{3a\left(21a^2-13b^2\right)e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^6d\sqrt{e\cos(c+dx)}} - \frac{9a\left(7a^4-9a^2b^2+2b^4\right)e^6\sqrt{\cos(c+dx)}\,\Pi}{8b^6\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d}$$

$$= \frac{9\left(7a^4-9a^2b^2+2b^4\right)e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{11/2}\left(-a^2+b^2\right)^{3/4}d} + \frac{9\left(7a^4-9a^2b^2+2b^4\right)e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e}}{\sqrt[4]{-a}}\right.}{8b^{11/2}\left(-a^2+b^2\right)^{3/4}d}$$

Mathematica [C] time = 27.5355, size = 2024, normalized size = 3.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(11/2)*Sec[c + d*x]^5*(-Cos[2*(c + d*x)]/(5*b^3) - (2*a*S
in[c + d*x])/b^4 - (-a^2 + b^2)^2/(2*b^5*(a + b*Sin[c + d*x])^2) + (17*a*(a
^2 - b^2))/(4*b^5*(a + b*Sin[c + d*x]))))/d + (3*(e*Cos[c + d*x])^(11/2)*((
-2*(30*a^2*b - 16*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)
]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4
```

```
, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^
2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2)))
- ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(
-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^
2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b
^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) +
((40*a^2*b - 14*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1
/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-
a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^
2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^
(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1
/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]
^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[
c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1
- Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2,
 (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, C
os[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5
/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c
+ d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*
Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]
] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2
+ b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c
 + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(
Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2
*(25*a^3 - 37*a*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*App
ellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]
*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4
, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b
^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
 b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c
+ d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2)))
 + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)
] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] -
Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]]
+ b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4
)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(40*b^5*d
*Cos[c + d*x]^(11/2))
```

Maple [C] time = 32.438, size = 85607, normalized size = 145.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.596 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=483

$$\frac{7e^{9/2}\left(5a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}}-\frac{7e^{9/2}\left(5a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}}+\frac{7ae^5\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{\ }{b-\ }\right.}{8b^5d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\,\mathrm{co}}}$$

```
[Out] (7*(5*a^2 - 2*b^2)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*b^(9/2)*(-a^2 + b^2)^(1/4)*d) - (7*(5*a^2 - 2*b^2)*
e^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/(8*b^(9/2)*(-a^2 + b^2)^(1/4)*d) - (35*a*e^4*Sqrt[e*Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2])/(4*b^4*d*Sqrt[Cos[c + d*x]]) + (7*a*(5*a^2 - 2*b^2)*e^
5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2,
2])/(8*b^5*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (7*a*(5*a^2 - 2
*b^2)*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c +
d*x)/2, 2])/(8*b^5*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*C
os[c + d*x])^(7/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (7*e^3*(e*Cos[c + d*x]
)^(3/2)*(5*a + 2*b*Sin[c + d*x]))/(12*b^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.07539, antiderivative size = 483, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2693, 2863, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{7e^{9/2}\left(5a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}}-\frac{7e^{9/2}\left(5a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}}+\frac{7ae^5\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{\ }{b-\ }\right.}{8b^5d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\,\mathrm{co}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (7*(5*a^2 - 2*b^2)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*b^(9/2)*(-a^2 + b^2)^(1/4)*d) - (7*(5*a^2 - 2*b^2)*
e^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/(8*b^(9/2)*(-a^2 + b^2)^(1/4)*d) - (35*a*e^4*Sqrt[e*Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2])/(4*b^4*d*Sqrt[Cos[c + d*x]]) + (7*a*(5*a^2 - 2*b^2)*e^
5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2,
2])/(8*b^5*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (7*a*(5*a^2 - 2
*b^2)*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c +
```

```
d*x)/2, 2])/(8*b^5*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*C
os[c + d*x])^(7/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (7*e^3*(e*Cos[c + d*x]
)^(3/2)*(5*a + 2*b*Sin[c + d*x]))/(12*b^3*d*(a + b*Sin[c + d*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{\left(7e^2\right)\int \frac{(e\cos(c+dx))^{5/2}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+2b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))} + \frac{\left(7e^4\right)\int \frac{\sqrt{e\cos(c+dx)}}{a+b}}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+2b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))} - \frac{\left(35ae^4\right)\int \sqrt{e\cos(}}{8b^4}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+2b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))} - \frac{\left(7a\left(5a^2-2b^2\right)e^5\right)}{}$$

$$= -\frac{35ae^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a}{12b^3d(a+b\sin}$$

$$= -\frac{35ae^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\sqrt{\cos(c+dx)}} + \frac{7a\left(5a^2-2b^2\right)e^5\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c-\right.}{8b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+dx)}}$$

$$= \frac{7\left(5a^2-2b^2\right)e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}d} - \frac{7\left(5a^2-2b^2\right)e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}d} - \frac{35ae^4}{}$$

Mathematica [C] time = 26.1235, size = 777, normalized size = 1.61

$$(e\cos(c+dx))^{9/2}\left(\frac{35a\left(a+b\sqrt{\sin^2(c+dx)}\right)\left(8b^{5/2}\cos^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{b^2-a^2}\right)+3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(-\log\left(-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(c+dx)}\right.\right.\right.}{12b^{9/2}\left(b^2-\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(9/2)*((-16*Cos[c + d*x]^(3/2))/(3*b^3) + (4*(a^2 - b^2)*
Cos[c + d*x]^(3/2))/(b^3*(a + b*Sin[c + d*x])^2) - (22*a*Cos[c + d*x]^(3/2)
)/(b^3*(a + b*Sin[c + d*x])) + (35*a*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4,
 Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*
Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x
]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/
(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4
)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*(a + b*Sqrt[Si
n[c + d*x]^2]))/(12*b^(9/2)*(-a^2 + b^2)*(a + b*Sin[c + d*x])) + (28*((a*Ap
pellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]
*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*
Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqr
t[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 +
I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[
Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x]*(a + b*Sqrt[
Sin[c + d*x]^2]))/(b^2*Sqrt[Sin[c + d*x]^2]*(a + b*Sin[c + d*x]))))/(8*d*Co
s[c + d*x]^(9/2))
```

Maple [C] time = 25.289, size = 85489, normalized size = 177.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.597 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=497

$$-\frac{5e^{7/2}\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}}-\frac{5e^{7/2}\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}}+\frac{5ae^4\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{}{b}\right.}{8b^4d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)}$$

```
[Out] (-5*(3*a^2 - 2*b^2)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*b^(7/2)*(-a^2 + b^2)^(3/4)*d) - (5*(3*a^2 - 2*b^2)
*e^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*b^(7/2)*(-a^2 + b^2)^(3/4)*d) - (15*a*e^4*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(4*b^4*d*Sqrt[e*Cos[c + d*x]]) + (5*a*(3*a^2 - 2*b^2)*e
^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2,
 2])/(8*b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (5*a
*(3*a^2 - 2*b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b
^2]), (c + d*x)/2, 2])/(8*b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos
[c + d*x]]) - (e*(e*Cos[c + d*x])^(5/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (
5*e^3*Sqrt[e*Cos[c + d*x]]*(3*a + 2*b*Sin[c + d*x]))/(4*b^3*d*(a + b*Sin[c
+ d*x]))
```

Rubi [A] time = 1.07856, antiderivative size = 497, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2693, 2863, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{5e^{7/2}\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}}-\frac{5e^{7/2}\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}}+\frac{5ae^4\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{}{b}\right.}{8b^4d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-5*(3*a^2 - 2*b^2)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*b^(7/2)*(-a^2 + b^2)^(3/4)*d) - (5*(3*a^2 - 2*b^2)
*e^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*b^(7/2)*(-a^2 + b^2)^(3/4)*d) - (15*a*e^4*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(4*b^4*d*Sqrt[e*Cos[c + d*x]]) + (5*a*(3*a^2 - 2*b^2)*e
^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2,
 2])/(8*b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (5*a
```

```
*(3*a^2 - 2*b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b
^2]), (c + d*x)/2, 2])/(8*b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos
[c + d*x]]) - (e*(e*Cos[c + d*x])^(5/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (
5*e^3*Sqrt[e*Cos[c + d*x]]*(3*a + 2*b*Sin[c + d*x]))/(4*b^3*d*(a + b*Sin[c
+ d*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{\left(5e^2\right)\int \frac{(e\cos(c+dx))^{3/2}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{5e^3\sqrt{e\cos(c+dx)}(3a+2b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))} + \frac{\left(5e^4\right)\int \frac{-b-\frac{3}{2}a\sin}{\sqrt{e\cos(c+dx)}(a+}}{4b^3}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{5e^3\sqrt{e\cos(c+dx)}(3a+2b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))} - \frac{\left(15ae^4\right)\int \frac{1}{\sqrt{e\cos(c+dx)}}}{8b^4}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{5e^3\sqrt{e\cos(c+dx)}(3a+2b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))} - \frac{\left(5a\left(3a^2-2b^2\right)e^4\right)\int}{16}$$

$$= -\frac{15ae^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{5e^3\sqrt{e\cos(c+dx)}(3a+2}{4b^3d(a+b\sin(c}$$

$$= -\frac{15ae^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\sqrt{e\cos(c+dx)}} + \frac{5a\left(3a^2-2b^2\right)e^4\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+}{8b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\cos(c+dx)}}$$

$$= -\frac{5\left(3a^2-2b^2\right)e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{5\left(3a^2-2b^2\right)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{15a}{}$$

Mathematica [C] time = 26.4244, size = 1954, normalized size = 3.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^3*((a^2 - b^2)/(2*b^3*(a + b*Sin[c + d
*x])^2) - (9*a)/(4*b^3*(a + b*Sin[c + d*x]))))/d - ((e*Cos[c + d*x])^(7/2)*
((-12*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/
2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c +
```

```
d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4,
Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4,
 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 +
b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
+ b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*
Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/
4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
 + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d
*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b
^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[
c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) + (4*b*(a + b*Sqr
t[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan
[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a
^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*
Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*
Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a
*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2
)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2
)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x
]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[
c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)
*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(
b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 +
2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) - (14*a*(a + b*Sqrt[1 - Cos[c + d*x
]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((
-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x
]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (
b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4,
Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 +
b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c
 + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d
*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b
*Sin[c + d*x]))))/(8*b^3*d*Cos[c + d*x]^(7/2))
```

Maple [C] time = 24.696, size = 65216, normalized size = 131.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(b*sin(d*x + c) + a)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.598 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=505

$$\frac{3e^{5/2}\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{5/2}d\left(b^2-a^2\right)^{5/4}}-\frac{3e^{5/2}\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{5/2}d\left(b^2-a^2\right)^{5/4}}+\frac{3ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{4b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}$$

```
[Out] (3*(a^2 - 2*b^2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2
)^(1/4)*Sqrt[e])])/(8*b^(5/2)*(-a^2 + b^2)^(5/4)*d) - (3*(a^2 - 2*b^2)*e^(5
/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(
8*b^(5/2)*(-a^2 + b^2)^(5/4)*d) + (3*a*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) - (3*a*(a^2 - 2*b^
2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x
)/2, 2])/(8*b^3*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]])
 - (3*a*(a^2 - 2*b^2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(8*b^3*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt
[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(3/2))/(2*b*d*(a + b*Sin[c + d*x])^
2) + (3*a*e*(e*Cos[c + d*x])^(3/2))/(4*b*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
)
```

Rubi [A] time = 1.10473, antiderivative size = 505, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2693, 2864, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{3e^{5/2}\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{5/2}d\left(b^2-a^2\right)^{5/4}}-\frac{3e^{5/2}\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{5/2}d\left(b^2-a^2\right)^{5/4}}+\frac{3ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{4b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*(a^2 - 2*b^2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2
)^(1/4)*Sqrt[e])])/(8*b^(5/2)*(-a^2 + b^2)^(5/4)*d) - (3*(a^2 - 2*b^2)*e^(5
/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(
8*b^(5/2)*(-a^2 + b^2)^(5/4)*d) + (3*a*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) - (3*a*(a^2 - 2*b^
2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x
)/2, 2])/(8*b^3*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]])
```

```
- (3*a*(a^2 - 2*b^2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(8*b^3*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt
[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(3/2))/(2*b*d*(a + b*Sin[c + d*x])^
2) + (3*a*e*(e*Cos[c + d*x])^(3/2))/(4*b*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
)
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} - \frac{\left(3e^2\right)\int \frac{\sqrt{e\cos(c+dx)}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} + \frac{3ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\left(3e^2\right)\int \frac{\sqrt{e\cos(c+dx)}\left(b+\frac{1}{2}a\sin(c+d\right.}{a+b\sin(c+dx)}}{4b\left(a^2-b^2\right)}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} + \frac{3ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\left(3ae^2\right)\int \sqrt{e\cos(c+dx)}\,dx}{8b^2\left(a^2-b^2\right)} -$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} + \frac{3ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\left(3a\left(a^2-2b^2\right)e^3\right)\int \frac{}{\sqrt{e\cos(c+d}}}{16b^3\left(a^2-\right.}$$

$$= \frac{3ae^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} + \frac{3ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+\right.}$$

$$= \frac{3ae^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{3a\left(a^2-2b^2\right)e^3\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\right|}{8b^3\left(a^2-b^2\right)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+dx)}}$$

$$= \frac{3\left(a^2-2b^2\right)e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d} - \frac{3\left(a^2-2b^2\right)e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d} + \frac{3ae^2\sqrt{e\,c}}{4b^2\left(a\right.}$$

Mathematica [C] time = 24.1576, size = 831, normalized size = 1.65

$$\frac{\sec^2(c+dx)\left(-\frac{3a\cos(c+dx)}{4b\left(b^2-a^2\right)(a+b\sin(c+dx))} - \frac{\cos(c+dx)}{2b(a+b\sin(c+dx))^2}\right)(e\cos(c+dx))^{5/2}}{d} + \frac{3\left(-\frac{a\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2c}{}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*(-Cos[c + d*x]/(2*b*(a + b*Sin[c + d
*x])^2) - (3*a*Cos[c + d*x])/(4*b*(-a^2 + b^2)*(a + b*Sin[c + d*x]))))/d +
(3*(e*Cos[c + d*x])^(5/2)*((-4*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((a*Appel
lF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Co
s[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqr
t[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b
]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[Sqr
t[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b
*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[
c + d*x]^2]*(a + b*Sin[c + d*x])) - (a*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*
b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-
a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1
- (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (S
qrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2]
 - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] +
 Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c +
d*x]^2)*(a + b*Sin[c + d*x]))))/(8*(a - b)*b*(a + b)*d*Cos[c + d*x]^(5/2))
```

Maple [C] time = 26.309, size = 63272, normalized size = 125.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(b*sin(d*x + c) + a)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(b*sin(d*x + c) + a)^3, x)
```

3.599 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=519

$$\frac{e^{3/2}\left(a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{3/2}d\left(b^2-a^2\right)^{7/4}}+\frac{e^{3/2}\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{3/2}d\left(b^2-a^2\right)^{7/4}}-\frac{ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}}+\frac{ae^2}{8b^2}$$

```
[Out] ((a^2 + 2*b^2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^
(1/4)*Sqrt[e])])/(8*b^(3/2)*(-a^2 + b^2)^(7/4)*d) + ((a^2 + 2*b^2)*e^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^
(3/2)*(-a^2 + b^2)^(7/4)*d) - (a*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2])/(4*b^2*(a^2 - b^2)*d*Sqrt[e*Cos[c + d*x]]) + (a*(a^2 + 2*b^2)*e^2*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])
/(8*b^2*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]
) + (a*(a^2 + 2*b^2)*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(8*b^2*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2
]))*d*Sqrt[e*Cos[c + d*x]]) - (e*Sqrt[e*Cos[c + d*x]])/(2*b*d*(a + b*Sin[c
+ d*x])^2) + (a*e*Sqrt[e*Cos[c + d*x]])/(4*b*(a^2 - b^2)*d*(a + b*Sin[c + d
*x]))
```

Rubi [A] time = 1.1384, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2693, 2864, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{e^{3/2}\left(a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{3/2}d\left(b^2-a^2\right)^{7/4}}+\frac{e^{3/2}\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{3/2}d\left(b^2-a^2\right)^{7/4}}-\frac{ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}}+\frac{ae^2}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^3,x]
```



```
[Out] ((a^2 + 2*b^2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^
(1/4)*Sqrt[e])])/(8*b^(3/2)*(-a^2 + b^2)^(7/4)*d) + ((a^2 + 2*b^2)*e^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^
(3/2)*(-a^2 + b^2)^(7/4)*d) - (a*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2])/(4*b^2*(a^2 - b^2)*d*Sqrt[e*Cos[c + d*x]]) + (a*(a^2 + 2*b^2)*e^2*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])
/(8*b^2*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]
```

```
) + (a*(a^2 + 2*b^2)*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(8*b^2*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2
]))*d*Sqrt[e*Cos[c + d*x]]) - (e*Sqrt[e*Cos[c + d*x]])/(2*b*d*(a + b*Sin[c
+ d*x])^2) + (a*e*Sqrt[e*Cos[c + d*x]])/(4*b*(a^2 - b^2)*d*(a + b*Sin[c + d
*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} - \frac{e^2\int \frac{\sin(c+dx)}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} + \frac{ae\sqrt{e\cos(c+dx)}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{e^2\int \frac{b-\frac{1}{2}a\sin(c+dx)}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}\,d}{4b\left(a^2-b^2\right)}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} + \frac{ae\sqrt{e\cos(c+dx)}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{\left(ae^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{8b^2\left(a^2-b^2\right)} + \frac{\left(\left(a^2\right.\right.}{}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} + \frac{ae\sqrt{e\cos(c+dx)}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\left(a\left(a^2+2b^2\right)e^2\right)\int \frac{}{\sqrt{e\cos(c+dx)}}}{16b^2\left(-a^2+\right.}$$

$$= -\frac{ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)d\sqrt{e\cos(c+dx)}} - \frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} + \frac{ae\sqrt{e\cos(c+dx)}}{4b\left(a^2-b^2\right)d(a+b\sin(c+a}$$

$$= -\frac{ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)d\sqrt{e\cos(c+dx)}} - \frac{a\left(a^2+2b^2\right)e^2\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8b^2\left(-a^2+b^2\right)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+dx)}}$$

$$= \frac{\left(a^2+2b^2\right)e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{3/2}\left(-a^2+b^2\right)^{7/4}d} + \frac{\left(a^2+2b^2\right)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{3/2}\left(-a^2+b^2\right)^{7/4}d} - \frac{ae^2\sqrt{\cos(c+}}{4b^2\left(a^2-b\right.}$$

Mathematica [C] time = 24.2999, size = 1211, normalized size = 2.33

$$\frac{(e\cos(c+dx))^{3/2}\sec(c+dx)\left(-\frac{a}{4b\left(b^2-a^2\right)(a+b\sin(c+dx))}-\frac{1}{2b(a+b\sin(c+dx))^2}\right)}{d} - \frac{(e\cos(c+dx))^{3/2}\left(\frac{4b\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(\frac{}{\sqrt{1-\cos^2(c+d}}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*Sec[c + d*x]*(-1/(2*b*(a + b*Sin[c + d*x])^2) - a/(
4*b*(-a^2 + b^2)*(a + b*Sin[c + d*x]))))/d - ((e*Cos[c + d*x])^(3/2)*((4*b*
(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/
4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(
Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c +
d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2,
 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*App
ellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])
*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*
(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*
ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[S
qrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I
*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4
)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x]
)/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (2*a*(a + b*Sqrt[1 - Co
s[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*
x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c +
d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2,
 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2
)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*S
qrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[
Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(
a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2]
 + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))
/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^
2)*(a + b*Sin[c + d*x]))))/(8*(a - b)*b*(a + b)*d*Cos[c + d*x]^(3/2))
```

Maple [C] time = 27.009, size = 45147, normalized size = 87.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos{(dx+c)})^{\frac{3}{2}}}{(b\sin{(dx+c)}+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(b*sin(d*x + c) + a)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos{(dx+c)})^{\frac{3}{2}}}{(b\sin{(dx+c)}+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(b*sin(d*x + c) + a)^3, x)
```

3.600 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=514

$$\frac{5ab(e\cos(c+dx))^{3/2}}{4de\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b(e\cos(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\sin(c+dx))^2}+\frac{\sqrt{e}\left(3a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8\sqrt{b}d\left(b^2-a^2\right)^{9/4}}-\frac{\sqrt{e}\left(3a^2\right.}{}$$

```
[Out] ((3*a^2 + 2*b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2
)^(1/4)*Sqrt[e])])/(8*Sqrt[b]*(-a^2 + b^2)^(9/4)*d) - ((3*a^2 + 2*b^2)*Sqrt
[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(
8*Sqrt[b]*(-a^2 + b^2)^(9/4)*d) + (5*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2])/(4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (a*(3*a^2 + 2*b^2)*e*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])
/(8*b*(a^2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (a*(3*
a^2 + 2*b^2)*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
(c + d*x)/2, 2])/(8*b*(a^2 - b^2)^2*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c +
 d*x]]) + (b*(e*Cos[c + d*x])^(3/2))/(2*(a^2 - b^2)*d*e*(a + b*Sin[c + d*x]
)^2) + (5*a*b*(e*Cos[c + d*x])^(3/2))/(4*(a^2 - b^2)^2*d*e*(a + b*Sin[c + d
*x]))
```

Rubi [A] time = 1.18696, antiderivative size = 514, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2694, 2864, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{5ab(e\cos(c+dx))^{3/2}}{4de\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b(e\cos(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\sin(c+dx))^2}+\frac{\sqrt{e}\left(3a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8\sqrt{b}d\left(b^2-a^2\right)^{9/4}}-\frac{\sqrt{e}\left(3a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] ((3*a^2 + 2*b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2
)^(1/4)*Sqrt[e])])/(8*Sqrt[b]*(-a^2 + b^2)^(9/4)*d) - ((3*a^2 + 2*b^2)*Sqrt
[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(
8*Sqrt[b]*(-a^2 + b^2)^(9/4)*d) + (5*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2])/(4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (a*(3*a^2 + 2*b^2)*e*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])
/(8*b*(a^2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (a*(3*
```

```
a^2 + 2*b^2)*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
(c + d*x)/2, 2])/(8*b*(a^2 - b^2)^2*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c +
 d*x]]) + (b*(e*Cos[c + d*x])^(3/2))/(2*(a^2 - b^2)*d*e*(a + b*Sin[c + d*x]
)^2) + (5*a*b*(e*Cos[c + d*x])^(3/2))/(4*(a^2 - b^2)^2*d*e*(a + b*Sin[c + d
*x]))
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^3}\,dx = \frac{b(e\cos(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} - \frac{\int \frac{\sqrt{e\cos(c+dx)}\left(-2a+\frac{1}{2}b\sin(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{5ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))} + \frac{\int \frac{\sqrt{e\cos(c+dx)}\left(\frac{1}{2}\left(4a^2-\right.\right.}{a+b\sin}}{2\left(a^2\right.}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{5ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))} + \frac{(5a)\int\sqrt{e\cos(c+}}{8\left(a^2-b^2\right)^2}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{5ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))} - \frac{\left(a\left(3a^2+2b^2\right)e\right)\int}{}$$

$$= \frac{5a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{b(e\cos(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{5ab(e\cos(c}{4\left(a^2-b^2\right)^2 de(a+}$$

$$= \frac{5a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{a\left(3a^2+2b^2\right)e\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right.}{8b\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+dx)}}$$

$$= \frac{\left(3a^2+2b^2\right)\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\left(-a^2+b^2\right)^{9/4}d} - \frac{\left(3a^2+2b^2\right)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\left(-a^2+b^2\right)^{9/4}d} + \frac{5a\sqrt{e\cos}}{4\left(a^2\right.}$$

Mathematica [C] time = 26.4621, size = 837, normalized size = 1.63

$$\frac{\sqrt{e\cos(c+dx)}\left(\frac{5ab\cos(c+dx)}{4\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\cos(c+dx)}{2\left(a^2-b^2\right)(a+b\sin(c+dx))^2}\right)}{d} + \frac{\sqrt{e\cos(c+dx)}\left(-\frac{5a\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+d\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*((b*Cos[c + d*x])/(2*(a^2 - b^2)*(a + b*Sin[c + d*x])
^2) + (5*a*b*Cos[c + d*x])/(4*(a^2 - b^2)^2*(a + b*Sin[c + d*x]))))/d + (Sq
rt[e*Cos[c + d*x]]*((-2*(8*a^2 + 2*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((
a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)
*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] +
Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]
] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1
 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (5*a*(a + b*Sqrt[1 - Cos[c + d*x
]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d
*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*
ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcT
an[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a
^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c
+ d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 -
 Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(8*(a - b)^2*(a + b)^2*d*Sqrt[Cos[
c + d*x]])
```

Maple [C] time = 27.897, size = 36688, normalized size = 71.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{\left(b\sin\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e \cos(dx+c)}}{(b \sin(dx+c)+a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a)^3, x)
```

3.601 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=520

$$\frac{7ab\sqrt{e\cos(c+dx)}}{4de\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\sqrt{e\cos(c+dx)}}{2de\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{3\sqrt{b}\left(5a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8d\sqrt{e}\left(b^2-a^2\right)^{11/4}}-\frac{3\sqrt{b}\left(5a\right.}{}$$

```
[Out] (-3*Sqrt[b]*(5*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(11/4)*d*Sqrt[e]) - (3*Sqrt[b]*(5*a^2
 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(8*(-a^2 + b^2)^(11/4)*d*Sqrt[e]) - (7*a*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2])/(4*(a^2 - b^2)^2*d*Sqrt[e*Cos[c + d*x]]) + (3*a*(5*a^2 + 2
*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)
/2, 2])/(8*(a^2 - b^2)^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c +
d*x]]) + (3*a*(5*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt
[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^2*(a^2 - b*(b + Sqrt[-a^2 +
b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (b*Sqrt[e*Cos[c + d*x]])/(2*(a^2 - b^2)*d*
e*(a + b*Sin[c + d*x])^2) + (7*a*b*Sqrt[e*Cos[c + d*x]])/(4*(a^2 - b^2)^2*d
*e*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.22608, antiderivative size = 520, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2694, 2864, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{7ab\sqrt{e\cos(c+dx)}}{4de\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\sqrt{e\cos(c+dx)}}{2de\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{3\sqrt{b}\left(5a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8d\sqrt{e}\left(b^2-a^2\right)^{11/4}}-\frac{3\sqrt{b}\left(5a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (-3*Sqrt[b]*(5*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(11/4)*d*Sqrt[e]) - (3*Sqrt[b]*(5*a^2
 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(8*(-a^2 + b^2)^(11/4)*d*Sqrt[e]) - (7*a*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2])/(4*(a^2 - b^2)^2*d*Sqrt[e*Cos[c + d*x]]) + (3*a*(5*a^2 + 2
*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)
/2, 2])/(8*(a^2 - b^2)^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c +
```

```
d*x]]) + (3*a*(5*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt
[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^2*(a^2 - b*(b + Sqrt[-a^2 +
b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (b*Sqrt[e*Cos[c + d*x]])/(2*(a^2 - b^2)*d*
e*(a + b*Sin[c + d*x])^2) + (7*a*b*Sqrt[e*Cos[c + d*x]])/(4*(a^2 - b^2)^2*d
*e*(a + b*Sin[c + d*x]))
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3}\,dx = \frac{b\sqrt{e\cos(c+dx)}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} - \frac{\int \frac{-2a+\frac{3}{2}b\sin(c+dx)}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{7ab\sqrt{e\cos(c+dx)}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))} + \cdots$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{7ab\sqrt{e\cos(c+dx)}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))} - \frac{(7a)\cdots}{\cdots}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{7ab\sqrt{e\cos(c+dx)}}{4\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))} - \frac{\left(3a\cdots\right.}{\cdots}$$

$$= -\frac{7a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^2 d\sqrt{e\cos(c+dx)}} + \frac{b\sqrt{e\cos(c+dx)}}{2\left(a^2-b^2\right)de(a+b\sin(c+dx))^2} + \frac{\cdots}{4\left(\cdots\right.}$$

$$= -\frac{7a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^2 d\sqrt{e\cos(c+dx)}} + \frac{3a\left(5a^2+2b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+\cdots}}\cdots\right.}{8\left(-a^2+b^2\right)^{5/2}\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{\cdots}}$$

$$= -\frac{3\sqrt{b}\left(5a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{11/4}d\sqrt{e}} - \frac{3\sqrt{b}\left(5a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos\cdots}}{\sqrt[4]{-a^2+\cdots}}\right.}{8\left(-a^2+b^2\right)^{11/4}d\sqrt{e}}$$

Mathematica [C] time = 24.9486, size = 1226, normalized size = 2.36

$$\frac{\cos(c+dx)\left(\frac{7ab}{4\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b}{2\left(a^2-b^2\right)(a+b\sin(c+dx))^2}\right)}{d\sqrt{e\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\left(\frac{14ab\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(\frac{\cdots}{\left(2\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(c+dx),\frac{b^2\cdots}{\cdots}\right.\right.\right.}\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (Cos[c + d*x]*(b/(2*(a^2 - b^2)*(a + b*Sin[c + d*x])^2) + (7*a*b)/(4*(a^2 -
 b^2)^2*(a + b*Sin[c + d*x]))))/(d*Sqrt[e*Cos[c + d*x]]) + (Sqrt[Cos[c + d*
x]]*((-2*(8*a^2 + 6*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)
*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1
/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*
b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
 b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c
 + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))
) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])
/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-
a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 +
 b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x]))
+ (14*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4,
-1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c
 + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1,
5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1
[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a
^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(
-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*A
rcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTa
n[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^
2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c +
 d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c
 + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c +
 d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(8*(a - b)^2*(a + b)
^2*d*Sqrt[e*Cos[c + d*x]])
```

Maple [C] time = 24.928, size = 25322, normalized size = 48.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e \cos(dx+c)}(b \sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^3), x)
```

3.602 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=596

$$\frac{5b^{3/2}\left(7a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{3/2}\left(b^2-a^2\right)^{13/4}}-\frac{5b^{3/2}\left(7a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{3/2}\left(b^2-a^2\right)^{13/4}}-\frac{a\left(8a^2+37b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e}}{4de^2\left(a^2-b^2\right)^3\sqrt{\cos(c+}}$$

```
[Out] (5*b^(3/2)*(7*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(13/4)*d*e^(3/2)) - (5*b^(3/2)*(7*a^2
+ 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*(-a^2 + b^2)^(13/4)*d*e^(3/2)) - (a*(8*a^2 + 37*b^2)*Sqrt[e*Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2])/(4*(a^2 - b^2)^3*d*e^2*Sqrt[Cos[c + d*x]])
 - (5*a*b*(7*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^
2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*e*Sqr
t[e*Cos[c + d*x]]) - (5*a*b*(7*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(
2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^3*(b + Sqrt[-a
^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) + b/(2*(a^2 - b^2)*d*e*Sqrt[e*Cos[c +
d*x]]*(a + b*Sin[c + d*x])^2) + (9*a*b)/(4*(a^2 - b^2)^2*d*e*Sqrt[e*Cos[c +
 d*x]]*(a + b*Sin[c + d*x])) - (5*b*(7*a^2 + 2*b^2) - a*(8*a^2 + 37*b^2)*Si
n[c + d*x])/(4*(a^2 - b^2)^3*d*e*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 1.59322, antiderivative size = 596, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {2694, 2864, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{5b^{3/2}\left(7a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{3/2}\left(b^2-a^2\right)^{13/4}}-\frac{5b^{3/2}\left(7a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{3/2}\left(b^2-a^2\right)^{13/4}}-\frac{a\left(8a^2+37b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e}}{4de^2\left(a^2-b^2\right)^3\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3),x]
```


```
[Out] (5*b^(3/2)*(7*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(13/4)*d*e^(3/2)) - (5*b^(3/2)*(7*a^2
+ 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*(-a^2 + b^2)^(13/4)*d*e^(3/2)) - (a*(8*a^2 + 37*b^2)*Sqrt[e*Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2])/(4*(a^2 - b^2)^3*d*e^2*Sqrt[Cos[c + d*x]])
```

```
 - (5*a*b*(7*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^
2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*e*Sqr
t[e*Cos[c + d*x]]) - (5*a*b*(7*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(
2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^3*(b + Sqrt[-a
^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) + b/(2*(a^2 - b^2)*d*e*Sqrt[e*Cos[c +
d*x]]*(a + b*Sin[c + d*x])^2) + (9*a*b)/(4*(a^2 - b^2)^2*d*e*Sqrt[e*Cos[c +
 d*x]]*(a + b*Sin[c + d*x])) - (5*b*(7*a^2 + 2*b^2) - a*(8*a^2 + 37*b^2)*Si
n[c + d*x])/(4*(a^2 - b^2)^3*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
```

```
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3}\,dx = \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2} - \frac{\int \frac{-2a+\frac{5}{2}b\sin(c+dx)}{(e\cos(c+dx))^{3/2}(a+b\sin(c+}}{2\left(a^2-b^2\right)}$$

$$= \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2} + \frac{9}{4\left(a^2-b^2\right)^2 de\sqrt{e\cos(c}}$$

$$= \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2} + \frac{9}{4\left(a^2-b^2\right)^2 de\sqrt{e\cos(c}}$$

$$= \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2} + \frac{9}{4\left(a^2-b^2\right)^2 de\sqrt{e\cos(c}}$$

$$= \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^2} + \frac{9}{4\left(a^2-b^2\right)^2 de\sqrt{e\cos(c}}$$

$$= -\frac{a\left(8a^2+37b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^3 de^2\sqrt{\cos(c+dx)}} + \frac{}{2\left(a^2-b^2\right)de\sqrt{e\cos(c}}$$

$$= -\frac{a\left(8a^2+37b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^3 de^2\sqrt{\cos(c+dx)}} - \frac{5ab\left(7a^2+2b^2\right)\sqrt{\cos(c}}{8\left(a^2-b^2\right)^3\left(b-\right.}$$

$$= \frac{5b^{3/2}\left(7a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}} - \frac{5b^{3/2}\left(7a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c}}{\sqrt[4]{-a}}\right.}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}}$$

Mathematica [C] time = 6.82397, size = 922, normalized size = 1.55

$$\frac{\cos^2(c+dx)\left(-\frac{13a\cos(c+dx)b^3}{4\left(a^2-b^2\right)^3(a+b\sin(c+dx))} - \frac{\cos(c+dx)b^3}{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2} + \frac{2\sec(c+dx)\left(\sin(c+dx)a^3-3ba^2+3b^2\sin(c+dx)a-b^3\right)}{\left(a^2-b^2\right)^3}\right)}{d(e\cos(c+dx))^{3/2}} - \frac{\cos^{\frac{3}{2}}(c+dx)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] -(Cos[c + d*x]^(3/2)*((-2*(8*a^4 + 72*a^2*b^2 + 10*b^4)*(a + b*Sqrt[1 - Cos
[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d
*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*
ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*Arc
Tan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt
[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*
Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*S
qrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c
 + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((8*a^3*b + 37*a
*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/
4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) +
3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d
*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]]
)/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sq
rt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]
^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(
8*(a - b)^3*(a + b)^3*d*(e*Cos[c + d*x])^(3/2)) + (Cos[c + d*x]^2*(-(b^3*Co
s[c + d*x])/(2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) - (13*a*b^3*Cos[c + d*
x])/(4*(a^2 - b^2)^3*(a + b*Sin[c + d*x])) + (2*Sec[c + d*x]*(-3*a^2*b - b^
3 + a^3*Sin[c + d*x] + 3*a*b^2*Sin[c + d*x]))/(a^2 - b^2)^3))/(d*(e*Cos[c +
 d*x])^(3/2))
```

Maple [C] time = 46.141, size = 46134, normalized size = 77.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^3), x)
```

3.603 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=614

$$-\frac{7b^{5/2}\left(9a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{5/2}\left(b^2-a^2\right)^{15/4}}-\frac{7b^{5/2}\left(9a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{5/2}\left(b^2-a^2\right)^{15/4}}+\frac{a\left(8a^2+69b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{12de^2\left(a^2-b^2\right)^3\sqrt{e\cos(c+}}$$

```
[Out] (-7*b^(5/2)*(9*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(15/4)*d*e^(5/2)) - (7*b^(5/2)*(9*a^2
 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(8*(-a^2 + b^2)^(15/4)*d*e^(5/2)) + (a*(8*a^2 + 69*b^2)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2])/(12*(a^2 - b^2)^3*d*e^2*Sqrt[e*Cos[c + d*x]
]) - (7*a*b^2*(9*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt
[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^3*(a^2 - b*(b - Sqrt[-a^2 +
b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) - (7*a*b^2*(9*a^2 + 2*b^2)*Sqrt[Cos[c +
d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b
^2)^3*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) + b/(2*(
a^2 - b^2)*d*e*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2) + (11*a*b)/(4
*(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])) - (7*b*(9*a
^2 + 2*b^2) - a*(8*a^2 + 69*b^2)*Sin[c + d*x])/(12*(a^2 - b^2)^3*d*e*(e*Cos
[c + d*x])^(3/2))
```

Rubi [A] time = 1.72145, antiderivative size = 614, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {2694, 2864, 2866, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{7b^{5/2}\left(9a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{5/2}\left(b^2-a^2\right)^{15/4}}-\frac{7b^{5/2}\left(9a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{5/2}\left(b^2-a^2\right)^{15/4}}+\frac{a\left(8a^2+69b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{12de^2\left(a^2-b^2\right)^3\sqrt{e\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^3),x]
```


```
[Out] (-7*b^(5/2)*(9*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(15/4)*d*e^(5/2)) - (7*b^(5/2)*(9*a^2
 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(8*(-a^2 + b^2)^(15/4)*d*e^(5/2)) + (a*(8*a^2 + 69*b^2)*Sqrt[Cos[c + d
```

```
*x]]*EllipticF[(c + d*x)/2, 2])/(12*(a^2 - b^2)^3*d*e^2*Sqrt[e*Cos[c + d*x]
]) - (7*a*b^2*(9*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt
[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^3*(a^2 - b*(b - Sqrt[-a^2 +
b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) - (7*a*b^2*(9*a^2 + 2*b^2)*Sqrt[Cos[c +
d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b
^2)^3*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) + b/(2*(
a^2 - b^2)*d*e*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2) + (11*a*b)/(4
*(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])) - (7*b*(9*a
^2 + 2*b^2) - a*(8*a^2 + 69*b^2)*Sin[c + d*x])/(12*(a^2 - b^2)^3*d*e*(e*Cos
[c + d*x])^(3/2))
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
```

```
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3}\,dx = \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} - \frac{\int \frac{-2a+\frac{7}{2}b\sin(c+dx)}{(e\cos(c+dx))^{5/2}(a+b\sin(c}}{2\left(a^2-b^2\right)}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2de(e\cos(c}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2de(e\cos(c}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2de(e\cos(c}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2de(e\cos(c}$$

$$= \frac{a\left(8a^2+69b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12\left(a^2-b^2\right)^3de^2\sqrt{e\cos(c+dx)}} + \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))}$$

$$= \frac{a\left(8a^2+69b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12\left(a^2-b^2\right)^3de^2\sqrt{e\cos(c+dx)}} + \frac{7ab^2\left(9a^2+2b^2\right)\sqrt{\cos(c+}}{8\left(-a^2+b^2\right)^{7/2}\left(b-\sqrt{-}\right.}$$

$$= -\frac{7b^{5/2}\left(9a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}} - \frac{7b^{5/2}\left(9a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\,c}}{\sqrt[4]{-a}}\right.}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}}$$

Mathematica [C] time = 24.0007, size = 1308, normalized size = 2.13

$$\frac{\left(-\frac{15ab^3}{4\left(a^2-b^2\right)^3(a+b\sin(c+dx))} - \frac{b^3}{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2} + \frac{2\sec^2(c+dx)\left(\sin(c+dx)a^3-3ba^2+3b^2\sin(c+dx)a-b^3\right)}{3\left(a^2-b^2\right)^3}\right)\cos^3(c+dx)}{d(e\cos(c+dx))^{5/2}} + \frac{\left(-\frac{2\left(8ba^3+69b^3a\right)}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*((-2*(8*a^4 - 120*a^2*b^2 - 42*b^4)*(a + b*Sqrt[1 - Cos
[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2,
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*
x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c
+ d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^
2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1,
9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a
^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)
*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] -
Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]
] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c +
 d*x]^2]*(a + b*Sin[c + d*x])) - (2*(8*a^3*b + 69*a*b^3)*(a + b*Sqrt[1 - Co
s[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*
x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c +
d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2,
 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2
)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*S
qrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[
Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(
a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2]
 + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))
/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^
2)*(a + b*Sin[c + d*x]))))/(24*(a - b)^3*(a + b)^3*d*(e*Cos[c + d*x])^(5/2)
) + (Cos[c + d*x]^3*(-b^3/(2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) - (15*a*
b^3)/(4*(a^2 - b^2)^3*(a + b*Sin[c + d*x])) + (2*Sec[c + d*x]^2*(-3*a^2*b -
 b^3 + a^3*Sin[c + d*x] + 3*a*b^2*Sin[c + d*x]))/(3*(a^2 - b^2)^3)))/(d*(e*
Cos[c + d*x])^(5/2))
```

Maple [C] time = 62.991, size = 32645, normalized size = 53.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(b\sin\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^3), x)
```

3.604 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=685

$$\frac{3\left(a\left(-64a^2b^2+8a^4-139b^4\right)\sin(c+dx)+15b^3\left(11a^2+2b^2\right)\right)}{20de^3\left(a^2-b^2\right)^4\sqrt{e\cos(c+dx)}}+\frac{9b^{7/2}\left(11a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{7/2}\left(b^2-a^2\right)^{17/4}}-\frac{9b^{7/2}\left(11a^2+\right.}{8}$$

```
[Out] (9*b^(7/2)*(11*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(17/4)*d*e^(7/2)) - (9*b^(7/2)*(11*a^
2 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
e])])/(8*(-a^2 + b^2)^(17/4)*d*e^(7/2)) - (3*a*(8*a^4 - 64*a^2*b^2 - 139*b^
4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(20*(a^2 - b^2)^4*d*e^4*
Sqrt[Cos[c + d*x]]) + (9*a*b^3*(11*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
Pi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^4*(b - Sqr
t[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + (9*a*b^3*(11*a^2 + 2*b^2)*Sqrt
[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8
*(a^2 - b^2)^4*(b + Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + b/(2*(a
^2 - b^2)*d*e*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2) + (13*a*b)/(4*
(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])) - (9*b*(11*a
^2 + 2*b^2) - a*(8*a^2 + 109*b^2)*Sin[c + d*x])/(20*(a^2 - b^2)^3*d*e*(e*Co
s[c + d*x])^(5/2)) + (3*(15*b^3*(11*a^2 + 2*b^2) + a*(8*a^4 - 64*a^2*b^2 -
139*b^4)*Sin[c + d*x]))/(20*(a^2 - b^2)^4*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 2.02813, antiderivative size = 685, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {2694, 2864, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{3\left(a\left(-64a^2b^2+8a^4-139b^4\right)\sin(c+dx)+15b^3\left(11a^2+2b^2\right)\right)}{20de^3\left(a^2-b^2\right)^4\sqrt{e\cos(c+dx)}}+\frac{9b^{7/2}\left(11a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8de^{7/2}\left(b^2-a^2\right)^{17/4}}-\frac{9b^{7/2}\left(11a^2+\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (9*b^(7/2)*(11*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(17/4)*d*e^(7/2)) - (9*b^(7/2)*(11*a^
2 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
```

```
e])])/(8*(-a^2 + b^2)^(17/4)*d*e^(7/2)) - (3*a*(8*a^4 - 64*a^2*b^2 - 139*b^
4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(20*(a^2 - b^2)^4*d*e^4*
Sqrt[Cos[c + d*x]]) + (9*a*b^3*(11*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
Pi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^4*(b - Sqr
t[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + (9*a*b^3*(11*a^2 + 2*b^2)*Sqrt
[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8
*(a^2 - b^2)^4*(b + Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + b/(2*(a
^2 - b^2)*d*e*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2) + (13*a*b)/(4*
(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])) - (9*b*(11*a
^2 + 2*b^2) - a*(8*a^2 + 109*b^2)*Sin[c + d*x])/(20*(a^2 - b^2)^3*d*e*(e*Co
s[c + d*x])^(5/2)) + (3*(15*b^3*(11*a^2 + 2*b^2) + a*(8*a^4 - 64*a^2*b^2 -
139*b^4)*Sin[c + d*x]))/(20*(a^2 - b^2)^4*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3}\,dx = \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} - \frac{\int \frac{-2a+\frac{9}{2}b\sin(c+dx)}{(e\cos(c+dx))^{7/2}(a+b\sin(c}}{2\left(a^2-b^2\right)}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2 de(e\cos(c}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2 de(e\cos(c}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2 de(e\cos(c}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2 de(e\cos(c}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right)^2 de(e\cos(c}$$

$$= -\frac{3a\left(8a^4-64a^2b^2-139b^4\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{20\left(a^2-b^2\right)^4 de^4\sqrt{\cos(c+dx)}} + \frac{}{2\left(a^2-b^2\right)de}$$

$$= -\frac{3a\left(8a^4-64a^2b^2-139b^4\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{20\left(a^2-b^2\right)^4 de^4\sqrt{\cos(c+dx)}} + \frac{9ab^3\left(11a^2+\right.}{8\left(a^2-\right.}$$

$$= \frac{9b^{7/2}\left(11a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{17/4}de^{7/2}} - \frac{9b^{7/2}\left(11a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e}}{\sqrt[4]{-a}}\right.}{8\left(-a^2+b^2\right)^{17/4}de^{7/2}}$$

Mathematica [C] time = 6.94991, size = 1014, normalized size = 1.48

$$\frac{\cos^4(c+dx)\left(\frac{21a\cos(c+dx)b^5}{4\left(a^2-b^2\right)^4(a+b\sin(c+dx))} + \frac{\cos(c+dx)b^5}{2\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2} + \frac{2\sec^3(c+dx)\left(\sin(c+dx)a^3-3ba^2+3b^2\sin(c+dx)a-b^3\right)}{5\left(a^2-b^2\right)^3} + \frac{2\sec(c+dx)\left(3\sin(c+d\right.}{}\right.}{d(e\cos(c+dx))^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (-3*Cos[c + d*x]^(7/2)*((-2*(8*a^6 - 64*a^4*b^2 - 304*a^2*b^4 - 30*b^6)*(a
+ b*Sqrt[1 - Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) +
((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2
)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(
1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^
2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^
2)^(1/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) -
((8*a^5*b - 64*a^3*b^3 - 139*a*b^5)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(
5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (
Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt
[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Lo
g[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x
]^2)*(a + b*Sin[c + d*x]))))/(40*(a - b)^4*(a + b)^4*d*(e*Cos[c + d*x])^(7/
2)) + (Cos[c + d*x]^4*((b^5*Cos[c + d*x])/(2*(a^2 - b^2)^3*(a + b*Sin[c + d
*x])^2) + (21*a*b^5*Cos[c + d*x])/(4*(a^2 - b^2)^4*(a + b*Sin[c + d*x])) +
(2*Sec[c + d*x]^3*(-3*a^2*b - b^3 + a^3*Sin[c + d*x] + 3*a*b^2*Sin[c + d*x]
))/(5*(a^2 - b^2)^3) + (2*Sec[c + d*x]*(50*a^2*b^3 + 10*b^5 + 3*a^5*Sin[c +
 d*x] - 24*a^3*b^2*Sin[c + d*x] - 39*a*b^4*Sin[c + d*x]))/(5*(a^2 - b^2)^4)
))/(d*(e*Cos[c + d*x])^(7/2))
```

Maple [C] time = 90.928, size = 49016, normalized size = 71.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(b*sin(d*x + c) + a)^3), x)
```

3.605 $\int \frac{(e\cos(c+dx))^{15/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=671

$$\frac{13e^7\sqrt{e\cos(c+dx)}\left(21a\left(11a^2-6b^2\right)-b\left(77a^2-20b^2\right)\sin(c+dx)\right)}{56b^7d}-\frac{39e^5(e\cos(c+dx))^{5/2}\left(77a^2+22ab\sin(c+dx)-2\right.}{280b^5d(a+b\sin(c+dx))}$$

```
[Out] (39*a*(11*a^4 - 17*a^2*b^2 + 6*b^4)*e^(15/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c +
 d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(16*b^(15/2)*(-a^2 + b^2)^(3/4)*d) +
 (39*a*(11*a^4 - 17*a^2*b^2 + 6*b^4)*e^(15/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
 + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(16*b^(15/2)*(-a^2 + b^2)^(3/4)*d)
 + (13*(231*a^4 - 203*a^2*b^2 + 20*b^4)*e^8*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2])/(56*b^8*d*Sqrt[e*Cos[c + d*x]]) - (39*a^2*(11*a^4 - 17*a^2*b
^2 + 6*b^4)*e^8*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
 (c + d*x)/2, 2])/(16*b^8*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c +
 d*x]]) - (39*a^2*(11*a^4 - 17*a^2*b^2 + 6*b^4)*e^8*Sqrt[Cos[c + d*x]]*Elli
pticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^8*(a^2 - b*(b +
 Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(13/2))/(
3*b*d*(a + b*Sin[c + d*x])^3) - (13*e^3*(e*Cos[c + d*x])^(9/2)*(11*a + 4*b*
Sin[c + d*x]))/(84*b^3*d*(a + b*Sin[c + d*x])^2) - (39*e^5*(e*Cos[c + d*x])
^(5/2)*(77*a^2 - 20*b^2 + 22*a*b*Sin[c + d*x]))/(280*b^5*d*(a + b*Sin[c + d
*x])) + (13*e^7*Sqrt[e*Cos[c + d*x]]*(21*a*(11*a^2 - 6*b^2) - b*(77*a^2 - 2
0*b^2)*Sin[c + d*x]))/(56*b^7*d)
```

Rubi [A] time = 1.83161, antiderivative size = 671, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {2693, 2863, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{13e^7\sqrt{e\cos(c+dx)}\left(21a\left(11a^2-6b^2\right)-b\left(77a^2-20b^2\right)\sin(c+dx)\right)}{56b^7d}-\frac{39e^5(e\cos(c+dx))^{5/2}\left(77a^2+22ab\sin(c+dx)-2\right.}{280b^5d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(15/2)/(a + b*Sin[c + d*x])^4,x]
```



```
[Out] (39*a*(11*a^4 - 17*a^2*b^2 + 6*b^4)*e^(15/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c +
 d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(16*b^(15/2)*(-a^2 + b^2)^(3/4)*d) +
```

```
(39*a*(11*a^4 - 17*a^2*b^2 + 6*b^4)*e^(15/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
 + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(16*b^(15/2)*(-a^2 + b^2)^(3/4)*d)
 + (13*(231*a^4 - 203*a^2*b^2 + 20*b^4)*e^8*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2])/(56*b^8*d*Sqrt[e*Cos[c + d*x]]) - (39*a^2*(11*a^4 - 17*a^2*b
^2 + 6*b^4)*e^8*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
 (c + d*x)/2, 2])/(16*b^8*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c +
 d*x]]) - (39*a^2*(11*a^4 - 17*a^2*b^2 + 6*b^4)*e^8*Sqrt[Cos[c + d*x]]*Elli
pticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^8*(a^2 - b*(b +
 Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(13/2))/(
3*b*d*(a + b*Sin[c + d*x])^3) - (13*e^3*(e*Cos[c + d*x])^(9/2)*(11*a + 4*b*
Sin[c + d*x]))/(84*b^3*d*(a + b*Sin[c + d*x])^2) - (39*e^5*(e*Cos[c + d*x])
^(5/2)*(77*a^2 - 20*b^2 + 22*a*b*Sin[c + d*x]))/(280*b^5*d*(a + b*Sin[c + d
*x])) + (13*e^7*Sqrt[e*Cos[c + d*x]]*(21*a*(11*a^2 - 6*b^2) - b*(77*a^2 - 2
0*b^2)*Sin[c + d*x]))/(56*b^7*d)
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
```

```
0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{15/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{\left(13e^2\right)\int \frac{(e\cos(c+dx))^{11/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} + \frac{\left(39e^4\right)\int \frac{(e\cos(c+dx}{(}}{}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} - \frac{39e^5(e\cos(c+dx)}{28}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} - \frac{39e^5(e\cos(c+dx)}{28}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} - \frac{39e^5(e\cos(c+dx)}{28}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} - \frac{39e^5(e\cos(c+dx)}{28}$$

$$= \frac{13\left(231a^4-203a^2b^2+20b^4\right)e^8\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{56b^8d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - 13$$

$$= \frac{13\left(231a^4-203a^2b^2+20b^4\right)e^8\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{56b^8d\sqrt{e\cos(c+dx)}} - \frac{39a^2\left(11a^4-17a^2b^2+6b^4\right)e}{16b^8\sqrt{-a^2+b^2}\,(b}$$

$$= \frac{39a\left(11a^4-17a^2b^2+6b^4\right)e^{15/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{15/2}\left(-a^2+b^2\right)^{3/4}d} + \frac{39a\left(11a^4-17a^2b^2+6b^4\right)e^{15/2}\tanh}{16b^{15/2}\left(-a^2+b^2\right)^{3/4}}$$

Mathematica [C] time = 27.7086, size = 2102, normalized size = 3.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(15/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(15/2)*((-2*(4410*a^3*b - 3418*a*b^3)*(a + b*Sqrt[1 - Cos
[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2,
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*
```

x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) + ((5600*a^3*b - 3472*a*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2*(3815*a^4 - 6251*a^2*b^2 + 1300*b^4)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(560*b^7*d*Cos[c + d*x]^(15/2)) + ((e*Cos[c + d*x])^(15/2)*Sec[c + d*x]^7*((-4*a*Cos[2*(c + d*x)])/(5*b^5) + ((-280*a^2 + 79*b^2)*Sin[c + d*x])/(42*b^6) - (-a^2 + b^2)^3/(3*b^7*(a + b*Sin[c + d*x])^3) - (37*a*(a^2 - b^2)^2)/(12*b^7*(a + b*Sin[c + d*x])^

```
2) + ((-a^2 + b^2)*(-393*a^2 + 76*b^2))/(24*b^7*(a + b*Sin[c + d*x])) + Sin
[3*(c + d*x)]/(14*b^4)))/d
```

Maple [C] time = 111.302, size = 300244, normalized size = 447.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(15/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(15/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.606 $\int \frac{(e\cos(c+dx))^{13/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=557

$$-\frac{77e^5(e\cos(c+dx))^{3/2}\left(15a^2+6ab\sin(c+dx)-4b^2\right)}{120b^5d(a+b\sin(c+dx))}+\frac{77ae^{13/2}\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{13/2}d\sqrt[4]{b^2-a^2}}-\frac{77ae^{13/2}\left(3a^2-2b^2\right.}{16b^{13/}}$$

```
[Out] (77*a*(3*a^2 - 2*b^2)*e^(13/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(16*b^(13/2)*(-a^2 + b^2)^(1/4)*d) - (77*a*(3*a^2
- 2*b^2)*e^(13/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4
)*Sqrt[e])])/(16*b^(13/2)*(-a^2 + b^2)^(1/4)*d) - (77*(15*a^2 - 4*b^2)*e^6*
Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(40*b^6*d*Sqrt[Cos[c + d*x]
]) + (77*a^2*(3*a^2 - 2*b^2)*e^7*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - S
qrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^7*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*
Cos[c + d*x]]) + (77*a^2*(3*a^2 - 2*b^2)*e^7*Sqrt[Cos[c + d*x]]*EllipticPi[
(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^7*(b + Sqrt[-a^2 + b^2
])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(11/2))/(3*b*d*(a + b*Sin[
c + d*x])^3) - (11*e^3*(e*Cos[c + d*x])^(7/2)*(9*a + 4*b*Sin[c + d*x]))/(60
*b^3*d*(a + b*Sin[c + d*x])^2) - (77*e^5*(e*Cos[c + d*x])^(3/2)*(15*a^2 - 4
*b^2 + 6*a*b*Sin[c + d*x]))/(120*b^5*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.36407, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2693, 2863, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{77e^5(e\cos(c+dx))^{3/2}\left(15a^2+6ab\sin(c+dx)-4b^2\right)}{120b^5d(a+b\sin(c+dx))}+\frac{77ae^{13/2}\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{13/2}d\sqrt[4]{b^2-a^2}}-\frac{77ae^{13/2}\left(3a^2-2b^2\right.}{16b^{13/}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(13/2)/(a + b*Sin[c + d*x])^4,x]
```



```
[Out] (77*a*(3*a^2 - 2*b^2)*e^(13/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(16*b^(13/2)*(-a^2 + b^2)^(1/4)*d) - (77*a*(3*a^2
- 2*b^2)*e^(13/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4
)*Sqrt[e])])/(16*b^(13/2)*(-a^2 + b^2)^(1/4)*d) - (77*(15*a^2 - 4*b^2)*e^6*
Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(40*b^6*d*Sqrt[Cos[c + d*x]
]) + (77*a^2*(3*a^2 - 2*b^2)*e^7*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - S
```

```
qrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^7*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*
Cos[c + d*x]]) + (77*a^2*(3*a^2 - 2*b^2)*e^7*Sqrt[Cos[c + d*x]]*EllipticPi[
(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^7*(b + Sqrt[-a^2 + b^2
])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(11/2))/(3*b*d*(a + b*Sin[
c + d*x])^3) - (11*e^3*(e*Cos[c + d*x])^(7/2)*(9*a + 4*b*Sin[c + d*x]))/(60
*b^3*d*(a + b*Sin[c + d*x])^2) - (77*e^5*(e*Cos[c + d*x])^(3/2)*(15*a^2 - 4
*b^2 + 6*a*b*Sin[c + d*x]))/(120*b^5*d*(a + b*Sin[c + d*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
```

```
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{13/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{\left(11e^2\right)\int \frac{(e\cos(c+dx))^{9/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+4b\sin(c+dx))}{60b^3d(a+b\sin(c+dx))^2} + \frac{\left(77e^4\right)\int \frac{(e\cos(c+d\cdots}{(\cdots}}{\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+4b\sin(c+dx))}{60b^3d(a+b\sin(c+dx))^2} - \frac{77e^5(e\cos(c+dx\cdots}{120\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+4b\sin(c+dx))}{60b^3d(a+b\sin(c+dx))^2} - \frac{77e^5(e\cos(c+dx\cdots}{120\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+4b\sin(c+dx))}{60b^3d(a+b\sin(c+dx))^2} - \frac{77e^5(e\cos(c+dx\cdots}{120\cdots}$$

$$= -\frac{77\left(15a^2-4b^2\right)e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{40b^6d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3(e\cos(\cdots}{60\cdots}$$

$$= -\frac{77\left(15a^2-4b^2\right)e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{40b^6d\sqrt{\cos(c+dx)}} + \frac{77a^2\left(3a^2-2b^2\right)e^7\sqrt{\cos(c+dx)}\,\Pi\left(\frac{\cdots}{b\cdots}\right.}{16b^7\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cdots}}$$

$$= \frac{77a\left(3a^2-2b^2\right)e^{13/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{13/2}\sqrt[4]{-a^2+b^2}\,d} - \frac{77a\left(3a^2-2b^2\right)e^{13/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{13/2}\sqrt[4]{-a^2+b^2}\,d} - \cdots$$

Mathematica [C] time = 27.0614, size = 937, normalized size = 1.68

$$\frac{(e\cos(c+dx))^{13/2}\sec^6(c+dx)\left(-\frac{8a\cos(c+dx)}{3b^5}+\frac{\sin(2(c+dx))}{5b^4}+\frac{20b^2\cos(c+dx)-71a^2\cos(c+dx)}{8b^5(a+b\sin(c+dx))}+\frac{9\left(a^3\cos(c+dx)-ab^2\cos(c+dx)\right)}{4b^5(a+b\sin(c+dx))^2}+\frac{-\cos(c+\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(13/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-77*(e*Cos[c + d*x])^(13/2)*((-12*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((a
*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 +
I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*
Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1
 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + L
og[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1
- Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((15*a^2 - 4*b^2)*(a + b*Sqrt[1 -
 Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b
^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^
2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/
4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)]
- Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]
] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/
4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2
 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(80*b^5*d*Cos[c + d*x]
^(13/2)) + ((e*Cos[c + d*x])^(13/2)*Sec[c + d*x]^6*((-8*a*Cos[c + d*x])/(3*
b^5) + (-(a^4*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*Cos[c + d*x])/(3
*b^5*(a + b*Sin[c + d*x])^3) + (9*(a^3*Cos[c + d*x] - a*b^2*Cos[c + d*x]))/
(4*b^5*(a + b*Sin[c + d*x])^2) + (-71*a^2*Cos[c + d*x] + 20*b^2*Cos[c + d*x
])/(8*b^5*(a + b*Sin[c + d*x])) + Sin[2*(c + d*x)]/(5*b^4)))/d
```

Maple [C] time = 78.119, size = 180834, normalized size = 324.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(13/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.607 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=571

$$-\frac{5e^5\sqrt{e\cos(c+dx)}\left(21a^2+14ab\sin(c+dx)-4b^2\right)}{8b^5d(a+b\sin(c+dx))}-\frac{15ae^{11/2}\left(7a^2-6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{11/2}d\left(b^2-a^2\right)^{3/4}}-\frac{15ae^{11/2}\left(7a^2-6b^2\right)}{16b^{11/2}d}$$

```
[Out] (-15*a*(7*a^2 - 6*b^2)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[e])])/(16*b^(11/2)*(-a^2 + b^2)^(3/4)*d) - (15*a*(7*a^2
 - 6*b^2)*e^(11/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/
4)*Sqrt[e])])/(16*b^(11/2)*(-a^2 + b^2)^(3/4)*d) - (5*(21*a^2 - 4*b^2)*e^6*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(8*b^6*d*Sqrt[e*Cos[c + d*x]]
) + (15*a^2*(7*a^2 - 6*b^2)*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sq
rt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*
d*Sqrt[e*Cos[c + d*x]]) + (15*a^2*(7*a^2 - 6*b^2)*e^6*Sqrt[Cos[c + d*x]]*El
lipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^6*(a^2 - b*(b
 + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(9/2))/
(3*b*d*(a + b*Sin[c + d*x])^3) - (e^3*(e*Cos[c + d*x])^(5/2)*(7*a + 4*b*Sin
[c + d*x]))/(4*b^3*d*(a + b*Sin[c + d*x])^2) - (5*e^5*Sqrt[e*Cos[c + d*x]]*
(21*a^2 - 4*b^2 + 14*a*b*Sin[c + d*x]))/(8*b^5*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.37786, antiderivative size = 571, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2693, 2863, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{5e^5\sqrt{e\cos(c+dx)}\left(21a^2+14ab\sin(c+dx)-4b^2\right)}{8b^5d(a+b\sin(c+dx))}-\frac{15ae^{11/2}\left(7a^2-6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{11/2}d\left(b^2-a^2\right)^{3/4}}-\frac{15ae^{11/2}\left(7a^2-6b^2\right)}{16b^{11/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^4,x]
```


```
[Out] (-15*a*(7*a^2 - 6*b^2)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[e])])/(16*b^(11/2)*(-a^2 + b^2)^(3/4)*d) - (15*a*(7*a^2
 - 6*b^2)*e^(11/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/
4)*Sqrt[e])])/(16*b^(11/2)*(-a^2 + b^2)^(3/4)*d) - (5*(21*a^2 - 4*b^2)*e^6*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(8*b^6*d*Sqrt[e*Cos[c + d*x]]
) + (15*a^2*(7*a^2 - 6*b^2)*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sq
```

```
rt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*
d*Sqrt[e*Cos[c + d*x]]) + (15*a^2*(7*a^2 - 6*b^2)*e^6*Sqrt[Cos[c + d*x]]*El
lipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^6*(a^2 - b*(b
 + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(9/2))/
(3*b*d*(a + b*Sin[c + d*x])^3) - (e^3*(e*Cos[c + d*x])^(5/2)*(7*a + 4*b*Sin
[c + d*x]))/(4*b^3*d*(a + b*Sin[c + d*x])^2) - (5*e^5*Sqrt[e*Cos[c + d*x]]*
(21*a^2 - 4*b^2 + 14*a*b*Sin[c + d*x]))/(8*b^5*d*(a + b*Sin[c + d*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
```

```
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{\left(3e^2\right)\int \frac{(e\cos(c+dx))^{7/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{2b}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e\cos(c+dx))^{5/2}(7a+4b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))^2} + \frac{\left(5e^4\right)\int \frac{(e\cos(c+dx))^{3/2}(\cdots}{(a+b\sin\cdots}}{4b^3\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e\cos(c+dx))^{5/2}(7a+4b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))^2} - \frac{5e^5\sqrt{e\cos(c+dx)}\left(21\cdots\right.}{8b^5d(a\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e\cos(c+dx))^{5/2}(7a+4b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))^2} - \frac{5e^5\sqrt{e\cos(c+dx)}\left(21\cdots\right.}{8b^5d(a\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e\cos(c+dx))^{5/2}(7a+4b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))^2} - \frac{5e^5\sqrt{e\cos(c+dx)}\left(21\cdots\right.}{8b^5d(a\cdots}$$

$$= -\frac{5\left(21a^2-4b^2\right)e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^6d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e\cos(c+dx\cdots}{4b^3d(a\cdots}$$

$$= -\frac{5\left(21a^2-4b^2\right)e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^6d\sqrt{e\cos(c+dx)}} + \frac{15a^2\left(7a^2-6b^2\right)e^6\sqrt{\cos(c+dx)}\Pi\left(\frac{2b\cdots}{b-\sqrt{-a\cdots}}\right.}{16b^6\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\cdots}}$$

$$= -\frac{15a\left(7a^2-6b^2\right)e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{11/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{15a\left(7a^2-6b^2\right)e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{11/2}\left(-a^2+b^2\right)^{3/4}d} - \cdots$$

Mathematica [C] time = 27.1574, size = 2020, normalized size = 3.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^4,x]

[Out] ((e*Cos[c + d*x])^(11/2)*Sec[c + d*x]^5*((2*Sin[c + d*x])/(3*b^4) - (-a^2 +
 b^2)^2/(3*b^5*(a + b*Sin[c + d*x])^3) + (25*a*(a^2 - b^2))/(12*b^5*(a + b*
Sin[c + d*x])^2) + (-165*a^2 + 52*b^2)/(24*b^5*(a + b*Sin[c + d*x]))))/d -
((e*Cos[c + d*x])^(11/2)*((-76*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2
)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^
2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2,
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c
+ d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[
c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c +
 d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
+ b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2]
+ (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]
))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c
 + d*x])) + (32*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/
2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a
^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2
)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(
3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/
2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^
(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c
 + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 -
 Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2,
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Co
s[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/
4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c +
 d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*L
og[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 +
 b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c
+ d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(S
qrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2*
(41*a^2 - 20*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*Appell
F1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sq
rt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -
1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*
AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d
*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) +
(a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] +
```

```
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log
[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b
*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))
*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(16*b^5*d*Co
s[c + d*x]^(11/2))
```

Maple [C] time = 91.461, size = 144252, normalized size = 252.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.608 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=591

$$\frac{7e^3\left(5a^2-4b^2\right)(e\cos(c+dx))^{3/2}}{8b^3d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{7ae^{9/2}\left(5a^2-6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{9/2}d\left(b^2-a^2\right)^{5/4}}-\frac{7ae^{9/2}\left(5a^2-6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{9/2}d\left(b^2-a^2\right)^{5/4}}$$

```
[Out] (7*a*(5*a^2 - 6*b^2)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
 b^2)^(1/4)*Sqrt[e])])/(16*b^(9/2)*(-a^2 + b^2)^(5/4)*d) - (7*a*(5*a^2 - 6*
b^2)*e^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(16*b^(9/2)*(-a^2 + b^2)^(5/4)*d) + (7*(5*a^2 - 4*b^2)*e^4*Sqrt[e*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(8*b^4*(a^2 - b^2)*d*Sqrt[Cos[c + d
*x]]) - (7*a^2*(5*a^2 - 6*b^2)*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b -
 Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^5*(a^2 - b^2)*(b - Sqrt[-a^2 + b
^2])*d*Sqrt[e*Cos[c + d*x]]) - (7*a^2*(5*a^2 - 6*b^2)*e^5*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^5*(a^2 -
b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(
7/2))/(3*b*d*(a + b*Sin[c + d*x])^3) + (7*(5*a^2 - 4*b^2)*e^3*(e*Cos[c + d*
x])^(3/2))/(8*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (7*e^3*(e*Cos[c + d
*x])^(3/2)*(5*a + 4*b*Sin[c + d*x]))/(12*b^3*d*(a + b*Sin[c + d*x])^2)
```

Rubi [A] time = 1.43009, antiderivative size = 591, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {2693, 2863, 2864, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{7e^3\left(5a^2-4b^2\right)(e\cos(c+dx))^{3/2}}{8b^3d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{7ae^{9/2}\left(5a^2-6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{9/2}d\left(b^2-a^2\right)^{5/4}}-\frac{7ae^{9/2}\left(5a^2-6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{9/2}d\left(b^2-a^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^4,x]
```


```
[Out] (7*a*(5*a^2 - 6*b^2)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
 b^2)^(1/4)*Sqrt[e])])/(16*b^(9/2)*(-a^2 + b^2)^(5/4)*d) - (7*a*(5*a^2 - 6*
b^2)*e^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(16*b^(9/2)*(-a^2 + b^2)^(5/4)*d) + (7*(5*a^2 - 4*b^2)*e^4*Sqrt[e*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(8*b^4*(a^2 - b^2)*d*Sqrt[Cos[c + d
```

```
*x]]) - (7*a^2*(5*a^2 - 6*b^2)*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b -
 Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^5*(a^2 - b^2)*(b - Sqrt[-a^2 + b
^2])*d*Sqrt[e*Cos[c + d*x]]) - (7*a^2*(5*a^2 - 6*b^2)*e^5*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^5*(a^2 -
b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(
7/2))/(3*b*d*(a + b*Sin[c + d*x])^3) + (7*(5*a^2 - 4*b^2)*e^3*(e*Cos[c + d*
x])^(3/2))/(8*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (7*e^3*(e*Cos[c + d
*x])^(3/2)*(5*a + 4*b*Sin[c + d*x]))/(12*b^3*d*(a + b*Sin[c + d*x])^2)
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
```

```
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} - \frac{(7e^2)\int \frac{(e\cos(c+dx))^{5/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+4b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))^2} + \frac{(7e^4)\int \frac{\sqrt{e\cos(c+dx)}(-}{(a+b\sin}}{4b^3}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} + \frac{7(5a^2-4b^2)e^3(e\cos(c+dx))^{3/2}}{8b^3(a^2-b^2)d(a+b\sin(c+dx))} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+4b}{12b^3d(a+b\sin(c+}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} + \frac{7(5a^2-4b^2)e^3(e\cos(c+dx))^{3/2}}{8b^3(a^2-b^2)d(a+b\sin(c+dx))} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+4b}{12b^3d(a+b\sin(c+}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} + \frac{7(5a^2-4b^2)e^3(e\cos(c+dx))^{3/2}}{8b^3(a^2-b^2)d(a+b\sin(c+dx))} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+4b}{12b^3d(a+b\sin(c+}$$

$$= \frac{7(5a^2-4b^2)e^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^4(a^2-b^2)d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} + \frac{7(5a^2-4b^2)e^3(}{8b^3(a^2-b^2)d(a}$$

$$= \frac{7(5a^2-4b^2)e^4\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^4(a^2-b^2)d\sqrt{\cos(c+dx)}} - \frac{7a^2(5a^2-6b^2)e^5\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+}}\right.}{16b^5(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\,c}}$$

$$= \frac{7a(5a^2-6b^2)e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{9/2}(-a^2+b^2)^{5/4}d} - \frac{7a(5a^2-6b^2)e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{9/2}(-a^2+b^2)^{5/4}d} + \frac{7(5}{}$$

Mathematica [C] time = 26.9477, size = 900, normalized size = 1.52

$$\frac{\sec^4(c+dx)\left(-\frac{5a\cos(c+dx)}{4b^3(a+b\sin(c+dx))^2} + \frac{12b^2\cos(c+dx)-19a^2\cos(c+dx)}{8b^3(b^2-a^2)(a+b\sin(c+dx))} + \frac{a^2\cos(c+dx)-b^2\cos(c+dx)}{3b^3(a+b\sin(c+dx))^3}\right)(e\cos(c+dx))^{9/2}}{d} + \frac{7\left(-\frac{(5a^2-4b^2)(a+b}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(9/2)*Sec[c + d*x]^4*((a^2*Cos[c + d*x] - b^2*Cos[c + d*x
])/(3*b^3*(a + b*Sin[c + d*x])^3) - (5*a*Cos[c + d*x])/(4*b^3*(a + b*Sin[c
+ d*x])^2) + (-19*a^2*Cos[c + d*x] + 12*b^2*Cos[c + d*x])/(8*b^3*(-a^2 + b^
2)*(a + b*Sin[c + d*x]))))/d + (7*(e*Cos[c + d*x])^(9/2)*((-4*a*b*(a + b*Sq
rt[1 - Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2
*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8
+ I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4
)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
- Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*
x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^
2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/
4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((5*a^
2 - 4*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2,
1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/
2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[
c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c +
d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^
2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c +
 d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x])
)))/(16*(a - b)*b^3*(a + b)*d*Cos[c + d*x]^(9/2))
```

Maple [C] time = 84.159, size = 237416, normalized size = 401.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.609 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=597

$$-\frac{5e^3\left(3a^2-4b^2\right)\sqrt{e\cos(c+dx)}}{24b^3d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{5ae^{7/2}\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{7/2}d\left(b^2-a^2\right)^{7/4}}-\frac{5ae^{7/2}\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{7/2}d\left(b^2-a^2\right)^{7/4}}$$

```
[Out] (-5*a*(a^2 - 2*b^2)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(16*b^(7/2)*(-a^2 + b^2)^(7/4)*d) - (5*a*(a^2 - 2*b^2
)*e^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(16*b^(7/2)*(-a^2 + b^2)^(7/4)*d) + (5*(3*a^2 - 4*b^2)*e^4*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2])/(24*b^4*(a^2 - b^2)*d*Sqrt[e*Cos[c + d*x
]]) - (5*a^2*(a^2 - 2*b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqr
t[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^4*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^
2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (5*a^2*(a^2 - 2*b^2)*e^4*Sqrt[Cos[c +
d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^4*(a^
2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*C
os[c + d*x])^(5/2))/(3*b*d*(a + b*Sin[c + d*x])^3) - (5*(3*a^2 - 4*b^2)*e^3
*Sqrt[e*Cos[c + d*x]])/(24*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])) + (5*e^3
*Sqrt[e*Cos[c + d*x]]*(3*a + 4*b*Sin[c + d*x]))/(12*b^3*d*(a + b*Sin[c + d*
x])^2)
```

Rubi [A] time = 1.52032, antiderivative size = 597, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {2693, 2863, 2864, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{5e^3\left(3a^2-4b^2\right)\sqrt{e\cos(c+dx)}}{24b^3d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{5ae^{7/2}\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{7/2}d\left(b^2-a^2\right)^{7/4}}-\frac{5ae^{7/2}\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{7/2}d\left(b^2-a^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^4,x]
```


```
[Out] (-5*a*(a^2 - 2*b^2)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(16*b^(7/2)*(-a^2 + b^2)^(7/4)*d) - (5*a*(a^2 - 2*b^2
)*e^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(16*b^(7/2)*(-a^2 + b^2)^(7/4)*d) + (5*(3*a^2 - 4*b^2)*e^4*Sqrt[Cos[c
```

```
+ d*x]]*EllipticF[(c + d*x)/2, 2])/(24*b^4*(a^2 - b^2)*d*Sqrt[e*Cos[c + d*x
]]) - (5*a^2*(a^2 - 2*b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqr
t[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^4*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^
2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (5*a^2*(a^2 - 2*b^2)*e^4*Sqrt[Cos[c +
d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^4*(a^
2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*C
os[c + d*x])^(5/2))/(3*b*d*(a + b*Sin[c + d*x])^3) - (5*(3*a^2 - 4*b^2)*e^3
*Sqrt[e*Cos[c + d*x]])/(24*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])) + (5*e^3
*Sqrt[e*Cos[c + d*x]]*(3*a + 4*b*Sin[c + d*x]))/(12*b^3*d*(a + b*Sin[c + d*
x])^2)
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(
p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
```

```
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{\left(5e^2\right)\int \frac{(e\cos(c+dx))^{3/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} + \frac{5e^3\sqrt{e\cos(c+dx)}(3a+4b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))^2} - \frac{\left(5e^4\right)\int \frac{-2b-\frac{3}{2}a\,\mathrm{si}}{\sqrt{e\cos(c+dx)}(a+}}{12b^3}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{5\left(3a^2-4b^2\right)e^3\sqrt{e\cos(c+dx)}}{24b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{5e^3\sqrt{e\cos(c+dx)}(3a+4}{12b^3d(a+b\sin(c-}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{5\left(3a^2-4b^2\right)e^3\sqrt{e\cos(c+dx)}}{24b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{5e^3\sqrt{e\cos(c+dx)}(3a+4}{12b^3d(a+b\sin(c-}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{5\left(3a^2-4b^2\right)e^3\sqrt{e\cos(c+dx)}}{24b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{5e^3\sqrt{e\cos(c+dx)}(3a+4}{12b^3d(a+b\sin(c-}$$

$$= \frac{5\left(3a^2-4b^2\right)e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24b^4\left(a^2-b^2\right)d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{5\left(3a^2-4b^2\right)e}{24b^3\left(a^2-b^2\right)d(}$$

$$= \frac{5\left(3a^2-4b^2\right)e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24b^4\left(a^2-b^2\right)d\sqrt{e\cos(c+dx)}} + \frac{5a^2\left(a^2-2b^2\right)e^4\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}}\right.}{16b^4\left(-a^2+b^2\right)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e}}$$

$$= -\frac{5a\left(a^2-2b^2\right)e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{7/2}\left(-a^2+b^2\right)^{7/4}d} - \frac{5a\left(a^2-2b^2\right)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{7/2}\left(-a^2+b^2\right)^{7/4}d} + \frac{5(3}{}$$

Mathematica [C] time = 24.97, size = 1263, normalized size = 2.12

$$\frac{\sec^3(c+dx)\left(-\frac{13a}{12b^3(a+b\sin(c+dx))^2} + \frac{28b^2-33a^2}{24b^3\left(b^2-a^2\right)(a+b\sin(c+dx))} + \frac{a^2-b^2}{3b^3(a+b\sin(c+dx))^3}\right)(e\cos(c+dx))^{7/2}}{d} + \frac{5\left(-\frac{2\left(3a^2-4b^2\right)\left(a+b\sqrt{1-\mathrm{co}}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^3*((a^2 - b^2)/(3*b^3*(a + b*Sin[c + d
*x])^3) - (13*a)/(12*b^3*(a + b*Sin[c + d*x])^2) + (-33*a^2 + 28*b^2)/(24*b
^3*(-a^2 + b^2)*(a + b*Sin[c + d*x]))))/d + (5*(e*Cos[c + d*x])^(7/2)*((-4*
a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1
, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]
])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[
c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2
, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)
*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt
[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
- 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + L
og[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^
(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c +
d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (2*(3*a^2 - 4*b^2)*
(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5
/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*S
qrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c
 + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2
, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*
AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2
)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 -
(Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + L
og[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
 b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/(
(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(48*(a - b)*b^3*(a + b)*d*Cos[
c + d*x]^(7/2))
```

Maple [C] time = 96.503, size = 192036, normalized size = 321.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.610 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=574

$$-\frac{ae^{5/2}\left(a^2-6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{5/2}d\left(b^2-a^2\right)^{9/4}}+\frac{ae^{5/2}\left(a^2-6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{5/2}d\left(b^2-a^2\right)^{9/4}}+\frac{e^2\left(a^2+4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(}}{8b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}$$

```
[Out] -(a*(a^2 - 6*b^2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^
2)^(1/4)*Sqrt[e])])/(16*b^(5/2)*(-a^2 + b^2)^(9/4)*d) + (a*(a^2 - 6*b^2)*e^
(5/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])
/(16*b^(5/2)*(-a^2 + b^2)^(9/4)*d) + ((a^2 + 4*b^2)*e^2*Sqrt[e*Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2])/(8*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) - (
a^2*(a^2 - 6*b^2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
b^2]), (c + d*x)/2, 2])/(16*b^3*(a^2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*Sqrt
[e*Cos[c + d*x]]) - (a^2*(a^2 - 6*b^2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2
*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^3*(a^2 - b^2)^2*(b + Sqr
t[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(3/2))/(3*b*d*
(a + b*Sin[c + d*x])^3) + (a*e*(e*Cos[c + d*x])^(3/2))/(4*b*(a^2 - b^2)*d*(
a + b*Sin[c + d*x])^2) + ((a^2 + 4*b^2)*e*(e*Cos[c + d*x])^(3/2))/(8*b*(a^2
 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.44708, antiderivative size = 574, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2693, 2864, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{ae^{5/2}\left(a^2-6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{5/2}d\left(b^2-a^2\right)^{9/4}}+\frac{ae^{5/2}\left(a^2-6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{5/2}d\left(b^2-a^2\right)^{9/4}}+\frac{e^2\left(a^2+4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(}}{8b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] -(a*(a^2 - 6*b^2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^
2)^(1/4)*Sqrt[e])])/(16*b^(5/2)*(-a^2 + b^2)^(9/4)*d) + (a*(a^2 - 6*b^2)*e^
(5/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])
/(16*b^(5/2)*(-a^2 + b^2)^(9/4)*d) + ((a^2 + 4*b^2)*e^2*Sqrt[e*Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2])/(8*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) - (
a^2*(a^2 - 6*b^2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
```

```
b^2]), (c + d*x)/2, 2])/(16*b^3*(a^2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*Sqrt
[e*Cos[c + d*x]]) - (a^2*(a^2 - 6*b^2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2
*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^3*(a^2 - b^2)^2*(b + Sqr
t[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(3/2))/(3*b*d*
(a + b*Sin[c + d*x])^3) + (a*e*(e*Cos[c + d*x])^(3/2))/(4*b*(a^2 - b^2)*d*(
a + b*Sin[c + d*x])^2) + ((a^2 + 4*b^2)*e*(e*Cos[c + d*x])^(3/2))/(8*b*(a^2
 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{2b}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}\left(2b-\frac{1}{2}a\sin(c+\right.}{(a+b\sin(c+dx))^2}}{4b\left(a^2-b^2\right)}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\left(a^2+4b^2\right)e(e\cos(c+dx}{8b\left(a^2-b^2\right)^2 d(a+b\sin(c}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\left(a^2+4b^2\right)e(e\cos(c+dx}{8b\left(a^2-b^2\right)^2 d(a+b\sin(c}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(e\cos(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\left(a^2+4b^2\right)e(e\cos(c+dx}{8b\left(a^2-b^2\right)^2 d(a+b\sin(c}$$

$$= \frac{\left(a^2+4b^2\right)e^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(e\cos(c}{4b\left(a^2-b^2\right)d(a+}$$

$$= \frac{\left(a^2+4b^2\right)e^2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} - \frac{a^2\left(a^2-6b^2\right)e^3\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\right.}{16b^3\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos}}$$

$$= -\frac{a\left(a^2-6b^2\right)e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{5/2}\left(-a^2+b^2\right)^{9/4}d} + \frac{a\left(a^2-6b^2\right)e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{5/2}\left(-a^2+b^2\right)^{9/4}d} + \frac{\left(a^2+\right.}{}$$

Mathematica [C] time = 26.8188, size = 892, normalized size = 1.55

$$\frac{\sec^2(c+dx)\left(-\frac{a\cos(c+dx)}{4b(b^2-a^2)(a+b\sin(c+dx))^2}-\frac{\cos(c+dx)}{3b(a+b\sin(c+dx))^3}+\frac{\cos(c+dx)a^2+4b^2\cos(c+dx)}{8b(b^2-a^2)^2(a+b\sin(c+dx))}\right)(e\cos(c+dx))^{5/2}}{d}+\frac{\left(-\frac{(a^2+4b^2)\left(a+b\sqrt{1-\cos^2(}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*(-Cos[c + d*x]/(3*b*(a + b*Sin[c + d
*x])^3) - (a*Cos[c + d*x])/(4*b*(-a^2 + b^2)*(a + b*Sin[c + d*x])^2) + (a^2
*Cos[c + d*x] + 4*b^2*Cos[c + d*x])/(8*b*(-a^2 + b^2)^2*(a + b*Sin[c + d*x]
))))/d + ((e*Cos[c + d*x])^(5/2)*((-20*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])
*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
+ b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((
1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2]
- (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]
 + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d
*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqr
t[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((a^2 + 4*b^2)*(a + b*Sqrt[1
- Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (
b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b
^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1
/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)]
 - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x
]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^
2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(16*(a - b)^2*b*(a +
b)^2*d*Cos[c + d*x]^(5/2))
```

Maple [C] time = 95.938, size = 179434, normalized size = 312.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(b\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(b*sin(d*x + c) + a)^4, x)
```

3.611 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=592

$$-\frac{ae^{3/2}\left(a^2+6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{3/2}d\left(b^2-a^2\right)^{11/4}}-\frac{ae^{3/2}\left(a^2+6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{3/2}d\left(b^2-a^2\right)^{11/4}}-\frac{e^2\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\cdots\right.}{24b^2d\left(a^2-b^2\right)^2\sqrt{e\cos(c+d\cdots}}$$

```
[Out] -(a*(a^2 + 6*b^2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^
2)^(1/4)*Sqrt[e])])/(16*b^(3/2)*(-a^2 + b^2)^(11/4)*d) - (a*(a^2 + 6*b^2)*e
^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])]
)/(16*b^(3/2)*(-a^2 + b^2)^(11/4)*d) - ((3*a^2 + 4*b^2)*e^2*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2])/(24*b^2*(a^2 - b^2)^2*d*Sqrt[e*Cos[c + d*x]]
) + (a^2*(a^2 + 6*b^2)*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a
^2 + b^2]), (c + d*x)/2, 2])/(16*b^2*(a^2 - b^2)^2*(a^2 - b*(b - Sqrt[-a^2
+ b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (a^2*(a^2 + 6*b^2)*e^2*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^2*(a^2 -
b^2)^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*Sqrt[e
*Cos[c + d*x]])/(3*b*d*(a + b*Sin[c + d*x])^3) + (a*e*Sqrt[e*Cos[c + d*x]])
/(12*b*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 + 4*b^2)*e*Sqrt[e*Co
s[c + d*x]])/(24*b*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.47103, antiderivative size = 592, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2693, 2864, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{ae^{3/2}\left(a^2+6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{3/2}d\left(b^2-a^2\right)^{11/4}}-\frac{ae^{3/2}\left(a^2+6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16b^{3/2}d\left(b^2-a^2\right)^{11/4}}-\frac{e^2\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\cdots\right.}{24b^2d\left(a^2-b^2\right)^2\sqrt{e\cos(c+d\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] -(a*(a^2 + 6*b^2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^
2)^(1/4)*Sqrt[e])])/(16*b^(3/2)*(-a^2 + b^2)^(11/4)*d) - (a*(a^2 + 6*b^2)*e
^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])]
)/(16*b^(3/2)*(-a^2 + b^2)^(11/4)*d) - ((3*a^2 + 4*b^2)*e^2*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2])/(24*b^2*(a^2 - b^2)^2*d*Sqrt[e*Cos[c + d*x]]
) + (a^2*(a^2 + 6*b^2)*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a
```

```
^2 + b^2]), (c + d*x)/2, 2])/(16*b^2*(a^2 - b^2)^2*(a^2 - b*(b - Sqrt[-a^2
+ b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (a^2*(a^2 + 6*b^2)*e^2*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^2*(a^2 -
b^2)^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*Sqrt[e
*Cos[c + d*x]])/(3*b*d*(a + b*Sin[c + d*x])^3) + (a*e*Sqrt[e*Cos[c + d*x]])
/(12*b*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 + 4*b^2)*e*Sqrt[e*Co
s[c + d*x]])/(24*b*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} - \frac{e^2\int \frac{\sin(c+dx)}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{ae\sqrt{e\cos(c+dx)}}{12b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{e^2\int \frac{2b-\frac{3}{2}a\sin(c+dx)}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))}}{12b\left(a^2-b^2\right)}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{ae\sqrt{e\cos(c+dx)}}{12b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\left(3a^2+4b^2\right)e\sqrt{e\cos(c+}}{24b\left(a^2-b^2\right)^2 d(a+b\sin(c}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{ae\sqrt{e\cos(c+dx)}}{12b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\left(3a^2+4b^2\right)e\sqrt{e\cos(c+}}{24b\left(a^2-b^2\right)^2 d(a+b\sin(c}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{ae\sqrt{e\cos(c+dx)}}{12b\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\left(3a^2+4b^2\right)e\sqrt{e\cos(c+}}{24b\left(a^2-b^2\right)^2 d(a+b\sin(c}$$

$$= -\frac{\left(3a^2+4b^2\right)e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24b^2\left(a^2-b^2\right)^2 d\sqrt{e\cos(c+dx)}} - \frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{ae\sqrt{e\cos}}{12b\left(a^2-b^2\right)d(a}$$

$$= -\frac{\left(3a^2+4b^2\right)e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24b^2\left(a^2-b^2\right)^2 d\sqrt{e\cos(c+dx)}} + \frac{a^2\left(a^2+6b^2\right)e^2\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\right.}{16b^2\left(-a^2+b^2\right)^{5/2}\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos}}$$

$$= -\frac{a\left(a^2+6b^2\right)e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{3/2}\left(-a^2+b^2\right)^{11/4}d} - \frac{a\left(a^2+6b^2\right)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{3/2}\left(-a^2+b^2\right)^{11/4}d} - \frac{\left(3a^2+\right.}{24}$$

Mathematica [C] time = 24.58, size = 1263, normalized size = 2.13

$$\frac{(e\cos(c+dx))^{3/2}\sec(c+dx)\left(-\frac{a}{12b(b^2-a^2)(a+b\sin(c+dx))^2}+\frac{3a^2+4b^2}{24b(b^2-a^2)^2(a+b\sin(c+dx))}-\frac{1}{3b(a+b\sin(c+dx))^3}\right)}{d}-\frac{(e\cos(c+dx))^{3/2}\Bigg(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*Sec[c + d*x]*(-1/(3*b*(a + b*Sin[c + d*x])^3) - a/(
12*b*(-a^2 + b^2)*(a + b*Sin[c + d*x])^2) + (3*a^2 + 4*b^2)/(24*b*(-a^2 + b
^2)^2*(a + b*Sin[c + d*x]))))/d - ((e*Cos[c + d*x])^(3/2)*((28*a*b*(a + b*S
qrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c
 + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 -
 Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2,
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Co
s[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/
4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c +
 d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTa
n[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1
 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2
 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c
 + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[C
os[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[
1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (2*(3*a^2 + 4*b^2)*(a + b*Sqrt[
1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c +
d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[
c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (
b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Co
s[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4
, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c +
d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqr
t[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2
- b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d
*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c +
 d*x]^2)*(a + b*Sin[c + d*x]))))/(48*(a - b)^2*b*(a + b)^2*d*Cos[c + d*x]^(
3/2))
```

Maple [C] time = 94.995, size = 138380, normalized size = 233.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(b\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(b*sin(d*x + c) + a)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos{(dx+c)})^{\frac{3}{2}}}{(b\sin{(dx+c)}+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(b*sin(d*x + c) + a)^4, x)
```

3.612 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=579

$$\frac{3ab(e\cos(c+dx))^{3/2}}{4de\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{b\left(11a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{8de\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{b(e\cos(c+dx))^{3/2}}{3de\left(a^2-b^2\right)(a+b\sin(c+dx))^3}-\frac{5a\sqrt{e}\left(a^2+\right.}{16}$$

```
[Out] (-5*a*(a^2 + 2*b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(16*Sqrt[b]*(-a^2 + b^2)^(13/4)*d) + (5*a*(a^2 + 2*b^
2)*Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
e])])/(16*Sqrt[b]*(-a^2 + b^2)^(13/4)*d) + ((11*a^2 + 4*b^2)*Sqrt[e*Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2])/(8*(a^2 - b^2)^3*d*Sqrt[Cos[c + d*x]]) +
(5*a^2*(a^2 + 2*b^2)*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
 b^2]), (c + d*x)/2, 2])/(16*b*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*Sqrt[
e*Cos[c + d*x]]) + (5*a^2*(a^2 + 2*b^2)*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*
b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b*(a^2 - b^2)^3*(b + Sqrt[-
a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (b*(e*Cos[c + d*x])^(3/2))/(3*(a^2 -
b^2)*d*e*(a + b*Sin[c + d*x])^3) + (3*a*b*(e*Cos[c + d*x])^(3/2))/(4*(a^2 -
 b^2)^2*d*e*(a + b*Sin[c + d*x])^2) + (b*(11*a^2 + 4*b^2)*(e*Cos[c + d*x])^
(3/2))/(8*(a^2 - b^2)^3*d*e*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.5317, antiderivative size = 579, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2694, 2864, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{3ab(e\cos(c+dx))^{3/2}}{4de\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{b\left(11a^2+4b^2\right)(e\cos(c+dx))^{3/2}}{8de\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{b(e\cos(c+dx))^{3/2}}{3de\left(a^2-b^2\right)(a+b\sin(c+dx))^3}-\frac{5a\sqrt{e}\left(a^2+\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^4,x]
```


```
[Out] (-5*a*(a^2 + 2*b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(16*Sqrt[b]*(-a^2 + b^2)^(13/4)*d) + (5*a*(a^2 + 2*b^
2)*Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
e])])/(16*Sqrt[b]*(-a^2 + b^2)^(13/4)*d) + ((11*a^2 + 4*b^2)*Sqrt[e*Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2])/(8*(a^2 - b^2)^3*d*Sqrt[Cos[c + d*x]]) +
(5*a^2*(a^2 + 2*b^2)*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
```

```
b^2]), (c + d*x)/2, 2])/(16*b*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*Sqrt[
e*Cos[c + d*x]]) + (5*a^2*(a^2 + 2*b^2)*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*
b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b*(a^2 - b^2)^3*(b + Sqrt[-
a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (b*(e*Cos[c + d*x])^(3/2))/(3*(a^2 -
b^2)*d*e*(a + b*Sin[c + d*x])^3) + (3*a*b*(e*Cos[c + d*x])^(3/2))/(4*(a^2 -
b^2)^2*d*e*(a + b*Sin[c + d*x])^2) + (b*(11*a^2 + 4*b^2)*(e*Cos[c + d*x])^
(3/2))/(8*(a^2 - b^2)^3*d*e*(a + b*Sin[c + d*x]))
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^4}\,dx = \frac{b(e\cos(c+dx))^{3/2}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} - \frac{\int \frac{\sqrt{e\cos(c+dx)}\left(-3a+\frac{3}{2}b\sin(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{3ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2de(a+b\sin(c+dx))^2} + \frac{\int \frac{\sqrt{e\cos(c+dx)}\left(3\left(2a^2\right.\right.}{(a+b\sin}}{6\left(a^2\right.}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{3ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2de(a+b\sin(c+dx))^2} + \frac{b\left(11a^2+4b^2\right)(e}{8\left(a^2-b^2\right)^3de(a\ }$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{3ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2de(a+b\sin(c+dx))^2} + \frac{b\left(11a^2+4b^2\right)(e}{8\left(a^2-b^2\right)^3de(a\ }$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{3ab(e\cos(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2de(a+b\sin(c+dx))^2} + \frac{b\left(11a^2+4b^2\right)(e}{8\left(a^2-b^2\right)^3de(a\ }$$

$$= \frac{\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8\left(a^2-b^2\right)^3d\sqrt{\cos(c+dx)}} + \frac{b(e\cos(c+dx))^{3/2}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{3}{4\left(a^2-\right.}$$

$$= \frac{\left(11a^2+4b^2\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8\left(a^2-b^2\right)^3d\sqrt{\cos(c+dx)}} + \frac{5a^2\left(a^2+2b^2\right)e\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\right.}{16b\left(a^2-b^2\right)^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos}}$$

$$= -\frac{5a\left(a^2+2b^2\right)\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16\sqrt{b}\left(-a^2+b^2\right)^{13/4}d} + \frac{5a\left(a^2+2b^2\right)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16\sqrt{b}\left(-a^2+b^2\right)^{13/4}d} + \frac{\left(11a\right.}{}$$

Mathematica [C] time = 6.73063, size = 900, normalized size = 1.55

$$\frac{\sqrt{e\cos(c+dx)}\left(\frac{3ab\cos(c+dx)}{4(a^2-b^2)^2(a+b\sin(c+dx))^2}+\frac{b\cos(c+dx)}{3(a^2-b^2)(a+b\sin(c+dx))^3}-\frac{-4\cos(c+dx)b^3-11a^2\cos(c+dx)b}{8(a^2-b^2)^3(a+b\sin(c+dx))}\right)}{d}+\frac{\sqrt{e\cos(c+dx)}\left(-\frac{(4b^3+11a^2b)(\cdots}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*((b*Cos[c + d*x])/(3*(a^2 - b^2)*(a + b*Sin[c + d*x])
^3) + (3*a*b*Cos[c + d*x])/(4*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) - (-11*
a^2*b*Cos[c + d*x] - 4*b^3*Cos[c + d*x])/(8*(a^2 - b^2)^3*(a + b*Sin[c + d*
x]))))/d + (Sqrt[e*Cos[c + d*x]]*((-2*(16*a^3 + 14*a*b^2)*(a + b*Sqrt[1 - C
os[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(
2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*A
rcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sq
rt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*
b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin
[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((11*a^2*b + 4
*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/
4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) +
3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d
*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]]
)/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sq
rt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]
^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(
16*(a - b)^3*(a + b)^3*d*Sqrt[Cos[c + d*x]])
```

Maple [C] time = 88.013, size = 112960, normalized size = 195.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos(dx+c)}}{(b\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos(dx+c)}}{(b\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(b*sin(d*x + c) + a)^4, x)
```

3.613 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=593

$$\frac{11ab\sqrt{e\cos(c+dx)}}{12de\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{b\left(57a^2+20b^2\right)\sqrt{e\cos(c+dx)}}{24de\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{b\sqrt{e\cos(c+dx)}}{3de\left(a^2-b^2\right)(a+b\sin(c+dx))^3}+\frac{7a\sqrt{b}\left(5\right.}{}$$

```
[Out] (7*a*Sqrt[b]*(5*a^2 + 6*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
 b^2)^(1/4)*Sqrt[e])])/(16*(-a^2 + b^2)^(15/4)*d*Sqrt[e]) + (7*a*Sqrt[b]*(5
*a^2 + 6*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[e])])/(16*(-a^2 + b^2)^(15/4)*d*Sqrt[e]) - ((57*a^2 + 20*b^2)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2])/(24*(a^2 - b^2)^3*d*Sqrt[e*Cos[c + d*x]
]) + (7*a^2*(5*a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-
a^2 + b^2]), (c + d*x)/2, 2])/(16*(a^2 - b^2)^3*(a^2 - b*(b - Sqrt[-a^2 + b
^2]))*d*Sqrt[e*Cos[c + d*x]]) + (7*a^2*(5*a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*(a^2 - b^2)^3*
(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (b*Sqrt[e*Cos[c
+ d*x]])/(3*(a^2 - b^2)*d*e*(a + b*Sin[c + d*x])^3) + (11*a*b*Sqrt[e*Cos[c
+ d*x]])/(12*(a^2 - b^2)^2*d*e*(a + b*Sin[c + d*x])^2) + (b*(57*a^2 + 20*b^
2)*Sqrt[e*Cos[c + d*x]])/(24*(a^2 - b^2)^3*d*e*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.57002, antiderivative size = 593, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {2694, 2864, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{11ab\sqrt{e\cos(c+dx)}}{12de\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{b\left(57a^2+20b^2\right)\sqrt{e\cos(c+dx)}}{24de\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{b\sqrt{e\cos(c+dx)}}{3de\left(a^2-b^2\right)(a+b\sin(c+dx))^3}+\frac{7a\sqrt{b}\left(5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^4),x]
```

```
[Out] (7*a*Sqrt[b]*(5*a^2 + 6*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
 b^2)^(1/4)*Sqrt[e])])/(16*(-a^2 + b^2)^(15/4)*d*Sqrt[e]) + (7*a*Sqrt[b]*(5
*a^2 + 6*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[e])])/(16*(-a^2 + b^2)^(15/4)*d*Sqrt[e]) - ((57*a^2 + 20*b^2)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2])/(24*(a^2 - b^2)^3*d*Sqrt[e*Cos[c + d*x]
]) + (7*a^2*(5*a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-
```

```
a^2 + b^2]), (c + d*x)/2, 2])/(16*(a^2 - b^2)^3*(a^2 - b*(b - Sqrt[-a^2 + b
^2]))*d*Sqrt[e*Cos[c + d*x]]) + (7*a^2*(5*a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*(a^2 - b^2)^3*
(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (b*Sqrt[e*Cos[c
+ d*x]])/(3*(a^2 - b^2)*d*e*(a + b*Sin[c + d*x])^3) + (11*a*b*Sqrt[e*Cos[c
+ d*x]])/(12*(a^2 - b^2)^2*d*e*(a + b*Sin[c + d*x])^2) + (b*(57*a^2 + 20*b^
2)*Sqrt[e*Cos[c + d*x]])/(24*(a^2 - b^2)^3*d*e*(a + b*Sin[c + d*x]))
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^4}\,dx = \frac{b\sqrt{e\cos(c+dx)}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} - \frac{\int \frac{-3a+\frac{5}{2}b\sin(c+dx)}{\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{11ab\sqrt{e\cos(c+dx)}}{12\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))^2} + \cdots$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{11ab\sqrt{e\cos(c+dx)}}{12\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))^2} + \frac{b}{24}\cdots$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{11ab\sqrt{e\cos(c+dx)}}{12\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))^2} + \frac{b}{24}\cdots$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{3\left(a^2-b^2\right)de(a+b\sin(c+dx))^3} + \frac{11ab\sqrt{e\cos(c+dx)}}{12\left(a^2-b^2\right)^2 de(a+b\sin(c+dx))^2} + \frac{b}{24}\cdots$$

$$= -\frac{\left(57a^2+20b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24\left(a^2-b^2\right)^3 d\sqrt{e\cos(c+dx)}} + \frac{b\sqrt{e\cos(c+dx)}}{3\left(a^2-b^2\right)de(a+b\sin(c+d\cdots}$$

$$= -\frac{\left(57a^2+20b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24\left(a^2-b^2\right)^3 d\sqrt{e\cos(c+dx)}} - \frac{7a^2\left(5a^2+6b^2\right)\sqrt{\cos(c+dx}\cdots}{16\left(-a^2+b^2\right)^{7/2}\left(b-\sqrt{-a\cdots}\right.}$$

$$= \frac{7a\sqrt{b}\left(5a^2+6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16\left(-a^2+b^2\right)^{15/4}d\sqrt{e}} + \frac{7a\sqrt{b}\left(5a^2+6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos\cdots}}{\sqrt[4]{-a^2+b\cdots}}\right.}{16\left(-a^2+b^2\right)^{15/4}d\sqrt{e}}$$

Mathematica [C] time = 25.2401, size = 1276, normalized size = 2.15

$$\frac{\cos(c+dx)\left(\frac{(57a^2+20b^2)b}{24(a^2-b^2)^3(a+b\sin(c+dx))}+\frac{11ab}{12(a^2-b^2)^2(a+b\sin(c+dx))^2}+\frac{b}{3(a^2-b^2)(a+b\sin(c+dx))^3}\right)}{d\sqrt{e\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\left(-\frac{2\left(-20b^3-57a^2b\right)\left(a+b\cdots\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^4),x]
```

```
[Out] (Cos[c + d*x]*(b/(3*(a^2 - b^2)*(a + b*Sin[c + d*x])^3) + (11*a*b)/(12*(a^2
 - b^2)^2*(a + b*Sin[c + d*x])^2) + (b*(57*a^2 + 20*b^2))/(24*(a^2 - b^2)^3
*(a + b*Sin[c + d*x]))))/(d*Sqrt[e*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*((-
2*(48*a^3 + 106*a*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)
]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4
, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^
2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2)))
- ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(
-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^
2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b
^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) -
(2*(-57*a^2*b - 20*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*
AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[
1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(
2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^
2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos
[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2
))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1
/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)]
 - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x
]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(48*(a
 - b)^3*(a + b)^3*d*Sqrt[e*Cos[c + d*x]])
```

Maple [C] time = 86.842, size = 85165, normalized size = 143.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^4), x)
```

3.614 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=674

$$-\frac{15ab^{3/2}\left(7a^2+6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16de^{3/2}\left(b^2-a^2\right)^{17/4}}+\frac{15ab^{3/2}\left(7a^2+6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16de^{3/2}\left(b^2-a^2\right)^{17/4}}-\frac{\left(151a^2b^2+16a^4+28b^4\right)E\left(\frac{1}{2}\right.}{8de^2\left(a^2-b^2\right)^4}$$

```
[Out] (-15*a*b^(3/2)*(7*a^2 + 6*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(16*(-a^2 + b^2)^(17/4)*d*e^(3/2)) + (15*a*b^(3/2)
*(7*a^2 + 6*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)
*Sqrt[e])])/(16*(-a^2 + b^2)^(17/4)*d*e^(3/2)) - ((16*a^4 + 151*a^2*b^2 + 2
8*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(8*(a^2 - b^2)^4*d*e
^2*Sqrt[Cos[c + d*x]]) - (15*a^2*b*(7*a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*(a^2 - b^2)^4*(b -
 Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) - (15*a^2*b*(7*a^2 + 6*b^2)*Sq
rt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/
(16*(a^2 - b^2)^4*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) + b/(3*(
a^2 - b^2)*d*e*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3) + (13*a*b)/(12*
(a^2 - b^2)^2*d*e*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2) + (b*(89*a^2
 + 28*b^2))/(24*(a^2 - b^2)^3*d*e*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])
) - (15*a*b*(7*a^2 + 6*b^2) - (16*a^4 + 151*a^2*b^2 + 28*b^4)*Sin[c + d*x])
/(8*(a^2 - b^2)^4*d*e*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 1.95235, antiderivative size = 674, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {2694, 2864, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{15ab^{3/2}\left(7a^2+6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16de^{3/2}\left(b^2-a^2\right)^{17/4}}+\frac{15ab^{3/2}\left(7a^2+6b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16de^{3/2}\left(b^2-a^2\right)^{17/4}}-\frac{\left(151a^2b^2+16a^4+28b^4\right)E\left(\frac{1}{2}\right.}{8de^2\left(a^2-b^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^4),x]
```

```
[Out] (-15*a*b^(3/2)*(7*a^2 + 6*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(16*(-a^2 + b^2)^(17/4)*d*e^(3/2)) + (15*a*b^(3/2)
*(7*a^2 + 6*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)
```

```
*Sqrt[e])])/(16*(-a^2 + b^2)^(17/4)*d*e^(3/2)) - ((16*a^4 + 151*a^2*b^2 + 2
8*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(8*(a^2 - b^2)^4*d*e
^2*Sqrt[Cos[c + d*x]]) - (15*a^2*b*(7*a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*(a^2 - b^2)^4*(b -
 Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) - (15*a^2*b*(7*a^2 + 6*b^2)*Sq
rt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/
(16*(a^2 - b^2)^4*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) + b/(3*(
a^2 - b^2)*d*e*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3) + (13*a*b)/(12*
(a^2 - b^2)^2*d*e*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2) + (b*(89*a^2
 + 28*b^2))/(24*(a^2 - b^2)^3*d*e*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])
) - (15*a*b*(7*a^2 + 6*b^2) - (16*a^4 + 151*a^2*b^2 + 28*b^4)*Sin[c + d*x])
/(8*(a^2 - b^2)^4*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^4}\,dx = \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3} - \frac{\int \frac{-3a+\frac{7}{2}b\sin(c+dx)}{(e\cos(c+dx))^{3/2}(a+b\sin(c+d}}{3\left(a^2-b^2\right)}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3} + \frac{13}{12\left(a^2-b^2\right)^2 de\sqrt{e\cos(c}}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3} + \frac{13}{12\left(a^2-b^2\right)^2 de\sqrt{e\cos(c}}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3} + \frac{13}{12\left(a^2-b^2\right)^2 de\sqrt{e\cos(c}}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3} + \frac{13}{12\left(a^2-b^2\right)^2 de\sqrt{e\cos(c}}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^3} + \frac{13}{12\left(a^2-b^2\right)^2 de\sqrt{e\cos(c}}$$

$$= -\frac{\left(16a^4+151a^2b^2+28b^4\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8\left(a^2-b^2\right)^4 de^2\sqrt{\cos(c+dx)}} + \frac{}{3\left(a^2-b^2\right)de\sqrt{}}$$

$$= -\frac{\left(16a^4+151a^2b^2+28b^4\right)\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8\left(a^2-b^2\right)^4 de^2\sqrt{\cos(c+dx)}} - \frac{15a^2b\left(7a^2+6\right.}{16\left(a^2-\right.}$$

$$= -\frac{15ab^{3/2}\left(7a^2+6b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16\left(-a^2+b^2\right)^{17/4}de^{3/2}} + \frac{15ab^{3/2}\left(7a^2+6b^2\right)\tanh^{-1}\left(\right.}{16\left(-a^2+b^2\right)^{17/4}de}$$

Mathematica [C] time = 7.02761, size = 996, normalized size = 1.48

$$\frac{\cos^2(c+dx)\left(-\frac{7a\cos(c+dx)b^3}{4\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2} - \frac{\cos(c+dx)b^3}{3\left(a^2-b^2\right)^2(a+b\sin(c+dx))^3} + \frac{2\sec(c+dx)\left(\sin(c+dx)a^4-4ba^3+6b^2\sin(c+dx)a^2-4b^3a+b^4\sin(c+dx)\right)}{\left(a^2-b^2\right)^4} + \frac{-1}{}\right.}{d(e\cos(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^4),x]
```

```
[Out] -(Cos[c + d*x]^(3/2)*((-2*(16*a^5 + 256*a^3*b^2 + 118*a*b^4)*(a + b*Sqrt[1
- Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[
c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8
)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] -
2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
 I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1
/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*
Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((16*a^4*b
+ 151*a^2*b^3 + 28*b^5)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF
1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos
[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt
[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*
Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 -
 b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*
x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*
Sin[c + d*x]))))/(16*(a - b)^4*(a + b)^4*d*(e*Cos[c + d*x])^(3/2)) + (Cos[c
 + d*x]^2*(-(b^3*Cos[c + d*x])/(3*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^3) - (
7*a*b^3*Cos[c + d*x])/(4*(a^2 - b^2)^3*(a + b*Sin[c + d*x])^2) + (-55*a^2*b
^3*Cos[c + d*x] - 12*b^5*Cos[c + d*x])/(8*(a^2 - b^2)^4*(a + b*Sin[c + d*x]
)) + (2*Sec[c + d*x]*(-4*a^3*b - 4*a*b^3 + a^4*Sin[c + d*x] + 6*a^2*b^2*Sin
[c + d*x] + b^4*Sin[c + d*x]))/(a^2 - b^2)^4))/(d*(e*Cos[c + d*x])^(3/2))
```

Maple [C] time = 152.143, size = 150599, normalized size = 223.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(b\sin\left(dx+c\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^4), x)
```

3.615 $\int \frac{1}{\sqrt{c\cos(e+fx)}\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=183

$$\frac{2\sqrt{2}\sqrt[4]{b-a}\sqrt{c\cos(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(1-\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{a+b}\sqrt{\frac{\cos(e+fx)+\sin(e+fx)+1}{\cos(e+fx)-\sin(e+fx)+1}}}{\sqrt[4]{b-a}}\right)\middle|-1\right)}{cf\sqrt[4]{a+b}\sqrt{\frac{\sin(e+fx)+\cos(e+fx)+1}{-\sin(e+fx)+\cos(e+fx)+1}}\sqrt{a+b\sin(e+fx)}}$$

```
[Out] (2*Sqrt[2]*(-a + b)^(1/4)*Sqrt[c*Cos[e + f*x]]*EllipticF[ArcSin[((a + b)^(1
/4)*Sqrt[(1 + Cos[e + f*x] + Sin[e + f*x])/(1 + Cos[e + f*x] - Sin[e + f*x]
)])/(-a + b)^(1/4)], -1]*Sqrt[(a + b*Sin[e + f*x])/((a - b)*(1 - Sin[e + f*
x])))])/((a + b)^(1/4)*c*f*Sqrt[(1 + Cos[e + f*x] + Sin[e + f*x])/(1 + Cos[e
 + f*x] - Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]])
```

Rubi [B] time = 0.427507, antiderivative size = 374, normalized size of antiderivative = 2.04, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2697, 220}

$$\frac{\sqrt{2}\sqrt[4]{a-b}\sqrt{c\cos(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(1-\sin(e+fx))}}\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(\sin(e)(-\cos(fx))-\cos(e)\sin(fx)+1)\left(\frac{\sqrt{a+b}(\sin(e+fx)+\cos(e+fx)+1)}{\sqrt{a-b}(-\sin(e+fx)+\cos(e+fx)+1)}+1\right)^2}}\left(\frac{\sqrt{a+b}(\sin(e+fx)+\cdots}{\sqrt{a-b}(-\sin(e+fx)+\cdots}\right.\cdots}{cf\sqrt[4]{a+b}\sqrt{\frac{\sin(e+fx)+\cos(e+fx)+1}{-\sin(e+fx)+\cos(e+fx)+1}}\sqrt{a+b\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(\sin(e)(-\cos(fx))-\cos\cdots}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(Sqrt[c*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]),x]
```

```
[Out] (Sqrt[2]*(a - b)^(1/4)*Sqrt[c*Cos[e + f*x]]*EllipticF[2*ArcTan[((a + b)^(1/
4)*Sqrt[(1 + Cos[e + f*x] + Sin[e + f*x])/(1 + Cos[e + f*x] - Sin[e + f*x])
])/(a - b)^(1/4)], 1/2]*Sqrt[(a + b*Sin[e + f*x])/((a - b)*(1 - Sin[e + f*x
]))]*Sqrt[(a + b*Sin[e + f*x])/((a - b)*(1 - Cos[f*x]*Sin[e] - Cos[e]*Sin[f
*x])*(1 + (Sqrt[a + b]*(1 + Cos[e + f*x] + Sin[e + f*x]))/(Sqrt[a - b]*(1 +
 Cos[e + f*x] - Sin[e + f*x])))^2)]*(1 + (Sqrt[a + b]*(1 + Cos[e + f*x] + S
in[e + f*x]))/(Sqrt[a - b]*(1 + Cos[e + f*x] - Sin[e + f*x]))))/((a + b)^(1
/4)*c*f*Sqrt[(1 + Cos[e + f*x] + Sin[e + f*x])/(1 + Cos[e + f*x] - Sin[e +
f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/((a - b)*(1 - Cos
[f*x]*Sin[e] - Cos[e]*Sin[f*x]))])
```

Rule 2697

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[(2*Sqrt[2]*Sqrt[g*Cos[e + f*x]]*Sqrt[(a + b*
Sin[e + f*x])/((a - b)*(1 - Sin[e + f*x]))])/(f*g*Sqrt[a + b*Sin[e + f*x]]*
Sqrt[(1 + Cos[e + f*x] + Sin[e + f*x])/(1 + Cos[e + f*x] - Sin[e + f*x])]),
 Subst[Int[1/Sqrt[1 + ((a + b)*x^4)/(a - b)], x], x, Sqrt[(1 + Cos[e + f*x]
 + Sin[e + f*x])/(1 + Cos[e + f*x] - Sin[e + f*x])]], x] /; FreeQ[{a, b, e,
 f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{c\cos(e+fx)}\sqrt{a+b\sin(e+fx)}}\,dx = \frac{\left(2\sqrt{2}\sqrt{c\cos(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(1-\sin(e+fx))}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(a+b)x^4}{a-b}}}\,dx, x, \sqrt{\frac{1+\cos(e+}{1+\cos(e+}}\right.}{cf\sqrt{\frac{1+\cos(e+fx)+\sin(e+fx)}{1+\cos(e+fx)-\sin(e+fx)}}\sqrt{a+b\sin(e+fx)}}$$

$$= \frac{\sqrt{2}\sqrt[4]{a-b}\sqrt{c\cos(e+fx)}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\sqrt{\frac{1+\cos(e+fx)+\sin(e+fx)}{1+\cos(e+fx)-\sin(e+fx)}}}{\sqrt[4]{a-b}}\right)|\frac{1}{2}\right)\sqrt{\frac{a+b\sin(}{(a-b)(1-\sin}}}{\sqrt[4]{a+b}cf\sqrt{\frac{1+\cos(e+fx)+\sin(e+fx)}{1+\cos(e+fx)-\sin(e+fx)}}}$$

Mathematica [C] time = 0.313366, size = 117, normalized size = 0.64

$$-\frac{2c(\sin(e+fx)-1)\left(\frac{(a+b)(\sin(e+fx)+1)}{(a-b)(\sin(e+fx)-1)}\right)^{3/4}\sqrt{a+b\sin(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{2(a+b\sin(e+fx))}{(a-b)(\sin(e+fx)-1)}\right)}{f(a+b)(c\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]),x]
```

```
[Out] (-2*c*Hypergeometric2F1[1/2, 3/4, 3/2, (-2*(a + b*Sin[e + f*x]))/((a - b)*(
-1 + Sin[e + f*x]))]*(-1 + Sin[e + f*x])*(((a + b)*(1 + Sin[e + f*x]))/((a
```

```
- b)*(-1 + Sin[e + f*x])))^(3/4)*Sqrt[a + b*Sin[e + f*x]])/((a + b)*f*(c*Co
s[e + f*x])^(3/2))
```

Maple [B] time = 0.514, size = 442, normalized size = 2.4

$$4\frac{\left(\cos\left(fx+e\right)+1\right)^{2}\left(-1+\cos\left(fx+e\right)\right)^{2}\left(b+\sqrt{-a^{2}+b^{2}}+a\right)\left(1+\sin\left(fx+e\right)\right)}{f\left(b+\sqrt{-a^{2}+b^{2}}-a\right)\sqrt{a+b\sin\left(fx+e\right)}\left(\sin\left(fx+e\right)\right)^{4}\sqrt{c\cos\left(fx+e\right)}}\mathrm{EllipticF}\left(\sqrt{\frac{\left(b+\sqrt{-a^{2}+b^{2}}-a\right)\left(-1\right.}{\left(b+\sqrt{-a^{2}+b^{2}}+a\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x)
```

```
[Out] 4/f/(b+(-a^2+b^2)^(1/2)-a)*EllipticF(((b+(-a^2+b^2)^(1/2)-a)/(b+(-a^2+b^2)^
(1/2)+a)*(-1+sin(f*x+e))/cos(f*x+e))^(1/2),((a-b+(-a^2+b^2)^(1/2))*(b+(-a^2
+b^2)^(1/2)+a)/(-b+(-a^2+b^2)^(1/2)-a)/(b+(-a^2+b^2)^(1/2)-a))^(1/2))*(1/(b
+(-a^2+b^2)^(1/2)+a)*(cos(f*x+e)*(-a^2+b^2)^(1/2)+a*sin(f*x+e)+b*cos(f*x+e)
+(-a^2+b^2)^(1/2)+b)/(1+cos(f*x+e)+sin(f*x+e)))^(1/2)*((b+(-a^2+b^2)^(1/2)-
a)/(b+(-a^2+b^2)^(1/2)+a)*(-1+sin(f*x+e))/cos(f*x+e))^(1/2)*(-1/(-b+(-a^2+b
^2)^(1/2)-a)*(a*sin(f*x+e)-cos(f*x+e)*(-a^2+b^2)^(1/2)+b*cos(f*x+e)-(-a^2+b
^2)^(1/2)+b)/(1+cos(f*x+e)+sin(f*x+e)))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+
e))^2*(b+(-a^2+b^2)^(1/2)+a)*(1+sin(f*x+e))/(a+b*sin(f*x+e))^(1/2)/sin(f*x+
e)^4/(c*cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{c\cos\left(fx+e\right)}\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c*cos(f*x + e))*sqrt(b*sin(f*x + e) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c\cos\left(fx+e\right)}\sqrt{b\sin\left(fx+e\right)+a}}{bc\cos\left(fx+e\right)\sin\left(fx+e\right)+ac\cos\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(c*cos(f*x + e))*sqrt(b*sin(f*x + e) + a)/(b*c*cos(f*x + e)*si
n(f*x + e) + a*c*cos(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{c\cos\left(e+fx\right)}\sqrt{a+b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*cos(e + f*x))*sqrt(a + b*sin(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{c\cos\left(fx+e\right)}\sqrt{b\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c*cos(f*x + e))*sqrt(b*sin(f*x + e) + a)), x)
```

3.616 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=229

$$-\frac{a\left(a^2(p+2)+3b^2\right)\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)(p+2)\sqrt{\sin^2(c+dx)}}-\frac{b\left(a^2\left(p^2+6p+11\right)+2b^2(p+2)\right)(e\,c}{de(p+1)(p+2)(p+3)}$$

```
[Out] -((b*(2*b^2*(2 + p) + a^2*(11 + 6*p + p^2))*(e*Cos[c + d*x])^(1 + p))/(d*e*
(1 + p)*(2 + p)*(3 + p))) - (a*(3*b^2 + a^2*(2 + p))*(e*Cos[c + d*x])^(1 +
p)*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d*x]^2]*Sin[c + d*x
])/(d*e*(1 + p)*(2 + p)*Sqrt[Sin[c + d*x]^2]) - (a*b*(5 + p)*(e*Cos[c + d*x
])^(1 + p)*(a + b*Sin[c + d*x]))/(d*e*(2 + p)*(3 + p)) - (b*(e*Cos[c + d*x]
)^(1 + p)*(a + b*Sin[c + d*x])^2)/(d*e*(3 + p))
```

Rubi [A] time = 0.369708, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2692, 2862, 2669, 2643}

$$-\frac{a\left(a^2(p+2)+3b^2\right)\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)(p+2)\sqrt{\sin^2(c+dx)}}-\frac{b\left(a^2\left(p^2+6p+11\right)+2b^2(p+2)\right)(e\,c}{de(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((b*(2*b^2*(2 + p) + a^2*(11 + 6*p + p^2))*(e*Cos[c + d*x])^(1 + p))/(d*e*
(1 + p)*(2 + p)*(3 + p))) - (a*(3*b^2 + a^2*(2 + p))*(e*Cos[c + d*x])^(1 +
p)*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d*x]^2]*Sin[c + d*x
])/(d*e*(1 + p)*(2 + p)*Sqrt[Sin[c + d*x]^2]) - (a*b*(5 + p)*(e*Cos[c + d*x
])^(1 + p)*(a + b*Sin[c + d*x]))/(d*e*(2 + p)*(3 + p)) - (b*(e*Cos[c + d*x]
)^(1 + p)*(a + b*Sin[c + d*x])^2)/(d*e*(3 + p))
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
```

```
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^3\,dx = -\frac{b(e\cos(c+dx))^{1+p}(a+b\sin(c+dx))^2}{de(3+p)} + \frac{\int (e\cos(c+dx))^p(a+b\sin(c+d}{}$$

$$= -\frac{ab(5+p)(e\cos(c+dx))^{1+p}(a+b\sin(c+dx))}{de(2+p)(3+p)} - \frac{b(e\cos(c+dx))^{1+p}(a+b\,\mathrm{s}}{de(3+p)}$$

$$= -\frac{b\left(2b^2(2+p)+a^2\left(11+6p+p^2\right)\right)(e\cos(c+dx))^{1+p}}{de(1+p)\left(6+5p+p^2\right)} - \frac{ab(5+p)(e\cos(c+}{de(2}$$

$$= -\frac{b\left(2b^2(2+p)+a^2\left(11+6p+p^2\right)\right)(e\cos(c+dx))^{1+p}}{de(1+p)\left(6+5p+p^2\right)} - \frac{a\left(a^2+\frac{3b^2}{2+p}\right)(e\cos(}{}$$

Mathematica [A] time = 54.7158, size = 290, normalized size = 1.27

$$\frac{8\sec^2(c+dx)^{p/2}(a+b\sin(c+dx))^3(e\cos(c+dx))^p\left(\frac{1}{3}a\left(a^2+3b^2\right)\tan^3(c+dx)\,{}_2F_1\left(\frac{3}{2},\frac{p+4}{2};\frac{5}{2};-\tan^2(c+dx)\right)-\frac{b\left(3a^2+b^2\right)}{}\right.}{d\left(2b\left(6a^2+b^2\right)\sin(2(c+dx))\sqrt{\sec^2(c+dx)}+8a^3-12ab^2\cos(2(c+\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (8*(e*Cos[c + d*x])^p*(Sec[c + d*x]^2)^(p/2)*(a + b*Sin[c + d*x])^3*((-3*a^
2*b*(Sec[c + d*x]^2)^(-3/2 - p/2))/(3 + p) + a^3*Hypergeometric2F1[1/2, (4
+ p)/2, 3/2, -Tan[c + d*x]^2]*Tan[c + d*x] + (a*(a^2 + 3*b^2)*Hypergeometri
c2F1[3/2, (4 + p)/2, 5/2, -Tan[c + d*x]^2]*Tan[c + d*x]^3)/3 - (b*(3*a^2 +
b^2)*(Sec[c + d*x]^2)^(-3/2 - p/2)*(2 + (3 + p)*Tan[c + d*x]^2))/((1 + p)*(
3 + p))))/(d*(8*a^3 + 12*a*b^2 - 12*a*b^2*Cos[2*(c + d*x)] + 2*b*(6*a^2 + b
^2)*Sqrt[Sec[c + d*x]^2]*Sin[2*(c + d*x)] - b^3*Sqrt[Sec[c + d*x]^2]*Sin[4*
(c + d*x)]))
```

Maple [F] time = 3.557, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p\,(a+b\sin(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^3\,(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,ab^2\cos\left(dx+c\right)^2-a^3-3\,ab^2+\left(b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3\right)\sin\left(dx+c\right)\right)\left(e\cos\left(dx+c\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*(e*cos(d*x + c))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(dx+c\right)+a\right)^3\left(e\cos\left(dx+c\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*(e*cos(d*x + c))^p, x)
```

3.617 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=157

$$-\frac{\left(a^2(p+2)+b^2\right)\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)(p+2)\sqrt{\sin^2(c+dx)}}-\frac{ab(p+3)(e\cos(c+dx))^{p+1}}{de(p+1)(p+2)}-\frac{b(a+b\sin}{}$$

```
[Out] -((a*b*(3 + p)*(e*Cos[c + d*x])^(1 + p))/(d*e*(1 + p)*(2 + p))) - ((b^2 + a
^2*(2 + p))*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[1/2, (1 + p)/2, (3 +
 p)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*e*(1 + p)*(2 + p)*Sqrt[Sin[c + d*x]
^2]) - (b*(e*Cos[c + d*x])^(1 + p)*(a + b*Sin[c + d*x]))/(d*e*(2 + p))
```

Rubi [A] time = 0.149241, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2692, 2669, 2643}

$$-\frac{\left(a^2(p+2)+b^2\right)\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)(p+2)\sqrt{\sin^2(c+dx)}}-\frac{ab(p+3)(e\cos(c+dx))^{p+1}}{de(p+1)(p+2)}-\frac{b(a+b\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -((a*b*(3 + p)*(e*Cos[c + d*x])^(1 + p))/(d*e*(1 + p)*(2 + p))) - ((b^2 + a
^2*(2 + p))*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[1/2, (1 + p)/2, (3 +
 p)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*e*(1 + p)*(2 + p)*Sqrt[Sin[c + d*x]
^2]) - (b*(e*Cos[c + d*x])^(1 + p)*(a + b*Sin[c + d*x]))/(d*e*(2 + p))
```


Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^2\,dx = -\frac{b(e\cos(c+dx))^{1+p}(a+b\sin(c+dx))}{de(2+p)}+\frac{\int (e\cos(c+dx))^p\left(b^2+a^2(2+p)\right.}{2+p}$$

$$= -\frac{ab(3+p)(e\cos(c+dx))^{1+p}}{de(1+p)(2+p)}-\frac{b(e\cos(c+dx))^{1+p}(a+b\sin(c+dx))}{de(2+p)}+\frac{\left(b^2\right.}{}$$

$$= -\frac{ab(3+p)(e\cos(c+dx))^{1+p}}{de(1+p)(2+p)}-\frac{\left(b^2+a^2(2+p)\right)(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2},\frac{1+}{2}\right.}{de(1+p)(2+p)\sqrt{\sin}}$$

Mathematica [C] time = 1.02257, size = 285, normalized size = 1.82

$$-\frac{(e\cos(c+dx))^p\left(-\frac{1}{2}a^2(p-1)\sin(2(c+dx))\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)+ab2^{-p}\left(1+e^{2i(c+dx)}\right)\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^p\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((e*Cos[c + d*x])^p*((a*b*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^p*(1 + E
^((2*I)*(c + d*x)))*(-(((-1 + p)*Hypergeometric2F1[1, (1 + p)/2, (1 - p)/2,
 -E^((2*I)*(c + d*x))])/E^(I*(c + d*x))) + E^(I*(c + d*x))*(1 + p)*Hypergeo
metric2F1[1, (3 + p)/2, (3 - p)/2, -E^((2*I)*(c + d*x))])*Sqrt[Sin[c + d*x]
^2])/(2^p*Cos[c + d*x]^p) - (b^2*(-1 + p)*Hypergeometric2F1[-1/2, (1 + p)/2
, (3 + p)/2, Cos[c + d*x]^2]*Sin[2*(c + d*x)])/2 - (a^2*(-1 + p)*Hypergeome
tric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d*x]^2]*Sin[2*(c + d*x)])/2))/((
```

```
d - d*p^2)*Sqrt[Sin[c + d*x]^2]))
```

Maple [F] time = 2.566, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p (a+b\sin(dx+c))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^2 (e\cos(dx+c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b^2\cos(dx+c)^2 - 2\,ab\sin(dx+c) - a^2 - b^2\right)(e\cos(dx+c))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*(e*cos(d*x
+ c))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^2(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*(e*cos(d*x + c))^p, x)
```

3.618 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)\sqrt{\sin^2(c+dx)}}-\frac{b(e\cos(c+dx))^{p+1}}{de(p+1)}$$

```
[Out] -((b*(e*Cos[c + d*x])^(1 + p))/(d*e*(1 + p))) - (a*(e*Cos[c + d*x])^(1 + p)
*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d*x]^2]*Sin[c + d*x])
/(d*e*(1 + p)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0520769, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2669, 2643}

$$-\frac{a\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)\sqrt{\sin^2(c+dx)}}-\frac{b(e\cos(c+dx))^{p+1}}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*(e*Cos[c + d*x])^(1 + p))/(d*e*(1 + p))) - (a*(e*Cos[c + d*x])^(1 + p)
*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d*x]^2]*Sin[c + d*x])
/(d*e*(1 + p)*Sqrt[Sin[c + d*x]^2])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))\,dx = -\frac{b(e\cos(c+dx))^{1+p}}{de(1+p)} + a\int (e\cos(c+dx))^p\,dx$$

$$= -\frac{b(e\cos(c+dx))^{1+p}}{de(1+p)} - \frac{a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\cos^2(c+dx)\right)\sin}{de(1+p)\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] time = 0.934504, size = 240, normalized size = 2.47

$$-\frac{(e\cos(c+dx))^p\left(-\frac{1}{2}a(p-1)\sin(2(c+dx))\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)+b2^{-p-1}\left(1+e^{2i(c+dx)}\right)\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^p\right.}{\left(d-dp^2\right)\sqrt{\mathrm{si}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((e*Cos[c + d*x])^p*((2^(-1 - p)*b*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))
^p*(1 + E^((2*I)*(c + d*x)))*(-(((-1 + p)*Hypergeometric2F1[1, (1 + p)/2, (
1 - p)/2, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x))) + E^(I*(c + d*x))*(1 + p)
*Hypergeometric2F1[1, (3 + p)/2, (3 - p)/2, -E^((2*I)*(c + d*x))])*Sqrt[Sin
[c + d*x]^2])/Cos[c + d*x]^p - (a*(-1 + p)*Hypergeometric2F1[1/2, (1 + p)/2
, (3 + p)/2, Cos[c + d*x]^2]*Sin[2*(c + d*x)])/2))/((d - d*p^2)*Sqrt[Sin[c
+ d*x]^2]))
```

Maple [F] time = 0.875, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p\,(a+b\sin(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sin(dx+c)+a)(e\cos(dx+c))^p\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(c+dx))^p\,(a+b\sin(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((e*cos(c + d*x))**p*(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] integrate((b*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

3.619 $\int \frac{(e\cos(c+dx))^p}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=158

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(1-p)}$$

```
[Out] -((e*AppellF1[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(1 - p)))
```

Rubi [A] time = 0.0793573, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2703}

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((e*AppellF1[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(1 - p)))
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{a+b\sin(c+dx)}\,dx = -\frac{eF_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\cdots\right.}{bd(1-p)}$$

Mathematica [B] time = 20.0596, size = 3819, normalized size = 24.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^p*Tan[c + d*x]*(a*Sqrt[Sec[c + d*x]^2] + b*Tan[c + d*x])*
(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*
x]^2)/a^2]*Tan[c + d*x]) + (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^
2, (-1 + b^2/a^2)*Tan[c + d*x]^2])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^
2)*Tan[c + d*x]^2)*(3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 +
 b^2/a^2)*Tan[c + d*x]^2] - (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[
c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2,
 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)))
)/(2*a^2*d*Sqrt[Sec[c + d*x]^2]*(a + b*Sin[c + d*x])*(a + (b*Tan[c + d*x])/
Sqrt[Sec[c + d*x]^2])*((Sqrt[Sec[c + d*x]^2]*(a*Sqrt[Sec[c + d*x]^2] + b*Ta
n[c + d*x])*(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]^2, ((-a^2 + b^2
)*Tan[c + d*x]^2)/a^2]*Tan[c + d*x]) + (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -T
an[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])/((Sec[c + d*x]^2)^(p/2)*(a^2
 + (a^2 - b^2)*Tan[c + d*x]^2)*(3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d
*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] - (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2
, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2
, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c
 + d*x]^2))))/(2*a^2*(a + (b*Tan[c + d*x])/Sqrt[Sec[c + d*x]^2])) - (Tan[c
+ d*x]^2*(a*Sqrt[Sec[c + d*x]^2] + b*Tan[c + d*x])*(-(b*AppellF1[1, (1 + p)
/2, 1, 2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Tan[c + d*x])
 + (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c
+ d*x]^2])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(3*a^
2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2
] - (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^
2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2
, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2))))/(2*a^2*Sqrt[Sec[c + d*
x]^2]*(a + (b*Tan[c + d*x])/Sqrt[Sec[c + d*x]^2])) + (Tan[c + d*x]*(b*Sec[c
 + d*x]^2 + a*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x])*(-(b*AppellF1[1, (1 + p)/2
, 1, 2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Tan[c + d*x]) +
```

(6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] - (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2))))/(2*a^2*Sqrt[Sec[c + d*x]^2]*(a + (b*Tan[c + d*x])/Sqrt[Sec[c + d*x]^2])) - (Tan[c + d*x]*(a*Sqrt[Sec[c + d*x]^2] + b*Tan[c + d*x])*(b*Sqrt[Sec[c + d*x]^2] - (b*Tan[c + d*x]^2)/Sqrt[Sec[c + d*x]^2])*(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Tan[c + d*x]) + (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] - (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2))))/(2*a^2*Sqrt[Sec[c + d*x]^2]*(a + (b*Tan[c + d*x])/Sqrt[Sec[c + d*x]^2])^2) + (Tan[c + d*x]*(a*Sqrt[Sec[c + d*x]^2] + b*Tan[c + d*x])*(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Sec[c + d*x]^2) - b*Tan[c + d*x]*(((-a^2 + b^2)*AppellF1[2, (1 + p)/2, 2, 3, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Sec[c + d*x]^2*Tan[c + d*x])/a^2 - ((1 + p)*AppellF1[2, 1 + (1 + p)/2, 1, 3, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Sec[c + d*x]^2*Tan[c + d*x])/2) - (12*a^5*(a^2 - b^2)*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*(Sec[c + d*x]^2)^(1 - p/2)*Tan[c + d*x])/((a^2 + (a^2 - b^2)*Tan[c + d*x]^2)^2*(3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] - (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)) - (6*a^5*p*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Tan[c + d*x])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] - (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)) + (6*a^5*(-(p*AppellF1[3/2, 1 + p/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/3 + (2*(-1 + b^2/a^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/3))/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] - (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)) - (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*(-2*(2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c

```
+ d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Sec[c + d*x]^2*Tan[c + d*x] + 3*a
^2*(-(p*AppellF1[3/2, 1 + p/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[
c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/3 + (2*(-1 + b^2/a^2)*AppellF1[3/2
, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]
^2*Tan[c + d*x])/3) - Tan[c + d*x]^2*(2*(a^2 - b^2)*((-3*p*AppellF1[5/2, 1
+ p/2, 2, 7/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]
^2*Tan[c + d*x])/5 + (12*(-1 + b^2/a^2)*AppellF1[5/2, p/2, 3, 7/2, -Tan[c +
 d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5) + a
^2*p*((6*(-1 + b^2/a^2)*AppellF1[5/2, (2 + p)/2, 2, 7/2, -Tan[c + d*x]^2, (
-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5 - (3*(2 + p)*A
ppellF1[5/2, 1 + (2 + p)/2, 1, 7/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c +
 d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/((Sec[c + d*x]^2)^(p/2)*(a^2 +
(a^2 - b^2)*Tan[c + d*x]^2)*(3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]
^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] - (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5
/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (
2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c +
d*x]^2)^2)))/(2*a^2*Sqrt[Sec[c + d*x]^2]*(a + (b*Tan[c + d*x])/Sqrt[Sec[c +
 d*x]^2])))
```

Maple [F] time = 0.628, size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\cos(dx+c))^p}{b\sin(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^p/(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{b\sin(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a), x)
```

3.620 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=170

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(2-p;\frac{1-p}{2},\frac{1-p}{2};3-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(2-p)(a+b\sin(c+dx))}$$

```
[Out] -((e*AppellF1[2 - p, (1 - p)/2, (1 - p)/2, 3 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(2 - p)*(a + b*Sin[c + d*x])))
```

Rubi [A] time = 0.0694385, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2703}

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(2-p;\frac{1-p}{2},\frac{1-p}{2};3-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(2-p)(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((e*AppellF1[2 - p, (1 - p)/2, (1 - p)/2, 3 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(2 - p)*(a + b*Sin[c + d*x])))
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^2}\,dx = -\frac{eF_1\left(2-p;\frac{1-p}{2},\frac{1-p}{2};3-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\Big(}{bd(2-p)(a+b\sin(c+dx))}$$

<u>Mathematica</u> [B] time = 25.5646, size = 4793, normalized size = 28.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^p*Tan[c + d*x]*(b*(a^2 - b^2)*AppellF1[1, (-1 + p)/2, 2,
2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Tan[c + d*x] + (3*a^
5*((-2*a^2*b^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Ta
n[c + d*x]^2)/a^2])/((-3*a^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-
1 + b^2/a^2)*Tan[c + d*x]^2] + (4*(a^2 - b^2)*AppellF1[3/2, p/2, 3, 5/2, -T
an[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)
/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2
)*(b^2*Tan[c + d*x]^2 - a^2*(1 + Tan[c + d*x]^2))^2) + ((a^2 + b^2)*AppellF
1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2])/((
-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d
*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b
^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d
*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(-(b^2*Tan[c + d*x]^
2) + a^2*(1 + Tan[c + d*x]^2)))))/(1 + Tan[c + d*x]^2)^(p/2)))/(a^3*(-a^2 +
 b^2)*d*(a + b*Sin[c + d*x])^2*((Sec[c + d*x]^2*(b*(a^2 - b^2)*AppellF1[1,
(-1 + p)/2, 2, 2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Tan[c
 + d*x] + (3*a^5*((-2*a^2*b^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (
(-a^2 + b^2)*Tan[c + d*x]^2)/a^2])/((-3*a^2*AppellF1[1/2, p/2, 2, 3/2, -Tan
[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (4*(a^2 - b^2)*AppellF1[3/2,
p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*Appell
F1[3/2, (2 + p)/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])
*Tan[c + d*x]^2)*(b^2*Tan[c + d*x]^2 - a^2*(1 + Tan[c + d*x]^2))^2) + ((a^2
 + b^2)*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d
*x]^2)/a^2])/((-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2
/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c +
d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1,
5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(-(b^
2*Tan[c + d*x]^2) + a^2*(1 + Tan[c + d*x]^2)))))/(1 + Tan[c + d*x]^2)^(p/2)
))/(a^3*(-a^2 + b^2)) + (Tan[c + d*x]*(b*(a^2 - b^2)*AppellF1[1, (-1 + p)/2
```

```
, 2, 2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Sec[c + d*x]^2
+ b*(a^2 - b^2)*Tan[c + d*x]*(-((-1 + p)*AppellF1[2, 1 + (-1 + p)/2, 2, 3,
-Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Sec[c + d*x]^2*Tan[c +
d*x])/2 + (2*(-a^2 + b^2)*AppellF1[2, (-1 + p)/2, 3, 3, -Tan[c + d*x]^2, ((
-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Sec[c + d*x]^2*Tan[c + d*x])/a^2) - 3*a^5*
p*Sec[c + d*x]^2*Tan[c + d*x]*(1 + Tan[c + d*x]^2)^(-1 - p/2)*((-2*a^2*b^2*
AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a
^2])/((-3*a^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Ta
n[c + d*x]^2] + (4*(a^2 - b^2)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2,
(-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 2, 5/2, -Ta
n[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(b^2*Tan[c +
d*x]^2 - a^2*(1 + Tan[c + d*x]^2))^2) + ((a^2 + b^2)*AppellF1[1/2, p/2, 1,
3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2])/((-3*a^2*AppellF1
[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^
2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c +
 d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^
2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(-(b^2*Tan[c + d*x]^2) + a^2*(1 + T
an[c + d*x]^2)))) + (3*a^5*((4*a^2*b^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c +
d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*(-2*a^2*Sec[c + d*x]^2*Tan[c + d
*x] + 2*b^2*Sec[c + d*x]^2*Tan[c + d*x]))/((-3*a^2*AppellF1[1/2, p/2, 2, 3/
2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (4*(a^2 - b^2)*AppellF
1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p
*AppellF1[3/2, (2 + p)/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d
*x]^2])*Tan[c + d*x]^2)*(b^2*Tan[c + d*x]^2 - a^2*(1 + Tan[c + d*x]^2))^3)
- (2*a^2*b^2*(-(p*AppellF1[3/2, 1 + p/2, 2, 5/2, -Tan[c + d*x]^2, ((-a^2 +
b^2)*Tan[c + d*x]^2)/a^2]*Sec[c + d*x]^2*Tan[c + d*x])/3 + (4*(-a^2 + b^2)*
AppellF1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a
^2]*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2)))/((-3*a^2*AppellF1[1/2, p/2, 2, 3
/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (4*(a^2 - b^2)*Appell
F1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*
p*AppellF1[3/2, (2 + p)/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c +
d*x]^2])*Tan[c + d*x]^2)*(b^2*Tan[c + d*x]^2 - a^2*(1 + Tan[c + d*x]^2))^2)
 - ((a^2 + b^2)*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*T
an[c + d*x]^2)/a^2]*(2*a^2*Sec[c + d*x]^2*Tan[c + d*x] - 2*b^2*Sec[c + d*x]
^2*Tan[c + d*x]))/((-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1
+ b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan
[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2
, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*
(-(b^2*Tan[c + d*x]^2) + a^2*(1 + Tan[c + d*x]^2))^2) + ((a^2 + b^2)*(-(p*A
ppellF1[3/2, 1 + p/2, 1, 5/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2
)/a^2]*Sec[c + d*x]^2*Tan[c + d*x])/3 + (2*(-a^2 + b^2)*AppellF1[3/2, p/2,
2, 5/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Sec[c + d*x]^2*
Tan[c + d*x])/(3*a^2)))/((-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2
, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2
, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2
```

```
+ p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*
x]^2)*(-(b^2*Tan[c + d*x]^2) + a^2*(1 + Tan[c + d*x]^2))) - ((a^2 + b^2)*Ap
pellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2
]*(2*(2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a
^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^
2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Sec[c + d*x]^2*Tan[c + d*x] - 3*a^2*(-(p
*AppellF1[3/2, 1 + p/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x
]^2]*Sec[c + d*x]^2*Tan[c + d*x])/3 + (2*(-1 + b^2/a^2)*AppellF1[3/2, p/2,
2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[
c + d*x])/3) + Tan[c + d*x]^2*(2*(a^2 - b^2)*((-3*p*AppellF1[5/2, 1 + p/2,
2, 7/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[
c + d*x])/5 + (12*(-1 + b^2/a^2)*AppellF1[5/2, p/2, 3, 7/2, -Tan[c + d*x]^2
, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5) + a^2*p*((
6*(-1 + b^2/a^2)*AppellF1[5/2, (2 + p)/2, 2, 7/2, -Tan[c + d*x]^2, (-1 + b^
2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5 - (3*(2 + p)*AppellF1
[5/2, 1 + (2 + p)/2, 1, 7/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2
]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/((-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -T
an[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2
, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*Appe
llF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2
])*Tan[c + d*x]^2)^2*(-(b^2*Tan[c + d*x]^2) + a^2*(1 + Tan[c + d*x]^2))) +
(2*a^2*b^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c
+ d*x]^2)/a^2]*(2*(4*(a^2 - b^2)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2
, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 2, 5/2, -
Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Sec[c + d*x]^2*Tan[c + d*x]
 - 3*a^2*(-(p*AppellF1[3/2, 1 + p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2
)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/3 + (4*(-1 + b^2/a^2)*Appell
F1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c
+ d*x]^2*Tan[c + d*x])/3) + Tan[c + d*x]^2*(4*(a^2 - b^2)*((-3*p*AppellF1[5
/2, 1 + p/2, 3, 7/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c
+ d*x]^2*Tan[c + d*x])/5 + (18*(-1 + b^2/a^2)*AppellF1[5/2, p/2, 4, 7/2, -T
an[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/
5) + a^2*p*((12*(-1 + b^2/a^2)*AppellF1[5/2, (2 + p)/2, 3, 7/2, -Tan[c + d*
x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5 - (3*(2
 + p)*AppellF1[5/2, 1 + (2 + p)/2, 2, 7/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*
Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/((-3*a^2*AppellF1[1/2, p
/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (4*(a^2 - b^2
)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2
] + a^2*p*AppellF1[3/2, (2 + p)/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*
Tan[c + d*x]^2])*Tan[c + d*x]^2)^2*(b^2*Tan[c + d*x]^2 - a^2*(1 + Tan[c + d
*x]^2))^2)))/(1 + Tan[c + d*x]^2)^(p/2)))/(a^3*(-a^2 + b^2))))
```

Maple [F] time = 0.526, size = 0, normalized size = 0.

$$\int \frac{(e\cos (dx+c))^{p}}{(a+b\sin (dx+c))^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos (dx+c))^{p}}{(b\sin (dx+c)+a)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(e\cos (dx+c))^{p}}{b^{2}\cos (dx+c)^{2}-2\,ab\sin (dx+c)-a^{2}-b^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(e*cos(d*x + c))^p/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
 - b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos{(dx+c)})^p}{(b\sin{(dx+c)}+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^2, x)
```

3.621 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=170

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(3-p;\frac{1-p}{2},\frac{1-p}{2};4-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(3-p)(a+b\sin(c+dx))^2}$$

```
[Out] -((e*AppellF1[3 - p, (1 - p)/2, (1 - p)/2, 4 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(3 - p)*(a + b*Sin[c + d*x])^2))
```

Rubi [A] time = 0.0689775, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2703}

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(3-p;\frac{1-p}{2},\frac{1-p}{2};4-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(3-p)(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((e*AppellF1[3 - p, (1 - p)/2, (1 - p)/2, 4 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(3 - p)*(a + b*Sin[c + d*x])^2))
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] && !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^3}\,dx = -\frac{eF_1\left(3-p;\frac{1-p}{2},\frac{1-p}{2};4-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\right.}{bd(3-p)(a+b\sin(c+dx))^2}$$

Mathematica [B] time = 28.951, size = 7904, normalized size = 46.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.744, size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(a+b\sin(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(e\cos(dx+c))^p}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(e*cos(d*x + c))^p/(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3
*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^3, x)
```

3.622 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=170

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(8-p;\frac{1-p}{2},\frac{1-p}{2};9-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(8-p)(a+b\sin(c+dx))^7}$$

```
[Out] -((e*AppellF1[8 - p, (1 - p)/2, (1 - p)/2, 9 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(8 - p)*(a + b*Sin[c + d*x])^7))
```

Rubi [A] time = 0.0704151, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2703}

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(8-p;\frac{1-p}{2},\frac{1-p}{2};9-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(8-p)(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -((e*AppellF1[8 - p, (1 - p)/2, (1 - p)/2, 9 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(8 - p)*(a + b*Sin[c + d*x])^7))
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^8}\,dx = -\frac{eF_1\left(8-p;\frac{1-p}{2},\frac{1-p}{2};9-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}}{bd(8-p)(a+b\sin(c+dx))^7}$$

Mathematica [F] time = 66.8614, size = 0, normalized size = 0.

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^8}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^8, x]
```

Maple [F] time = 4.677, size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(a+b\sin(dx+c))^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{}{b^8 \cos(dx+c)^8 + a^8 + 28\,a^6b^2 + 70\,a^4b^4 + 28\,a^2b^6 + b^8 - 4\left(7\,a^2b^6 + b^8\right)\cos(dx+c)^6 + 2\left(35\,a^4b^4 + 42\,a^2b^6 + \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^p/(b^8*cos(d*x + c)^8 + a^8 + 28*a^6*b^2 + 70*a^4
*b^4 + 28*a^2*b^6 + b^8 - 4*(7*a^2*b^6 + b^8)*cos(d*x + c)^6 + 2*(35*a^4*b^
4 + 42*a^2*b^6 + 3*b^8)*cos(d*x + c)^4 - 4*(7*a^6*b^2 + 35*a^4*b^4 + 21*a^2
*b^6 + b^8)*cos(d*x + c)^2 - 8*(a*b^7*cos(d*x + c)^6 - a^7*b - 7*a^5*b^3 -
7*a^3*b^5 - a*b^7 - (7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 + (7*a^5*b^3 + 14*
a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^8, x)
```

3.623 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=156

$$\frac{2e(a+b\sin(c+dx))^{7/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{7}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{9}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{7bd}$$

```
[Out] (2*e*AppellF1[7/2, (1 - p)/2, (1 - p)/2, 9/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d*x
])^(7/2)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c +
 d*x])/(a + b))^((1 - p)/2))/(7*b*d)
```

Rubi [A] time = 0.133758, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2704, 138}

$$\frac{2e(a+b\sin(c+dx))^{7/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{7}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{9}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*e*AppellF1[7/2, (1 - p)/2, (1 - p)/2, 9/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d*x
])^(7/2)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c +
 d*x])/(a + b))^((1 - p)/2))/(7*b*d)
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^{5/2}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\text{Subst}\left(\int(a\right.}{d}$$

$$= \frac{2eF_1\left(\frac{7}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{9}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}(a+b\sin(c}{7bd}$$

Mathematica [A] time = 7.81917, size = 187, normalized size = 1.2

$$\frac{2e(a+b\sin(c+dx))^{7/2}(e\cos(c+dx))^{p-1}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{\sqrt{b^2}-a}\right)^{\frac{1-p}{2}}F_1\left(\frac{7}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{9}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right.}{7bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*e*AppellF1[7/2, (1 - p)/2, (1 - p)/2, 9/2, (a + b*Sin[c + d*x])/(a - Sqr
t[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*((
Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*(a + b*Sin[c + d*x
])^(7/2)*((Sqrt[b^2] + b*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(7*b*
d)
```

Maple [F] time = 0.176, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p(a+b\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin (dx+c)+a)^{\frac{5}{2}}(e\cos (dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b^2\cos (dx+c)^2-2\,ab\sin (dx+c)-a^2-b^2\right)\sqrt{b\sin (dx+c)+a}\,(e\cos (dx+c))^p\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(b*sin(
d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{5}{2}}(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*(e*cos(d*x + c))^p, x)
```

3.624 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=156

$$\frac{2e(a+b\sin(c+dx))^{5/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{5}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{7}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{5bd}$$

```
[Out] (2*e*AppellF1[5/2, (1 - p)/2, (1 - p)/2, 7/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d*x
])^(5/2)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c +
 d*x])/(a + b))^((1 - p)/2))/(5*b*d)
```

Rubi [A] time = 0.115663, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2704, 138}

$$\frac{2e(a+b\sin(c+dx))^{5/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{5}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{7}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*e*AppellF1[5/2, (1 - p)/2, (1 - p)/2, 7/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d*x
])^(5/2)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c +
 d*x])/(a + b))^((1 - p)/2))/(5*b*d)
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^{3/2}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\mathrm{Subst}\left(\int (a\right.}{d}$$

$$= \frac{2eF_1\left(\frac{5}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{7}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}(a+b\sin(c}{5bd}$$

Mathematica [A] time = 0.820552, size = 187, normalized size = 1.2

$$\frac{2e(a+b\sin(c+dx))^{5/2}(e\cos(c+dx))^{p-1}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{\sqrt{b^2}-a}\right)^{\frac{1-p}{2}}F_1\left(\frac{5}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{7}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right)}{5bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*e*AppellF1[5/2, (1 - p)/2, (1 - p)/2, 7/2, (a + b*Sin[c + d*x])/(a - Sqr
t[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*((
Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*(a + b*Sin[c + d*x
])^(5/2)*((Sqrt[b^2] + b*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(5*b*
d)
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p(a+b\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}(e\cos(dx+c))^p\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sin(dx+c)+a)^{\frac{3}{2}}(e\cos(dx+c))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^(3/2)*(e*cos(d*x + c))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^{\frac{3}{2}}(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*(e*cos(d*x + c))^p, x)
```

3.625 $\int (e\cos(c+dx))^p\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{2e(a+b\sin(c+dx))^{3/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{5}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{3bd}$$

```
[Out] (2*e*AppellF1[3/2, (1 - p)/2, (1 - p)/2, 5/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d*x
])^(3/2)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c +
 d*x])/(a + b))^((1 - p)/2))/(3*b*d)
```

Rubi [A] time = 0.106051, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2704, 138}

$$\frac{2e(a+b\sin(c+dx))^{3/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{5}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*Sqrt[a + b*Sin[c + d*x]],x]
```


```
[Out] (2*e*AppellF1[3/2, (1 - p)/2, (1 - p)/2, 5/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d*x
])^(3/2)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c +
 d*x])/(a + b))^((1 - p)/2))/(3*b*d)
```


Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^p\sqrt{a+b\sin(c+dx)}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\text{Subst}\left(\int\sqrt{a+\right.}{d}$$

$$= \frac{2eF_1\left(\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{5}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}(a+b\sin(c+}{3bd}$$

Mathematica [A] time = 1.08305, size = 187, normalized size = 1.2

$$\frac{2e(a+b\sin(c+dx))^{3/2}(e\cos(c+dx))^{p-1}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{\sqrt{b^2}-a}\right)^{\frac{1-p}{2}}F_1\left(\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{5}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right.}{3bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*e*AppellF1[3/2, (1 - p)/2, (1 - p)/2, 5/2, (a + b*Sin[c + d*x])/(a - Sqr
t[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*((
Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*(a + b*Sin[c + d*x
])^(3/2)*((Sqrt[b^2] + b*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(3*b*
d)
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p\sqrt{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)+a}\,(e\cos(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sin(dx+c)+a}\,(e\cos(dx+c))^p\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(c+dx))^p\sqrt{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**p*sqrt(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin (dx + c) + a} \, (e \cos (dx + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*(e*cos(d*x + c))^p, x)
```

3.626 $\int \frac{(e\cos(c+dx))^p}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=154

$$\frac{2e\sqrt{a+b\sin(c+dx)}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{3}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd}$$

```
[Out] (2*e*AppellF1[1/2, (1 - p)/2, (1 - p)/2, 3/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*Sqrt[a + b*Sin[c +
 d*x]]*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c + d
*x])/(a + b))^((1 - p)/2))/(b*d)
```

Rubi [A] time = 0.116675, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2704, 138}

$$\frac{2e\sqrt{a+b\sin(c+dx)}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{3}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*e*AppellF1[1/2, (1 - p)/2, (1 - p)/2, 3/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*Sqrt[a + b*Sin[c +
 d*x]]*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c + d
*x])/(a + b))^((1 - p)/2))/(b*d)
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\operatorname{Subst}\left(\int \frac{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{\frac{1}{2}(-1+p)}\left(\frac{b}{a+b}\right.}{\sqrt{a+bx}}\right.}{d}$$

$$= \frac{2eF_1\left(\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{3}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}\sqrt{a+b\sin(c+dx)}\left(1-\frac{a+b\sin}{a}\right.}{bd}$$

Mathematica [A] time = 1.09462, size = 185, normalized size = 1.2

$$\frac{2e\sqrt{a+b\sin(c+dx)}(e\cos(c+dx))^{p-1}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{\sqrt{b^2}-a}\right)^{\frac{1-p}{2}}F_1\left(\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{3}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right)}{bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*e*AppellF1[1/2, (1 - p)/2, (1 - p)/2, 3/2, (a + b*Sin[c + d*x])/(a - Sqr
t[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*((
Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*Sqrt[a + b*Sin[c +
 d*x]]*((Sqrt[b^2] + b*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(b*d)
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p\frac{1}{\sqrt{a+b\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(e\cos(dx+c))^p}{\sqrt{b\sin(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^p/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(c+dx))^p}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**p/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{\sqrt{b\sin(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/sqrt(b*sin(d*x + c) + a), x)
```

3.627 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(-\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd\sqrt{a+b\sin(c+dx)}}$$

```
[Out] (-2*e*AppellF1[-1/2, (1 - p)/2, (1 - p)/2, 1/2, (a + b*Sin[c + d*x])/(a - b
), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(1 - (a + b*Sin[
c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^((1 - p)/
2))/(b*d*Sqrt[a + b*Sin[c + d*x]])
```

Rubi [A] time = 0.11651, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2704, 138}

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(-\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^(3/2),x]
```


```
[Out] (-2*e*AppellF1[-1/2, (1 - p)/2, (1 - p)/2, 1/2, (a + b*Sin[c + d*x])/(a - b
), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(1 - (a + b*Sin[
c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^((1 - p)/
2))/(b*d*Sqrt[a + b*Sin[c + d*x]])
```


Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\text{Subst}\left(\int \frac{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{\frac{1}{2}(-1+p)}\left(\right.}{(a+bx)^{3}}\right.}{d}$$

$$= -\frac{2eF_1\left(-\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1\right.}{bd\sqrt{a+b\sin(c+dx)}}$$

Mathematica [A] time = 2.92009, size = 185, normalized size = 1.2

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{\sqrt{b^2}-a}\right)^{\frac{1-p}{2}}F_1\left(-\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{1}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right)}{bd\sqrt{a+b\sin(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*AppellF1[-1/2, (1 - p)/2, (1 - p)/2, 1/2, (a + b*Sin[c + d*x])/(a - S
qrt[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*
((Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*((Sqrt[b^2] + b*
Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(b*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [F] time = 0.129, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p (a+b\sin(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,(e\cos(dx+c))^p}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*(e*cos(d*x + c))^p/(b^2*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**p/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^(3/2), x)
```

3.628 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(-\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{3bd(a+b\sin(c+dx))^{3/2}}$$

```
[Out] (-2*e*AppellF1[-3/2, (1 - p)/2, (1 - p)/2, -1/2, (a + b*Sin[c + d*x])/(a -
b), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(1 - (a + b*Sin
[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^((1 - p)
/2))/(3*b*d*(a + b*Sin[c + d*x])^(3/2))
```

Rubi [A] time = 0.117712, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2704, 138}

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(-\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*e*AppellF1[-3/2, (1 - p)/2, (1 - p)/2, -1/2, (a + b*Sin[c + d*x])/(a -
b), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(1 - (a + b*Sin
[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^((1 - p)
/2))/(3*b*d*(a + b*Sin[c + d*x])^(3/2))
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\operatorname{Subst}\left(\int \frac{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{\frac{1}{2}(-1+p)}\left(\right.}{(a+bx)^{5}}\right.}{d}$$

$$= -\frac{2eF_1\left(-\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(\right.}{3bd(a+b\sin(c+dx))^{3/2}}$$

Mathematica [A] time = 3.07107, size = 187, normalized size = 1.2

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{\sqrt{b^2}-a}\right)^{\frac{1-p}{2}}F_1\left(-\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right)}{3bd(a+b\sin(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*e*AppellF1[-3/2, (1 - p)/2, (1 - p)/2, -1/2, (a + b*Sin[c + d*x])/(a -
Sqrt[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)
*((Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*((Sqrt[b^2] + b
*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(3*b*d*(a + b*Sin[c + d*x])^(
3/2))
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p (a+b\sin(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^p}{(b\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{b\sin(dx+c)+a}\,(e\cos(dx+c))^p}{3\,ab^2\cos(dx+c)^2-a^3-3\,ab^2+\left(b^3\cos(dx+c)^2-3\,a^2b-b^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*(e*cos(d*x + c))^p/(3*a*b^2*cos(d*x + c)
^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos{(dx+c)})^p}{(b\sin{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p/(b*sin(d*x + c) + a)^(5/2), x)
```

3.629 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=158

$$\frac{e(e\cos(c+dx))^{p-1}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(m+1;\frac{1-p}{2},\frac{1-p}{2};m+2;\frac{a+b\sin(c+dx)}{a-b}\right.}{bd(m+1)}$$

```
[Out] (e*AppellF1[1 + m, (1 - p)/2, (1 - p)/2, 2 + m, (a + b*Sin[c + d*x])/(a - b
), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d
*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin
[c + d*x])/(a + b))^((1 - p)/2))/(b*d*(1 + m))
```

Rubi [A] time = 0.0985978, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2704, 138}

$$\frac{e(e\cos(c+dx))^{p-1}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(m+1;\frac{1-p}{2},\frac{1-p}{2};m+2;\frac{a+b\sin(c+dx)}{a-b}\right.}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^m,x]
```


```
[Out] (e*AppellF1[1 + m, (1 - p)/2, (1 - p)/2, 2 + m, (a + b*Sin[c + d*x])/(a - b
), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d
*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin
[c + d*x])/(a + b))^((1 - p)/2))/(b*d*(1 + m))
```


Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\mathrm{Subst}\left(\int(a+\right.}{d}$$

$$= \frac{eF_1\left(1+m;\frac{1-p}{2},\frac{1-p}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}(a+b}{bd(1+m)}$$

Mathematica [F] time = 2.36251, size = 0, normalized size = 0.

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^m, x]
```

Maple [F] time = 0.863, size = 0, normalized size = 0.

$$\int \left(e\cos\left(dx+c\right)\right)^{p}\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p (b\sin(dx+c)+a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^p*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((e\cos(dx+c))^p (b\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^p*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(c+dx))^p (a+b\sin(c+dx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Integral((e*cos(c + d*x))**p*(a + b*sin(c + d*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^p (b\sin(dx+c)+a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^p*(b*sin(d*x + c) + a)^m, x)
```

3.630 $\int \cos^7(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=254

$$-\frac{(a^2-b^2)^3(a+b\sin(c+dx))^{m+1}}{b^7d(m+1)}+\frac{6a(a^2-b^2)^2(a+b\sin(c+dx))^{m+2}}{b^7d(m+2)}-\frac{3(-6a^2b^2+5a^4+b^4)(a+b\sin(c+dx))^{m+}}{b^7d(m+3)}$$

```
[Out] -(((a^2 - b^2)^3*(a + b*Sin[c + d*x])^(1 + m))/(b^7*d*(1 + m))) + (6*a*(a^2
 - b^2)^2*(a + b*Sin[c + d*x])^(2 + m))/(b^7*d*(2 + m)) - (3*(5*a^4 - 6*a^2
*b^2 + b^4)*(a + b*Sin[c + d*x])^(3 + m))/(b^7*d*(3 + m)) + (4*a*(5*a^2 - 3
*b^2)*(a + b*Sin[c + d*x])^(4 + m))/(b^7*d*(4 + m)) - (3*(5*a^2 - b^2)*(a +
 b*Sin[c + d*x])^(5 + m))/(b^7*d*(5 + m)) + (6*a*(a + b*Sin[c + d*x])^(6 +
m))/(b^7*d*(6 + m)) - (a + b*Sin[c + d*x])^(7 + m)/(b^7*d*(7 + m))
```

Rubi [A] time = 0.16399, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{(a^2-b^2)^3(a+b\sin(c+dx))^{m+1}}{b^7d(m+1)}+\frac{6a(a^2-b^2)^2(a+b\sin(c+dx))^{m+2}}{b^7d(m+2)}-\frac{3(-6a^2b^2+5a^4+b^4)(a+b\sin(c+dx))^{m+}}{b^7d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((a^2 - b^2)^3*(a + b*Sin[c + d*x])^(1 + m))/(b^7*d*(1 + m))) + (6*a*(a^2
 - b^2)^2*(a + b*Sin[c + d*x])^(2 + m))/(b^7*d*(2 + m)) - (3*(5*a^4 - 6*a^2
*b^2 + b^4)*(a + b*Sin[c + d*x])^(3 + m))/(b^7*d*(3 + m)) + (4*a*(5*a^2 - 3
*b^2)*(a + b*Sin[c + d*x])^(4 + m))/(b^7*d*(4 + m)) - (3*(5*a^2 - b^2)*(a +
 b*Sin[c + d*x])^(5 + m))/(b^7*d*(5 + m)) + (6*a*(a + b*Sin[c + d*x])^(6 +
m))/(b^7*d*(6 + m)) - (a + b*Sin[c + d*x])^(7 + m)/(b^7*d*(7 + m))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int (a+x)^m\left(b^2-x^2\right)^3\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int\left(-\left(a^2-b^2\right)^3(a+x)^m+6a\left(a^2-b^2\right)^2(a+x)^{1+m}-3\left(5a^4-6a^2b^2+b\right.\right.\right.}{}$$

$$= -\frac{\left(a^2-b^2\right)^3(a+b\sin(c+dx))^{1+m}}{b^7d(1+m)}+\frac{6a\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{2+m}}{b^7d(2+m)}-\frac{3\left(5\right.}{}$$

Mathematica [A] time = 6.11701, size = 459, normalized size = 1.81

$$6\left(\left(b^2-a^2\right)\left(\frac{4\left(\left(b^2-a^2\right)\left(-\frac{\left(a^2-b^2\right)(a+b\sin(c+dx))^{m+1}}{m+1}+\frac{2a(a+b\sin(c+dx))^{m+2}}{m+2}-\frac{(a+b\sin(c+dx))^{m+3}}{m+3}\right)+a\left(-\frac{\left(a^2-b^2\right)(a+b\sin(c+dx))^{m+2}}{m+2}+\frac{2a(a+b\sin(c+dx))^{m+3}}{m+3}-\frac{(a+b\sin(c+dx))^{m+4}}{m+4}\right)\right)}{m+5}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((b^6*Cos[c + d*x]^6*(a + b*Sin[c + d*x])^(1 + m))/(7 + m) + (6*((-a^2 + b^
2)*((b^4*Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(1 + m))/(5 + m) + (4*((-a^2 +
 b^2)*(-(((a^2 - b^2)*(a + b*Sin[c + d*x])^(1 + m))/(1 + m)) + (2*a*(a + b*
Sin[c + d*x])^(2 + m))/(2 + m) - (a + b*Sin[c + d*x])^(3 + m)/(3 + m)) + a*
(-(((a^2 - b^2)*(a + b*Sin[c + d*x])^(2 + m))/(2 + m)) + (2*a*(a + b*Sin[c
+ d*x])^(3 + m))/(3 + m) - (a + b*Sin[c + d*x])^(4 + m)/(4 + m))))/(5 + m))
 + a*((b^4*Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(2 + m))/(6 + m) + (4*((-a^2
 + b^2)*(-(((a^2 - b^2)*(a + b*Sin[c + d*x])^(2 + m))/(2 + m)) + (2*a*(a +
b*Sin[c + d*x])^(3 + m))/(3 + m) - (a + b*Sin[c + d*x])^(4 + m)/(4 + m)) +
a*(-(((a^2 - b^2)*(a + b*Sin[c + d*x])^(3 + m))/(3 + m)) + (2*a*(a + b*Sin[
c + d*x])^(4 + m))/(4 + m) - (a + b*Sin[c + d*x])^(5 + m)/(5 + m))))/(6 + m
))))/(7 + m))/(b^7*d)
```

Maple [F] time = 0.3, size = 0, normalized size = 0.

$$\int (\cos (dx + c))^7 (a + b \sin (dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.6926, size = 1801, normalized size = 7.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -(720*a^7 - 3024*a^5*b^2 + 5040*a^3*b^4 - 5040*a*b^6 - (a*b^6*m^6 + 15*a*b^
6*m^5 + 85*a*b^6*m^4 + 225*a*b^6*m^3 + 274*a*b^6*m^2 + 120*a*b^6*m)*cos(d*x
 + c)^6 - 6*(2*a*b^6*m^5 - (5*a^3*b^4 - 23*a*b^6)*m^4 - 2*(15*a^3*b^4 - 44*
a*b^6)*m^3 - (55*a^3*b^4 - 133*a*b^6)*m^2 - 6*(5*a^3*b^4 - 11*a*b^6)*m)*cos
(d*x + c)^4 - 192*(a^3*b^4 + a*b^6)*m^3 + 288*(a^5*b^2 - 2*a^3*b^4 - 7*a*b^
6)*m^2 - 24*((a^3*b^4 + 3*a*b^6)*m^4 - 6*(a^3*b^4 - 5*a*b^6)*m^3 + (15*a^5*
b^2 - 55*a^3*b^4 + 84*a*b^6)*m^2 + 3*(5*a^5*b^2 - 16*a^3*b^4 + 19*a*b^6)*m)
*cos(d*x + c)^2 - 192*(3*a^5*b^2 - 13*a^3*b^4 + 32*a*b^6)*m - (2304*b^7 + (
b^7*m^6 + 21*b^7*m^5 + 175*b^7*m^4 + 735*b^7*m^3 + 1624*b^7*m^2 + 1764*b^7*
m + 720*b^7)*cos(d*x + c)^6 + 6*(144*b^7 + (a^2*b^5 + b^7)*m^5 + 2*(5*a^2*b
```

```
^5 + 8*b^7)*m^4 + 5*(7*a^2*b^5 + 19*b^7)*m^3 + 10*(5*a^2*b^5 + 26*b^7)*m^2
+ 12*(2*a^2*b^5 + 27*b^7)*m)*cos(d*x + c)^4 + 48*(a^4*b^3 + 6*a^2*b^5 + b^7
)*m^3 - 576*(a^4*b^3 - 4*a^2*b^5 - b^7)*m^2 + 24*(48*b^7 + (3*a^2*b^5 + b^7
)*m^4 - (5*a^4*b^3 - 24*a^2*b^5 - 13*b^7)*m^3 - (15*a^4*b^3 - 51*a^2*b^5 -
56*b^7)*m^2 - 2*(5*a^4*b^3 - 15*a^2*b^5 - 46*b^7)*m)*cos(d*x + c)^2 + 48*(1
5*a^6*b - 58*a^4*b^3 + 87*a^2*b^5 + 44*b^7)*m)*sin(d*x + c))*(b*sin(d*x + c
) + a)^m/(b^7*d*m^7 + 28*b^7*d*m^6 + 322*b^7*d*m^5 + 1960*b^7*d*m^4 + 6769*
b^7*d*m^3 + 13132*b^7*d*m^2 + 13068*b^7*d*m + 5040*b^7*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17609, size = 4832, normalized size = 19.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] -((b*sin(d*x + c) + a)^7*(b*sin(d*x + c) + a)^m*m^6 - 6*(b*sin(d*x + c) + a
)^6*(b*sin(d*x + c) + a)^m*a*m^6 + 15*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c
) + a)^m*a^2*m^6 - 20*(b*sin(d*x + c) + a)^4*(b*sin(d*x + c) + a)^m*a^3*m^6
 + 15*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a^4*m^6 - 6*(b*sin(d*x
+ c) + a)^2*(b*sin(d*x + c) + a)^m*a^5*m^6 + (b*sin(d*x + c) + a)*(b*sin(d*
x + c) + a)^m*a^6*m^6 - 3*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*b^2
*m^6 + 12*(b*sin(d*x + c) + a)^4*(b*sin(d*x + c) + a)^m*a*b^2*m^6 - 18*(b*s
in(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a^2*b^2*m^6 + 12*(b*sin(d*x + c)
+ a)^2*(b*sin(d*x + c) + a)^m*a^3*b^2*m^6 - 3*(b*sin(d*x + c) + a)*(b*sin(d
*x + c) + a)^m*a^4*b^2*m^6 + 3*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^
m*b^4*m^6 - 6*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a*b^4*m^6 + 3*(
b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^4*m^6 - (b*sin(d*x + c) +
```

a)*(b*sin(d*x + c) + a)^m*b^6*m^6 + 21*(b*sin(d*x + c) + a)^7*(b*sin(d*x +
c) + a)^m*m^5 - 132*(b*sin(d*x + c) + a)^6*(b*sin(d*x + c) + a)^m*a*m^5 + 3
45*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*a^2*m^5 - 480*(b*sin(d*x +
c) + a)^4*(b*sin(d*x + c) + a)^m*a^3*m^5 + 375*(b*sin(d*x + c) + a)^3*(b*s
in(d*x + c) + a)^m*a^4*m^5 - 156*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a
)^m*a^5*m^5 + 27*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^6*m^5 - 69*(
b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*b^2*m^5 + 288*(b*sin(d*x + c)
+ a)^4*(b*sin(d*x + c) + a)^m*a*b^2*m^5 - 450*(b*sin(d*x + c) + a)^3*(b*sin
(d*x + c) + a)^m*a^2*b^2*m^5 + 312*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) +
a)^m*a^3*b^2*m^5 - 81*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*b^2*
m^5 + 75*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*b^4*m^5 - 156*(b*sin
(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a*b^4*m^5 + 81*(b*sin(d*x + c) + a)
*(b*sin(d*x + c) + a)^m*a^2*b^4*m^5 - 27*(b*sin(d*x + c) + a)*(b*sin(d*x +
c) + a)^m*b^6*m^5 + 175*(b*sin(d*x + c) + a)^7*(b*sin(d*x + c) + a)^m*m^4 -
1140*(b*sin(d*x + c) + a)^6*(b*sin(d*x + c) + a)^m*a*m^4 + 3105*(b*sin(d*x
+ c) + a)^5*(b*sin(d*x + c) + a)^m*a^2*m^4 - 4520*(b*sin(d*x + c) + a)^4*(
b*sin(d*x + c) + a)^m*a^3*m^4 + 3705*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c)
+ a)^m*a^4*m^4 - 1620*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^5*m^
4 + 295*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^6*m^4 - 621*(b*sin(d*
x + c) + a)^5*(b*sin(d*x + c) + a)^m*b^2*m^4 + 2712*(b*sin(d*x + c) + a)^4*
(b*sin(d*x + c) + a)^m*a*b^2*m^4 - 4446*(b*sin(d*x + c) + a)^3*(b*sin(d*x +
c) + a)^m*a^2*b^2*m^4 + 3240*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m
*a^3*b^2*m^4 - 885*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*b^2*m^4
+ 741*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*b^4*m^4 - 1620*(b*sin(d
x + c) + a)^2(b*sin(d*x + c) + a)^m*a*b^4*m^4 + 885*(b*sin(d*x + c) + a)*
(b*sin(d*x + c) + a)^m*a^2*b^4*m^4 - 295*(b*sin(d*x + c) + a)*(b*sin(d*x +
c) + a)^m*b^6*m^4 + 735*(b*sin(d*x + c) + a)^7*(b*sin(d*x + c) + a)^m*m^3 -
4920*(b*sin(d*x + c) + a)^6*(b*sin(d*x + c) + a)^m*a*m^3 + 13875*(b*sin(d*
x + c) + a)^5*(b*sin(d*x + c) + a)^m*a^2*m^3 - 21120*(b*sin(d*x + c) + a)^4
*(b*sin(d*x + c) + a)^m*a^3*m^3 + 18285*(b*sin(d*x + c) + a)^3*(b*sin(d*x +
c) + a)^m*a^4*m^3 - 8520*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^5
m^3 + 1665(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^6*m^3 - 2775*(b*s
in(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*b^2*m^3 + 12672*(b*sin(d*x + c) +
a)^4*(b*sin(d*x + c) + a)^m*a*b^2*m^3 - 21942*(b*sin(d*x + c) + a)^3*(b*si
n(d*x + c) + a)^m*a^2*b^2*m^3 + 17040*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c
) + a)^m*a^3*b^2*m^3 - 4995*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4
*b^2*m^3 + 3657*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*b^4*m^3 - 852
0*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a*b^4*m^3 + 4995*(b*sin(d*x
+ c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^4*m^3 - 1665*(b*sin(d*x + c) + a)*(
b*sin(d*x + c) + a)^m*b^6*m^3 + 1624*(b*sin(d*x + c) + a)^7*(b*sin(d*x + c)
+ a)^m*m^2 - 11094*(b*sin(d*x + c) + a)^6*(b*sin(d*x + c) + a)^m*a*m^2 + 3
2160*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*a^2*m^2 - 50900*(b*sin(d
x + c) + a)^4(b*sin(d*x + c) + a)^m*a^3*m^2 + 46680*(b*sin(d*x + c) + a)^
3*(b*sin(d*x + c) + a)^m*a^4*m^2 - 23574*(b*sin(d*x + c) + a)^2*(b*sin(d*x
+ c) + a)^m*a^5*m^2 + 5104*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^6*

m^2 - 6432*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*b^2*m^2 + 30540*(b*sin(d*x + c) + a)^4*(b*sin(d*x + c) + a)^m*a*b^2*m^2 - 56016*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a^2*b^2*m^2 + 47148*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^3*b^2*m^2 - 15312*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*b^2*m^2 + 9336*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*b^4*m^2 - 23574*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a*b^4*m^2 + 15312*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^4*m^2 - 5104*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^6*m^2 + 1764*(b*sin(d*x + c) + a)^7*(b*sin(d*x + c) + a)^m*m - 12228*(b*sin(d*x + c) + a)^6*(b*sin(d*x + c) + a)^m*a*m + 36180*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*a^2*m - 59040*(b*sin(d*x + c) + a)^4*(b*sin(d*x + c) + a)^m*a^3*m + 56940*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a^4*m - 31644*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^5*m + 8028*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^6*m - 7236*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*b^2*m + 35424*(b*sin(d*x + c) + a)^4*(b*sin(d*x + c) + a)^m*a*b^2*m - 68328*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a^2*b^2*m + 63288*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^3*b^2*m - 24084*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*b^2*m + 11388*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*b^4*m - 31644*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a*b^4*m + 24084*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^4*m - 8028*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^6*m + 720*(b*sin(d*x + c) + a)^7*(b*sin(d*x + c) + a)^m - 5040*(b*sin(d*x + c) + a)^6*(b*sin(d*x + c) + a)^m*a + 15120*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*a^2 - 25200*(b*sin(d*x + c) + a)^4*(b*sin(d*x + c) + a)^m*a^3 + 25200*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a^4 - 15120*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^5 + 5040*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^6 - 3024*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*b^2 + 15120*(b*sin(d*x + c) + a)^4*(b*sin(d*x + c) + a)^m*a*b^2 - 30240*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a^2*b^2 + 30240*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^3*b^2 - 15120*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*b^2 + 5040*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*b^4 - 15120*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a*b^4 + 15120*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^4 - 5040*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^6)/((b^6*m^7 + 28*b^6*m^6 + 322*b^6*m^5 + 1960*b^6*m^4 + 6769*b^6*m^3 + 13132*b^6*m^2 + 13068*b^6*m + 5040*b^6)*b*d)

3.631 $\int \cos^5(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=167

$$\frac{(a^2-b^2)^2(a+b\sin(c+dx))^{m+1}}{b^5d(m+1)}-\frac{4a(a^2-b^2)(a+b\sin(c+dx))^{m+2}}{b^5d(m+2)}+\frac{2(3a^2-b^2)(a+b\sin(c+dx))^{m+3}}{b^5d(m+3)}-\frac{4a(a+b}{b^5}$$

```
[Out] ((a^2 - b^2)^2*(a + b*Sin[c + d*x])^(1 + m))/(b^5*d*(1 + m)) - (4*a*(a^2 -
b^2)*(a + b*Sin[c + d*x])^(2 + m))/(b^5*d*(2 + m)) + (2*(3*a^2 - b^2)*(a +
b*Sin[c + d*x])^(3 + m))/(b^5*d*(3 + m)) - (4*a*(a + b*Sin[c + d*x])^(4 + m
))/(b^5*d*(4 + m)) + (a + b*Sin[c + d*x])^(5 + m)/(b^5*d*(5 + m))
```

Rubi [A] time = 0.112014, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{(a^2-b^2)^2(a+b\sin(c+dx))^{m+1}}{b^5d(m+1)}-\frac{4a(a^2-b^2)(a+b\sin(c+dx))^{m+2}}{b^5d(m+2)}+\frac{2(3a^2-b^2)(a+b\sin(c+dx))^{m+3}}{b^5d(m+3)}-\frac{4a(a+b}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^m,x]
```


```
[Out] ((a^2 - b^2)^2*(a + b*Sin[c + d*x])^(1 + m))/(b^5*d*(1 + m)) - (4*a*(a^2 -
b^2)*(a + b*Sin[c + d*x])^(2 + m))/(b^5*d*(2 + m)) + (2*(3*a^2 - b^2)*(a +
b*Sin[c + d*x])^(3 + m))/(b^5*d*(3 + m)) - (4*a*(a + b*Sin[c + d*x])^(4 + m
))/(b^5*d*(4 + m)) + (a + b*Sin[c + d*x])^(5 + m)/(b^5*d*(5 + m))
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int (a+x)^m\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2(a+x)^m-4\left(a^3-ab^2\right)(a+x)^{1+m}+2\left(3a^2-b^2\right)(a+x)^{2+}\right.\right.}{b^5d}$$
$$= \frac{\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{1+m}}{b^5d(1+m)} - \frac{4a\left(a^2-b^2\right)(a+b\sin(c+dx))^{2+m}}{b^5d(2+m)} + \frac{2\left(3a^2\right.}{}$$

Mathematica [A] time = 0.914049, size = 169, normalized size = 1.01

$$\frac{(a+b\sin(c+dx))^{m+1}\left(4\left(b^2-a^2\right)\left(\frac{b^2-a^2}{m+1}-\frac{(a+b\sin(c+dx))^2}{m+3}+\frac{2a(a+b\sin(c+dx))}{m+2}\right)+4a(a+b\sin(c+dx))\left(\frac{b^2-a^2}{m+2}-\frac{(a+b\sin(c+dx))^2}{m+4}\right.\right.}{b^5d(m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a + b*Sin[c + d*x])^(1 + m)*(b^4*Cos[c + d*x]^4 + 4*(-a^2 + b^2)*((-a^2 +
 b^2)/(1 + m) + (2*a*(a + b*Sin[c + d*x]))/(2 + m) - (a + b*Sin[c + d*x])^2
/(3 + m)) + 4*a*(a + b*Sin[c + d*x])*((-a^2 + b^2)/(2 + m) + (2*a*(a + b*Si
n[c + d*x]))/(3 + m) - (a + b*Sin[c + d*x])^2/(4 + m))))/(b^5*d*(5 + m))
```

Maple [F] time = 0.238, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^5(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.069, size = 825, normalized size = 4.94

$$\left(24\,a^5 - 80\,a^3b^2 + 120\,ab^4 + \left(ab^4m^4 + 6\,ab^4m^3 + 11\,ab^4m^2 + 6\,ab^4m\right)\cos\left(dx+c\right)^4 + 8\left(a^3b^2 + 3\,ab^4\right)m^2 + 4\left(2\,ab^4m^3 -\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (24*a^5 - 80*a^3*b^2 + 120*a*b^4 + (a*b^4*m^4 + 6*a*b^4*m^3 + 11*a*b^4*m^2
+ 6*a*b^4*m)*cos(d*x + c)^4 + 8*(a^3*b^2 + 3*a*b^4)*m^2 + 4*(2*a*b^4*m^3 -
3*(a^3*b^2 - 3*a*b^4)*m^2 - (3*a^3*b^2 - 7*a*b^4)*m)*cos(d*x + c)^2 - 24*(a
^3*b^2 - 5*a*b^4)*m + (64*b^5 + (b^5*m^4 + 10*b^5*m^3 + 35*b^5*m^2 + 50*b^5
*m + 24*b^5)*cos(d*x + c)^4 + 8*(3*a^2*b^3 + b^5)*m^2 + 4*(8*b^5 + (a^2*b^3
 + b^5)*m^3 + (3*a^2*b^3 + 7*b^5)*m^2 + 2*(a^2*b^3 + 7*b^5)*m)*cos(d*x + c)
^2 - 24*(a^4*b - 3*a^2*b^3 - 2*b^5)*m)*sin(d*x + c))*(b*sin(d*x + c) + a)^m
/(b^5*d*m^5 + 15*b^5*d*m^4 + 85*b^5*d*m^3 + 225*b^5*d*m^2 + 274*b^5*d*m + 1
20*b^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14003, size = 1904, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] ((b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*m^4 - 4*(b*sin(d*x + c) + a)
^4*(b*sin(d*x + c) + a)^m*a*m^4 + 6*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c)
+ a)^m*a^2*m^4 - 4*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^3*m^4 +
(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*m^4 - 2*(b*sin(d*x + c) + a
)^3*(b*sin(d*x + c) + a)^m*b^2*m^4 + 4*(b*sin(d*x + c) + a)^2*(b*sin(d*x +
c) + a)^m*a*b^2*m^4 - 2*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^2
*m^4 + (b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^4*m^4 + 10*(b*sin(d*x
+ c) + a)^5*(b*sin(d*x + c) + a)^m*m^3 - 44*(b*sin(d*x + c) + a)^4*(b*sin(d
*x + c) + a)^m*a*m^3 + 72*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a^2
*m^3 - 52*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^3*m^3 + 14*(b*sin
(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*m^3 - 24*(b*sin(d*x + c) + a)^3*(
b*sin(d*x + c) + a)^m*b^2*m^3 + 52*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) +
 a)^m*a*b^2*m^3 - 28*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^2*m^
3 + 14*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^4*m^3 + 35*(b*sin(d*x
+ c) + a)^5*(b*sin(d*x + c) + a)^m*m^2 - 164*(b*sin(d*x + c) + a)^4*(b*sin(
d*x + c) + a)^m*a*m^2 + 294*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a
^2*m^2 - 236*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^3*m^2 + 71*(b*
sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*m^2 - 98*(b*sin(d*x + c) + a)^
3*(b*sin(d*x + c) + a)^m*b^2*m^2 + 236*(b*sin(d*x + c) + a)^2*(b*sin(d*x +
c) + a)^m*a*b^2*m^2 - 142*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b
^2*m^2 + 71*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^4*m^2 + 50*(b*sin
(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*m - 244*(b*sin(d*x + c) + a)^4*(b*s
in(d*x + c) + a)^m*a*m + 468*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*
a^2*m - 428*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^3*m + 154*(b*si
n(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*m - 156*(b*sin(d*x + c) + a)^3*(
b*sin(d*x + c) + a)^m*b^2*m + 428*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) +
a)^m*a*b^2*m - 308*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^2*m +
154*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^4*m + 24*(b*sin(d*x + c)
+ a)^5*(b*sin(d*x + c) + a)^m - 120*(b*sin(d*x + c) + a)^4*(b*sin(d*x + c)
+ a)^m*a + 240*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a^2 - 240*(b*s
in(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^3 + 120*(b*sin(d*x + c) + a)*(b
*sin(d*x + c) + a)^m*a^4 - 80*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m
*b^2 + 240*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a*b^2 - 240*(b*sin
(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^2 + 120*(b*sin(d*x + c) + a)*(b
*sin(d*x + c) + a)^m*b^4)/((b^4*m^5 + 15*b^4*m^4 + 85*b^4*m^3 + 225*b^4*m^2
```

```
+ 274*b^4*m + 120*b^4)*b*d)
```

3.632 $\int \cos^3(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=92

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^{m+1}}{b^3d(m+1)}+\frac{2a(a+b\sin(c+dx))^{m+2}}{b^3d(m+2)}-\frac{(a+b\sin(c+dx))^{m+3}}{b^3d(m+3)}$$

```
[Out] -(((a^2 - b^2)*(a + b*Sin[c + d*x])^(1 + m))/(b^3*d*(1 + m))) + (2*a*(a + b
*Sin[c + d*x])^(2 + m))/(b^3*d*(2 + m)) - (a + b*Sin[c + d*x])^(3 + m)/(b^3
*d*(3 + m))
```

Rubi [A] time = 0.0719789, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^{m+1}}{b^3d(m+1)}+\frac{2a(a+b\sin(c+dx))^{m+2}}{b^3d(m+2)}-\frac{(a+b\sin(c+dx))^{m+3}}{b^3d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((a^2 - b^2)*(a + b*Sin[c + d*x])^(1 + m))/(b^3*d*(1 + m))) + (2*a*(a + b
*Sin[c + d*x])^(2 + m))/(b^3*d*(2 + m)) - (a + b*Sin[c + d*x])^(3 + m)/(b^3
*d*(3 + m))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int (a+x)^m\left(b^2-x^2\right)\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^m + 2a(a+x)^{1+m} - (a+x)^{2+m}\right)\,dx, x, b\sin(c+d\right.}{b^3d}$$
$$= -\frac{\left(a^2-b^2\right)(a+b\sin(c+dx))^{1+m}}{b^3d(1+m)} + \frac{2a(a+b\sin(c+dx))^{2+m}}{b^3d(2+m)} - \frac{(a+b\sin(c+}{b^3d(3+}$$

Mathematica [A] time = 0.263323, size = 74, normalized size = 0.8

$$\frac{(a+b\sin(c+dx))^{m+1}\left(\frac{b^2-a^2}{m+1} - \frac{(a+b\sin(c+dx))^2}{m+3} + \frac{2a(a+b\sin(c+dx))}{m+2}\right)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a + b*Sin[c + d*x])^(1 + m)*((-a^2 + b^2)/(1 + m) + (2*a*(a + b*Sin[c + d
*x]))/(2 + m) - (a + b*Sin[c + d*x])^2/(3 + m)))/(b^3*d)
```

Maple [F] time = 0.204, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^3 (a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44343, size = 311, normalized size = 3.38

$$\frac{\left(4\,ab^2m-2\,a^3+6\,ab^2+\left(ab^2m^2+ab^2m\right)\cos\left(dx+c\right)^2+\left(4\,b^3+\left(b^3m^2+3\,b^3m+2\,b^3\right)\cos\left(dx+c\right)^2+2\left(a^2b+b^3\right)m\right)\sin\left(dx+c\right)\right)\left(b\sin\left(dx+c\right)+a\right)^m}{b^3dm^3+6\,b^3dm^2+11\,b^3dm+6\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (4*a*b^2*m - 2*a^3 + 6*a*b^2 + (a*b^2*m^2 + a*b^2*m)*cos(d*x + c)^2 + (4*b^
3 + (b^3*m^2 + 3*b^3*m + 2*b^3)*cos(d*x + c)^2 + 2*(a^2*b + b^3)*m)*sin(d*x
 + c))*(b*sin(d*x + c) + a)^m/(b^3*d*m^3 + 6*b^3*d*m^2 + 11*b^3*d*m + 6*b^3
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1189, size = 504, normalized size = 5.48

$$-\frac{\left(b\sin\left(dx+c\right)+a\right)^3\left(b\sin\left(dx+c\right)+a\right)^m m^2-2\left(b\sin\left(dx+c\right)+a\right)^2\left(b\sin\left(dx+c\right)+a\right)^m am^2+\left(b\sin\left(dx+c\right)+a\right)\left(b\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] -((b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*m^2 - 2*(b*sin(d*x + c) + a
)^2*(b*sin(d*x + c) + a)^m*a*m^2 + (b*sin(d*x + c) + a)*(b*sin(d*x + c) + a
)^m*a^2*m^2 - (b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^2*m^2 + 3*(b*si
n(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*m - 8*(b*sin(d*x + c) + a)^2*(b*si
n(d*x + c) + a)^m*a*m + 5*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*m
 - 5*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^2*m + 2*(b*sin(d*x + c)
+ a)^3*(b*sin(d*x + c) + a)^m - 6*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) +
a)^m*a + 6*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2 - 6*(b*sin(d*x +
 c) + a)*(b*sin(d*x + c) + a)^m*b^2)/((b^2*m^3 + 6*b^2*m^2 + 11*b^2*m + 6*b
^2)*b*d)
```

3.633 $\int \cos(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=26

$$\frac{(a+b\sin(c+dx))^{m+1}}{bd(m+1)}$$

```
[Out] (a + b*Sin[c + d*x])^(1 + m)/(b*d*(1 + m))
```

Rubi [A] time = 0.026876, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 32}

$$\frac{(a+b\sin(c+dx))^{m+1}}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (a + b*Sin[c + d*x])^(1 + m)/(b*d*(1 + m))
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^m\,dx&=\frac{\operatorname{Subst}\left(\int (a+x)^m\,dx,x,b\sin(c+dx)\right)}{bd}\\&=\frac{(a+b\sin(c+dx))^{1+m}}{bd(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0248363, size = 26, normalized size = 1.

$$\frac{(a+b\sin(c+dx))^{m+1}}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (a + b*Sin[c + d*x])^(1 + m)/(b*d*(1 + m))
```

Maple [A] time = 0.009, size = 27, normalized size = 1.

$$\frac{(a+b\sin(dx+c))^{1+m}}{bd(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] (a+b*sin(d*x+c))^(1+m)/b/d/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40809, size = 80, normalized size = 3.08

$$\frac{(b\sin(dx+c)+a)(b\sin(dx+c)+a)^{m}}{bdm+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m/(b*d*m + b*d)
```

Sympy [A] time = 2.64656, size = 99, normalized size = 3.81

$$\begin{cases} \frac{x\cos(c)}{a} & \text{for } b = 0 \wedge d = 0 \wedge m = -1 \\ \frac{a^m \sin(c+dx)}{d} & \text{for } b = 0 \\ x\left(a + b\sin(c)\right)^m \cos(c) & \text{for } d = 0 \\ \frac{\log\left(\frac{a}{b}+\sin(c+dx)\right)}{bd} & \text{for } m = -1 \\ \frac{a(a+b\sin(c+dx))^m}{bdm+bd} + \frac{b(a+b\sin(c+dx))^m \sin(c+dx)}{bdm+bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*cos(c)/a, Eq(b, 0) & Eq(d, 0) & Eq(m, -1)), (a**m*sin(c + d*x)
/d, Eq(b, 0)), (x*(a + b*sin(c))**m*cos(c), Eq(d, 0)), (log(a/b + sin(c + d
*x))/(b*d), Eq(m, -1)), (a*(a + b*sin(c + d*x))**m/(b*d*m + b*d) + b*(a + b
*sin(c + d*x))**m*sin(c + d*x)/(b*d*m + b*d), True))
```

Giac [A] time = 1.10451, size = 35, normalized size = 1.35

$$\frac{(b\sin(dx+c)+a)^{m+1}}{bd(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] (b*sin(d*x + c) + a)^(m + 1)/(b*d*(m + 1))
```

3.634 $\int \sec(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=115

$$\frac{(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a+b}\right)}{2d(m+1)(a+b)}-\frac{(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)}{2d(m+1)(a-b)}$$

```
[Out] -(Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a - b)]*(a + b*S
in[c + d*x])^(1 + m))/(2*(a - b)*d*(1 + m)) + (Hypergeometric2F1[1, 1 + m,
2 + m, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m))/(2*(a +
b)*d*(1 + m))
```

Rubi [A] time = 0.114643, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2668, 712, 68}

$$\frac{(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a+b}\right)}{2d(m+1)(a+b)}-\frac{(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)}{2d(m+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^m,x]
```


```
[Out] -(Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a - b)]*(a + b*S
in[c + d*x])^(1 + m))/(2*(a - b)*d*(1 + m)) + (Hypergeometric2F1[1, 1 + m,
2 + m, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m))/(2*(a +
b)*d*(1 + m))
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+b\sin(c+dx))^m\,dx &= \frac{b\,\mathrm{Subst}\left(\int \frac{(a+x)^m}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{b\,\mathrm{Subst}\left(\int \left(\frac{(a+x)^m}{2b(b-x)}+\frac{(a+x)^m}{2b(b+x)}\right)dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{\mathrm{Subst}\left(\int \frac{(a+x)^m}{b-x}\,dx, x, b\sin(c+dx)\right)}{2d}+\frac{\mathrm{Subst}\left(\int \frac{(a+x)^m}{b+x}\,dx, x, b\sin(c+dx)\right)}{2d}\\ &= -\frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a-b}\right)(a+b\sin(c+dx))^{1+m}}{2(a-b)d(1+m)}+\frac{{}_2F_1\left(1,1+m;2+m\right.}{}\end{aligned}$$

Mathematica [A] time = 0.11361, size = 99, normalized size = 0.86

$$-\frac{(a+b\sin(c+dx))^{m+1}\left((a+b)\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)+(b-a)\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a+b}\right)\right)}{2d(m+1)(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((a + b)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a - b)]
 + (-a + b)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)
])*(a + b*Sin[c + d*x])^(1 + m))/(2*(a - b)*(a + b)*d*(1 + m))
```

Maple [F] time = 0.632, size = 0, normalized size = 0.

$$\int \sec(dx+c)\,(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^m \sec(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sin(dx+c)+a)^m \sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sin(c+dx))^m \sec(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**m*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^m\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

3.635 $\int \sec^3(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=183

$$-\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{m+1}}{2d\left(a^2-b^2\right)}-\frac{(a-b(1-m))(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)}{4d(m+1)(a-b)^2}$$

```
[Out] -((a - b*(1 - m))*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(
a - b)]*(a + b*Sin[c + d*x])^(1 + m))/(4*(a - b)^2*d*(1 + m)) + ((a + b - b
*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)]*(a + b
*Sin[c + d*x])^(1 + m))/(4*(a + b)^2*d*(1 + m)) - (Sec[c + d*x]^2*(b - a*Si
n[c + d*x])*(a + b*Sin[c + d*x])^(1 + m))/(2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.226047, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2668, 741, 831, 68}

$$-\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{m+1}}{2d\left(a^2-b^2\right)}-\frac{(a-b(1-m))(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)}{4d(m+1)(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -((a - b*(1 - m))*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(
a - b)]*(a + b*Sin[c + d*x])^(1 + m))/(4*(a - b)^2*d*(1 + m)) + ((a + b - b
*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)]*(a + b
*Sin[c + d*x])^(1 + m))/(4*(a + b)^2*d*(1 + m)) - (Sec[c + d*x]^2*(b - a*Si
n[c + d*x])*(a + b*Sin[c + d*x])^(1 + m))/(2*(a^2 - b^2)*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
```

```
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{b^3\,\mathrm{Subst}\left(\int \frac{(a+x)^m}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{2\left(a^2-b^2\right)d} + \frac{b\,\mathrm{Subst}\left(\int \frac{(a+x)^m\left(a^2-b^2\right.}{b^2-}\right.}{2\left(\right.}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{2\left(a^2-b^2\right)d} + \frac{b\,\mathrm{Subst}\left(\int\left(\frac{\left(b\left(a^2-b^2(1-\right.\right.}{2}\right.\right.}{}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{2\left(a^2-b^2\right)d} + \frac{((a+b)(a-b(1-m)))}{}$$

$$= -\frac{(a-b(1-m))\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a-b}\right)(a+b\sin(c+dx))^{1+m}}{4(a-b)^2d(1+m)} + \frac{(a+}{}$$

Mathematica [A] time = 0.566822, size = 157, normalized size = 0.86

$$\frac{(a+b\sin(c+dx))^{m+1}\left(\frac{b\left((a+b)^2(a+b(m-1))\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)-(a-b)^2(a-bm+b)\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a+b}\right)\right)}{(m+1)(a-b)(a+b)}+2b\sec^2(c+dx)(\right.}{4bd\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a + b*Sin[c + d*x])^(1 + m)*((b*((a + b)^2*(a + b*(-1 + m))*Hypergeometri
c2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a - b)] - (a - b)^2*(a + b - b*
m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)]))/((a -
 b)*(a + b)*(1 + m)) + 2*b*Sec[c + d*x]^2*(b - a*Sin[c + d*x])))/(4*b*(-a^2
 + b^2)*d)
```

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^3\,(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^m\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sin(dx+c)+a)^m\sec(dx+c)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^m\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

3.636 $\int \sec^5(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=305

$$-\frac{\left(3a^2-3ab(2-m)+b^2\left(m^2-4m+3\right)\right)(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)}{16d(m+1)(a-b)^3}+\frac{\left(3a^2+3ab(2-m)\right.}{}$$

```
[Out] -((3*a^2 - 3*a*b*(2 - m) + b^2*(3 - 4*m + m^2))*Hypergeometric2F1[1, 1 + m,
 2 + m, (a + b*Sin[c + d*x])/(a - b)]*(a + b*Sin[c + d*x])^(1 + m))/(16*(a
- b)^3*d*(1 + m)) + ((3*a^2 + 3*a*b*(2 - m) + b^2*(3 - 4*m + m^2))*Hypergeo
metric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d*x
])^(1 + m))/(16*(a + b)^3*d*(1 + m)) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x])
*(a + b*Sin[c + d*x])^(1 + m))/(4*(a^2 - b^2)*d) + (Sec[c + d*x]^2*(a + b*S
in[c + d*x])^(1 + m)*(b*(b^2*(3 - m) - a^2*(1 + m)) + a*(3*a^2 - b^2*(5 - 2
*m))*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.420683, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2668, 741, 823, 831, 68}

$$-\frac{\left(3a^2-3ab(2-m)+b^2\left(m^2-4m+3\right)\right)(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)}{16d(m+1)(a-b)^3}+\frac{\left(3a^2+3ab(2-m)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^m,x]
```


```
[Out] -((3*a^2 - 3*a*b*(2 - m) + b^2*(3 - 4*m + m^2))*Hypergeometric2F1[1, 1 + m,
 2 + m, (a + b*Sin[c + d*x])/(a - b)]*(a + b*Sin[c + d*x])^(1 + m))/(16*(a
- b)^3*d*(1 + m)) + ((3*a^2 + 3*a*b*(2 - m) + b^2*(3 - 4*m + m^2))*Hypergeo
metric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d*x
])^(1 + m))/(16*(a + b)^3*d*(1 + m)) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x])
*(a + b*Sin[c + d*x])^(1 + m))/(4*(a^2 - b^2)*d) + (Sec[c + d*x]^2*(a + b*S
in[c + d*x])^(1 + m)*(b*(b^2*(3 - m) - a^2*(1 + m)) + a*(3*a^2 - b^2*(5 - 2
*m))*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```


Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{b^5\,\mathrm{Subst}\left(\int \frac{(a+x)^m}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{4(a^2-b^2)\,d} + \frac{b^3\,\mathrm{Subst}\left(\int \frac{(a+x)^m(3a}{}\right.}{}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{4(a^2-b^2)\,d} + \frac{\sec^2(c+dx)(a+b\,\mathrm{si}}{}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{4(a^2-b^2)\,d} + \frac{\sec^2(c+dx)(a+b\,\mathrm{si}}{}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{4(a^2-b^2)\,d} + \frac{\sec^2(c+dx)(a+b\,\mathrm{si}}{}$$

$$= -\frac{\left(3a^2-3ab(2-m)+b^2\left(3-4m+m^2\right)\right)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a-b}\right)(a}{16(a-b)^3d(1+m)}$$

Mathematica [A] time = 4.06833, size = 260, normalized size = 0.85

$$\frac{(a+b\sin(c+dx))^{m+1}\left(\frac{(a+b)^3\left(3a^2+3ab(m-2)+b^2\left(m^2-4m+3\right)\right)\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)-(a-b)^3\left(3a^2-3ab(m-2)+b^2\left(m^2-4m+3\right)\right)\,{}_2F_1\left(1,m+1;m+\right.}{(m+1)(a-b)(a+b)\left(a^2-b^2\right)}\right.}{16d\left(b^2-a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a + b*Sin[c + d*x])^(1 + m)*(((a + b)^3*(3*a^2 + 3*a*b*(-2 + m) + b^2*(3
- 4*m + m^2))*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a -
b)] - (a - b)^3*(3*a^2 - 3*a*b*(-2 + m) + b^2*(3 - 4*m + m^2))*Hypergeometr
ic2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)])/((a - b)*(a + b)*(a^2
 - b^2)*(1 + m)) + 4*Sec[c + d*x]^4*(b - a*Sin[c + d*x]) + (2*Sec[c + d*x]^
2*(b^3*(-3 + m) + a^2*b*(1 + m) - a*(3*a^2 + b^2*(-5 + 2*m))*Sin[c + d*x]))
/(a^2 - b^2)))/(16*(-a^2 + b^2)*d)
```

Maple [F] time = 0.563, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{5} (a+b\sin (dx+c))^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin (dx+c)+a)^{m} \sec (dx+c)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sin (dx+c)+a)^{m} \sec (dx+c)^{5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^m \sec(dx+c)^5\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

3.637 $\int \cos^4(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=129

$$\frac{\cos^3(c+dx)(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{3}{2},-\frac{3}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}$$

```
[Out] (AppellF1[1 + m, -3/2, -3/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(1 + m))/(b*d*(1 +
 m)*(1 - (a + b*Sin[c + d*x])/(a - b))^(3/2)*(1 - (a + b*Sin[c + d*x])/(a +
 b))^(3/2))
```

Rubi [A] time = 0.0940427, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2704, 138}

$$\frac{\cos^3(c+dx)(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{3}{2},-\frac{3}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, -3/2, -3/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(1 + m))/(b*d*(1 +
 m)*(1 - (a + b*Sin[c + d*x])/(a - b))^(3/2)*(1 - (a + b*Sin[c + d*x])/(a +
 b))^(3/2))
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\cos^3(c+dx)\,\mathrm{Subst}\left(\int (a+bx)^m\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{3/2}\left(\frac{b}{a+b}-\frac{bx}{a+b}\right)^{3/2}dx,x,\sin(c+d}{d\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}$$

$$= \frac{F_1\left(1+m;-\frac{3}{2},-\frac{3}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\cos^3(c+dx)(a+b\sin(c+}{bd(1+m)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}$$

Mathematica [F] time = 4.11705, size = 0, normalized size = 0.

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^m, x]
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^4\,(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^m\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sin(dx+c)+a)^m\cos(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)+a)^m\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

3.638 $\int \cos^2(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=127

$$\frac{\cos(c+dx)(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] (AppellF1[1 + m, -1/2, -1/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^(1 + m))/(b*d*(1 + m
)*Sqrt[1 - (a + b*Sin[c + d*x])/(a - b)]*Sqrt[1 - (a + b*Sin[c + d*x])/(a +
 b)])
```

Rubi [A] time = 0.0889027, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2704, 138}

$$\frac{\cos(c+dx)(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, -1/2, -1/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^(1 + m))/(b*d*(1 + m
)*Sqrt[1 - (a + b*Sin[c + d*x])/(a - b)]*Sqrt[1 - (a + b*Sin[c + d*x])/(a +
 b)])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\cos(c+dx)\operatorname{Subst}\left(\int (a+bx)^m\sqrt{-\frac{b}{a-b}-\frac{bx}{a-b}}\sqrt{\frac{b}{a+b}-\frac{bx}{a+b}}\,dx,x,\sin(c+dx)\right)}{d\sqrt{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

$$= \frac{F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\cos(c+dx)(a+b\sin(c+d}{bd(1+m)\sqrt{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [F] time = 5.86056, size = 0, normalized size = 0.

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^m, x]
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2\,(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sin (dx + c) + a)^m \cos (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sin (dx + c) + a)^m \cos (dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sin (dx + c) + a)^m \cos (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

3.639 $\int \sec^2(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=129

$$\frac{\sec^3(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{3}{2},\frac{3}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)}$$

```
[Out] (AppellF1[1 + m, 3/2, 3/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[
c + d*x])/(a + b)]*Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*
Sin[c + d*x])/(a - b))^(3/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(3/2))/(b*d
*(1 + m))
```

Rubi [A] time = 0.086305, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2704, 138}

$$\frac{\sec^3(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{3}{2},\frac{3}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 3/2, 3/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[
c + d*x])/(a + b)]*Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*
Sin[c + d*x])/(a - b))^(3/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(3/2))/(b*d
*(1 + m))
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\left(\sec^3(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}\right)\mathrm{Subst}\left(\int \frac{(a+bx)}{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{3/2}\left(\right.}\right.}{d}$$

$$= \frac{F_1\left(1+m;\frac{3}{2},\frac{3}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec^3(c+dx)(a+b\sin(c+dx))}{bd(1+m)}$$

Mathematica [F] time = 2.08265, size = 0, normalized size = 0.

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^m, x]
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin (dx+c)+a)^m \sec (dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sin (dx+c)+a)^m \sec (dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin (dx+c)+a)^m \sec (dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

3.640 $\int \sec^4(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=129

$$\frac{\sec^5(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{5}{2},\frac{5}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)}$$

```
[Out] (AppellF1[1 + m, 5/2, 5/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[
c + d*x])/(a + b)]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*
Sin[c + d*x])/(a - b))^(5/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(5/2))/(b*d
*(1 + m))
```

Rubi [A] time = 0.0859064, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2704, 138}

$$\frac{\sec^5(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{5}{2},\frac{5}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 5/2, 5/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[
c + d*x])/(a + b)]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*
Sin[c + d*x])/(a - b))^(5/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(5/2))/(b*d
*(1 + m))
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\left(\sec^5(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{(a+bx)}{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{5/2}\left(\cdots\right.}\right.}{d}$$

$$= \frac{F_1\left(1+m;\frac{5}{2},\frac{5}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec^5(c+dx)(a+b\sin(c+dx))\cdots}{bd(1+m)}$$

Mathematica [F] time = 3.88072, size = 0, normalized size = 0.

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^m, x]
```

Maple [F] time = 0.229, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^4 (a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin (dx+c)+a)^m \sec (dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sin (dx+c)+a)^m \sec (dx+c)^4 , x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sin (dx+c)+a)^m \sec (dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

3.641 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=134

$$\frac{e(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{3}{4},-\frac{3}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}}$$

```
[Out] (e*AppellF1[1 + m, -3/4, -3/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*
Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^(1 + m))
/(b*d*(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^(3/4)*(1 - (a + b*Sin[c +
d*x])/(a + b))^(3/4))
```

Rubi [A] time = 0.100995, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2704, 138}

$$\frac{e(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{3}{4},-\frac{3}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^m,x]
```


```
[Out] (e*AppellF1[1 + m, -3/4, -3/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*
Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^(1 + m))
/(b*d*(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^(3/4)*(1 - (a + b*Sin[c +
d*x])/(a + b))^(3/4))
```


Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{3/2}\right)\operatorname{Subst}\left(\int (a+bx)^m\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{3/4}\left(\frac{b}{a+b}-\frac{bx}{a+b}\right)^{3/4}dx,\right.}{d\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}}$$

$$= \frac{eF_1\left(1+m;-\frac{3}{4},-\frac{3}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{3/2}(a+}{bd(1+m)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}}$$

Mathematica [F] time = 56.6606, size = 0, normalized size = 0.

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^m, x]
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^m e^2\cos(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m*e^2*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(b*sin(d*x + c) + a)^m, x)
```

3.642 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=134

$$\frac{e\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{1}{4},-\frac{1}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] (e*AppellF1[1 + m, -1/4, -1/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*
Sin[c + d*x])/(a + b)]*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^(1 + m))/(
b*d*(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^(1/4)*(1 - (a + b*Sin[c + d*
x])/(a + b))^(1/4))
```

Rubi [A] time = 0.0990883, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2704, 138}

$$\frac{e\sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{1}{4},-\frac{1}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^m,x]
```


```
[Out] (e*AppellF1[1 + m, -1/4, -1/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*
Sin[c + d*x])/(a + b)]*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^(1 + m))/(
b*d*(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^(1/4)*(1 - (a + b*Sin[c + d*
x])/(a + b))^(1/4))
```


Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^m\,dx = \frac{\left(e\sqrt{e\cos(c+dx)}\right)\mathrm{Subst}\left(\int (a+bx)^m\sqrt[4]{-\frac{b}{a-b}-\frac{bx}{a-b}}\sqrt[4]{\frac{b}{a+b}-\frac{bx}{a+b}}\,dx,x,\sin(c}{d\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

$$= \frac{eF_1\left(1+m;-\frac{1}{4},-\frac{1}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sqrt{e\cos(c+dx)}(a+b\,\mathrm{s}}{bd(1+m)\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [F] time = 5.84385, size = 0, normalized size = 0.

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^m, x]
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a+b\sin(dx+c))^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}\,(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^m e\cos(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m*e*cos(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}\,(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(b*sin(d*x + c) + a)^m, x)
```

3.643 $\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=134

$$\frac{e\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{1}{4},\frac{1}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt{e\cos(c+dx)}}$$

```
[Out] (e*AppellF1[1 + m, 1/4, 1/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(1/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(1/4))/(b*d*(1 + m)*Sqrt
[e*Cos[c + d*x]])
```

Rubi [A] time = 0.0905221, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2704, 138}

$$\frac{e\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{1}{4},\frac{1}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, 1/4, 1/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(1/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(1/4))/(b*d*(1 + m)*Sqrt
[e*Cos[c + d*x]])
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^m\,dx = \frac{\left(e\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}\right)\text{Subst}\left(\int \frac{(a+bx)^m}{\sqrt[4]{-\frac{b}{a-b}-\frac{bx}{a-b}}\sqrt[4]{\frac{b}{a+b}-\frac{bx}{a+b}}}\,dx, x, \sin\right.}{d\sqrt{e\cos(c+dx)}}$$

$$= \frac{eF_1\left(1+m;\frac{1}{4},\frac{1}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\sqrt[4]{1-}}{bd(1+m)\sqrt{e\cos(c+dx)}}$$

Mathematica [F] time = 1.98565, size = 0, normalized size = 0.

$$\int \sqrt{e\cos(c+dx)}(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^m, x]
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}\,(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))*(a + b*sin(c + d*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m, x)
```

3.644 $\int \frac{(a+b\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=134

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{3}{4},\frac{3}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{3/2}}$$

```
[Out] (e*AppellF1[1 + m, 3/4, 3/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(3/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(3/4))/(b*d*(1 + m)*(e*C
os[c + d*x])^(3/2))
```

Rubi [A] time = 0.0943236, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2704, 138}

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{3}{4},\frac{3}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (e*AppellF1[1 + m, 3/4, 3/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(3/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(3/4))/(b*d*(1 + m)*(e*C
os[c + d*x])^(3/2))
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx = \frac{\left(e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}\right)\operatorname{Subst}\left(\int \frac{(a+bx)^m}{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{3/4}\left(\frac{b}{a+b}-\frac{bx}{a+b}\right)^{3/4}}\,dx, x, \sin(c+\right.}{d(e\cos(c+dx))^{3/2}}$$

$$= \frac{eF_1\left(1+m;\frac{3}{4},\frac{3}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/}}{bd(1+m)(e\cos(c+dx))^{3/2}}$$

Mathematica [F] time = 1.75679, size = 0, normalized size = 0.

$$\int \frac{(a+b\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]], x]
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int (a+b\sin(dx+c))^m \frac{1}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sin(dx+c)+a)^m}{\sqrt{e \cos(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m/sqrt(e*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e \cos(dx+c)}(b \sin(dx+c)+a)^m}{e \cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m/(e*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b \sin(c+dx))^m}{\sqrt{e \cos(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**m/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((a + b*sin(c + d*x))**m/sqrt(e*cos(c + d*x)), x)
```

<u>**Giac**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^m}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m/sqrt(e*cos(d*x + c)), x)
```

3.645 $\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{5}{4},\frac{5}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{5/2}}$$

```
[Out] (e*AppellF1[1 + m, 5/4, 5/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(5/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(5/4))/(b*d*(1 + m)*(e*C
os[c + d*x])^(5/2))
```

Rubi [A] time = 0.099244, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2704, 138}

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{5}{4},\frac{5}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (e*AppellF1[1 + m, 5/4, 5/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(5/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(5/4))/(b*d*(1 + m)*(e*C
os[c + d*x])^(5/2))
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx = \frac{\left(e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/4}\right)\mathrm{Subst}\left(\int \frac{(a+bx)^m}{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{5/4}\left(\frac{b}{a+b}-\frac{bx}{a+b}\right)^{5/4}}\,dx,x,\sin(c+\right.}{d(e\cos(c+dx))^{5/2}}$$

$$= \frac{eF_1\left(1+m;\frac{5}{4},\frac{5}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/}}{bd(1+m)(e\cos(c+dx))^{5/2}}$$

Mathematica [F] time = 1.9744, size = 0, normalized size = 0.

$$\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2), x]
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int (a+b\sin(dx+c))^m (e\cos(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^m}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^m}{e^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m/(e^2*cos(d*x + c)^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**m/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^m}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(3/2), x)
```

3.646 $\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{7/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{7/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{7}{4},\frac{7}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{7/2}}$$

```
[Out] (e*AppellF1[1 + m, 7/4, 7/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(7/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(7/4))/(b*d*(1 + m)*(e*C
os[c + d*x])^(7/2))
```

Rubi [A] time = 0.100349, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2704, 138}

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{7/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{7/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{7}{4},\frac{7}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2),x]
```



```
[Out] (e*AppellF1[1 + m, 7/4, 7/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(7/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(7/4))/(b*d*(1 + m)*(e*C
os[c + d*x])^(7/2))
```


Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx = \frac{\left(e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{7/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{7/4}\right)\text{Subst}\left(\int \frac{(a+bx)^m}{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{7/4}\left(\frac{b}{a+b}-\frac{bx}{a+b}\right)^{7/4}}\,dx, x, \sin(c+\right.}{d(e\cos(c+dx))^{7/2}}$$
$$= \frac{eF_1\left(1+m;\frac{7}{4},\frac{7}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{7/}}{bd(1+m)(e\cos(c+dx))^{7/2}}$$

Mathematica [F] time = 2.13406, size = 0, normalized size = 0.

$$\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2),x]

[Out] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2), x]
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int (a+b\sin(dx+c))^m (e\cos(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^m}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\cos(dx+c)}(b\sin(dx+c)+a)^m}{e^3\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*cos(d*x + c))*(b*sin(d*x + c) + a)^m/(e^3*cos(d*x + c)^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**m/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^m}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^(5/2), x)
```

3.647 $\int (e\cos(c+dx))^{-4-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=598

$$-\frac{a2^{-\frac{m}{2}-\frac{1}{2}}\left(a^2(m+2)+2ab-b^2\right)(1-\sin(c+dx))^2(e\cos(c+dx))^{-m-3}\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+3}{2}}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1-}{}\right.}{de(1-m)(m+3)(a-b)(a+b)^3}$$

```
[Out] -(((e*Cos[c + d*x])^(-3 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*e*(3
+ m))) + (2*b*(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a -
 b)^2*d*e^3*(1 + m)*(3 + m)) + (a*(e*Cos[c + d*x])^(-3 - m)*(1 + Sin[c + d*
x])*(a + b*Sin[c + d*x])^(1 + m))/((a^2 - b^2)*d*e*(3 + m)) + (a*(3*b + a*(
2 + m))*(e*Cos[c + d*x])^(-3 - m)*(1 - Sin[c + d*x])*(1 + Sin[c + d*x])*(a
+ b*Sin[c + d*x])^(1 + m))/((a - b)*(a + b)^2*d*e*(1 + m)*(3 + m)) - (2^(3/
2 - m/2)*a*b*(e*Cos[c + d*x])^(-1 - m)*Hypergeometric2F1[(-1 - m)/2, (1 + m
)/2, (1 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*Sin[c + d*x]))]*(((a
 + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x]))^((1 + m)/2)*(a + b*Sin[c +
d*x])^(1 + m))/((a - b)^2*(a + b)*d*e^3*(1 + m)*(3 + m)) - (2^(-1/2 - m/2)*
a*(2*a*b - b^2 + a^2*(2 + m))*(e*Cos[c + d*x])^(-3 - m)*Hypergeometric2F1[(
1 - m)/2, (3 + m)/2, (3 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*Sin[
c + d*x]))]*(1 - Sin[c + d*x])^2*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c
 + d*x]))^((3 + m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*(a + b)^3*d*e*
(1 - m)*(3 + m))
```

Rubi [A] time = 1.01811, antiderivative size = 598, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2700, 2699, 2920, 132, 129, 155, 12}

$$-\frac{a2^{-\frac{m}{2}-\frac{1}{2}}\left(a^2(m+2)+2ab-b^2\right)(1-\sin(c+dx))^2(e\cos(c+dx))^{-m-3}\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+3}{2}}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1-}{}\right.}{de(1-m)(m+3)(a-b)(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-4 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-3 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*e*(3
+ m))) + (2*b*(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a -
 b)^2*d*e^3*(1 + m)*(3 + m)) + (a*(e*Cos[c + d*x])^(-3 - m)*(1 + Sin[c + d*
x])*(a + b*Sin[c + d*x])^(1 + m))/((a^2 - b^2)*d*e*(3 + m)) + (a*(3*b + a*(
2 + m))*(e*Cos[c + d*x])^(-3 - m)*(1 - Sin[c + d*x])*(1 + Sin[c + d*x])*(a
```

```
+ b*Sin[c + d*x])^(1 + m))/((a - b)*(a + b)^2*d*e*(1 + m)*(3 + m)) - (2^(3/
2 - m/2)*a*b*(e*Cos[c + d*x])^(-1 - m)*Hypergeometric2F1[(-1 - m)/2, (1 + m
)/2, (1 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*Sin[c + d*x]))]*(((a
 + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x]))^((1 + m)/2)*(a + b*Sin[c +
d*x])^(1 + m))/((a - b)^2*(a + b)*d*e^3*(1 + m)*(3 + m)) - (2^(-1/2 - m/2)*
a*(2*a*b - b^2 + a^2*(2 + m))*(e*Cos[c + d*x])^(-3 - m)*Hypergeometric2F1[(
1 - m)/2, (3 + m)/2, (3 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*Sin[
c + d*x]))]*(1 - Sin[c + d*x])^2*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c
 + d*x]))^((3 + m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*(a + b)^3*d*e*
(1 - m)*(3 + m))
```

Rule 2700

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1))/(f*g*(a - b)*(p + 1)), x] + (-Dist[(b*(m + p + 2))/(g^2*(a - b)*(
p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m, x], x] + Dist
[a/(g^2*(a - b)), Int[((g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m)/(1
- Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^
2, 0] && ILtQ[m + p + 2, 0]
```

Rule 2699

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1))/(f*g*(a - b)*(p + 1)), x] + Dist[a/(g^2*(a - b)), Int[((g*Cos[e +
 f*x])^(p + 2)*(a + b*Sin[e + f*x])^m)/(1 - Sin[e + f*x]), x], x] /; FreeQ[
{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && EqQ[m + p + 2, 0]
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
```

```
p + 2, 0] &&  !IntegerQ[n]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int (e\cos(c+dx))^{-4-m}(a+b\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{a\int \frac{(e\cos(c+dx))^{-2-m}(a+b\sin(c}{1-\sin(c+dx)}}{(a-b)e^2}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+dx))^{-1-m}(a+}{(a-b)^2de^3(1+m}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+dx))^{-1-m}(a+}{(a-b)^2de^3(1+m}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+dx))^{-1-m}(a+}{(a-b)^2de^3(1+m}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+dx))^{-1-m}(a+}{(a-b)^2de^3(1+m}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+dx))^{-1-m}(a+}{(a-b)^2de^3(1+m}$$

Mathematica [A] time = 6.09413, size = 826, normalized size = 1.38

$$\frac{\cos(c+dx)(a+b\sin(c+dx))^{m+1}(e\cos(c+dx))^{-m-4}}{(a-b)d(-m-3)} + \frac{2b\cos^{m+4}(c+dx)\left(\frac{2^{\frac{1}{2}(-m-1)+1}a\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{m+1}{2};\frac{1}{2}(-m-1)+1;\frac{(a-b)(1-\sin(c}{2(a+b\sin(c+}\right)}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-4 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (Cos[c + d*x]*(e*Cos[c + d*x])^(-4 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a -
 b)*d*(-3 - m)) + (2*b*Cos[c + d*x]^(4 + m)*(e*Cos[c + d*x])^(-4 - m)*((Cos
[c + d*x]^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*(-1 - m)) + (2^
(1 + (-1 - m)/2)*a*Cos[c + d*x]^(-1 - m)*Hypergeometric2F1[(-1 - m)/2, (1 +
 m)/2, 1 + (-1 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*Sin[c + d*x])
)]*(1 - Sin[c + d*x])^((-1 - m)/2 + (1 + m)/2)*(1 + Sin[c + d*x])^((-1 - m)
```

```
/2 + (1 + m)/2)*(-(((-a - b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x])))^((1
 + m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((-a - b)*(a - b)*d*(-1 - m))))/((a
- b)*(-3 - m)) + (a*Cos[c + d*x]*(e*Cos[c + d*x])^(-4 - m)*(1 - Sin[c + d*x
])^((3 + m)/2)*(1 + Sin[c + d*x])^((3 + m)/2)*(((1 - Sin[c + d*x])^((-3 - m
)/2)*(1 + Sin[c + d*x])^(1 + (-3 - m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((-a
 - b)*(-3 - m)) - (-((3*b + a*(2 + m))*(1 - Sin[c + d*x])^(1 + (-3 - m)/2)*
(1 + Sin[c + d*x])^(1 + (-3 - m)/2)*(a + b*Sin[c + d*x])^(1 + m))/(2*(-a -
b)*(1 + (-3 - m)/2)) - (2^(-1 + (-3 - m)/2)*(1 + m)*(2*a*b - b^2 + a^2*(2 +
 m))*Hypergeometric2F1[2 + (-3 - m)/2, (3 + m)/2, 3 + (-3 - m)/2, ((a - b)*
(1 - Sin[c + d*x]))/(2*(a + b*Sin[c + d*x]))]*(1 - Sin[c + d*x])^(2 + (-3 -
 m)/2)*(1 + Sin[c + d*x])^((-3 - m)/2)*(-(((-a - b)*(1 + Sin[c + d*x]))/(a
+ b*Sin[c + d*x])))^((3 + m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((-a - b)^2*(
1 + (-3 - m)/2)*(2 + (-3 - m)/2)))/((-a - b)*(-3 - m))))/((a - b)*d)
```

Maple [F] time = 0.202, size = 0, normalized size = 0.

$$\int \left(e \cos \left(dx+c\right)\right)^{-4-m} \left(a+b \sin \left(dx+c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e \cos \left(dx+c\right)\right)^{-m-4} \left(b \sin \left(dx+c\right)+a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 4)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e \cos \left(dx+c\right)\right)^{-m-4} \left(b \sin \left(dx+c\right)+a\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-m - 4)*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-4-m)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m-4} (b\sin(dx+c)+a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 4)*(b*sin(d*x + c) + a)^m, x)
```

3.648 $\int (e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=311

$$-\frac{\left(a^2(m+1)-b^2\right)(\sin(c+dx)+1)^3\sec^4(c+dx)(e\cos(c+dx))^{-m}(a+b\sin(c+dx))^{m+1}\left(\frac{(a+b)(\sin(c+dx)+1)}{(a-b)(\sin(c+dx)-1)}\right)^{\frac{m-2}{2}}{}_2F_1\left(\frac{m}{2},m\right.}{de^3m(m+1)(a-b)^3}$$

```
[Out] (Sec[c + d*x]^4*(-1 + Sin[c + d*x])*(1 + Sin[c + d*x])*(a + b*Sin[c + d*x])
^(1 + m))/((a - b)*d*e^3*(2 + m)*(e*Cos[c + d*x])^m) + ((-2*b + a*(2 + m))*
Sec[c + d*x]^4*(-1 + Sin[c + d*x])*(1 + Sin[c + d*x])^2*(a + b*Sin[c + d*x]
)^(1 + m))/((a - b)^2*d*e^3*m*(2 + m)*(e*Cos[c + d*x])^m) - ((-b^2 + a^2*(1
 + m))*Hypergeometric2F1[m/2, 1 + m, 2 + m, (-2*(a + b*Sin[c + d*x]))/((a -
 b)*(-1 + Sin[c + d*x]))]*Sec[c + d*x]^4*(1 + Sin[c + d*x])^3*(((a + b)*(1
+ Sin[c + d*x]))/((a - b)*(-1 + Sin[c + d*x])))^((-2 + m)/2)*(a + b*Sin[c +
 d*x])^(1 + m))/((a - b)^3*d*e^3*m*(1 + m)*(e*Cos[c + d*x])^m)
```

Rubi [A] time = 0.511881, antiderivative size = 420, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2700, 2698, 2920, 96, 132}

$$-\frac{b(1-\sin(c+dx))(e\cos(c+dx))^{-m-2}\left(-\frac{(a-b)(1-\sin(c+dx))}{(a+b)(\sin(c+dx)+1)}\right)^{m/2}(a+b\sin(c+dx))^{m+1}{}_2F_1\left(m+1,\frac{m+2}{2};m+2;\frac{2(a+b\sin(c+dx)}{(a+b)(\sin(c+dx)+}\right.}{de(m+1)(m+2)\left(a^2-b^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(e*Cos[c + d*x])^(-3 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-2 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*e*(2
+ m))) - (b*(e*Cos[c + d*x])^(-2 - m)*Hypergeometric2F1[1 + m, (2 + m)/2, 2
 + m, (2*(a + b*Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))]*(1 - Sin[c + d
*x])*(-(((a - b)*(1 - Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))))^(m/2)*(
a + b*Sin[c + d*x])^(1 + m))/((a^2 - b^2)*d*e*(1 + m)*(2 + m)) + (a*(e*Cos[
c + d*x])^(-2 - m)*(1 + Sin[c + d*x])*(a + b*Sin[c + d*x])^(1 + m))/((a^2 -
 b^2)*d*e*(2 + m)) + (a*(a + b + a*m)*(e*Cos[c + d*x])^(-2 - m)*Hypergeomet
ric2F1[-m/2, (2 + m)/2, (2 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*S
in[c + d*x]))]*(1 - Sin[c + d*x])*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Sin[
c + d*x]))^((2 + m)/2)*(a + b*Sin[c + d*x])^(1 + m))/(2^(m/2)*(a - b)*(a +
b)^2*d*e*m*(2 + m))
```

Rule 2700

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1))/(f*g*(a - b)*(p + 1)), x] + (-Dist[(b*(m + p + 2))/(g^2*(a - b)*(
p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m, x], x] + Dist
[a/(g^2*(a - b)), Int[((g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m)/(1
- Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^
2, 0] && ILtQ[m + p + 2, 0]
```

Rule 2698

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(1 - Sin[e + f*x])
*(a + b*Sin[e + f*x])^(m + 1)*(-(((a - b)*(1 - Sin[e + f*x]))/((a + b)*(1 +
 Sin[e + f*x]))))^(m/2)*Hypergeometric2F1[m + 1, m/2 + 1, m + 2, (2*(a + b*
Sin[e + f*x]))/((a + b)*(1 + Sin[e + f*x]))])/(f*(a + b)*(m + 1)), x] /; Fr
eeQ[{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && EqQ[m + p + 1, 0]
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
```

```
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(2+m)} + \frac{a\int \frac{(e\cos(c+dx))^{-1-m}(a+b\sin(c+d}{1-\sin(c+dx)}}{(a-b)e^2}$$

$$= -\frac{(e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(2+m)} - \frac{b(e\cos(c+dx))^{-2-m}\,{}_2F_1\Big(1}{}$$

$$= -\frac{(e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(2+m)} - \frac{b(e\cos(c+dx))^{-2-m}\,{}_2F_1\Big(1}{}$$

$$= -\frac{(e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(2+m)} - \frac{b(e\cos(c+dx))^{-2-m}\,{}_2F_1\Big(1}{}$$

Mathematica [A] time = 5.05326, size = 319, normalized size = 1.03

$$\frac{\sec^2(c+dx)(e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\left(\frac{b(\sin(c+dx)+1)(a+b\sin(c+dx))\left(\frac{(a+b)(\sin(c+dx)+1)}{(a-b)(\sin(c+dx)-1)}\right)^{m/2}{}_2F_1\left(m+1,\frac{m+2}{2};m+2;-\frac{2(a+b\sin(c+dx))}{(a-b)(\sin(c+dx)-1)}\right.}{(m+1)(a-b)}\right.}{de^3(m+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-3 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (Sec[c + d*x]^2*(a + b*Sin[c + d*x])^m*(-a - b*Sin[c + d*x] + (b*Hypergeome
tric2F1[1 + m, (2 + m)/2, 2 + m, (-2*(a + b*Sin[c + d*x]))/((a - b)*(-1 + S
in[c + d*x]))]*(1 + Sin[c + d*x])*(((a + b)*(1 + Sin[c + d*x]))/((a - b)*(-
1 + Sin[c + d*x])))^(m/2)*(a + b*Sin[c + d*x]))/((a - b)*(1 + m)) + (a*(1 -
 Sin[c + d*x])*(1 + Sin[c + d*x])*(((a + b + a*m)*Hypergeometric2F1[-m/2, (
2 + m)/2, 1 - m/2, -((a - b)*(-1 + Sin[c + d*x]))/(2*(a + b*Sin[c + d*x]))]
*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x]))^(m/2))/2^(m/2) - (m*(a
 + b*Sin[c + d*x]))/(-1 + Sin[c + d*x])))/((a + b)*m)))/((a - b)*d*e^3*(2 +
 m)*(e*Cos[c + d*x])^m)
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-3-m}(a+b\sin(dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m-3}(b\sin(dx+c)+a)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 3)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((e\cos(dx+c))^{-m-3}(b\sin(dx+c)+a)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-m - 3)*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-3-m)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m-3}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 3)*(b*sin(d*x + c) + a)^m, x)
```

3.649 $\int (e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=201

$$\frac{a2^{\frac{1}{2}-\frac{m}{2}}(e\cos(c+dx))^{-m-1}\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+1}{2}}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{m+1}{2};\frac{1-m}{2};\frac{(a-b)(1-\sin(c+dx))}{2(a+b\sin(c+dx))}\right)}{de(m+1)\left(a^2-b^2\right)}-\frac{(e\cos}{}$$

```
[Out] -(((e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*e*(1
+ m))) + (2^(1/2 - m/2)*a*(e*Cos[c + d*x])^(-1 - m)*Hypergeometric2F1[(-1 -
 m)/2, (1 + m)/2, (1 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*Sin[c +
 d*x]))]*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x]))^((1 + m)/2)*(a
 + b*Sin[c + d*x])^(1 + m))/((a^2 - b^2)*d*e*(1 + m))
```

Rubi [A] time = 0.292773, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2699, 2920, 132}

$$\frac{a2^{\frac{1}{2}-\frac{m}{2}}(e\cos(c+dx))^{-m-1}\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+1}{2}}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{m+1}{2};\frac{1-m}{2};\frac{(a-b)(1-\sin(c+dx))}{2(a+b\sin(c+dx))}\right)}{de(m+1)\left(a^2-b^2\right)}-\frac{(e\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-2 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*e*(1
+ m))) + (2^(1/2 - m/2)*a*(e*Cos[c + d*x])^(-1 - m)*Hypergeometric2F1[(-1 -
 m)/2, (1 + m)/2, (1 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*Sin[c +
 d*x]))]*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x]))^((1 + m)/2)*(a
 + b*Sin[c + d*x])^(1 + m))/((a^2 - b^2)*d*e*(1 + m))
```

Rule 2699

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1))/(f*g*(a - b)*(p + 1)), x] + Dist[a/(g^2*(a - b)), Int[((g*Cos[e +
 f*x])^(p + 2)*(a + b*Sin[e + f*x])^m)/(1 - Sin[e + f*x]), x], x] /; FreeQ[
{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && EqQ[m + p + 2, 0]
```

Rule 2920

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a
^m*g*(g*Cos[e + f*x])^(p - 1))/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
 + f*x])^((p - 1)/2)), Subst[Int[(1 + (b*x)/a)^(m + (p - 1)/2)*(1 - (b*x)/a
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(1+m)} + \frac{a\int \frac{(e\cos(c+dx))^{-m}(a+b\sin(c+dx)}{1-\sin(c+dx)}}{(a-b)e^2}$$

$$= -\frac{(e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(1+m)} + \frac{\left(a(e\cos(c+dx))^{-1-m}(1-\mathrm{s}\right.}{}$$

$$= -\frac{(e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(1+m)} + \frac{2^{\frac{1}{2}-\frac{m}{2}}a(e\cos(c+dx))^{-1-m}\,{}_2}{}$$

Mathematica [A] time = 0.940747, size = 168, normalized size = 0.84

$$-\frac{2^{\frac{1}{2}(-m-1)}(e\cos(c+dx))^{-m-1}(a+b\sin(c+dx))^{m+1}\left(2^{\frac{m+1}{2}}(a+b)-2a\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+1}{2}}{}_2F_1\left(\frac{1}{2}(-m-1),\frac{m+1}{2};\frac{1-m}{2};-\frac{(a}{}\right.\right.}{de(m+1)(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-2 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -((2^((-1 - m)/2)*(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m)*(2
^((1 + m)/2)*(a + b) - 2*a*Hypergeometric2F1[(-1 - m)/2, (1 + m)/2, (1 - m)
/2, -((a - b)*(-1 + Sin[c + d*x]))/(2*(a + b*Sin[c + d*x]))]*(((a + b)*(1 +
```

```
Sin[c + d*x]))/(a + b*Sin[c + d*x]))^((1 + m)/2)))/((a - b)*(a + b)*d*e*(1
+ m)))
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-2-m}(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m-2}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 2)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((e\cos(dx+c))^{-m-2}(b\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-m - 2)*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-2-m)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m-2}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 2)*(b*sin(d*x + c) + a)^m, x)
```

3.650 $\int (e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=132

$$\frac{e(1-\sin(c+dx))(e\cos(c+dx))^{-m-2}\left(-\frac{(a-b)(1-\sin(c+dx))}{(a+b)(\sin(c+dx)+1)}\right)^{m/2}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(m+1,\frac{m+2}{2};m+2;\frac{2(a+b\sin(c+dx))}{(a+b)(\sin(c+dx)+1)}\right)}{d(m+1)(a+b)}$$

```
[Out] (e*(e*Cos[c + d*x])^(-2 - m)*Hypergeometric2F1[1 + m, (2 + m)/2, 2 + m, (2*
(a + b*Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))]*(1 - Sin[c + d*x])*(-((
(a - b)*(1 - Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))))^(m/2)*(a + b*Sin
[c + d*x])^(1 + m))/((a + b)*d*(1 + m))
```

Rubi [A] time = 0.0654538, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2698}

$$\frac{e(1-\sin(c+dx))(e\cos(c+dx))^{-m-2}\left(-\frac{(a-b)(1-\sin(c+dx))}{(a+b)(\sin(c+dx)+1)}\right)^{m/2}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(m+1,\frac{m+2}{2};m+2;\frac{2(a+b\sin(c+dx))}{(a+b)(\sin(c+dx)+1)}\right)}{d(m+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^m,x]
```


```
[Out] (e*(e*Cos[c + d*x])^(-2 - m)*Hypergeometric2F1[1 + m, (2 + m)/2, 2 + m, (2*
(a + b*Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))]*(1 - Sin[c + d*x])*(-((
(a - b)*(1 - Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))))^(m/2)*(a + b*Sin
[c + d*x])^(1 + m))/((a + b)*d*(1 + m))
```


Rule 2698

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(1 - Sin[e + f*x])
*(a + b*Sin[e + f*x])^(m + 1)*(-(((a - b)*(1 - Sin[e + f*x]))/((a + b)*(1 +
 Sin[e + f*x]))))^(m/2)*Hypergeometric2F1[m + 1, m/2 + 1, m + 2, (2*(a + b*
Sin[e + f*x]))/((a + b)*(1 + Sin[e + f*x]))])/(f*(a + b)*(m + 1)), x] /; Fr
eeQ[{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && EqQ[m + p + 1, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^m\,dx = \frac{e(e\cos(c+dx))^{-2-m}\,{}_2F_1\left(1+m,\frac{2+m}{2};2+m;\frac{2(a+b\sin(c+dx))}{(a+b)(1+\sin(c+dx))}\right)(1-\sin(c+}{(a+b)d(1+m)}$$

Mathematica [A] time = 0.371724, size = 132, normalized size = 1.

$$-\frac{e(\sin(c+dx)+1)(e\cos(c+dx))^{-m-2}\left(\frac{(a+b)(\sin(c+dx)+1)}{(a-b)(\sin(c+dx)-1)}\right)^{m/2}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(m+1,\frac{m+2}{2};m+2;-\frac{2(a+b\sin(c+dx)}{(a-b)(\sin(c+dx)-}\right.}{d(m+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -((e*(e*Cos[c + d*x])^(-2 - m)*Hypergeometric2F1[1 + m, (2 + m)/2, 2 + m, (
-2*(a + b*Sin[c + d*x]))/((a - b)*(-1 + Sin[c + d*x]))]*(1 + Sin[c + d*x])*
(((a + b)*(1 + Sin[c + d*x]))/((a - b)*(-1 + Sin[c + d*x])))^(m/2)*(a + b*S
in[c + d*x])^(1 + m))/((a - b)*d*(1 + m)))
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-1-m}(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m-1}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 1)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e\cos\left(dx+c\right)\right)^{-m-1}\left(b\sin\left(dx+c\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-m - 1)*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-1-m)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e\cos\left(dx+c\right)\right)^{-m-1}\left(b\sin\left(dx+c\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m - 1)*(b*sin(d*x + c) + a)^m, x)
```

3.651 $\int (e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=152

$$\frac{e(e\cos(c+dx))^{-m-1}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{m+1}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{m+1}{2}}F_1\left(m+1;\frac{m+1}{2},\frac{m+1}{2};m+2;\frac{a+b\sin(c+dx)}{a-b}\right.}{bd(m+1)}$$

```
[Out] (e*AppellF1[1 + m, (1 + m)/2, (1 + m)/2, 2 + m, (a + b*Sin[c + d*x])/(a - b
), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d
*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 + m)/2)*(1 - (a + b*Sin
[c + d*x])/(a + b))^((1 + m)/2))/(b*d*(1 + m))
```

Rubi [A] time = 0.103209, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2704, 138}

$$\frac{e(e\cos(c+dx))^{-m-1}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{m+1}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{m+1}{2}}F_1\left(m+1;\frac{m+1}{2},\frac{m+1}{2};m+2;\frac{a+b\sin(c+dx)}{a-b}\right.}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, (1 + m)/2, (1 + m)/2, 2 + m, (a + b*Sin[c + d*x])/(a - b
), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d
*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 + m)/2)*(1 - (a + b*Sin
[c + d*x])/(a + b))^((1 + m)/2))/(b*d*(1 + m))
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{-1-m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1+m}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1+m}{2}}\right)\text{Subst}\left(\int\right.}{d}$$

$$= \frac{eF_1\left(1+m;\frac{1+m}{2},\frac{1+m}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1-m}(}{bd(1+m)}$$

Mathematica [F] time = 1.58231, size = 0, normalized size = 0.

$$\int (e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^m,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^m, x]
```

Maple [F] time = 0.389, size = 0, normalized size = 0.

$$\int \frac{(a+b\sin(dx+c))^m}{(e\cos(dx+c))^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x)
```

```
[Out] int((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sin(dx+c)+a)^m}{(e\cos(dx+c))^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sin(dx+c)+a)^m}{(e\cos(dx+c))^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**m/((e*cos(d*x+c))**m),x)
```

```
[Out] Integral((e*cos(c + d*x))**(-m)*(a + b*sin(c + d*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sin{(dx+c)} + a)^m}{(e \cos{(dx+c)})^m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m/(e*cos(d*x + c))^m, x)
```

3.652 $\int (e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=142

$$\frac{e(e\cos(c+dx))^{-m}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{m/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{m/2}F_1\left(m+1;\frac{m}{2},\frac{m}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b}{}\right.}{bd(m+1)}$$

```
[Out] (e*AppellF1[1 + m, m/2, m/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(m/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(m/2))/(b*d*(1 + m)*(e*C
os[c + d*x])^m)
```

Rubi [A] time = 0.0966598, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2704, 138}

$$\frac{e(e\cos(c+dx))^{-m}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{m/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{m/2}F_1\left(m+1;\frac{m}{2},\frac{m}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b}{}\right.}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(1 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, m/2, m/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(m/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(m/2))/(b*d*(1 + m)*(e*C
os[c + d*x])^m)
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{-m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{m/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{m/2}\right)\text{Subst}\left(\int (a$$

$$\overline{d}$$

$$= \frac{eF_1\left(1+m;\frac{m}{2},\frac{m}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-m}(a+b}{bd(1+m)}$$

Mathematica [F] time = 5.25377, size = 0, normalized size = 0.

$$\int (e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^(1 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] Integrate[(e*Cos[c + d*x])^(1 - m)*(a + b*Sin[c + d*x])^m, x]
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{1-m}(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m+1}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m + 1)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((e\cos(dx+c))^{-m+1}(b\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-m + 1)*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1-m)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m+1}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m + 1)*(b*sin(d*x + c) + a)^m, x)
```

3.653 $\int (e\cos(c+dx))^{2-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=152

$$\frac{e(e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{m-1}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{m-1}{2}}F_1\left(m+1;\frac{m-1}{2},\frac{m-1}{2};m+2;\frac{a+b\sin(c+d}{a-b}\right.}{bd(m+1)}$$

```
[Out] (e*AppellF1[1 + m, (-1 + m)/2, (-1 + m)/2, 2 + m, (a + b*Sin[c + d*x])/(a -
 b), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(1 - m)*(a + b*Sin[c +
d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^((-1 + m)/2)*(1 - (a + b*S
in[c + d*x])/(a + b))^((-1 + m)/2))/(b*d*(1 + m))
```

Rubi [A] time = 0.103763, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2704, 138}

$$\frac{e(e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{m-1}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{m-1}{2}}F_1\left(m+1;\frac{m-1}{2},\frac{m-1}{2};m+2;\frac{a+b\sin(c+d}{a-b}\right.}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(2 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, (-1 + m)/2, (-1 + m)/2, 2 + m, (a + b*Sin[c + d*x])/(a -
 b), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(1 - m)*(a + b*Sin[c +
d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^((-1 + m)/2)*(1 - (a + b*S
in[c + d*x])/(a + b))^((-1 + m)/2))/(b*d*(1 + m))
```

Rule 2704

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(1 - (a + b*Si
n[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1
)/2)), Subst[Int[(-(b/(a - b)) - (b*x)/(a - b))^((p - 1)/2)*(b/(a + b) - (b
*x)/(a + b))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a,
 b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{2-m}(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{1-m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1}{2}(-1+m)}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1}{2}(-1+m)}\right)\mathrm{S}}{d}$$

$$= \frac{eF_1\left(1+m;\frac{1}{2}(-1+m),\frac{1}{2}(-1+m);2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos}{}$$

Mathematica [F] time = 4.23534, size = 0, normalized size = 0.

$$\int (e\cos(c+dx))^{2-m}(a+b\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^(2 - m)*(a + b*Sin[c + d*x])^m,x]

[Out] Integrate[(e*Cos[c + d*x])^(2 - m)*(a + b*Sin[c + d*x])^m, x]
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{2-m}(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m+2}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(-m + 2)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((e\cos(dx+c))^{-m+2}(b\sin(dx+c)+a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((e*cos(d*x + c))^(-m + 2)*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(2-m)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{-m+2}(b\sin(dx+c)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(-m + 2)*(b*sin(d*x + c) + a)^m, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```